# Non-perturbative renormalization in quenched QCD with Iwasaki action

CP-PACS Collaboration :
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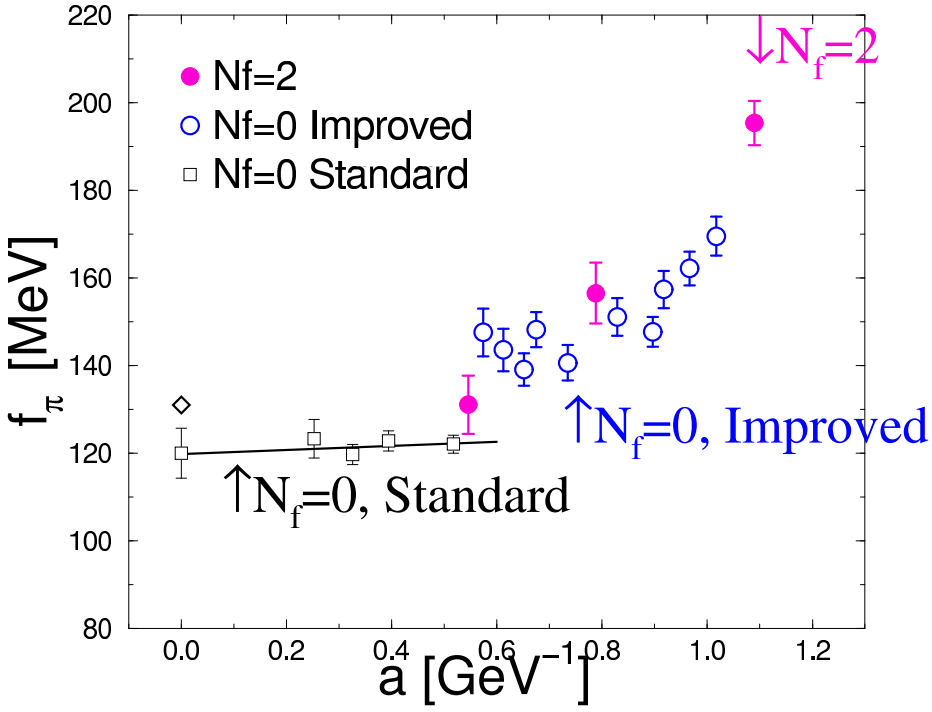
1. Motivation
2. Simulations
3. Results of Z-factors, $Z_V$ , $Z_A$
4. Scaling property of meson decay constants
5. Summary

# 1 Motivation

CP-PACS data




meson decay constants determined with an improved action on coarse lattices have large scaling violation

One possibility of the large scaling violation

Z-factors

tadpole improved 1-loop perturbation

purpose of this work

to see how the scaling behavior is changed, when we use Z-factors estimated non-perturbatively by the SF method in focus on quenched QCD this work

We have investigated

- feasibility of the SF method for this action
  (reported at LATTICE 2001)

In this work

- exceptional configurations which appear at small $\beta$
- Z-factors normalized at $L \to \infty$
- scaling property of decay constants

# 2 Simulation

## 2.1 Simulation detail

- the same action comb. as CP-PACS

  RG gauge action

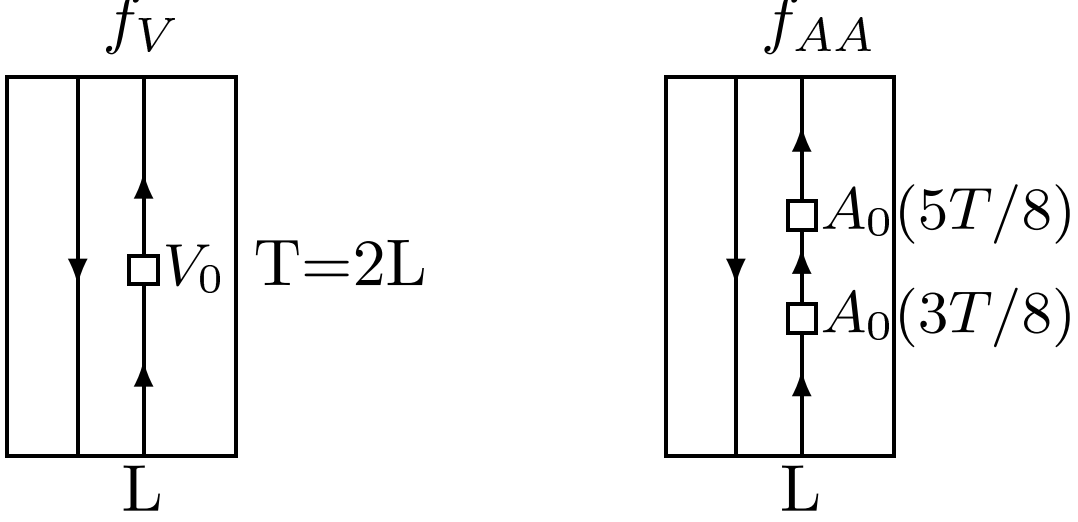
  $S_g = \beta\Big\{c_0 U_{\rm plaq} + c_1 U_{\rm rect}\Big\}$, $c_1 = -0.331$

  tadpole improved clover quark action

  $$c_{SW} = \left(1 - \frac{0.8412}{\beta}\right)^{-\frac{3}{4}}$$

  determined perturbatively

- this combination of actions has $O(a)$ error




Similarly $f_1$, $f_A$ and $f_P$

$$Z_V = \frac{f_1}{f_V}, \quad Z_A = \sqrt{\frac{f_1}{f_{AA}}},$$

$$m_q = \frac{\frac{1}{2}(\partial_0^* + \partial_0) f_A + c_A a \partial_0^* \partial_0 f_P}{2 f_P}$$

Physical size

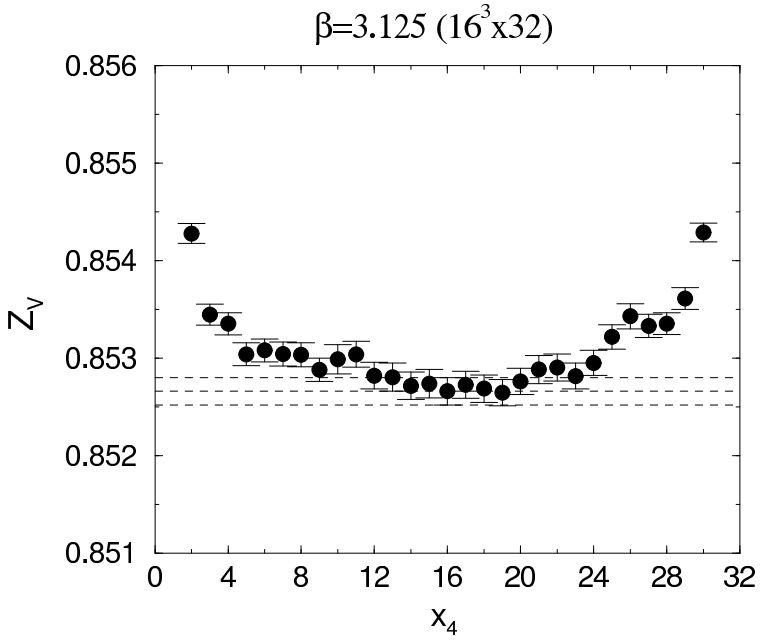
String tension is used to set physical volume

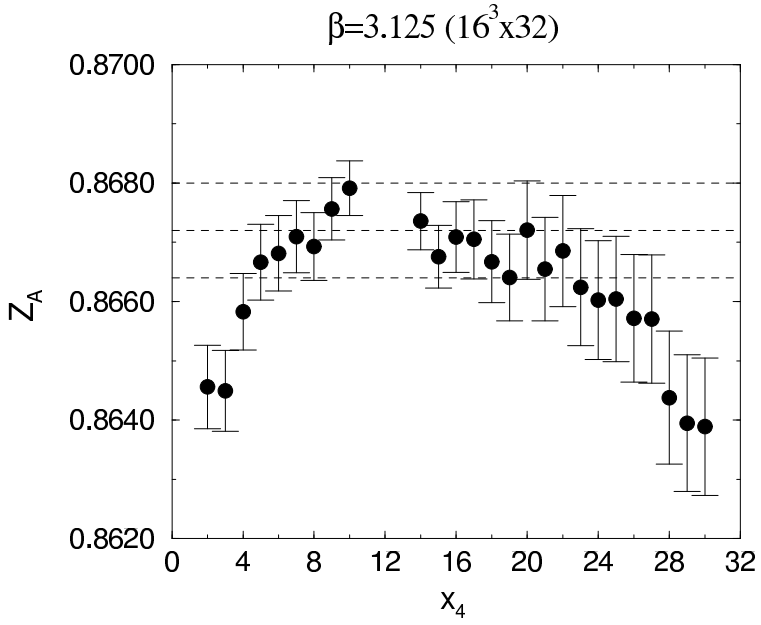
Z-factors normalized at

- $8^3$ lattice at $\beta = 2.6$ $L \approx 0.8$ fm
- $L = \infty$

apply the Schrödinger functional method to our action combination

plot Z-factors vs. time slice



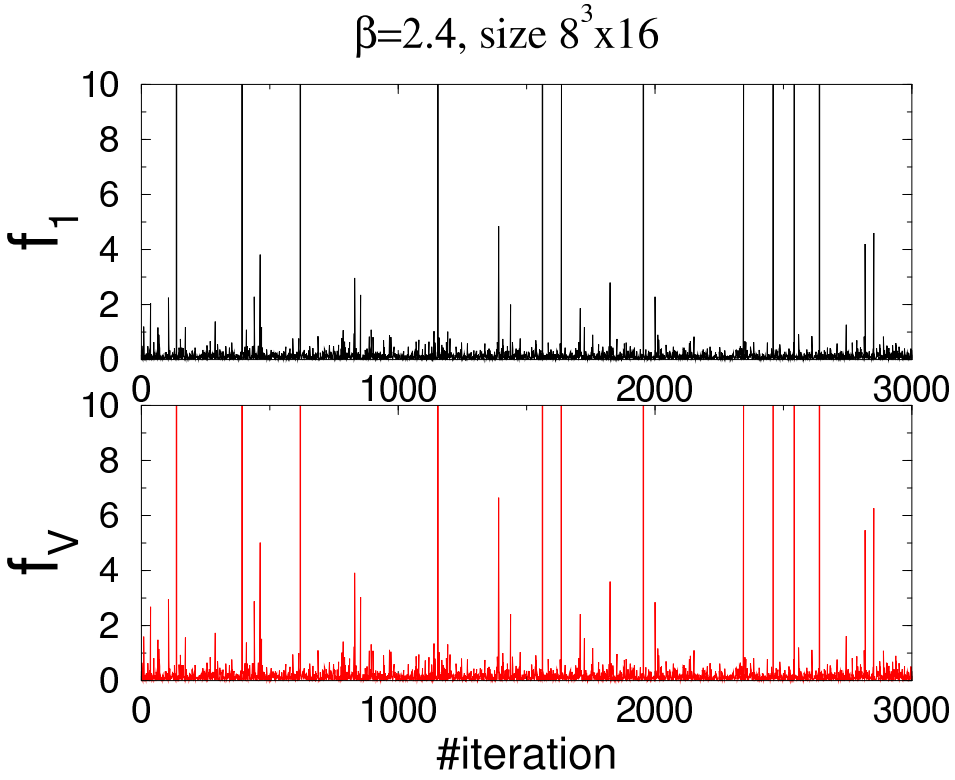

reasonable plateaus of Z-factors appear

Schrödinger functional method can be safely applied our action combination

## 2.2 Exceptional configuration

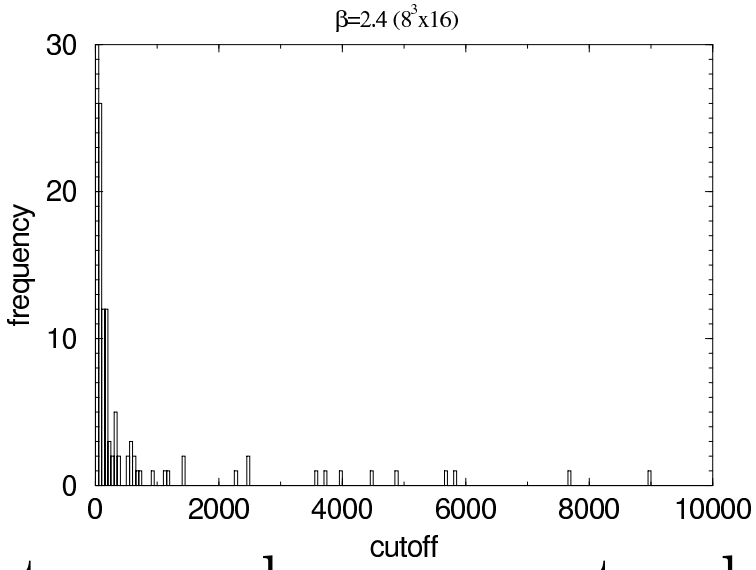
straight-forward to calculate Z-factors for a range of high $\beta$ values, $\beta \geq 2.6$

However, at $\beta = 2.2, 2.4$ on $8^3 \times 16$ lattice, anomalously large values appear in the ensembles of $f_A$, $f_P$, $f_1$, $f_V$ and $f_{AA}$




CP-PACS data of $f_\pi$, $f_\rho$ exist

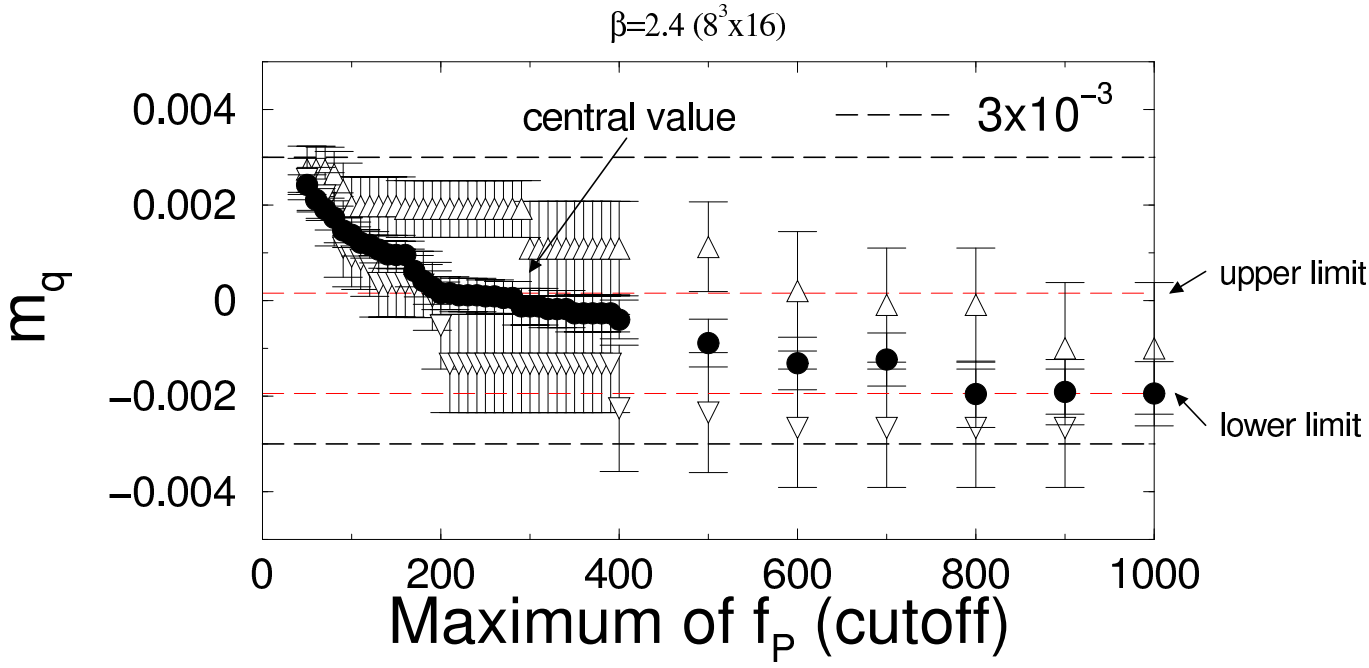
for $2.187 \leq \beta \leq 2.575$




- histogram drags on up to a large values

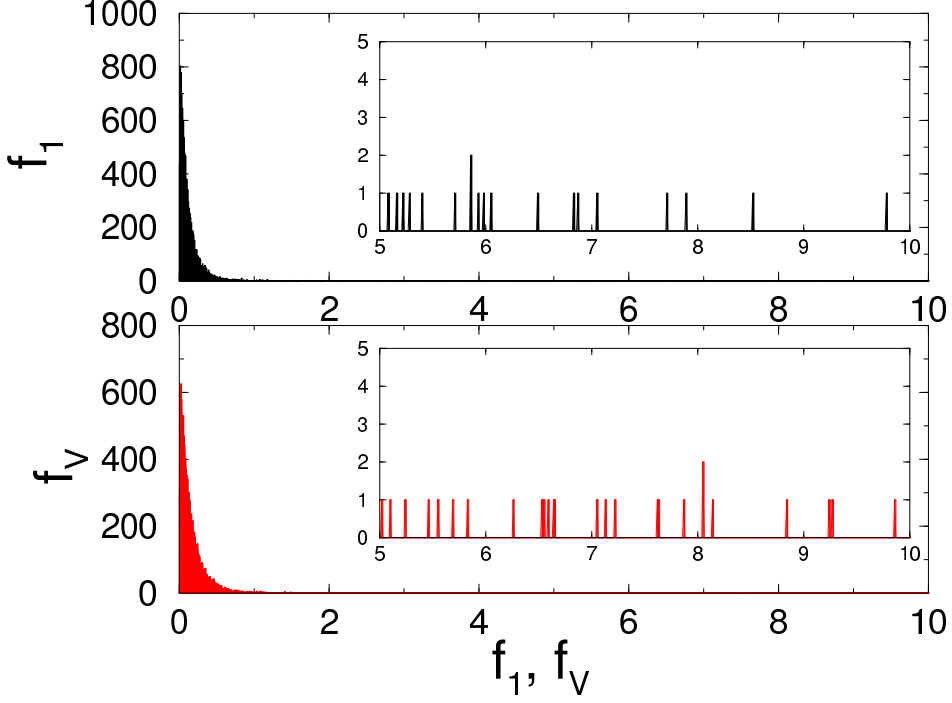
- Difficult to determine $\kappa_c$ precisely

$\Rightarrow$ discard large values from ensembles




central value from cut=300

Effects of uncertainty of quark mass on Z-factors later

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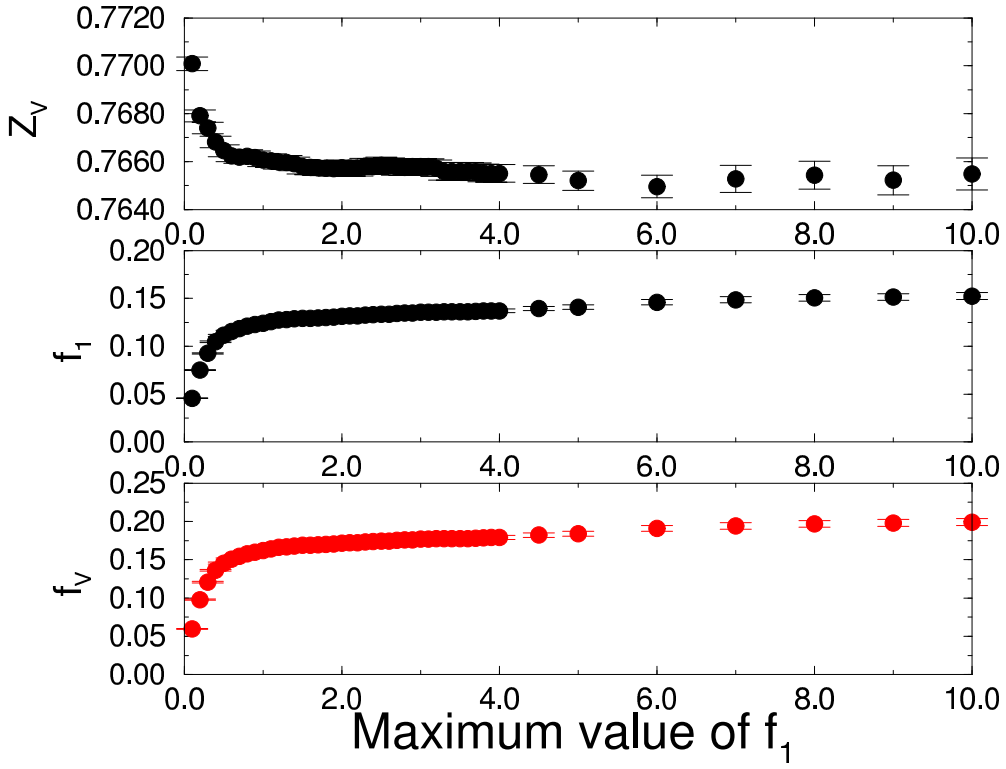
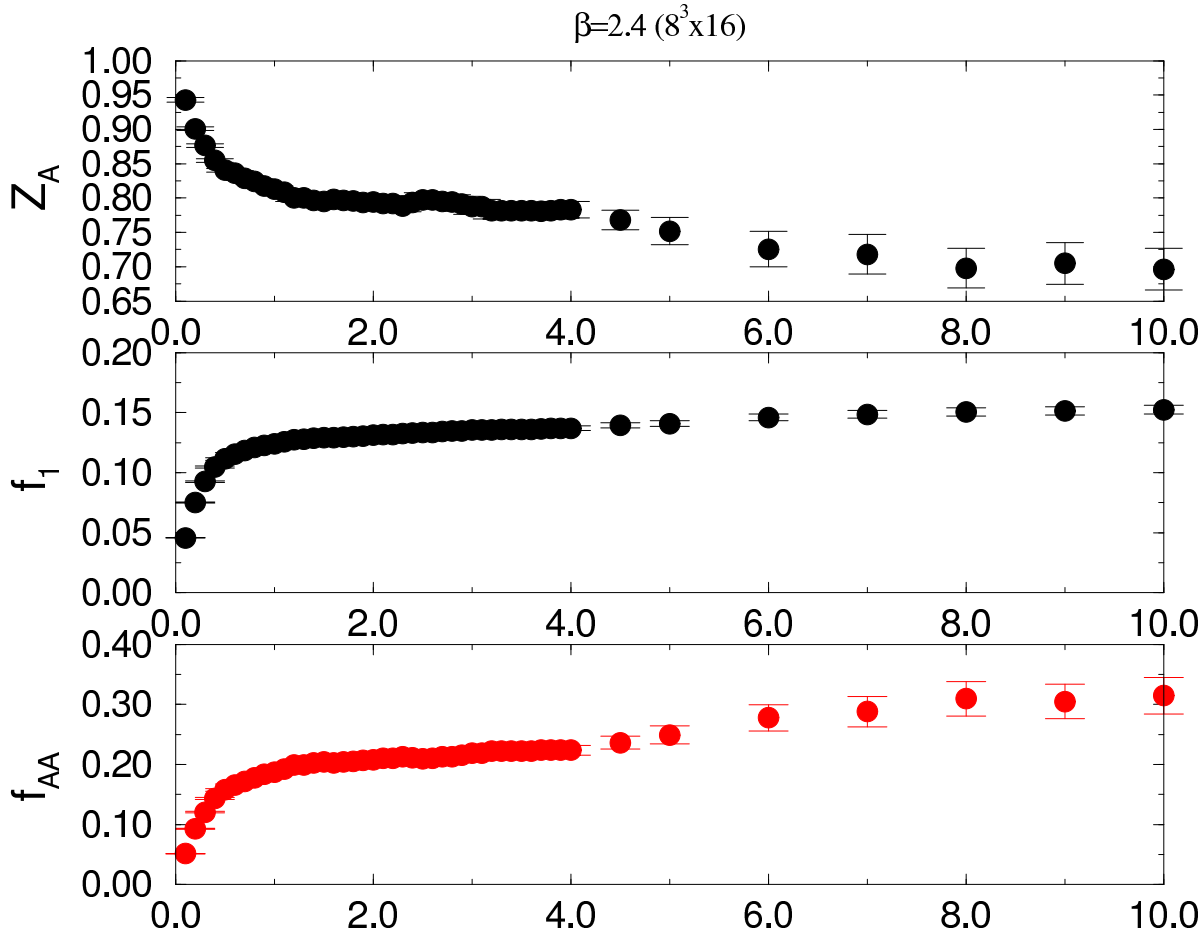
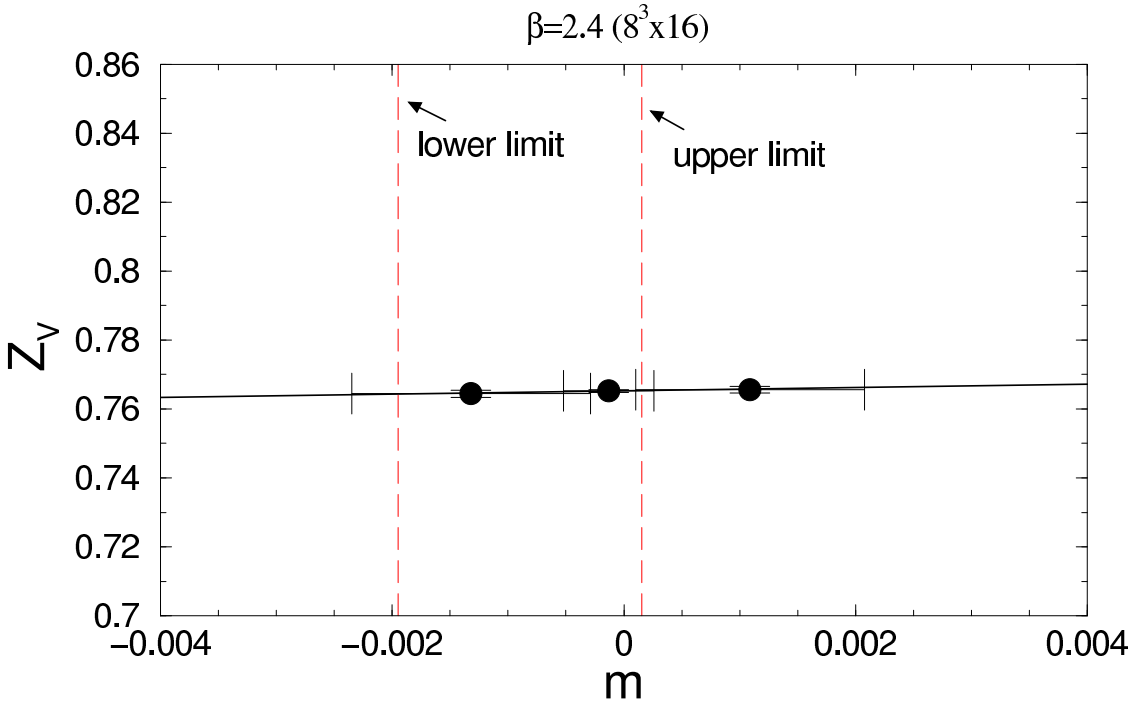
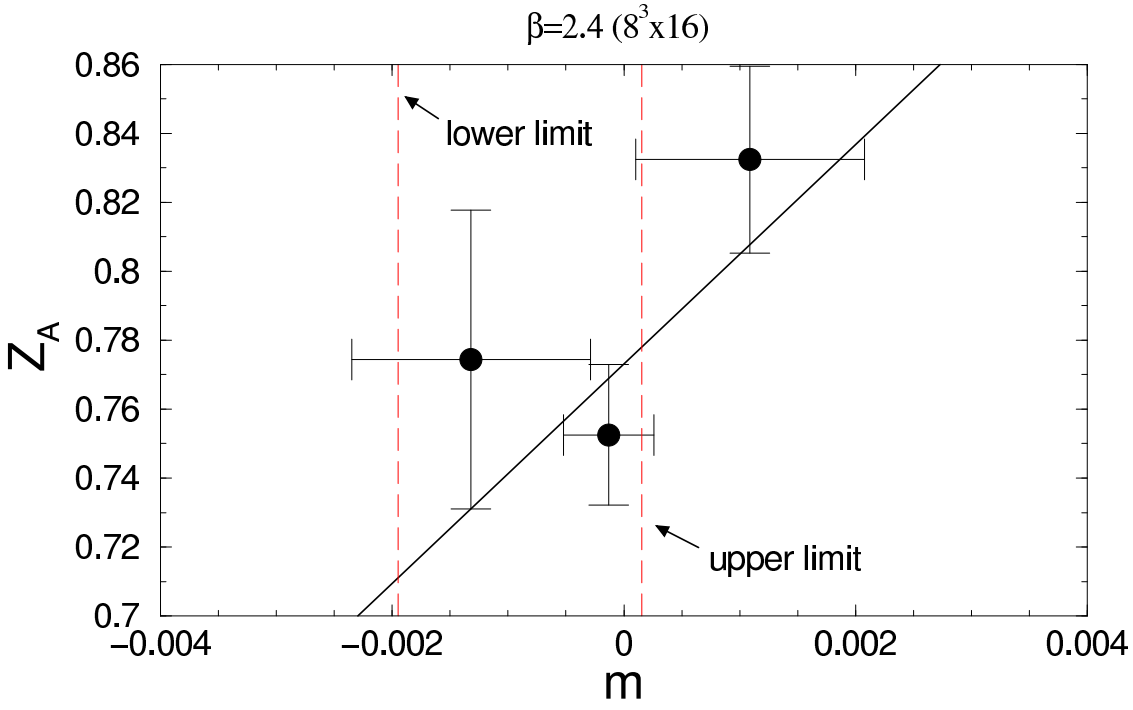
- $f_1, f_V, f_{AA}$ depend strongly on "cut"
- $Z_V = \frac{f_1}{f_V}$, $Z_A = \sqrt{\frac{f_1}{f_{AA}}}$ are stable against the change of "cut"



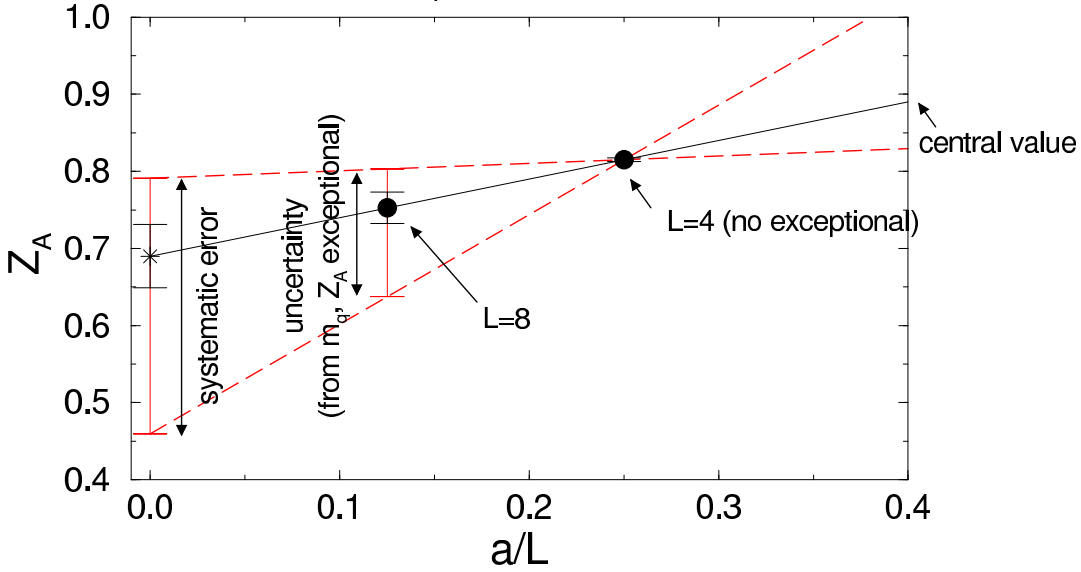

β=2.4 (8^3x16)
Z_A
f_1
f_AA
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.20
0.15
0.10
0.05
0.00
0.40
0.30
0.20
0.10
0.00
0.0
2.0
4.0
6.0
8.0
10.0




Change of $Z_V$ is little for range on uncertainty of $m_q$
a typical $m_q$ dependence of $Z_A$

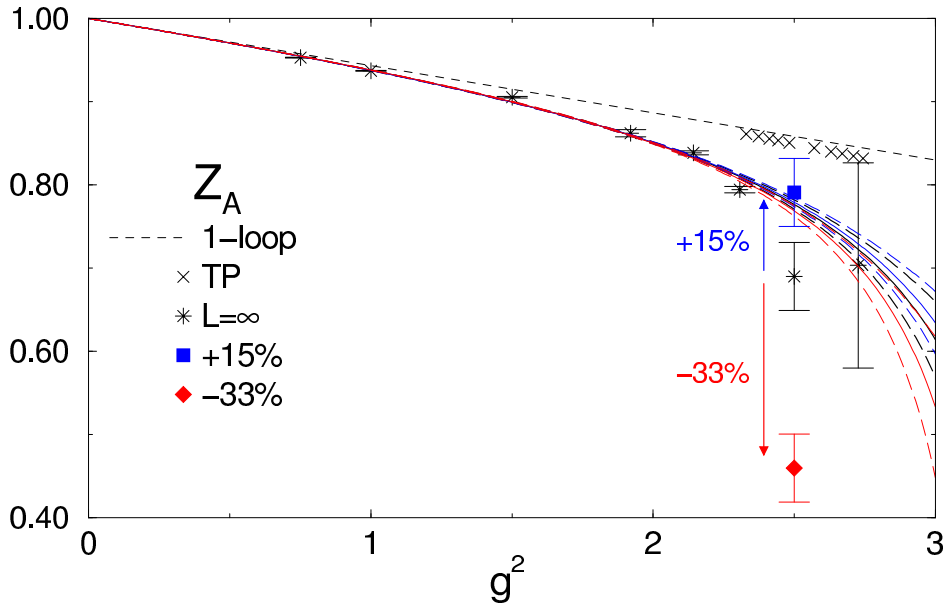



Change of $Z_A$ is not small for the range



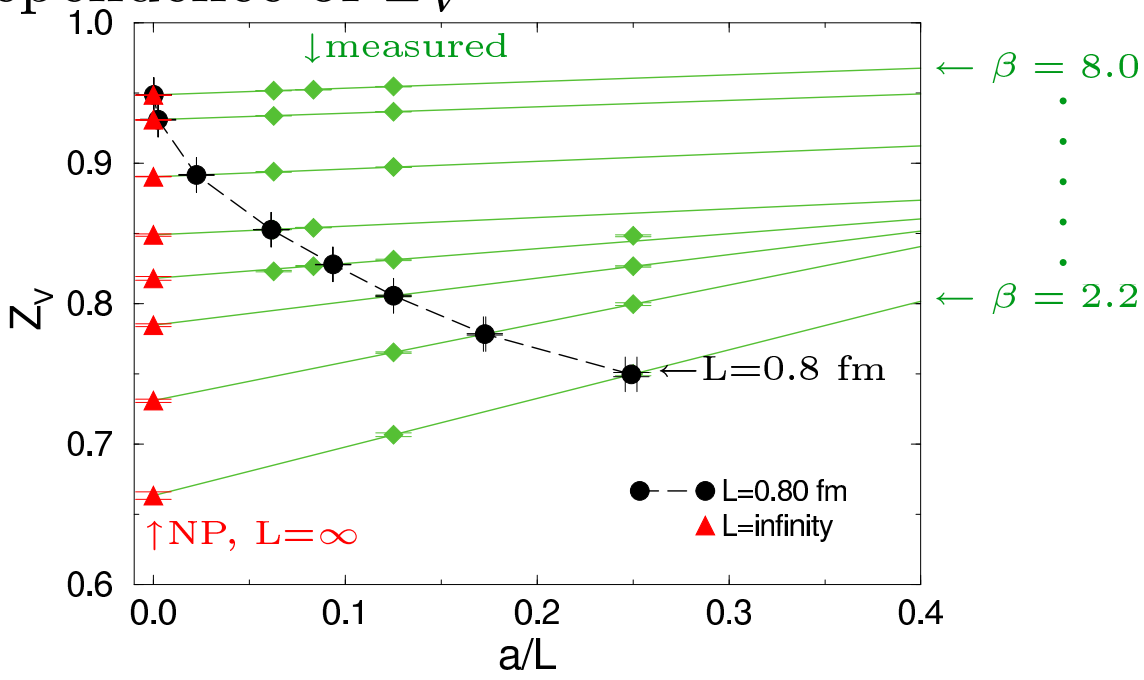

uncertainty of $m_q = 0$ point leads to about $+15 \sim -33$ % systematic error in $Z_A$ ($L = \infty$)

Pad'e fit unchanged

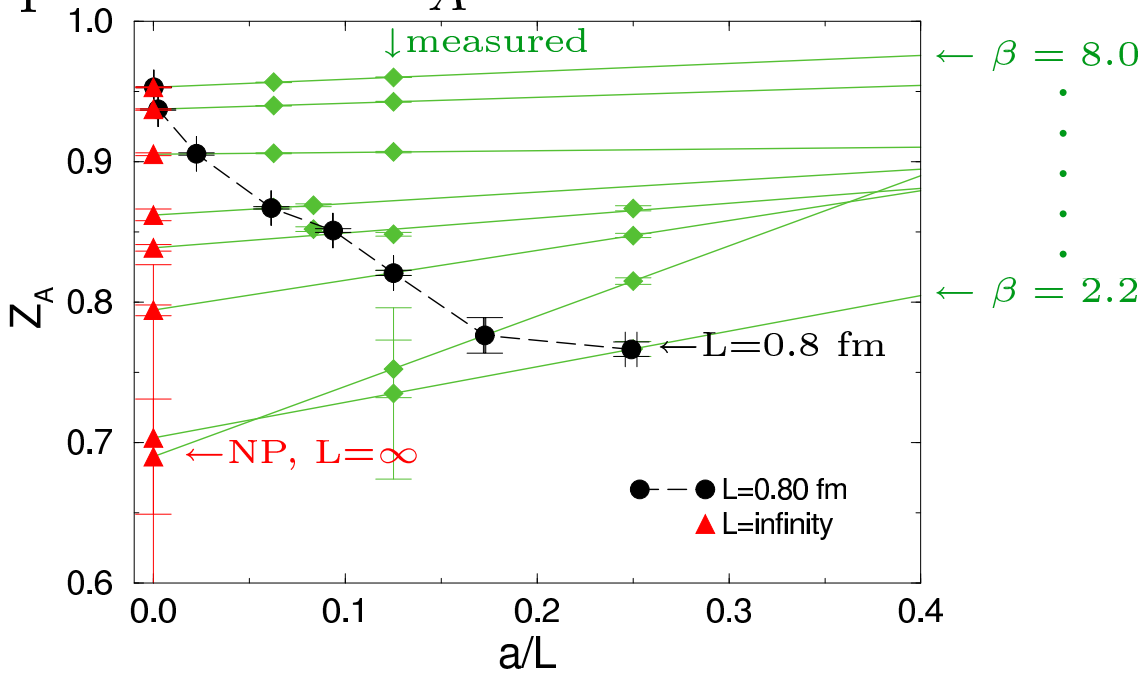


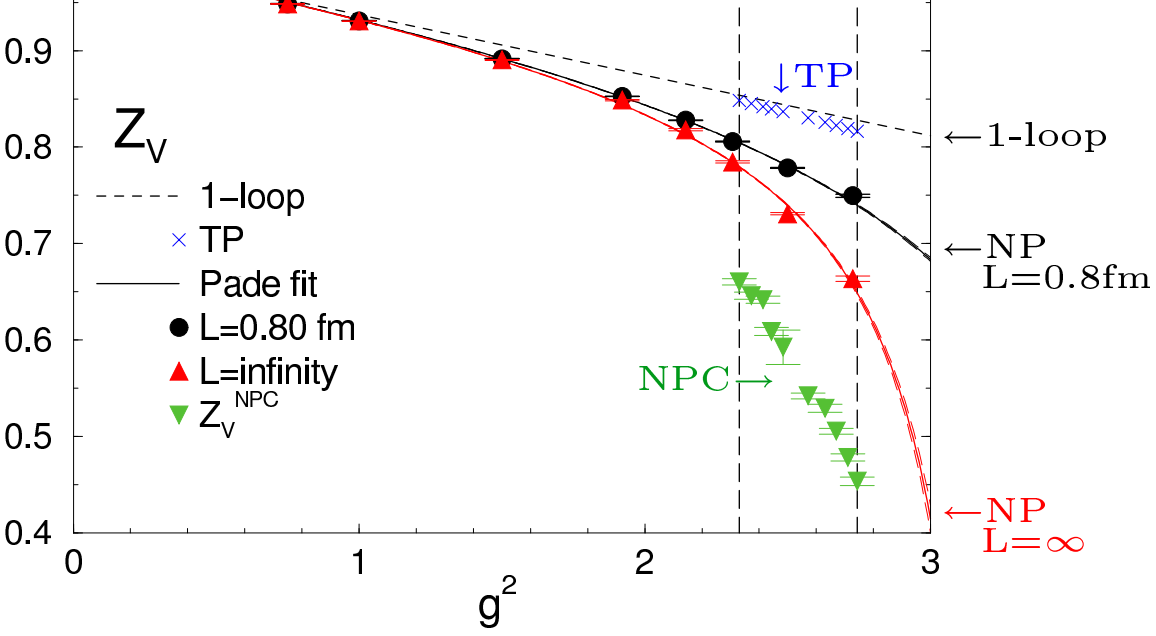

Pad'e fit for $Z_A$ is determined mainly by data at high $\beta$ values ($\beta \geq 2.6$)
The ambiguity in mq dose not affect our final results

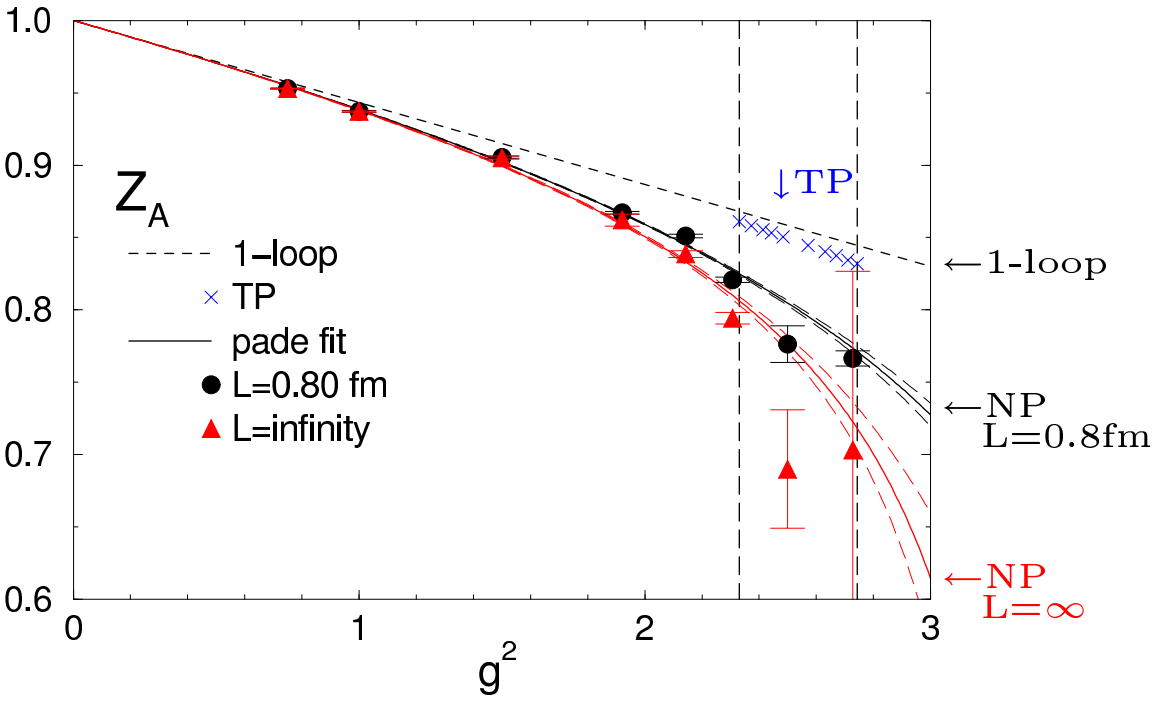
# Size dependence of $Z_V$




# Size dependence of $Z_A$






non-perturbative values of Z-factors ($L = \infty$) are smaller than perturbative ones by about

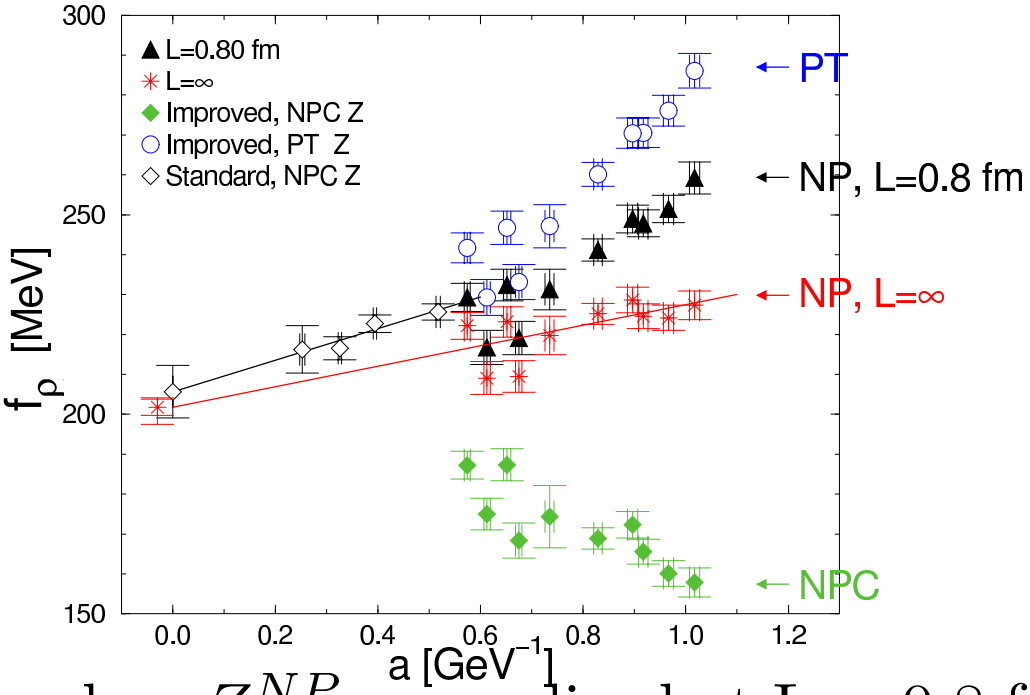
20% for vector current

15% for axial-vector current

at the largest coupling of the CP-PACS simulation, $\beta$=2.187.

# meson decay constants

$f_\rho$

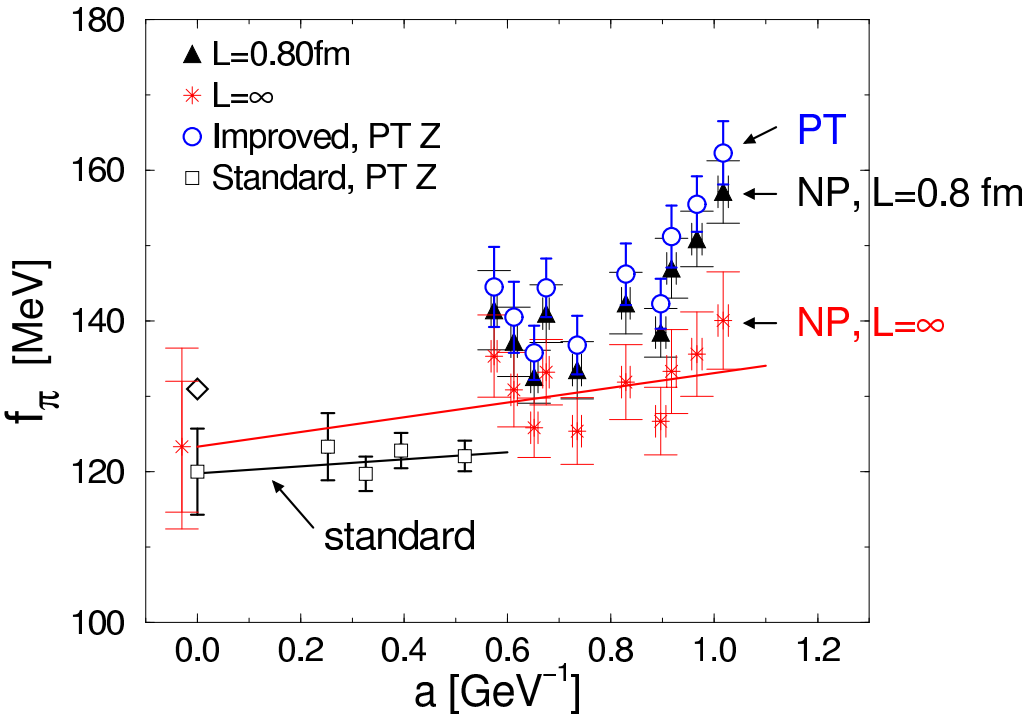



- when $Z_V^{NP}$ normalized at L = 0.8 fm is used, scaling violation is reduced. and when that normalized at L = $\infty$ is used, scaling violation is further reduced

- $f_\rho$ in the continuum limit is consistent with that from standard action

- when we use $Z_V^{NPC}$ from conserved current, large scaling violation remains because conserved vector current is not $O(a)$ improved with tensor operator

$f_\pi$




- Non-perturbative Z-factor reduces magnitude of scaling violation also for $f_\pi$
- the continuum limit of $f_\pi$ is consistent with that from standard action

# 5 Summary

For the RG improved gauge and tadpole improved clover quark action,
scaling violations of meson decay constants are significantly reduced
by using Z-factors estimated non-perturbatively rather than those estimated by perturbation theory.

Scaling property is best improved, when Z-factors are normalized at infinite volume.
We think that this is achieved
because $O(a/L)$ error in Z-factors are completely removed.

TABLE I.

| $\beta$ | $L^3 \times T$ | $\kappa_c$ | $m_q$ | $Z_V$ | # of conf for $Z_V$ | $Z_A$ | # of conf for $Z_A$ |
|---|---|---|---|---|---|---|---|
| 2.2 | $4^3 \times 8$ | 0.139281(56) | -0.000529(495) | 0.7499(11) | 2000 | 0.7667(52) | 2000 |
| | $8^3 \times 16$ | 0.140570(15) | 0.001493(399) | 0.7067(12) | 20000 | 0.7350(610) | 20000 |
| 2.4 | $4^3 \times 8$ | 0.136933(21) | -0.000764(337) | 0.7995(10) | 1000 | 0.8150(24) | 2000 |
| | $8^3 \times 16$ | 0.137481(04) | -0.000130(395) | 0.7652(04) | 10000 | 0.7525(204) | 10000 |
| 2.6 | $4^3 \times 8$ | 0.135558(08) | -0.000276(341) | 0.8265(05) | 2000 | 0.8474(16) | 2000 |
| | $8^3 \times 16$ | 0.135701(10) | -0.000464(154) | 0.8056(05) | 500 | 0.8208(18) | 2000 |
| 2.8 | $4^3 \times 8$ | 0.134532(10) | 0.000002(396) | 0.8482(06) | 1000 | 0.8667(18) | 1000 |
| | $8^3 \times 16$ | 0.134515(07) | -0.000105(139) | 0.8312(04) | 500 | 0.8482(13) | 1000 |
| | $12^3 \times 24$ | 0.134554(08) | 0.000525(111) | 0.8268(03) | 500 | 0.8520(17) | 1000 |
| | $16^3 \times 32$ | 0.134587(09) | 0.000663(139) | 0.8231(06) | 500 | | |
| 3.125 | $12^3 \times 24$ | 0.133209(01) | -0.000039(091) | 0.8540(02) | 500 | 0.8689(09) | 500 |
| | $16^3 \times 32$ | 0.133219(05) | -0.000092(060) | 0.8527(01) | 500 | 0.8672(08) | 500 |
| 4.0 | $8^3 \times 16$ | 0.131094(04) | -0.000302(086) | 0.8972(02) | 500 | 0.9068(06) | 500 |
| | $16^3 \times 32$ | 0.131083(01) | 0.000160(044) | 0.8938(01) | 500 | 0.9060(04) | 300 |
| 6.0 | $8^3 \times 16$ | 0.128898(03) | -0.000191(061) | 0.9366(01) | 300 | 0.9425(03) | 300 |
| | $16^3 \times 32$ | 0.128891(01) | -0.000060(023) | 0.9337(01) | 300 | 0.9398(02) | 300 |
| 8.0 | $8^3 \times 16$ | 0.127869(02) | 0.000138(041) | 0.9545(01) | 300 | 0.9600(02) | 300 |
| | $12^3 \times 24$ | 0.127870(01) | 0.000576(331) | 0.9523(01) | 300 | | |
| | $16^3 \times 32$ | 0.127870(01) | 0.000021(017) | 0.95151(3) | 300 | 0.9565(01) | 200 |

TABLE II. Z-factors at fixed physical volumes

| $\beta$ | $Z_V$ normalized at $L = 0.8$ fm | $Z_V$ normalized at $L = \infty$ | $Z_A$ normalized at $L = 0.8$ fm | $Z_A$ normalized at $L = \infty$ |
|---|---|---|---|---|
| 2.2 | 0.7495(15) | 0.6635(26) | 0.7664(52) | 0.7033(1221) |
| 2.4 | 0.7783(05) | 0.7309(13) | 0.7763(127) | 0.6900(409) |
| 2.6 | 0.8056(05) | 0.7847(11) | 0.8208(18) | 0.7942(39) |
| 2.8 | 0.8279(02) | 0.8180(12) | 0.8511(13) | 0.8386(22) |
| 3.125 | 0.8526(01) | 0.8488(07) | 0.8671(08) | 0.8621(42) |
| 4.0 | 0.8917(02) | 0.8904(03) | 0.9055(08) | 0.9052(10) |
| 6.0 | 0.9309(02) | 0.9308(02) | 0.9372(05) | 0.9371(05) |
| 8.0 | 0.9485(01) | 0.9485(01) | 0.9530(03) | 0.9530(03) |

TABLE III.

| | $\delta m_q$ | $\delta Z^{(m_q)}$ | $\delta Z^{(except.)}$ | $\delta Z$ | $\delta Z^{(L=0.8\text{ fm})}$ | $\delta Z^{(L=\infty)}$ |
|---|---|---|---|---|---|---|
| $Z_V(\beta = 2.2)$ | +0.00344-0.00170 | +0.04%-0.02% | +0.31%-0.06% | +0.35%-0.08% | +0.0013%-0.0058% | +0.75%-0.17% |
| $Z_V(\beta = 2.4)$ | +0.00028-0.00182 | +0.01%-0.12% | +0.08%-0.03% | +0.09%-0.15% | +0.044%-0.10% | +0.19%-0.31% |
| $Z_A(\beta = 2.2)$ | +0.00344-0.00170 | +3.2% -1.5% | +18.1%-2.1% | +21.3%-3.6% | +0.17%-0.25% | +44.5%-8.9% |
| $Z_A(\beta = 2.4)$ | +0.00028-0.00182 | +0.6% -8.0% | +6.1%-7.3% | +6.7%-15.3% | +4.0%-9.2% | +14.6%-33.4% |

TABLE IV. $f_\rho$ for various choices of $Z_V$.

| $\beta$ | $a^{-1}[GeV]$ | $Z_V$ (TP) | $Z_V$ (NP,$L = 0.8$ fm) | $Z_V$ (NP,$L = \infty$) | $f_\rho$ (TP) | $f_\rho$ (NP,$L = 0.8$ fm) | $f_\rho$ (NP,$L = \infty$) |
|---|---|---|---|---|---|---|---|
| 2.187 | 1.017(10) | 0.81657 | 0.7400(08)(+04-10) | 0.6488(24)(+48-27) | 0.2861(44) | 0.2593(43)(+01-04) | 0.2273(43)(+17-09) |
| 2.214 | 0.966(10) | 0.81923 | 0.7462(07)(+04-09) | 0.6651(20)(+40-23) | 0.2761(38) | 0.2515(37)(+01-03) | 0.2242(38)(+13-08) |
| 2.247 | 0.917(09) | 0.82237 | 0.7532(06)(+03-08) | 0.6825(16)(+32-19) | 0.2706(37) | 0.2478(36)(+01-03) | 0.2246(36)(+11-06) |
| 2.281 | 0.896(10) | 0.82548 | 0.7601(06)(+03-07) | 0.6981(13)(+26-16) | 0.2704(38) | 0.2490(37)(+01-02) | 0.2287(36)(+09-05) |
| 2.334 | 0.829(08) | 0.83009 | 0.7698(04)(+02-06) | 0.7187(10)(+20-13) | 0.2601(30) | 0.2412(29)(+01-02) | 0.2252(29)(+06-04) |
| 2.416 | 0.734(09) | 0.83673 | 0.7832(03)(+02-04) | 0.7441(07)(+13-09) | 0.2471(54) | 0.2313(51)(+01-01) | 0.2197(50)(+04-03) |
| 2.456 | 0.674(06) | 0.83978 | 0.7891(03)(+01-03) | 0.7544(06)(+10-08) | 0.2332(44) | 0.2191(42)(+00-01) | 0.2095(41)(+03-02) |
| 2.487 | 0.652(07) | 0.84205 | 0.7934(03)(+01-03) | 0.7617(06)(+09-07) | 0.2467(42) | 0.2324(40)(+00-01) | 0.2232(40)(+03-02) |
| 2.528 | 0.612(06) | 0.84496 | 0.7988(02)(+01-03) | 0.7705(05)(+07-06) | 0.2293(45) | 0.2168(43)(+00-01) | 0.2091(42)(+02-02) |
| 2.575 | 0.574(06) | 0.84816 | 0.8046(02)(+01-02) | 0.7796(05)(+06-05) | 0.2417(37) | 0.2293(36)(+00-01) | 0.2222(35)(+02-01) |

TABLE V. $f_\pi$ for various choice of $Z_A$

| $\beta$ | $a^{-1}[GeV]$ | $Z_A$ (TP) | $Z_A$ (NP,$L = 0.8$ fm) | $Z_A$ (NP,$L = \infty$) | $f_\pi$ (TP) | $f_\pi$ (NP,$L = 0.8$ fm) | $f_\pi$ (NP,$L = \infty$) |
|---|---|---|---|---|---|---|---|
| 2.187 | 1.017(10) | 0.83204 | 0.7712(38)(+27-54) | 0.7180(146)(+125-236) | 0.1623(42) | 0.1504(46)(+05-11) | 0.1401(65)(+24-46) |
| 2.214 | 0.966(10) | 0.83449 | 0.7762(34)(+24-49) | 0.7276(127)(+109-203) | 0.1555(37) | 0.1446(41)(+04-09) | 0.1356(56)(+20-38) |
| 2.247 | 0.917(09) | 0.83737 | 0.7819(30)(+21-43) | 0.7383(108)(+093-171) | 0.1512(41) | 0.1412(44)(+04-08) | 0.1333(56)(+17-31) |
| 2.281 | 0.896(10) | 0.84022 | 0.7875(27)(+19-38) | 0.7482(092)(+080-143) | 0.1423(33) | 0.1334(36)(+03-06) | 0.1267(45)(+14-24) |
| 2.334 | 0.829(08) | 0.84446 | 0.7956(22)(+16-31) | 0.7619(073)(+063-111) | 0.1462(41) | 0.1377(42)(+03-05) | 0.1319(50)(+11-19) |
| 2.416 | 0.734(09) | 0.85057 | 0.8068(17)(+12-23) | 0.7797(052)(+045-076) | 0.1368(39) | 0.1298(40)(+02-04) | 0.1254(44)(+07-12) |
| 2.456 | 0.674(06) | 0.85336 | 0.8118(15)(+10-20) | 0.7873(045)(+038-064) | 0.1444(39) | 0.1374(40)(+02-03) | 0.1332(44)(+06-11) |
| 2.487 | 0.652(07) | 0.85546 | 0.8155(14)(+09-18) | 0.7927(040)(+034-056) | 0.1358(36) | 0.1295(37)(+01-03) | 0.1258(40)(+05-09) |
| 2.528 | 0.612(06) | 0.85813 | 0.8201(12)(+08-15) | 0.7994(034)(+029-047) | 0.1405(47) | 0.1343(47)(+01-02) | 0.1309(49)(+05-08) |
| 2.575 | 0.574(06) | 0.86107 | 0.8250(11)(+07-13) | 0.8064(029)(+024-039) | 0.1445(53) | 0.1384(53)(+01-02) | 0.1353(55)(+04-06) |