

D meson spectroscopy with the domain-wall fermions

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for the RBC collaboration

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Introduction

- Domain-wall fermions (DWF) :
 - better control of chiral extrapolation
 - negligible $O(a)$ error
- In simulations of heavy-light system, major uncertainties come from
 - chiral extrapolations
 - ⇒ DWF for light quarks
 - scaling violation [$O(am_Q) \not\ll 1$]
 - ⇒ DWF for heavy quarks



Using DWF to describe both heavy and light quarks is promising.

Massive DWF?

RBC, in preparation?

There exists $am_{f,\max}$ up to which

- the behavior of the wave function in 5th dimension does not differ much from that for light quarks,
- am_{res} is still fairly small compared to $O(a\Lambda_{\text{QCD}})$ and $O(a^2m_f^2)$.

No peculiar behavior is observed in hadronic observables, *e.g.* m_{PS} for $am_f < 1$.

The value of $am_{f,\max}$ depends on the lattice parameters.

- DBW2 gauge action
- weaker coupling ($1/a \sim 3$ GeV)
- $L_s = 10$

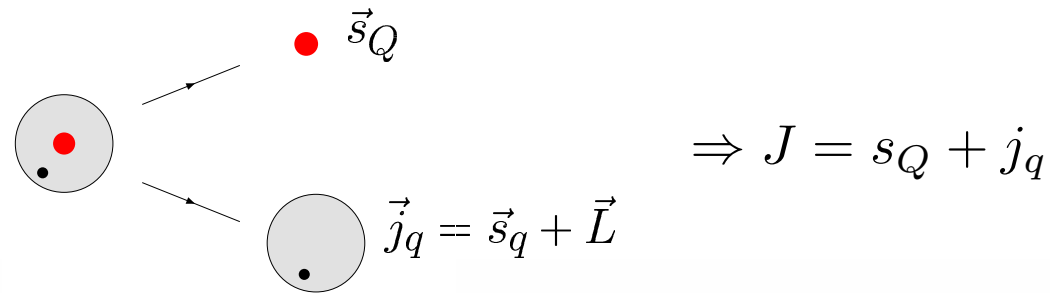
\Rightarrow

$$am_{f,\max} \sim 0.4\text{--}0.5,$$
$$am_c \sim 0.35$$

Allows to simulate charm with DWF.

Classification of HL mesons

Heavy quark spin decouples from the other.



L	j_q	J^P			
S	1/2	{	0^-	pseudoscalar	$D_{(s)}$
			1^-	vector	$D_{(s)}^*$
P	1/2	{	0^+	scalar	$D_{(s)0}^*$
			1^+	pseudovector	$D'_{(s)1}$
	3/2	{	1^+	pseudovector	$D_{(s)1}$
			2^+	tensor	$D_{(s)2}^*$

Hereafter concentrate on 0^- , 1^- , 0^+ , 1^+ .

Experiments

BABAR, BELLE, CLEO, FOCUS

charm-strange mesons

J^P	Mass [MeV]
$D_s (0^-)$	1968.5(6)
$D_s^* (1^-)$	2112.4(7)
$D_{s0}^* (0^+)$	2317.0(4)
$D'_{s1} (1^+)$	2458.2(1.0)

\Rightarrow

Splittings between different parities: Δ_{qJ}

q	$0^+ - 0^-$ [MeV]	$1^+ - 1^-$ [MeV]
s	348.4(9)	345.9(1.2)
u, d	444(36)	420(36)

- $\Delta_{q0} \approx \Delta_{q1} \Rightarrow$ insensitive to J
 \Leftrightarrow universal Δ_{hf}
- depends on m_q , and $\Delta_{udJ} > \Delta_{sJ}$

charm-nonstrange mesons

J^P	Mass [MeV]
$D (0^-)$	1869.3(5)
$D^* (1^-)$	2010.0(5)
$D_0^* (0^+)$	2308(17)(15)(28)
$D'_1 (1^+)$	2427(26)(20)(15)

Hyperfine splittings: Δ_{hf}

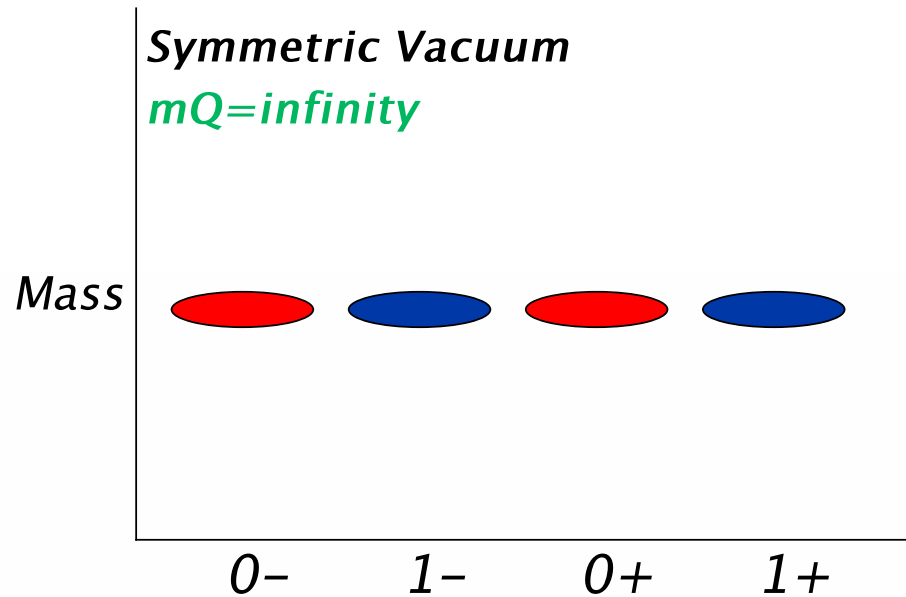
q	$1^- - 0^-$ [MeV]	$1^+ - 0^+$ [MeV]
s	143.8(4)	141
u, d	140.64(10)	~ 120

Δ_{hf} looks independent of m_l and parity.

† Some of them need confirmation.

Parity Doubling Model

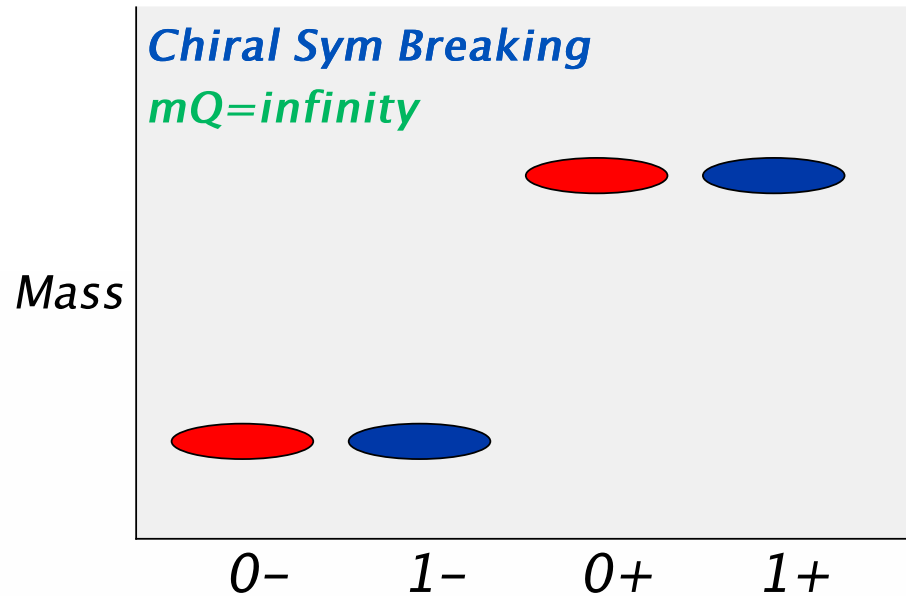
Bardeen and Hill (1994), Bardeen,Eichten,Hill(2003)



- Start with hypothetical, symmetric vacuum, and impose HQ and χ Sym
 \Rightarrow Four degenerate heavy-light mesons (any splittings = 0)

Parity Doubling Model

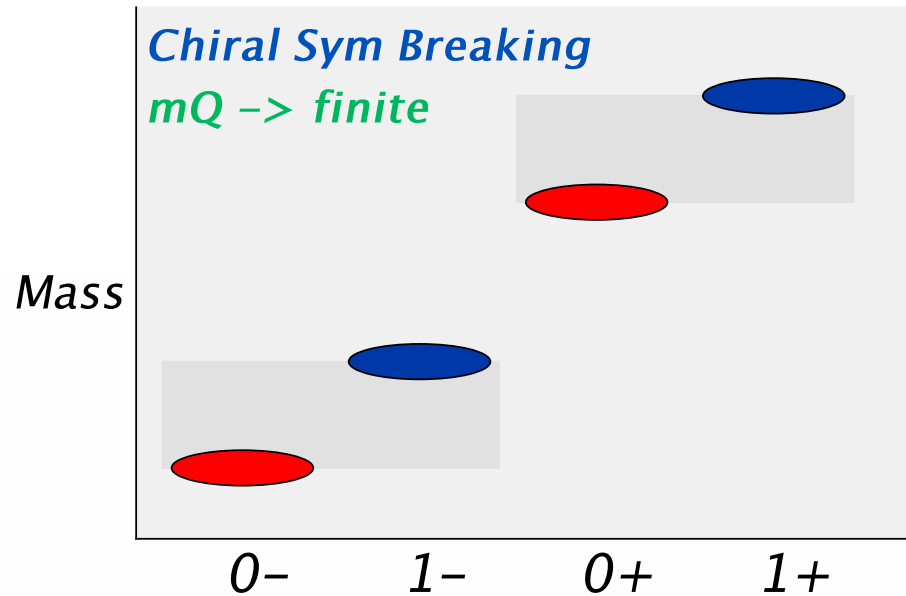
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 \Rightarrow Splitting between + and -
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 - Spontaneous χ sym. breaking $\rightarrow \langle \sigma \rangle$,
 \Rightarrow Splitting between + and - (Goldberger-Treiman relation $\Delta M = g_\pi f_\pi$)
 - $m_Q \rightarrow \text{finite}$
 \Rightarrow Hyperfine Splitting
-
- “ $\Delta_{q0} \approx \Delta_{q1}$ ” \rightarrow understandable
 - does not tell much about $\Delta_s < \Delta_{u,d}$.

Previous Lattice calculations

- In previous works,
 - Heavy : Clover/NRQCD/Static
 - Light : Wilson/Clover

Most results $> \Delta_{\text{exp}} \sim 350 \text{ MeV}$

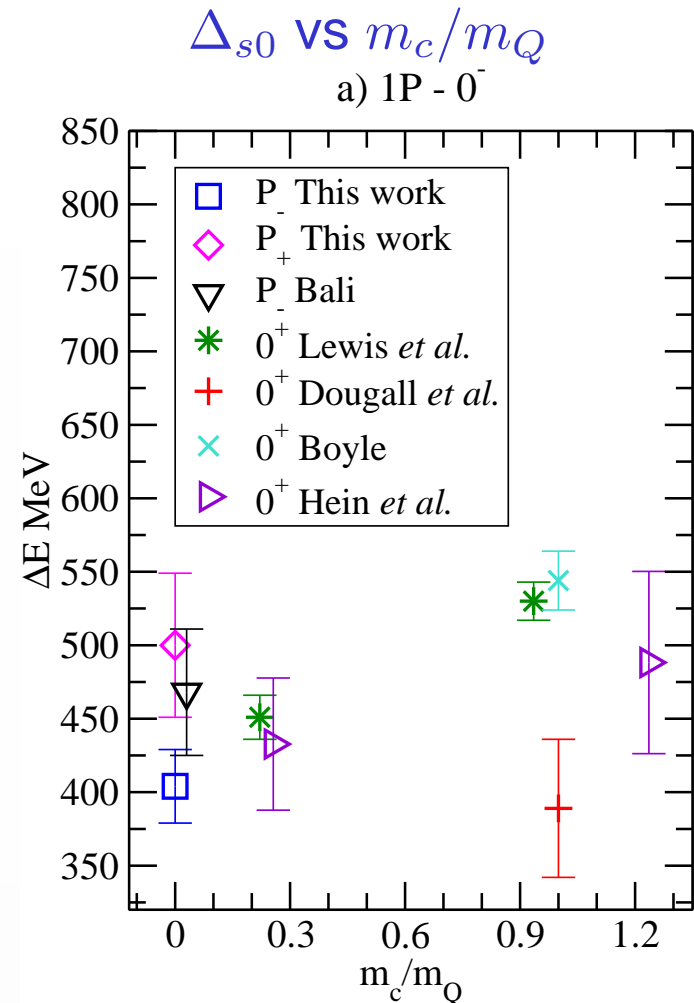
† Most of potential models as well

- No systematic study on the light quark mass dependence

When using DWF, it is interesting to see

- † what is the absolute value of the splitting,
- † heavy quark mass dependence below $\sim m_c$,
- † light quark mass dependence.

UKQCD (Green *et al.*), Phys.Rev.D69(2004)



Previous Lattice calculations (2)

How about the degeneracy between

Δ_{s0} and Δ_{s1} ?

According to UKQCD (Clover/Clover),
the degeneracy looks good at fine lattice.

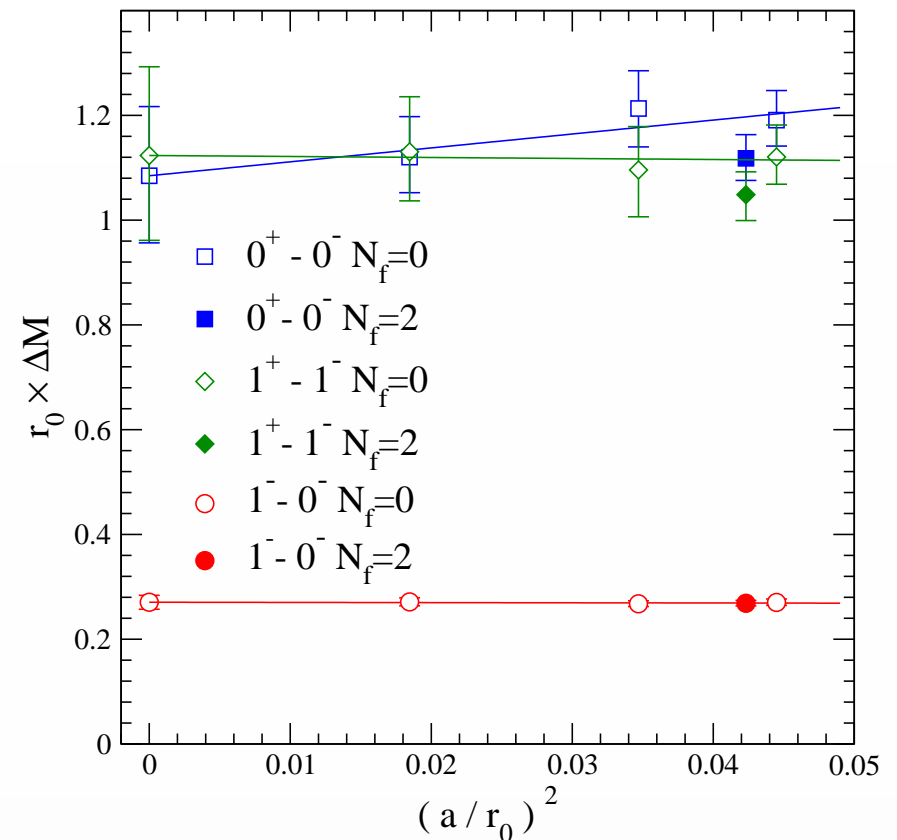
When using DWF, it is interesting to see

† the degeneracy between

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UKQCD (Dougall *et al.*), Phys.Lett.B669(2003)

Δ_{s0} and Δ_{s1} vs a^2/r_0



Simulation parameters

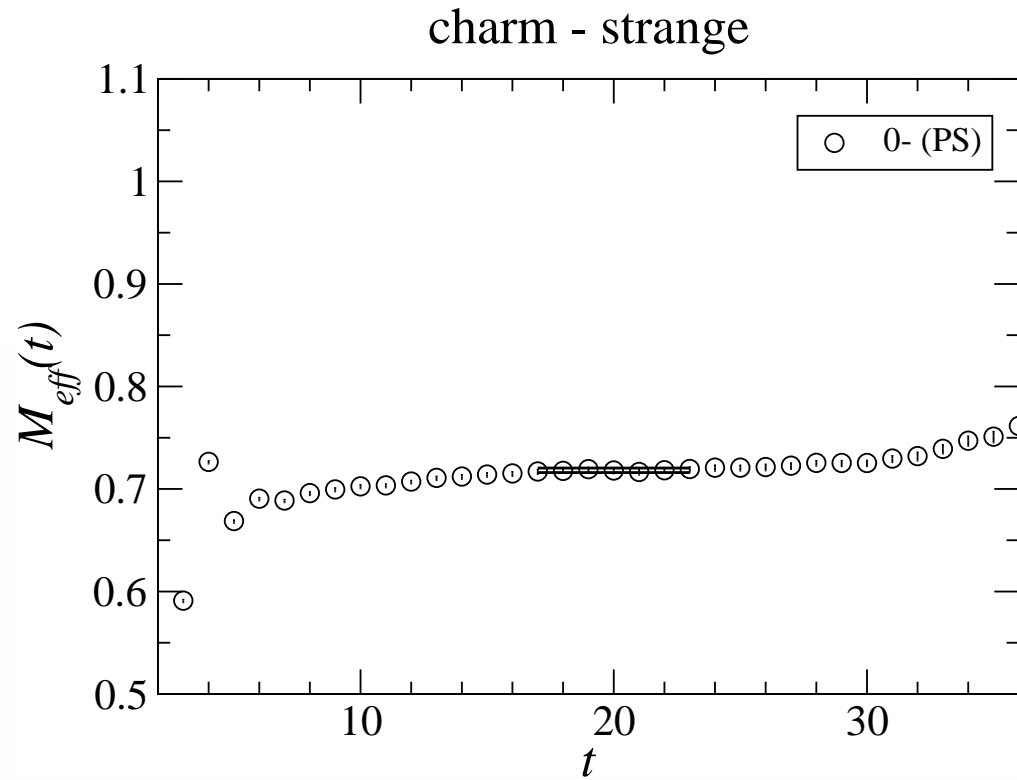
gauge action	quenched DBW2
quark action	DWF
β	1.22
size	$24^3 \times 48 \times 10$ (~ 1.6 fm) ³
M_5	1.65
m_{light}	0.008, 0.016, 0.024, 0.032, 0.040
m_{heavy}	0.1, 0.2, 0.3, 0.4, 0.5
# of config.	64

- $1/a = 2.91(5)$ GeV with M_ρ input
($1/a = 3.11(2)$ GeV for r_0 input)
- $\{M_K, M_{D_s}\} \rightarrow$ inputs to set $\{m_s, m_c\}$

$m_{\text{strange}} \approx 0.032$, $m_{\text{charm}} \approx 0.35$ are covered.

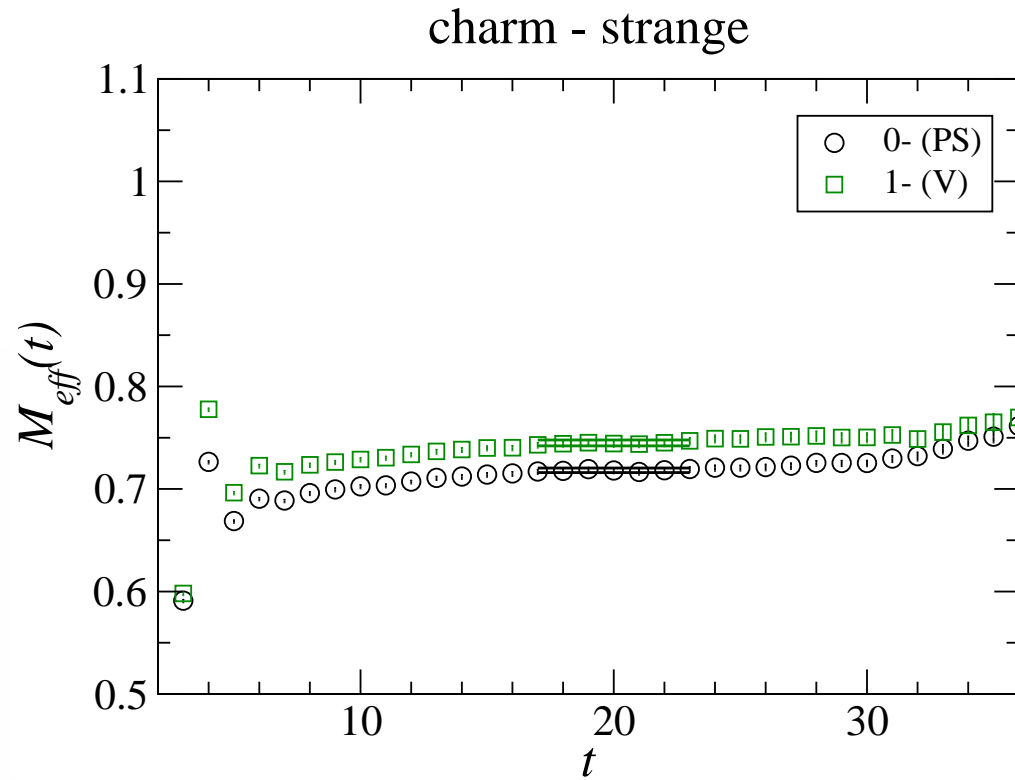
The calculation is in progress.

Effective mass plots



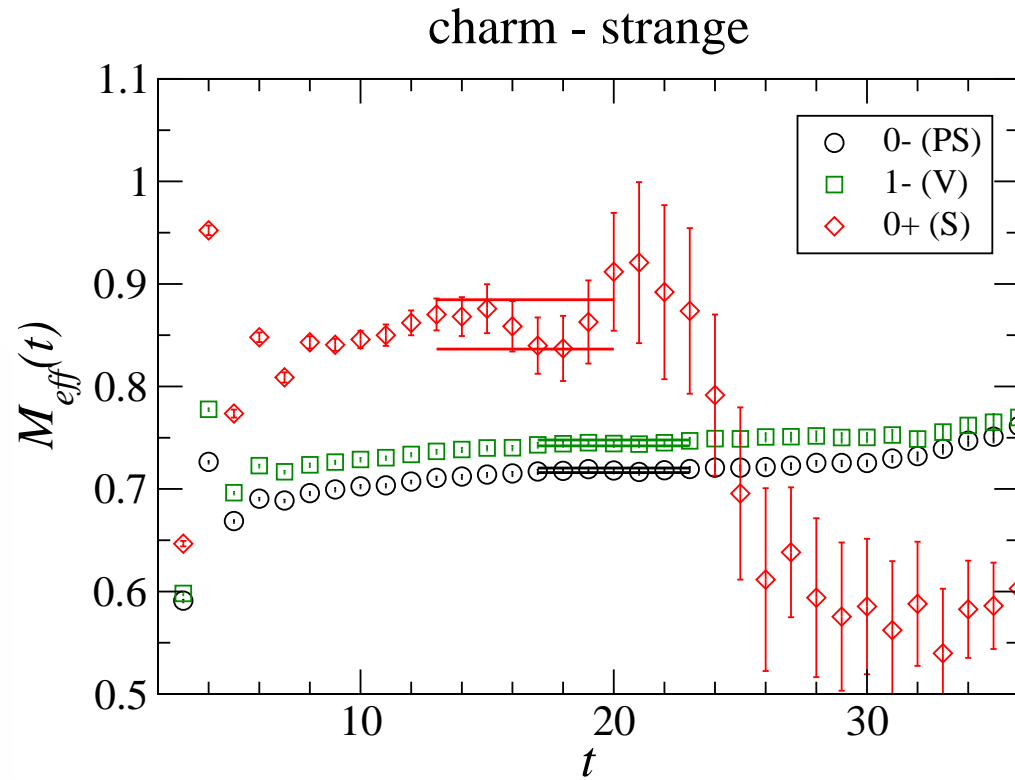
- Pseudo-scalar

Effective mass plots



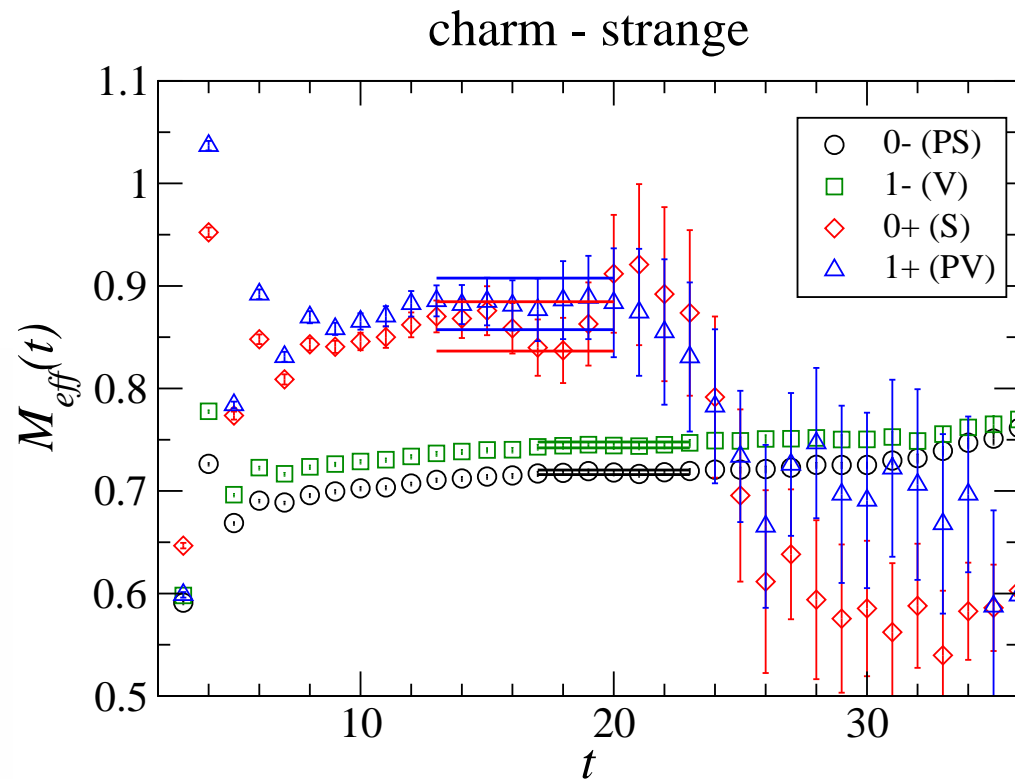
- Pseudo-scalar
- Vector

Effective mass plots



- Pseudo-scalar
- Vector
- Scalar

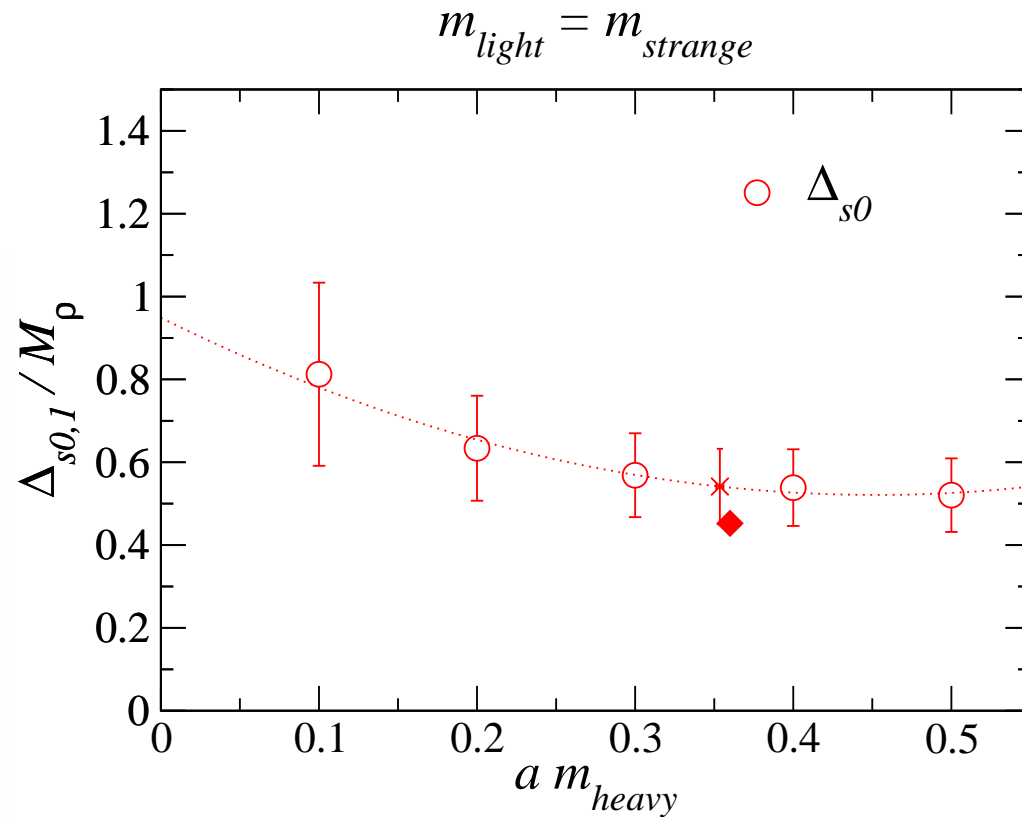
Effective mass plots



- Pseudo-scalar
- Vector
- Scalar
- Pseudo-vector

Reasonable plateaus for all mesons allows us to extract masses.

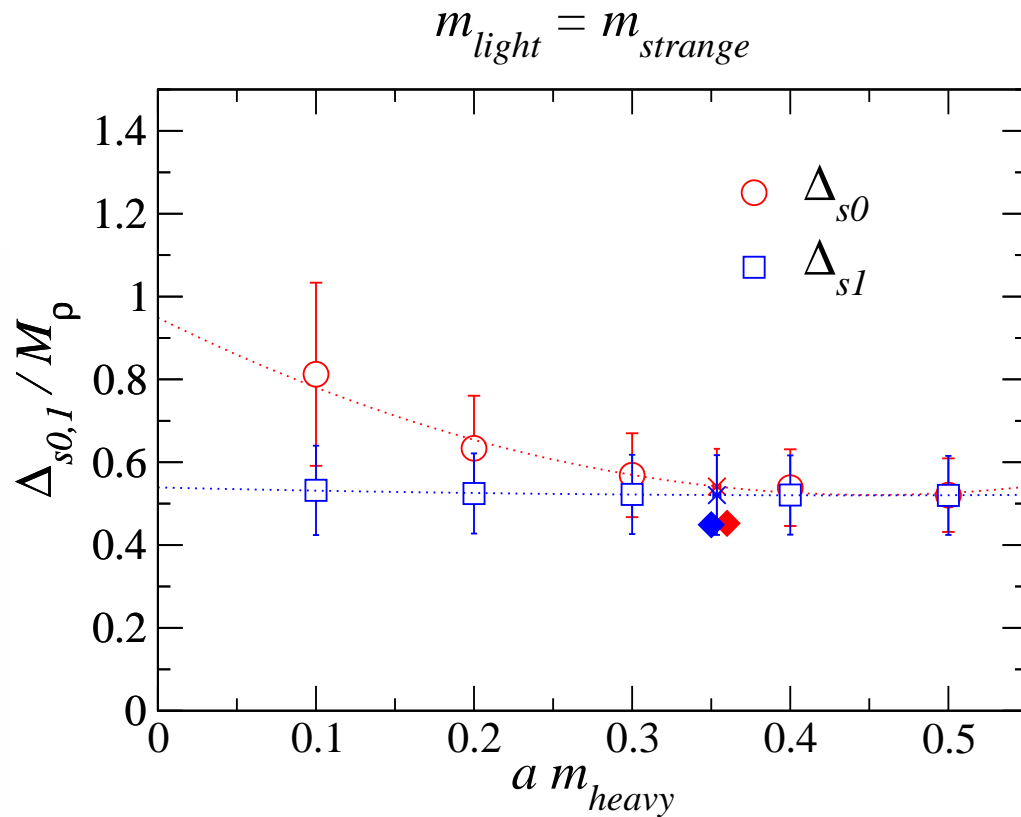
Results for D_{sJ}



- $\Delta_{s0} = M_{0+} - M_{0-}$
increase as $m_{heavy} \rightarrow 0$?
constant for $m_{heavy} \gtrsim m_c$

$$\Delta_{s0} = 417(70) \text{ MeV}$$

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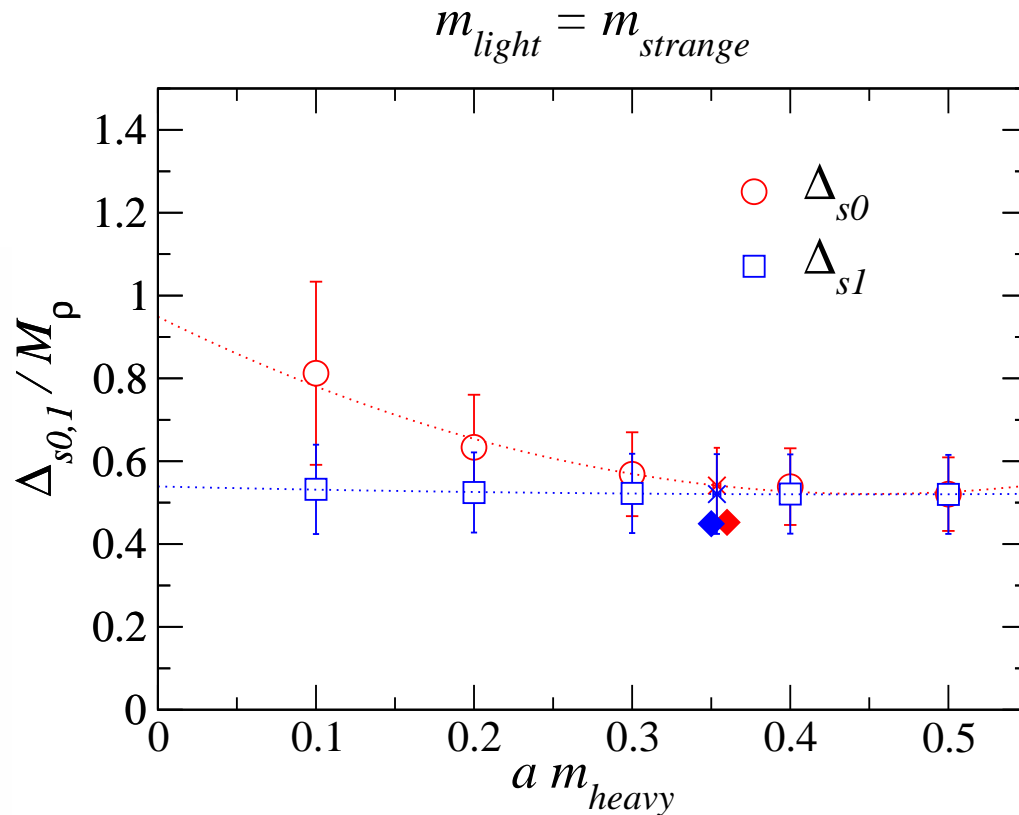
$$\Delta_{s0} = 417(70) \text{ MeV}$$

- $\Delta_{s1} = M_{1+} - M_{1-}$
constant for all m_{heavy}

$$\Delta_{s1} = 401(74) \text{ MeV}$$

– Consistent with experiments within large statistical uncertainty.

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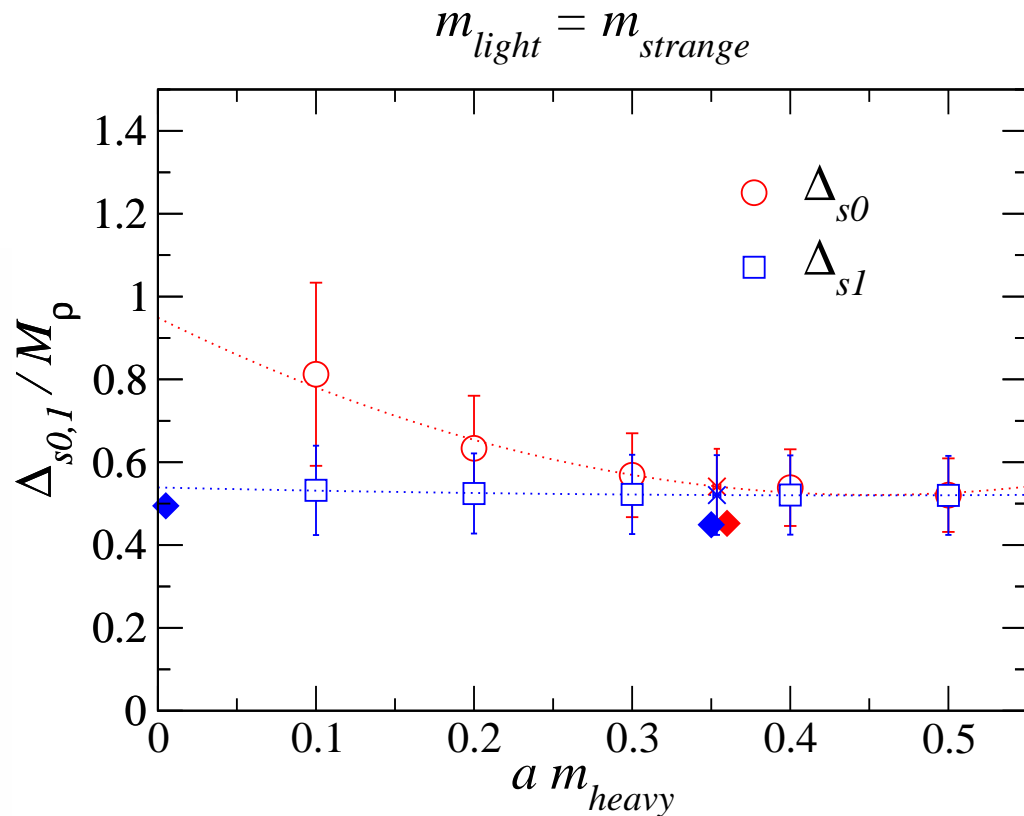
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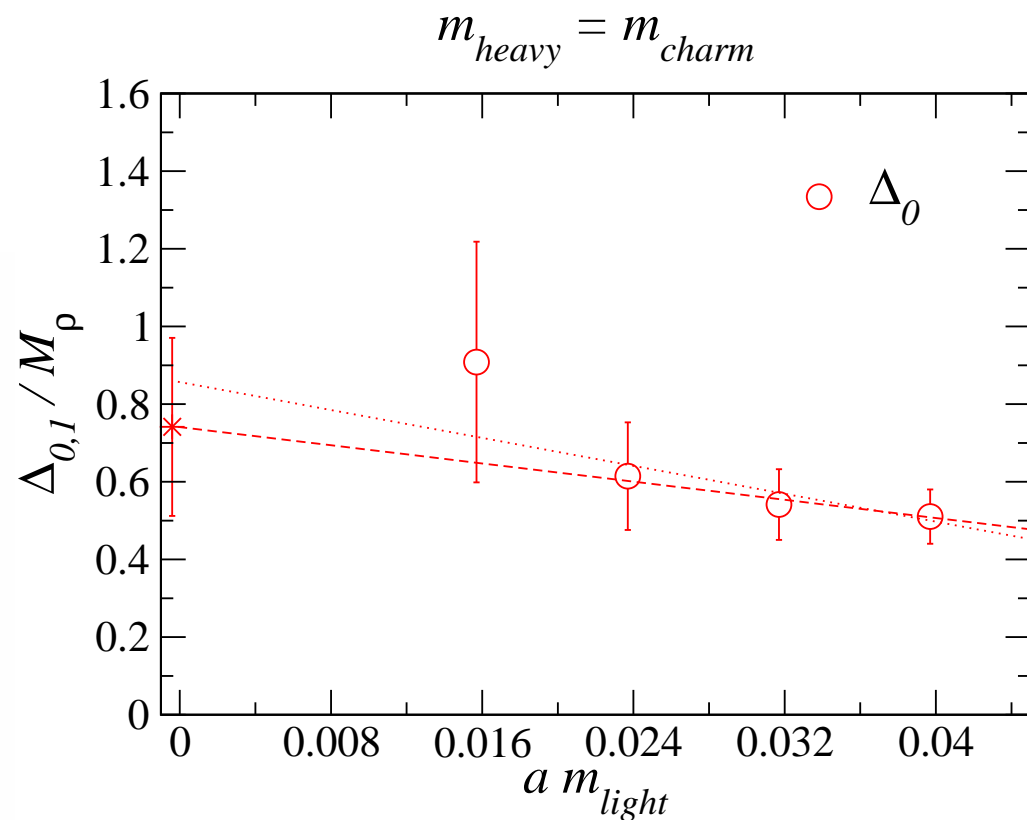
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- Consistent with experiments within large statistical uncertainty.
- Degeneracy between Δ_{s0} and Δ_{s1} takes place in $m_Q > m_c$.
- $K_1(1270) - K^*(892)$ splitting seems to support $\Delta_{s1} = \text{constant}$.

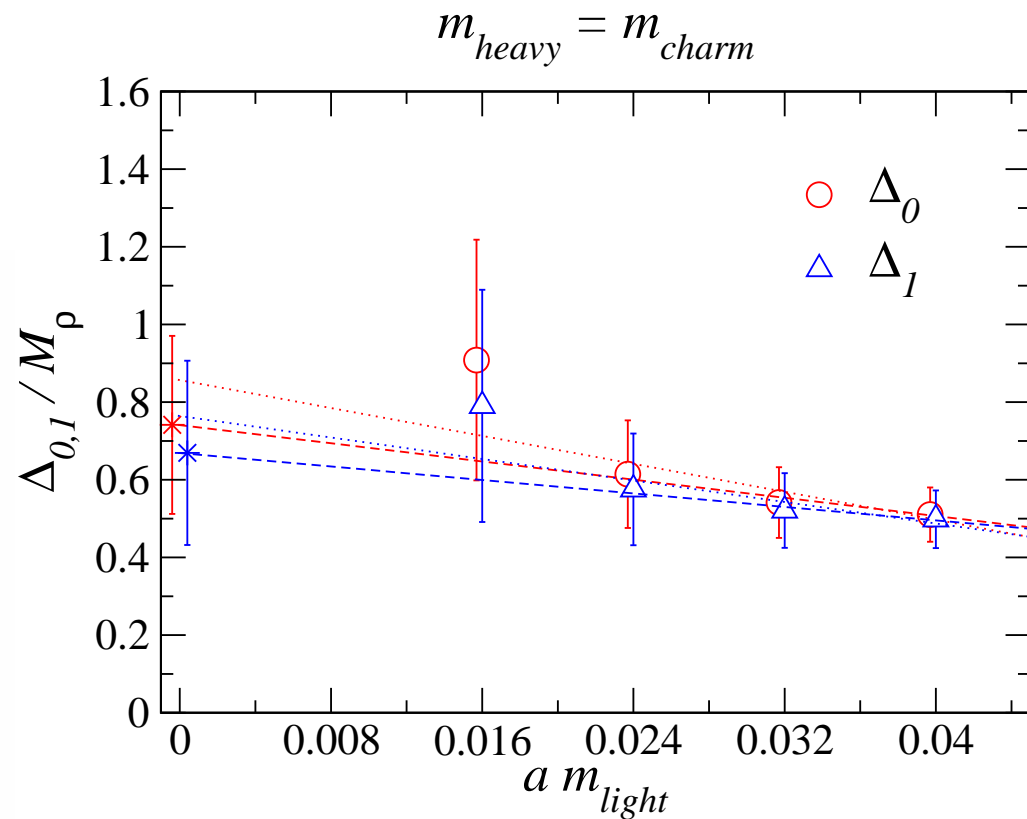
Chiral extrapolation of $\Delta_{q0,1}$



- $\Delta_0 = M_{0+} - M_{0-}$
increase as $m_{light} \rightarrow 0$?
With 3 data points,

$$\Delta_0 = 571(177) \text{ MeV}$$

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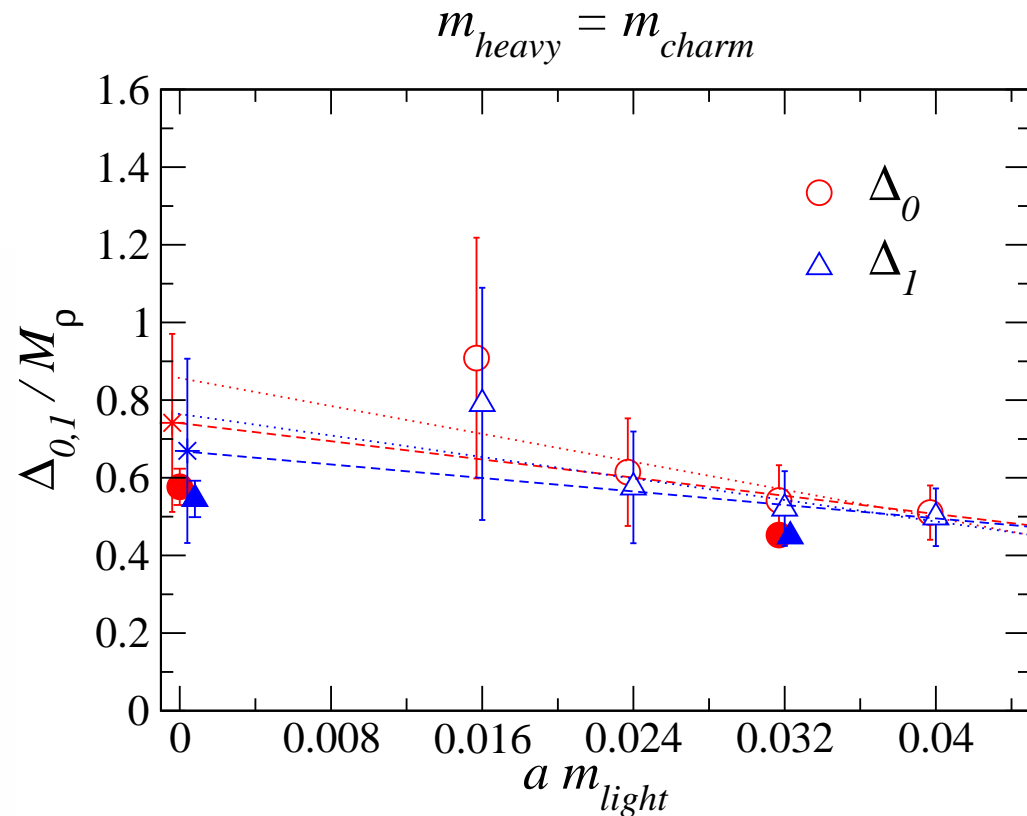
With 3 data points,

$$\Delta_0 = 571(177) \text{ MeV}$$

- $\Delta_1 = M_{1+} - M_{1-}$
With 3 data points,

$$\Delta_1 = 515(183) \text{ MeV}$$

Chiral extrapolation of $\Delta_{q0,1}$



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increase as $m_{light} \rightarrow 0$?

With 3 data points,

$$\Delta_0 = 571(177) \text{ MeV}$$

- $\Delta_1 = M_{1+} - M_{1-}$

With 3 data points,

$$\Delta_1 = 515(183) \text{ MeV}$$

Consistent with experiments within large uncertainty.

Especially the moderate slope in exp. is well reproduced.

Leptonic decay constant

f_D : *General remarks*

- CLEO-c is expected to determine $f_{D_{(s)}}$ in a few % soon.
Currently, $f_{D_s} = 266 \pm 32$ MeV [PDG'04],
 $f_D = 230 \pm 42 \pm 10$ MeV [CLEO-c@Lat'04]

In the following, $f_{D_{(s)}}$ are briefly discussed.

Z_A for the heavy-light current of DWF is not available.

⇒ **very preliminary**

Proper normalization for Massive DWF

NY, Aoki, Kuramashi (2004)

Defining Z_{dw} by $q_R = Z_{\text{dw}}^{1/2} q_{\text{lat}}$, the proper tree level normalization is given by

$$Z_{\text{dw}}(am_f, \omega) = \frac{am_f (1 + (am_f)^2) \cosh(am_p) - 2(am_f)^2 \omega \sinh(am_p)}{(1 - (am_f)^2) \sinh(am_p)},$$

where $\omega = 2 - M_5$ and

$$am_p = \ln \left| \frac{-(am_f)\omega^2 + \sqrt{(1 + (am_f)^2)^2 + (am_f)^2\omega^2(\omega^2 - 4)}}{1 + (am_f)^2 - 2\omega(am_f)} \right|.$$

In the massless limit, $Z_{\text{dw}} \rightarrow 1/\omega (2 - \omega)$, and $m_p \rightarrow \omega (2 - \omega) m_f$.

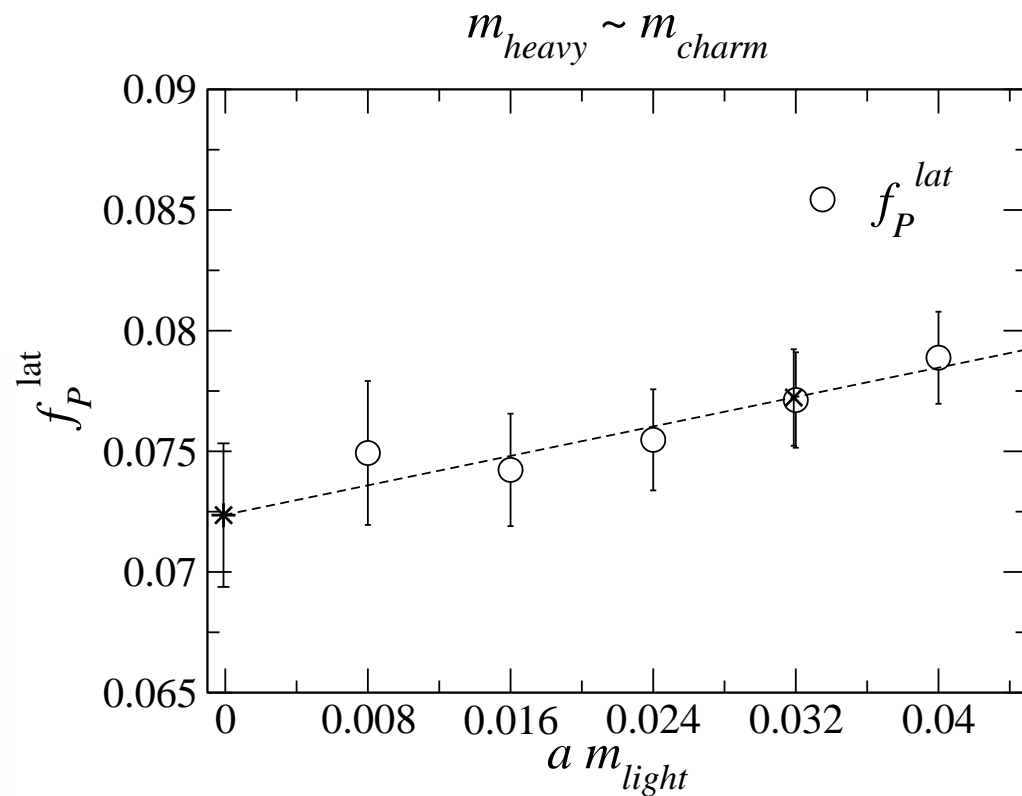
With $M_5=1.65$, $am_c=0.35$ $\sqrt{Z_{\text{dw}}^{\text{H}}(m_c)/Z_{\text{dw}}^{\text{L}}(0)}=1.179$,

$Z_A^{\text{LL, NP}} = 0.888$ for light-light currents is available.

Thus,

$$Z_A^{\text{HL}} = Z_A^{\text{LL, NP}} \times \sqrt{Z_{\text{dw}}(am_c)/Z_{\text{dw}}(0)} = 1.047$$

Results of $f_{D(s)}$



With $Z_A^{HL}=1.047$,
the preliminary results:

$$f_{D_s} = 228(6) \text{ MeV},$$

$$f_D = 212(9) \text{ MeV},$$

$$f_{D_s}/f_D = 1.07(3).$$

(statistical error only)

Summary

- DWF is applied to charmed-light system
- Splittings between different parities are consistent with exp within large uncertainty, though the central values look a little higher than exp like the previous lattice calculations do.
- In $m_Q \gtrsim m_c$, degeneracy of splittings between different parities is observed.
- Experimental observation of $\Delta_{s0,1} < \Delta_{ud0,1}$ is likely to happen.
- To make sure all the above, we need more statistics.

Backup Slides

Domain-wall fermion

Kaplan(1992), Shamir(1993), Furman and Shamir(1995)

$$S_{\text{DW}} = \sum_{x,y} \sum_{s,t} \bar{\psi}_s(x) D_{\text{DW}}(x, s; y, t) \psi_t(y),$$

$$D_{\text{DW}}(x, s; y, t) = D_{\text{W}}(x, y) \delta_{s,t} + \delta_{x,y} D_5(s, t),$$

$$D_{\text{W}}(x, y) = \sum_{\mu} \gamma_{\mu} D_{\mu} - \frac{a}{2} \sum_{\mu} D_{\mu}^2 - M_5 \delta_{x,y},$$

$$D_5(s, t) = \delta_{s,t} - P_L \delta_{s+1,t} - P_R \delta_{s-1,t} \\ + m_f [P_L \delta_{s,N_5} \delta_{1,t} + P_R \delta_{s,1} \delta_{N_5,t}],$$

$$\text{physical quark field : } \begin{cases} q(x) = P_L \psi_1(x) + P_R \psi_{N_5}(x) \\ \bar{q}(x) = \bar{\psi}_1(x) P_R + \bar{\psi}_{N_5}(x) P_L \end{cases},$$

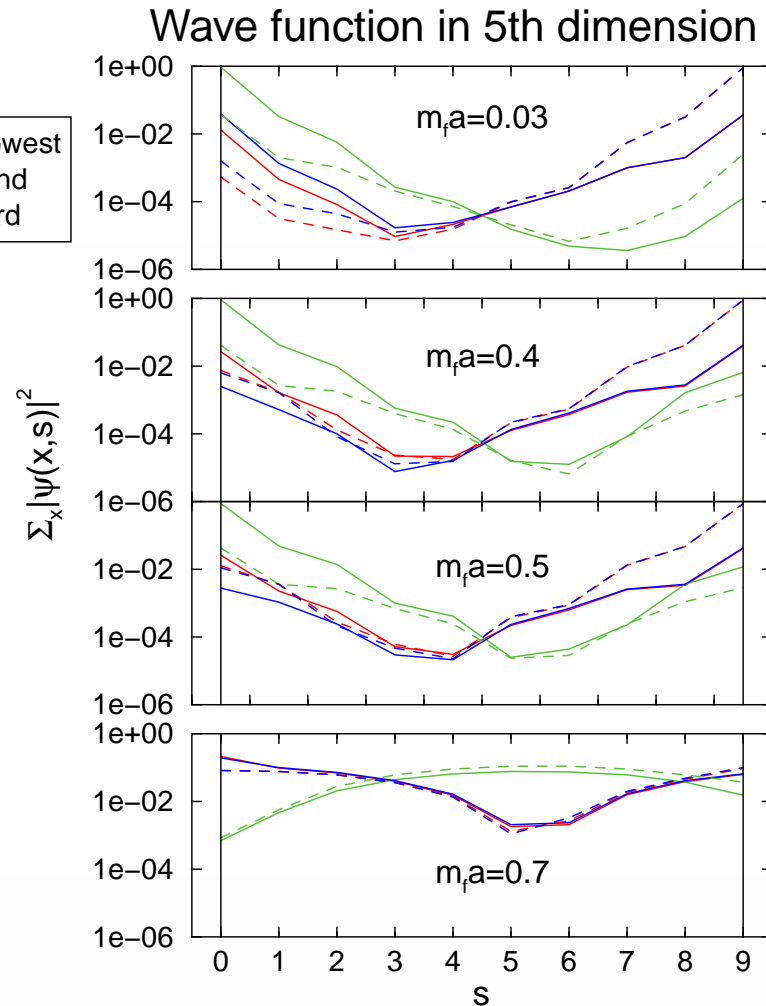
$$P_{R/L} = (1 \pm \gamma_5)/2,$$

$$1 \leq s, t \leq N_5 \quad (N_5 \rightarrow \infty)$$

- Exact chiral symmetry at the finite lattice spacing
- Leading scaling violation $\sim O(a^2)$ without any tuning
- The space-time rotational symmetry

Wavefunction in 5th dimension

RBC collaboration



- The behavior of the wave function in fifth dimension does not change much if $am_f \lesssim 0.4-5$.
- $O(a)$ error $\sim am_{\text{res}} \ll 10^{-2}$