

# Unquenched QCD project by CP-PACS and JLQCD collaboration

**Tomomi Ishikawa (CCP, Univ. of Tsukuba)  
for CP-PACS and JLQCD collaboration**

[tomomi@rccp.tsukuba.ac.jp](mailto:tomomi@rccp.tsukuba.ac.jp)

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# Introduction

- **Light hadron spectrum**
  - Direct test of QCD at low energy scale
  - Determination of fundamental parameters  
quark masses, etc.
- **Systematic studies by CP-PACS and JLQCD collab.**
  - quenched QCD
    - ◆ plaquette gauge + Wilson quark (CP-PACS, 2003)
    - ◆ RG-improved gauge + clover quark (tad.imp.  $c_{SW}$ ) (CP-PACS, 2001)  
⇒ **systematic deviation from experiment**
  - $N_f = 2$  QCD
    - ◆ RG-improved gauge + clover quark (tad.imp.  $c_{SW}$ ) (CP-PACS, 2001)
    - ◆ plaquette gauge + clover quark (NP  $c_{SW}$ ) (JLQCD, 2003)  
⇒ **deviation is reduced**
  - **Next (and final) step :**  $N_f = 3$  QCD

## □ Computing facilities



machines	GF/ node	total		for LQCD	
		# Node	GFlops	# Node	GFlops
SR8000/F1 @KEK	12	100	1200	~ 64	~ 768
CP-PACS @CCP, U.Tsukuba	0.3	2048	614	2048	614
SR8000/G1 @CCP, U.Tsukuba	14.4	12	173	12	173
VPP5000 @SIPC, U.Tsukuba	9.6	80	768	~ 24	~ 230
Earth Simulator @JAMSTEC	64	640	40960	~ 10	~ 640

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# Simulation parameters

- with degenerate up and down quarks and strange quark
- Algorithm
  - dynamical u, d, s quarks    $\longleftrightarrow$  odd flavor algorithm is needed.
    - ◆ HMC for ud quarks
    - ◆ **PHMC** for strange quark
- Lattice action
  - gauge : RG improved action
  - quark : non-perturbatively  $\mathcal{O}(a)$  improved Wilson action
- $\beta = 1.9$ ,  $c_{SW} = 1.715$ , ( $a^{-1} \sim 2GeV$ )
- Lattice size :  $20^3 \times 40$  ( $La \simeq 2.0fm$ )  
small for baryons    $\longrightarrow$  concentrate on meson sector

## ◻ $\kappa$ parameters

### ▫ 5 ud quark masses

$$m_{PS,LL}/m_{V,LL} \sim 0.62 - 0.77$$

$$(m_\pi/m_\rho = 0.18)$$

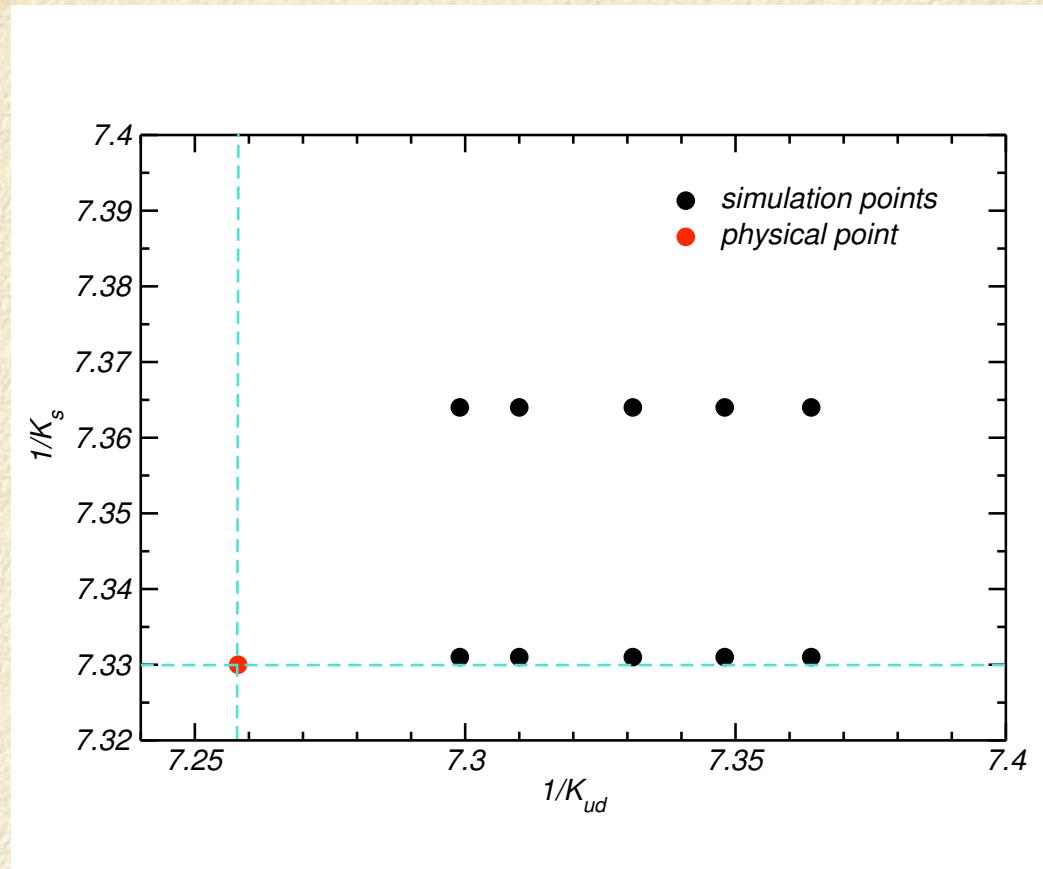
### ▫ 2 strange quark masses

$$m_{PS,SS}/m_{V,SS} \sim 0.71, 0.77$$

$$(m_{\eta_s}/m_\phi = 0.68 : \text{ChPT})$$

## ◻ Statics

- 5000 traj at each simulation point
- measure hadron masses every 10 trajectories
- statistical error  $\leftarrow$  jack-knife with bin size of 100 traj



# Analysis

- chiral extrapolation
  - fitting functions (VWI)

$$\begin{aligned} m_{PS}(K_{ud}, K_s; K_{val,1}, K_{val,2})^2 \\ = B_S^{PS}(2m_{ud}^{VWI} + m_s^{VWI}) + B_V^{PS}(m_{val,1}^{VWI} + m_{val,2}^{VWI}) \\ + D_{VS}^{PS}(2m_{ud}^{VWI} + m_s^{VWI})(m_{val,1}^{VWI} + m_{val,2}^{VWI}) \\ + C_V^{PS}((m_{val,1}^{VWI})^2 + (m_{val,2}^{VWI})^2) + 2D_{VV}^{PS}m_{val,1}^{VWI}m_{val,2}^{VWI} \end{aligned}$$

$$\begin{aligned} m_V(K_{ud}, K_s; K_{val,1}, K_{val,2}) \\ = A^{VK} + B_S^{VK}(2m_{ud}^{VWI} + m_s^{VWI}) + B_V^{VK}(m_{val,1}^{VWI} + m_{val,2}^{VWI}) \\ + D_{VS}^{VK}(2m_{ud}^{VWI} + m_s^{VWI})(m_{val,1}^{VWI} + m_{val,2}^{VWI}) \\ + C_V^{VK}((m_{val,1}^{VWI})^2 + (m_{val,2}^{VWI})^2) \end{aligned}$$

$$m_{ud}^{VWI} = \frac{1}{2} \left( \frac{1}{K_{ud}} - \frac{1}{K_c} \right), \quad m_s^{VWI} = \frac{1}{2} \left( \frac{1}{K_s} - \frac{1}{K_c} \right), \quad m_{val,i}^{VWI} = \frac{1}{2} \left( \frac{1}{K_{val,i}} - \frac{1}{K_c} \right)$$

## □ fitting functions (AWI)

$$\begin{aligned} m_{PS}(K_{ud}, K_s; K_{val,1}, K_{val,2})^2 &= B_V^{PS}(m_{val,1}^{AWI} + m_{val,2}^{AWI}) \\ &\quad + D_{VS}^{PS}(2m_{ud}^{AWI} + m_s^{AWI})(m_{val,1}^{AWI} + m_{val,2}^{AWI}) \\ &\quad + C_V^{PS}((m_{val,1}^{AWI})^2 + (m_{val,2}^{AWI})^2) \end{aligned}$$

$$\begin{aligned} m_V(K_{ud}, K_s; K_{val,1}, K_{val,2}) &= A^{VK} + B_V^{VK}(2m_{ud}^{AWI} + m_s^{AWI}) \\ &\quad + D_{VS}^{VK}(2m_{ud}^{AWI} + m_s^{AWI})(m_{val,1}^{AWI} + m_{val,2}^{AWI}) \\ &\quad + C_V^{VK}((m_{val,1}^{AWI})^2 + (m_{val,2}^{AWI})^2) \end{aligned}$$

$$m^{AWI} = \frac{\langle 0 | \nabla_4 A_4 | PS \rangle}{2 \langle 0 | P | PS \rangle}$$

## □ Input to fix $m_{ud}$ and $m_s$

### □ K-input

$$\frac{m_{PS,LL}(K_{ud}, K_s)}{m_{V,LL}(K_{ud}, K_s)} = \frac{m_\pi}{m_\rho}, \quad \frac{m_{PS,LS}(K_{ud}, K_s)}{m_{V,LL}(K_{ud}, K_s)} = \frac{m_K}{m_\rho}$$

### □ $\phi$ -input

$$\frac{m_{PS,LL}(K_{ud}, K_s)}{m_{V,LL}(K_{ud}, K_s)} = \frac{m_\pi}{m_\rho}, \quad \frac{m_{V,SS}(K_{ud}, K_s)}{m_{V,LL}(K_{ud}, K_s)} = \frac{m_\phi}{m_\rho}$$

## □ Input to fix $a^{-1} \frac{\text{---}}{m_\rho}$

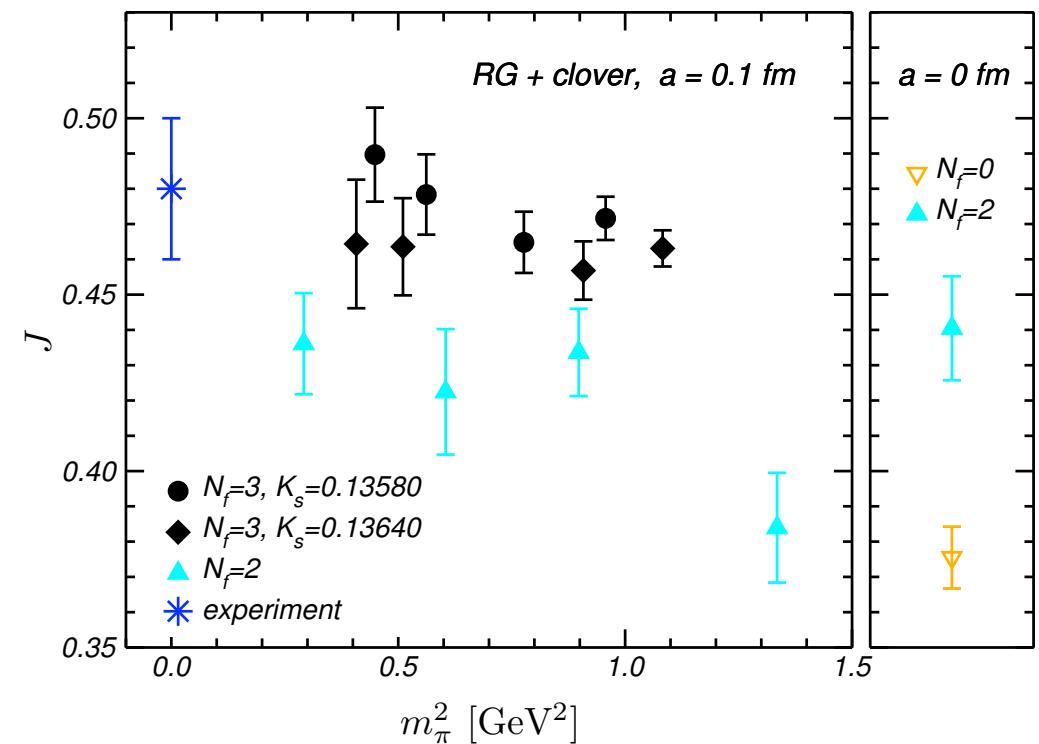
$$a^{-1} = \begin{cases} 1.98(4) \text{ GeV} & (\text{K-input}) \\ 1.98(4) \text{ GeV} & (\phi\text{-input}) \end{cases}$$

# Meson spectrum

## □ J parameter

$$J = m_V \frac{dm_V}{dm_{PS}^2} \quad \left( \text{at} \quad \frac{m_{PS}}{m_V} = \frac{m_K}{m_{K^*}} \right) \simeq m_{K^*} \frac{m_{K^*} - m_\rho}{m_K^2 - m_\pi^2} = 0.48(2)$$

- quenched  
smaller than experiment
- 2-flavor  
deviation is reduced
- 3-flavor  
consistent with experiment

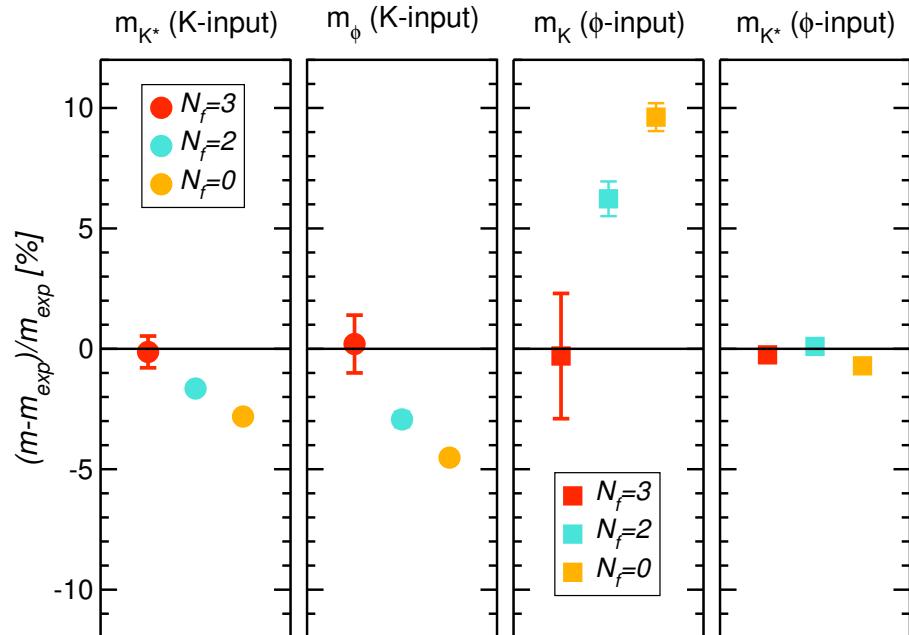
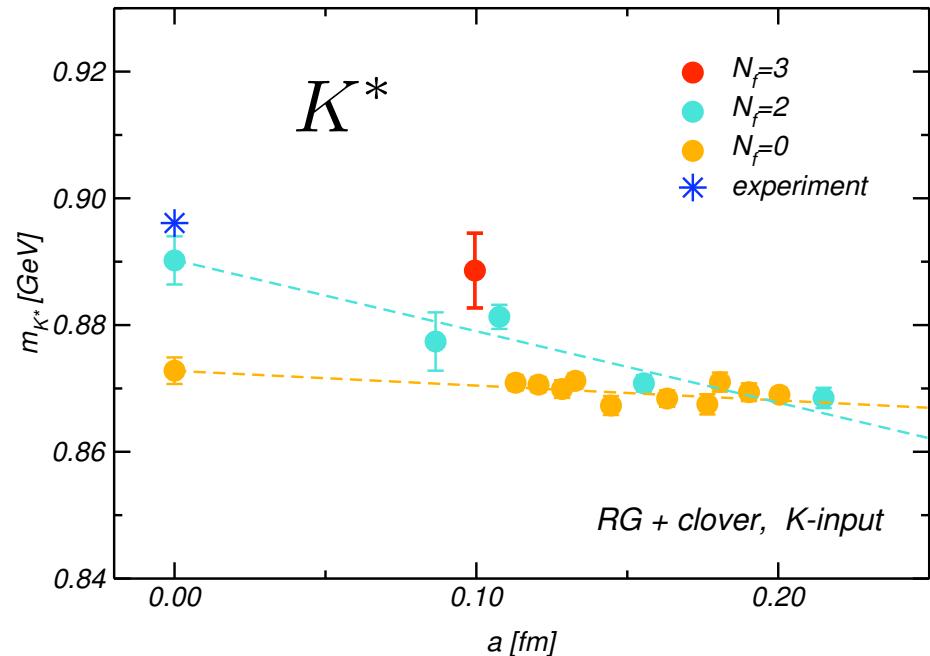


## Light meson masses

- At  $a \sim 0.1$  fm
  - ◆ consistent with experiment
  - ◆ K-input and  $\phi$ -input agree
- NP  $c_{SW}$ 
  - ◆ small scaling violation (?)



The consistency with experiment maintains even in the continuum limit.



# Quark masses

## □ VWI quark masses

- **define**  $K_{c,L}(K_s)$        $\Leftarrow m_{PS,LL}(K_{c,L}, K_s) = 0$
- $K_{c,L}(K_s) > K_c$  (**due to lack of chiral symmetry**)  
     $\Rightarrow K_{ud,phys} > K_c \Rightarrow$  **negative**  $m_{ud}^{VWI}$
- **define**  $m_q^{VWI}$  **as**  
$$m_q^{VWI} = \frac{1}{2} \left( \frac{1}{K_q} - \frac{1}{K_{c,L}(K_{s,phys})} \right) > 0$$

## □ AWI quark masses

- no such problems as in VWI quark masses

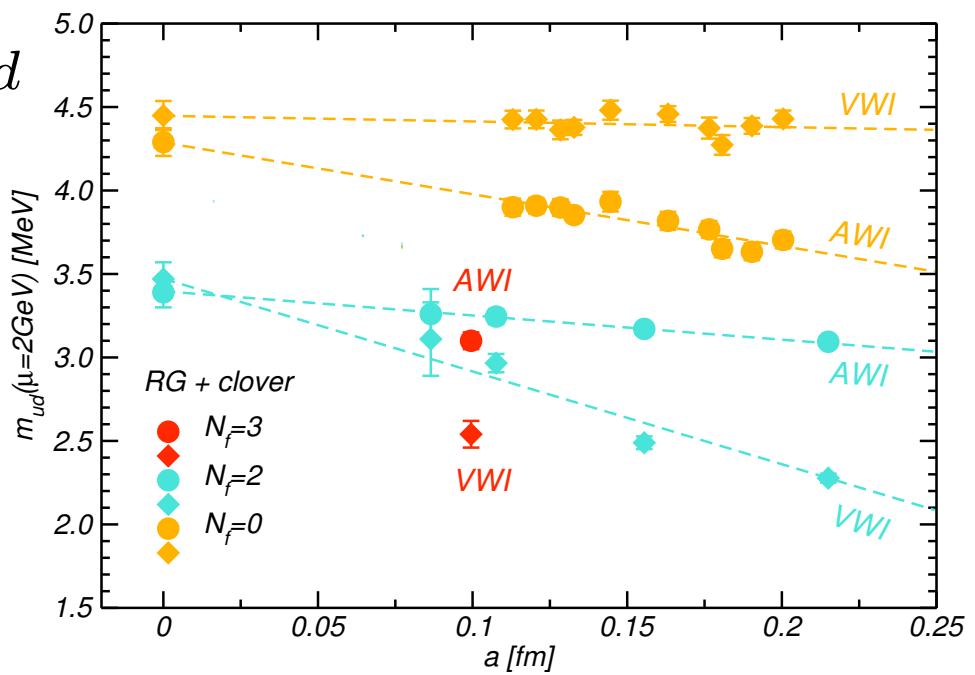
## □ renormalization

- MF-improved 1-loop matching with  $\overline{\text{MS}}$  at  $\mu = a^{-1}$
- 4-loop running to  $\mu = 2 \text{ GeV}$

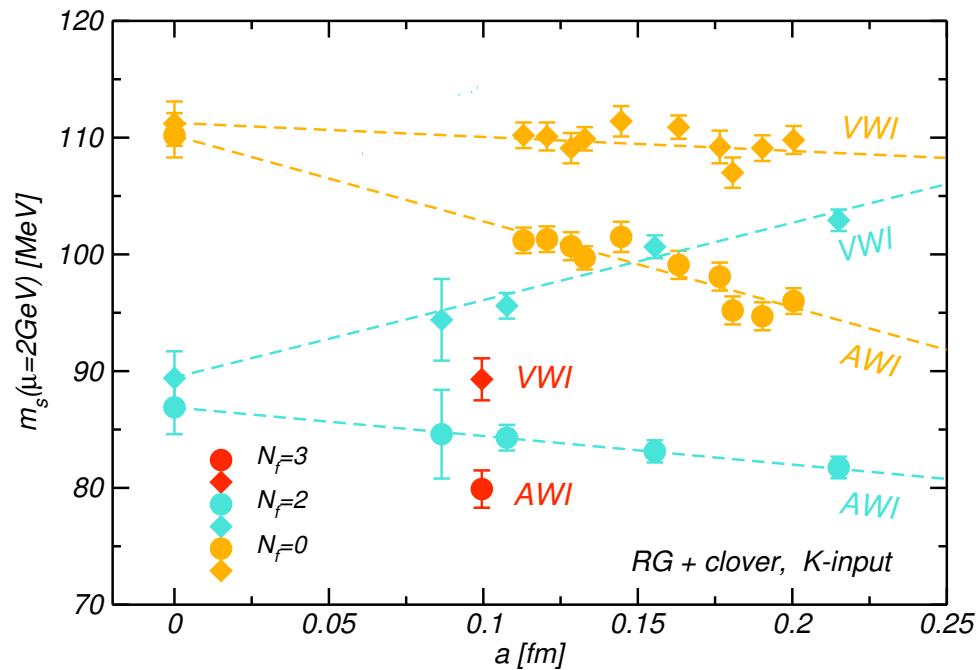
## Results

□ significant deviation  
between  
AWI and VWI masses

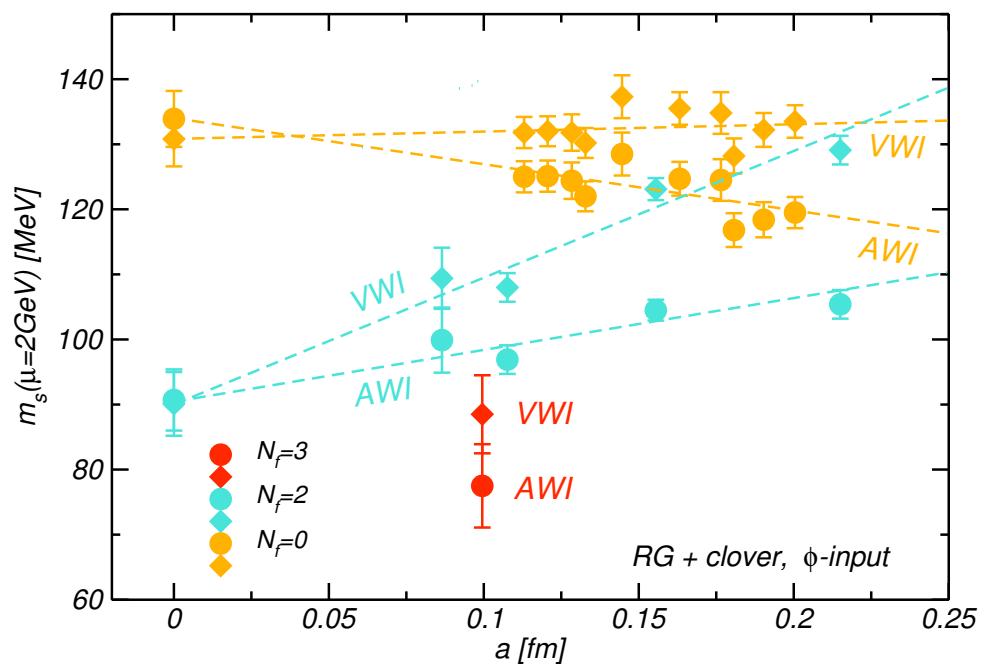
$m_{ud}$



$m_s(K - \text{input})$



$m_s(\phi - \text{input})$



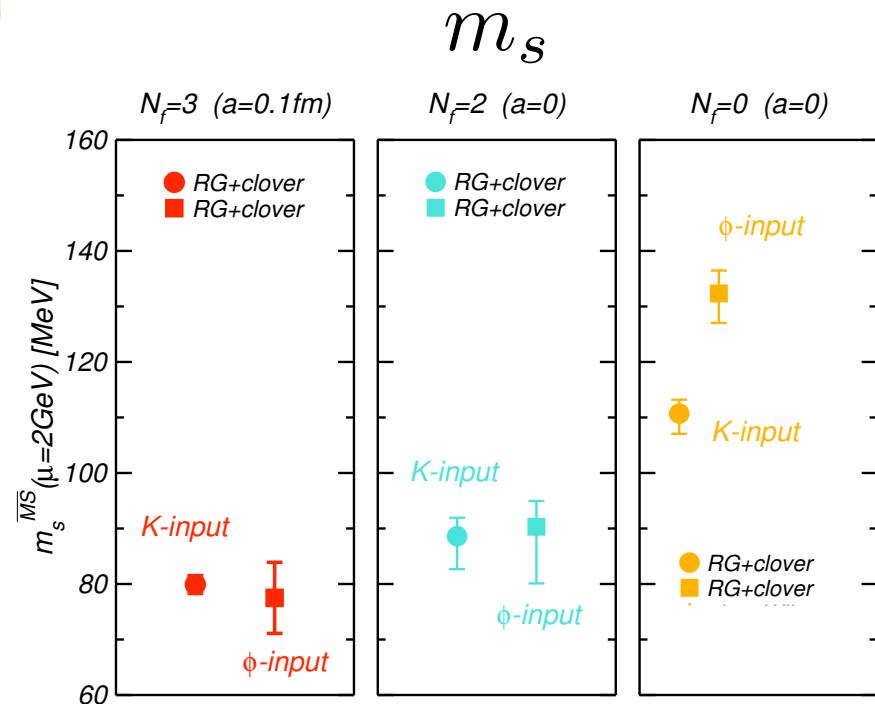
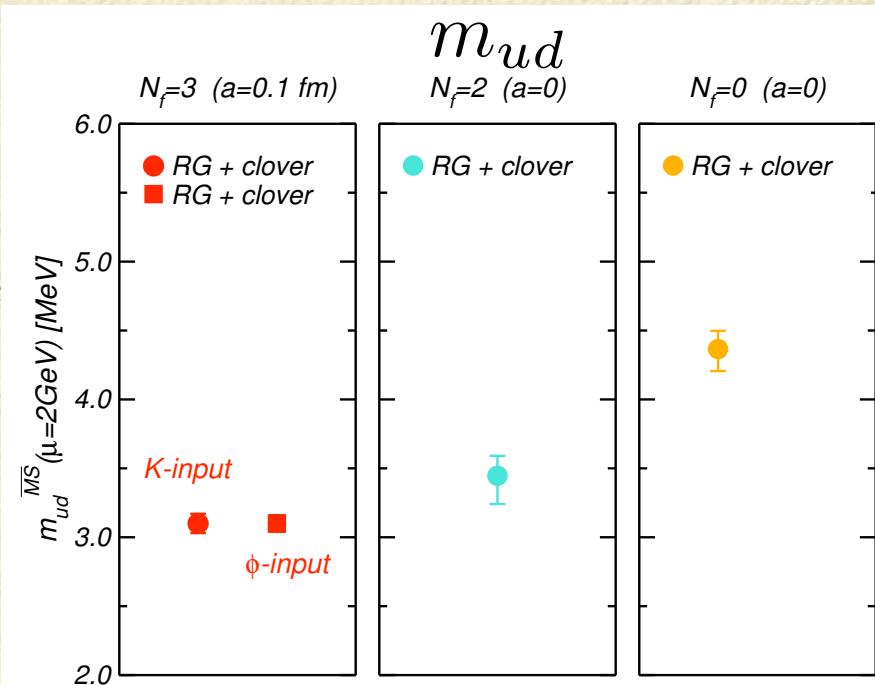
**Assuming small scaling  
violation in  $m_q^{AWI}$ ,  
in the  $\overline{\text{MS}}$  scheme at  $\mu = 2 \text{ GeV}$**

$$m_{ud} = 3.10(7) \text{ MeV}$$

$$m_s = 78.7(3.3) \text{ MeV}$$

$$m_s/m_{ud} = 25.4(1.2)$$

- $m_{ud}, m_s$  :  
**10-15% smaller than in  $N_f = 2$**
- $m_s/m_{ud}$  :  
**consistent with 1-loop ChPT,  
24.4(1.5)**



# Finite size effect

## □ Comparison with $16^3 \times 32$ result

- $16^3 \times 32$  lattice,  $\beta = 1.9$ , 3000 traj (T.Kaneko et al., Lat 03)
- results on  $20^3 \times 40$  lattice

$m_{ud}$  : 10% larger than on  $16^3 \times 32$  lattice

$m_s$  : 4% larger than on  $16^3 \times 32$  lattice

	$20^3 \times 40$	$16^3 \times 32$
$m_{ud}$	3.10(7)	2.89(6)
$m_s$	78.7(3.3)	75.6(3.4)
$m_s/m_{ud}$	25.4(1.2)	26.2(1.0)

More analysis of the finite size effect is needed.

# Conclusions and future plans

- $N_f = 3$  QCD project of CP-PACS+JLQCD
  - $20^3 \times 40$  lattice,  $a \sim 0.1$  fm
  - RG-improved gauge action + NP improved clover quark
- Light meson spectrum
  - consistent with experiment already at  $a \sim 0.1$  fm
- Quark mass
  - 10-15% smaller than in  $N_f = 2$
- Next
  - other lattice spacing ( ↪ investigation of scaling violation )
  - finite size effect