Lattice Perturbation Theory for Heavy Quark Physics

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ILFT Workshop

Shuzenji, Izu, Japan

Sept 23, 2004

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Outline

- The need for lattice perturbation theory
- The Fermilab action
- Techniques for lattice PT
- Current matching calculations

Need for Lattice Perturbation Theory

- Monte Carlo simulation times scale as $\frac{1}{a^6}$
- Use improved actions to reduce cutoff effects

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- Conversion between α_V and $\alpha_{\overline{MS}}$
- Matching LEFT operators to QCD one

$$\mathcal{O}^{\text{Cont}} = Z_{\mathcal{O}} \left[\mathcal{O}_0^{\text{latt}} + \sum_n c_n \mathcal{O}_n \right]$$

- Short distance coefficients, can match perturbativly if $\alpha_V(\pi/a)$ is not too large.
- Important for determining decay constants, form factors, etc.

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$$\begin{aligned} S &= a^4 \sum_x \bar{\psi}(x) \left\{ m_0 + \frac{1}{2} \left[(1+\gamma_0) D_0^- - (1-\gamma_0) D_0^+ \right] \right. \\ &+ \left. \zeta \vec{\gamma} \cdot \vec{D} - \frac{a r_s \zeta}{2} \triangle^{(3)} - \frac{i a c_B \zeta}{2} \vec{\Sigma} \cdot \vec{B} - \frac{a c_E \zeta}{2} \vec{\alpha} \cdot \vec{E} \right\} \psi(x) \end{aligned}$$

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- At tree level $\zeta = r_s = c_B = 1$, and $c_E = \frac{1 + \frac{m_0 (2+m_0)}{4(1+m_0)}}{1+m_0} \approx 1$

Lattice Perturbation Theory Techniques

- The lattice cutoff means that Feynman rules are much more complicated
- Non-linear connection $U_\mu(x) = \mathrm{e}^{igaA_\mu(x)}$ produces an infinite tower of extra vertices
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- Use of highly improved gauge and quark actions adds additional operators
- Automated perturbation theory addresses these issues
- Generate vertex functions (and diagrams) automatically
- Perform loop integrals using VEGAS

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- Correct $\mathcal{O}(a)$ errors by using rotated spinors

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- Matching condition: $\bar{\psi}_c\Gamma_{\mu}\psi_c - Z_{\Gamma}\bar{\Psi}_l\Gamma\Psi_l = 0 + \mathcal{O}(a^2)$

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$$Z_{\Gamma}^{h\ell} = \sqrt{Z_{V_4}^{hh} Z_{V_4}^{\ell\ell}} \rho_{\Gamma}^{h\ell}$$

- $Z_{V_4}^{qq}$ can be determined non-perturbativly
- Expect that the perturbative corrections to ρ will be small, for $m_0^h a$ not too large
- Verified (to one loop) by Harada *et.al.* for Wilson glue and clover light quarks
- Improved glue? Asqtad light quarks?

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- Fermilab approach means $Z_{\Gamma_4} \neq Z_{\Gamma_i}$
- To compute ρ for a given mass pair we need to evaluate six Z factors ($\Gamma = \gamma_4$, $\Gamma = \gamma_i$, $\Gamma = \gamma_5\gamma_4$ and $\Gamma = \gamma_5\gamma_i$)

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- Wilson-like fermions have a non-trivial wavefunction renormalization at tree-level

$$Z_{\mathcal{O}}^{(0)} = \sqrt{(1+m_q)(1+m_Q)}$$

Computing $Z^{(1)}$ (cont)

- Account for dominant mass dependence by computing $\mathcal{Z} = Z/Z^{(0)}$

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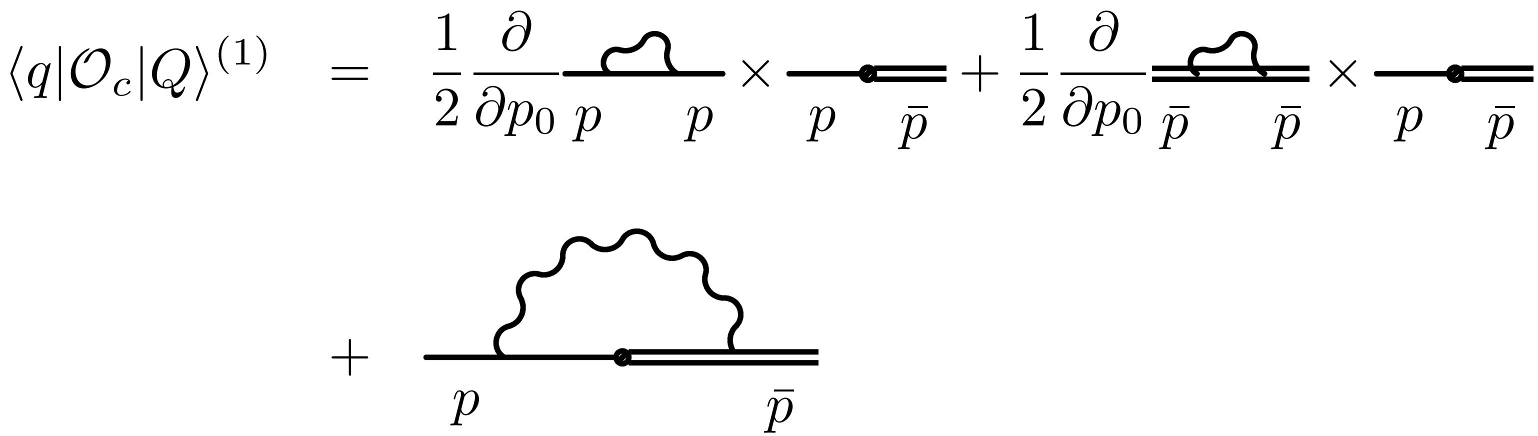
- Important to use the same infrared regulator for both lattice and continuum theories
- Results presented here use a gluon mass
- Could also use twisted periodic boundary conditions
- Lattice to lattice matching is possible for these currents

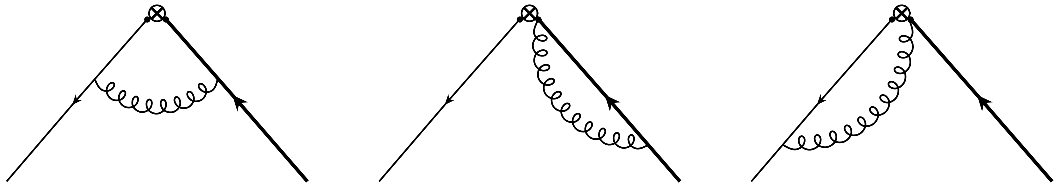
Calculation Details (continuum)

- Continuum calculation uses a gluon mass IR regulator
- Pauli-Villars regulator Λ for the UV
- Continuum result does not depend on Λ
- Three diagrams



Calculation Details (lattice)

- Lattice calculation uses a gluon mass IR regulator
- Five diagrams without the rotations
- For clover light, and Fermilab heavy, the rotations add 12 further diagrams
- For Asqtad light the rotations add 3 diagrams



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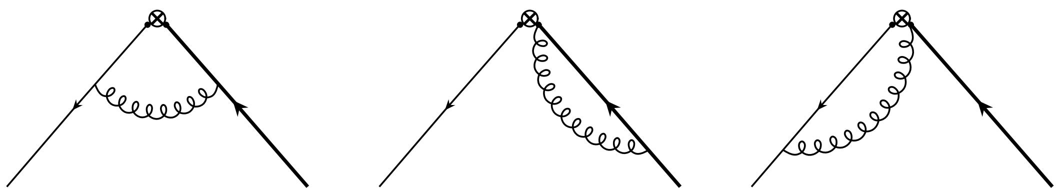

- evaluate diagrams by brute force VEGAS integration

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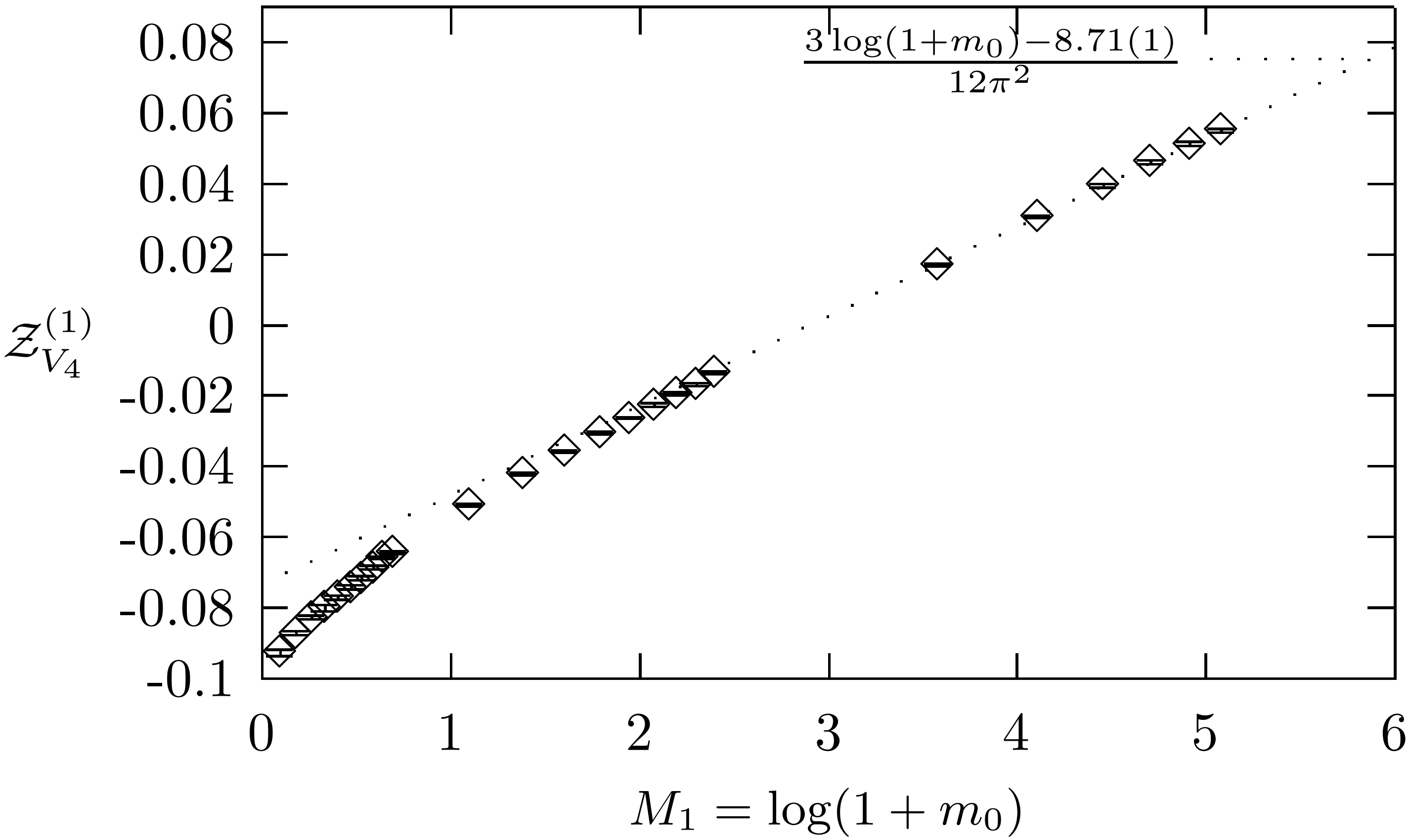
- All these checks are satisfied

$\mathcal{Z}_{V_4}^{h\ell}$ with Improved glue



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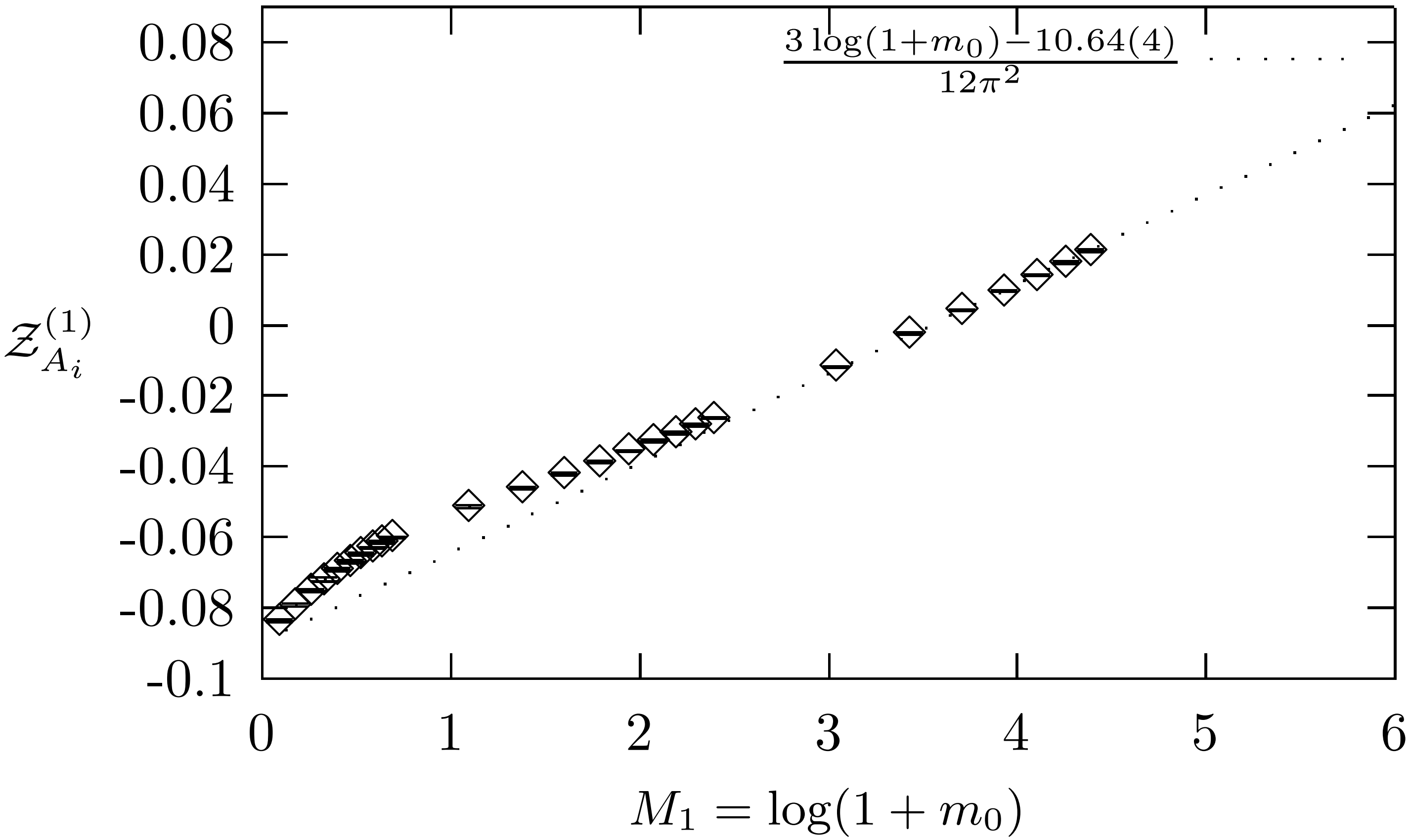
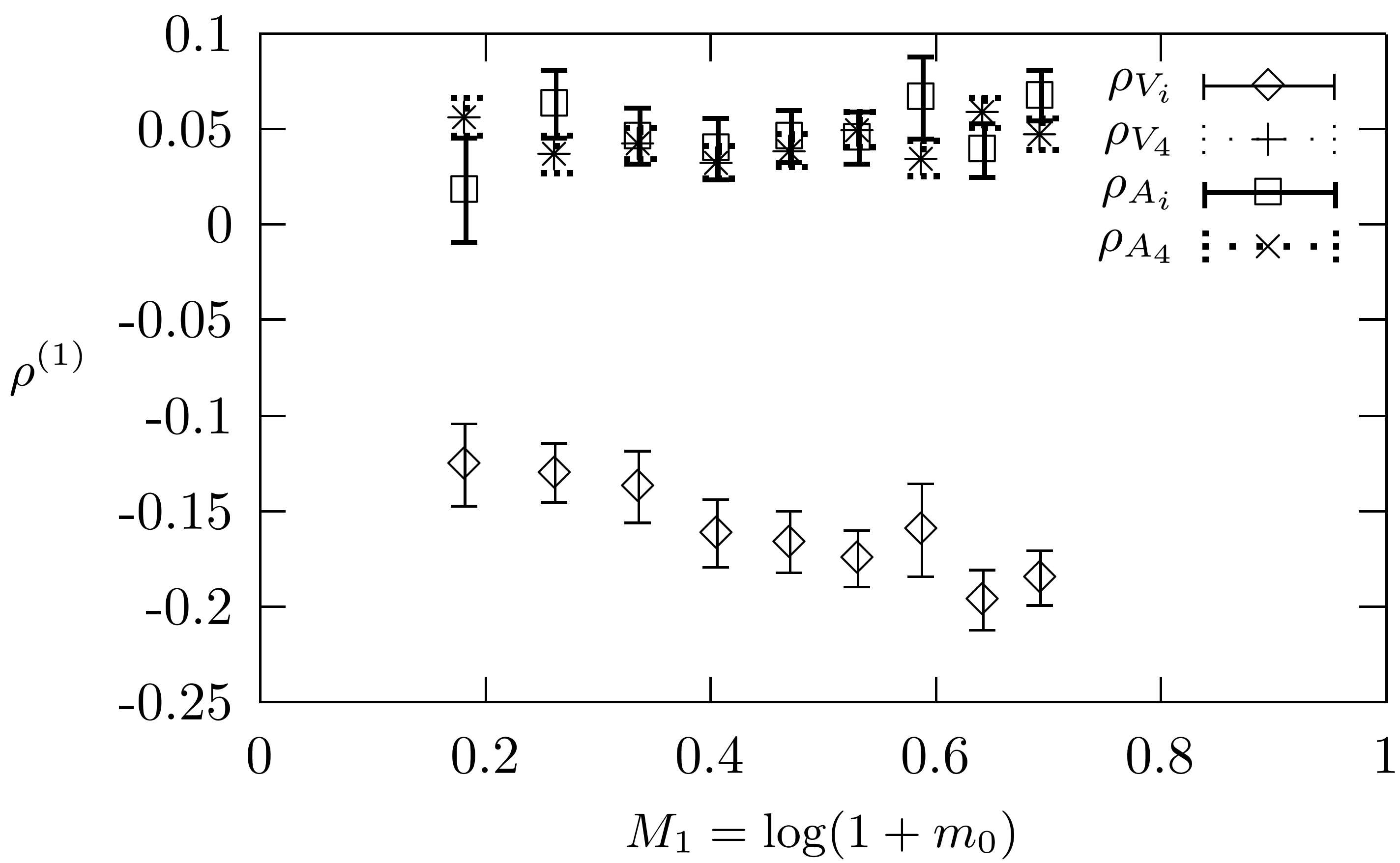
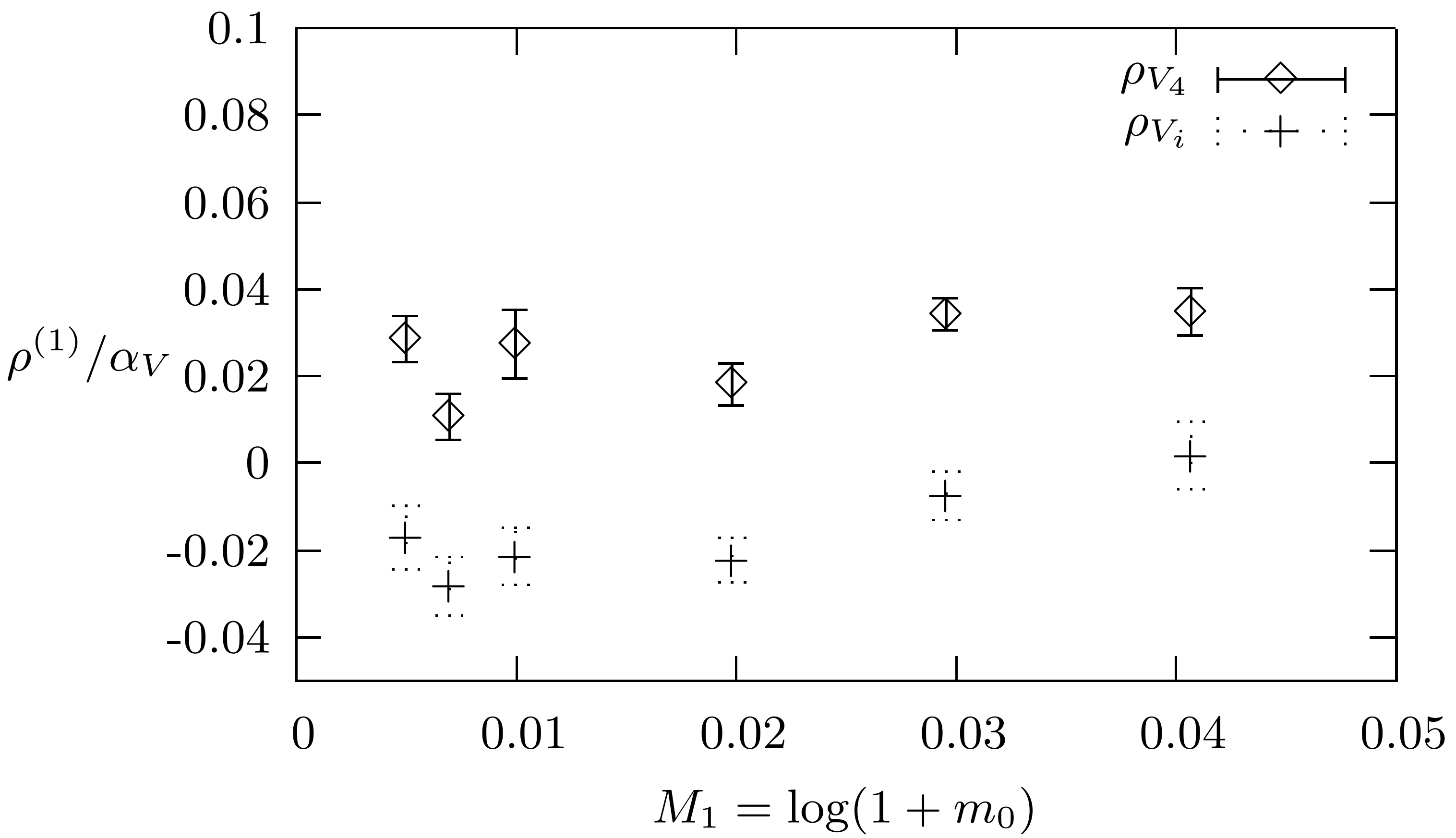


ρ factors for light clover quarks and improved glue



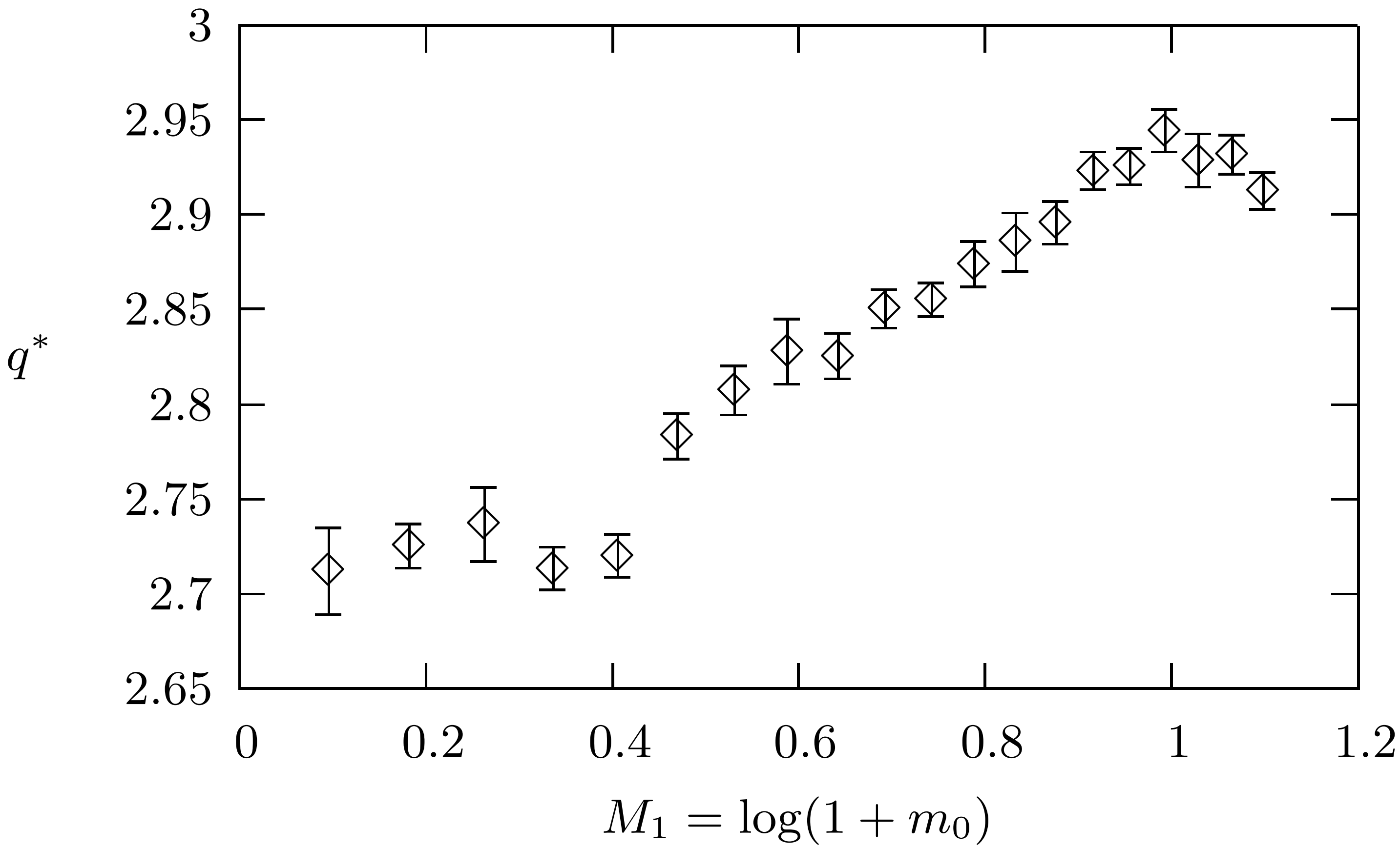
$m_{0,\ell} = 0.01$

ρ factors for light Asqtad quarks and improved glue



$m_{0,c} = 0.575$

q^* for Z_{V_4}



$m_{0,\ell} = 0.001$

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Use of ρ factors greatly reduces perturbative errors due to current matching

Conclusions and Outlook

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- Smallness of one loop terms means that two loop PT is not needed
- Does a similar factorization work for NRQCD?
- Dominant error is now matching of parameters in the action
- These calculations are nearing completion (Fermilab and NRQCD)