Charmed meson spectrum and decay constants with the one-loop $O(a)$ improved Relativistic heavy quark action

in collaboration with

Y. Kayaba, S. Aoki and Y. Kuramashi

WORKSHOP @TSUKUBA

Feb 2-4, 2004

[0. Introduction]

Precise determination of weak matrix elements and spectrum for B and D mesons.

$\Downarrow$

Lattice QCD.

Heavy Quarks on the Lattice

· $m_b a \sim 1 - 2$ in quenched
· $m_b a \sim 2 - 3$ in full QCD

$\Longrightarrow$ Large cutoff effects $\mathcal{O}((m_b a)^n)$, $\mathcal{O}((m_b a)^n (\Lambda_{QCD} a)^m)$

$\Downarrow$

We want to reduce these cutoff effects !!

· NRQCD
· HQET $\longrightarrow$ Effctive theories.
· FNAL $\cdots$ and more

Kuramashi-san's Talk :

$O(a\Lambda_{QCD})$ improved heavy quark action for $m_Q > \Lambda_{QCD}$

(Aoki, Kuramashi and Tominaga;2001)

$$S_q = \sum_x \Big[m_0\bar{q}(x)q(x) + \bar{q}(x)\gamma_0 D_0 q(x) + \nu \sum_i \bar{q}(x)\gamma_i D_i q(x) \\ -\frac{r_t a}{2}\bar{q}(x)D_0^2 q(x) - r_s\frac{a}{2}\sum_i \bar{q}(x)D_i^2 q(x) \\ -\frac{iga}{2}c_E \sum_i \bar{q}(x)\sigma_{0i}F_{0i}q(x) - \frac{iga}{4}c_B \sum_{i,j}\bar{q}(x)\sigma_{ij}F_{ij}q(x)\Big]$$

$$r_t = 1$$

⇓

$\cdot\mathcal{O}(f_1(m_Q a))$ errors can be absorbed in Z_q, Z_m.

$\cdot\mathcal{O}(f_2(m_Q a)a\Lambda_{QCD})$ errors can be removed adjusting

$\nu(m_q, g^2), r_s(m_q, g^2), c_E(m_q, g^2), c_B(m_q, g^2)$

$m_q \to 0$ Clover action : $\nu = r_s = 1,\ c_E = c_B = c_{SW}$

We calculated $O(a\Lambda_{QCD})$ improvement coefficients up to 1-loop level by ordinary perturbation theory.

⇓

Remaining cutoff effects are

$\mathcal{O}((a\Lambda_{QCD})^2)$ or $O(g^4 a\Lambda_{QCD})$!!

⇓

Do the Continuum Dispersion relation retain on the Lattice?

$O(a\Lambda_{QCD})$ improved Axial vector current for $m_Q > \Lambda_{QCD}$

$\Gamma_\mu = \gamma_\mu\gamma_5$

$$A^R_\mu(x) = Z_A\,[\,\bar{q}(x)\Gamma_\mu Q(x) - g^2 c^L_{A_\mu}(\vec{\partial}_i\bar{q}(x))\gamma_i\Gamma_\mu Q(x) - g^2 c^H_{A_\mu}\bar{q}(x)\Gamma_\mu\gamma_i(\vec{\partial}_i Q(x)) - g^2 c^+_{A_\mu}\partial^+_\mu(\bar{q}(x)\gamma_5 Q(x)) - g^2 c^-_{A_\mu}\partial^-_\mu(\bar{q}(x)\gamma_5 Q(x))\,]$$

where $\quad \partial^+_\mu = \vec{\partial}_\mu + \overleftarrow{\partial}_\mu\ , \quad \partial^-_\mu = \vec{\partial}_\mu - \overleftarrow{\partial}_\mu$

$\Downarrow$

$\cdot\mathcal{O}(f_1(m_Q a))$ errors can be absorbed in $Z_{A_\mu}(m_q a, m_Q a, g^2)$.
$\cdot\mathcal{O}(f_2(m_Q a)a\Lambda_{QCD})$ errors can be removed adjusting $c^{+,-,H,L}_{A_\mu}(m_q a, m_Q a, g^2)$

$m_q,\ m_Q \to 0 \quad c^+_{A_\mu} = c_{A_\mu},\ c^{-,H,L}_{A_\mu} = 0$

We calculated $O(a\Lambda_{QCD})$ improvement coefficients up to 1-loop level by ordinary perturbation theory.

$\Downarrow$

Remaining cutoff effects are $\mathcal{O}((a\Lambda_{QCD})^2)$ or $O(g^4 a\Lambda_{QCD})$!!

$\Downarrow$

Space-time symmetry $A_4 \leftrightarrow A_k$ retains on the Lattice?

$O(a\Lambda_{QCD})$ improved vector current can be obtained in same way.

Quenched QCD simulation :
We measured recovery of Space-time symmetry and dispersion relation using Relativistic heavy quark action.

We calculated charmed meson spectrum and decay constants.

Contents

0. Introduction.
1. Simulation details.
2. Numerical test of the relativistic heavy quark action :
   Space-time symmetry and Dispersion relation.
3. Charmed meson spectrum and decay constant.
4. Summary.

[1. Simulation details]

Quenched QCD simulation.

Light quark : Clover action
Heavy qaurk : Relativistic Heavy quark action
Gauge : Plaquette action.

$$\beta = 6/g^2 = 6.0, \quad a(r_0)^{-1} = 2.12[GeV]$$

$$\text{volume} = 24^3 * 48(\sim 2.2 fm), \quad \#\text{Conf} = 300$$

flavor	$name$	κ	M_{PS}/M_V	ν	r_s	c_B	$\omega = c_E/c_B$
light	$k1$	0.1345	0.543(6)	1.0	1.0	1.769	1.0
	$k2$	0.1338	0.698(2)	1.0	1.0	1.769	1.0
	$k3$	0.1330	0.778(2)	1.0	1.0	1.769	1.0
heavy	$k4$	0.1146	0.9335(5)	1.0416	1.1603	2.0125	0.9223
	$k5$	0.1019	0.9700(2)	1.0830	1.2620	2.1715	0.8857
	$k6$	0.0950	0.9801(2)	1.1128	1.3284	2.2783	0.8644
	$k7$	0.0749	0.9905(1)	1.2387	1.5826	2.7001	0.7999

$k_{crit} = 0.13523(2), \quad k_{strange} = 0.1338(1)(m_K^2 \text{input})$
$k2, k3 \sim k_{strange},\ k5, k6 \sim k_{charm}.$
$c_{SW}^{NP} = 1.769(\text{ALPHA collab.})$

c_B and c_E are shifted by c_{SW}^{NP} :

$$c_B = [c_B^{(0)}(m_{pole}) + g^2_{\overline{MS}} c_B^{(1)}(m_{pole}) - (c_B^{(0)}(0) + \alpha * c_B^{(1)}(0))] + c_{SW}^{NP}$$

$$c_E = [c_E^{(0)}(m_{pole}) + g^2_{\overline{MS}} c_E^{(1)}(m_{pole}) - (c_E^{(0)}(0) + \alpha * c_E^{(1)}(0))] + c_{SW}^{NP}$$

Light current :
$Z_A^{NP} = 0.807$, $c_A^{NP} = 0.037$(LANL).

Heavy-light and heavy Current :

Z_{A_4},Z_{A_k}, $c_{A_4}^{(+,-,H,L)}$,$c_{A_k}^{(+,-,H,L)}$
where $c_{A_4}^{+}$, $c_{A_k}^{+}$ are shifted by c_A^{NP}.

For Comparison : Heavy quark with Clover action.

flavor	*name*	κ	M_{PS}/M_V	ν	r_s	c_B	$\omega = c_E/c_B$
light	$k3$	0.1330	0.778(2)	1.0	1.0	1.769	1.0
heavy	$k4$	0.1278	0.9358(7)	1.0	1.0	1.769	1.0
	$k5$	0.1190	0.9770(3)	1.0	1.0	1.769	1.0
	$k6$	0.1148	0.9833(2)	1.0	1.0	1.769	1.0

where $k3 \sim k_{strange}$, $k5, k6 \sim k_{charm}$.

[2. Numerical test of the relativistic heavy quark action.]

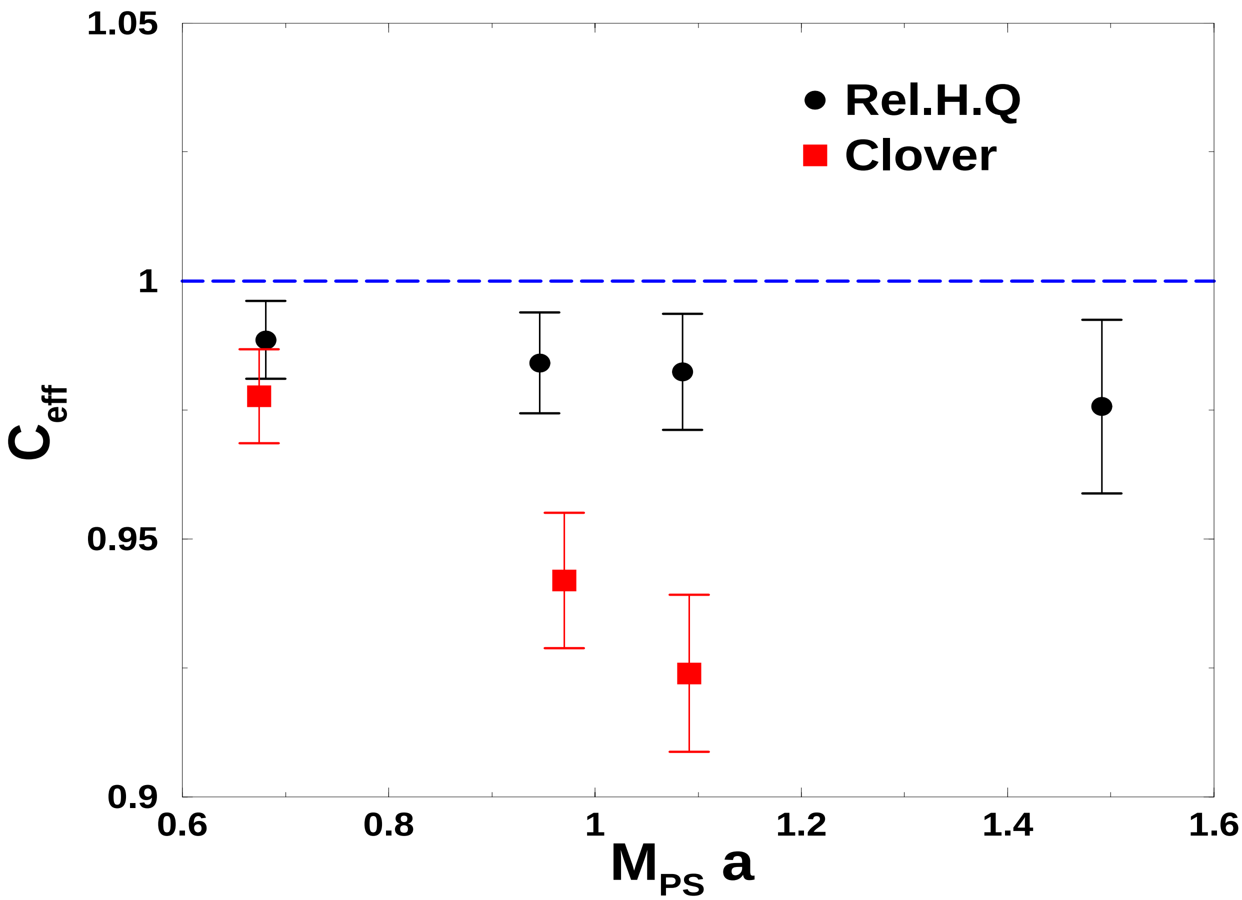
Does the RHQ action retains Space-time symmetry and dispersion relation on the lattice?

Effective speed of light :

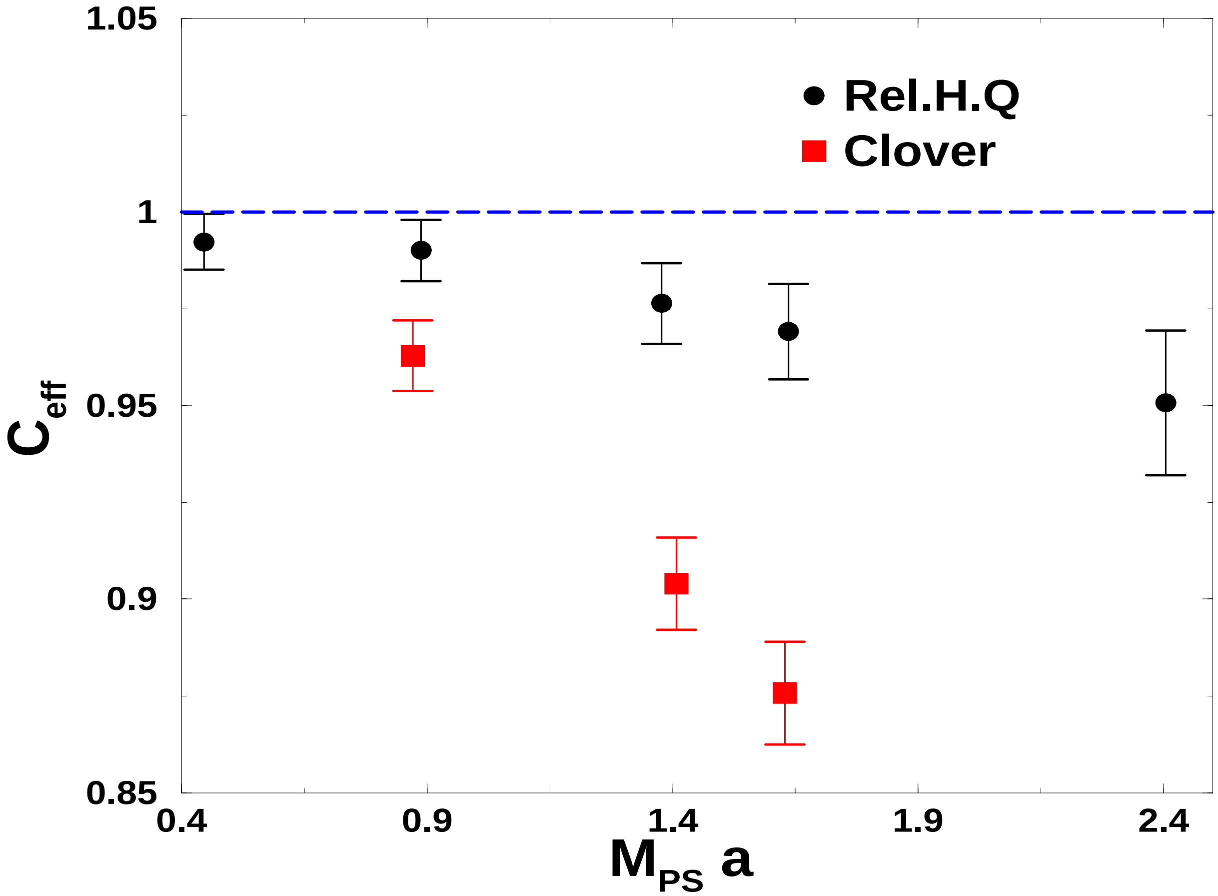
$$c_{eff}(P_s) = \sqrt{\frac{E(P_s)^2 - E(P_s=0)^2}{P_s^2}} \quad \xrightarrow{a\to 0} \quad 1$$

from PS meson Energy with spatial momentum mode :

$$|P_s| = 0, 1, \sqrt{2} \text{ in } \frac{2\pi}{L} \text{ unit}$$



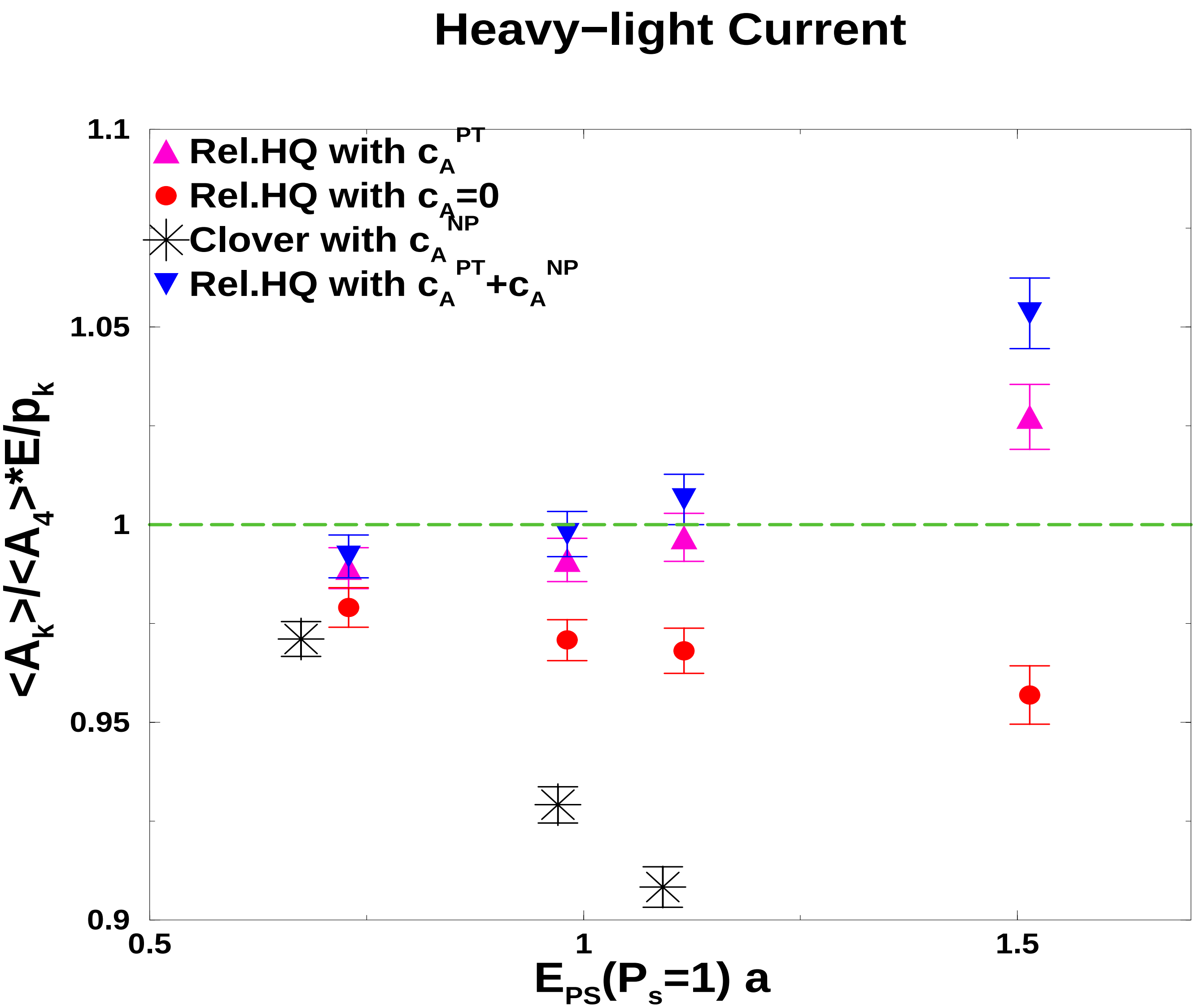
Effective speed of light in Heavy-Light meson case.



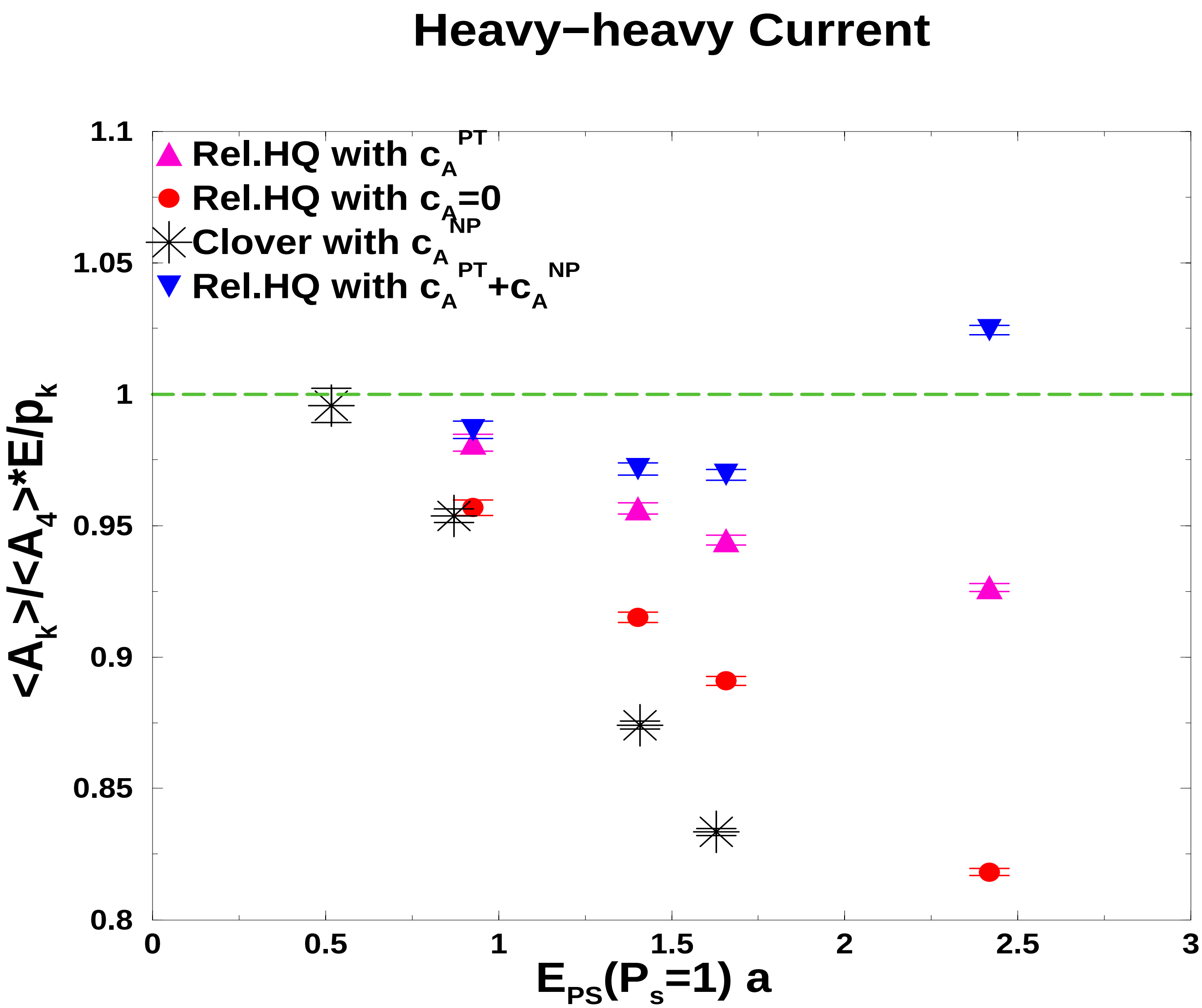
Effective speed of light in Heavy-heavy meson case.

Space-time symmetry from Axial vector current A_k and A_4 :

$$i\frac{< 0|A_k^R|PS >}{< 0|A_4^R|PS >} * \frac{E}{|p_s|} \quad \xrightarrow{a\to 0} \quad 1$$



Space-time symmetry from Heavy-Light current.

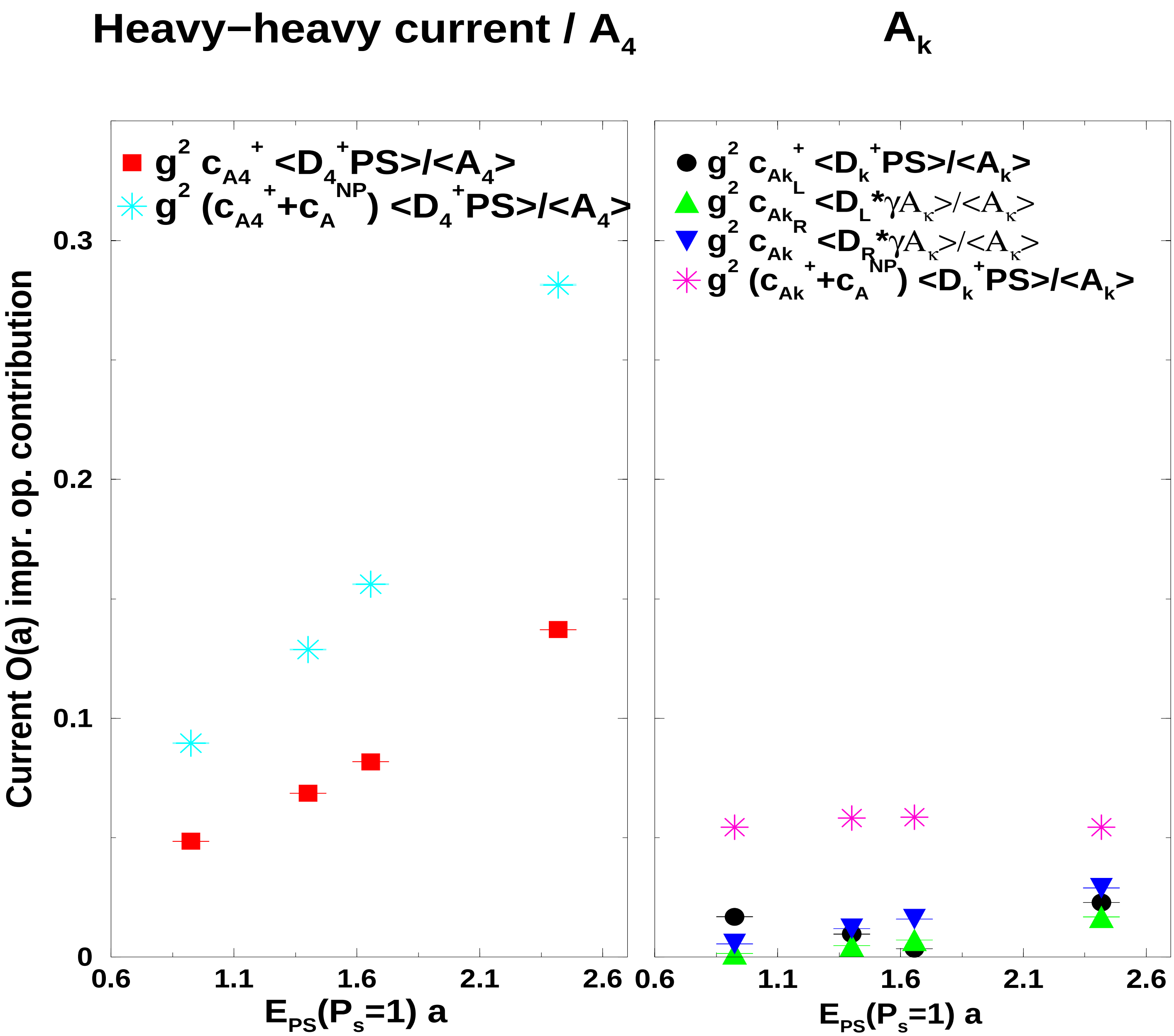


Space-time symmetry from Heavy-heavy current.

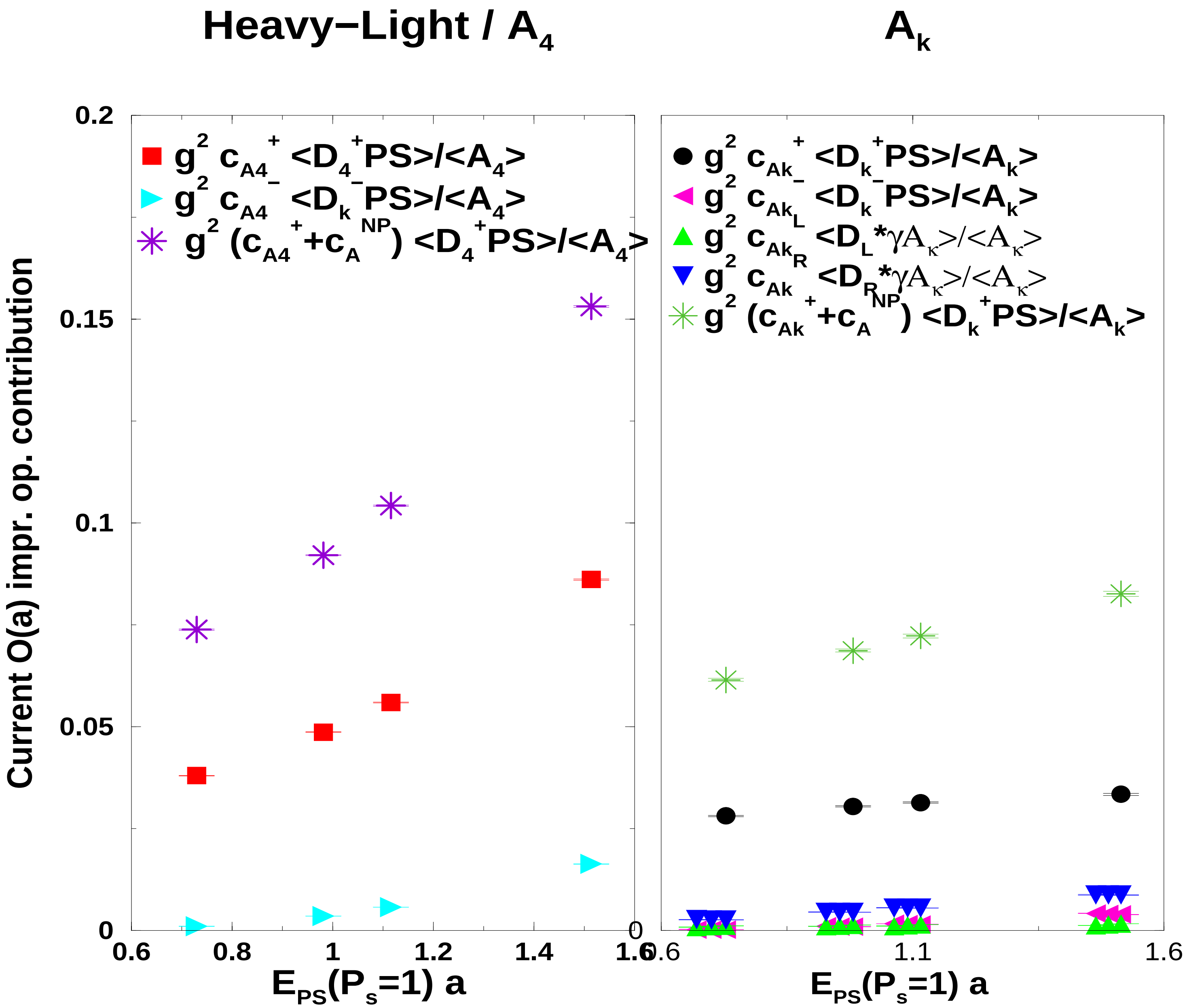
Clover action breaks Space-time symmetry and Dispertion relation to an $O(f_1(m_Q a)(a\Lambda_{QCD}))$.

Relativistic heavy quark action holds them within $O(f_2(am_Q)(a\Lambda_{QCD})^2)$ errors.

Which is the operator that gives dominant correction to Axial vector current?



Contributions to the heavy-heavy current from each $O(a)$ improvement operator.



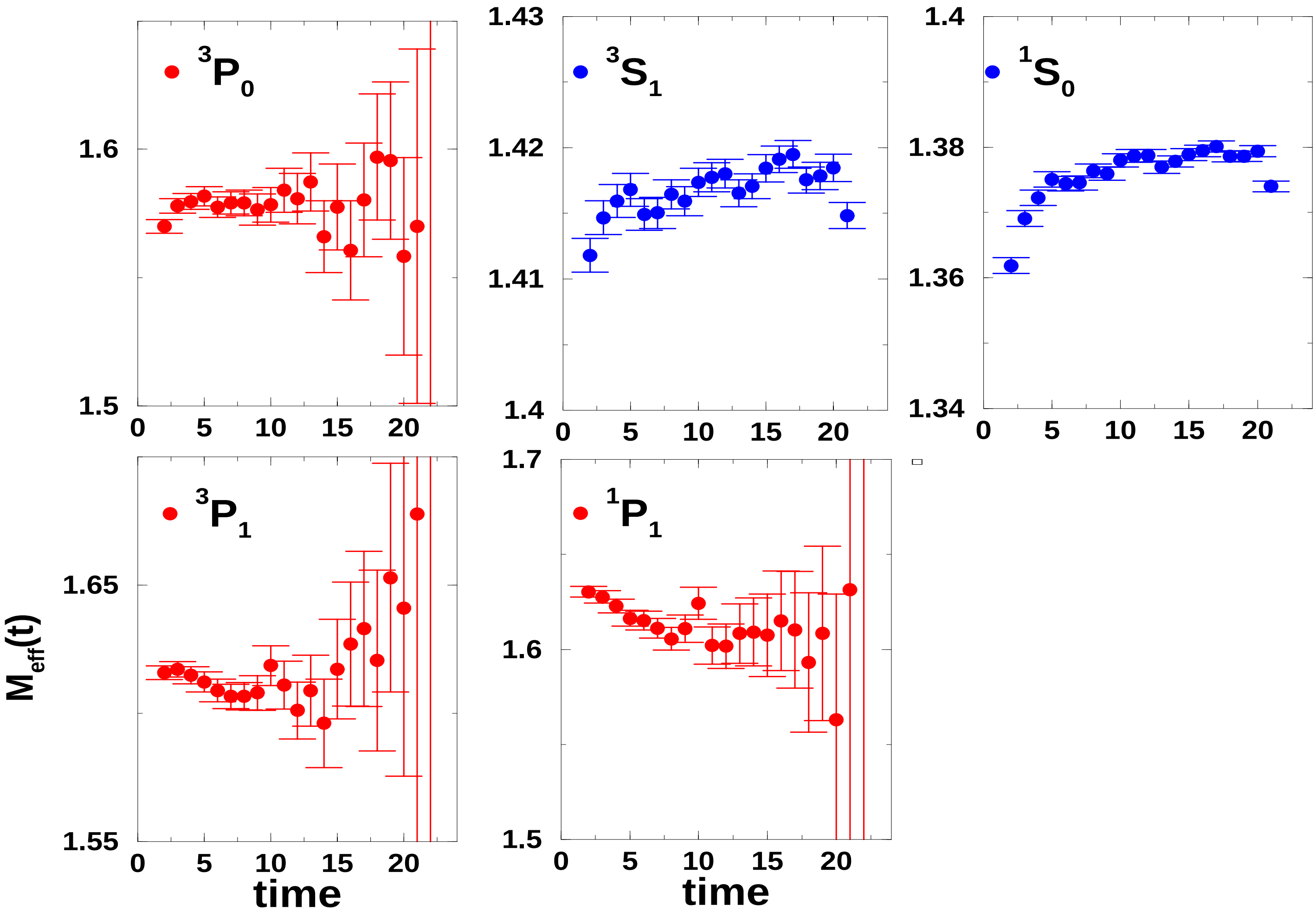
Contributions to the heavy-light current from each $O(a)$ improvement operator.

Dominant contribution comes from total derivative terms $c^{+}_{A_\mu}\partial^{+}_{\mu}(\bar{q}(x)\gamma_5 Q(x))$.

[3. Chamed meson spectrum and decay constant (preliminary results)]

Chamed meson spectrum

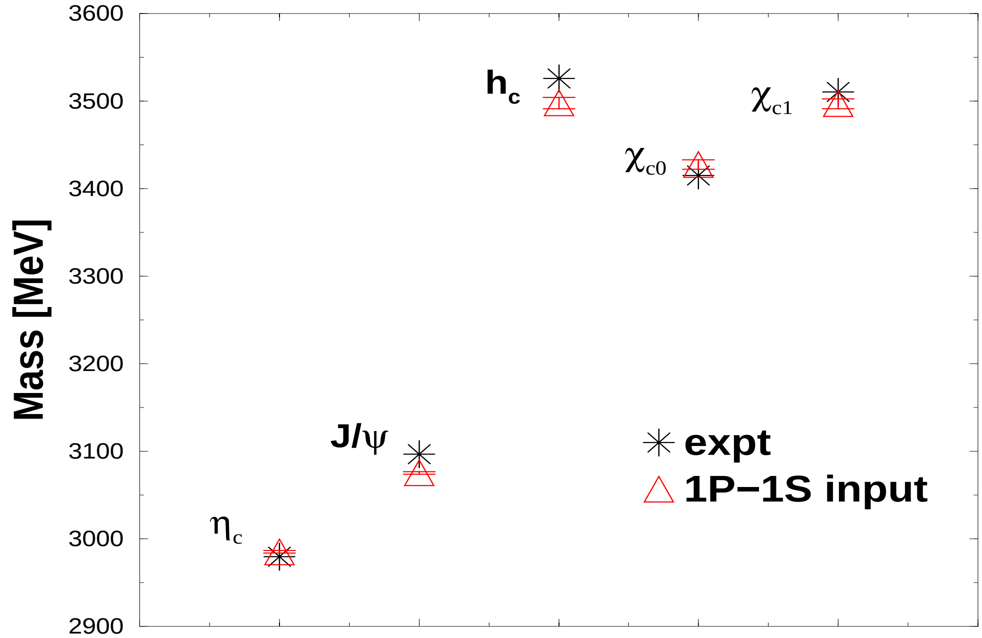
$^{2S+1}L_J$	J^{PC}	name	Γ operator
$^1S_0$	0^{-+}	η_c	$\bar{\psi}\gamma_5\psi$
$^3S_1$	1^{--}	J/ψ	$\bar{\psi}\gamma_i\psi$
$^1P_1$	1^{+-}	h_c	$\bar{\psi}\sigma_{ij}\psi$
$^3P_0$	0^{++}	χ_{c0}	$\bar{\psi}\psi$
$^3P_1$	1^{++}	χ_{c1}	$\bar{\psi}\gamma_i\gamma_5\psi$



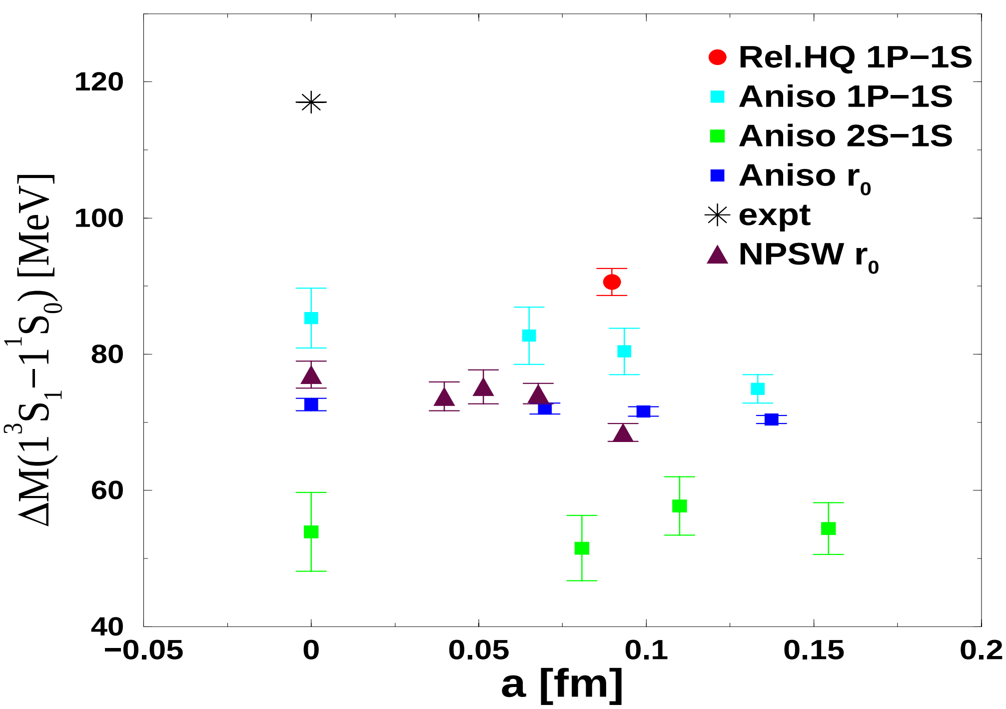
Effective mass of charmonium meson($k5 - k5$ case).

$$k_{charm} = 0.1023(35) \text{ from } \frac{M(1\overline{S})}{\Delta M(1\overline{P}-1\overline{S})}$$

$$a^{-1}(1\overline{P} - 1\overline{S}) = 2.195(23)[GeV]$$



Charmonium Masses.

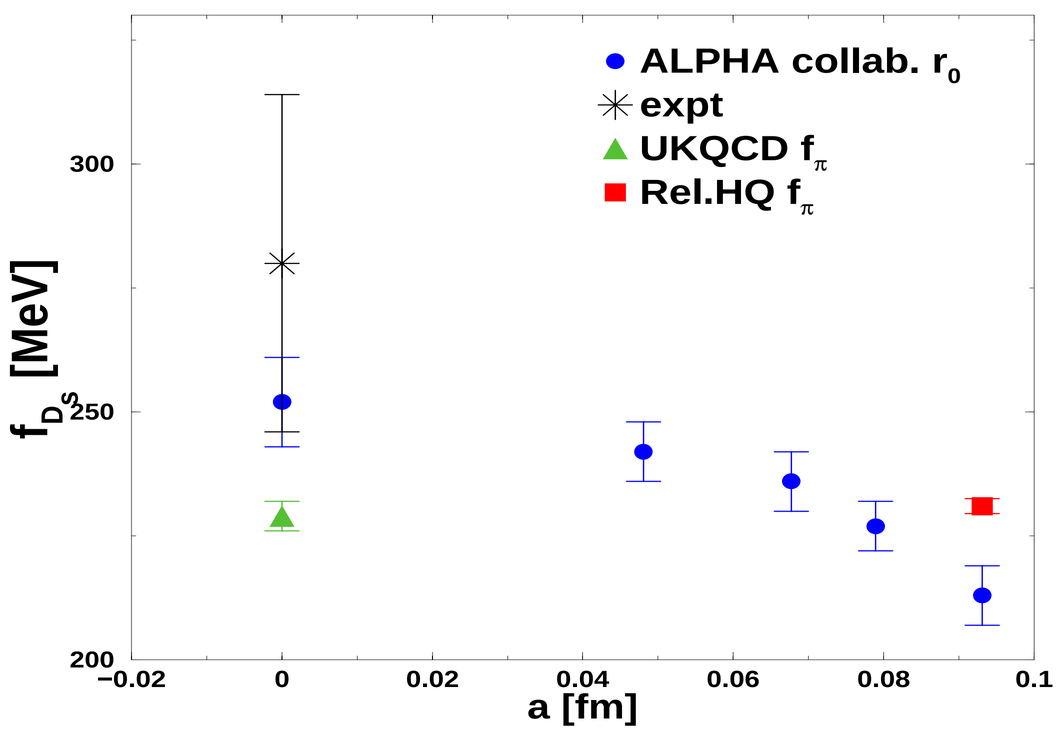


S-state hyper-fine splitting.

D_s meson decay constant.

$$< 0|A^R_\mu|PS >= i p_\mu f_{PS}$$

$$a^{-1}(f_\pi) = 1.91(6)[GeV]$$



D_s meson decay constant.

[4. Summary]

$O(a\Lambda_{QCD})$ improved heavy quark action for $m_Q > \Lambda_{QCD}$

$$S_q = \sum_x [m_0 \bar{q}(x)q(x) + \bar{q}(x)\gamma_0 D_0 q(x) + \nu \sum_i \bar{q}(x)\gamma_i D_i q(x)$$
$$-\frac{r_t a}{2}\bar{q}(x)D_0^2 q(x) - r_s \frac{a}{2}\sum_i \bar{q}(x)D_i^2 q(x)$$
$$-\frac{iga}{2}c_E \sum_i \bar{q}(x)\sigma_{0i}F_{0i}q(x) - \frac{iga}{4}c_B \sum_{i,j}\bar{q}(x)\sigma_{ij}F_{ij}q(x)\Big]$$

$r_t = 1$

$O(a\Lambda_{QCD})$ improved Axial vector current for $m_Q > \Lambda_{QCD}$

$\Gamma_\mu = \gamma_\mu\gamma_5$

$$A_\mu^{lat,R}(x) = Z_A^{lat.}[\,\bar{q}(x)\Gamma_\mu Q(x)$$
$$-g^2 c_{A_\mu}^L(\vec{\partial}_i \bar{q}(x))\gamma_i\Gamma_\mu Q(x) - g^2 c_{A_\mu}^H \bar{q}(x)\Gamma_\mu\gamma_i(\vec{\partial}_i Q(x))$$
$$-g^2 c_{A_\mu}^{+}\partial_\mu^{+}(\bar{q}(x)\gamma_5 Q(x)) - g^2 c_{A_\mu}^{-}\partial_\mu^{-}(\bar{q}(x)\gamma_5 Q(x))\,\Big]$$

$\mathcal{O}(a\Lambda_{QCD})$ improvement coefficients up to 1-loop level are calculated.

From Quenched simulation results, we confirmed that

- Clover action breaks Space-time symmetry and dispertion relation to an $O(f_1(m_Q a)(a\Lambda_{QCD}))$.
- Relativistic heavy quark action retains them within $O(f_2(am_Q)(a\Lambda_{QCD})^2)$ errors.

Dominant corrections to Axial vector currents comes from total derivative terms $c^{+}_{A_\mu}\partial^{+}_{\mu}(\bar{q}(x)\gamma_5 Q(x))$.

We have calculated Charmonium spectrum, S-state hyper-fine splitting and D_s meson decay constant at $\beta = 6.0$.

[Future work]

- Scaling behavior of f_{D_s}, $M(J/\psi - \eta_c)$, $\cdots$.
- Non-perturbative determination of $O(a)$ improvement coefficients.
- $N_f = 2, 3$ full QCD simulation.