Non–perturbative renormalization in two flavor QCD: coupling, quark mass and axial current

ALPHA Collaboration

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work in collaboration with
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Motivations

- α_s , m_i fundamental parameters describing the theory of strong interactions, QCD ,
- ratio of light quark masses predicted by χPT. Their magnitude is not accessible by χPT combined with experimental data $\Rightarrow$ $m_{\rm strange}$ from quenched lattice QCD + $K-$mesons masses as input:

$$\overline{m}_{\rm s}(2\ {\rm GeV}) = 97(4)\ {\rm MeV} \qquad [\overline{\rm MS}\ scheme]$$

ALPHA+UKQCD '99

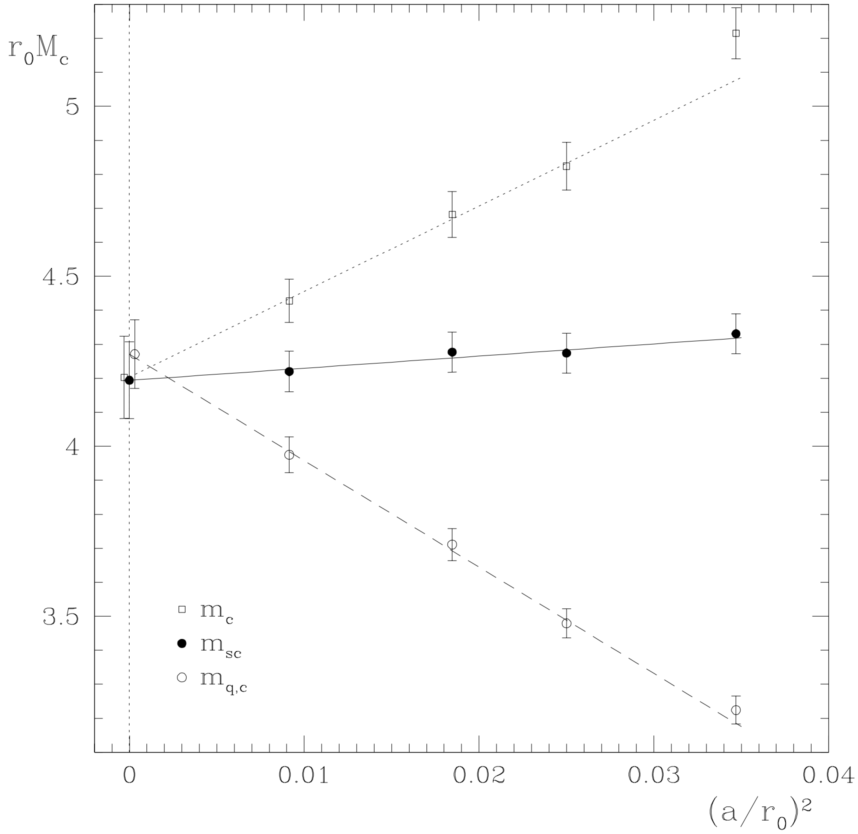
- running quark masses enter the strong interactions corrections to weak processes involving quarks (e.g. D and B decays)

m_c from PDG: $1.0 \leq \overline{m}_c(\overline{m}_c) \leq\ 1.4{\rm GeV}$

quenched result: $\overline{m}_c(\overline{m}_c) = 1.301(34)\ {\rm GeV}$

ALPHA 2002

The quenched charm quark mass



- D_s meson mass used as input (electromagnetic mass shifts are negligible)
- 3 different lattice definitions of the quark mass extrapolated to the continuum limit (using previously computed, mass independent, Z factors)
- $\Rightarrow$ $m_c^{\mathrm{SF}}(1.436 r_0)$ evolved to m_c^{RGI} and then in the $\overline{\mathrm{MS}}$ scheme at the scale m_c

g_0 and m_i^{bare}, $i = 1 \ldots N_{\rm f}$ are the free parameters of QCD. Renormalized parameters depend on some energy scale μ :

- In a mass independent renormalization scheme running is given by RGE's

$$\mu \frac{\mathrm{d}\overline{g}}{\mathrm{d}\mu} = \beta(\overline{g}) \,, \quad \mu \frac{\mathrm{d}\overline{m}_i}{\mathrm{d}\mu} = \tau(\overline{g})\overline{m}_i$$

$$\beta(\overline{g}) \underset{\overline{g}\to 0}{\sim} -\overline{g}^3\{b_0 + b_1\overline{g}^2 + b_2\overline{g}^4 + \ldots\}$$
$$\tau(\overline{g}) \underset{\overline{g}\to 0}{\sim} -\overline{g}^2\{d_0 + d_1\overline{g}^2 + \ldots\}$$

- $\overline{g}$, $\overline{m}_i$ and β, τ are renormalization scheme dependent

- β and τ are non-perturbatively defined functions, if the coupling $\overline{g}$ is non-perturbative

Renormalization Group Invariants

$$\Lambda = \mu(b_0\overline{g}^2)^{-b_1/2b_0^2} \exp\left\{-\frac{1}{2b_0\overline{g}^2}\right\}$$
$$\times \exp\left\{-\int_0^{\overline{g}} \mathrm{d}x \left[\frac{1}{\beta(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x}\right]\right\}$$
$$M_i = \overline{m}_i(2b_0\overline{g}^2)^{-d_0/2b_0} \exp\left\{-\int_0^{\overline{g}} \mathrm{d}x \left[\frac{\tau(x)}{\beta(x)} - \frac{d_0}{b_0 x}\right]\right\}$$
$$\overline{g} = \overline{g}(\mu)\,, \quad \overline{m}_i = \overline{m}_i(\mu)$$

- exact, defined beyond perturbation theory
- $\Lambda_{\mathrm{scheme}}/\Lambda_{\mathrm{scheme}'}$ = number, exact at 1–loop
- $(M_{\mathrm{scheme}})_i = (M_{\mathrm{scheme}'})_i = M_i$
- $\overline{m}_i/M_i$ independent of flavour i

Λ, M_i fundamental parameters of QCD may be computed on the lattice, inputting quantities like hadron masses or F_π (low energy hadronic scheme).

$\Rightarrow$ The running of parameters must be non–perturbatively traced to low energy scales.

$$\begin{bmatrix} F_\pi \\ \\ m_{\mathrm{K}} \end{bmatrix} \quad \begin{matrix} \text{Lattice QCD} \\ \Longleftrightarrow \\ g_0,\, m_i^{\mathrm{bare}},\, i = 1, ..., N_{\mathrm{f}} \end{matrix} \quad \begin{bmatrix} \Lambda_{\mathrm{QCD}} \\ \\ M_{\mathrm{s}} \end{bmatrix}$$

we adopt the Schrödinger functional renormalization scheme:

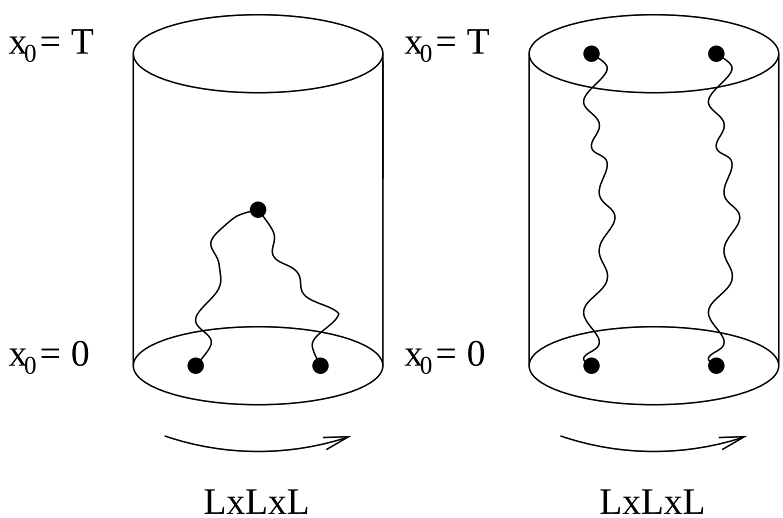
$$\mathcal{Z} = e^{-\Gamma} = \int_{\mathrm{TxL}^3} D[U, \psi\bar{\psi}] e^{-S}$$

+ periodic boundary conditions in space and Dirichlet boundary conditions in time (angle η and Grassman values $\rho, \bar{\rho}$)

- coupling:

$$\bar{g}^2(L) = k / \frac{\partial \Gamma}{\partial \eta} = k / \langle \frac{\partial S}{\partial \eta} \rangle$$

- quark mass. Gauge invariant correlators:



$T = L$
$C = C' = 0$
$\theta = 1/2$

Renormalized and O(a) improved quark mass can be defined via the PCAC relation :

$$\overline{m}_i(\mu) = \frac{Z_{\mathrm{A}}}{Z_{\mathrm{P}}(L)} \frac{(1 + b_{\mathrm{A}} m_{\mathrm{q}}) \tilde{\partial}_0 f_{\mathrm{A}}^{\mathrm{I}}}{(1 + b_{\mathrm{P}} m_{\mathrm{q}}) f_{\mathrm{P}}}, \quad \mu = \frac{1}{L}$$

$$f_{\mathrm{A}}^{\mathrm{I}} = f_{\mathrm{A}} + c_{\mathrm{A}} a \partial_0 f_{\mathrm{P}}$$

Z_{A} by current algebra, scale independent.
We define :

$$Z_{\mathrm{P}}(L) = \frac{\sqrt{3f_1}}{f_{\mathrm{P}}(L/2)} \quad \text{at} \quad m_i = 0\,, \quad i = 1, ..., N_{\mathrm{f}}$$

and we use 1 loop value for c_{A}.

- Running: step scaling functions $\sigma(u)$, $\sigma_{\mathrm{P}}(u)$

$$\sigma(u) = \bar{g}^2(2L)|_{\bar{g}^2(L)=u, m_i=0}\,,$$

$$\sigma_{\mathrm{P}}(u) = \frac{\overline{m}(L)}{\overline{m}(2L)} = \left.\frac{Z_{\mathrm{P}}(2L)}{Z_{\mathrm{P}}(L)}\right|_{\bar{g}^2(L)=u, m_i=0}$$

$\Rightarrow$ integrated form of the β and τ–functions.

- Starting from L_{max} s.t. $\bar{g}^2(L_{\mathrm{max}}) = u_0$ and solving k times the recursions:

$$\begin{cases} u_0 = \overline{g}^2(L_{\mathrm{max}}) = 3.3 \\ \sigma(u_{k+1}) = u_k \end{cases} \qquad \begin{cases} v_0 = 1 \\ v_{k+1} = \dfrac{v_k}{\sigma_{\mathrm{P}}(u_k)} \end{cases}$$

$$\Rightarrow u_k = \overline{g}^2(2^{-k}L_{\mathrm{max}}) \qquad \Rightarrow v_k = \frac{\overline{m}(2L_{\mathrm{max}})}{\overline{m}(2^{-k+1}L_{\mathrm{max}})}$$

until the coupling $\bar{g}^2(2{-}kL_{\mathrm{max}})$ is perturbative, one can compute ΛL_{max} and $M/\overline{m}(2L_{\mathrm{max}})$ using 2(τ)/ 3(β) loop functions.

O(a) improved Wilson fermions ($N_\mathrm{f} = 2$) in SF.

coupling: u and $\sigma(u)$ for lattice resolutions $L/a = 4, 5, 6, 8$ and $L/a = 8, 10, 12, 16$.

L/a	u	Σ	u	Σ
4	0.9793(7)	1.0643(34)	1.5031(12)	1.720(5)
5	0.9793(6)	1.0721(39)	1.5033(26)	1.737(10)
6	0.9793(11)	1.0802(44)	1.5031(30)	1.730(12)
8	0.9807(17)	1.0745(55)	1.5077(43)	1.716(14)
4	1.1814(5)	1.3154(55)	2.0142(24)	2.481(17)
5	1.1807(12)	1.3287(59)	2.0142(44)	2.438(19)
6	1.1814(15)	1.3253(67)	2.0146(56)	2.508(26)
8	1.1818(29)	1.3338(58)	2.0142(102)	2.475(31)
4	1.5031(10)	1.731(6)	2.4792(34)	3.251(28)
5	1.5031(20)	1.758(11)	2.4792(73)	3.336(50)
6	1.5031(25)	1.745(12)	2.4792(82)	3.156(55)
8			2.4792(128)	3.348(48)
4	1.7319(11)	2.058(7)	3.334(11)	5.513(42)
5	1.7333(32)	2.086(21)	3.334(15)	5.41(12)
6	1.7319(34)	2.058(20)	3.326(20)	5.62(9)
8			3.334(19)	5.48(12)

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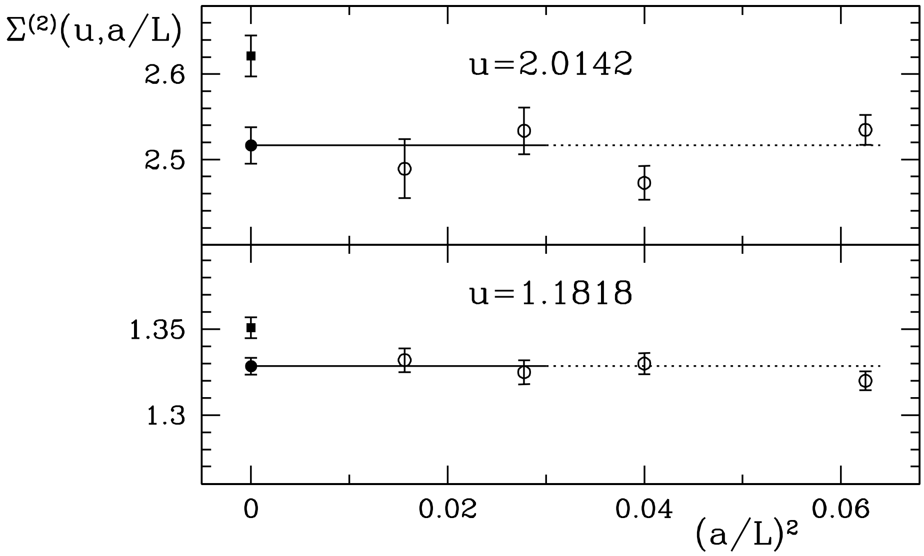
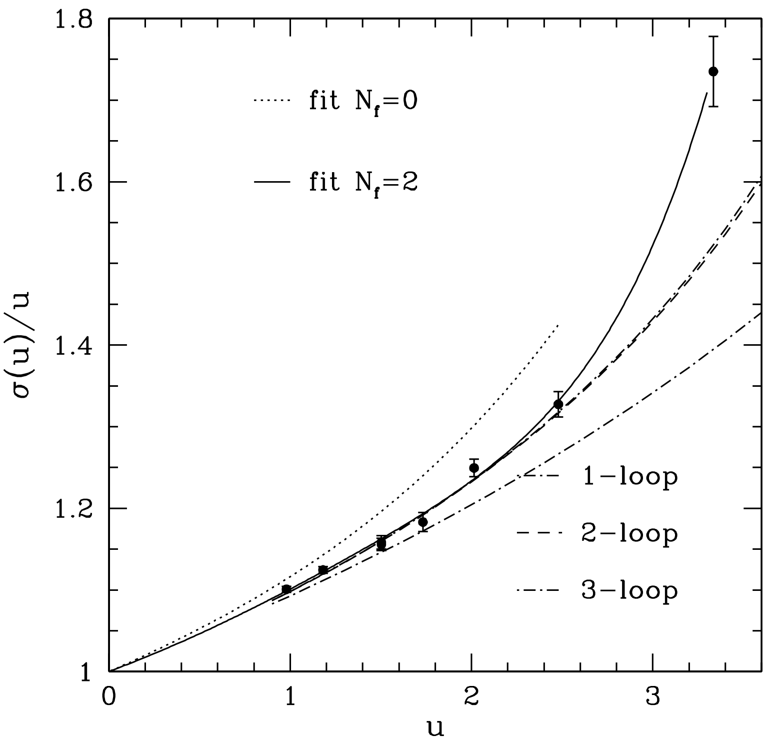
hep-lat/0105003

continuum limit from $L/a = 6, 8$, $L/a = 5$ used to estimate systematic uncertainty,

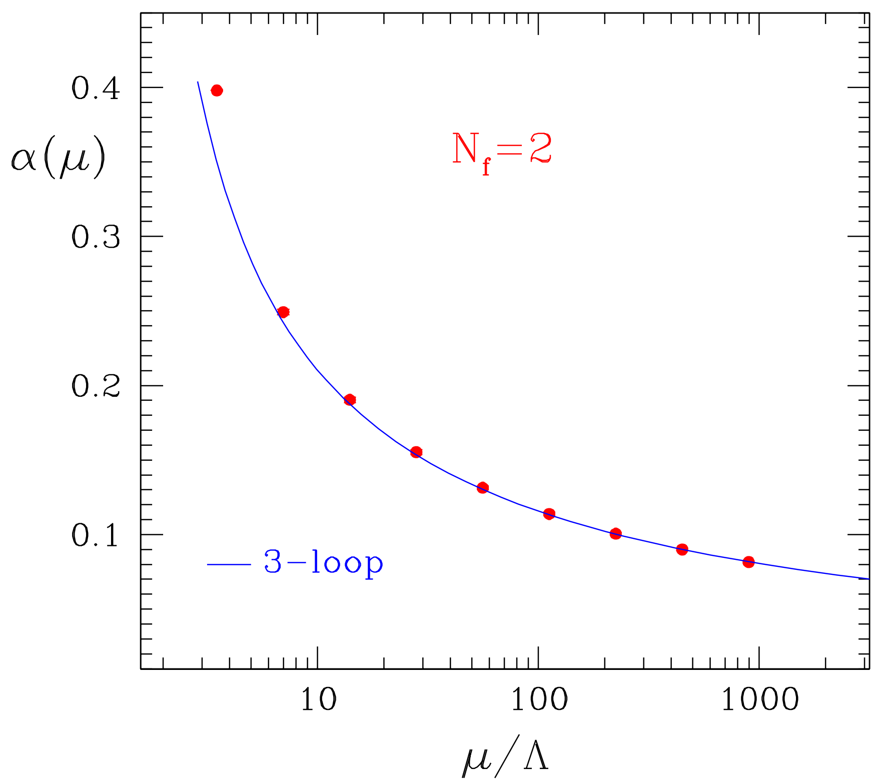




Non–perturbative evolution $L_{\rm max} \to 2^{-n} L_{\rm max}$,
perturbative evolution $2^{-n} L_{\rm max} \to 0$ (∞ energy):

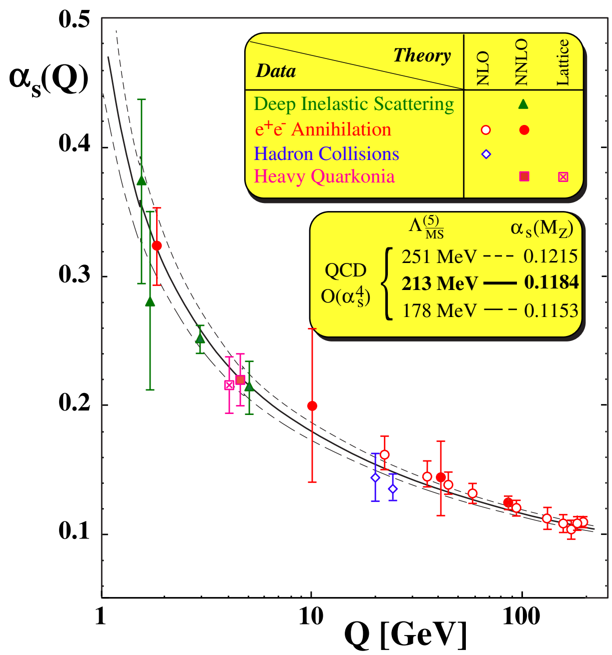
n	continuum 6/8	$L/a = 5$	n	continuum 6/8	$L/a = 5$
5	1.82(5)	1.87	6	1.23(5)	1.27
6	1.84(6)	1.89	7	1.25(6)	1.28
7	1.85(7)	1.91	8	1.26(7)	1.30

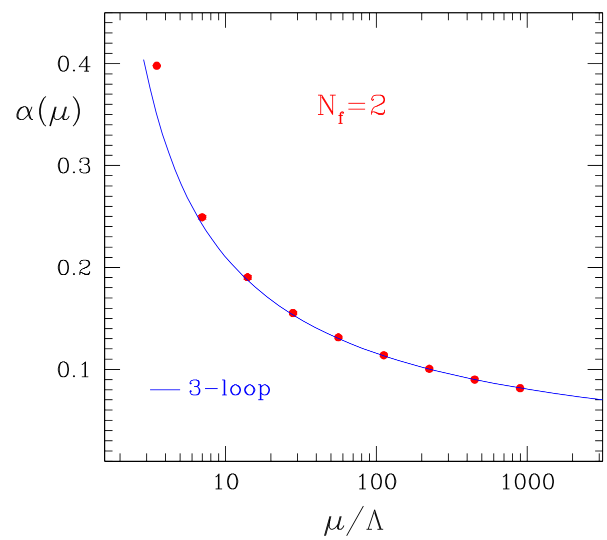
$$\ln(\Lambda L_{\rm max}) = -1.85(13) \quad [\bar{g}^2(L_{\rm max}) = 3.3]$$
$$\ln(\Lambda L'_{\rm max}) = -1.26(11) \quad [\bar{g}^2(L'_{\rm max}) = 5]\,,$$



Comparison with experimental results

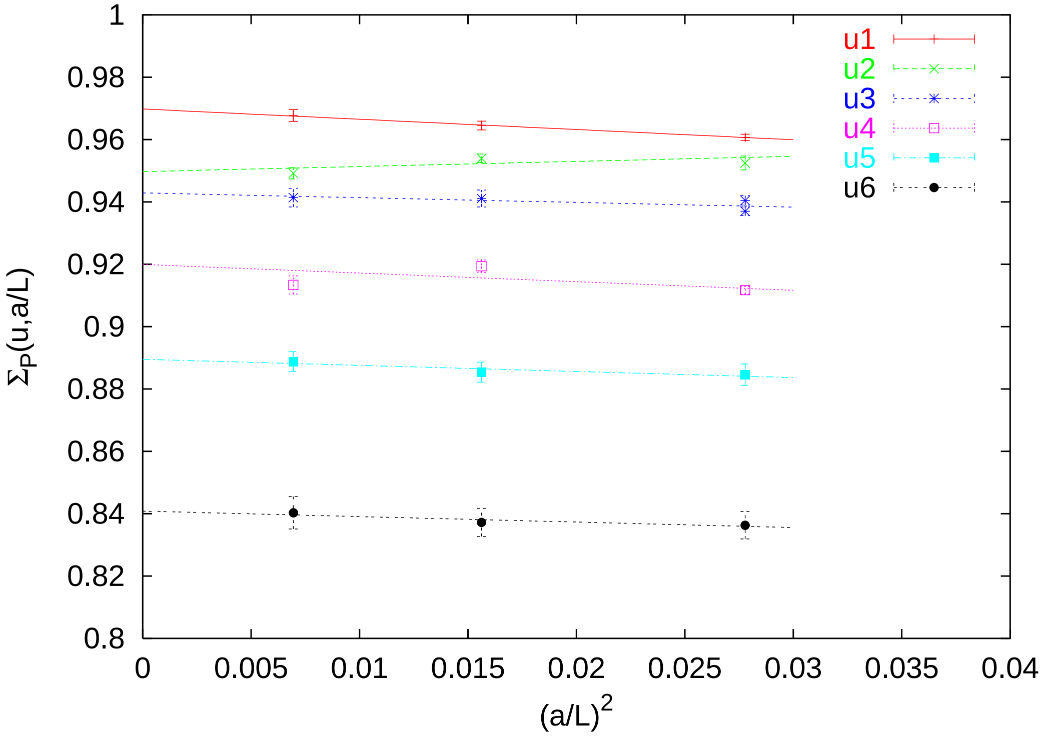
(Bethke 2000)

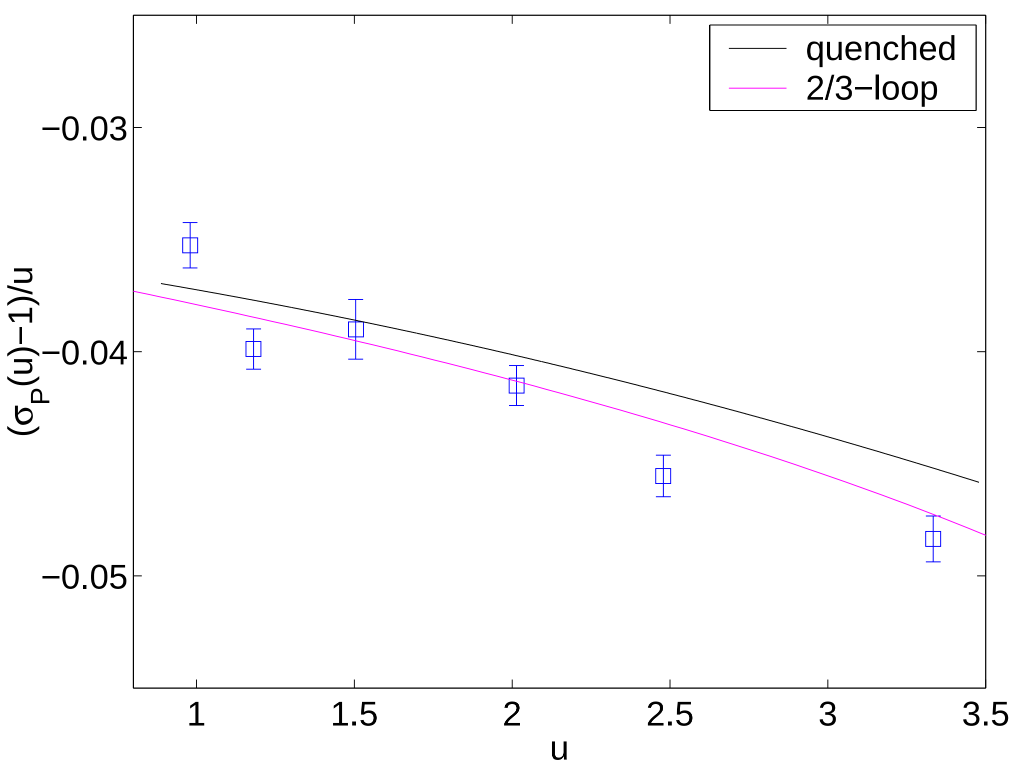




- results from experiments compared with perturbation theory predictions $\Rightarrow$ only short distance regime accessible
- non perturbative assumptions or inputs needed for estimates at lower energies (factorization, sum rules ...)

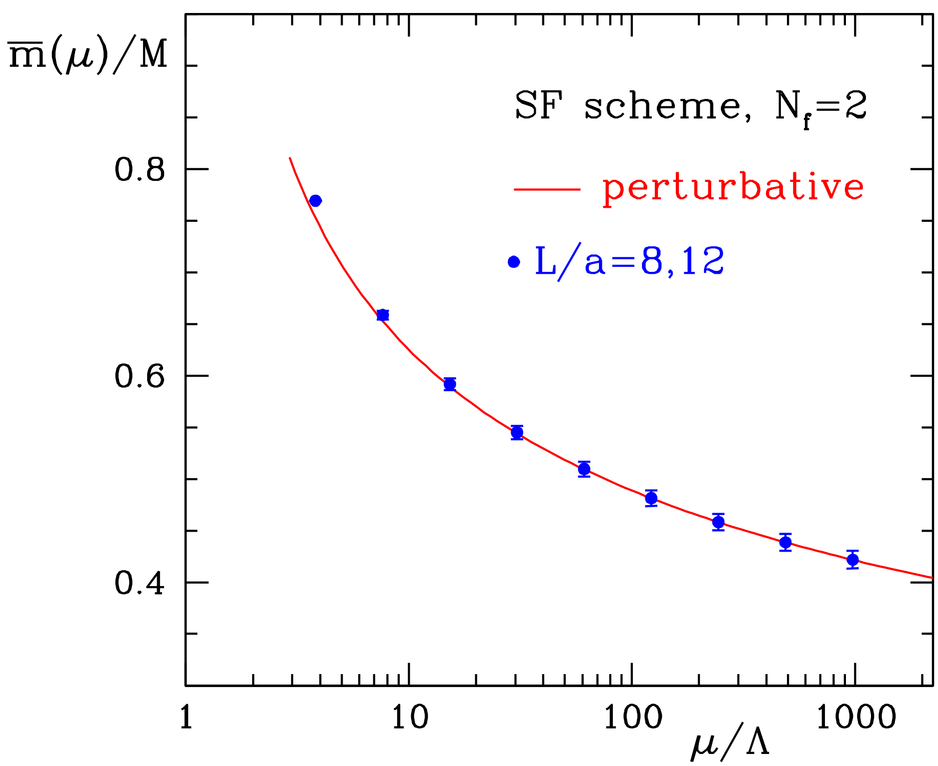
running mass: at the same couplings we computed $Z_{\rm P}(L)$ and $Z_{\rm P}(2L)$ for lattice resolutions $L/a = 6, 8, 12$ and $L/a = 12, 16, 24$





Non–perturbative evolution $L_{\max} \to 2^{-7}L_{\max}$, $\bar{g}^2(L_{\max}) = 4.68$ taking c.l. from $L/a = 8$ and $L/a = 12$ data. Perturbative evolution $2^{-7}L_{\max} \to 0$ (∞ energy) using 2(τ)/3(β) loop functions.

$$\frac{M}{\overline{m}(L_{\max})} = 1.30(3)$$



The axial current renormalization constant

It enters the computation of F_π and pseudoscalar mesons decay constants (e.g. $F_{\mathrm{D_s}}$)

We want to use the full PCAC relation to derive a renormalization condition

internal operator

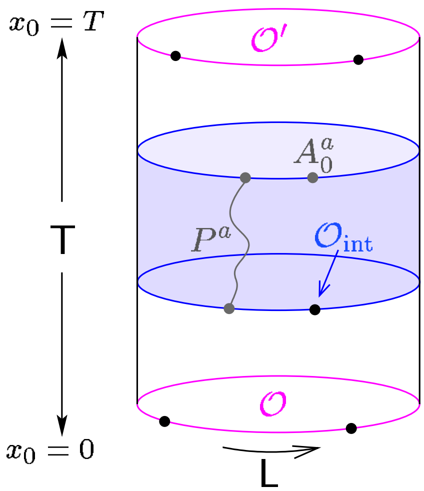
variation of internal operator

$$\int \mathrm{d}^3\mathbf{y}\,\mathrm{d}^3\mathbf{x}\,\epsilon^{abc}\Big\langle A_0^a(y_0+t,\mathbf{x})A_0^b(y)\mathcal{O}_{\text{ext}}\Big\rangle$$

$$-2m\int \mathrm{d}^3\mathbf{y}\,\mathrm{d}^3\mathbf{x}\int_{y_0}^{y_0+t}\mathrm{d}x_0\,\epsilon^{abc}\Big\langle P^a(x)A_0^b(y)\mathcal{O}_{\text{ext}}\Big\rangle \;=\; i\int \mathrm{d}^3\mathbf{y}\,\Big\langle V_0^c(y)\mathcal{O}_{\text{ext}}\Big\rangle$$

i.e., including the volume integral.

This is in view of dynamical results. In this case the cost of the simulations rapidly increases when decreasing the quark mass.



Schrödinger Functional:

- QCD on a space-time cylinder $L^3 \times T$
- periodic b.c.'s in spatial directions
- fixed (Dirichlet) b.c.'s in time direction
- $\mathcal{O}^c_{\rm ext} = -\frac{1}{6L^6}\epsilon^{cde}\mathcal{O}'^d\mathcal{O}^e$
- $\mathcal{O}$ ($\mathcal{O}'$): zero-momentum pseudo–scalar states at $x_0=0$ ($x_0=T$)

With insertions at a physical distance $3/4L$ among each other. Introducing the correlators

$$f_{A^A}(x_0, y_0) = -\frac{a^6}{6L^6}\sum_{\mathbf{x},\mathbf{y}}\epsilon^{abc}\epsilon^{cde}\left\langle \mathcal{O}'^d (A_I)^a_0(x)(A_I)^b_0(y)\mathcal{O}^e\right\rangle$$

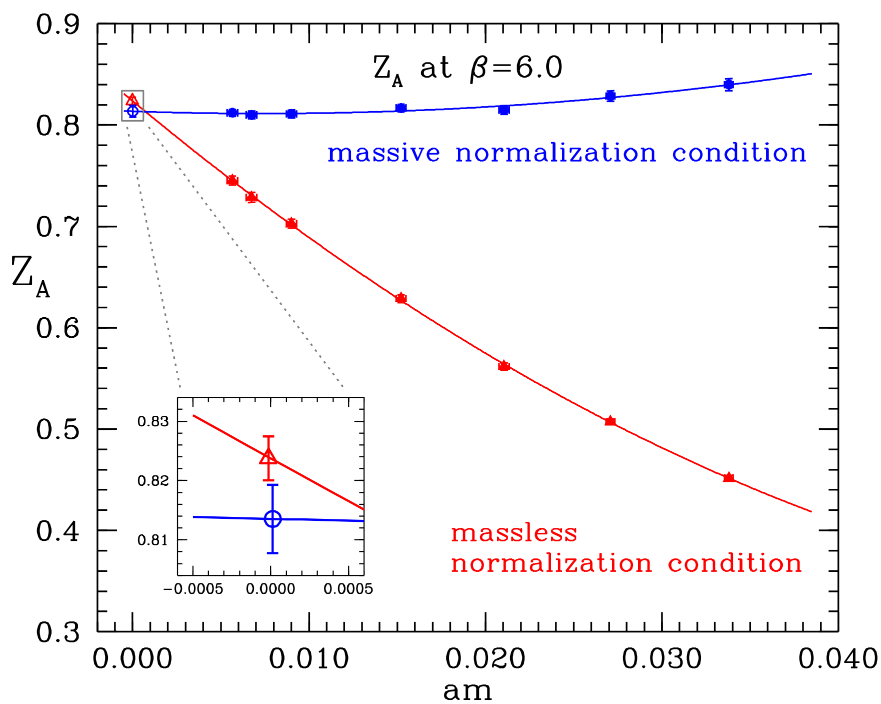
$$f_{PA}(y_0+t, y_0) = -\frac{a^7}{6L^6}\sum_{x_0=y_0}^{y_0+t}\sum_{\mathbf{x},\mathbf{y}}\epsilon^{abc}\epsilon^{cde}\left\langle \mathcal{O}'^d P^a(x)(A_I)^b_0(y)\mathcal{O}^e\right\rangle$$

contact terms!

The WI reads

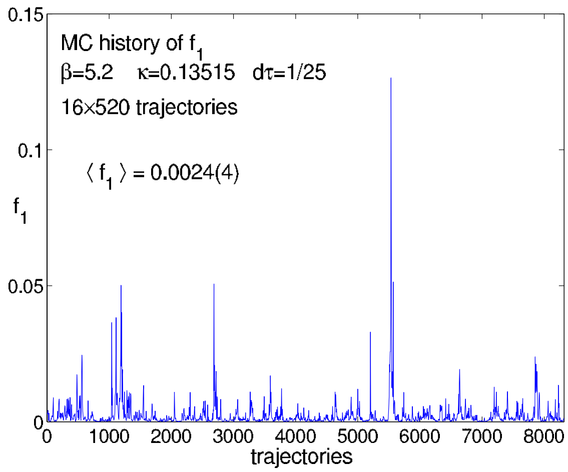
$$Z_A^2(1+b_A a m_q)^2(f_{AA} - 2m f_{PA}) = f_1$$

- Excluding the volume integral would be wrong (for a finite mass) !! We would have errors $O(r_0 m)$ instead of errors $O(am)$.
- The contact terms introduce $O(a)$ (improvement doesn't work). They seem to be integrable, after extrapolation the result is consistent with the one when the volume integral is neglected.



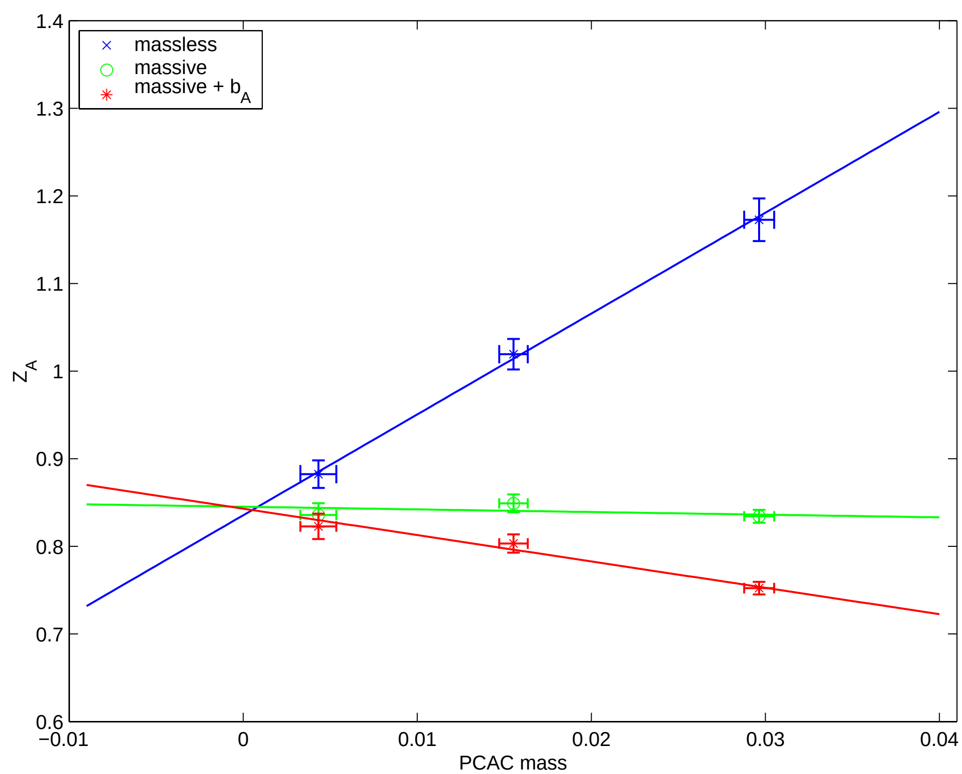
$N_{\rm f} = 2$

In order to keep under control $\mathrm{O}(a^2)$ ambiguities we want to compute Z_A on a line of constant physics (for c_A they are $\mathrm{O}(a)$!!).



- Correlation between spikes in f_1 and small eigenvalues of the Dirac operator?
- Can we sample the spikes with their proper weight?

$\Rightarrow$ PHMC (?)



The axial current improvement constant c_A (in collaboration with CP-PACS and JLQCD)

- Large cut-off effects have been observed in $N_f = 2$ simulations with Wilson fermions

action	β	$N_{\rm f}$	$\Delta m[\mathrm{MeV}] \times \frac{a^{-1}}{2\,\mathrm{GeV}}$
W/SW	6.0	0	12(1)
W/SW	5.2	2	28(1)
W/W	5.5	2	96(4)
I/SW	2.2	2	−2(4)

$$\Delta m = m(\infty) - m(L/a = 8)$$

- improvement coefficients are ambiguous by O(a)
- computing c_A on a line of constant physics, these ambiguities are not removed, but smoothly vanish ($\propto a$) when approaching the region of applicability of PT

$$(A_0^a)_I(x) = A^a(x) + ac_A\frac{\partial_0 + \partial_0^*}{2}P^a(x)$$

in the SF we introduce wavefunctions at the boundaries

$$O_{ij} = \frac{a^6}{L^3}\sum_{\mathbf{u},\mathbf{v}} \bar{\zeta}_i(\mathbf{u})\gamma_5\zeta_j(\mathbf{v})\ \omega(\mathbf{u}-\mathbf{v})$$

$$f_X(x_0,T,L) = -\frac{L^3}{2}\langle X(x)\,O\rangle$$

The improved quark mass $m = r + ac_A s$ is

$$r(x_0) = \frac{\frac{1}{2}(\partial_0+\partial_0^*)f_A(x_0)}{2f_P(x_0)}$$

$$s(x_0) = \frac{\partial_0\partial_0^* f_P(x_0)}{2f_P(x_0)}$$

We require m to be constant when changing

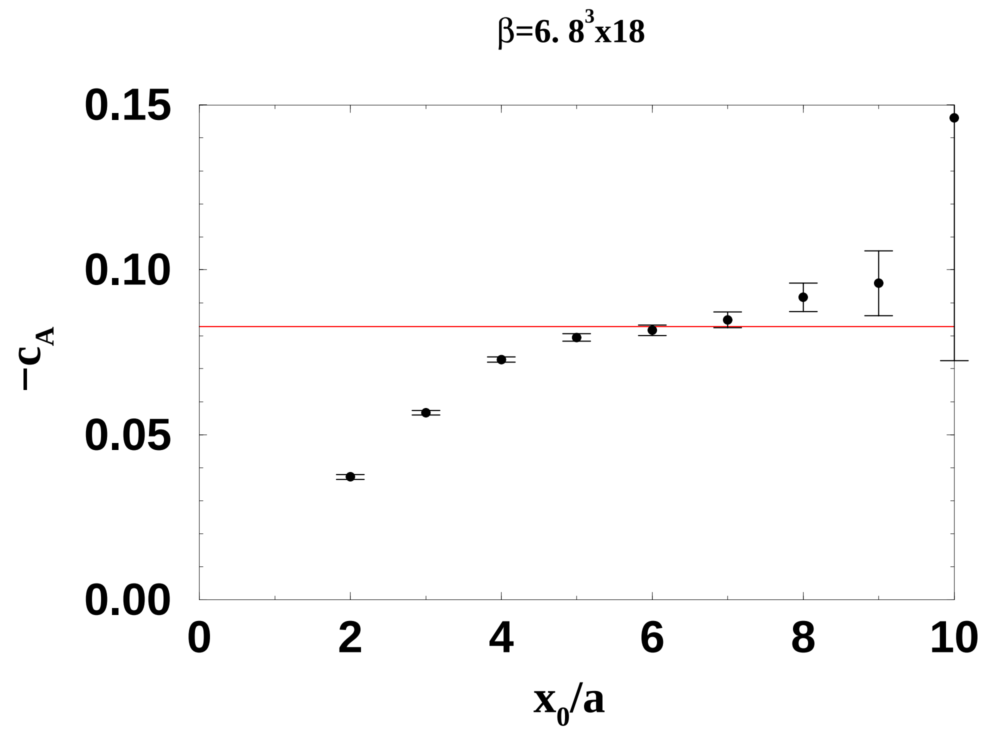
(0) θ at fixed $x_0 = T/2$, constant ω (old ALPHA)

(1) x_0 at fixed θ , constant ω ("slope criterion")

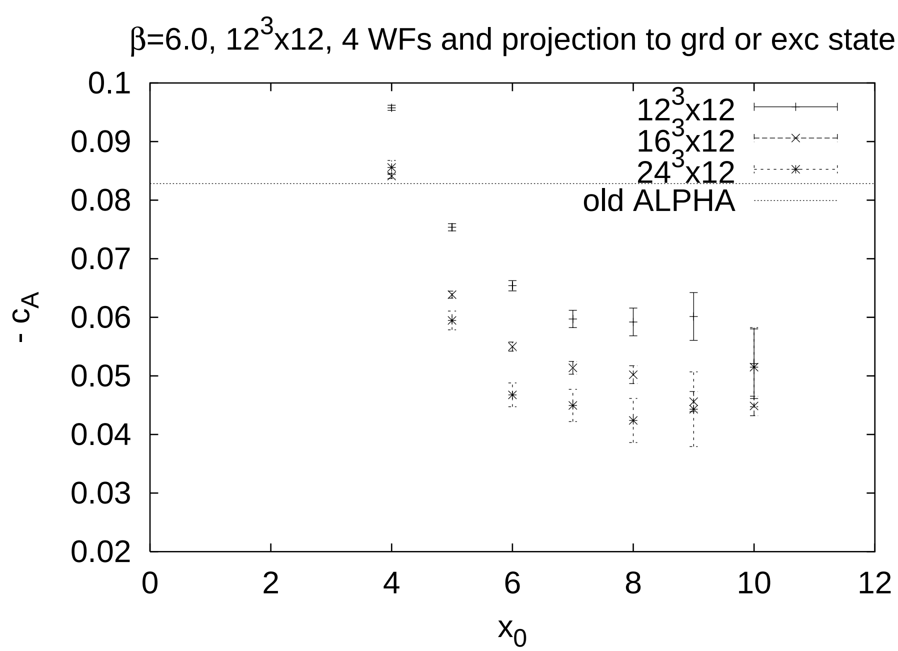
(2) state produced by $O[\omega]$ ($\simeq$ UKQCD, "gap criterion")

$N_f = 0$ results

slope method; $\theta = \pi$



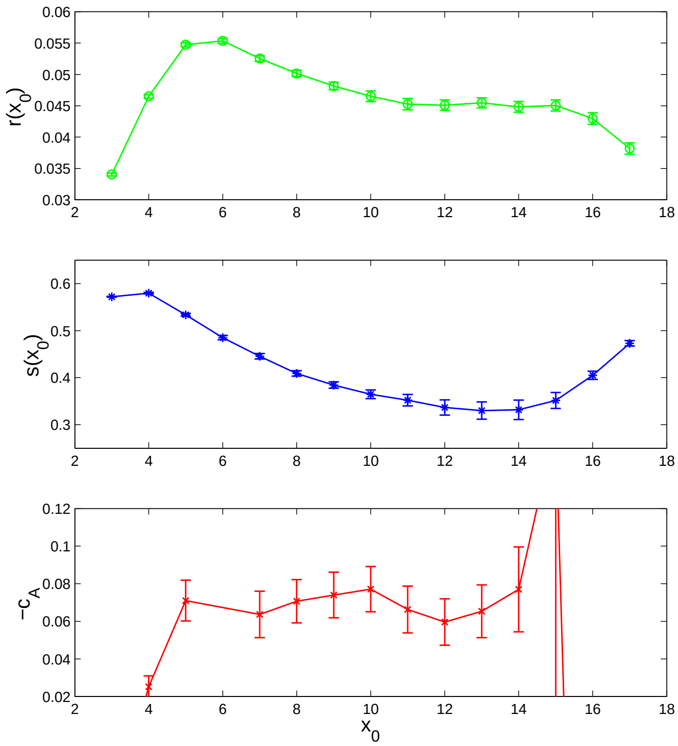
gap method; excited and ground pion states



- at larger values of β the two methods give consistent results

$N_f = 2$ PRELIMINARY results @ $\beta = 5.2$

slope method; $\theta = \pi$



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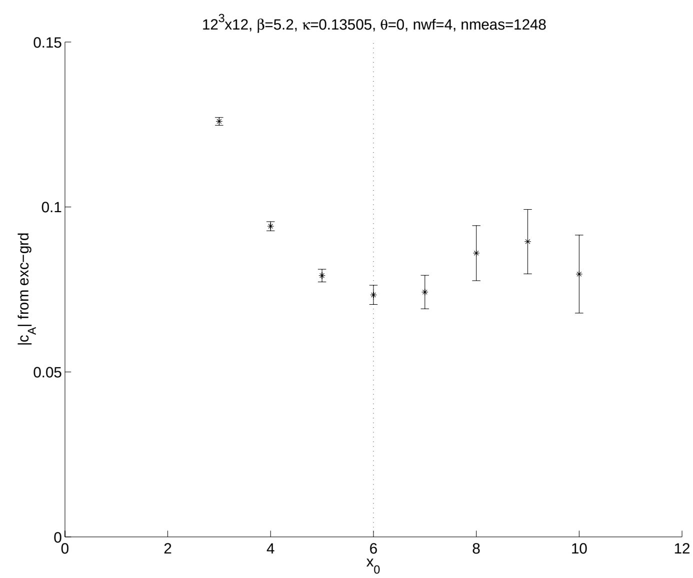

- The methods, surprisingly give consistent results. On the other hand the old ALPHA method for $\theta = 0,\ \theta' = \pi,\ V = 8^3 \times 18$ would give $c_A = -0.036(6)$.

- We have a preference for the gap method
  - The interpretation is clearer, results can be understood by a two state analysis of the correlator,
  - by definition it doesn't involve high-energy states,
  - good sensitivity $\propto (m_{\pi^*}^2 - m_\pi^2)$
  - no large lattices needed (12^4 @ $\beta = 5.2$) and good signal.

- Δm for W/SW is essentially unchanged if the NP value of c_A is used !! Large O(a^2) ?

Conclusions:

- accurate studies of cutoff dependence in unquenched ($N_\mathrm{f} = 2$) simulations
- continuum limit for $\sigma(u)$ and ΛL_max
- Z_P up to lattices $L/a = 12$ and $L/a = 24$, continuum limit for σ_P

$$\Rightarrow \frac{M}{\overline{m}(L_\mathrm{max})}$$

Outlook:

- hadronic scheme and chiral perturbation theory

$$\Rightarrow \quad F_\mathrm{PS} L_\mathrm{max}$$

$$\overline{m}(2L_\mathrm{max})(F_\mathrm{PS})^{-1} \quad at \quad m_\mathrm{PS} \approx m_\mathrm{K}$$

- target/result

$$\Lambda (F_\mathrm{PS})^{-1}$$

$$M_\mathrm{strange}(F_\mathrm{PS})^{-1}$$

- Z_A and c_A for two dynamical Wilson quarks. Preliminary results @ $\beta = 5.2$. To keep cut-off effects (observed to be quite large) under control, it is important to stay on a line fo renormalized parameters, especially for c_A.