Light Hadron Spectroscopy in Two-flavor Lattice QCD with Small Sea Quark Masses

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1. Introduction
2. Simulations
3. Results(Chiral extrapolations)
4. Conclusion

. Introduction

urposes for hadron spectrum calculations

- **Verification of QCD at low energies**
- **Basis of the other lattice calculations (matrix elements,finite temperature/density)**
- **Quark masses**

eatures of lattice QCD

dvantage]

- **Non-perturbative calculations**

isadvantage]

- **Statistical and systematic errors**
  - **Continuum extrapolation($a \to 0$)**
    $\to$ improvement of actions and operators
  - **Renormalization factor**
    $\to$ non-perturbative renormalization method
  - **Finite size effect(studied well)**
  - **Chiral extrapolation**

Chiral extrapolation]

n lattice QCD, sea quark masses
n two-flavor cases, $m_{ud} \equiv (m_u + m_d)/2$)
re heavier than those in our real world
ecause of computational costs

lattice data must be extrapolated
to the physical point.

ttice m_{PS}/m_V → real $m_\pi/m_\rho = 0.17$

$_{PS}/m_V$	**0.17**	**0.35**	**0.60**	**0.80**
$^{AWI}_{quark}$[MeV]	~ 5	**13**	**63**	**91**

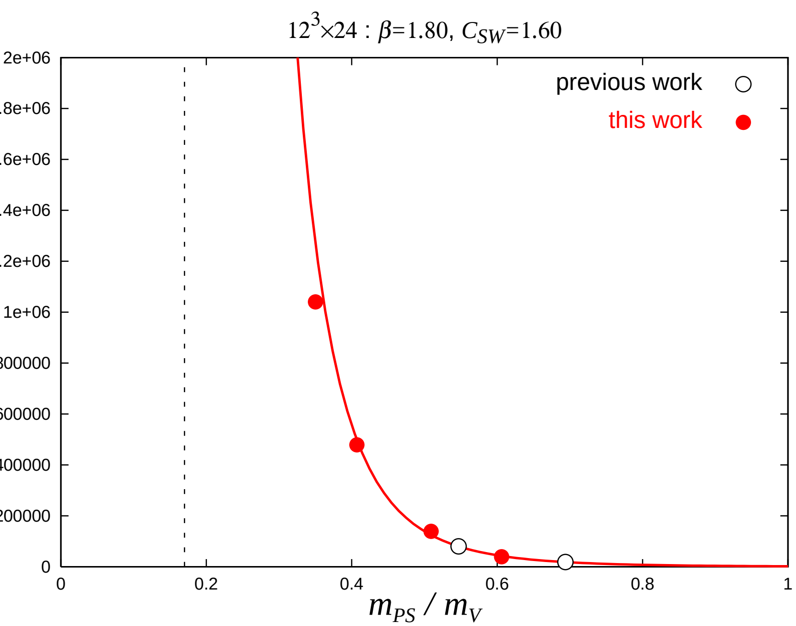
- **Long extrapolations involve sizable systematic errors(as we see later)**
  → **more realistic simulations are desirable**

f.1 [Computational cost]

ost increases extremely
s sea quark masses become small

- **CPU time** $(\sim N_{inv}/dt) \propto (m_{PS}/m_V)^{-6}$

  N_{inv} **: # of inversions for the quark matrix**
  dt **: step size in molecular dynamics**



. Simulations

$_f = 2$ **full QCD simulations**

ction and run parameters]

- **Small sea quark masses**
  $(m_{PS}/m_V = 0.60 - 0.35$, $m_{quark} = 63 - 13$ **MeV**)
- **Coarse lattices** ($a = 0.2$[**fm**],$L = 2.4 - 3.2$[**fm**])
- **RG gauge + tadpole-improved Clover quark**
- **HMC with the simple leapfrog scheme**
- **Measurements : every 5 trajectory (1 conf.)**

[Statistics]

combined with our previous work, CP-PACS,2002

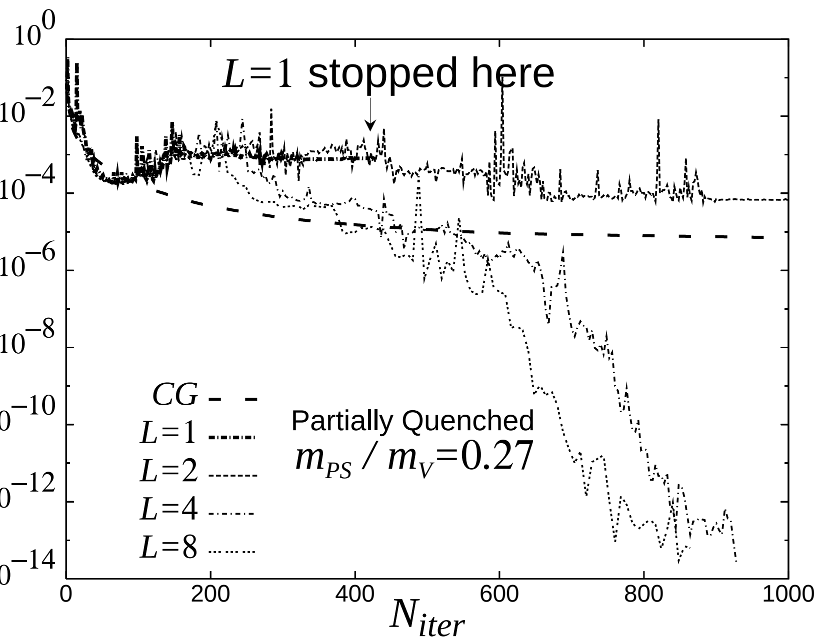
$[12^3 \times 24]$

κ_{sea}	m_{PS}/m_V	**# traj.**
0.14090	**0.80**	**6250**
0.14300	**0.75**	**5000**
0.14450	**0.70**	**7000**
0.14585	**0.60**	**4000**
0.14640	**0.55**	**5250**
0.14660	**0.50**	**4000**
0.14705	**0.40**	**4000**
0.14720	**0.35**	**1400**

f.2 [Issues on algorithms]

wo problems in small sea quark mass regions

- **BiCGStab for quark inversions often does not work :** $Ax = b \rightarrow x = A^{-1}b$
  $\rightarrow$ **Solved by BiCGStab(L)**
  L **: the order of the minimal residual polynomial**
  S.Itoh and YN,2003



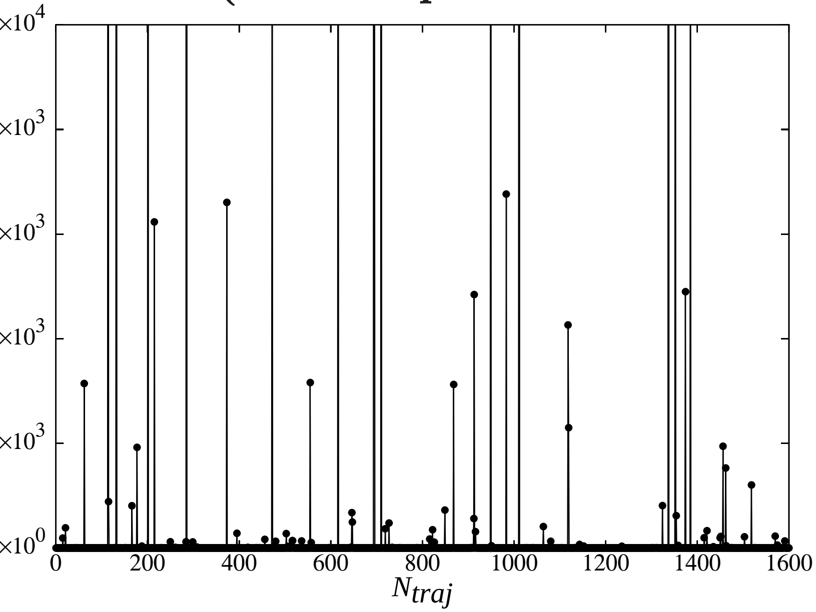
- **Efficiency decrease by huge $\Delta H \equiv H_{trial} - H_{start}$**
  Jansen and Sommer,1998

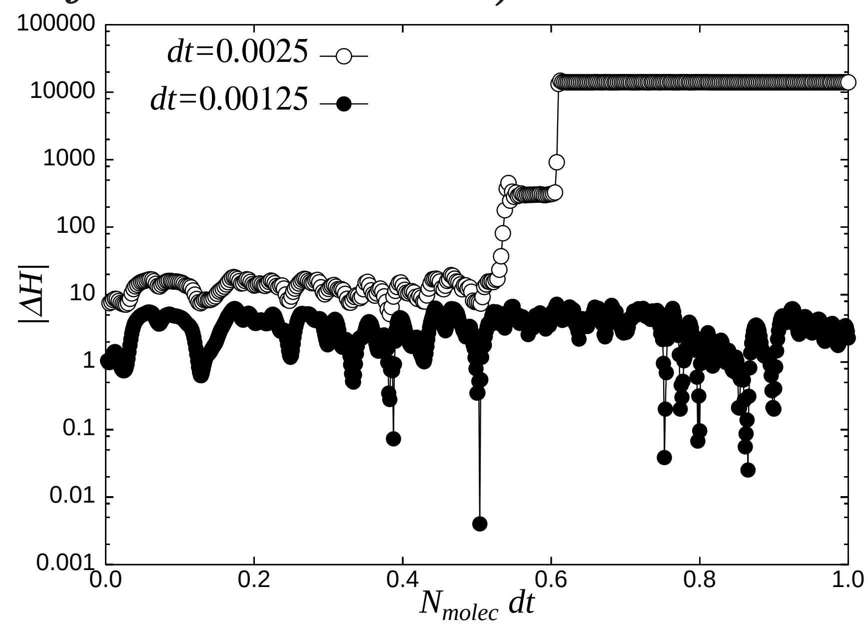
  – **As sea quark masses become smaller, more frequently huge ΔH appears**

$$P_{acc} = \mathbf{erfc}\left(\frac{1}{2}\sqrt{\langle \Delta H \rangle}\right) \sim \exp\left(-\sqrt{\frac{\langle \Delta H \rangle}{\pi}} - \frac{\langle \Delta H \rangle}{2\pi}\right)$$

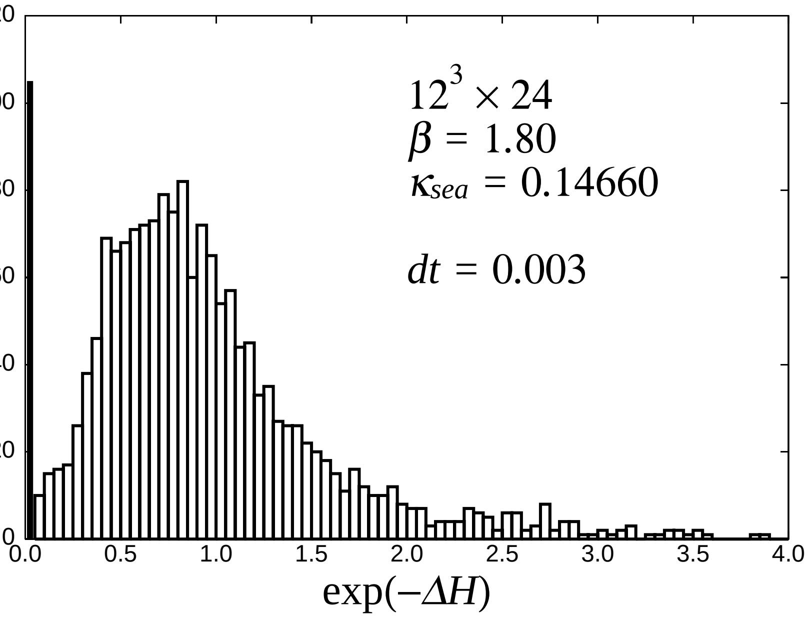
$$\langle \Delta H \rangle \sim O((dt)^4 V) \sim O(1)$$

$\rightarrow$ **The analysis has not been finished yet.**
The prescription is employing small dt.
(dt : step size of molecular dynamics in HMC)





he histogram of spikes



. Results

Chiral extrapolation]

arameterize lattice data O with m_{quark}

r chiral extrapolations

$$O = f(m_{quark})$$

hoice of the extrapolation function f

- **polynomial(conventional method)**
- **Chiral perturbation(ChPT) formulae**
- **Wilson ChPT(WChPT) formulae**

. The physical quark mass $m_{quark}^{physical}$ is obtained by

$$m_{quark}^{physical} = f^{-1}(O^{physical})$$

olynomial (conventional) extrapolation

- **Check the validity of the previous quadratic fit from $m_{PS}/m_V = 0.80 - 0.55$ by comparing with new small sea quark mass data $m_{PS}/m_V = 0.50 - 0.35$**

[ector meson mass]
e parameterize vector meson masses m_V
a function of pseudoscalar meson masses m_{PS}

$$m_V = A + Bm_{PS}^2 + Dm_{PS}^4 + Fm_{PS}^6 \quad \text{for } m_{PS}/m_V = 0.80 - 0.35$$

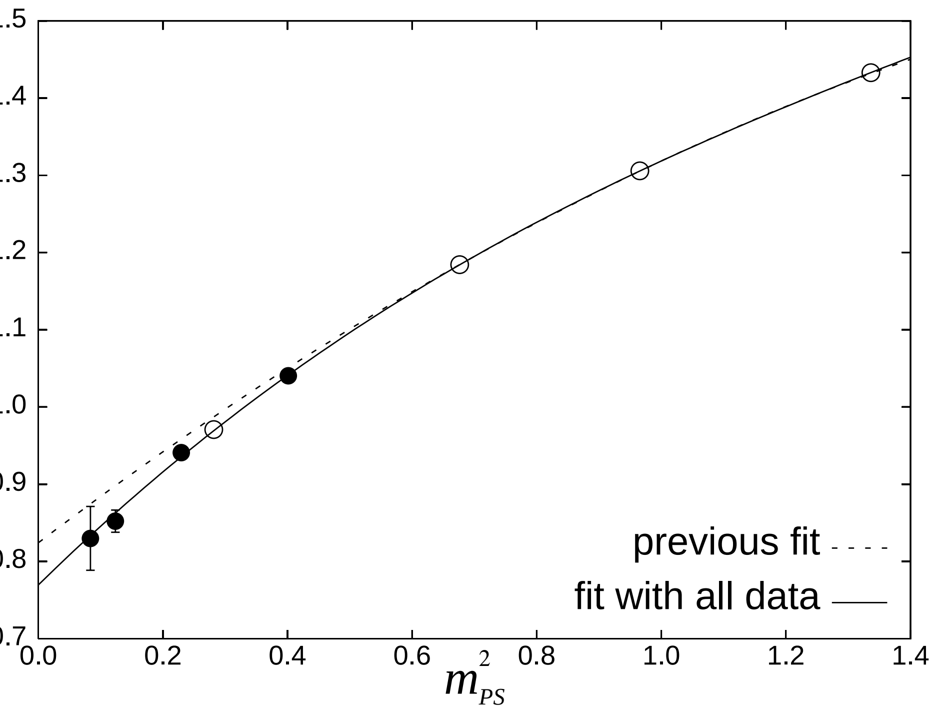
$$m_V = A + Bm_{PS}^2 + Dm_{PS}^4 \quad \text{for } m_{PS}/m_V = 0.80 - 0.55$$

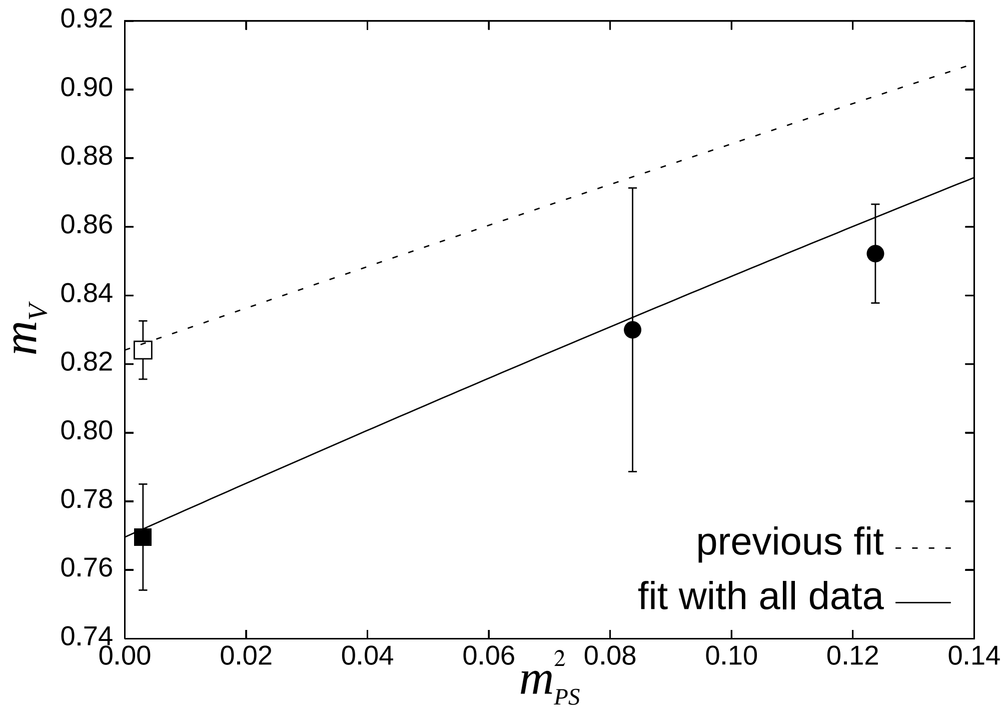
(previous fit) CP-PACS,2002

- **We use m_{PS}^2 instead of m_{quark} for comparison with the previous work (We parameterize m_{PS}^2 as a function of m_{quark}, as explained later)**

- Systematic deviations from the previous fit are observed in small sea quark mass region
  → Sea quark mass dependence was under-estimated by the previous quadratic extrapolation from $m_{PS}/m_V = 0.80 - 0.55$

  The deviation in the chiral limit is $7\%(3.5\sigma)$





[Pseudoscalar meson mass]

We parameterize pseudoscalar meson masses m_{PS} as a function of VWI quark masses m_{quark}^{VWI}

$$m_{PS}^2 = Bm_{quark}^{VWI} + C(m_{quark}^{VWI})^2 + D(m_{quark}^{VWI})^3 + E(m_{quark}^{VWI})^4$$

for $m_{PS}/m_V = 0.80 - 0.35$

$$m_{PS}^2 = Bm_{quark}^{VWI} + C(m_{quark}^{VWI})^2$$

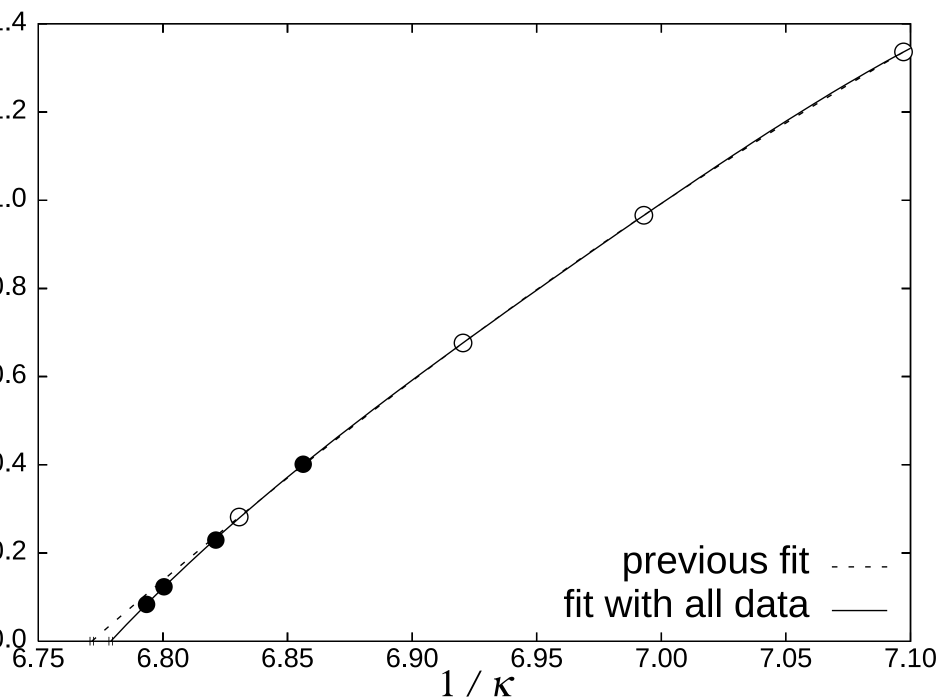
for $m_{PS}/m_V = 0.80 - 0.55$**(previous fit)** **CP-PACS,2002**

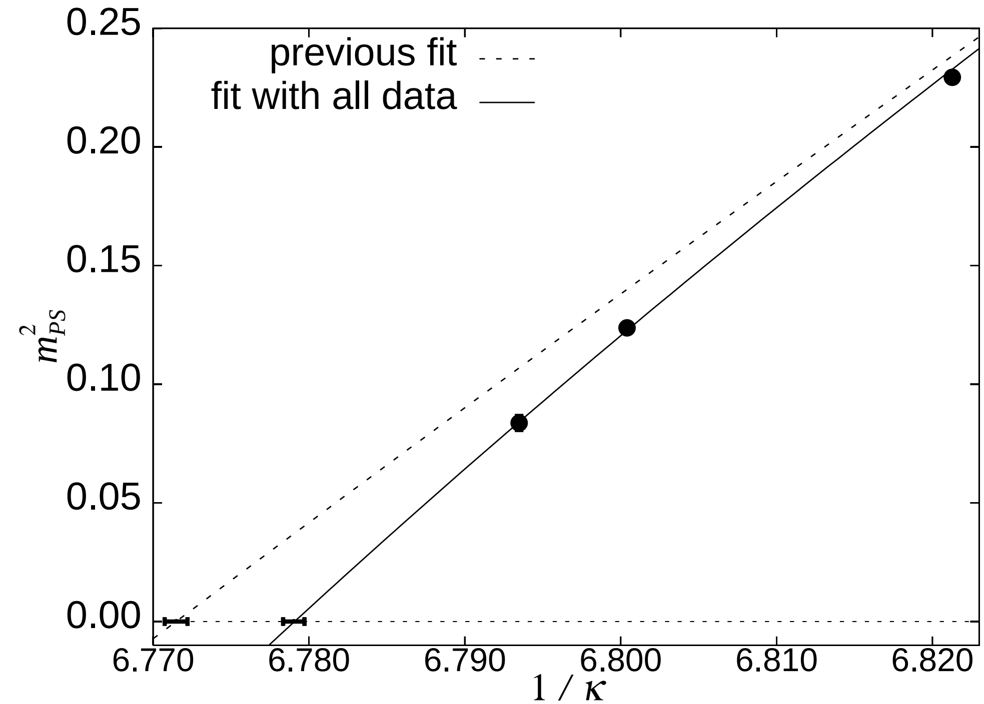
where

$$m_{quark}^{VWI} = \frac{1}{2}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)$$

- **Large deviations from the previous fit are also observed in small sea quark mass region (10σ difference in κ_c)**

- **The quadratic fit form can not describe our data of** $m_{PS}/m_V = 0.80 - 0.35(\chi^2/dof = 8)$
  cf. $m_{PS}/m_V = 0.80 - 0.55(\chi^2/dof = 0.1)$

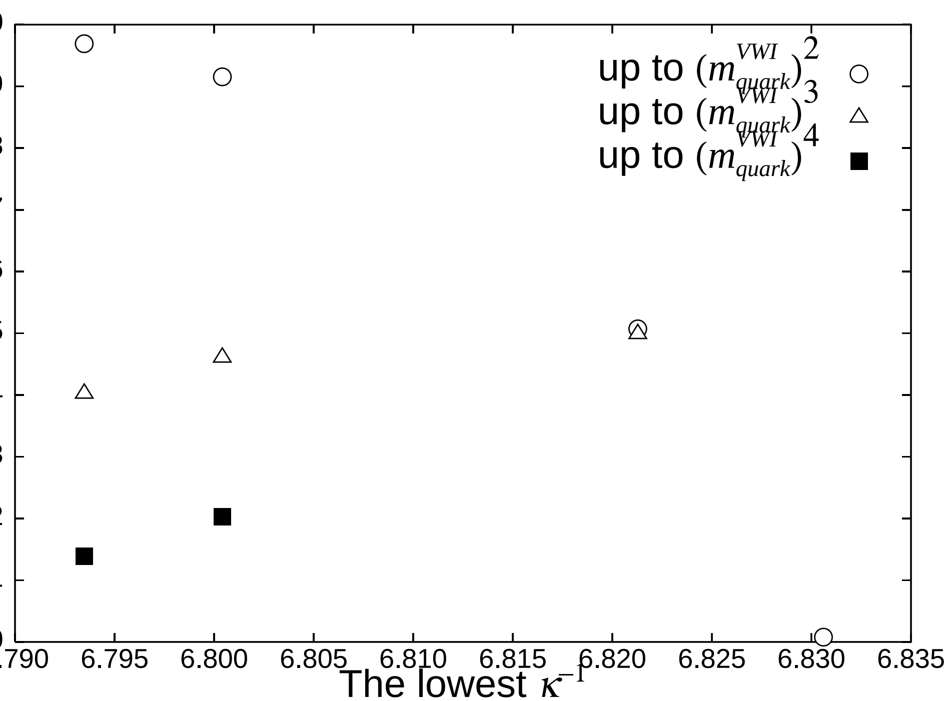


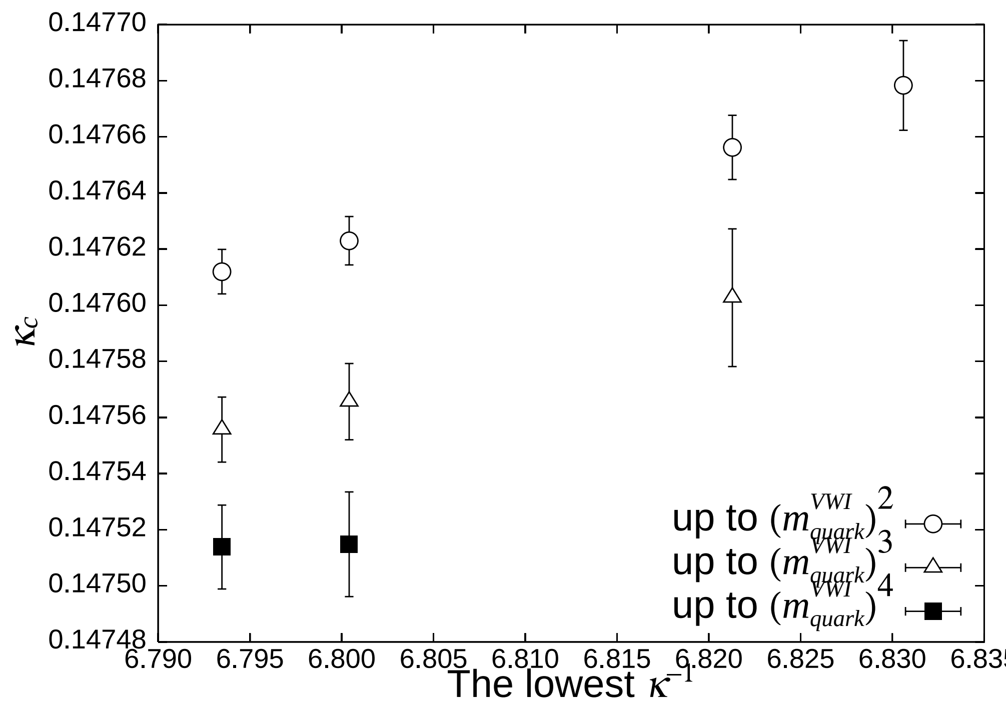


.3 How do the sea quark mass dependence
n m_{PS}^2 increase?

- **With** $m_{PS}/m_V = 0.55 - 0.80$ **data,**
  the quadratic fit works($\chi^2/dof = 0.1$)

- **As the smaller sea quark mass data**
  is added to the fit, χ^2/dof
  increases and κ_c **changes.**



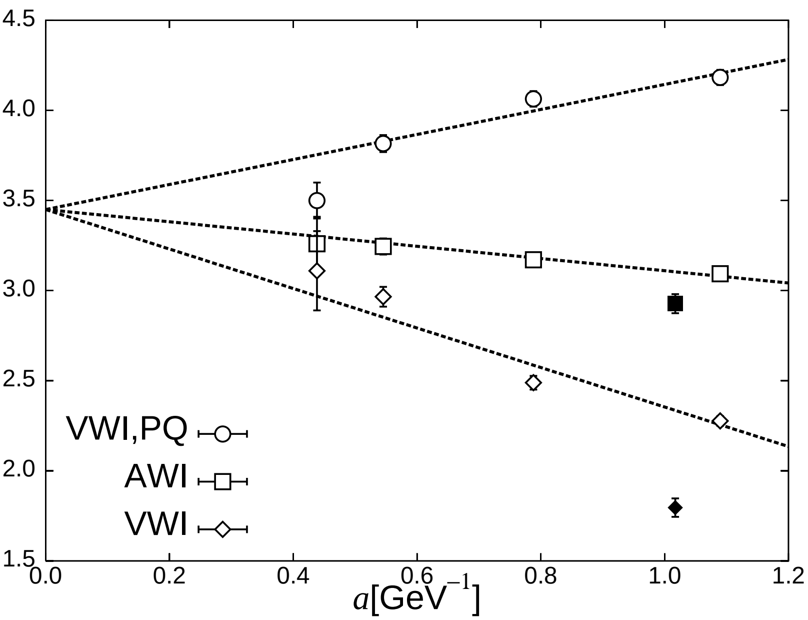


Results of polynomial extrapolations]

- **Smaller sea quark mass data require higher order polynomials towards the chiral limit in addition to the quadratic term**

uark masses in physical units from $m_{PS}/m_V = 0.80 - 0.35$]

- $m_{ud,R}^{AWI,\overline{\mathbf{MS}}}(\mu = 2\mathbf{GeV})$ **decreases by 8%**(4σ)
- $m_{ud,R}^{VWI,\overline{\mathbf{MS}}}(\mu = 2\mathbf{GeV})$ **decreases by 23%**(10σ)



hiral extrapolations based on ChPT

- **In the case of polynomial extrapolations, more higher order terms may be needed if smaller sea quark mass data are available**
  $\rightarrow$ ChPT may give a guide for extrapolations, which is expected to describe the sea quark mass dependence around the physical point.

hPT : a low energy effective theory of QCD

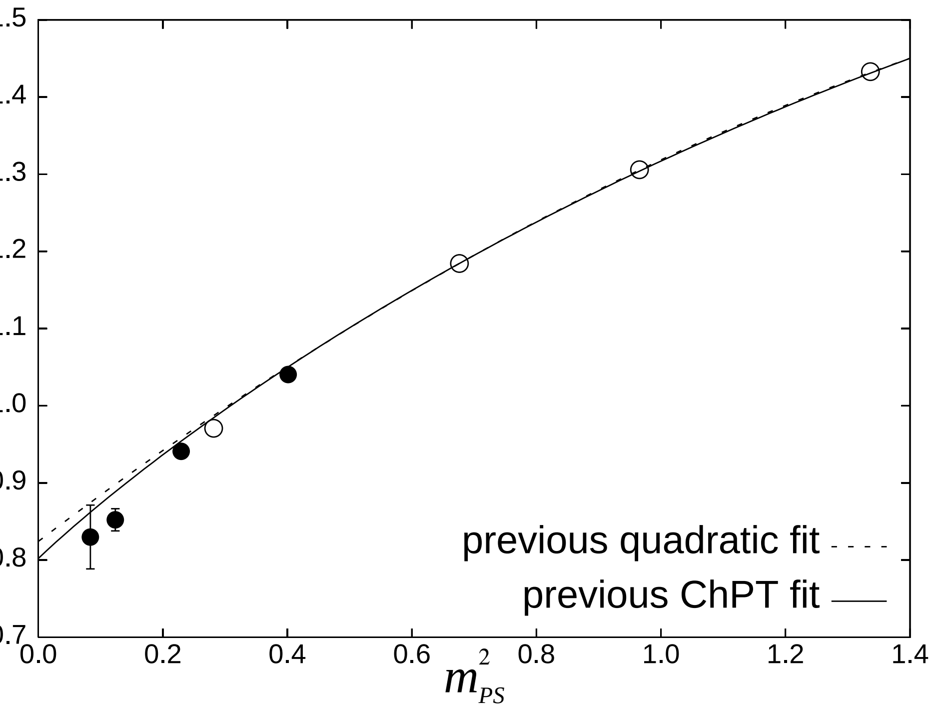
einberg,1979;Gasser and Leutwyler,1984,1985

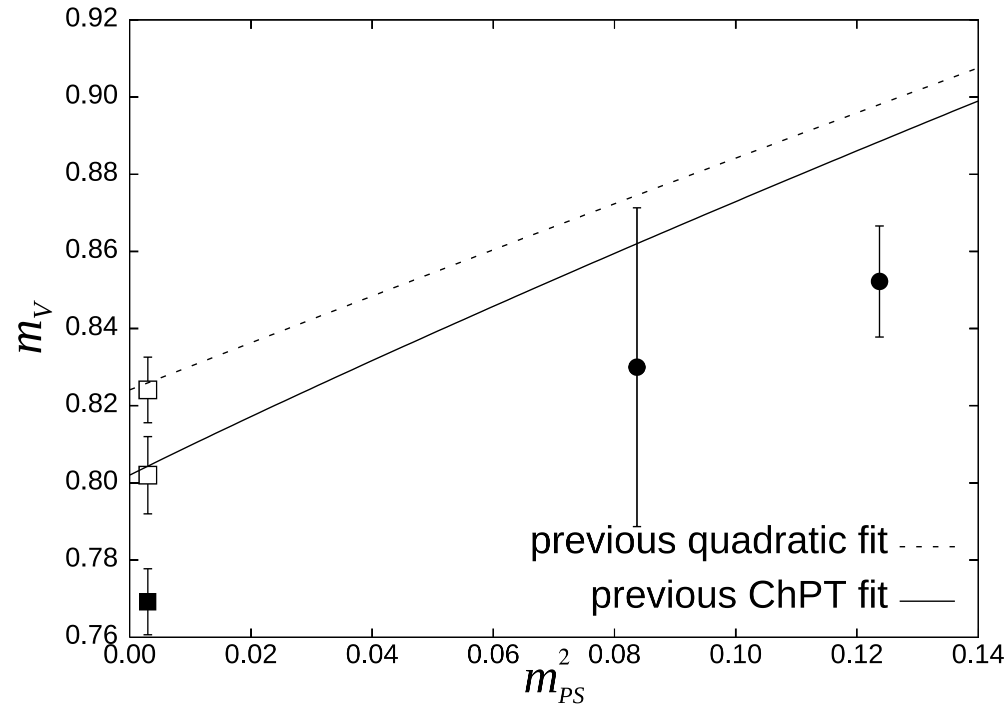
$_{PS}/m_V$	**0**	**0.17**	**0.35**	**0.60**	**0.80**
$^{AWI}_{quark}$[MeV]	**0**	~ 5	**13**	**63**	**91**

[Vector meson mass with ChPT in the static limit]

$$m_V = A + Bm_{PS}^2 + Cm_{PS}^3$$ **Jenkins *et.al.*,1995**

- **ChPT fit with the previous data $m_{PS}/m_V = 0.80 - 0.55$ predicted the new $m_{PS}/m_V = 0.50 - 0.35$ data better than the quadratic fit**





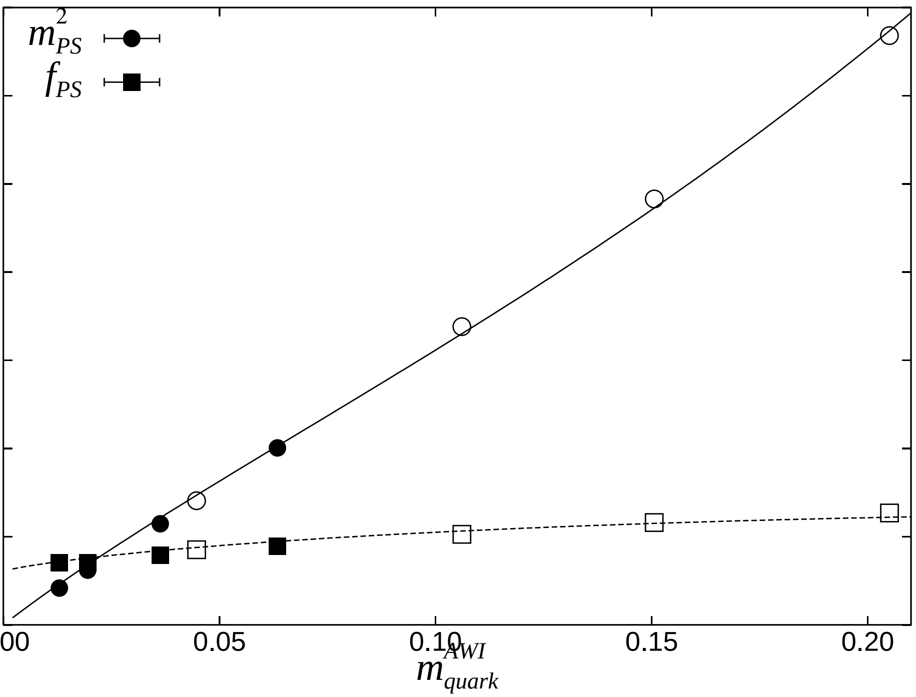
]One-loop ChPT fit to m_{PS}^2 and f_{PS}]

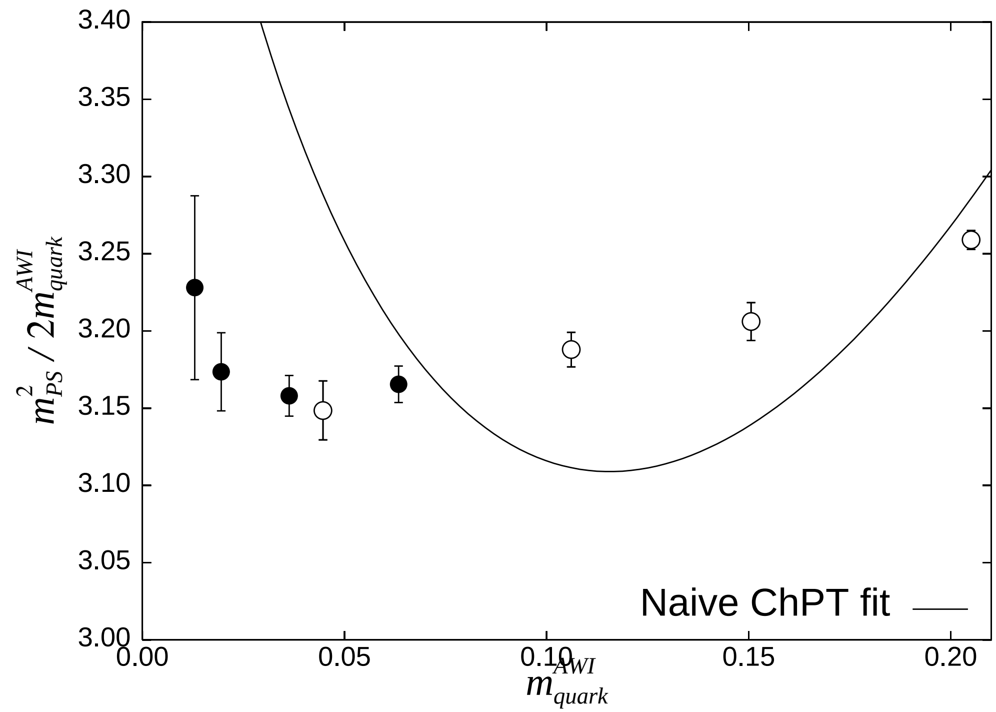
$$m_{PS}^2 = 2m_{quark}B_0\left(1+\frac{B_0 m_{quark}}{(4\pi f)^2}\log\frac{2B_0 m_{quark}}{\Lambda_3}\right)$$

$$f_{PS} = f\left(\frac{2B_0 m_{quark}}{(4\pi f)^2}\log\frac{2B_0 m_{quark}}{\Lambda_4}\right)$$

- **Fitting parameters :** $B_0, f, \Lambda_3, \Lambda_4$

hPT fails to describe our data of $m_{PS}/m_V = 0.80 - 0.35$
$[m_{quark}^{AWI}$ **:** $\chi^2/dof = 71]$





[Reasons of failure of the naive ChPT formulae]

- **Sea quark masses are still too heavy for the one-loop formulae (a fit dropping the heavy data $m_{PS}/m_V = 0.60 - 0.35$ gives $\chi^2/dof \sim 4$)**
- **Chiral symmetry is explicitly broken for the Wilson-type fermions**
  $\rightarrow$ Modify ChPT for Wilson-type fermions(WChPT)
  Sharpe and Singleton,1998; Rupak and Shoresh,2002; O.Baer *et.al.*,2003; S.Aoki,2003

$$\mathcal{L}_{WChPT} = \frac{f^2}{4}\left(1 + c_0(S^0 - 1)\right)\mathbf{Tr}\left(\partial_\mu U \partial^\mu U\right) + c_1 S^0 + c_2 (S^0)^2$$

$$S_0 \equiv \mathbf{Tr}(U + U^\dagger), \qquad U \equiv \exp\left(i \sum_{a=1}^{3} \frac{\pi_a \sigma^a}{f}\right) \in SU(N_f = 2)$$

$$\begin{aligned} c_0 &= W_0 a, \\ c_1 &= W_1 a + B_1 m_{quark}, \\ c_2 &= W_2 a^2 + V_2 (m_{quark} a) + O(m_{quark}^2). \end{aligned}$$

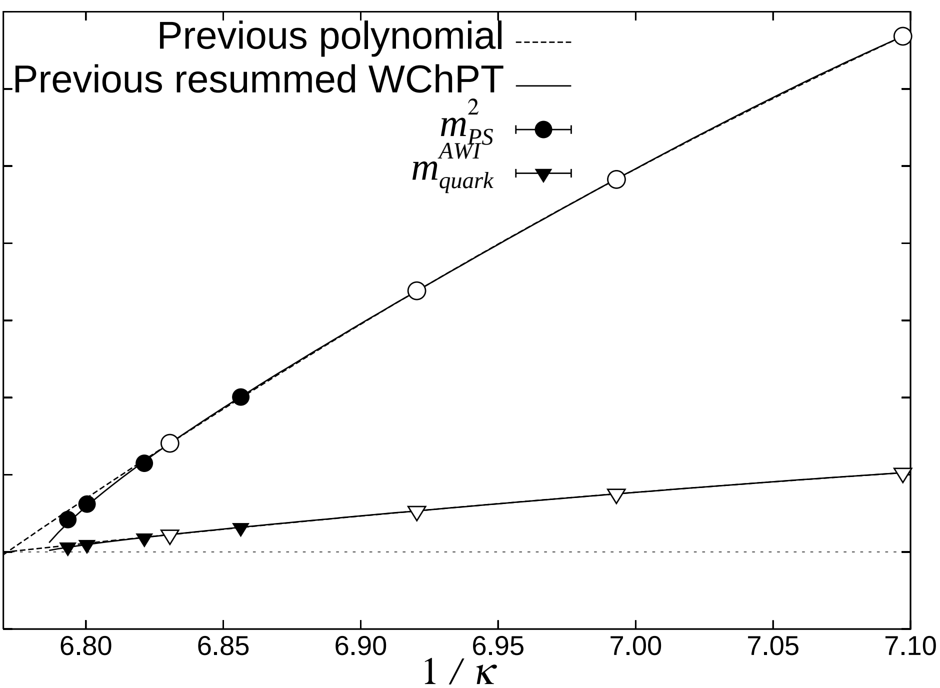
where $W = V = 0$ recovers the naive ChPT Lagrangian

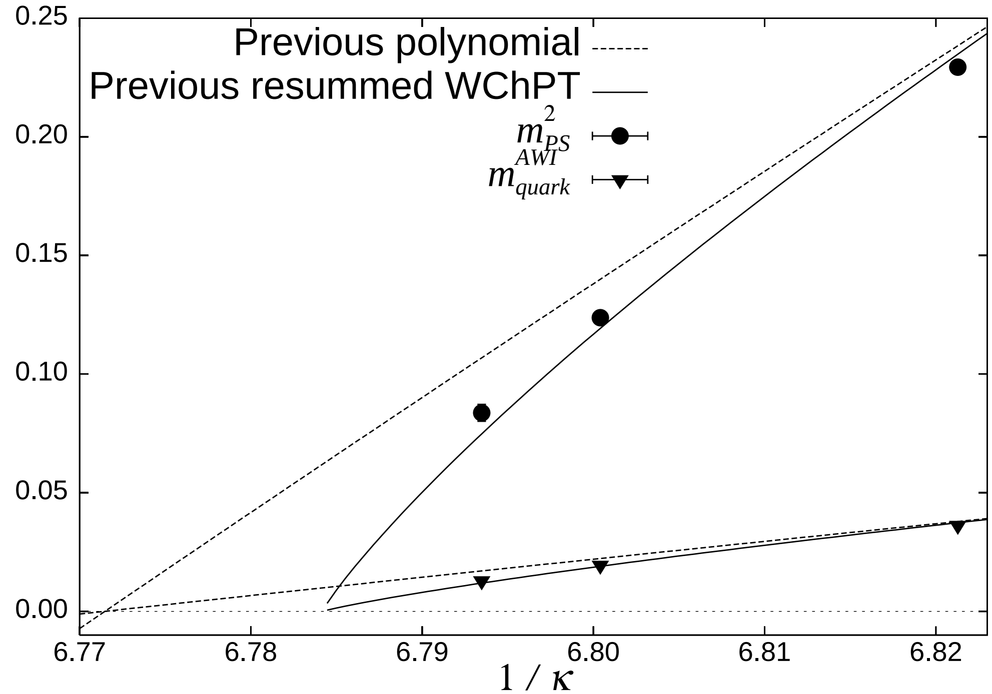
[Resummed WChPT formulae at one-loop] S.Aoki,2003

$$m_{PS}^2 = A m_{quark}^{VWI}\left(-\log\left(\frac{m_{quark}^{VWI}}{\Lambda_0}\right)\right)^{\omega_0}\left(1+\omega_1^{PS} m_{quark}^{VWI}\log\left(\frac{A m_{quark}^{VWI}}{\Lambda_3^2}\right)\right)$$

$$m_{quark}^{AWI} = m_{quark}^{VWI}\left(-\log\left(\frac{m_{quark}^{VWI}}{\Lambda_0}\right)\right)^{\omega_0}\left(1+\omega_1^{AWI} m_{quark}^{VWI}\log\left(\frac{A m_{quark}^{VWI}}{\Lambda_{3,AWI}^2}\right)\right)$$

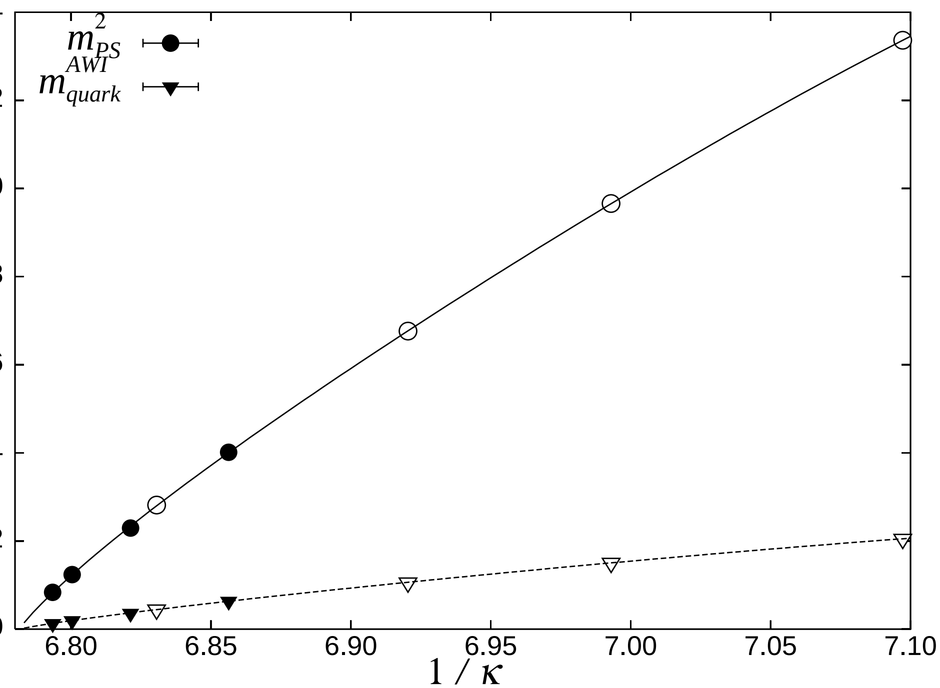
- **The WChPT formulae with the previous data $m_{PS}/m_V = 0.80 - 0.55$ predicted the data of $m_{PS}/m_V = 0.50 - 0.35$ better than polynomials**

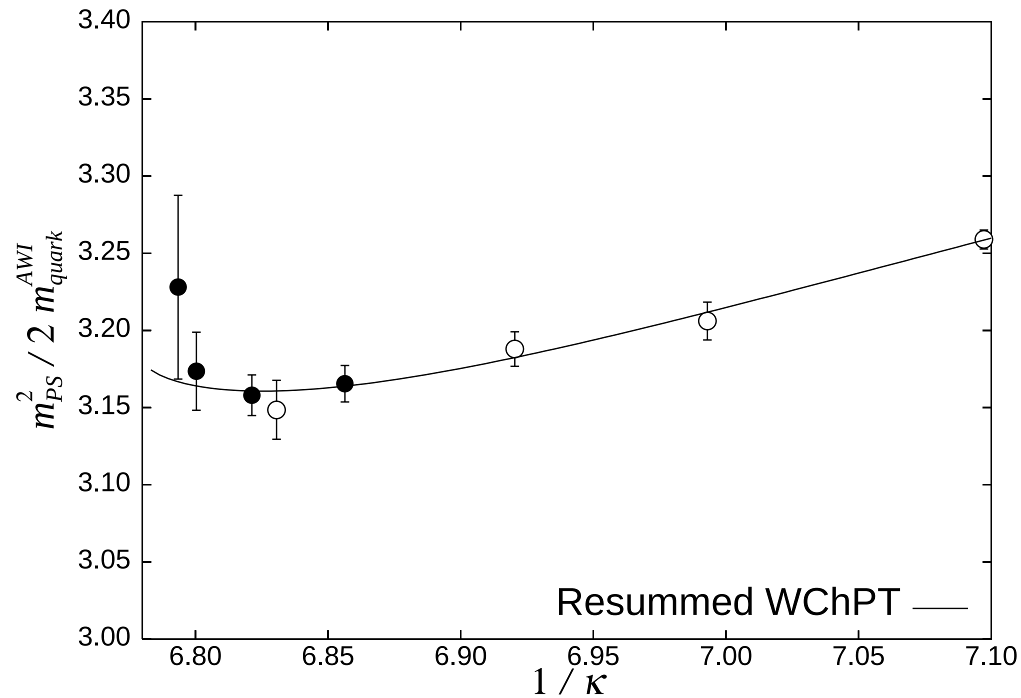




[WChPT fit with our all data of $m_{PS}/m_V = 0.80 - 0.35$]

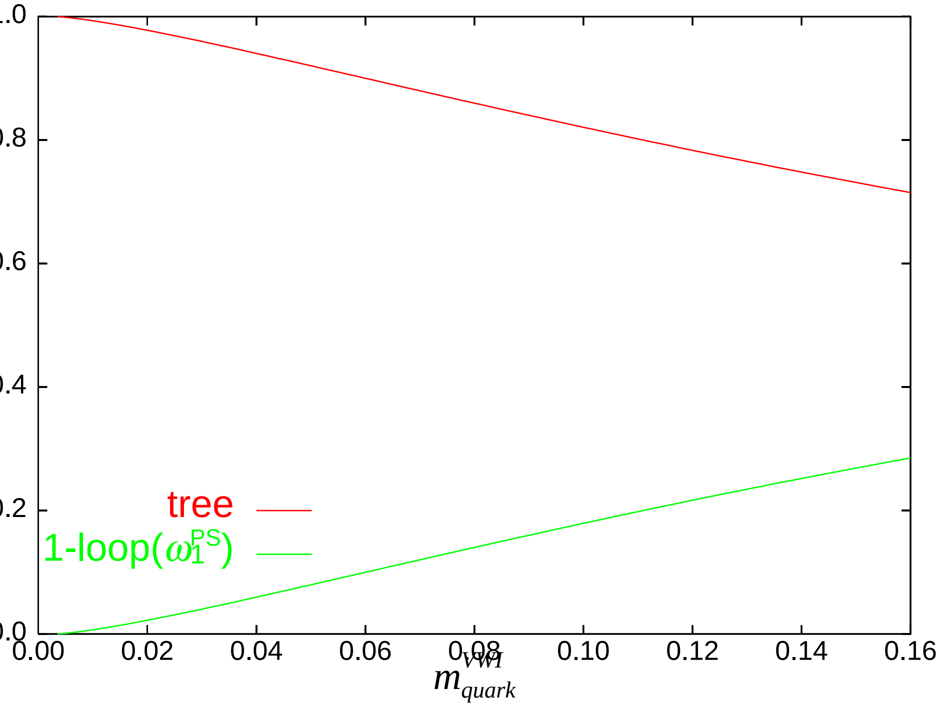
- **The WChPT formulae describe our data well($\chi^2/dof = 1.6$), in contrast to the naive ChPT case.**
- $\Lambda_3 = 0.15$**[GeV] is comparable with the phenomenological estimate(**$0.2 < \Lambda_3 < 2$**[GeV]** Gasser and Leutwyler,1984 **)**
- $O(a)$ **contribution** $(\omega_1^{PS} - \omega_1^{AWI})$ **suppresses the curvature in** $m_{PS}^2/2m_{quark}^{AWI}$ **to approximately** 10% **of the naive ChPT**





[Convergence check of the resummed WChPT]
at the heaviest point,

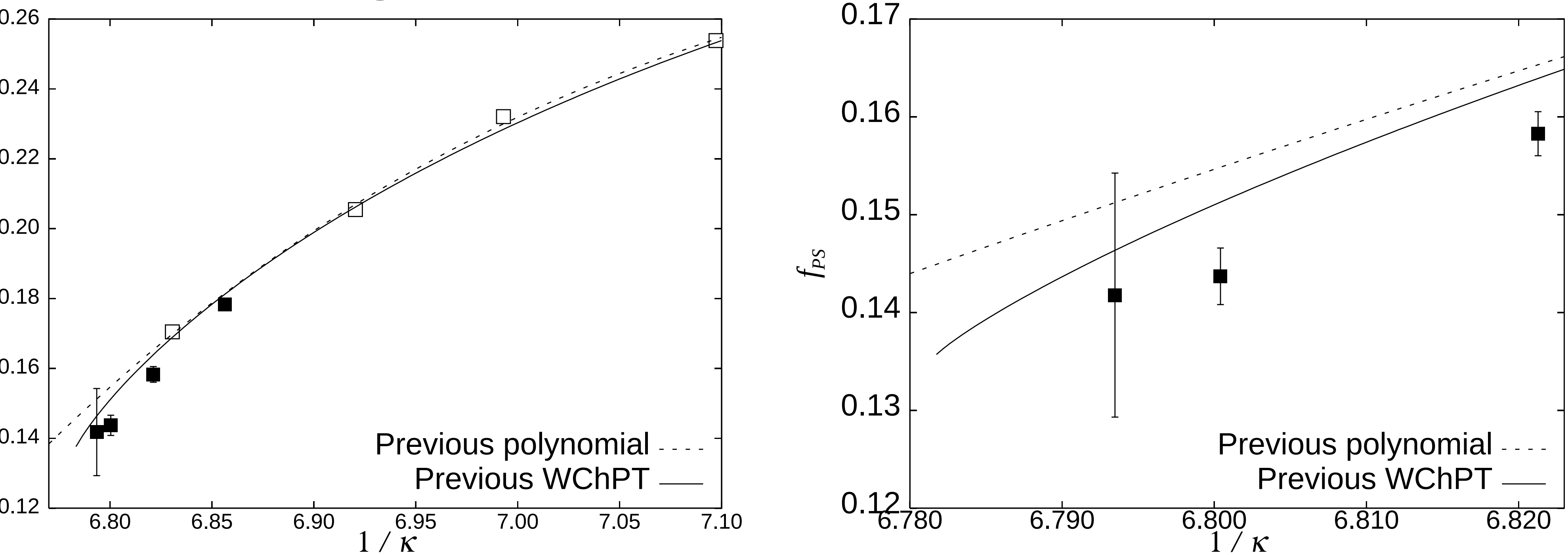
- **Tree : 71%**
- **one-loop : 29%**



[WChPT formula for the pseudoscalar decay constant]

$$f_{PS} = f\left(1 - \omega_1^{f_{PS}} m_{quark}^{VWI} \log\left(\frac{A m_{quark}^{VWI}}{\Lambda_4^2}\right)\right)$$

- **WChPT formula with the previous data $m_{PS}/m_V = 0.80 - 0.55$ predicted $m_{PS}/m_V = 0.50 - 0.35$ data better than polynomial**
- **The WChPT describes our data of $m_{PS}/m_V = 0.80 - 0.35$ ($\chi^2/dof = 3.6$)**
- $\Lambda_4 = 2.44(13)$ **is comparable with(a little larger than) the phenomenological estimate ($\Lambda_4 = 1.26 \pm 0.14$[GeV] Colangelo *et al.*,2001)**
  $\rightarrow$ The scaling violation leads to the deviation!?



Conclusion

'e studied light hadron spectrum using several
iiral extrapolations with our data of
$_{PS}/m_V = 0.80 - 0.35$. Our results imply that
is better to employ WChPT formulae instead of
onventional polynomials for chiral extrapolations.
eeasons]

- WChPT has a concrete theoretical background
- WChPT formulae from $m_{PS}/m_V = 0.80 - 0.55$ predicted $m_{PS}/m_V = 0.50 - 0.35$ data better than polynomials
- WChPT formulae describe our data of $m_{PS}/m_V = 0.80 - 035$ in contrast to the naive ChPT formulae
- Convergence of one-loop WChPT formulae is reasonable

rediction by WChPT formulae]

$$m_{ud,R}^{VWI,AWI}(\overline{\mathbf{MS}}, \mu = 2\mathrm{GeV}) = 1.230(94), 2.725(38)[\mathrm{MeV}]$$

$$m_{ud,R}^{VWI,AWI}(\overline{\mathbf{MS}}, \mu = 2\mathrm{GeV}) = 1.749(53), 2.851(58)[\mathrm{MeV}] \quad \text{cf. polynomial}$$

uture works]

- **Check the validity of WChPT formulae (smaller sea quark masses, scaling)**
- **WChPT formulae for vector mesons and baryons**

.4 [Finite size effect]

$= 2.4$fm vs $L = 3.2$fm

- **No finite size effects in the meson quantities** $(m_{PS}, m_V, m_{quark}^{AWI})$
- $1-3\%(0.8-2.3\sigma)$ **decreases in the baryon quantities**(m_N, m_Δ)