Non-perturbative renormalization in quenched QCD with Iwasaki action

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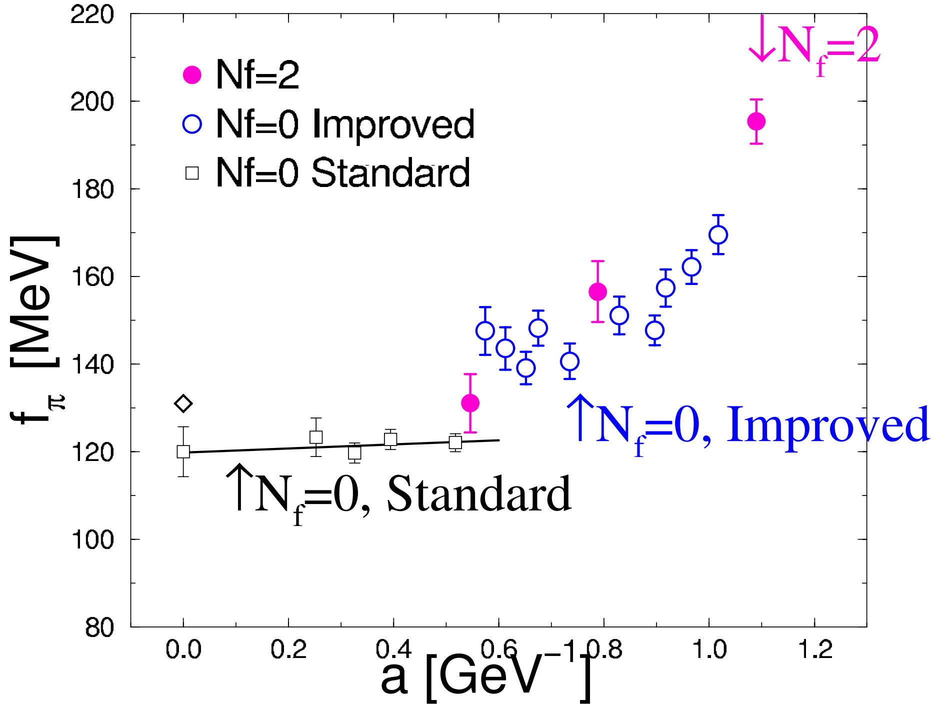
1 Motivation

CP-PACS data



meson decay constants determined with an improved action on coarse lattices have large scaling violation

One possibility of the large scaling violation

Z-factors

tadpole improved 1-loop perturbation

purpose of this work

to see how the scaling behavior is changed, when we use Z-factors estimated non-perturbatively by the SF method in focus on quenched QCD this work

We have investigated

- feasibility of the SF method for this action
  (reported at LATTICE 2001)

In this work

- exceptional configurations which appear at small β
- Z-factors normalized at $L \to \infty$
- scaling property of decay constants

2 Simulation

2.1 Simulation detail

- the same action comb. as CP-PACS

  RG gauge action

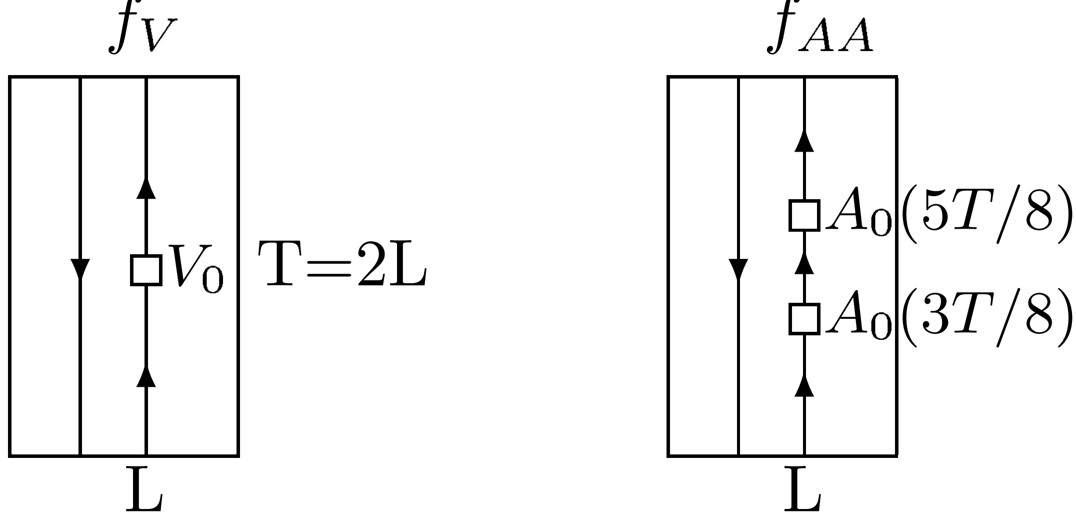
  $S_g = \beta \Big\{ c_{\bar{0}} U_{\rm plaq} + c_1 U_{\rm rect} \Big\},\ c_1 = -0.331$

  tadpole improved clover quark action

  $$c_{SW} = \left(1 - \frac{0.8412}{\beta}\right)^{-\frac{3}{4}}$$

  determined perturbatively

- this combination of actions has $O(a)$ error



Similarly f_1, f_A and f_P

$$Z_V = \frac{f_1}{f_V}, \qquad Z_A = \sqrt{\frac{f_1}{f_{AA}}},$$

$$m_q = \frac{\frac{1}{2}(\partial_0^* + \partial_0) f_A + c_A a \partial_0^* \partial_0 f_P}{2 f_P}$$

Physical size

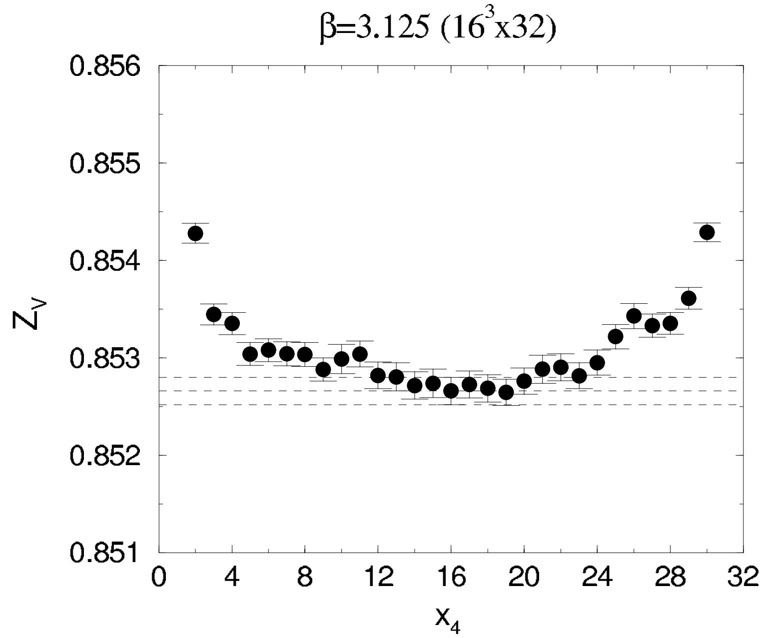
String tension is used to set physical volume

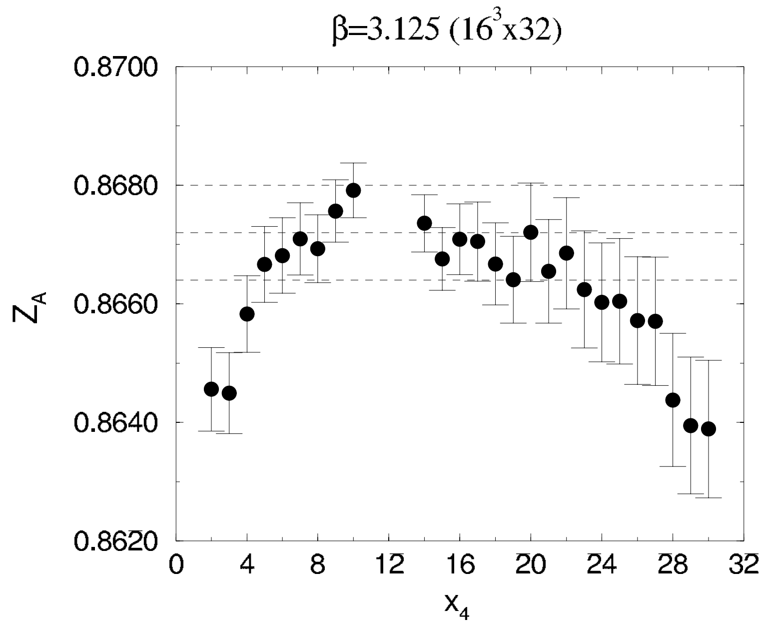
Z-factors normalized at

- 8^3 lattice at $\beta = 2.6$ $L \approx 0.8$ fm
- $L = \infty$

apply the Schrödinger functional method to our action combination

plot Z-factors vs. time slice





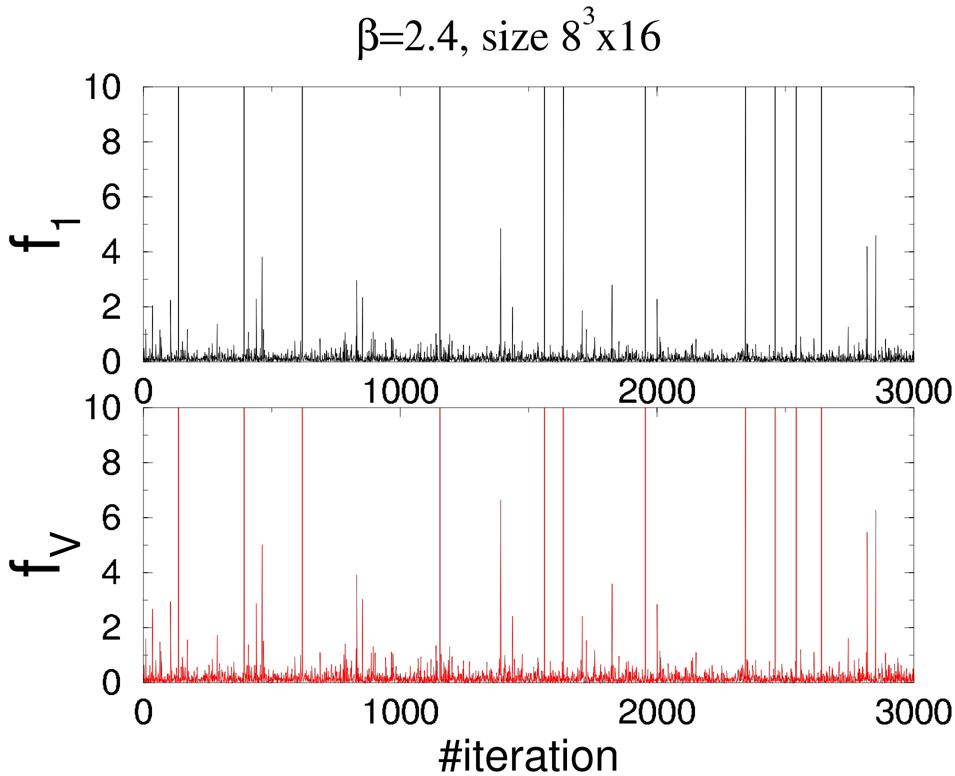
reasonable plateaus of Z-factors appear

Schrödinger functional method can be safely applied our action combination

2.2 Exceptional configuration

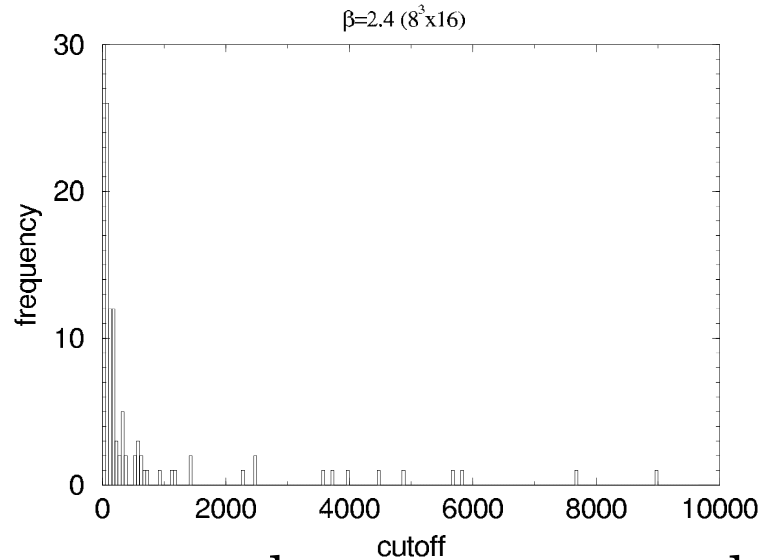
straight-forward to calculate Z-factors for a range of high β values, $\beta \geq 2.6$

However, at $\beta = 2.2, 2.4$ on $8^3 \times 16$ lattice, anomalously large values appear in the ensembles of f_A, f_P, f_1, f_V and f_{AA}



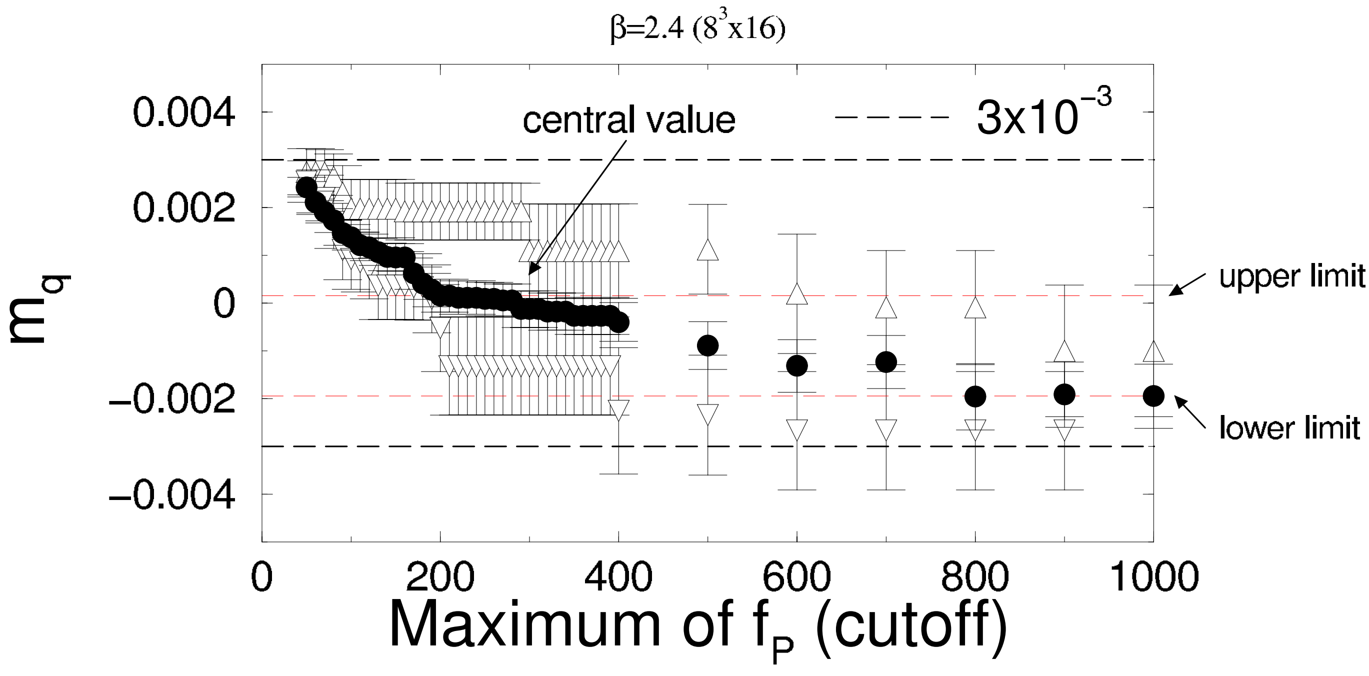
CP-PACS data of f_π, f_ρ exist

for $2.187 \leq \beta \leq 2.575$

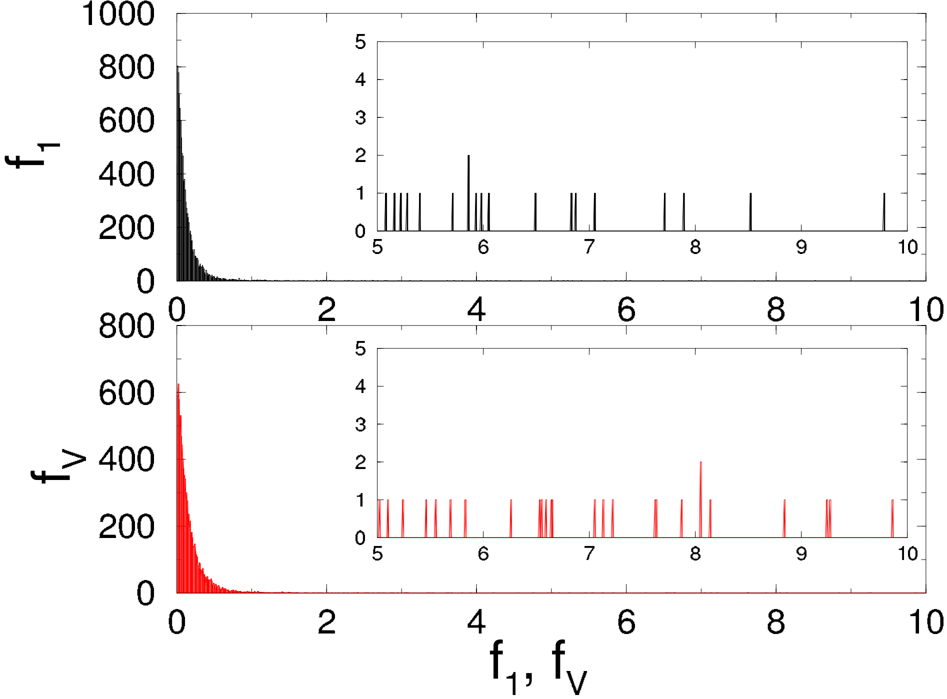


- histogram drags on up to a large values
- Difficult to determine κ_c precisely

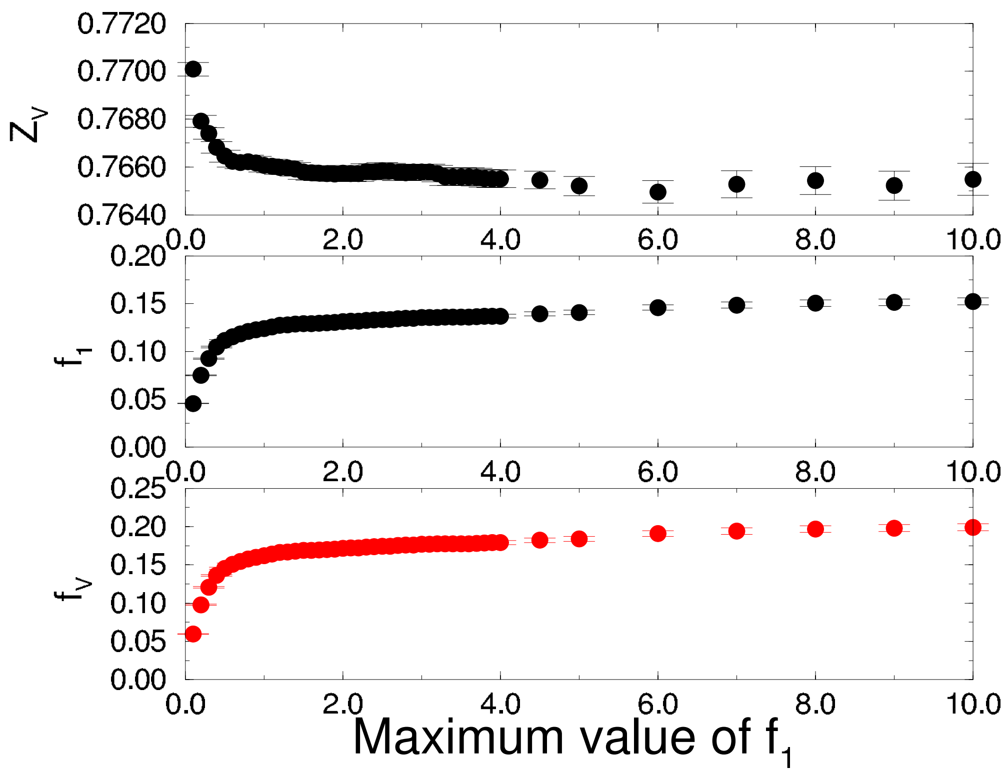
$\Rightarrow$ discard large values from ensembles

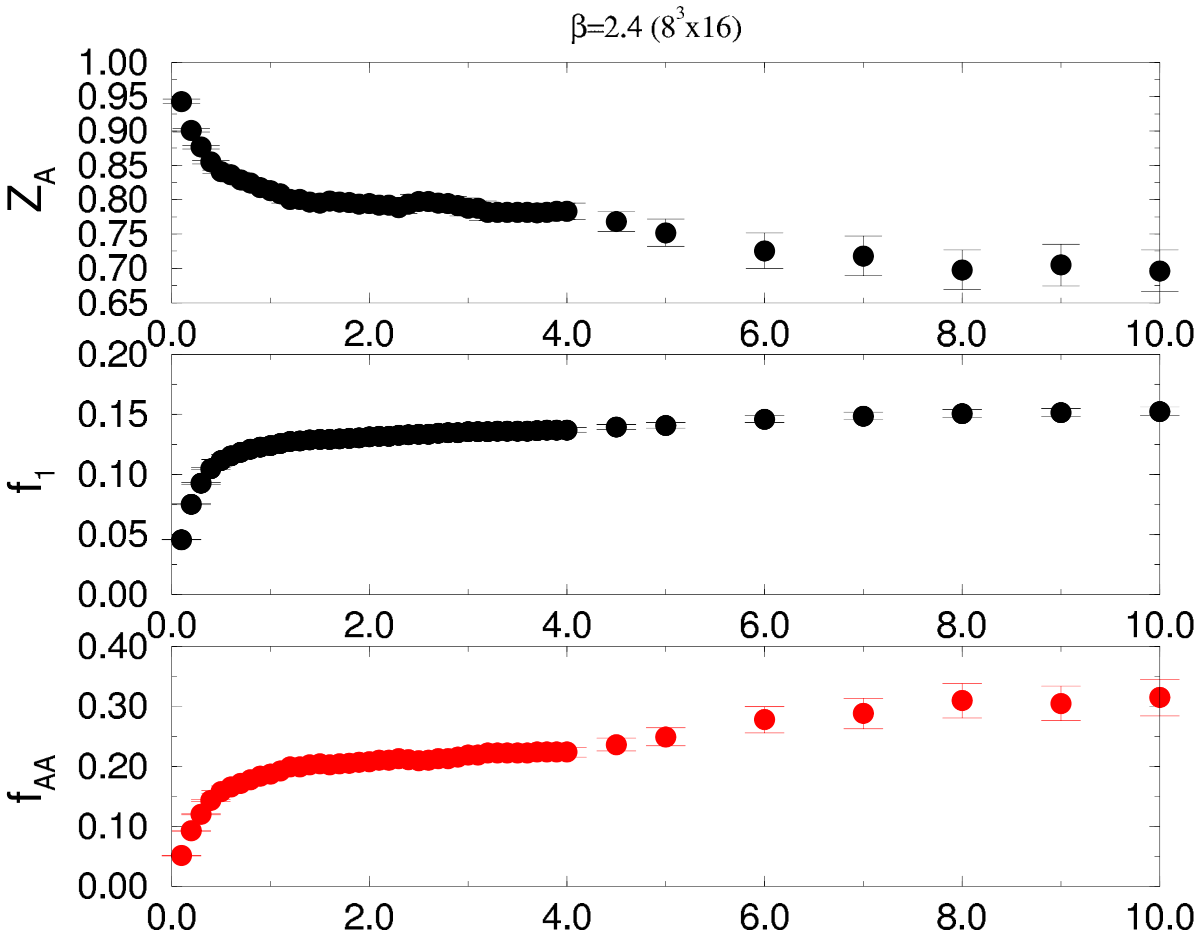


central value from cut=300
Effects of uncertainty of quark mass on Z-factors later

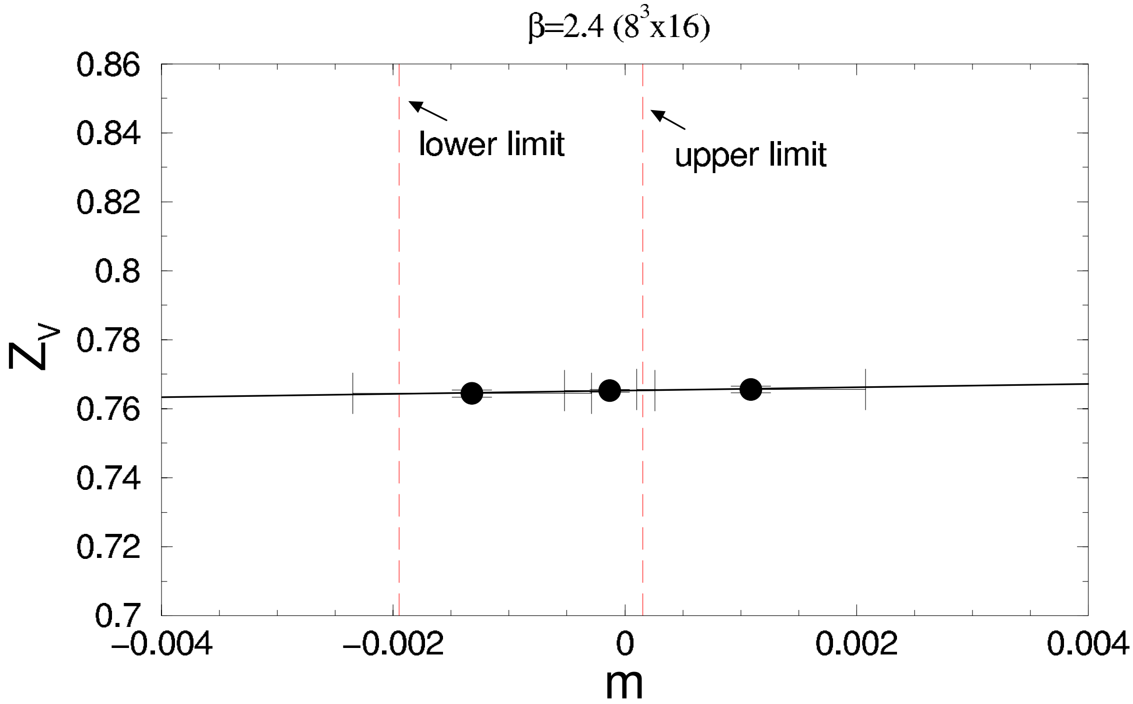


- f_1,f_V,f_{AA} depend strongly on "cut"
- $Z_V = \frac{f_1}{f_V}$, $Z_A = \sqrt{\frac{f_1}{f_{AA}}}$ are stable against the change of "cut"

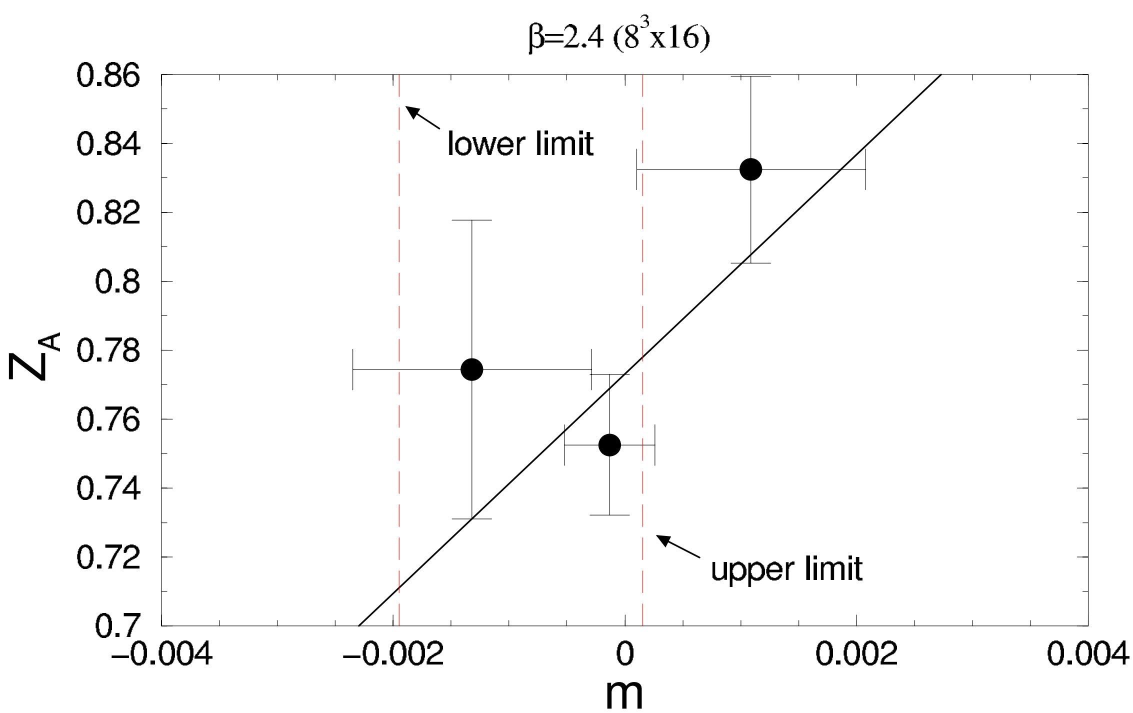




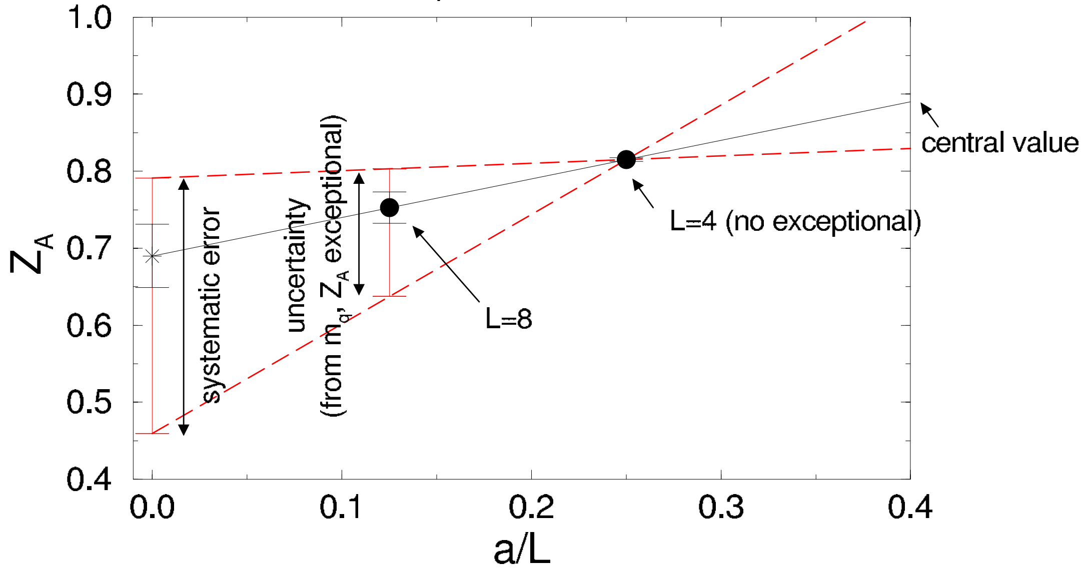
β=2.4 (8^3x16)
Z_A
f_1
f_{AA}



Change of Z_V is little for range on uncertainty of m_q
a typical m_q dependence of Z_A

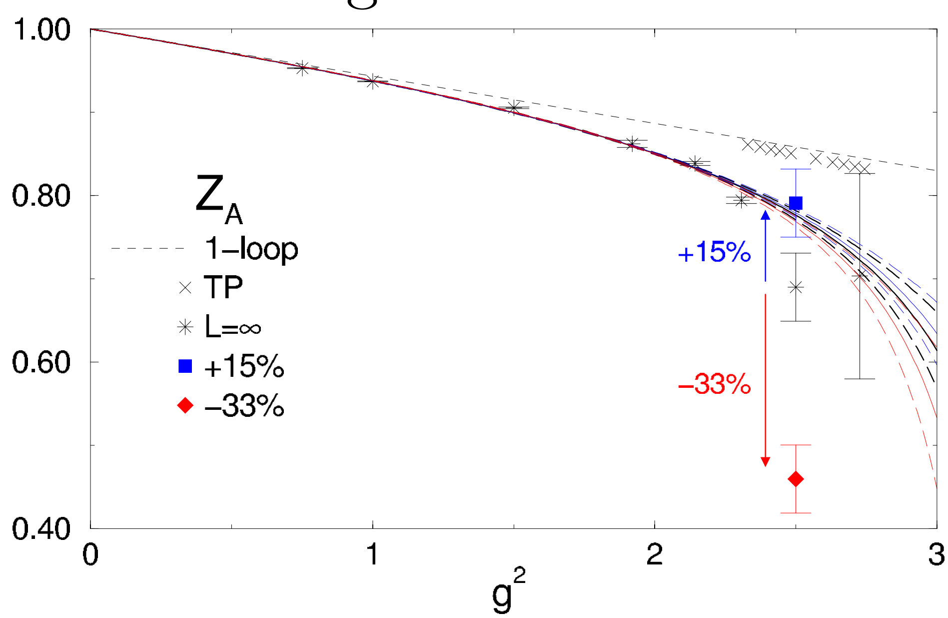


Change of Z_A is not small for the range



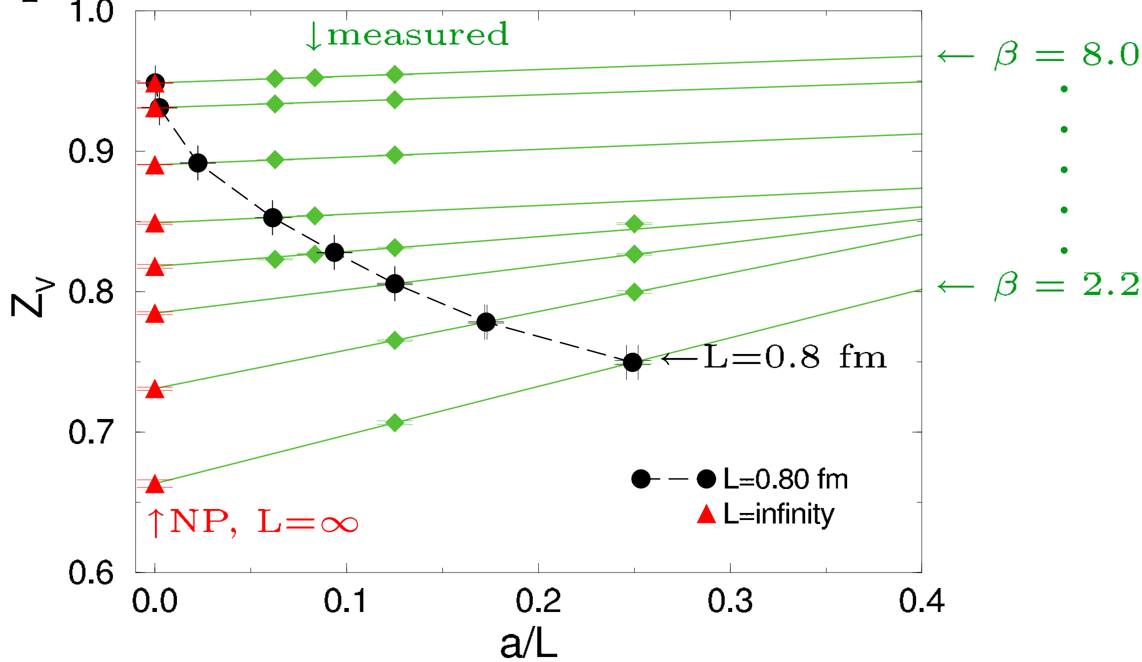
uncertainty of $m_q = 0$ point leads to about $+15 \sim -33$ % systematic error in Z_A ($L = \infty$)

Pad'e fit unchanged

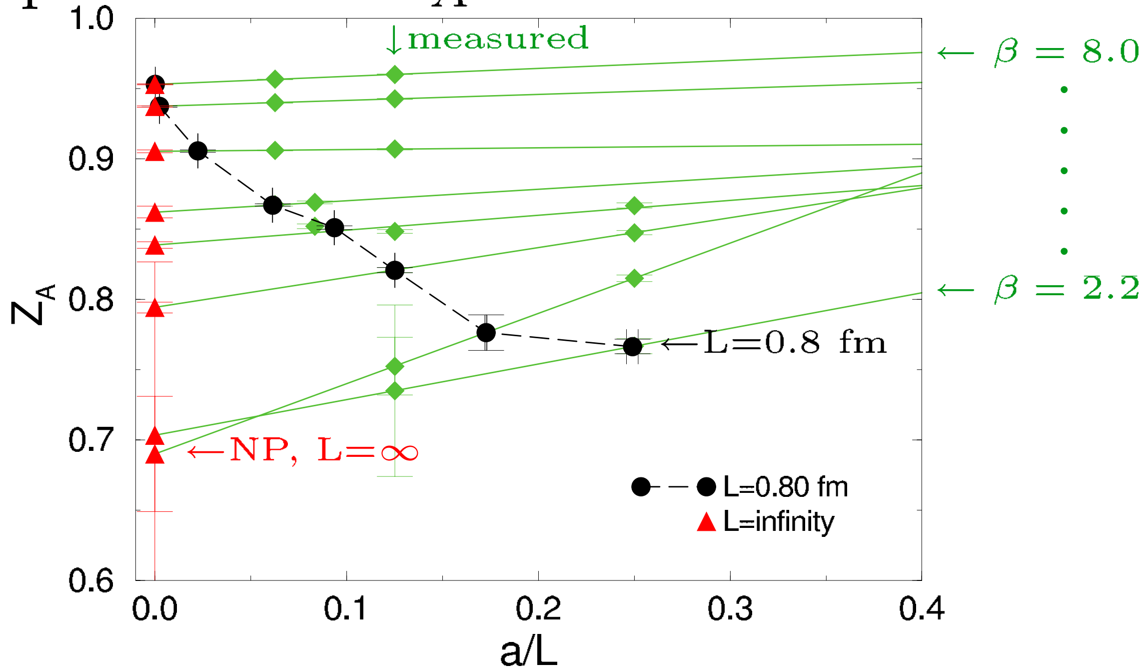


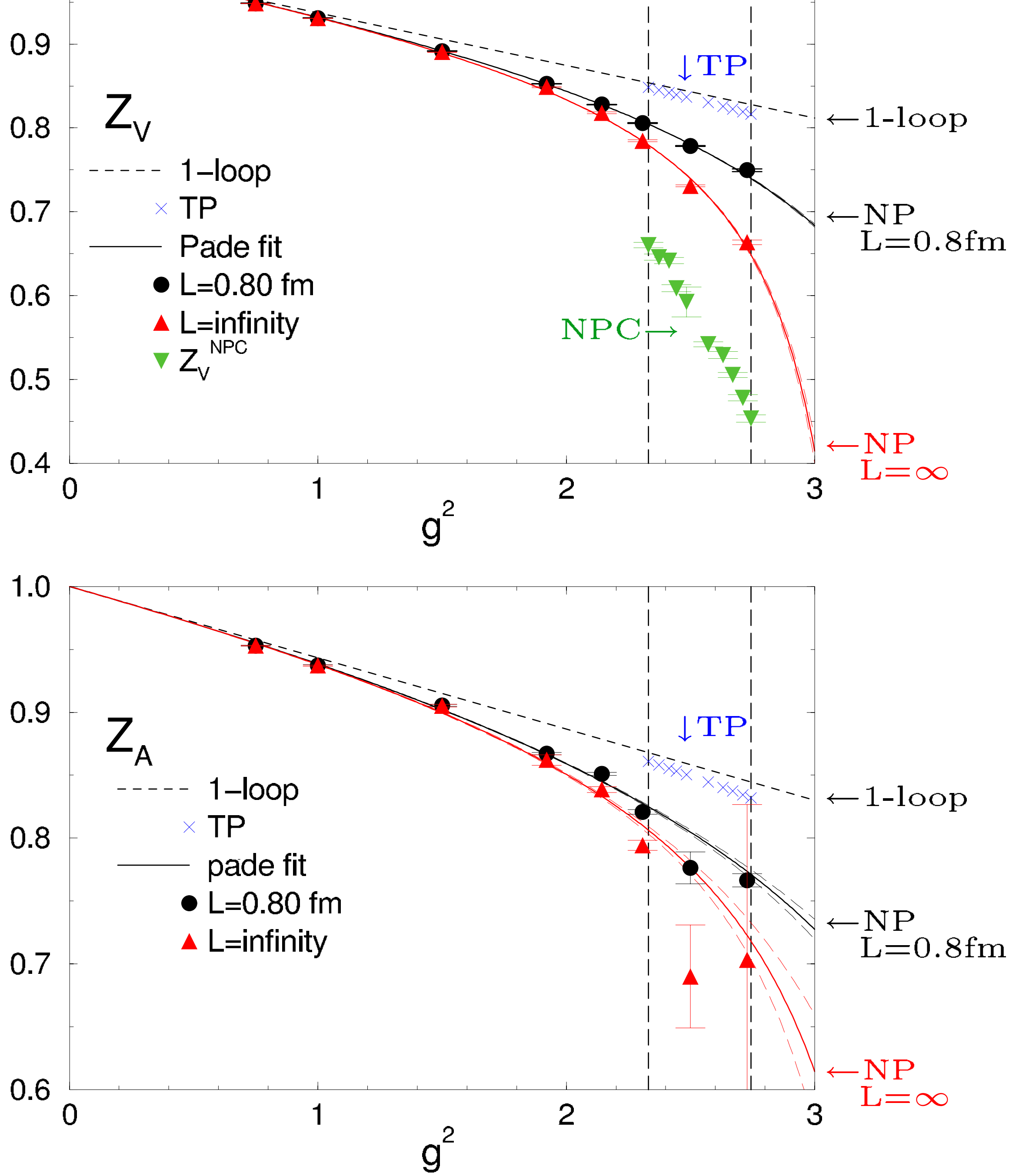
Pad'e fit for Z_A is determined mainly by data at high β values ($\beta \geq 2.6$)
The ambiguity in mq dose not affect our final results

Size dependence of Z_V



Size dependence of Z_A





non-perturbative values of Z-factors ($L = \infty$) are smaller than perturbative ones by about

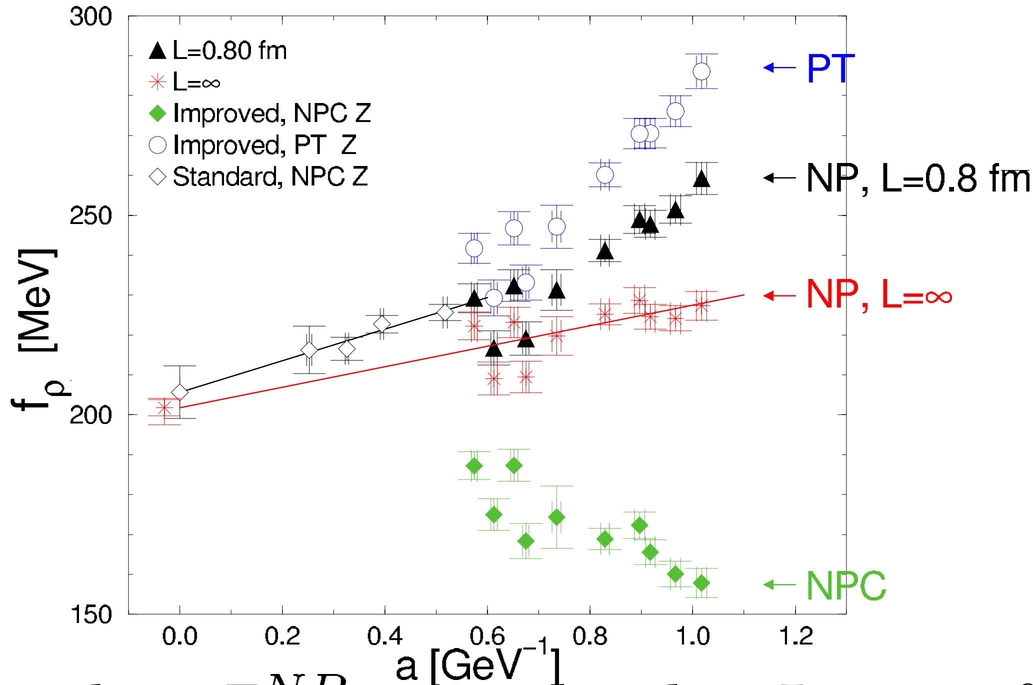
20% for vector current

15% for axial-vector current

at the largest coupling of the CP-PACS simulation, β=2.187.

meson decay constants

f_ρ

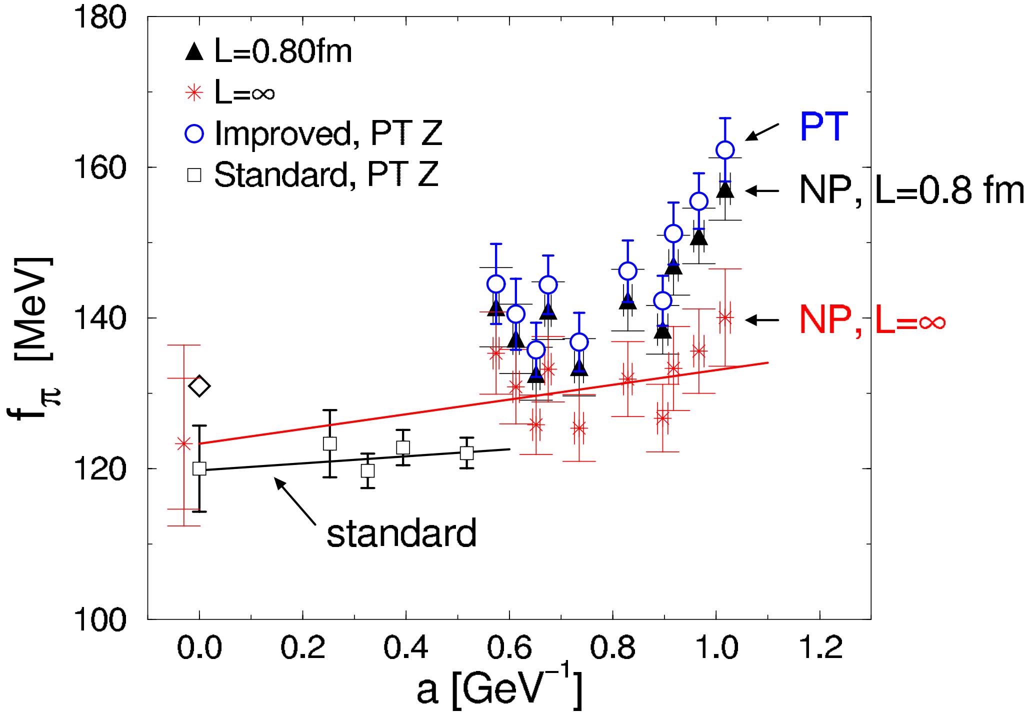


- when Z_V^{NP} normalized at L = 0.8 fm is used, scaling violation is reduced. and when that normalized at L = ∞ is used, scaling violation is further reduced

- f_ρ in the continuum limit is consistent with that from standard action

- when we use Z_V^{NPC} from conserved current, large scaling violation remains because conserved vector current is not $O(a)$ improved with tensor operator

f_π



- Non-perturbative Z-factor reduces magnitude of scaling violation also for f_π

- the continuum limit of f_π is consistent with that from standard action

5 Summary

For the RG improved gauge and tadpole improved clover quark action,
scaling violations of meson decay constants are significantly reduced
by using Z-factors estimated non-perturbatively rather than those estimated by perturbation theory.

Scaling property is best improved, when Z-factors are normalized at infinite volume.
We think that this is achieved
because $O(a/L)$ error in Z-factors are completely removed.

TABLE I.

β	$L^3 \times T$	κ_c	m_q	Z_V	# of conf for Z_V	Z_A	# of conf for Z_A
2.2	$4^3 \times 8$	0.139281(56)	-0.000529(495)	0.7499(11)	2000	0.7667(52)	2000
	$8^3 \times 16$	0.140570(15)	0.001493(399)	0.7067(12)	20000	0.7350(610)	20000
2.4	$4^3 \times 8$	0.136933(21)	-0.000764(337)	0.7995(10)	1000	0.8150(24)	2000
	$8^3 \times 16$	0.137481(04)	-0.000130(395)	0.7652(04)	10000	0.7525(204)	10000
2.6	$4^3 \times 8$	0.135558(08)	-0.000276(341)	0.8265(05)	2000	0.8474(16)	2000
	$8^3 \times 16$	0.135701(10)	-0.000464(154)	0.8056(05)	500	0.8208(18)	2000
2.8	$4^3 \times 8$	0.134532(10)	0.000002(396)	0.8482(06)	1000	0.8667(18)	1000
	$8^3 \times 16$	0.134515(07)	-0.000105(139)	0.8312(04)	500	0.8482(13)	1000
	$12^3 \times 24$	0.134554(08)	0.000525(111)	0.8268(03)	500	0.8520(17)	1000
	$16^3 \times 32$	0.134587(09)	0.000663(139)	0.8231(06)	500		
3.125	$12^3 \times 24$	0.133209(01)	-0.000039(091)	0.8540(02)	500	0.8689(09)	500
	$16^3 \times 32$	0.133219(05)	-0.000092(060)	0.8527(01)	500	0.8672(08)	500
4.0	$8^3 \times 16$	0.131094(04)	-0.000302(086)	0.8972(02)	500	0.9068(06)	500
	$16^3 \times 32$	0.131083(01)	0.000160(044)	0.8938(01)	500	0.9060(04)	300
6.0	$8^3 \times 16$	0.128898(03)	-0.000191(061)	0.9366(01)	300	0.9425(03)	300
	$16^3 \times 32$	0.128891(01)	-0.000060(023)	0.9337(01)	300	0.9398(02)	300
8.0	$8^3 \times 16$	0.127869(02)	0.000138(041)	0.9545(01)	300	0.9600(02)	300
	$12^3 \times 24$	0.127870(01)	0.000576(331)	0.9523(01)	300		
	$16^3 \times 32$	0.127870(01)	0.000021(017)	0.95151(3)	300	0.9565(01)	200

TABLE II. Z-factors at fixed physical volumes

β	Z_V normalized at $L = 0.8$ fm	Z_V normalized at $L = \infty$	Z_A normalized at $L = 0.8$ fm	Z_A normalized at $L = \infty$
2.2	0.7495(15)	0.6635(26)	0.7664(52)	0.7033(1221)
2.4	0.7783(05)	0.7309(13)	0.7763(127)	0.6900(409)
2.6	0.8056(05)	0.7847(11)	0.8208(18)	0.7942(39)
2.8	0.8279(02)	0.8180(12)	0.8511(13)	0.8386(22)
3.125	0.8526(01)	0.8488(07)	0.8671(08)	0.8621(42)
4.0	0.8917(02)	0.8904(03)	0.9055(08)	0.9052(10)
6.0	0.9309(02)	0.9308(02)	0.9372(05)	0.9371(05)
8.0	0.9485(01)	0.9485(01)	0.9530(03)	0.9530(03)

TABLE III.

	δm_q	$\delta Z^{(m_q)}$	$\delta Z^{(except.)}$	δZ	$\delta Z^{(L=0.8\text{ fm})}$	$\delta Z^{(L=\infty)}$
$Z_V(\beta = 2.2)$	+0.00344-0.00170	+0.04%-0.02%	+0.31%-0.06%	+0.35%-0.08%	+0.0013%-0.0058%	+0.75%-0.17%
$Z_V(\beta = 2.4)$	+0.00028-0.00182	+0.01%-0.12%	+0.08%-0.03%	+0.09%-0.15%	+0.044%-0.10%	+0.19%-0.31%
$Z_A(\beta = 2.2)$	+0.00344-0.00170	+3.2% -1.5%	+18.1%-2.1%	+21.3%-3.6%	+0.17%-0.25%	+44.5%-8.9%
$Z_A(\beta = 2.4)$	+0.00028-0.00182	+0.6% -8.0%	+6.1%-7.3%	+6.7%-15.3%	+4.0%-9.2%	+14.6%-33.4%

TABLE IV. f_ρ for various choices of Z_V.

β	$a^{-1}[GeV]$	Z_V (TP)	Z_V (NP,$L=0.8$ fm)	Z_V (NP,$L=\infty$)	f_ρ (TP)	f_ρ (NP,$L=0.8$ fm)	f_ρ (NP,$L=\infty$)
2.187	1.017(10)	0.81657	0.7400(08)(+04-10)	0.6488(24)(+48-27)	0.2861(44)	0.2593(43)(+01-04)	0.2273(43)(+17-09)
2.214	0.966(10)	0.81923	0.7462(07)(+04-09)	0.6651(20)(+40-23)	0.2761(38)	0.2515(37)(+01-03)	0.2242(38)(+13-08)
2.247	0.917(09)	0.82237	0.7532(06)(+03-08)	0.6825(16)(+32-19)	0.2706(37)	0.2478(36)(+01-03)	0.2246(36)(+11-06)
2.281	0.896(10)	0.82548	0.7601(06)(+03-07)	0.6981(13)(+26-16)	0.2704(38)	0.2490(37)(+01-02)	0.2287(36)(+09-05)
2.334	0.829(08)	0.83009	0.7698(04)(+02-06)	0.7187(10)(+20-13)	0.2601(30)	0.2412(29)(+01-02)	0.2252(29)(+06-04)
2.416	0.734(09)	0.83673	0.7832(03)(+02-04)	0.7441(07)(+13-09)	0.2471(54)	0.2313(51)(+01-01)	0.2197(50)(+04-03)
2.456	0.674(06)	0.83978	0.7891(03)(+01-03)	0.7544(06)(+10-08)	0.2332(44)	0.2191(42)(+00-01)	0.2095(41)(+03-02)
2.487	0.652(07)	0.84205	0.7934(03)(+01-03)	0.7617(06)(+09-07)	0.2467(42)	0.2324(40)(+00-01)	0.2232(40)(+03-02)
2.528	0.612(06)	0.84496	0.7988(02)(+01-03)	0.7705(05)(+07-06)	0.2293(45)	0.2168(43)(+00-01)	0.2091(42)(+02-02)
2.575	0.574(06)	0.84816	0.8046(02)(+01-02)	0.7796(05)(+06-05)	0.2417(37)	0.2293(36)(+00-01)	0.2222(35)(+02-01)

TABLE V. f_π for various choice of Z_A

β	$a^{-1}[GeV]$	Z_A (TP)	Z_A (NP,$L=0.8$ fm)	Z_A (NP,$L=\infty$)	f_π (TP)	f_π (NP,$L=0.8$ fm)	f_π (NP,$L=\infty$)
2.187	1.017(10)	0.83204	0.7712(38)(+27-54)	0.7180(146)(+125-236)	0.1623(42)	0.1504(46)(+05-11)	0.1401(65)(+24-46)
2.214	0.966(10)	0.83449	0.7762(34)(+24-49)	0.7276(127)(+109-203)	0.1555(37)	0.1446(41)(+04-09)	0.1356(56)(+20-38)
2.247	0.917(09)	0.83737	0.7819(30)(+21-43)	0.7383(108)(+093-171)	0.1512(41)	0.1412(44)(+04-08)	0.1333(56)(+17-31)
2.281	0.896(10)	0.84022	0.7875(27)(+19-38)	0.7482(092)(+080-143)	0.1423(33)	0.1334(36)(+03-06)	0.1267(45)(+14-24)
2.334	0.829(08)	0.84446	0.7956(22)(+16-31)	0.7619(073)(+063-111)	0.1462(41)	0.1377(42)(+03-05)	0.1319(50)(+11-19)
2.416	0.734(09)	0.85057	0.8068(17)(+12-23)	0.7797(052)(+045-076)	0.1368(39)	0.1298(40)(+02-04)	0.1254(44)(+07-12)
2.456	0.674(06)	0.85336	0.8118(15)(+10-20)	0.7873(045)(+038-064)	0.1444(39)	0.1374(40)(+02-03)	0.1332(44)(+06-11)
2.487	0.652(07)	0.85546	0.8155(14)(+09-18)	0.7927(040)(+034-056)	0.1358(36)	0.1295(37)(+01-03)	0.1258(40)(+05-09)
2.528	0.612(06)	0.85813	0.8201(12)(+08-15)	0.7994(034)(+029-047)	0.1405(47)	0.1343(47)(+01-02)	0.1309(49)(+05-08)
2.575	0.574(06)	0.86107	0.8250(11)(+07-13)	0.8064(029)(+024-039)	0.1445(53)	0.1384(53)(+01-02)	0.1353(55)(+04-06)