Twisted mass QCD towards the chiral limit

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for the χ L F coll.

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Outline

- Introduction
- Mesonic correlation functions
- Scaling tests
- Neutral pion
- Strange quark mass
- Conclusions and outlooks

Introduction

$$S_{tm} = a^4 \sum_x \bar{\psi}(x)\{\frac{1}{2}[\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - a\nabla^*_\mu \nabla_\mu] + m_0 + i\mu\gamma_5\tau^3\}\psi(x)$$

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- These kind of actions have a long history (S. Aoki,..)
- It was introduced as a lattice QCD action to remove exceptional configuration (R. Frezzotti, S. Sint)
- Later it was realized that it has more profound properties (R. Frezzotti, G.C. Rossi)
  - $O(a)$ improved correlation functions could be obtained
  - It has a residual chiral symmetry (renormalization pattern could be simplified)

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- Automatic $O(a)$ improvement
- It is possible to reach low quark masses (p-regime)
- It is computationally cheaper than “chiral” fermions
- Unquenching is definitively possible

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- $\kappa = \kappa_c$ determined with Wilson action ($\mu = 0$) requiring the pion mass to vanish
- Masses and decay constants are $O(a)$ improved (R. Frezzotti, G.C. Rossi hep-lat/0306014)
- Special care should be taken at very low quark masses (S. Aoki, O. Bär hep-lat/0409006)

Simulation parameters

β	T/a	L/a	L/r_0	$N_{\rm meas}$
5.7	32	12	4.095	600
5.85	32	16	3.443	378
6.0	32	16	2.981	388
6.1	40	20	3.162	300
6.2	48	24	3.261	223
6.45	64	32	3.060	

Observables

$$P^a(x) = \bar{\psi}(x)\gamma_5 \frac{\tau^a}{2}\psi(x) \qquad a = 1, 2$$

$$A^a_\mu(x) = \bar{\psi}(x)\gamma_\mu\gamma_5 \frac{\tau^a}{2}\psi(x) \qquad a = 1, 2$$

- am_{PS} and aF_{PS} from

$$f^a_P(t) = a^3 \sum_{\mathbf{x}} \langle P^a(\mathbf{x}, t) P^a(0) \rangle \qquad a = 1, 2$$

- am_V from

$$f^a_A(t) = \frac{a^3}{3} \sum_{k=1}^{3} \sum_{\mathbf{x}} \langle A^a_k(\mathbf{x}, t) A^a_k(0) \rangle \qquad a = 1, 2$$

Observables

$$a^3 \sum_{\mathbf{x}} \langle P^a(x) P^a(0) \rangle = \frac{|\langle 0|P^a|PS\rangle|^2}{m_{PS}} e^{-m_{PS}\frac{T}{2}} \cosh\left[m_{PS} \left(x_0 - \frac{T}{2} \right) \right]$$

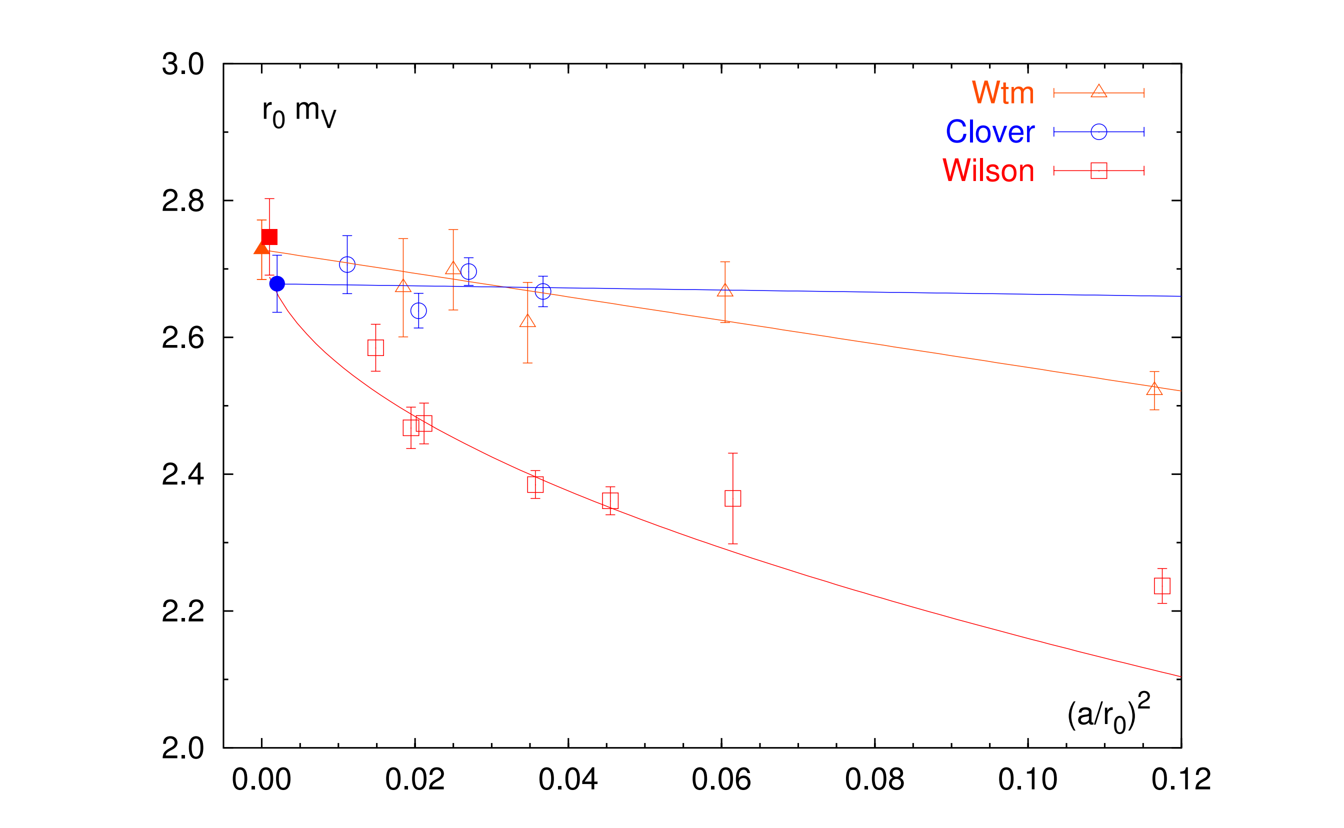
$$2\mu \langle 0|P^1|PS\rangle = \partial^*_\mu \langle 0|V^2_\mu|PS\rangle = f_{PS} m^2_{PS} \qquad Z_P = Z_\mu^{-1}$$

$$f_{PS} = \frac{2\mu}{m^2_{PS}} \langle 0|P^1|PS\rangle$$

R. Frezzotti, S. Sint NPB Proc. 106 (2002) 814

M. Della Morte, R. Frezzotti, J. Heitger NPB Proc. 106 (2002) 260

Scaling test (1)



$$(r_0 m_{PS})^2 = 3.3 \Rightarrow m_{PS} = 720MeV$$

(K. Jansen, A.S., C. Urbach, I. Wetzorke Phys. Lett. B586 (2004))

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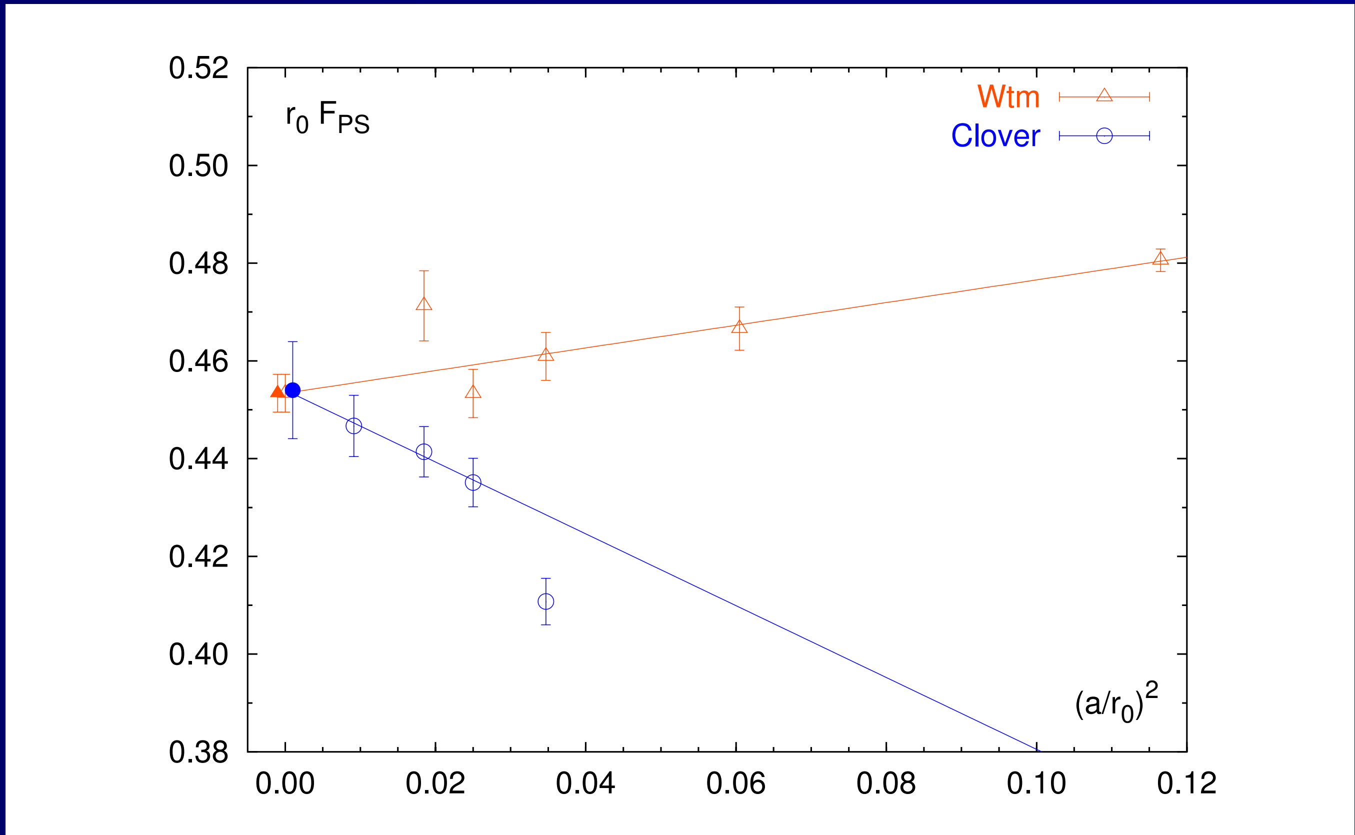

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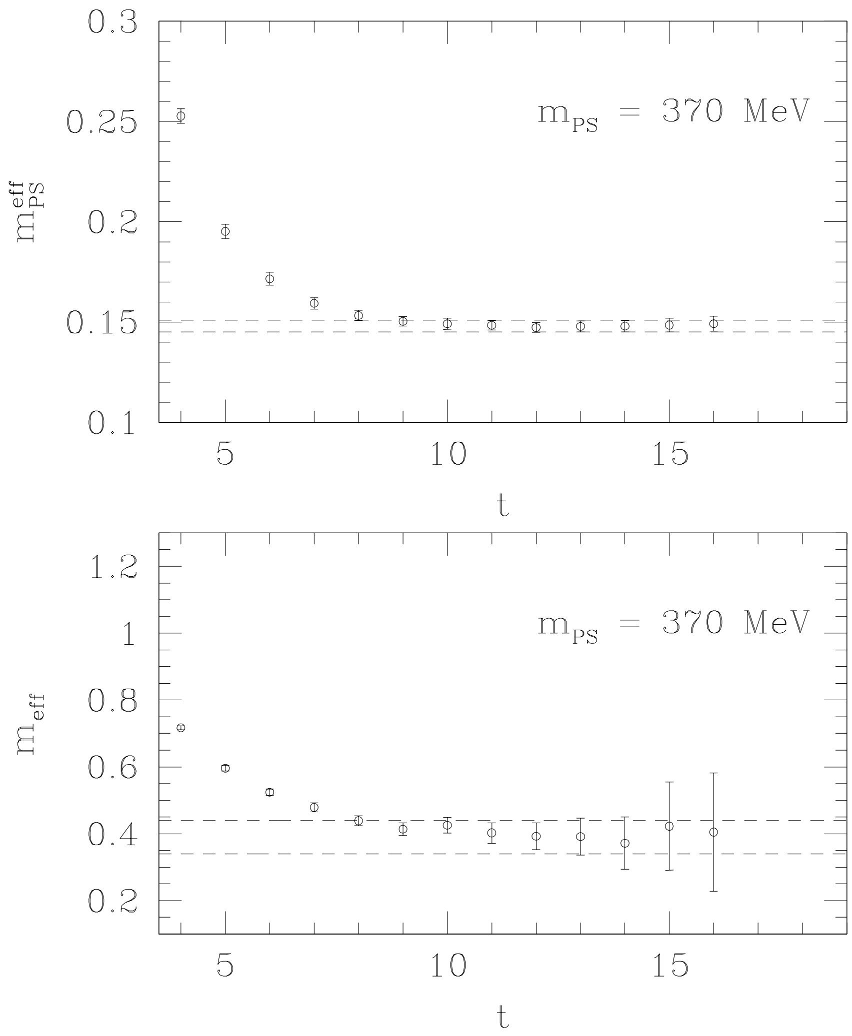
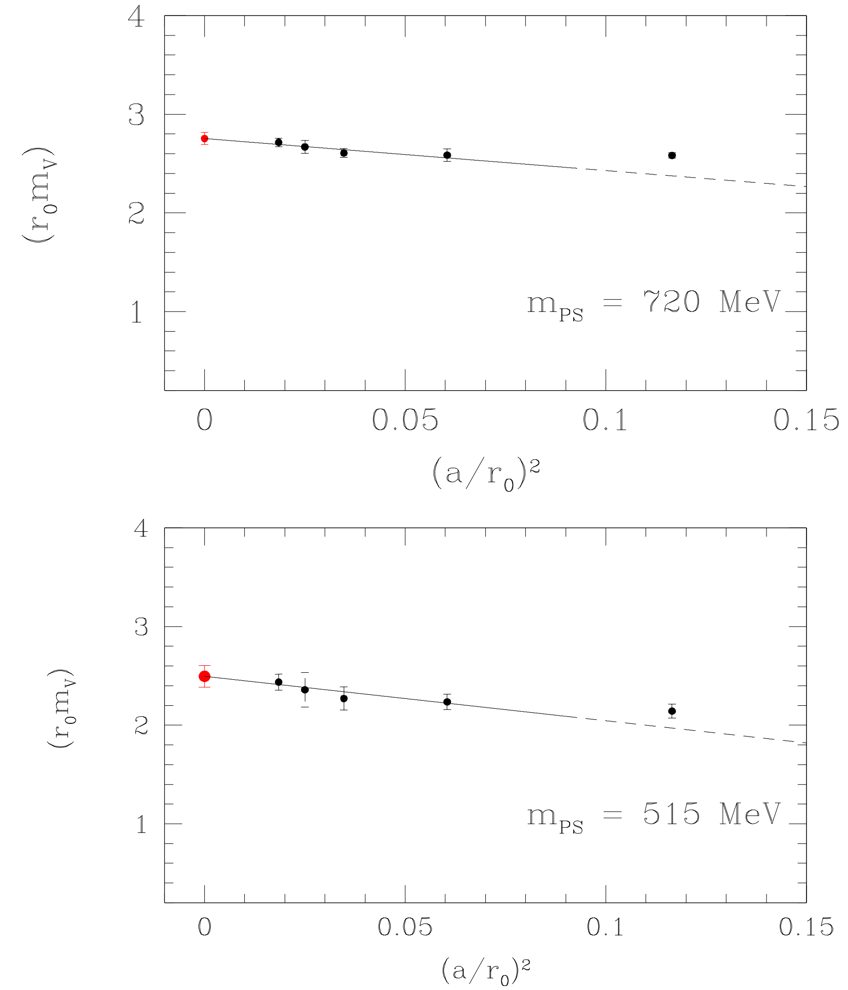
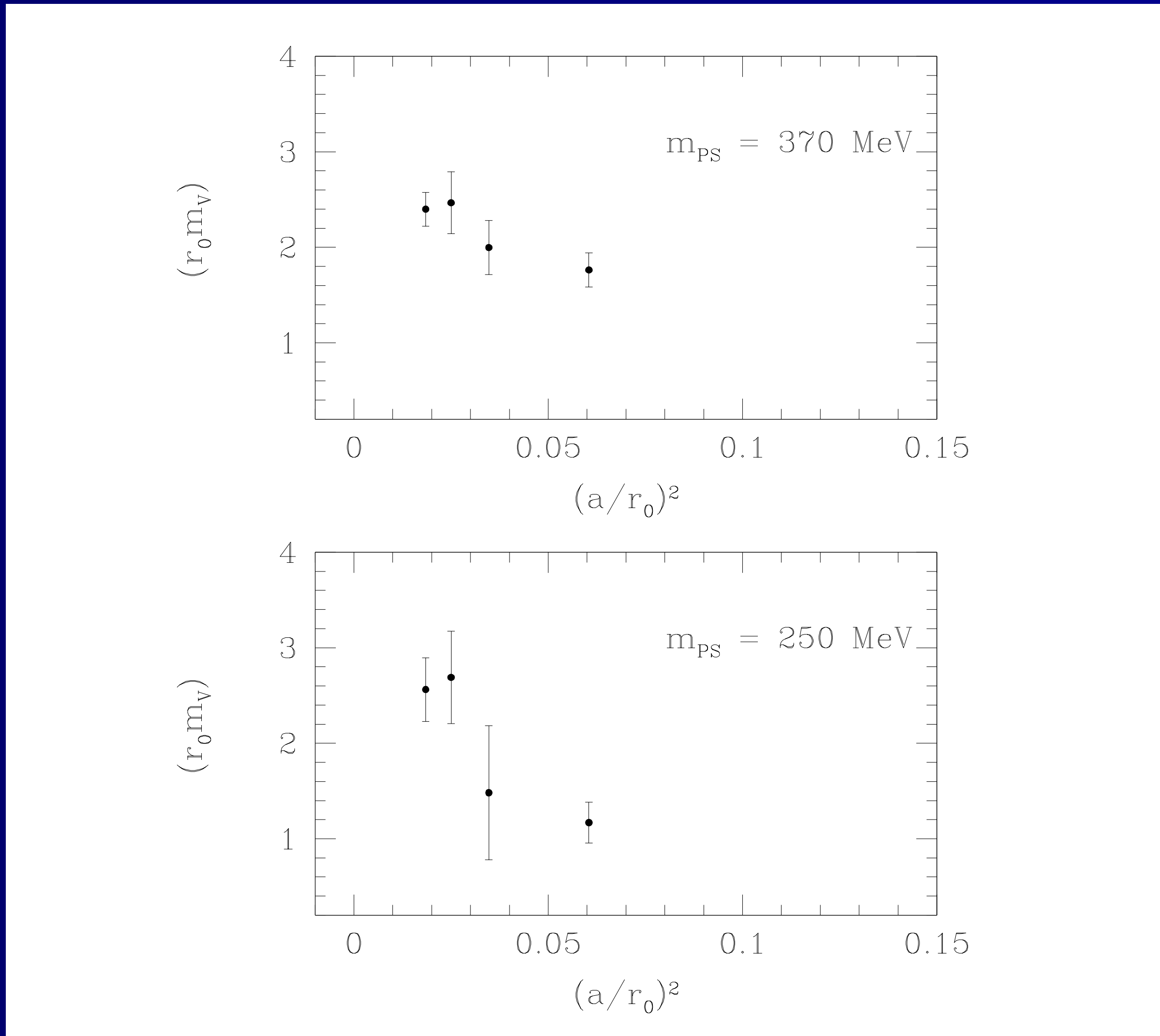
Effective masses



Scaling test (2)



Scaling test (2)



(Work in progress – PRELIMINARY)

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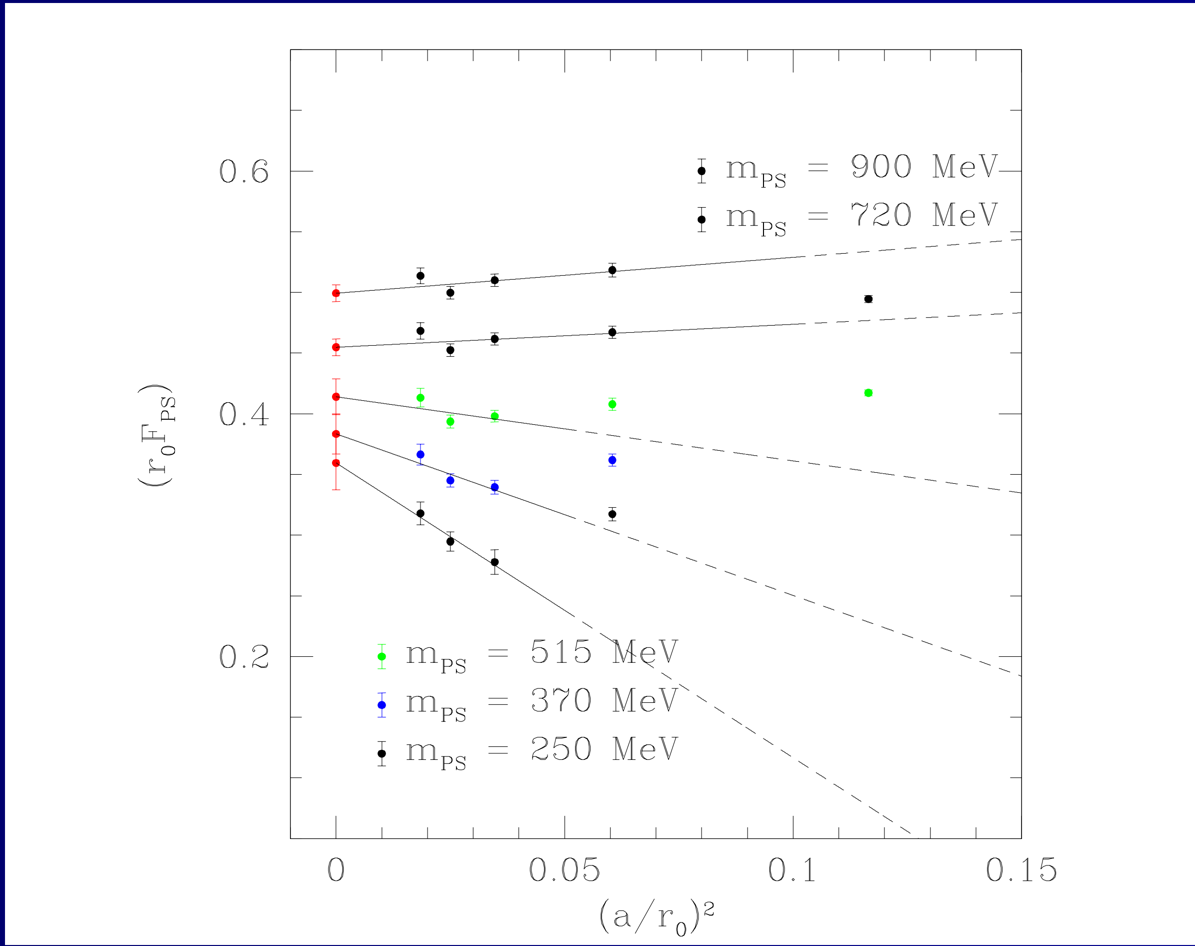
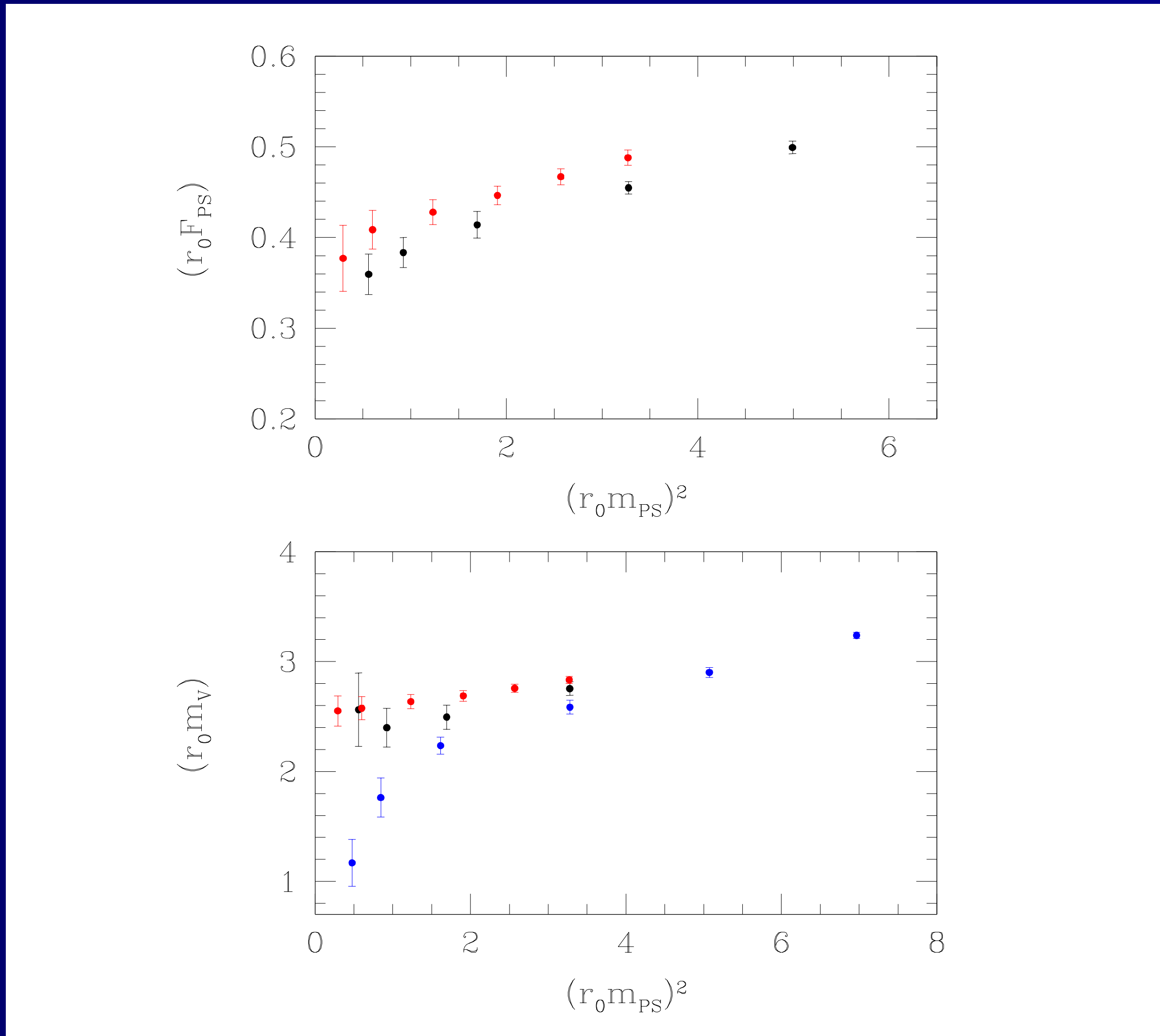


(Work in progress – PRELIMINARY)

Comparison overlap-tm



Comments

- Warning (R. Frezzotti, G.C. Rossi hep-lat/0306014)

$$a\Lambda^5 \ll \mu\Lambda^3 \Rightarrow (a\Lambda)^2 \ll a\mu$$

for $O(a)$ improved quantities

$$(a\Lambda)^3 \ll a\mu$$

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- It is mandatory to understand from numerical and analytical works in which region of a there is a scaling depending on the quark mass (Lat04)

Comments

- S. Aoki, O. Bär hep-lat/0409006
- For κ_c computed from the pseudoscalar mass

$$m_{PS}^2 \propto \mu + O(a^2/\mu) \qquad a\mu \gg (a\Lambda)^2$$

$$m_{PS}^2 \propto (a\mu)^{2/3} \qquad a\mu \ll (a\Lambda)^2$$

- Using alternative definitions of κ_c it is maybe possible to have an earlier scaling in a^2 also for low quark masses (in the Aoki scenario)

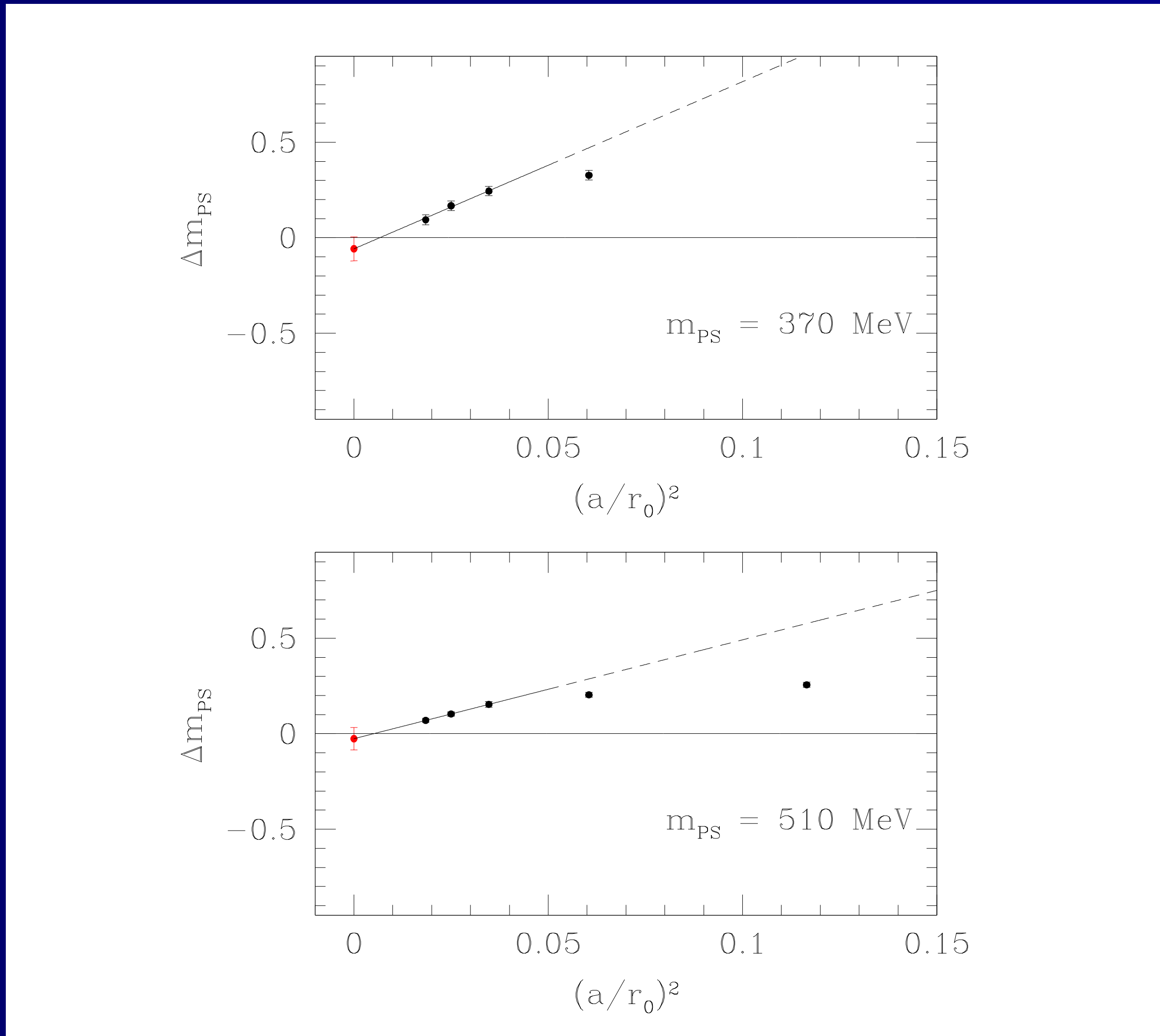
Neutral pion

$$S^3(x) = \bar{\psi}(x)\frac{\tau^3}{2}\psi(x)$$

$$f_S^a(t) = a^3 \sum_{\mathbf{x}} \langle S^3(\mathbf{x},t) S^3(0) \rangle \qquad a = 1,2$$

- We have analized only the connected diagram
- Using the OS action one can proove that it is a sensible definition of the neutral pion mass
- The disconnected diagram is an $O(a^2)$

Neutral pion



Strange quark mass

$$M_s + \hat{M} = Z_\mu(\mu_s + \hat{\mu}), \qquad \hat{M} = \frac{1}{2}(M_u + M_d)$$

$$(r_0 m_{PS})^2 = A + B(r_0\mu)$$

$$m_{PS}(\mu_{ref}, \mu_{ref}) = m_K \qquad (r_0 m_{PS})^2 = 1.5736 \Rightarrow r_o(\mu_s + \hat{\mu}) = 2 r_0 \mu_{ref}$$

Renormalization factor

(P. Hernandez, K. Jansen, L. Lellouch and H. Wittig JHEP 0107 (2001))

$$O^R = Z_O^r(g_0)O^r(g_0), \qquad O'^R(x) = Z_{O'}^r(g_0)O'^r(g_0,x)$$

$$O'^R(x_{ref}) = \lim_{a \longrightarrow 0} Z_{O'}^{r'}(g_0')O'^{r'}(g_0', x_{ref}) = U_{O'}(x_{ref})$$

My renormalization scheme is defined by

$$Z_{O'}^r(g_0) = \frac{U_{O'}(x_{ref})}{O'^r(g_0, x_{ref})}$$

$Z_O^r(g_0)$ must have a “relation” with $Z_{O'}^r(g_0)$

Renormalization factor

Examples:

1) $r' \longrightarrow$ NP clover improved, $r \longrightarrow$ tm

$$O'^{R} = r_0 M_{RGI}, \qquad O'^{r}(g_0, x) = r_0 \mu$$

$$x = (r_0 m_{PS})^2$$

2) $r' \longrightarrow$ NP clover improved, $r \longrightarrow$ tm

$$O'^{R} = r_0^2 |\langle 0|P^{RGI}|PS\rangle|, \qquad O'^{r}(g_0, x) = r_0^2 |\langle 0|P|PS\rangle|$$

$$x = (r_0 m_{PS})^2, \qquad O'^{r'} = (1 + a b_P m_q) r_0^2 |\langle 0|P|PS\rangle|$$

Renormalization factor

$$U_P = \lim_{a \to 0} Z_P^{cl}(g_0^2)(1 + ab_P m_q) r_0^2 |\langle 0|P|PS\rangle \qquad x = x_{ref}$$

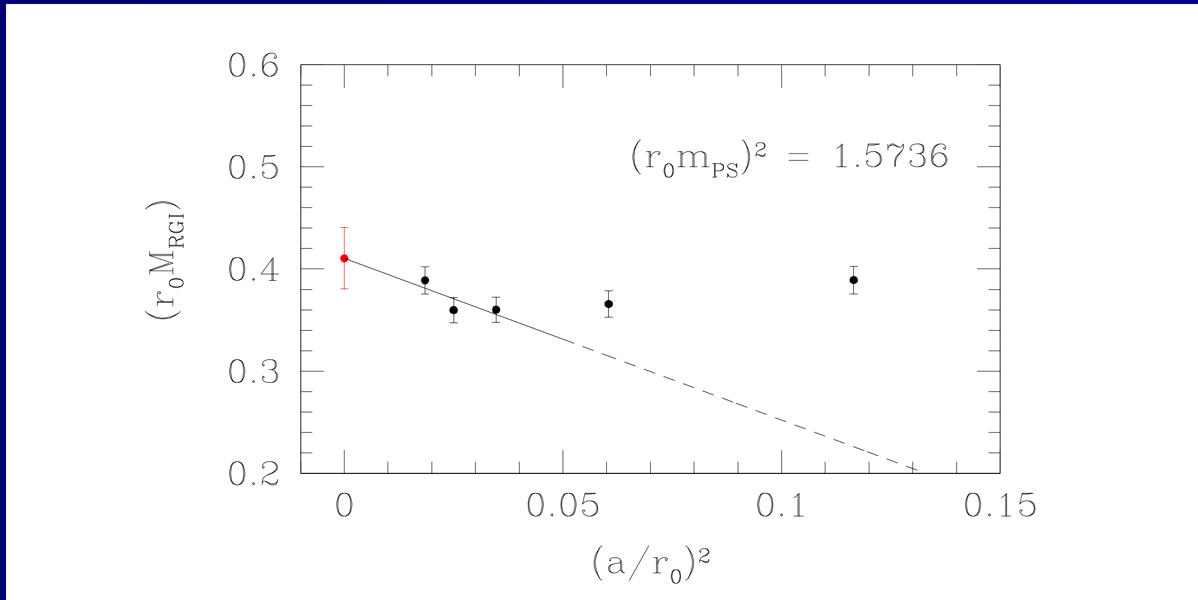
(J. Garden, J. Heitger, R. Sommer, H. Wittig Nucl. Phys B571 (2000))

$$Z_P^{tm}(g_0^2) = \frac{U_P(x_{ref})}{r_0^2 |\langle 0|P|PS\rangle|(g_0^2, x_{ref})}$$

$$Z_P^{tm}(g_0^2) = \frac{1}{Z_\mu^{tm}(g_0^2)}$$

$$r_0(M_s + \hat{M}) = \lim_{a \to 0} Z_\mu^{tm}(g_0^2) 2 r_0 \mu_{ref}$$

Strange quark mass



$$r_0(M_s + \hat{M}) = 0.410(30) \Rightarrow (M_s + \hat{M}) = 161(12)MeV$$

$$\frac{M_s}{\hat{M}} = 24.4 \pm 1.5 \Rightarrow \bar{m}_s(2GeV) = 116(11)MeV$$

Conclusions

- Detailed analysis of quenched Wtm at low quark masses
- We can reach pseudoscalar masses of the order of $250 MeV$ $m_{PS}/m_V \simeq 0.29$
- Going towards small quark masses the cutoff effects $O(a^2)$ increase (for f_{PS}) and the scaling window gets narrow (for f_{PS} and m_V)
- It is possible to follow the scaling $O(a^2)$ at low masses ($m_{PS} \leq 500$ MeV $\beta \geq 6.0$)
- The pion splitting is under control
- Renormalization factors through suitable matching conditions
- Determination of the strange quark mass in the continuum

Outlooks

- Study the dependence of the scaling on the choice of κ_c
- Perform simulation at $\beta = 6.45$ and conclude the scaling tests
- Study the contribution of the disconnected diagrams in the neutral-charged pion splitting
- For the unquenched simulation see the talk of I. Montvay