Schrödinger functional coupling with improved gauge actions in SU(3) gauge theory

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"Non-perturbative improvement and renormalization"

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Introduction

- **Motivation**

  High energy experiments $\overline{\mathrm{MS}}$**-scheme**

$$\alpha_{\overline{\mathrm{MS}}}(m_Z = 91.19\mathrm{GeV}) \approx 0.12$$

- **Problem**
  - **Non-perturbative evolution of running coupling**

$$\Uparrow$$

Step scaling function(SSF), Schrödinger Functional (SF)

$\sim$ **ALPHA Collaboration**

Strategy

Output	**PT matching**	**SF scheme**
$\alpha_{\overline{\mathrm{MS}}}$	$\longleftarrow$	$\alpha_{\mathrm{SF}}(q = 2/L)$
		$\uparrow$
		$\alpha_{\mathrm{SF}}(q = 1/L)$
		$\bullet$
energy $\uparrow$		**NP evolution**
		$\bullet$
		$\alpha_{\mathrm{SF}}(q = 2/L_{\mathrm{max}})$
Input		$\uparrow$
Hadronic scheme	$\longrightarrow$	$\alpha_{\mathrm{SF}}(q = 1/L_{\mathrm{max}})$
m_π, f_π, r_0	**matching** L_{max}/r_0	

$N_f = 3$ project

- $N_f = 3$ **QCD simulation**

  CP-PACS/JLQCD Collaborations

Ultimate goal **: evaluation of** $\alpha_{\overline{\rm MS}}$ **from** $N_f = 3$ **QCD simulation**

- **Why do we choose Iwasaki action ?**

  Ref. JLQCD Collab., Nucl. Phys. B (Proc. Suppl.) 106, 263 (2002)

plaquette action + $O(a)$ improved Wilson fermion	Iwasaki action + $O(a)$ improved Wilson fermion
$\downarrow$	$\downarrow$
strong lattice artifacts associated with phase transition	such lattice artifacts are absent

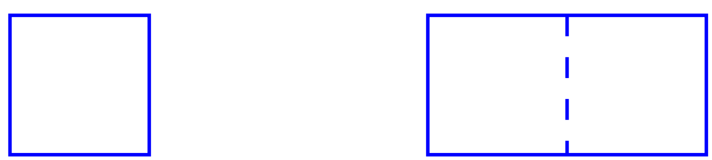
$$\Downarrow$$

We decided to adopt the Iwasaki action as gauge part.

Purpose of this talk

Cut off dependence and universality check of SSF and low energy scale ratio with improved gauge actions in SU(3) gauge theory.

$\sim$ **toward** $N_f = 2, 3$ **simulations**

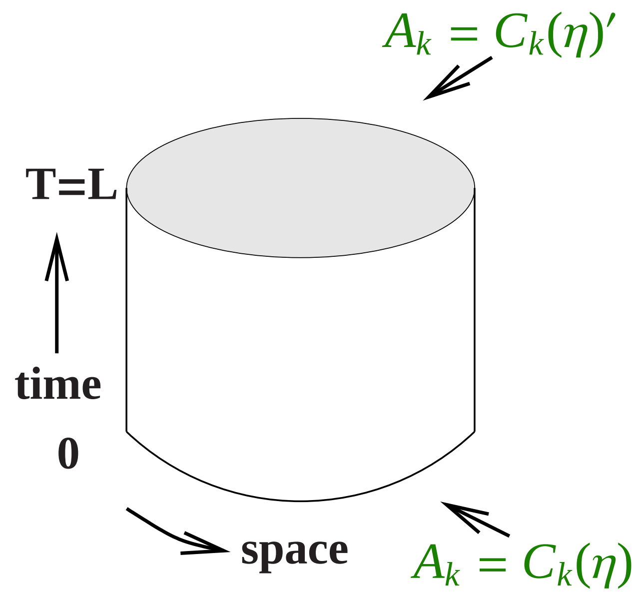


Outline

- **SF set up and SF coupling**
- **O(a) boundary improvement coefficients**
- **Non-perturbative evolution of running coupling**
- **Simulation results**
  - **SSF at $u = 0.9944$ (weak coupling region)**
  - **SSF at $u = 2.4484$ (strong coupling region)**
  - L_{max}/r_0 **(low energy scale ratio)**
- **Conclusion**

SF set up and SF coupling

- **SF set up**



- **Definition of SF coupling**

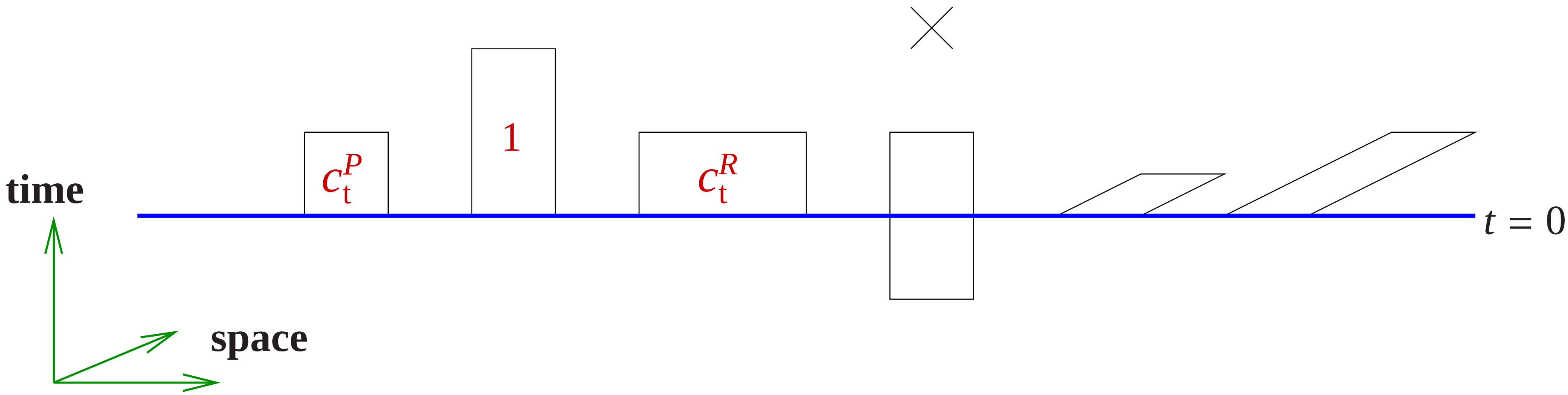
$$\bar{g}^2_{\mathrm{SF}}(L) = k/\Gamma'|_{\eta=\nu=0} = k/\left\langle \frac{\partial S}{\partial \eta} \right\rangle\Bigg|_{\eta=\nu=0},$$

$$e^{-\Gamma} = \mathcal{Z} = \int D[U] e^{-S[U]},$$

where $S[U] = S_{\mathrm{imp}}[U]$ **: improved gauge action.**

O(a) boundary improvement coefficients

- **O(a) boundary improvement coefficients**



$$
\begin{aligned}
c_t^P(g_0^2) &= 1 + c_t^{P(1)} g_0^2 + O(g_0^4), \\
c_t^R(g_0^2) &= 3/2 + c_t^{R(1)} g_0^2 + O(g_0^4),
\end{aligned}
$$

Ref. S. Aoki et al., NPB 540 (1999) 501, S. Takeda et al., PRD 68 (2003) 014505

	Iwasaki action	**LW action**	**DBW2 action**
$c_t^{P(1)}$	0.15180(13)	–0.002970(1)	0.448(26)

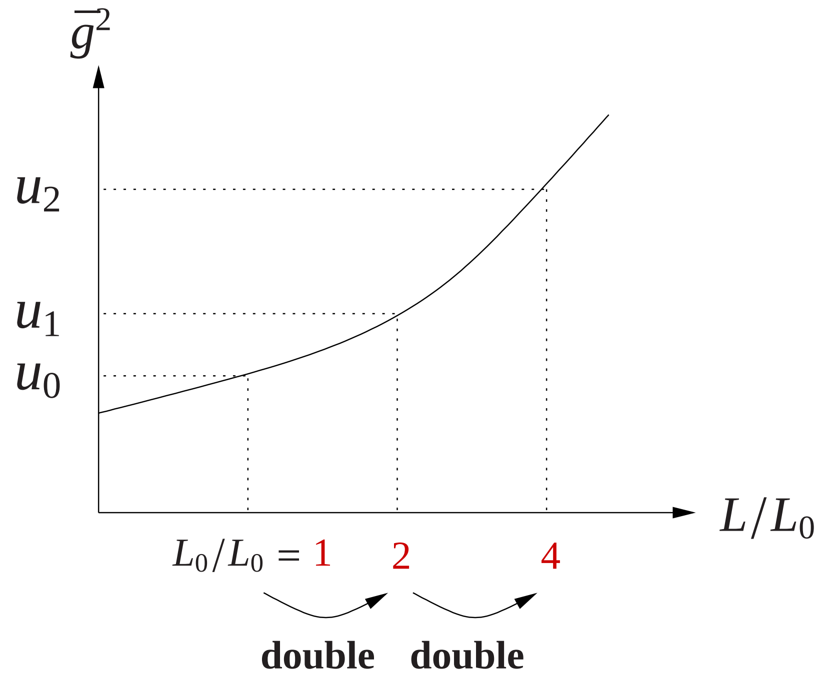
with $c_t^{R(1)} = 2c_t^{P(1)}$

Non-perturbative evolution of running coupling

- **Step scaling function (SSF)**
  - **Continuum SSF**

$$\sigma(2,u) = \bar{g}^2(2L)\big|_{\bar{g}^2(L)=u}$$

$$\begin{aligned} u_0 &= \bar{g}^2(L_0) \\ u_1 &= \sigma(2,u_0) \\ u_2 &= \sigma(2,u_1) \\ &\vdots \end{aligned}$$



  - **Lattice SSF**

$\Sigma(2,u,a/L)$: **SSF calculated on the lattice**

Continuum limit

$$\sigma(2,u) = \lim_{a/L\to 0} \Sigma(2,u,a/L)$$

- **Low energy scale**

$$u_i = \bar{g}^2(L_i) \qquad i = 0, 1, 2, \ldots, n$$

$$L_i = 2^{i-n} L_n = 2^{i-n} L_{\mathrm{max}} \quad (L_n = L_{\mathrm{max}})$$

  - **Def of L_{max}**

$$\bar{g}^2_{\mathrm{SF}}(L_{\mathrm{max}}) = 3.480$$

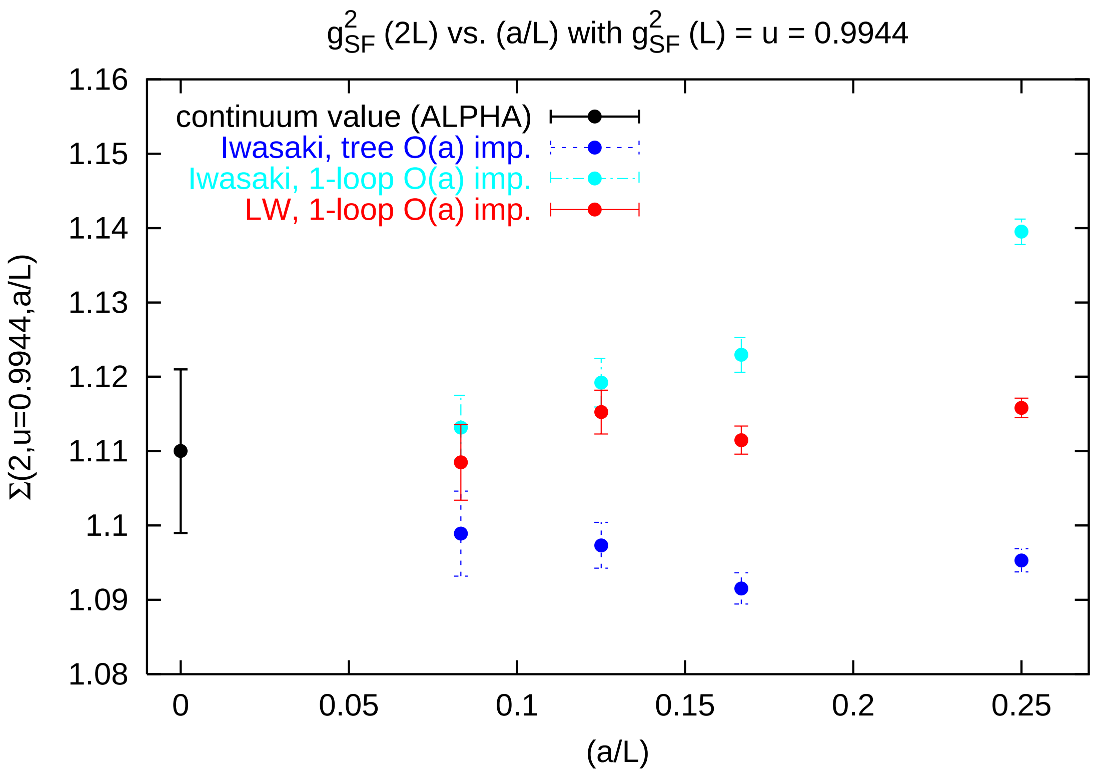
  - **Continuum limit of low energy scale ratio**

$$L_{\mathrm{max}}/r_0 = \lim_{a/L_{\mathrm{max}} \to 0} \underbrace{(L_{\mathrm{max}}/a)}_{\textbf{SF scheme}} \times \underbrace{(a/r_0)}_{\textbf{Hadronic scheme}}$$

$$r_0 = 0.5\ \textbf{fm : sommer scale}$$

Simulation results

- **SSF at $u = 0.9944$ (weak coupling region)**



Ref. ALPHA Collaboration, NPB 544 (1999)

Perturbative improvement

$k = 0, 1$: **tree level, 1-loop O(a) improvement case**

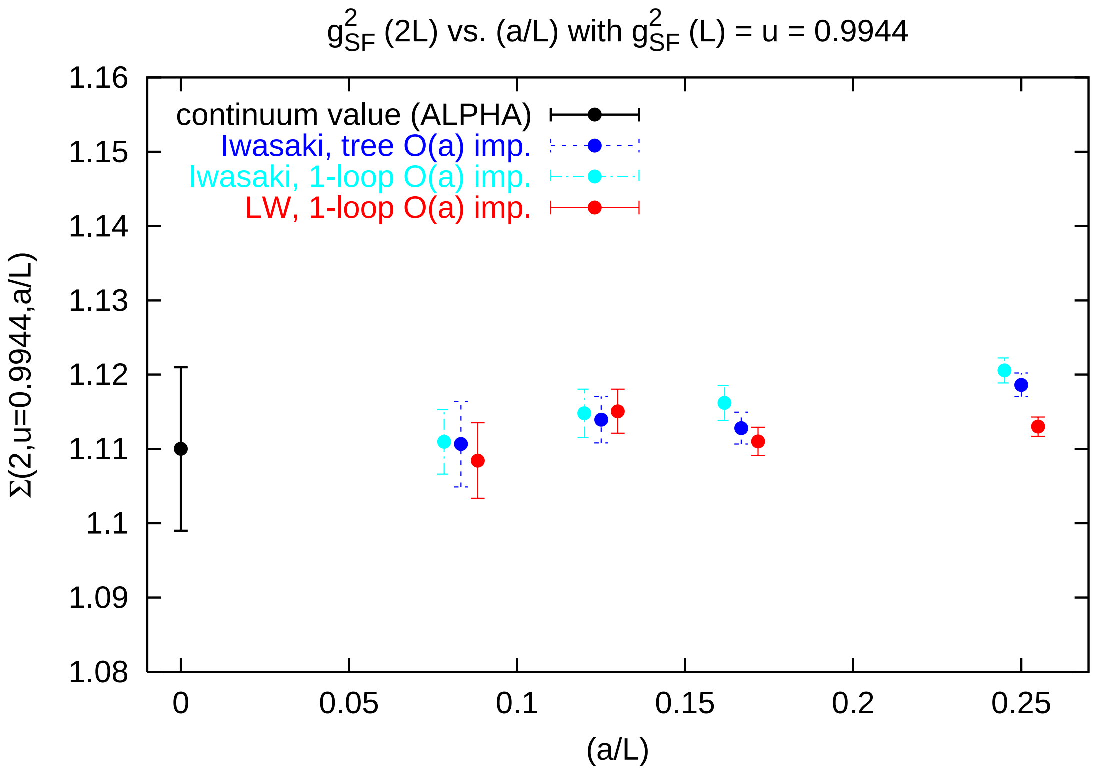
$$\Sigma_1^{(k)}(u, a/L) = \frac{\Sigma^{(k)}(u, a/L)}{1 + \delta_1{}^{(k)}(a/L)u},$$

$\Sigma^{(k)}(u, a/L)$: **raw data**

$\delta_1{}^{(k)}(a/L)$: **1-loop deviation**

SSF at $u = 0.9944$ (weak coupling region)

perturbative improvement : no 1-loop order lattice artifact

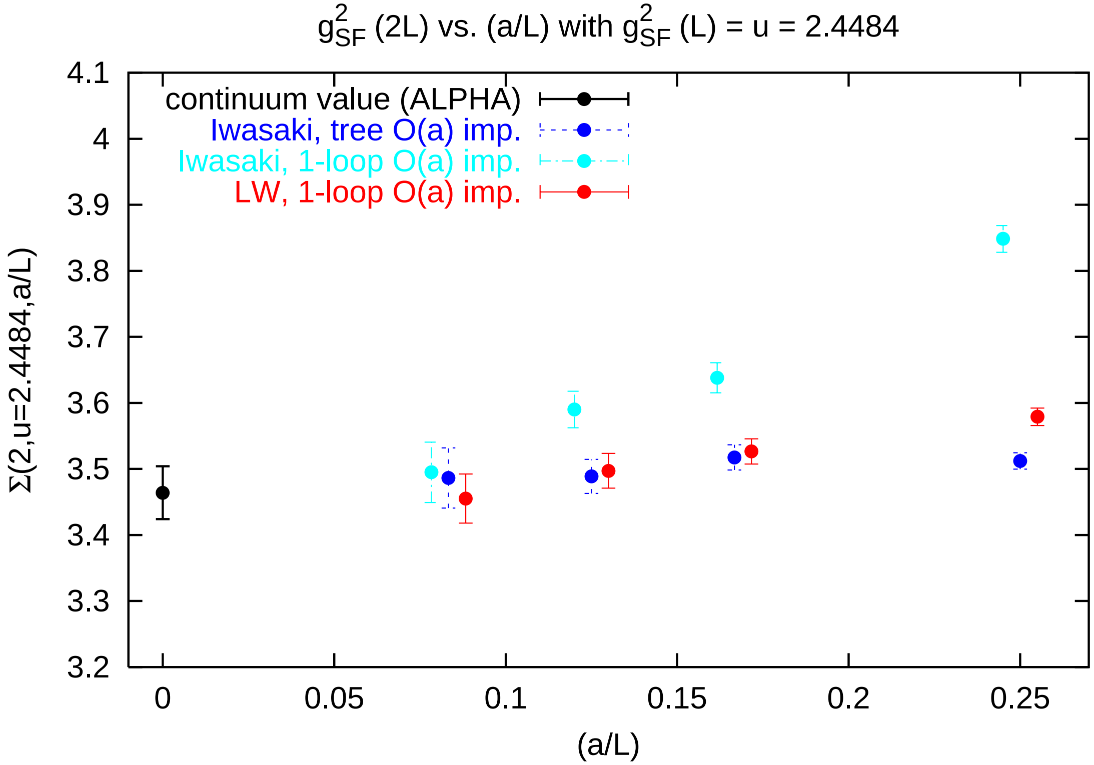


linear fit

action	Iwasaki	LW	plaquette (ALPHA)
$\sigma(2, u)$	**1.106(4)**	**1.111(4)**	**1.110(11)**

SSF at $u = 2.4484$ (strong coupling region)

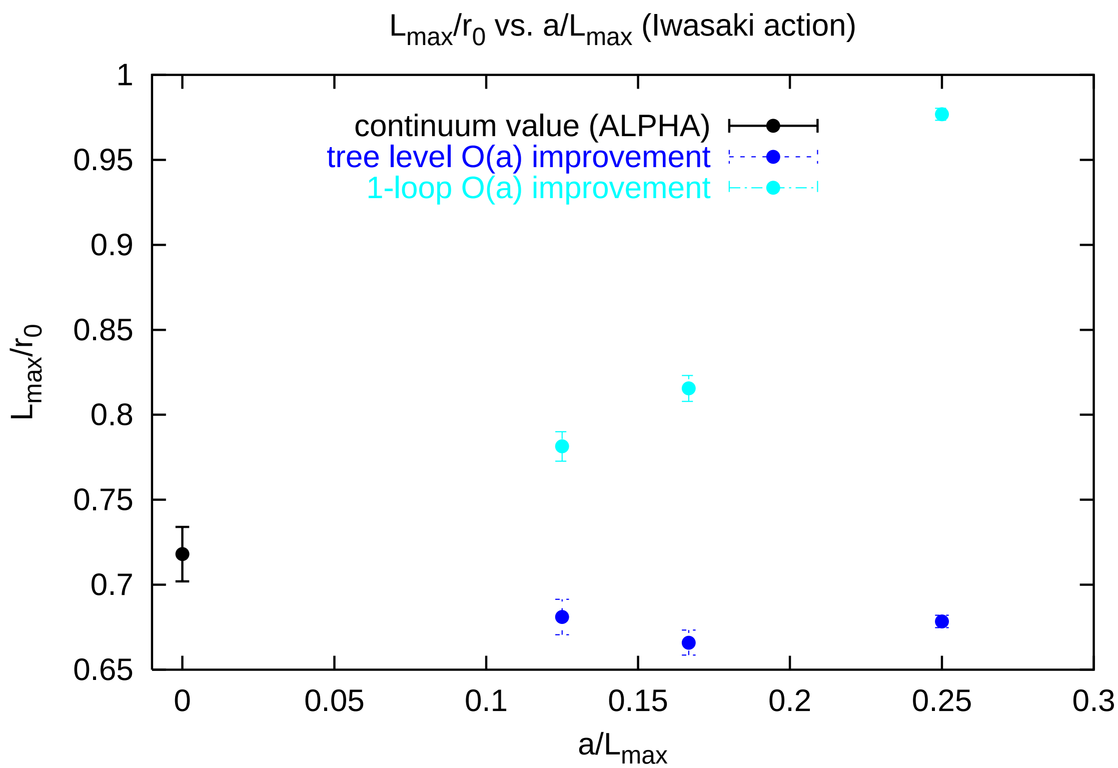
perturbative improvement is implemented



linear fit

action	Iwasaki	LW	plaquette (ALPHA)
$\sigma(2,u)$	**3.486(37)**	**3.409(35)**	**3.464(40)**

Low energy scale ratio $L_{\max}/r_0$



Conclusion

- **SSF** **tree level O(a) improved Iwasaki action**
  pertutbative improvement is implemented

	Iwasaki	plaquette (ALPHA)
$\sigma(2, u = 0.9944)$	**1.106(4)**	**1.110(11)**
$\sigma(2, u = 2.4484)$	**3.486(37)**	**3.464(40)**

$\Longrightarrow$ **universality OK**

- **Concerning low energy scale ratio, the universality will be confirmed soon.**
- **For tree level O(a) improved Iwasaki action, perturbative improvement is very efficient reducing the lattice artifact of SSF.**

We are going to investigate $N_f = 2, 3$ cases