Toward $O(a)$-improvement of axial current in lattice QCD

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for the ALPHA/CP-PACS/JLQCD collaborations

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I. Introduction

CP-PACS/JLQCD project in three-flavor QCD

(T.Ishikawa's talk for details)

- RG-improved gauge + NP $O(a)$-improved Wilson quark
- use all machines available : 2.3 TFLOPS in total

  CP-PACS(0.3), SR8K/G1(0.2), VPP-5000(0.3) @ Univ.of Tsukuba
  SR8K/F1(0.8) @ KEK
  Earth Simulator (0.8) @ JAMSTEC

- one-loop value for c_A, $Z_{\{A,P,m\}}$, $b_{\{A,P,m\}}$

$\Downarrow$

- $m_q \Rightarrow$ fundamental parameter : $O(ag^4)$ scaling violation

$\Uparrow$

NP determination of c_A, $Z_{\{A,P,m\}}$, $b_{\{A,P,m\}}$

ALPHA

- $O(a)$-improvement
- plaquette gauge + NP $O(a)$-improved Wilson quarks
- non-perturbative dertmination of improvement and renormalization constants in quenched and two-flavor QCD

$\Downarrow$

collaboration in computaion of c_A, ... in full QCD

ALPHA : c_A in two-flavor QCD $\longrightarrow$ talk by Della Morte

CP-PACS/JLQCD : a/L dependence of c_A in quenched QCD
with plaquette gauge action $\longrightarrow$ this talk

contents :

II. Topics on determination of c_A

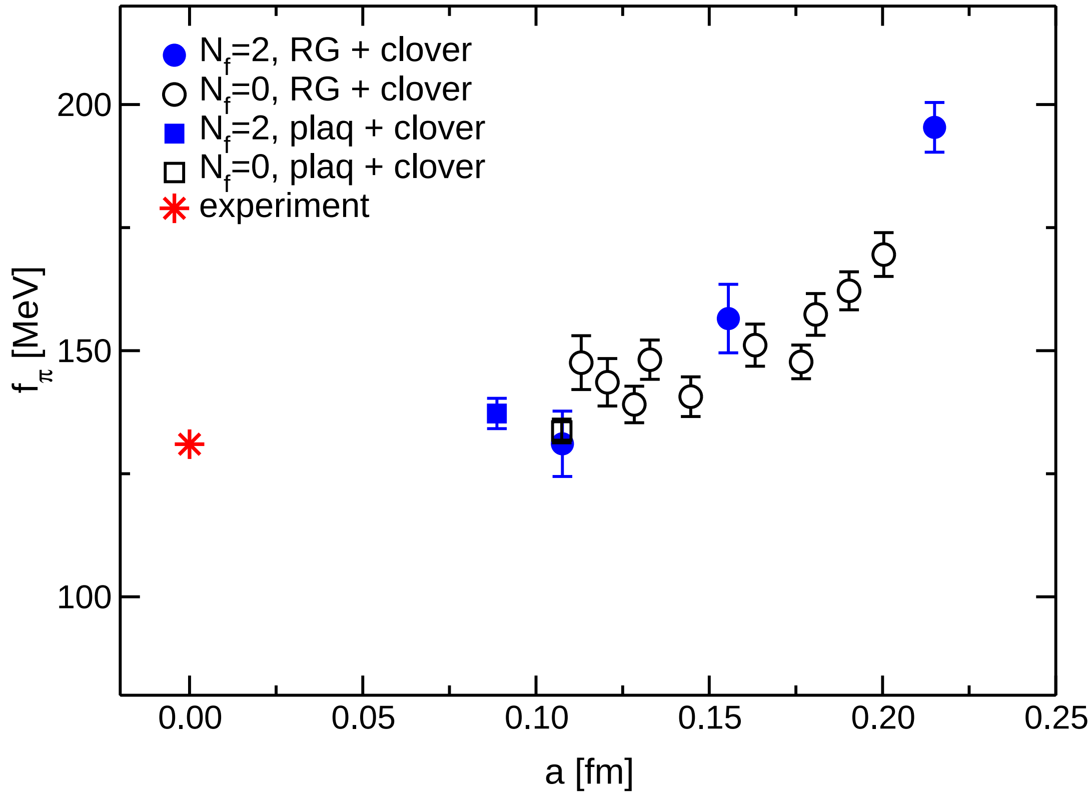
contents :

II-1 motivation

CP-PACS study in two-flavor QCD

f_{PS} (using one-loop c_A, Z_A, b_A) $\Rightarrow$ large scaling violation

f_π vs a



- scaling violation :

  $\sim O(a)$

  $\Rightarrow$ NP determination of c_A

  (or $O(a^2)$?)

- renormalization factor Z_A

  $\Rightarrow$ talk by K.Ide

JLQCD data in quenched QCD

- with plaq. gauge + NP $O(a)$-improved Wilson quarks
- AWI $m_q = \langle A_4 P\rangle / (2\langle PP\rangle)$

$$m_s^{\mathrm{AWI}} = \begin{cases} 104.1(1.6)\ \text{MeV} & \text{with one-loop } c,\ Z,\ b \\ 91.9(1.4)\ \text{MeV} & \text{with NP } c,\ Z,\ b \text{ by ALPHA} \end{cases}$$

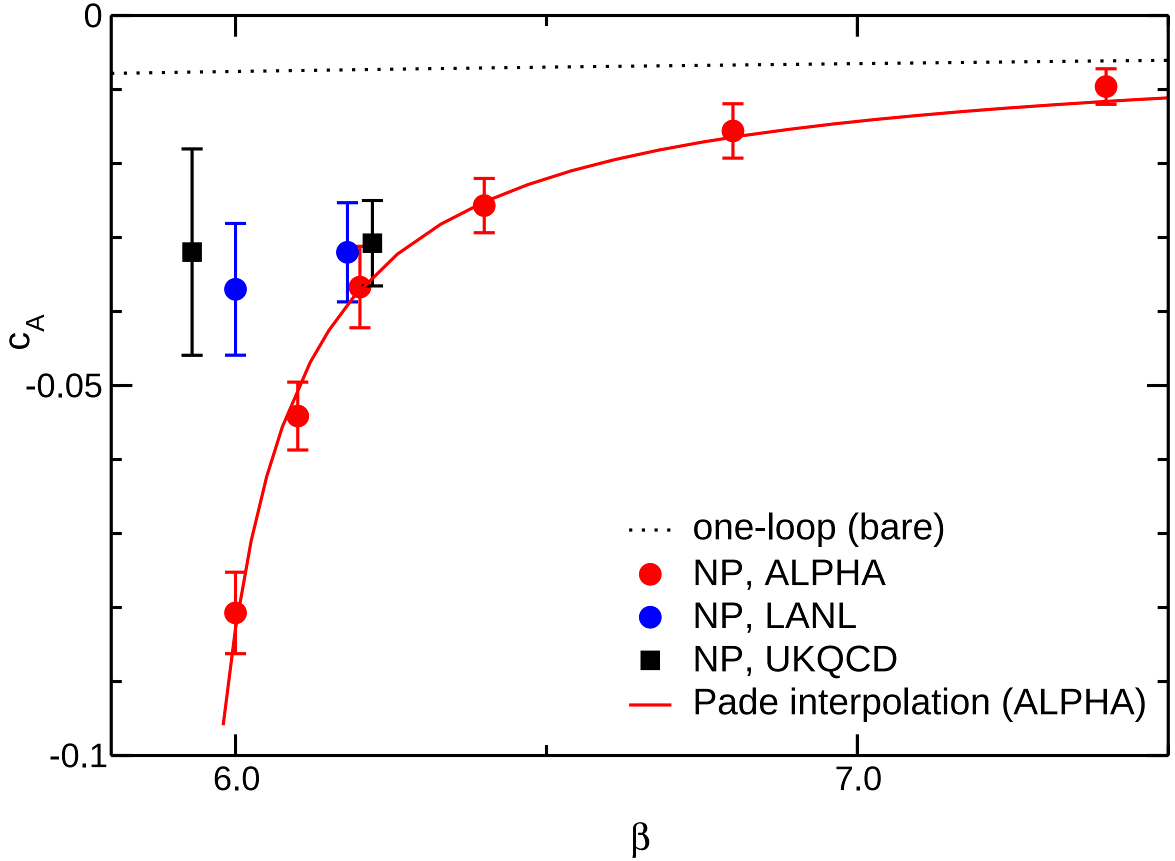
$\Rightarrow$ 12% difference

- $(Z_A/Z_P)_{\text{one-loop}} = 1.19 \Leftrightarrow (Z_A/Z_P)_{\text{NP}} = 1.22$
- $(b_A - b_P)_{\text{one-loop}} = -0.013 \Leftrightarrow (b_A - b_P)_{\text{NP}} = 0.171$: but small $O(am_q)$
- $c_{A,\text{one-loop}} = -0.013 \Leftrightarrow c_{A,\text{NP}} = -0.083$

II-2 $O(a)$ effects in c_A

previous results of NP c_A in quenched QCD

NP c_A vs β



⇒ $O(a)$ effects in c_A is not small ?

⇒ large difference in $O(a^2)$ scaling property?

⇒ study of a/L dependence is important

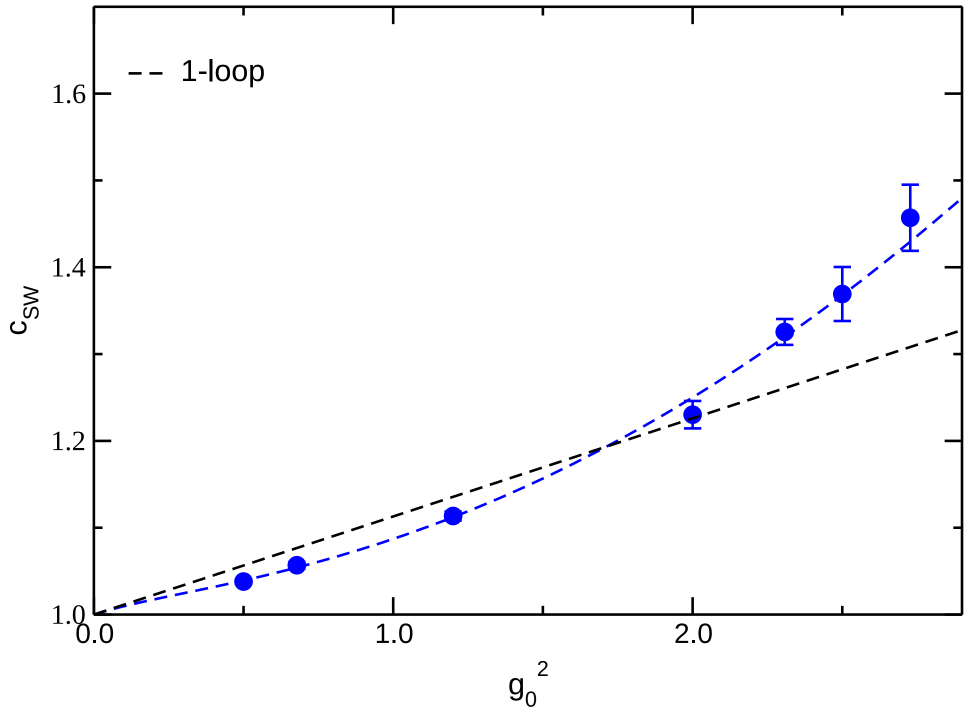
CP-PACS/JLQCD determination of $c_{\rm SW}$

- in three-flavor QCD with RG improd guage

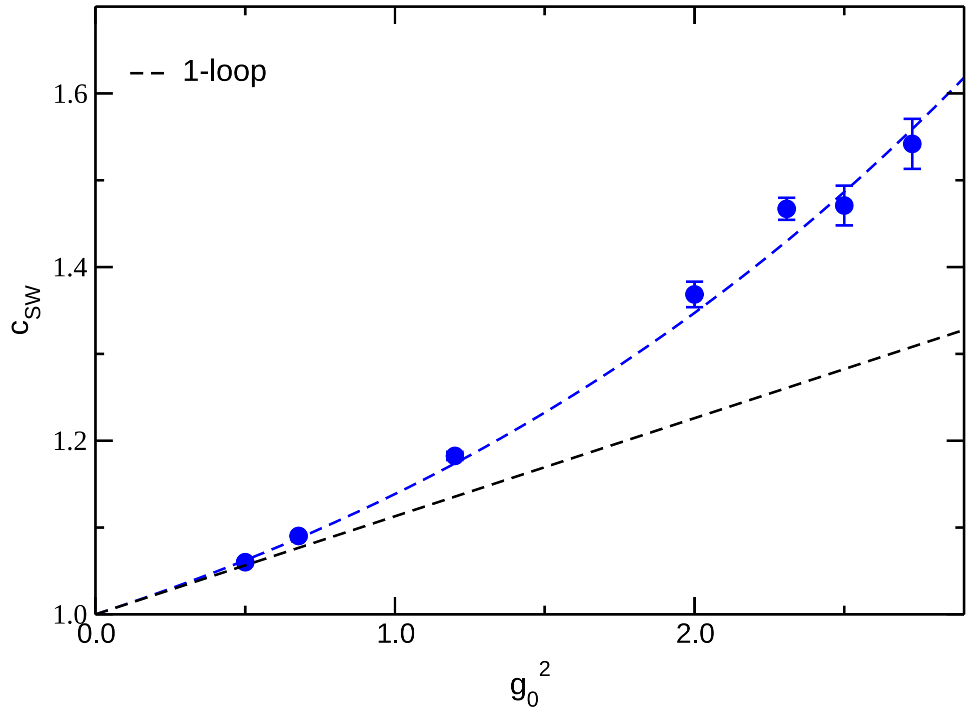
  $\Rightarrow$ non-negligible a/L dependence of $c_{\rm SW}$

  $\Rightarrow$ correct NP $c_{\rm SW}$ to $L = L^*$ $\quad (L^* = 6a(\beta = 1.9))$

$L/a = 8$



interpolated to $L = L^*$



III. Old method

contents :

III-1 Method

method to determine c_A

1) calculate quark masses with two values of θ

$$\psi(x+L\hat{k}) = e^{i\theta_k}\psi(x), \quad \bar{\psi}(x+L\hat{k}) = \bar{\psi}(x)e^{-i\theta_k}$$

$$f_A(x_0) = -\frac{1}{3}\langle A_0^a \mathcal{O}^a\rangle, \quad f_P(x_0) = -\frac{1}{3}\langle P^a \mathcal{O}^a\rangle$$

$$m(\theta, x_0) = r(\theta, x_0) + c_A\, s(\theta, x_0),$$

$$r(x_0) = \frac{1}{4}(\partial_0 + \partial_0^*)\, f_A(x_0)/f_P(x_0), \quad s(x_0) = \frac{1}{2}\, a\, \partial_0\, \partial_0^*\, f_P(x_0)/f_P(x_0),$$

2) determine c_A by imposing that quark masses coincide with each other

$$m(\theta', x_0) - m(\theta, x_0) = \Delta m^{(0)},$$

where we take $\theta = 0,\ \theta' = 1,\ x_0 = T/2$

simulation parameters

- in quenched QCD with plaq. gauge + $O(a)$-improved Wilson quarks
- use HMC algorithm on SR8K at KEK
- take NP c_{SW} from ALPHA
- take K_c on $16^3 \times 32$ from ALPHA
  - exceptional conf ⇒ heavier mass at $\beta = 6.0$
  - mass dependence of c_A is checked at $\beta = 12.0$ and 6.2 on $8^3 \times 16$

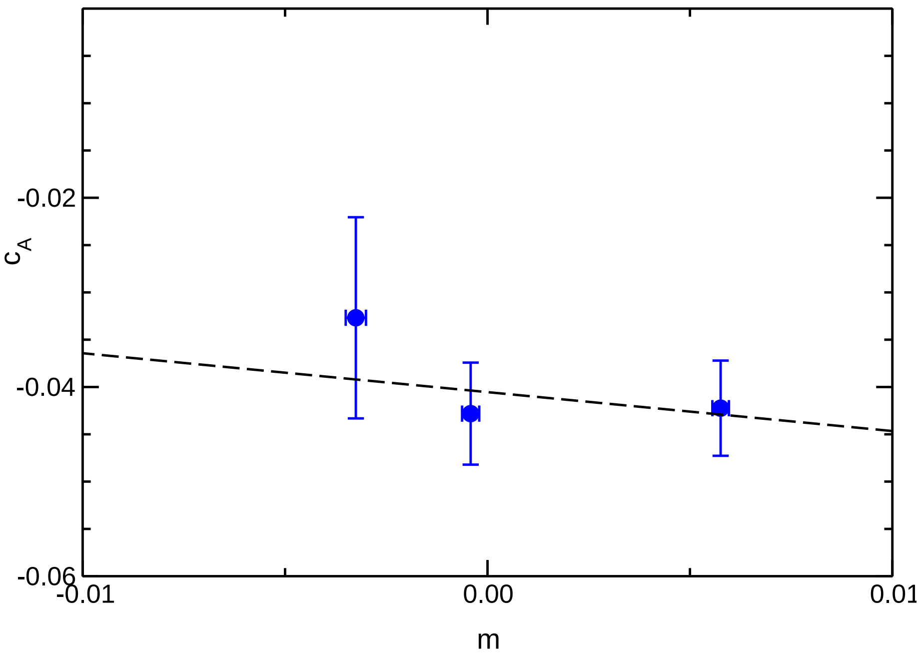
statistics [traj]

	$6^3 \times 12$	$8^3 \times 16$	$10^3 \times 20$	$12^3 \times 24$	$14^3 \times 28$
β=12.0	40000	40000	37000	40000	40000
8.0	60000	35000	40000	48000	40000
6.8	80000	45000	–	40000	40500
6.4	–	55000	–	60000	–
6.2	100000	80000	–	80000	–
6.0	–	40000	–	–	–

III-2 Mass dependence of c_A

at $\beta = 6.2$ on $8^3 \times 16$

c_A vs m



- small m dependence of c_A

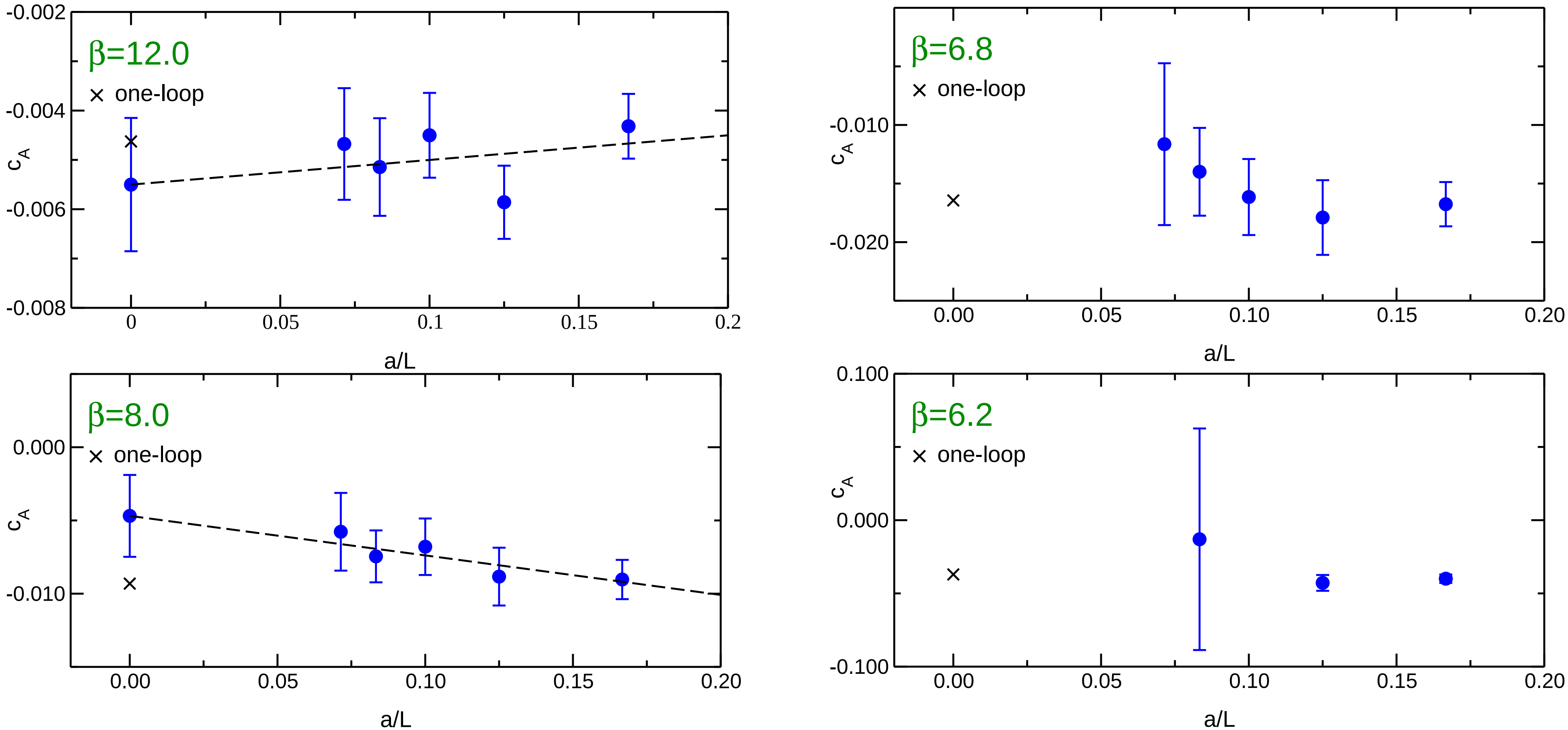
  $\Leftarrow$ confirm ALPHA's observation

- $K_c(16^3 \times 32)$ is close to $K_c(8^3 \times 16)$
  - $m = -0.00042(21)$ at $\beta = 6.2$ on $8^3 \times 16$
  - $m = -0.000050(34)$ at $\beta = 12.0$ on $8^3 \times 16$

$\Rightarrow$ ignore m dependence of c_A in the following

III-3 volume dependence of c_A

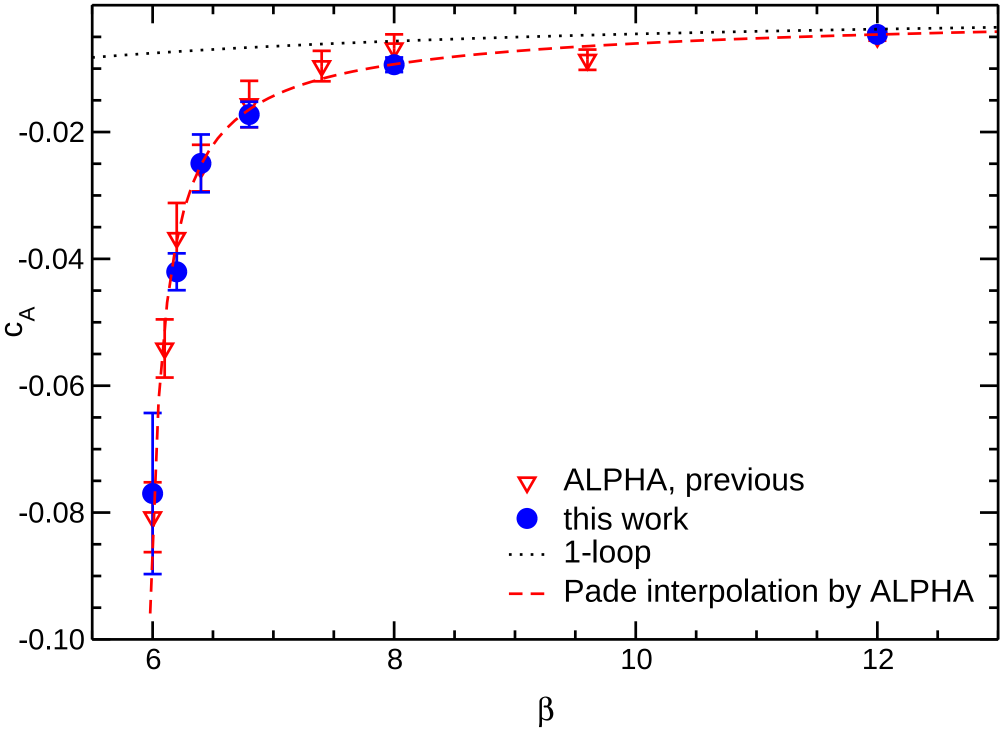
c_A vs a/L

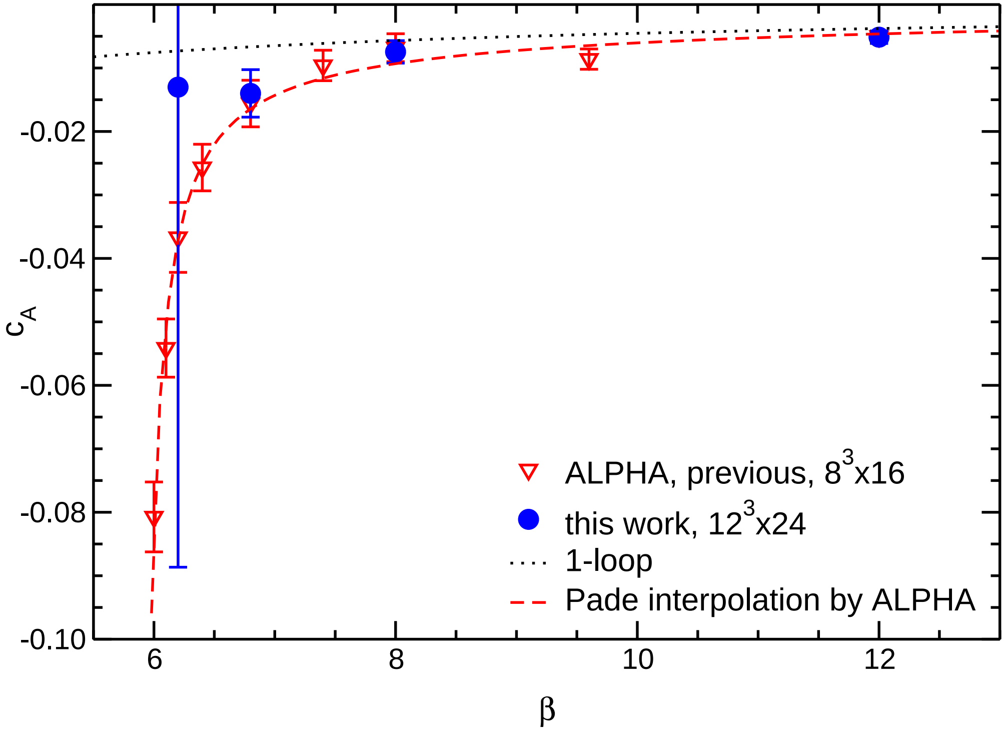


error of c_A increases for larger L/a

c_A vs β

on $8^3 \times 16$ on $12^3 \times 24$





large error obscures β dependence

sensitivity

$$c_A = \frac{\Delta m^{(0)} - (r(\theta', x_0) - r(\theta, x_0))}{s(\theta', x_0) - s(\theta, x_0)}$$

larger $L \Rightarrow \theta$ has smaller effects ($p_k = \theta_k / L$)
$\Rightarrow$ smaller difference between $s(\theta')$ and $s(\theta)$
$\Rightarrow$ c_A loses its sensitivity to the improvement condition

IV. New method

contents :

IV-1 Method

"slope method" to determine c_A

- improvement condition

$$m' - m = \Delta m^{(0)}$$

old method : difference between $\theta \Leftrightarrow \theta' = 1$ at fixed x_0 $(= T/2)$

new method : difference between $x_0 \Leftrightarrow x_{0,\mathrm{ref}}$ with fixed θ $(= \pi)$

$\Rightarrow$ simulations with a single value of θ

- choice of x_0 and $x_{0,\mathrm{ref}}$

x_0 : contribution of the ground state dominates $(= T/2)$

$x_{0,\mathrm{ref}}$: contribution of the excited state dominates $(= T/4)$

simulation parameters

- in quenched QCD with plaq. gauge $+O(a)$-improved Wilson quarks
- use HMC algorithm on SR8K at KEK
- take NP $c_{\rm SW}$ from ALPHA
- take K_c on $16^3 \times 32$ from ALPHA
    - exceptional conf $\Rightarrow$ heavier mass at β= 6.0 and 6.2
    - mass dependence of c_A is checked at β= 12.0 and 6.4
- T/L=2
    - T/L=3 is tested at $\beta = 12.0$

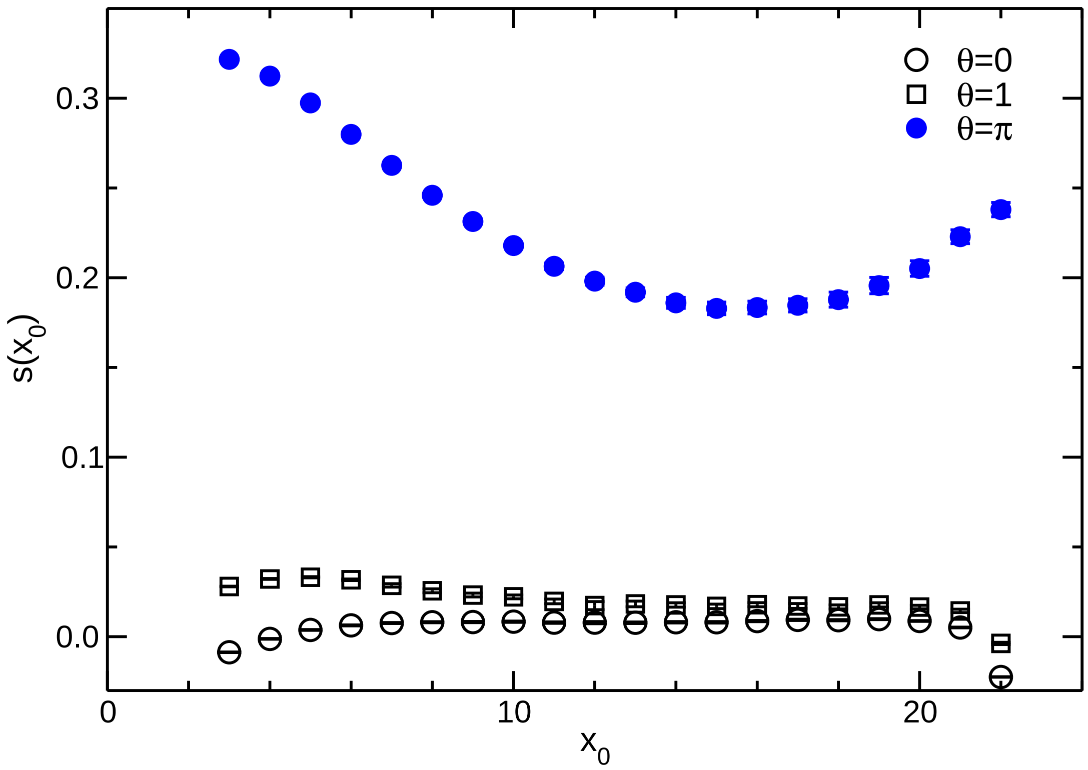
statistics [traj]

	$\beta = 12.0$	8.0	6.8	6.4	6.2	6.0
$8^3 \times 16$	40000	40000	60000	40000	30000	60000
$12^3 \times 24$	18000	80000	40000	40000	38000	46000
$16^3 \times 32$	30000	40000	40000	40000	40000	40000

IV-2 Results of s and c_A

$s(x_0)$ on $12^3 \times 24$

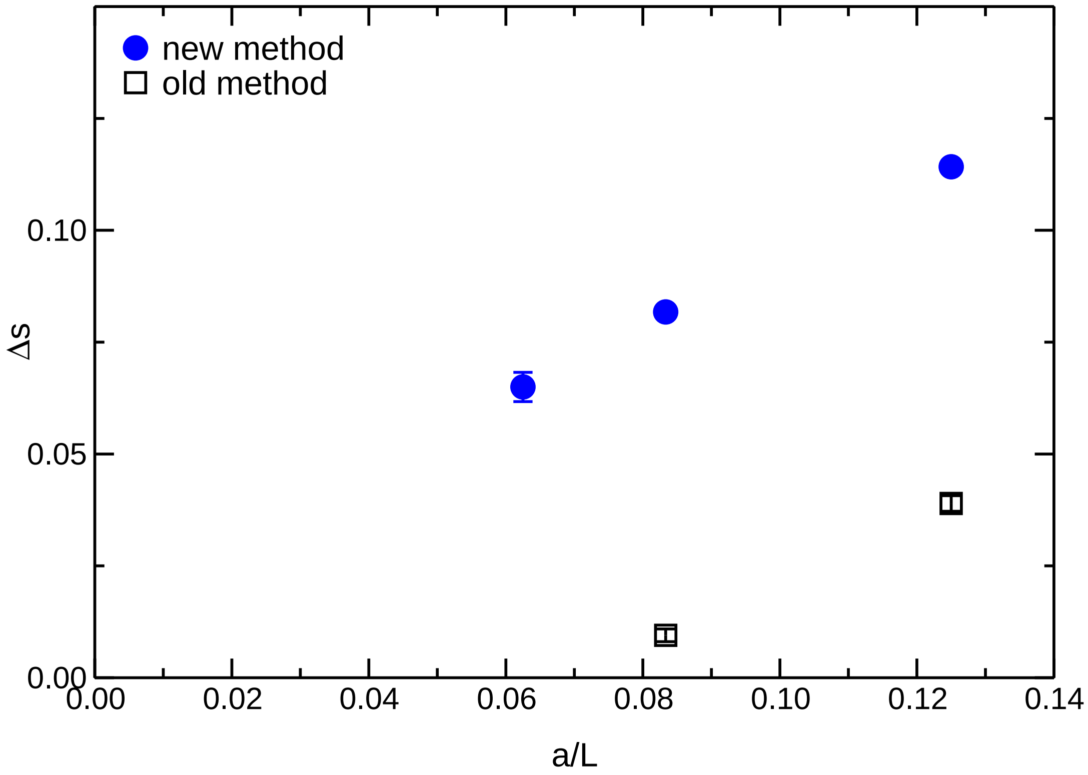
β=6.4, c_{SW}=1.52562, 12^3x24, K=0.13572



- small difference between $s(T/2)$ with $\theta = 0$ and 1
- much larger difference between $s(T/2)$ and $s(T/4)$ with $\theta = \pi$

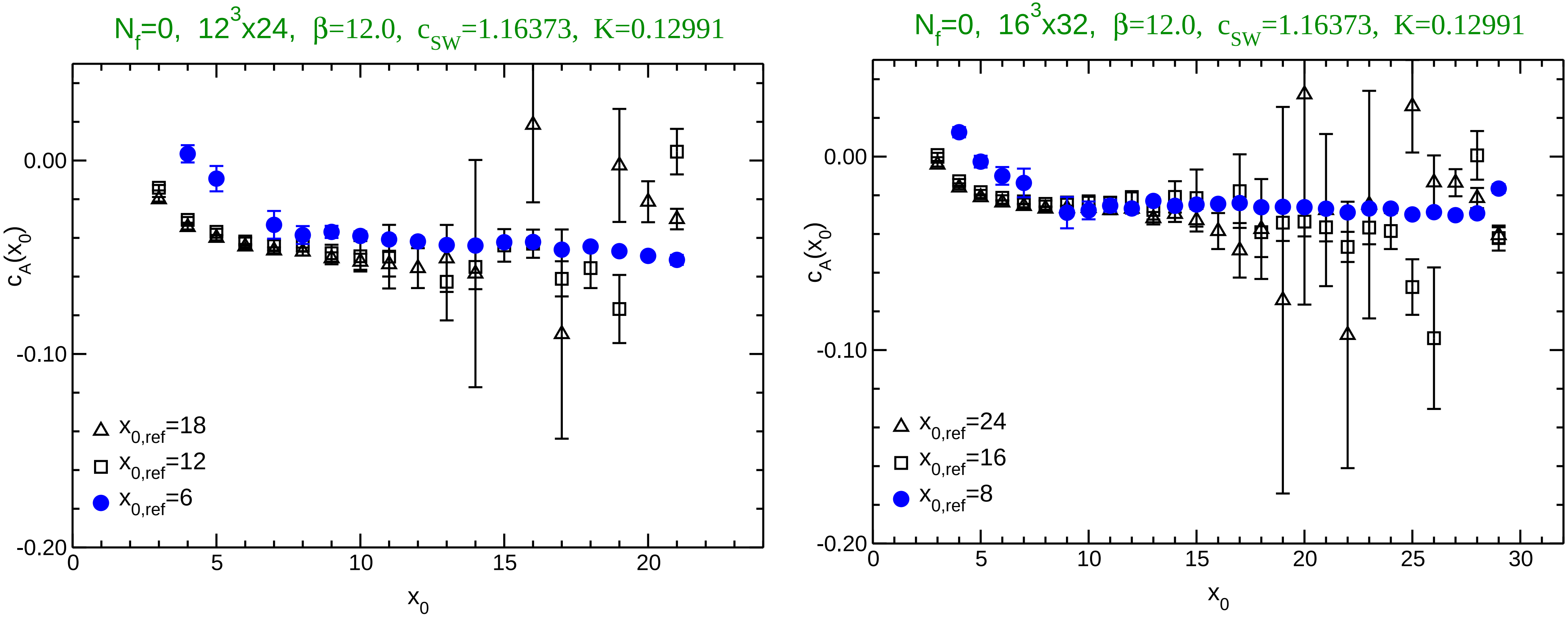
$$c_A = \frac{\Delta m^{(0)} - (r' - r)}{s' - s}$$

Δs at β=6.4



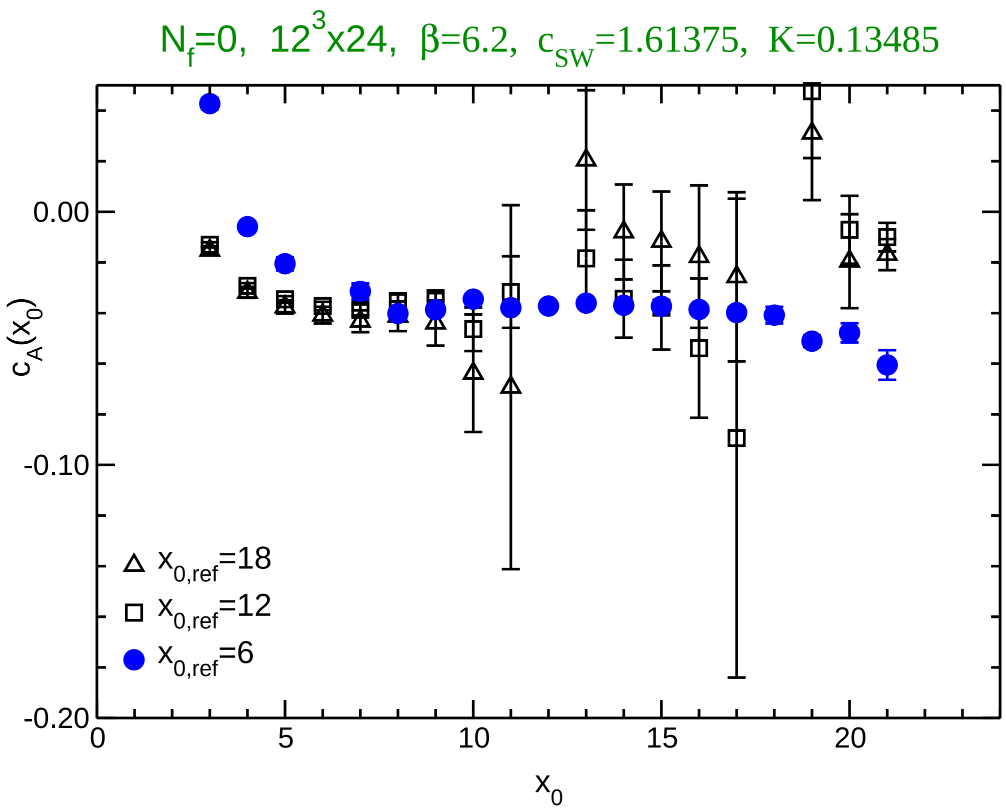
- old method : Δs almost vanishes at $L/a \sim 12$
- new method :much larger Δs

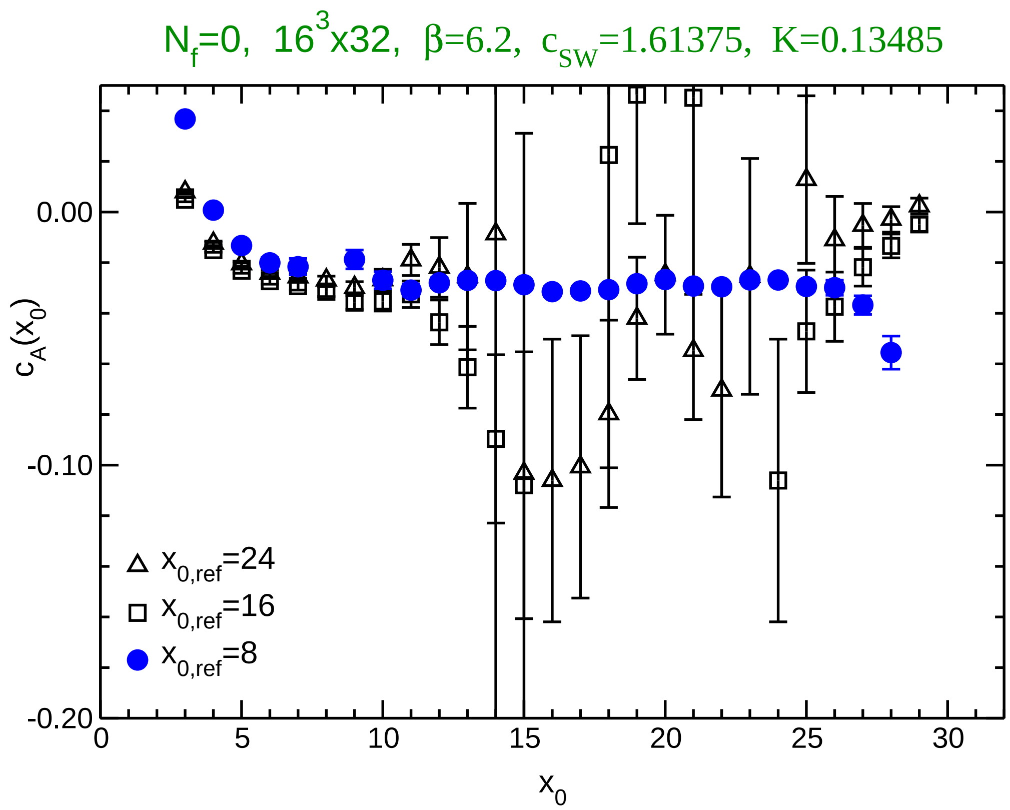
$c_A(x_0)$ at β =12.0



- x_0, $x_{0,\mathrm{ref}}$ dependence of c_A is not large
- take $x_0 = T/2$, $x_{0,\mathrm{ref}} = T/4$

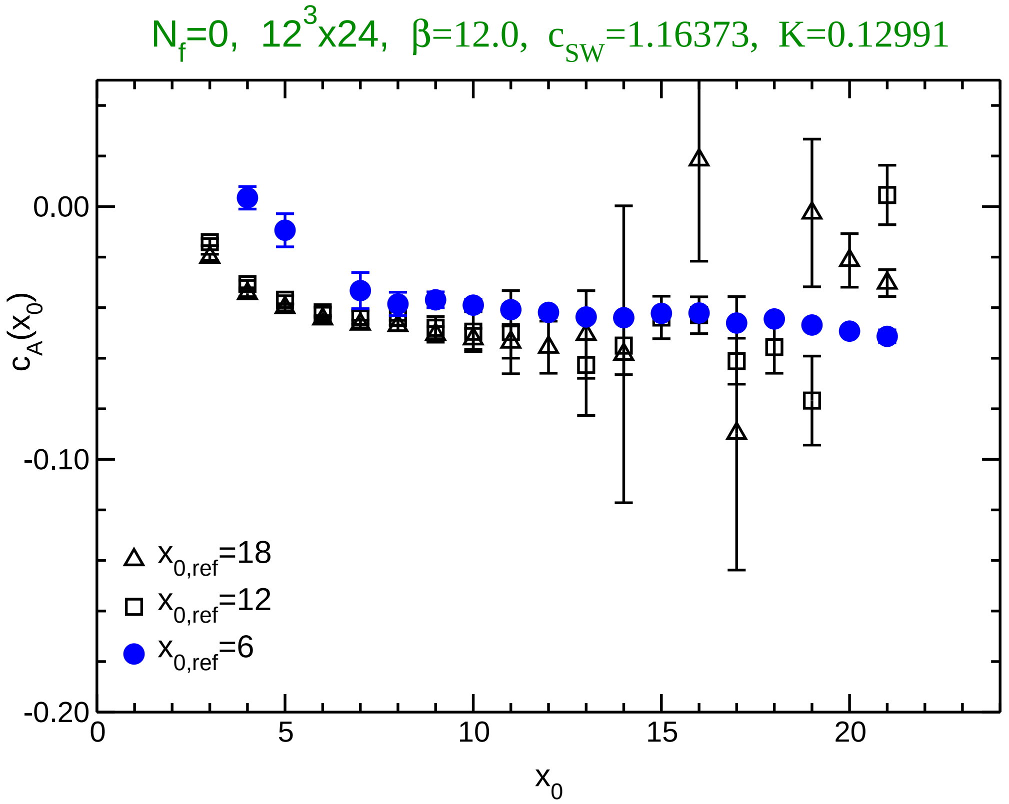
$c_A(x_0)$ at $\beta = 6.2$

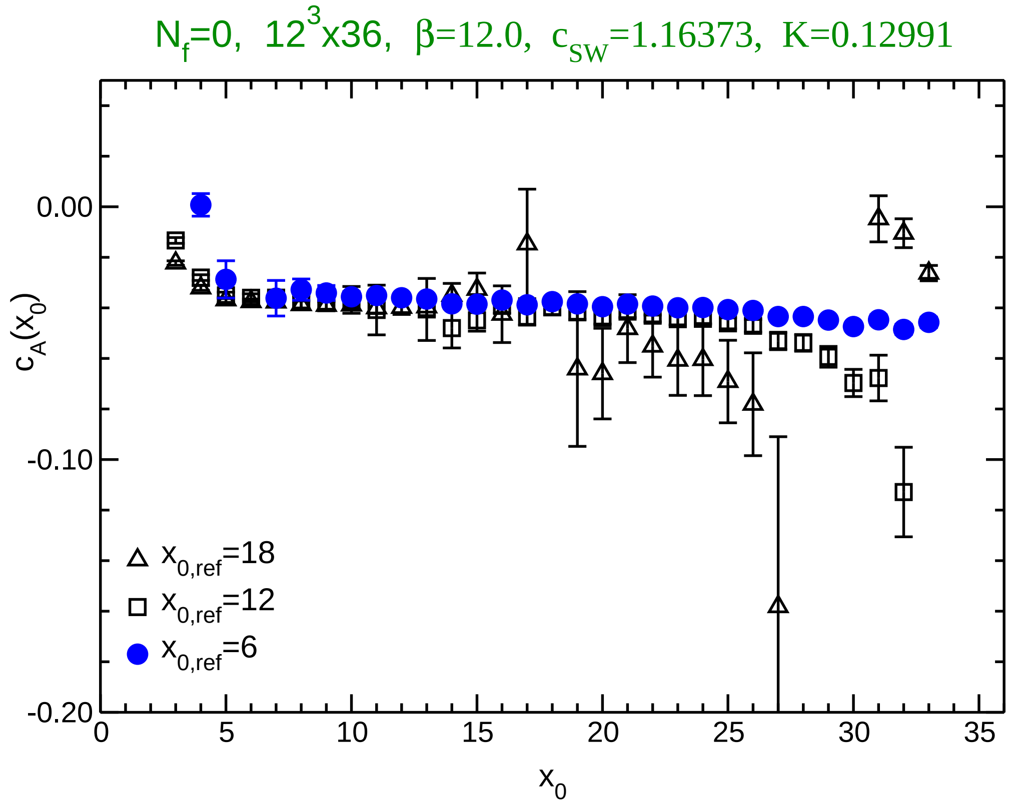




- exceptional conf appears at $\beta = 6.4$ and 6.2 $\Rightarrow$ filtered out

$T/L = 2$ or $T/L = 3$





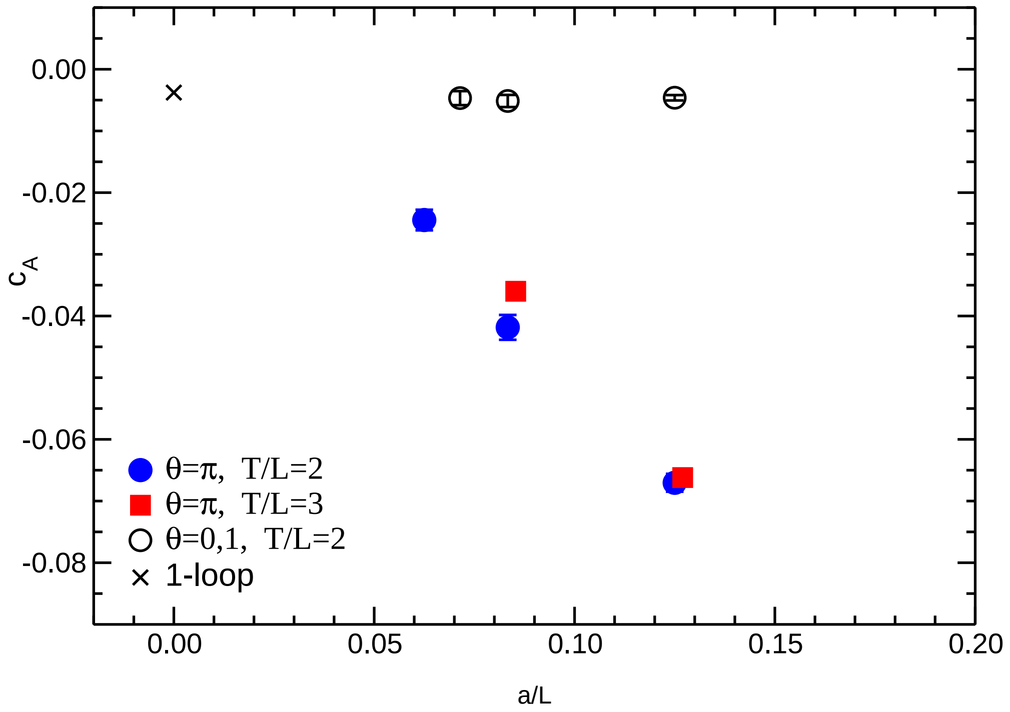
- give consistent c_A with each other

  $\Rightarrow$ take $T/L = 2$ to reduce CPU time

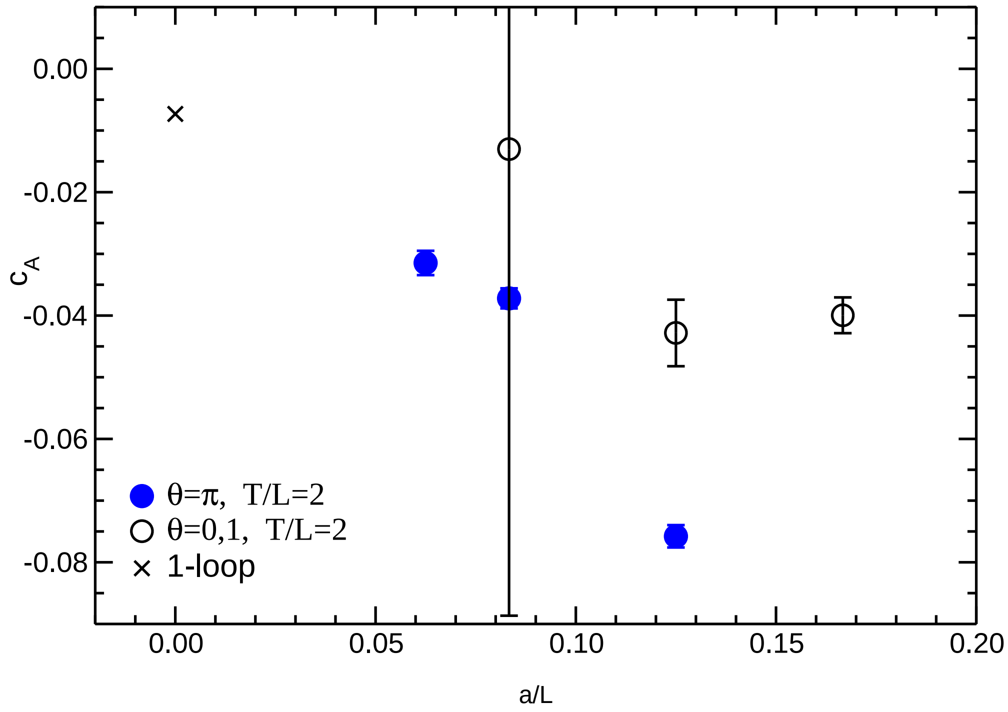
IV-3 a/L dependence of c_A

c_A vs a/L

N_f=0, β=12.0, c_{SW}=1.16373, K=0.12991

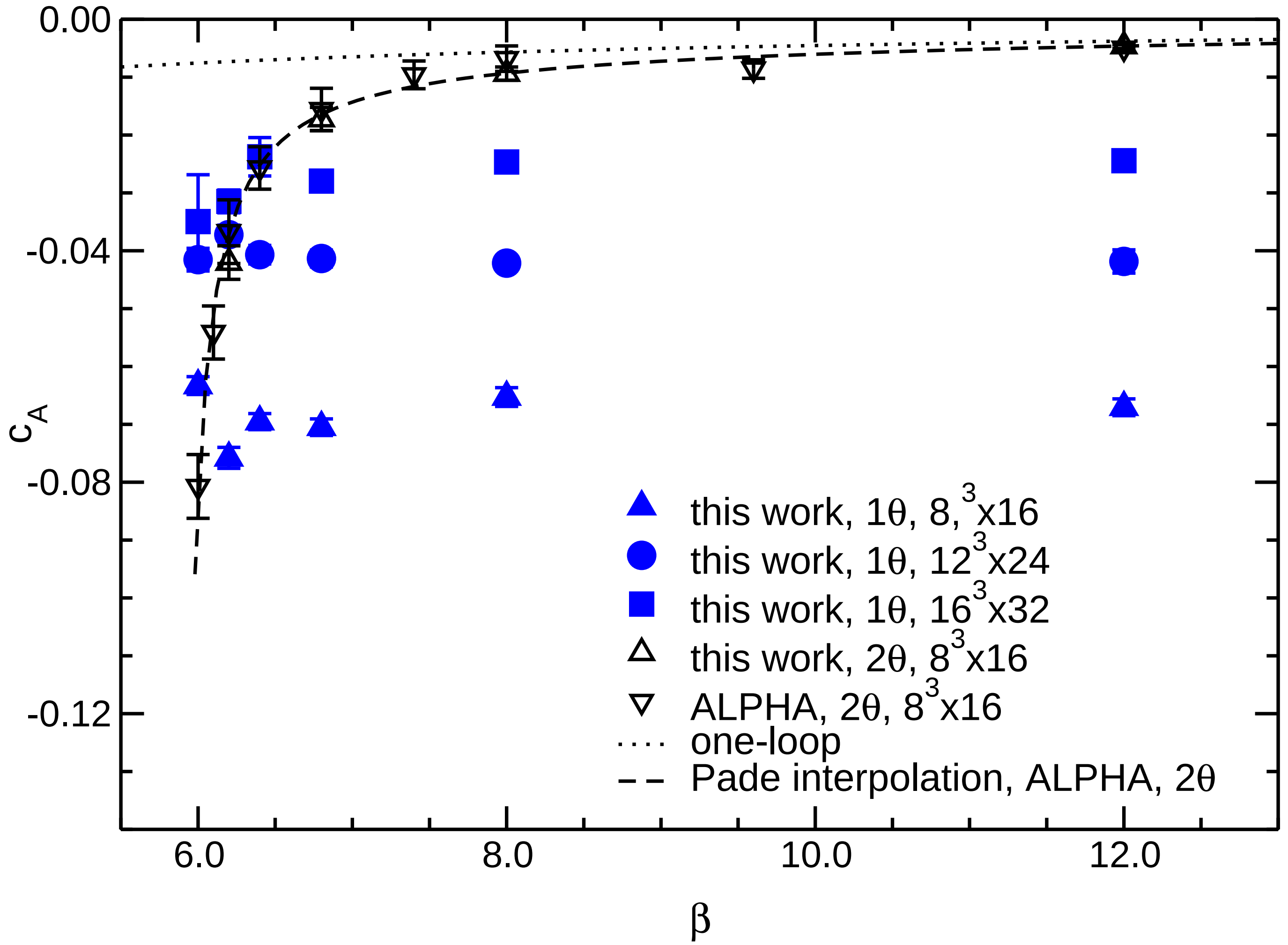


N_f=0, β=6.2, c_{SW}=1.61375, K=0.13485



- c_A has large a/L dependence both at weak and strong couplings

c_A vs β

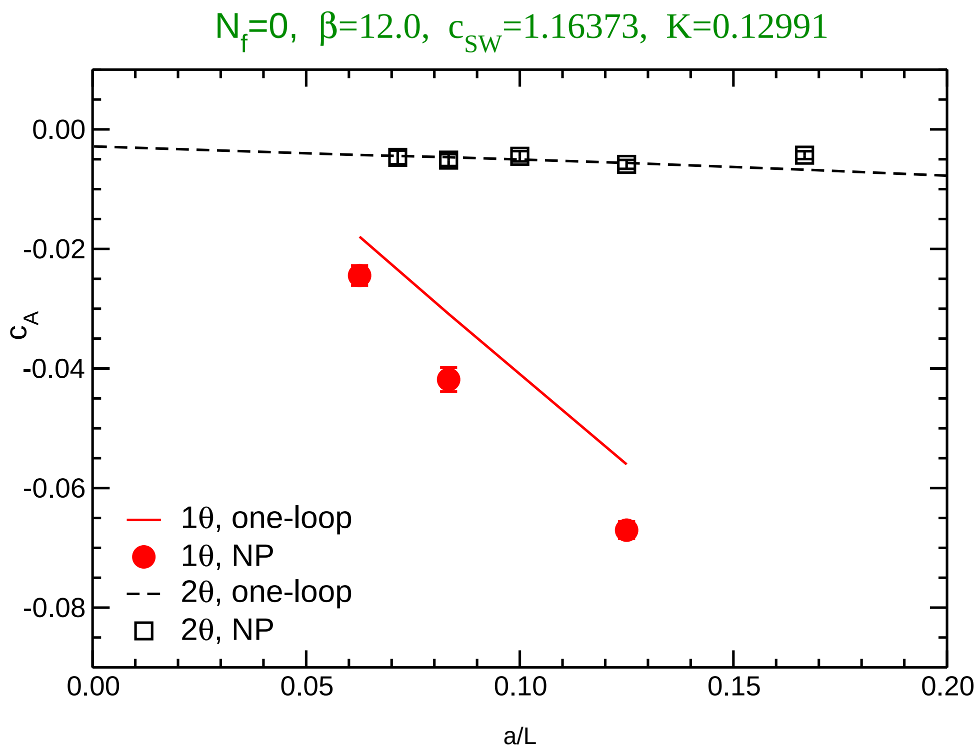


- small g^2 dependence of c_A with L/a fixed

perturbation

- at tree-level
  - $r(x_0)$, $s(x_0)$ does not depends on x_0
    $\Rightarrow$ c_A is ill-determined at tree-level

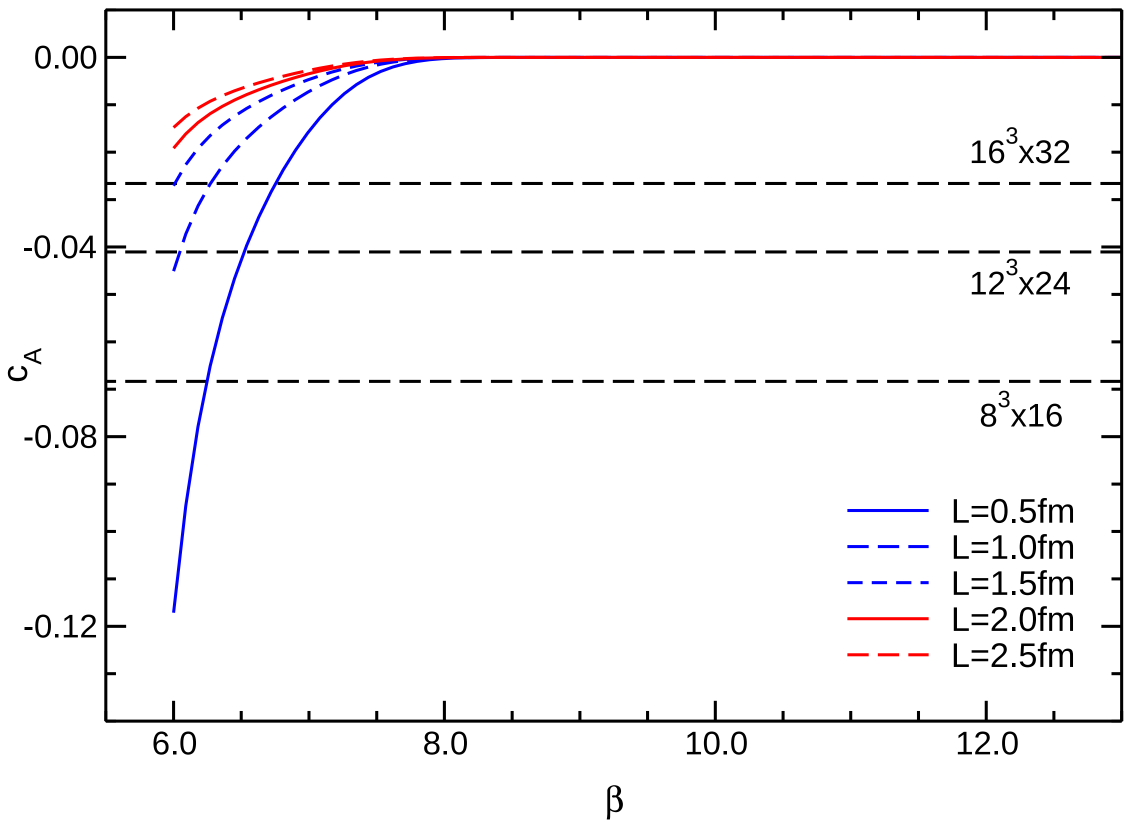
$$c_A = \frac{\Delta m^{(0)} - (r(\theta, x_0) - r(\theta, x_{0,\mathrm{ref}}))}{s(\theta, x_0) - s(\theta, x_{0,\mathrm{ref}})}$$



- at one-loop
  - $c_A \sim O(1)$
    $\Leftarrow$ one-loop calc. at g_0^2 =1 and 0.5 give the same c_A (Aoki, 2003)

on a constant physics line

- assume c_A doesn't depend on g^2
- fit c_A as a function of a/L
- use ALPHA's interpolation formula for r_0 (Guagnelli *et al.*,1998)



- $c_A \rightarrow 0$ as $g^2 \rightarrow 0$
- strongly depends on the choice of L at $\beta \sim 6$.

  $L =$0.5 – 2.0 fm

  $\Rightarrow \Delta m_q^{\mathrm{AWI}} \sim$ 15%

V. Summary

ALPHA/CP-PACS/JLQCD

- collaboration on NP computation of c_A

  this talk : a/L dependence in quenched QCD with plaq. gauge

old method

- loses sensitivity to improvement condition

  $\Rightarrow$ error of c_A rapidly blow up as $L \to \infty$

"slope criterion" method

- good sensitivity
- fixed a/L : independent of g^2 (?), strongly depends on a/L
- fixed L : large volume dependence at low β

"gap criterion" method

- a/L dependence should be studied