Nucleon Matrix Elements with Domain Wall Fermions

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RBC calculations of nucleon structure: form factors, moments of structure functions and nucleon decay matrix elements,

- using DWF and DBW2 actions.
- Based on the works by Yasumichi Aoki, Tom Blum, Kostas Orginos, Shoichi Sasaki, ...

Domain wall fermions (DWF) preserves almost exact chiral symmetry on the lattice:

- by introducing a fictitious fifth dimension in which the symmetry violation is exponentially suppressed.

DBW2 ("doubly blocked Wilson 2") action improves approach to the continuum:

- by adding rectangular (2 × 1) Wilson loops to the action.

By combining the two, the “residual mass,” which controls low energy chiral behavior, is driven to

- \( am_{\text{res}} \sim O(10^{-4}) \) or \( m_{\text{res}} < \text{MeV} \).

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Numerical calculation: we like to have

- good chiral behavior, \textit{i.e.} close enough to the continuum, and
- big enough volume.

Improved gauge actions help both. DBW$^{2}$, in particular,

\[ S_{G} = \beta [c_0 \sum W_{1,1} + c_1 \sum W_{1,2}], \]

with \( c_0 + 8c_1 = 1 \), and \( c_1 = -1.4069 \).

Fermion action: DWF,

Quenched calculation: about 400 lattices, complete,

- \( \beta = 0.87 \), at the chiral limit, \( am_{\rho} = 0.592(9) \) (so \( a^{-1} \sim 1.3\text{GeV} \)),
- \( L_s = 16, M_5 = 1.8, am_{\text{res}} \sim 5 \times 10^{-3} \),
- \( 8^3 \times 24 \times 16 (\sim (1.2\text{fm})^3) \) and \( 16^3 \times 32 \times 16 (\sim (2.4\text{fm})^3) \) volumes,
- \( m_N/m_{\rho} \sim 1.3 \).

Dynamical calculation \((N_f = 2)\): about 50 lattice at each of \( m_f a = 0.04, 0.03, \) and \( 0.02 \), ongoing,

- \( \beta = 0.8 \) \((m_{\rho} \) and Sommer scales agree with \( a^{-1} \sim 1.7\text{GeV} \)),
- \( L_s = 12, M_5 = 1.8, m_{\text{res}} \sim 2.5 \text{MeV} \),
- \( 16^3 \times 32 \times 16 (\sim (2.0\text{fm})^3) \) volume,
- \( m_N/m_{\rho} \sim 1.35 \).

Axial charge: from neutron $\beta$ decay, we know $g_V = G_F \cos \theta_c$ and $g_A/g_V = 1.2670(30)^2$:

- $g_V \propto \lim_{q^2 \to 0} g_V(q^2)$ with $\langle n|V_\mu^- (x)|p\rangle = i\bar{u}_n[\gamma_\mu g_V(q^2) + q_\lambda \sigma_{\lambda\mu} g_M(q^2)]u_p e^{-iqx}$,

- $g_A \propto \lim_{q^2 \to 0} g_A(q^2)$ with $\langle n|A_\mu^- (x)|p\rangle = i\bar{u}_n \gamma_5[\gamma_\mu g_A(q^2) + q_\mu g_P(q^2)]u_p e^{-iqx}$.

On the lattice, in general, we calculate the relevant matrix elements of these currents:

- with a lattice cutoff, $a^{-1} \sim 1$-2 GeV,
- and extrapolate to the continuum, $a \to 0$,

introducing lattice renormalization: $g_{V,A}^{\text{renormalized}} = Z_{V,A} g_{V,A}^{\text{lattice}}$.

Also, unwanted lattice artefact may result in unphysical mixing of chirally distinct operators.

DWF makes $g_A/g_V$ particularly easy, because:

- the chiral symmetry is almost exact, and
- maintains $Z_A = Z_V$, so that $g_A^{\text{lattice}}/g_V^{\text{lattice}}$ directly yields the renormalized value.

\[^2\]The Particle Data Group.
Historically

- NR quark model gives $5/3$,
- MIT bag model gives $1.07$,
- lattice calculations with Wilson or clover fermions typically underestimates by up to $25\%$:

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<th>type</th>
<th>group</th>
<th>fermion</th>
<th>lattice</th>
<th>$\beta$</th>
<th>volume</th>
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<th>$g_A$</th>
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<td>KEK$^a$</td>
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<td>(1.5fm)$^3$</td>
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<td>$\geq 4.5$</td>
<td>0.907(20)</td>
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$^d$D. Dolgov et al, hep-lat/0201021.

– with $Z_A \neq Z_V$ and other renormalization complications.
Our formulation follows the standard one,

- Two-point function: \( G_N(t) = \text{Tr}[(1 + \gamma_t) \sum_{\vec{x}} \langle TB_1(x)B_1(0) \rangle] \), using \( B_1 = \epsilon_{abc}(u_a^T C \gamma_5 d_b) u_c \) for proton,

- Three-point functions,
  - vector: \( G_{uv}^{u,d}(t, t') = \text{Tr}[(1 + \gamma_t) \sum_{\vec{x}'} \sum_{\vec{x}} \langle TB_1(x')V_{t}^{u,d}(x)B_1(0) \rangle] \),
  - axial: \( G_{A}^{u,d}(t, t') = \frac{1}{3} \sum_{i=x,y,z} \text{Tr}[(1 + \gamma_t)\gamma_i \gamma_5 \sum_{\vec{x}'} \sum_{\vec{x}} \langle TB_1(x')A_{i}^{u,d}(x)B_1(0) \rangle] \).

with fixed \( t' = t_{\text{source}} - t_{\text{sink}} \) and \( t < t' \).

- From the lattice estimate
  \[
g_{\Gamma}^{\text{lattice}} = \frac{G_{u}^{u}(t, t') - G_{d}^{d}(t, t')}{G_{N}(t)},
\]
with \( \Gamma = V \) or \( A \), the renormalized value
\[
g_{\Gamma}^{\text{ren}} = Z_{\Gamma} g_{\Gamma}^{\text{lattice}},
\]
is obtained.

- Non-perturbative renormalizations, defined by
\[
[\bar{u}\Gamma d]_{\text{ren}} = Z_{\Gamma} [\bar{u}\Gamma d]_{0},
\]
satisfies \( Z_A = Z_V \) well, so that
\[
\left( \frac{g_A}{g_V} \right)^{\text{ren}} = \left( \frac{G_{u}^{u}(t, t') - G_{d}^{d}(t, t')}{G_{V}^{u}(t, t') - G_{V}^{d}(t, t')} \right)^{\text{lattice}}.
\]
g_{A} is also described as \( \Delta u - \Delta d \).
Numerical calculations with Wilson (single plaquette) gauge action:
- RIKEN-BNL-Columbia QCDSP,
- 400 gauge configurations, using a heat-bath algorithm,
- \( \beta = 6.0, \ 16^3 \times 32 \times 16, \ M_5 = 1.8, \)
- source at \( t = 5, \) sink at 21, current insertions in between.

\( Z_V = 1/g_{\text{v}}^{\text{lattice}} \) is well-behaved,

- the value 0.764(2) at \( m_f = 0.02 \) agrees well with \( Z_A = 0.7555(3) \) from
  - \( \langle A_{\mu}^{\text{conserved}}(t) \bar{q} \gamma_5 q(0) \rangle = Z_A \langle A_{\mu}^{\text{local}}(t) \bar{q} \gamma_5 q(0) \rangle \) (RBC hep-lat/0007038, to appear in Phys. Rev. D),
- linear fit gives \( Z_V = 0.760(7) \) at \( m_f = 0, \) and quadratic fit, 0.761(5).
\(\Delta u, \Delta d, \) and \(g_A/g_V\) (averaged in \(10 \leq t \leq 16\)):  

- linear extrapolation yields 0.81(11) at \(m_f = 0\), and similarly small values for  
  - \(\Delta q/g_V = 0.49(12)\) and  
  - \((\delta q/g_V)_{\text{lattice}} = 0.47(10)\) (with a preliminary \(Z_T \sim 1.1\)).  
- While relevant three-point functions are well behaved in DWF, and \(Z_V = Z_A\) is well satisfied, 0.760(7) and 0.7555(3).
Why so small?

- finite lattice volume $^3$,
- excited states (small separation between $t_{\text{source}}$ and $t_{\text{sink}}$),
- quenching (zero modes, absent pion cloud, ...).

To investigate size-dependence, we simultaneously need

- good chiral behavior, \textit{i.e.} close enough to the continuum, and
- big enough volume.

Improved gauge actions help both. DBW$^2$$^4$, in particular,

$$S_G = \beta [c_0 \sum W_{1,1} + c_1 \sum W_{1,2}],$$

with $c_0 + 8c_1 = 1$ and $c_1 = -1.4069$:

- very small residual chiral symmetry breaking, $am_{\text{res}} < 10^{-3}$,
- at the chiral limit, $am_\rho = 0.592(9)$ (so $a^{-1} \sim 1.3\text{GeV}$), $m_\rho/m_N \sim 0.8$,
- $m_\pi(m_f = 0.02) \sim 0.3a^{-1}$.


DBW2 calculations are performed at $a \sim 0.15$ fm ($\beta = 0.87$) with both wall and sequential sources on
- $8^3 \times 24 \times 16 \ (\sim (1.2 \text{fm})^3)$, 400 configurations (wall) and 160 (sequential),
- $16^3 \times 32 \times 16 \ (\sim (2.4 \text{fm})^3)$, 100 configurations (wall and sequential),
- source-sink separation of about 1.5 fm,
- $m_f = 0.02, 0.04, \ldots$: $m_\pi \geq 390 \text{MeV}$, $m_\pi L \geq 4.8$ and 2.4.

Renormalization factors: $\mathcal{O}^{\text{ren}}(\mu) = Z_\mathcal{O}(a\mu) \mathcal{O}^{\text{lattice}}(a)$.

- $Z_V$ shows slight quadratic dependence on $m_f$ as expected: $V^{\text{conserved}}_\mu = Z_V V^{\text{local}}_\mu + \mathcal{O}(m_f^2 a^2)$,
  - yielding a value $Z_V = 0.784(15)$,
  - agrees well with $Z_A = 0.77759(45)$ \(^5\).

\(^5\)RBC Collaboration, in preparation: this value is obtained from a relation $\langle A^{\text{conserved}}_\mu(t)[\bar{q}\gamma_5 q](0) \rangle = Z_A \langle A^{\text{local}}_\mu(t)[\bar{q}\gamma_5 q](0) \rangle$. 
Bare $g_A^{\text{lattice}}$ from wall source show volume dependence at medium $m_f ((2.4\text{fm})^3 \text{ (filled)}$ and $(1.2\text{fm})^3 \text{ (open)}$ volumes):
Bare $\Delta u^{\text{lattice}}$ and $\Delta d^{\text{lattice}}$ from sequential source ($(2.4\text{fm})^3$):
Bare $g_V^{\text{lattice}}$ from sequential source ($(2.4\text{fm})^3$):
$g_A/g_V^{\text{lattice}}$ from sequential source ($(2.4 \text{fm})^3$):
\[(g_A/g_V)^{\text{lattice}} = (g_A/g_V)^{\text{ren}}: m_f \text{ and volume dependence in bare and physical scales (} m_\rho \text{ and Sommer):}

- Clear volume dependence is seen between \((2.4\text{fm})^3\) and \((1.2\text{fm})^3\) volumes.
- The large volume results (sequential)
  - show a very mild \(m_f\) dependence,
  - extrapolate to about 8% under estimation, \(g_A = 1.15(11)\).
Alternatively we can use $g_A^{\text{lattice}} \times Z_A$:

agree well with $(g_A/g_V)^{\text{lattice}}$ in the chiral limit, and and an expected difference seen away from there.
New, this year, of axial charge: dynamical result seems to follow the quenched $^6$.

$^6$Note the lattice scales obtained from $m_\rho$ and Sommer scale agree, with $a^{-1} \sim 1.7$ GeV.
Structure functions: measured in deep inelastic scatterings (and RHIC/Spin):

- DIS

\[
\left| \frac{A}{4\pi} \right|^2 = \frac{\alpha^2}{Q^4} I_{\mu\nu} W_{\mu\nu} \\
W_{\mu\nu} = W^{[\mu\nu]} + W^{\{\mu\nu\}}
\]

\[
W^{[\mu\nu]}(x, Q^2) = \left( -g^{\mu\nu} + \frac{q^{\mu} q^{\nu}}{q^2} \right) F_1(x, Q^2) + \left( P^\mu - \frac{\nu}{q^2} q^{\mu} \right) \left( P^\nu - \frac{\nu}{q^2} q^{\nu} \right) \frac{F_2(x, Q^2)}{\nu}
\]

\[
W^{\{\mu\nu\}}(x, Q^2) = ie^{\mu\nu\rho\sigma} q_\rho \left( \frac{S^\sigma}{\nu} (g_1(x, Q^2) + g_2(x, Q^2)) - \frac{q \cdot S P^\sigma}{\nu^2} g_2(x, Q^2) \right)
\]

with \( \nu = q \cdot P, S^2 = -M^2, x = Q^2/2\nu. \)

- The same structure functions appear in RHIC/Spin (which also provides \( h_1(x, Q^2) \)).

Moments of the structure functions are accessible on the lattice:

\[
2 \int_0^1 dx x^{n-1} F_1(x, Q^2) = \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu)) \langle x^n \rangle_q(\mu) + \mathcal{O}(1/Q^2),
\]

\[
\int_0^1 dx x^{n-2} F_2(x, Q^2) = \sum_{f=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu)) \langle x^n \rangle_q(\mu) + \mathcal{O}(1/Q^2),
\]

\[
2 \int_0^1 dx x^n g_1(x, Q^2) = \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu)) \langle x^n \rangle_{\Delta q}(\mu) + \mathcal{O}(1/Q^2),
\]

\[
2 \int_0^1 dx x^n g_2(x, Q^2) = \frac{1}{2n+1} \sum_{q=u,d} [e_{2,n}^{(q)}(\mu^2/Q^2, g(\mu)) d_n^q(\mu) - 2e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu)) \langle x^n \rangle_{\Delta q}(\mu)] + \mathcal{O}(1/Q^2)
\]

- \( c_1, c_2, e_1, \) and \( e_2 \) are the Wilson coefficients (perturbative),

- \( \langle x^n \rangle_q(\mu), \langle x^n \rangle_{\Delta q}(\mu) \) and \( d_n \) are forward nucleon matrix elements of certain local operators.
Lattice operators:

- Unpolarized (\( F_1/F_2 \)):

\[
\frac{1}{2} \sum_s \langle P, S | \mathcal{O}_{\mu_1 \mu_2 \cdots \mu_n}^q | P, S \rangle = 2 \langle x_{n-1}^q \rangle (\mu) [P_{\mu_1} P_{\mu_2} \cdots P_{\mu_n} + \cdots - \text{(trace)}]
\]

\[
\mathcal{O}_{\mu_1 \mu_2 \cdots \mu_n}^q = \bar{q} \left[ \left( \frac{i}{2} \right)^{n-1} \gamma_{\mu_1} \overset{\rightarrow}{D}_{\mu_2} \cdots \overset{\rightarrow}{D}_{\mu_n} - \text{(trace)} \right] q
\]

On the lattice we can measure: \( \langle x \rangle_q, \langle x^2 \rangle_q \) and \( \langle x^3 \rangle_q \).

- Polarized \( (g_1/g_2) \) and transversity \( (h_1) \):

\[
- \langle P, S | \mathcal{O}_{[\sigma \mu_1 \mu_2 \cdots \mu_n]}^{\delta q} | P, S \rangle = \frac{2}{n+1} \langle x^n \rangle \Delta_q (\mu) [(S_\sigma P_{\mu_1} - S_{\mu_1} P_\sigma)P_{\mu_2} \cdots P_{\mu_n} + \cdots - \text{(traces)}]
\]

\[
\mathcal{O}_{[\sigma \mu_1 \mu_2 \cdots \mu_n]}^{\delta q} = \bar{q} \left[ \left( \frac{i}{2} \right)^n \gamma_5 \gamma_{\sigma} \overset{\rightarrow}{D}_{\mu_1} \cdots \overset{\rightarrow}{D}_{\mu_n} - \text{(traces)} \right] q
\]

\[
\langle P, S | \mathcal{O}_{[\rho \nu \mu_1 \mu_2 \cdots \mu_n]}^{\sigma q} | P, S \rangle = \frac{2}{m_N} \langle x^n \rangle \delta_q [(S_\rho P_\nu - S_\nu P_\rho)P_{\mu_1} P_{\mu_2} \cdots P_{\mu_n} + \cdots - \text{(traces)}]
\]

\[
\mathcal{O}_{[\rho \nu \mu_1 \mu_2 \cdots \mu_n]}^{\sigma q} = \bar{q} \left[ \left( \frac{i}{2} \right)^n \gamma_5 \gamma_{\rho} \overset{\rightarrow}{D}_{\mu_1} \cdots \overset{\rightarrow}{D}_{\mu_n} - \text{(traces)} \right] q
\]

On the lattice we can measure: \( \langle 1 \rangle_{\Delta q} (g_A) \), \( \langle x \rangle_{\Delta q} \), \( \langle x^2 \rangle_{\Delta q} \), \( d_1 \), \( d_2 \), \( \langle 1 \rangle_{\delta q} \) and \( \langle x \rangle_{\delta q} \).

- Higher moment operators mix with lower dimensional ones.

- Only \( \langle x \rangle_q, \langle 1 \rangle_{\Delta q}, \langle x \rangle_{\Delta q}, d_1, d_2, \langle 1 \rangle_{\delta q} \) and \( \langle x \rangle_{\delta q} \) can be measured with \( \vec{P} = 0 \).
Renormalization: $\mathcal{O}^{\text{ren}} = Z_\mathcal{O}(a\mu)\mathcal{O}^{\text{lat}}(a)$,

- lattice complications: operator mixing from broken Lorentz or chiral symmetry,
- NPR is required when mixing with lower dimensional operator occurs.

We calculate $Z_\mathcal{O}(a\mu)$ non-perturbatively in RI/MOM scheme\textsuperscript{7} with perturbative matching to $\overline{\text{MS}}$.

- compute off-shell matrix element of the operator, $\mathcal{O}$, in Landau gauge,
- impose a MOM scheme condition $\text{Tr} \, V_\mathcal{O}(p^2)\Gamma|_{p^2=\mu^2} \frac{Z_\mathcal{O}}{Z_q} = 1$,
  - $V_\mathcal{O}(p^2)$ is the relevant amputated vertex,
  - $\Gamma$ is an appropriate projector,
- extrapolate to the chiral limit, defining the RI scheme,
- in an appropriate window, $\Lambda_{\text{QCD}} \ll \mu^2 \ll a^{-1}$, a scale invariant
  $$Z_{\text{rgi}} = \frac{Z(\mu^2)}{C(\mu^2)}$$

  is obtained, with the operator running $C(\mu^2)$ in the continuum perturbation theory.
- Now we can perturbatively match to e.g. $\overline{\text{MS}}$.

Works nicely with DWF.

Quark density \( \langle x \rangle_{u-d} \), calculated with \( \mathcal{O}_4^q = \sqrt{q} \left[ \gamma_4 \, \overrightarrow{D}_4 - \frac{1}{3} \sum_{k=1}^{3} \gamma_k \, \overrightarrow{D}_k \right] q \).

- Quenched calculation complete with NPR,
  - \( Z = 1.02(10) \), with \( \overline{\text{MS}} \) 2 GeV, 2-loop running,
  - no curvature seen in the chiral limit,
  - \( \langle x \rangle_u / \langle x \rangle_d = 2.41(4) \) at the chiral limit.

- Dynamical calculation ongoing, lacks NPR,
Polarization, \( \langle x \rangle_{\Delta u - \Delta d} \), calculated with \( \mathcal{O}^{5q}_{34} = \frac{1}{4} q \gamma_5 \left[ \gamma_3 \overleftrightarrow{D_4} + \gamma_4 \overleftrightarrow{D_3} \right] q \).

- Quenched calculation complete with NPR,

- \( Z = 1.02(9) \), with \( \overline{\text{MS}} \) 2 GeV, 2-loop running,
- no curvature seen in the chiral limit.

- Dynamical calculation ongoing.
Transversity, $\langle 1 \rangle_{\delta u-\delta d}$, calculated with $O_{34}^{\sigma q} = \bar{q} \gamma_5 \sigma_{34} q$.

- Quenched calculation complete with NPR,
  
  \[ \langle 1 \rangle_{\delta u-\delta d} = 1.193(30), \overline{\text{MS}} \ (2 \text{ GeV}) \ 2\text{-loop running}, \]
  
  - QCDSF (quenched continuum): $\langle 1 \rangle_{\delta u-\delta d} = 1.214(40), \overline{\text{MS}} \ (1 \text{ GeV}) \ 1\text{-loop perturbative}$. 

- Dynamical calculation ongoing, lacks NPR,
$d_1$: twist-3 part of $g_2$ ($\langle x \rangle_{\Delta q}$ is twist-2),

$$2 \int_0^1 dx^n g_2(x, Q^2) = \frac{1}{2n+1} \sum_{q=u,d} [e_{2,n}^q(\mu^2/Q^2, g(\mu)) d_n^q(\mu) - 2e_{1,n}^q(\mu^2/Q^2, g(\mu)) \langle x^n \rangle_{\Delta q}(\mu)],$$

calculated with $\mathcal{O}_{[\sigma \mu_1] \mu_2 \cdots \mu_n}^{[5]q} = \bar{q} \left[ \left( \frac{i}{2} \right)^n \gamma_5 \gamma_{[\sigma} \overset{\leftrightarrow}{D}_{\mu_1}] \cdots \overset{\leftrightarrow}{D}_{\mu_n} - \text{traces} \right] q$.

- negligible in Wandzura-Wilczek relation, $g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$,
- but need not be small in a confining theory (Jaffe and Ji, Phys. Rev. D43, 91),

quenched unrenormalized,

dynamical unrenormalized,

- small in the chiral limit (no power divergent mixing),
- disagree with Wilson fermion results (which suffer from power divergent mixing)?
Nucleon decay (Yasumichi Aoki): proton decay with dimension 6 operators such as

\[ \langle \pi^0 e^+ | q q l | p \rangle \]

or more precisely the hadronic matrix elements in general take the form of

\[ \langle \pi^0 | i \epsilon_{ijk} (u^{iT} C P_{L/R} d^j) P_L u^k | p \rangle \]

(SUSY) GUT processes: classification by \( SU(3) \times SU(2) \times U(1) \) leads to a complete set of operators. Relevant for \( p/n \) decay are,

\[ \langle \pi^0 | i \epsilon_{ijk} (u^{iT} C P_{L/R} d^j) P_L u^k | p \rangle, \]
\[ \langle K^0 | i \epsilon_{ijk} (u^{iT} C P_{L/R} s^j) P_L u^k | p \rangle, \]
\[ \langle K^+ | i \epsilon_{ijk} (u^{iT} C P_{L/R} d^j) P_L s^k | p \rangle, \]
\[ \langle \eta | i \epsilon_{ijk} (u^{iT} C P_{L/R} d^j) P_L u^k | p \rangle, \]
and those obtained through the exchange of \( u \) and \( d \).

Lattice methods:

- indirect: chiral perturbation (tree level) + low-energy constant (lattice), \( ie \)

\[ \mathcal{L}_\chi(\text{mesons and baryons}: D, F, f_{\text{meson}}, m_{\text{baryon}}) + \text{(baryon decay interaction: } \alpha, \beta) \],

- direct: calculate all the relevant 2- and 4-point functions on the lattice.
Issues:

- direct method is about 10 times more expensive,
- indirect and direct results disagree (Gavela et al (1989)),
- $|\text{indirect}| = |\text{direct}| + \sim 50\%$ (JLQCD (2000)).

Direct method:

$$
\langle \pi^0 | 2 \epsilon_{ijk}(u^T C P_{L/R} d^j) P_L u^k | p \rangle = P_L [W_0(q^2) - W_q(q^2)i(\gamma q)] u_p,
$$

where $q$ is the momentum transfer of $p \rightarrow \pi^0$.

- as $i(\gamma q)v_e \sim m ev_e$ is negligible, we need to extract $W_0$,
- yet the mixing of $W_q$ is inevitable because we also need to project to positive parity proton,

$$
\text{tr} \left( P_L [W_0 - W_q i(\gamma q)] \frac{1 + \gamma_4}{2} \right) = W_0 - iq_4 W_q,
$$

- we go around this by injecting finite momentum (JLQCD, PRD 62, 014506 (2000)),

$$
\text{tr} \left( P_L [W_0 - W_q i(\gamma q)] \frac{1 + \gamma_4}{2} i\gamma_j \right) = q_j W_q.
$$

Slightly different sequential propagators are used.
Remaining problems:

- chiral symmetry,
  - previous studies used Wilson fermions which explicitly break chiral symmetry,
    \[ O_{RL}^{\text{cont}} = Z O_{RL}^{\text{latt}} + Z_{\text{mix}} O_{LL}^{\text{latt}} + Z'_{\text{mix}} O_{\gamma L}^{\text{latt}} \]
  - so the results need not match the chiral perturbation,
  - with DWF better chiral symmetry, the indirect method may work.

- \( \mathcal{O}(a) \) scaling violation,

- quenched approximation.

DWF:

- good chiral symmetry, \( O_{RL}^{\text{cont}} = Z O_{RL}^{\text{latt}} \),
  - should match the chiral perturbation at finite \( a \),
  - if the low-energy coefficients are calculated on the lattice,
  - note \( f_\pi \) and \( g_A (= D + F) \) are consistent with experiment within a few \% even at finite \( a \),

- scaling violation starts at \( \mathcal{O}(a^2) \),
Renormalization: NPR works well, one-loop matching from MOM to $\overline{\text{MS}}$(NDR), two-loop running to 2 GeV.

Quenched results: the direct and indirect methods disagree with each other. We have to

- follow through the direct method, or
- work out higher order chiral pertubation.
Quenching error: estimated in the indirect method, appear small from $\frac{1}{2}m_s \leq m_{\text{sea}} \leq m_s$.

The dynamical result shows stronger dependence on $m_{\pi}$, but the extrapolation to the chiral limit is consistent with that of the quenched within $\sim 20\%$ error.

Summary of the low energy parameter of nucleon decay at the renormalization scale $\mu = 2$ GeV. Quoted errors for DWF are statistical only. $\alpha + \beta = 0$ within the error.

<table>
<thead>
<tr>
<th>Fermion</th>
<th>Wilson$^a$</th>
<th>DWF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_f$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$a$ [fm]</td>
<td>0</td>
<td>0.15</td>
</tr>
<tr>
<td>$</td>
<td>\alpha</td>
<td>$ [GeV$^3$]</td>
</tr>
<tr>
<td>$</td>
<td>\beta</td>
<td>$ [GeV$^3$]</td>
</tr>
</tbody>
</table>

$^a$Tsutsui et al., [CP-PACS Collaboration], arXiv:hep-lat/0402026.

Need to explore much lighter quark mass with dynamical flavors. The direct method is favored.
New, this year, are

- **axial charge**
  - dynamical result seems to follow the quenched,

- **quark density** $\langle x \rangle_{u-d}$,
  - quenched calculation complete with NPR (no curvature seen in the chiral limit),
  - dynamical calculation ongoing, lacks NPR,

- **polarization** $\langle x \rangle_{\Delta u-\Delta d}$,
  - quenched calculation complete with NPR (no curvature seen in the chiral limit),
  - dynamical calculation ongoing, lacks NPR,

- **transversity** $\langle x \rangle_{\delta u-\delta d}$,
  - quenched calculation complete with NPR,
  - dynamical calculation ongoing, lacks NPR,

- **$d_1$: twist-3 part of $g_2$ ($\langle x \rangle_{\Delta q}$ is twist-2),**
  - negligible in Wandzura-Wilczek relation of $g_1$ and $g_2$,
  - but need not be small in a confining theory (Jaffe and Ji, Phys. Rev. D43, 91),
  - small in the chiral limit in both quenched and dynamical (unrenormalized),
  - disagree with quenched Wilson fermion results (which suffer from power divergent mixing)?

- **Nucleon decay**:
  - quenched calculation complete with NPR, in favor of the direct method,
  - dynamical calculation well under way.
Conclusions

• Quenched calculations are almost complete with NPR.
• $N_f = 2$ dynamical calculations are well under way.
• Axial charge: dynamical result seems to follow the quenched,
  – seem to agree well with the experiment,
  – no curvature seen down to 390 MeV pion mass.
• Moments of structure functions: quenched results almost complete with NPR,
  – no curvature seen in $\langle x \rangle_{u-d}$, $\langle x \rangle_{\Delta u-\Delta d}$ and $\langle 1 \rangle_{\delta u-\delta d}$ down to 390 MeV pion mass,
  dynamical calculations are ongoing,
  – $d_1$ in the chiral limit seems small in both quenched and dynamical.
• Nucleon decay: quenched calculation almost complete with NPR,
  – favors the direct method,
  dynamical calculation well under way.

Immediate future

• Publish quenched results for structure functions and nucleon decay.
• Finish ongoing dynamical calculations (QCDSP/QCDOC).
• Explore lighter quark mass and (2+1)-flavor dynamical (QCDOC).
• Turn on observables with finite momentum: some form factors, e.g. $F_1$, $F_2$, $g_P$ and electric dipole and higher
  moments of the structure functions.