B_K from Quenched and Dynamical Domain-wall QCD

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for RIKEN BNL Columbia Collaboration

Introduction

- RBC's longstanding project for kaon physics (2002–)

  Domain-wall fermion + DBW2 gauge action

spectrums	decay constants	B-parameter	$\Delta I = 1/2$ rule	ϵ'/ϵ
$m_{PS},\ m_\rho, m_{\rm res},\ Z_A$	$f_\pi,\ f_K$	B_K		

stage 1 $\longrightarrow$ stage 2 $\longrightarrow$ stage 3

  – Quenched calc. : $a^{-1} \approx$ 2 GeV, 3 GeV — scaling behavior ?

  – Dynamical calc. : $N_f = 2,\ a^{-1} \approx$ 1.7 GeV, three $m_{\rm sea}$ — quenching effect ?

  T. Izubuchi's talk

- In this talk –

  – Detail of our numerical simulation (mainly quenched)

  – Quenched results — stage 1: $f_\pi,\ f_K$, NPR, B_K

  – Dynamical results

  – Conclusion

Simulation Parameters

QUENCHED

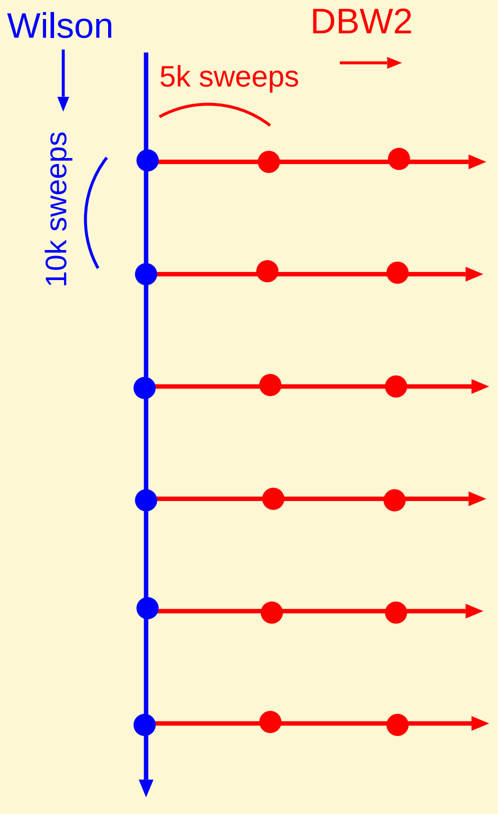
Two lattice scales are examined

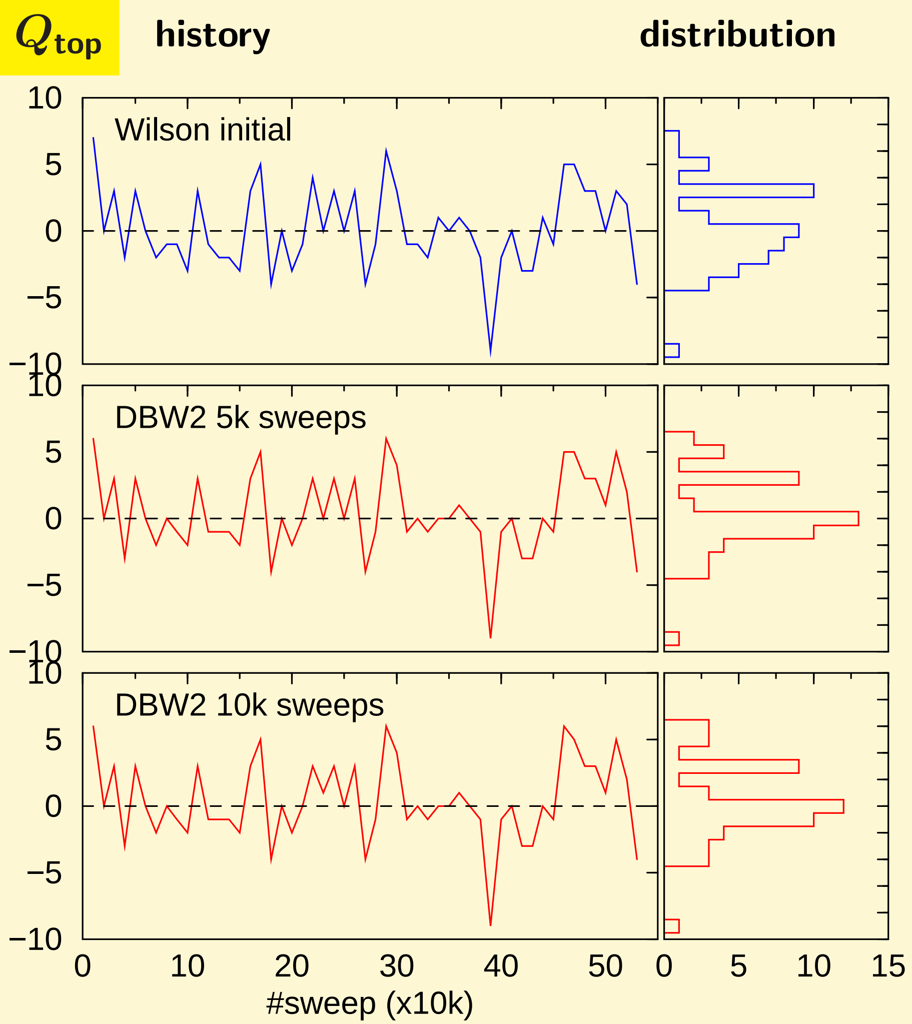
$N_f = 2$ DYNAMICAL

$m_{\text{sea}} = 0.02, 0.03, 0.04$

	$\beta = 1.22$	$\beta = 1.04$	$\beta = 0.80$
size	$24^3 \times 48$	$16^3 \times 32$	$16^3 \times 32$
L_s	10	16	12
M_5	1.65	1.70	1.80
wall-source	$t = 7,\ 41$	$t = 5,\ 27$	$t = 4,\ 28$
statistics for ME	106	202	94
NPR	53	50	40
m_f	$0.008 \times 1, 2, 3, 4, 5$	$0.01 \times (1), 2, 3, 4, 5$	$0.01 \times 1, 2, 3, 4, 5$
$m_s/2$	0.01595(78)	0.02246(38)	$\simeq 0.021$
m_{res}	$0.9722(27) \cdot 10^{-4}$	$1.86(12) \cdot 10^{-5}$	$1.372(44) \cdot 10^{-3}$
$a^{-1}(m_\rho)$	2.914(54) GeV	1.982(21) GeV	1.690(53) GeV
Phys. volume	$(1.65\ \text{fm})^3$	$(1.61\ \text{fm})^3$	$(1.89\ \text{fm})^3$

- **Gauge generation for $\beta = 1.22$ for topological charge changing**





"$\mathcal{O}(a^2)$ modification" of the action

5k + 10k sweeps

well-distributed: $\langle Q_{top} \rangle = 0.38 \pm 0.29$

Quenched Results

- **Decay Constants**

wall-point AP: $\sum_{\mathbf{x}} \left\langle A_4^{\text{point}}(\mathbf{x}, t) P^{\text{wall}\,\dagger}(t_0) \right\rangle$

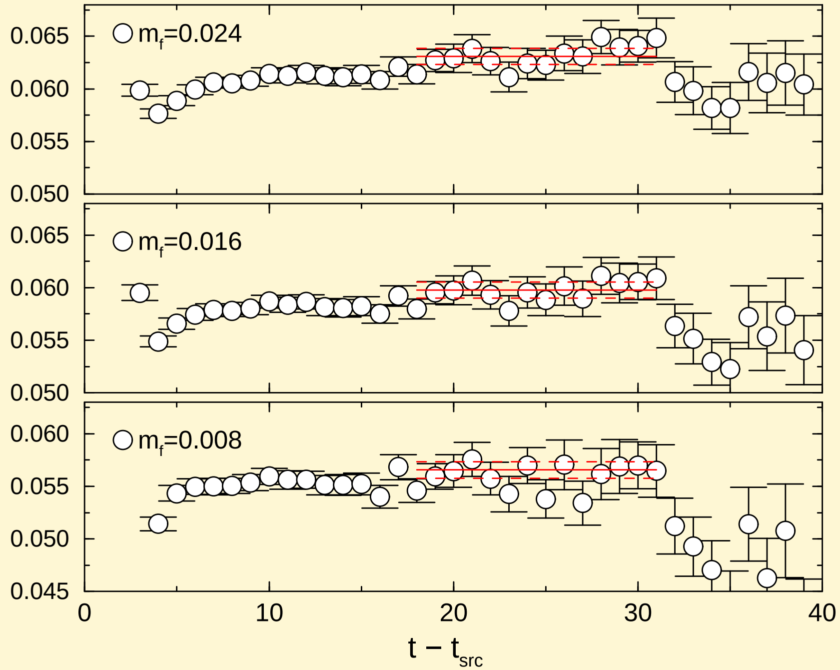
wall-wall PP: $\left\langle P^{\text{wall}}(t) P^{\text{wall}\,\dagger}(t_0) \right\rangle$

decay constant on the lattice

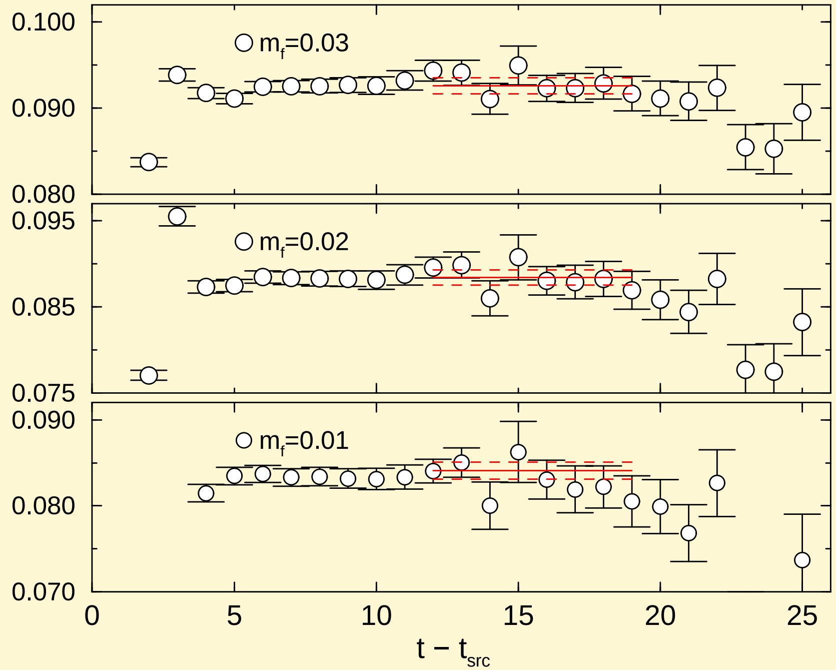
$$f_{PS}^{(\text{latt})} = \frac{\text{Amp.}[\text{wall-point AP}]}{\sqrt{\frac{m_{PS}}{2} \cdot V \cdot \text{Amp.}[\text{wall-wall PP}]}}$$

Amp's from simultaneous fit

$\beta = 1.22$



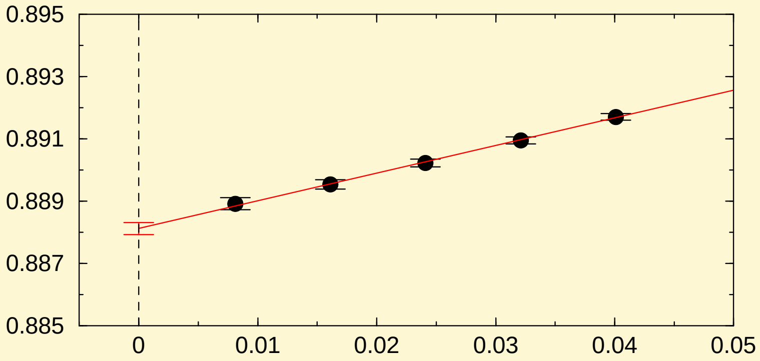
$\beta = 1.04$



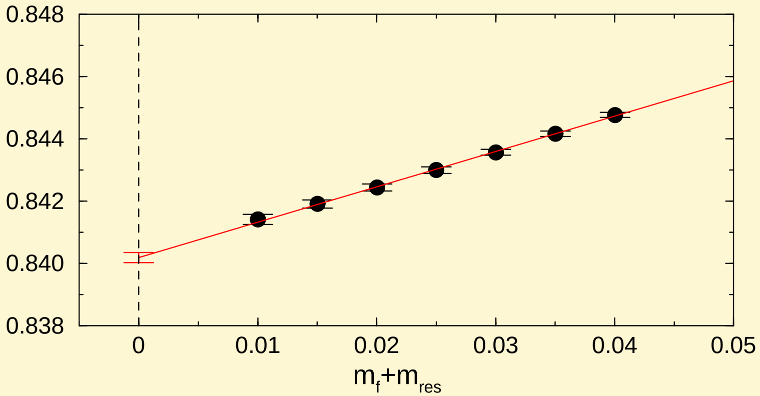
Renormalized Values

$$Z_A = \frac{\left\langle \mathcal{A}_4^{(\text{conserved})} P^\dagger \right\rangle}{\left\langle A_4 P^\dagger \right\rangle}$$

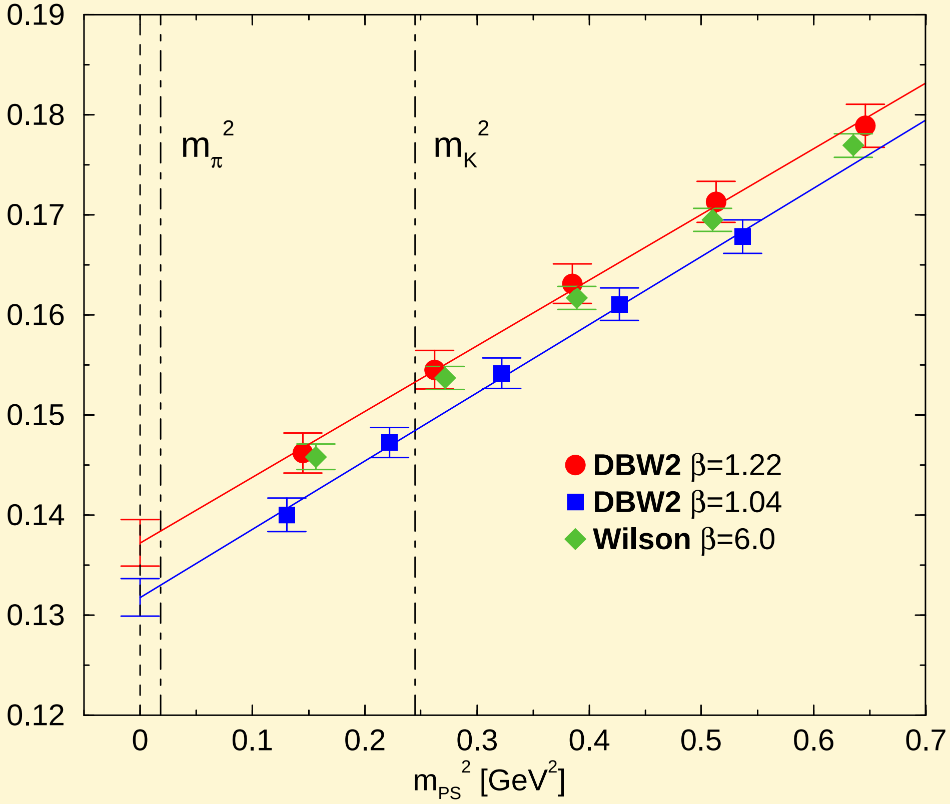
Z_A vs m_f, $\beta = 1.22$



$\beta = 1.04$



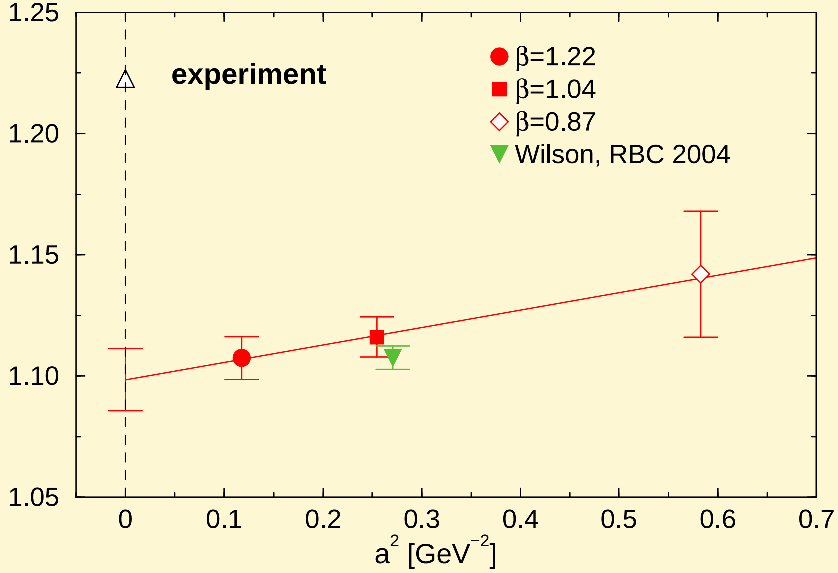
$f_{PS} = Z_A f_{PS}^{(\text{latt})}$ vs m_{PS}^2



GREEN: RBC DWF+Wilson, 2003

● Scaling Property of Decay Constants

f_K / f_π



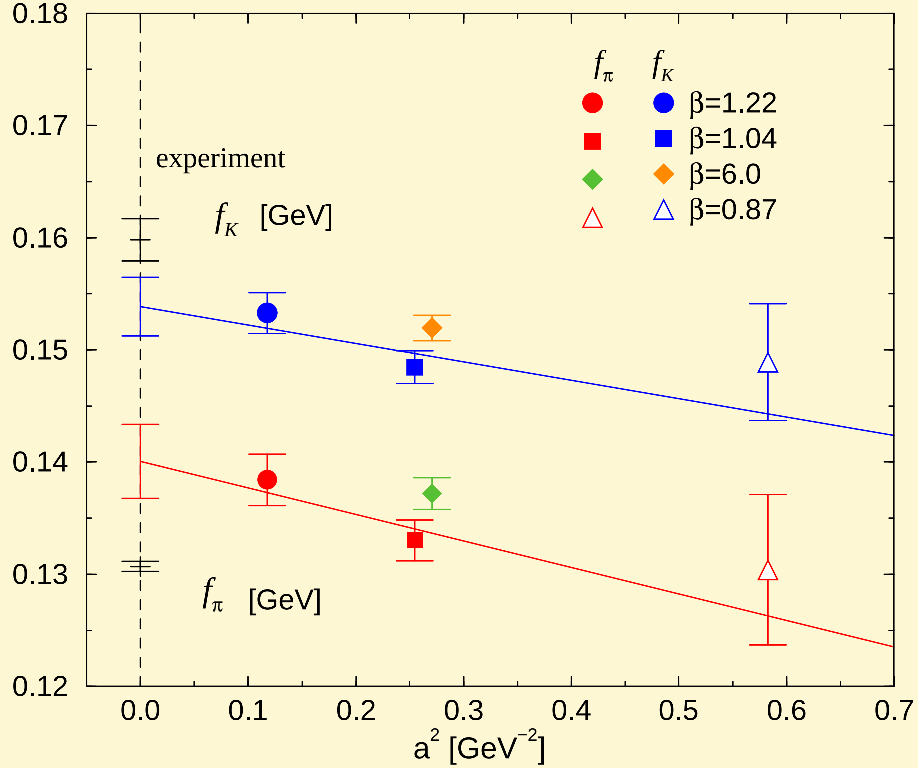
– **Continuum limit**

$$f_K / f_\pi = 1.098(13)$$

Roughly consistent with qChPT

Bernard & Golterman, 1992

f_π , f_K



– **Continuum limit** f_K : **4% smaller**

f_π : **7% larger**

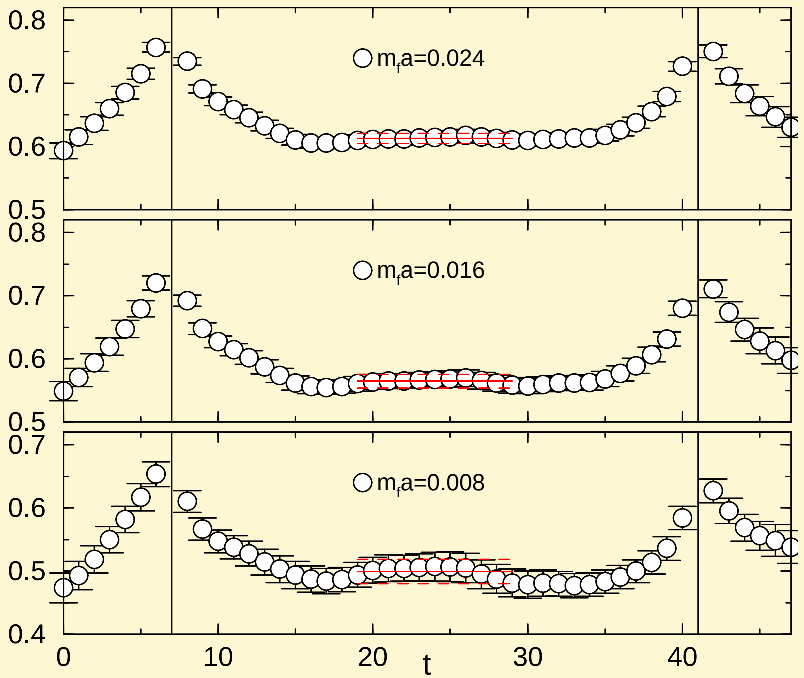
NNLO term in the chiral extapolation?

- **Kaon B-parameter B_K (bare value)**

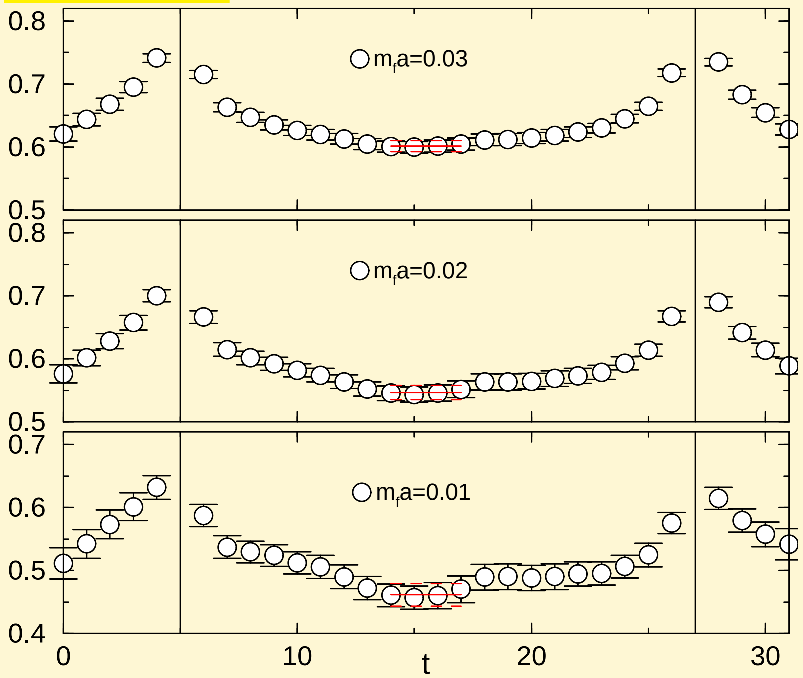
$$B_{PS}^{(\text{latt})} = \left.\frac{\sum_{\mathbf{x}} \langle P^{\text{wall}}(t_0') Q_{\Delta S=2}(\mathbf{x}, t) P^{\text{wall}} \rangle}{\frac{8}{3}\sum_{\mathbf{x},\mathbf{y}} \langle P^{\text{wall}}(t_0') A^{\text{point}}(\mathbf{x}, t)\rangle \langle A^{\text{point}}(\mathbf{y}, t) P^{\text{wall}}(t_0)\rangle}\right|_{t_0' \ll t \ll t_0}$$

Fairly good signals

$\beta = 1.22$

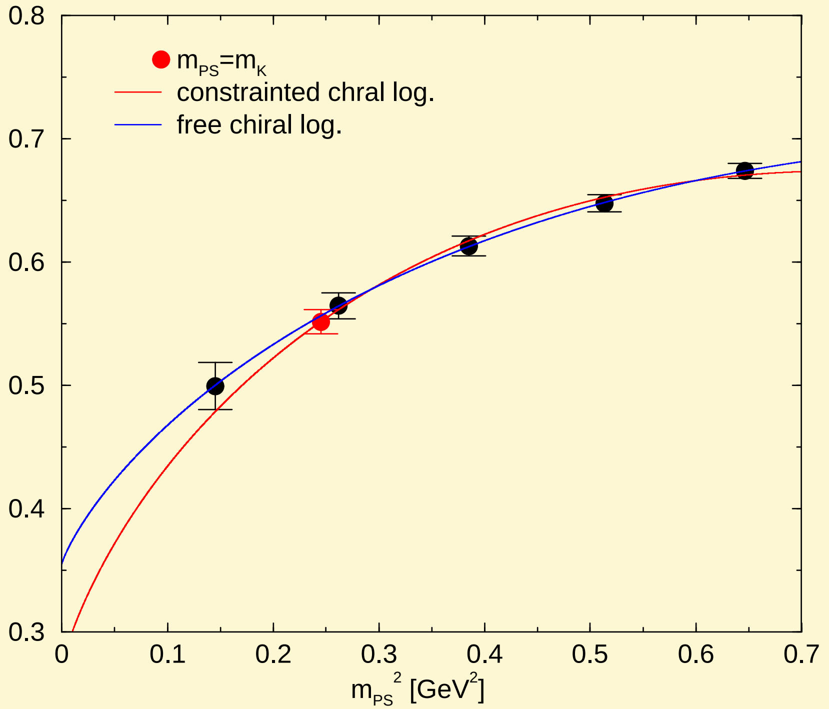


$\beta = 1.04$

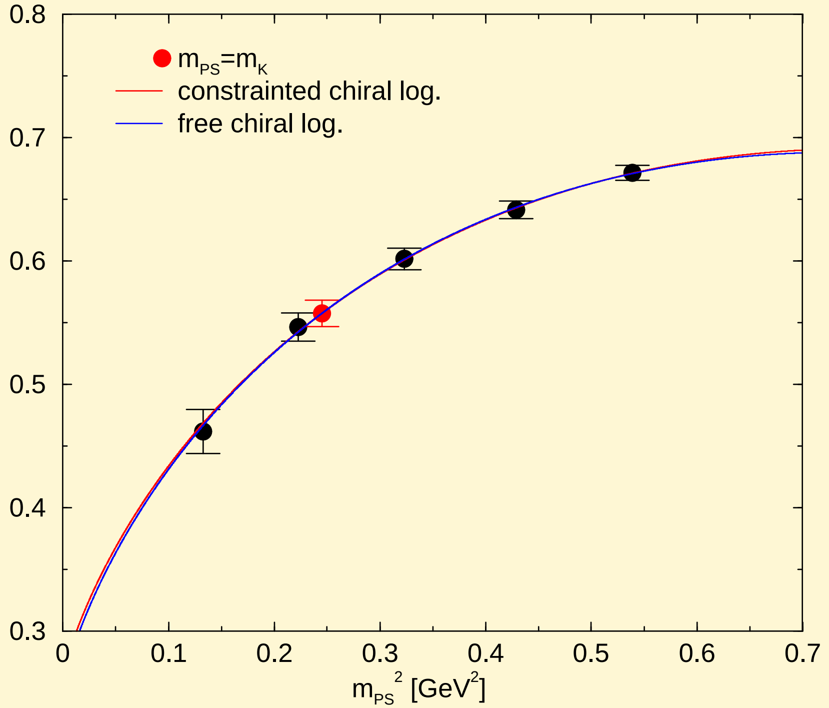


- **Chiral expansion** **RED:** $m_{PS} = m_K$

$B^{(\text{latt})}_{PS}$ vs m^2_{PS}, $\beta = 1.22$



$\beta = 1.04$



red line: $B_{PS} = \xi_0 \left[1 - \frac{6}{(4\pi f)^2} m^2_{PS} \ln m^2_{PS}\right] + \xi_2 \cdot m^2_{PS}$ **Sharpe, 1992**

blue line: $B_{PS} = \xi_0 + \xi_1 \cdot m^2_{PS} \ln m^2_{PS} + \xi_2 \cdot m^2_{PS}$

– $\beta = 1.22$: $\xi_1/\xi_0 \approx 50\%$ of $-6/(4\pi f)^2$, B_K differs by $< 1\%$

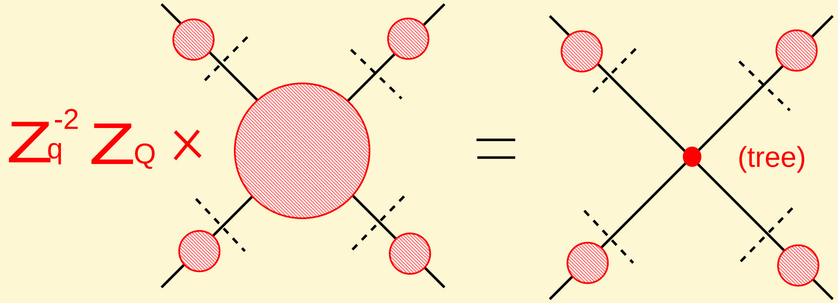
– $\beta = 1.04$: ξ_1/ξ_0 consistent with $-6/(4\pi f)^2$

- **Non-perturbative Renormalization (NPR) for B_K** Martinelli *et al.*, 1995

  renorm. condition: $Z_q^{-2} Z_Q \Gamma^{(4)}_{\text{latt}}(p, m_0, g_0^2; 1/a) = \Gamma^{(4)}_{\text{tree}}(p, m, g^2)$

  $\Gamma^{(n)}(p, m, g)$: **n-point Green function (momentum space)**

  $\Rightarrow \; Z_{B_K} = Z_Q / Z_A^2$



- **operator mixing**

  – **parity conserved part:** $\langle K | Q_{\Delta S=2} | \overline{K} \rangle = \langle K | \underbrace{(\bar{s}d)_V (\bar{s}d)_V + (\bar{s}d)_A (\bar{s}d)_A}_{\mathcal{O}_{VV+AA}} | \overline{K} \rangle$

  – **renormalized op:**

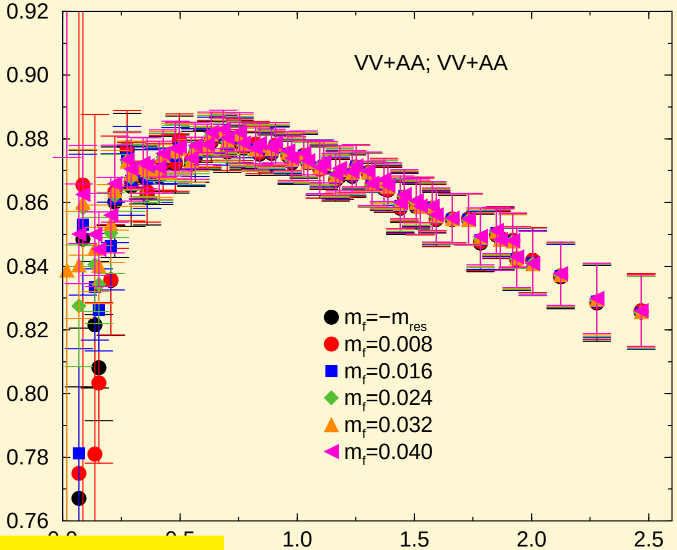
$$Q^{(\text{ren})}_{\Delta S=2} = Z_{VV+AA,\, VV+AA} \cdot \mathcal{O}_{VV+AA} + Z_{VV+AA,\, VV-AA} \cdot \mathcal{O}_{VV-AA} + Z_{VV+AA,\, SS-PP} \cdot \mathcal{O}_{SS-PP} + Z_{VV+AA,\, SS+PP} \cdot \mathcal{O}_{SS+PP} + Z_{VV+AA,\, TT} \cdot \mathcal{O}_{TT}$$

  operator mixing due to chiral symm. breaking

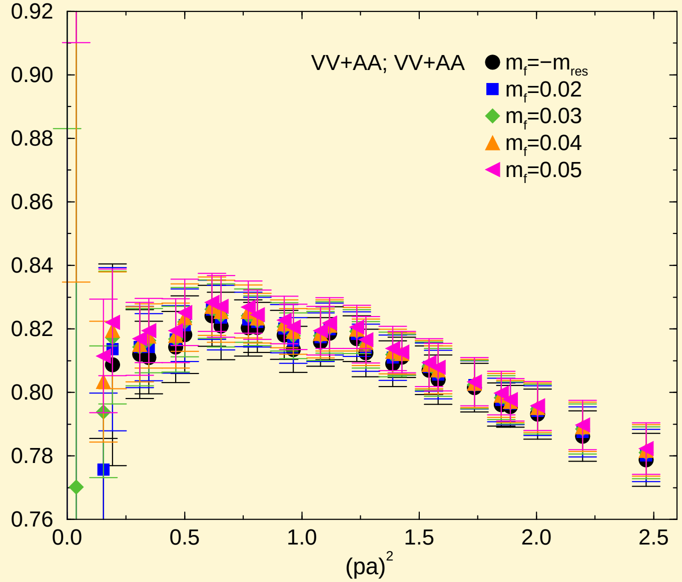
N.B. $L \leftrightarrow R$ **occurs twice in the mixing** $\Rightarrow \mathcal{O}(m_{\text{res}}^2)$ **effect**

- **renormalization condition to solve mixing:** $Z_q^{-2} Z_{ij} \Gamma^{\text{latt}}_{\mathcal{O}_j}(p;\ a^{-1}) = \Gamma^{\text{tree}}_{\mathcal{O}_i}$

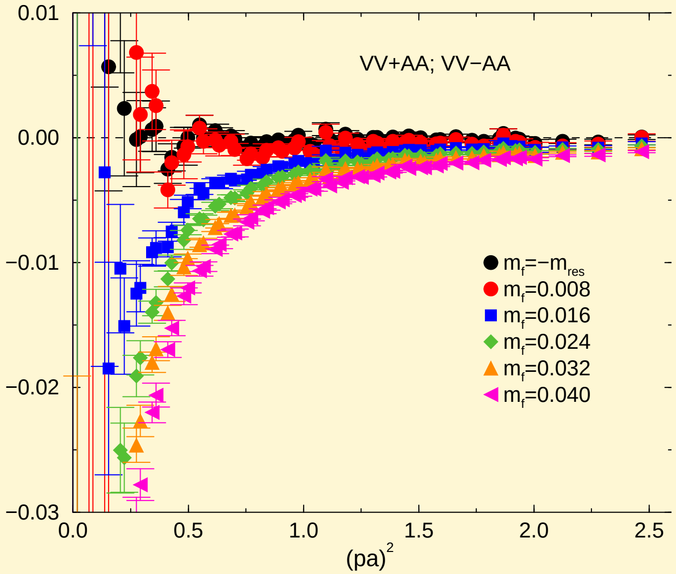
$Z_q^{-2}Z$ VV+AA;VV+AA $\beta = 1.22$



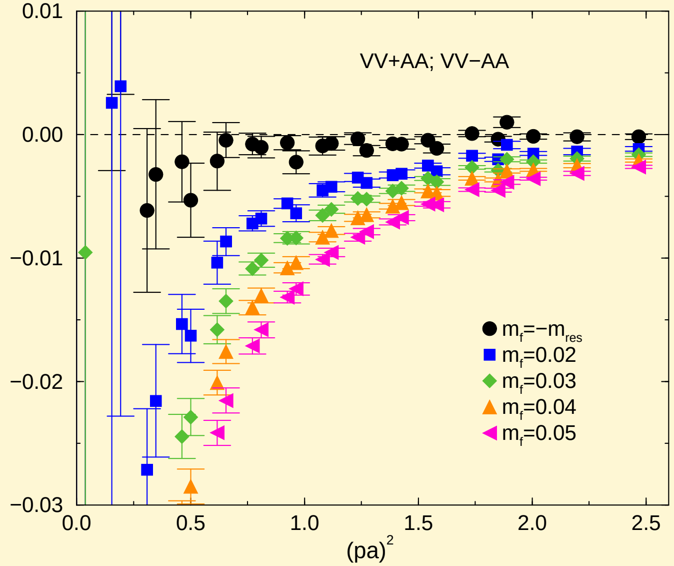
$\beta = 1.04$



$Z_q^{-2}Z$ VV+AA;VV-AA $\beta = 1.22$

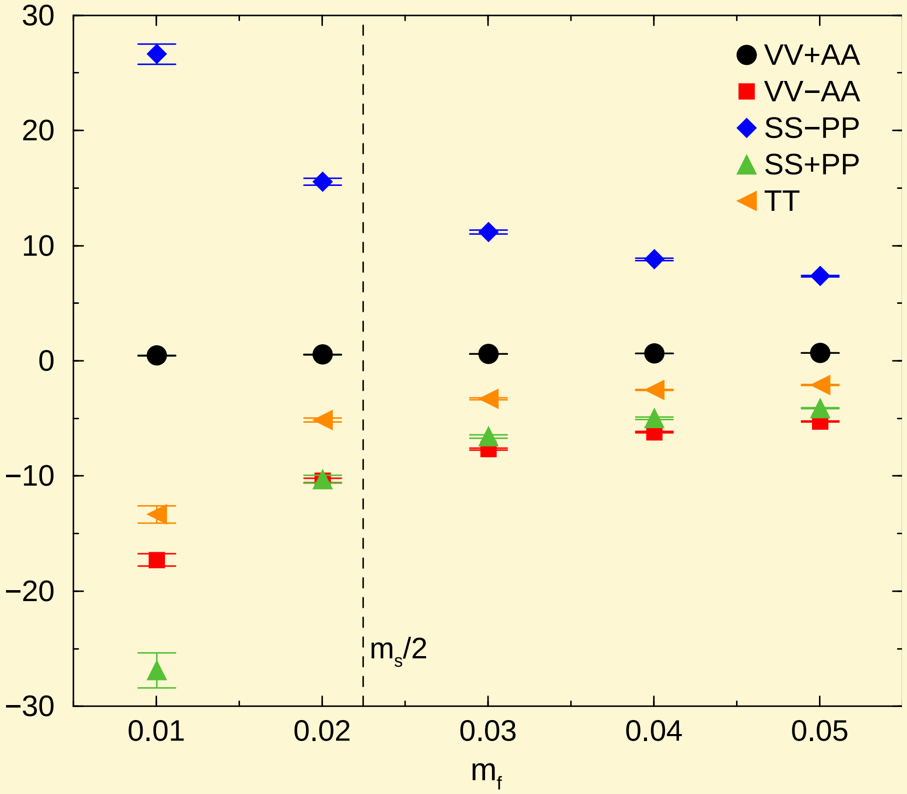


$\beta = 1.04$



- **Estimation of the operator mixing**
  - mass-pole for $SS \mp PP$ and TT $\Rightarrow$ Z_{ij} at largest p and heaviest m_f
  - B-parametes from all mixing operators $\Rightarrow$ B_i at $m_f = 0.02$

B-parameters, $\beta = 1.04$



$$\frac{Z_{VV+AA,VV+AA}}{Z_A^2} \cdot B^{(\text{latt})}_{VV+AA} = 0.518(20)$$

$$\frac{Z_{VV+AA,VV-AA}}{Z_A^2} \cdot B^{(\text{latt})}_{VV-AA} = 0.0022(29)$$

$$\frac{Z_{VV+AA,SS-PP}}{Z_A^2} \cdot B^{(\text{latt})}_{SS-PP} = -0.0085(12)$$

$$\frac{Z_{VV+AA,SS+PP}}{Z_A^2} \cdot B^{(\text{latt})}_{SS+PP} = 0.00669(89)$$

$$\frac{Z_{VV+AA,TT}}{Z_A^2} \cdot B^{(\text{latt})}_{TT} = -0.00047(36)$$

Negrigible!

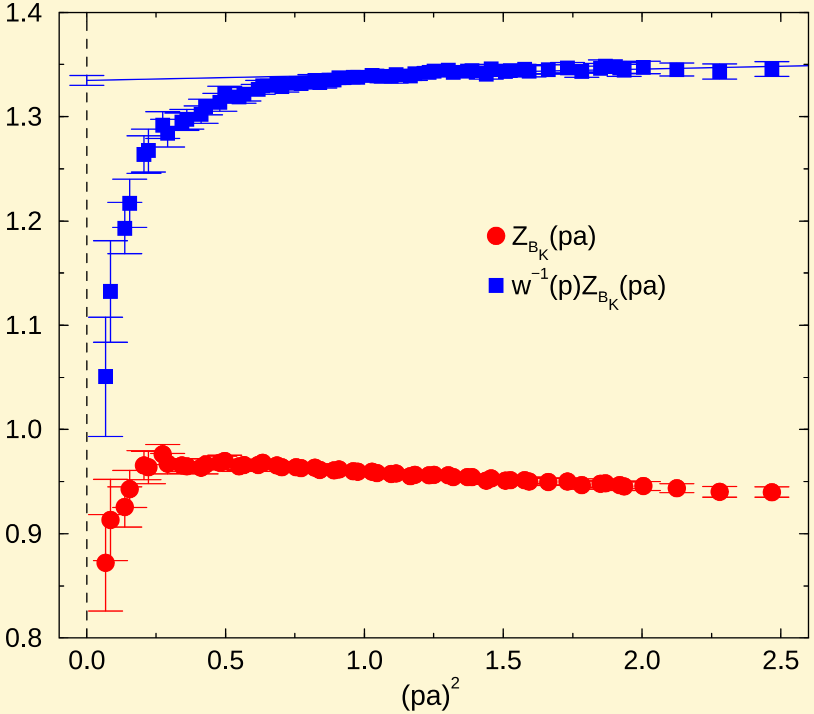
$$Z^{\text{RI/MOM}}_{B_K}(p) = \frac{Z_{VV+AA,VV+AA}(p)}{Z_A(p)^2}$$

- **Matching**

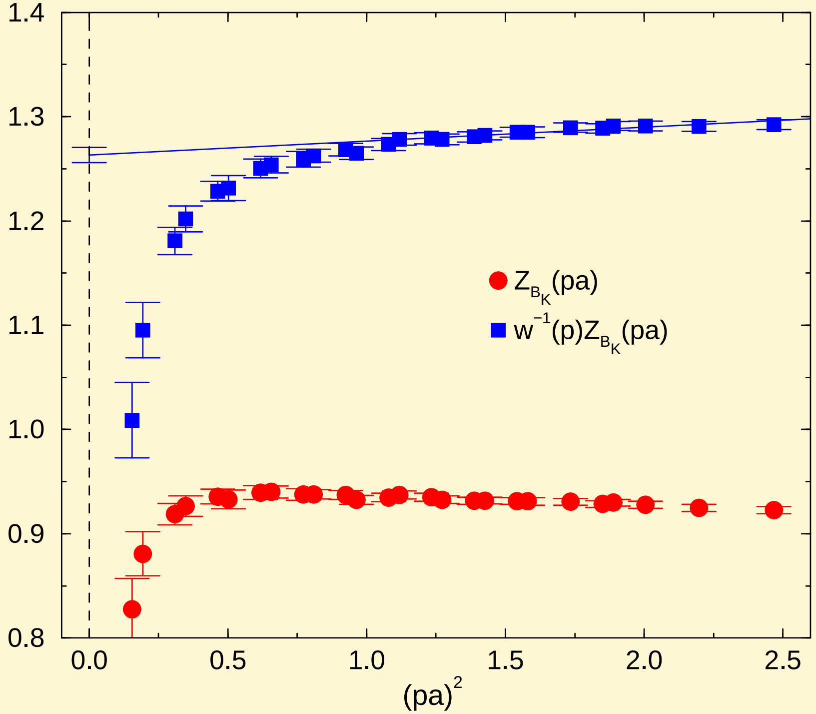
RGI value : $\hat{B}_K = \hat{Z}_{B_K} \cdot B_K^{(\mathrm{latt})}$

$$\hat{Z}_{B_K} = w^{-1}_{\mathrm{RI/MOM}}(\mu = p_{\mathrm{latt}}) \cdot Z^{\mathrm{RI/MOM}}_{B_K}(p_{\mathrm{latt}})$$ Ciuchini *et al.*, 1999

$\beta = 1.22$



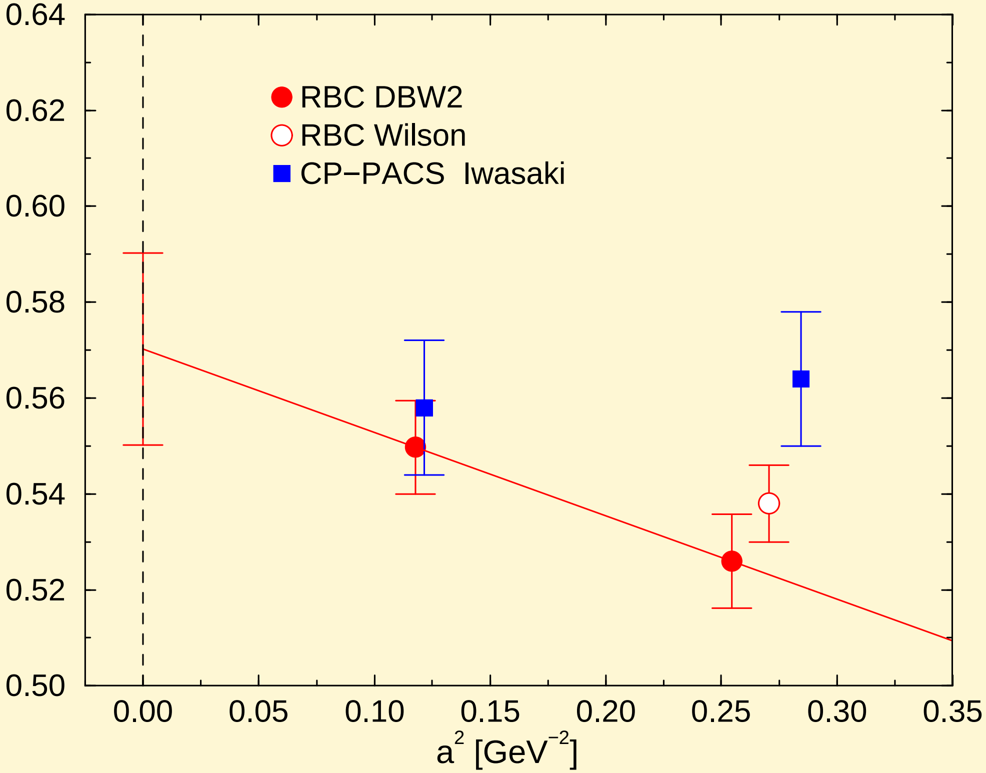
$\beta = 1.04$



$\overline{\mathrm{MS}}$ NDR value : $Z^{\overline{\mathrm{MS}}}_{B_K}(\mu = 2\ \mathrm{GeV}) = w^{-1}_{\overline{\mathrm{MS}}}(\mu = 2\ \mathrm{GeV}) \cdot \hat{Z}_{B_K}$

- **Scaling Property of B_K**

$B_K^{\overline{\text{MS}},\ \text{NDR}}(\mu = 2\ \text{GeV})$, comparison with previous works



Previous works:

(■) CP-PACS, 2001 (Iwasaki, PR)

– $16^3 \times 40,\ L_s = 16,\ a^{-1} = 1.88$ GeV

– $24^3 \times 60,\ L_s = 16,\ a^{-1} = 2.87$ GeV

(○) RBC, 2003 (Wilson, NPR)

– $16^3 \times 32,\ L_s = 16,\ a^{-1} = 1.92$ GeV

– difference at corse lattice, agreement at fine lattice

– continuum limit: $B_K^{\overline{\text{MS}},\ \text{NDR}}(\mu = 2\ \text{GeV}) = 0.569(21)$

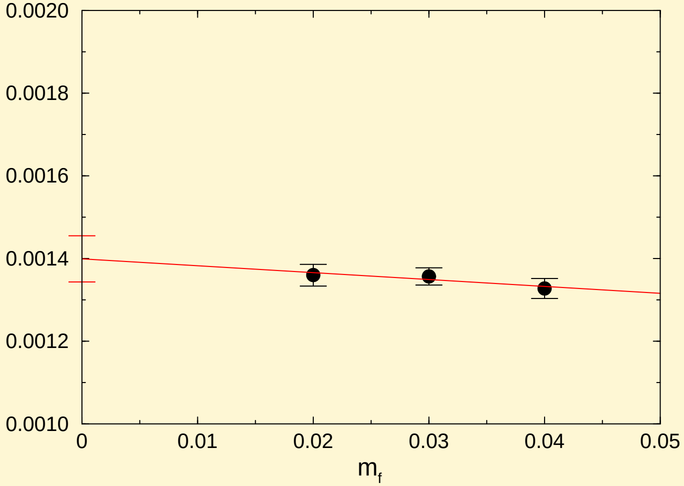
$\hat{B}_K = 0.762(27)$ **a few % (statistical) error**

Dynamical Results

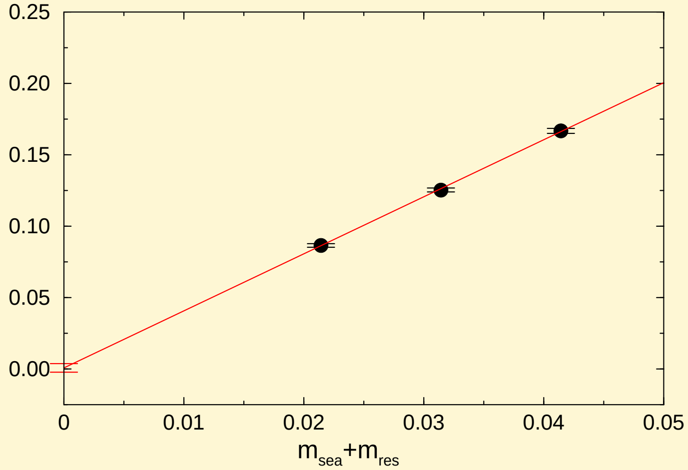
w/ degenerate masses, preliminary

- **Spectrums:** $m_{\text{sea}} = m_{\text{valence}}$ **(degenerate)** $\longrightarrow 0$

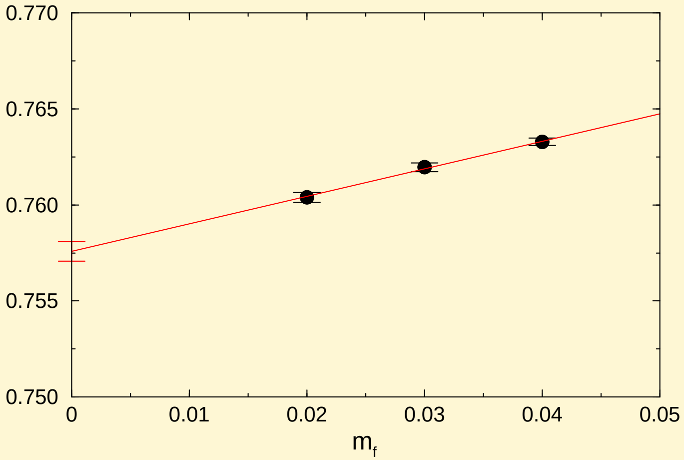
m_{res} vs m_f



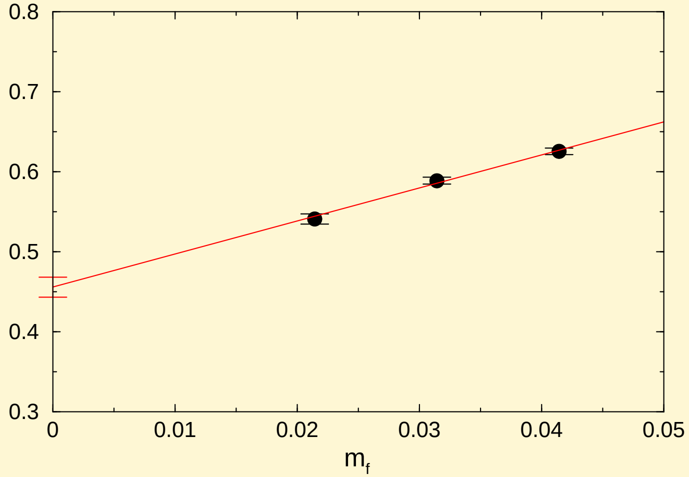
m^2_{PS} vs $m_f + m_{\text{res}}$



Z_A vs m_f



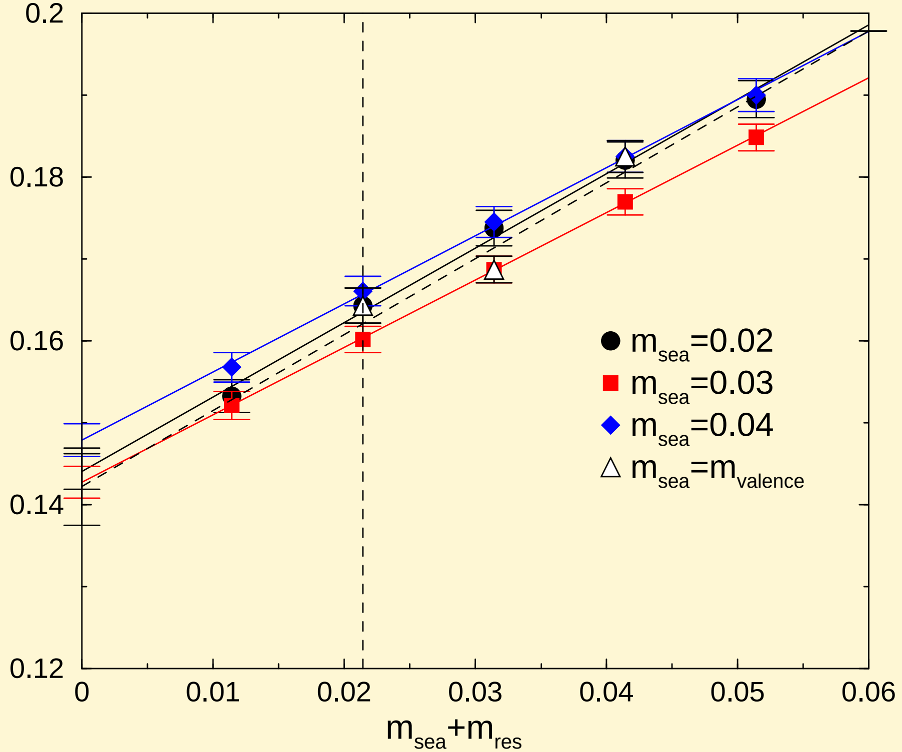
m_V vs $m_f + m_{\text{res}}$



$m_\rho = 770$ **MeV** $\Rightarrow \ a^{-1} = 1.690(53)$ **GeV**

- **Decay constants**

f_{PS} [GeV] vs $m_{\text{sea}} + m_{\text{res}}$



f_π : $m_{\text{valence}} = m_{\text{sea}} \longrightarrow 0$

linear fit: $142.2(4.7)$ **MeV**

$m_{\text{valence}} \longrightarrow 0, \;\; m_{\text{sea}} \longrightarrow 0$

linear fit: $138.7(4.6)$ **MeV**

f_K : $m_{\text{valence}} = m_s/2, \;\; m_{\text{sea}} \longrightarrow 0$

linear fit: $159.2(4.6)$ **MeV**

NOTE: PQChPT Golterman & Leung, 1998

$m_\pi^2 \equiv 2B_0 m_{\text{valence}} = 2B_0 m_{\text{sea}}$

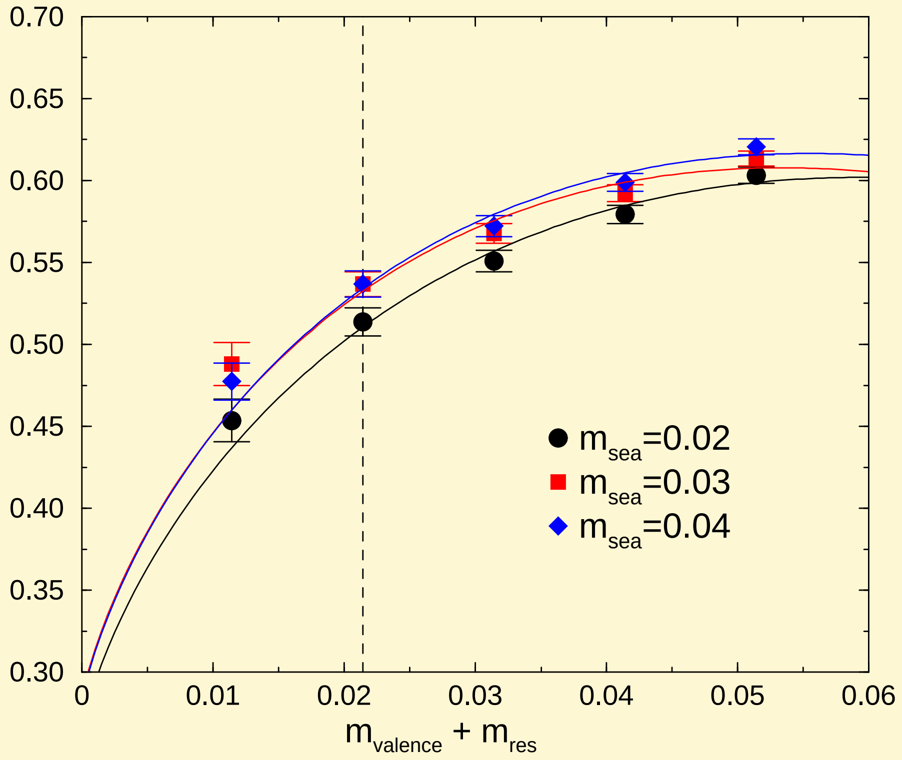
$$f_\pi = f\left[1 - \frac{N_f}{(4\pi f)^2} m_\pi^2 \ln\left(\frac{m_\pi^2}{\Lambda_\chi^2}\right)\right] + \mathcal{O}(m^2)$$

No evidence of the curvature

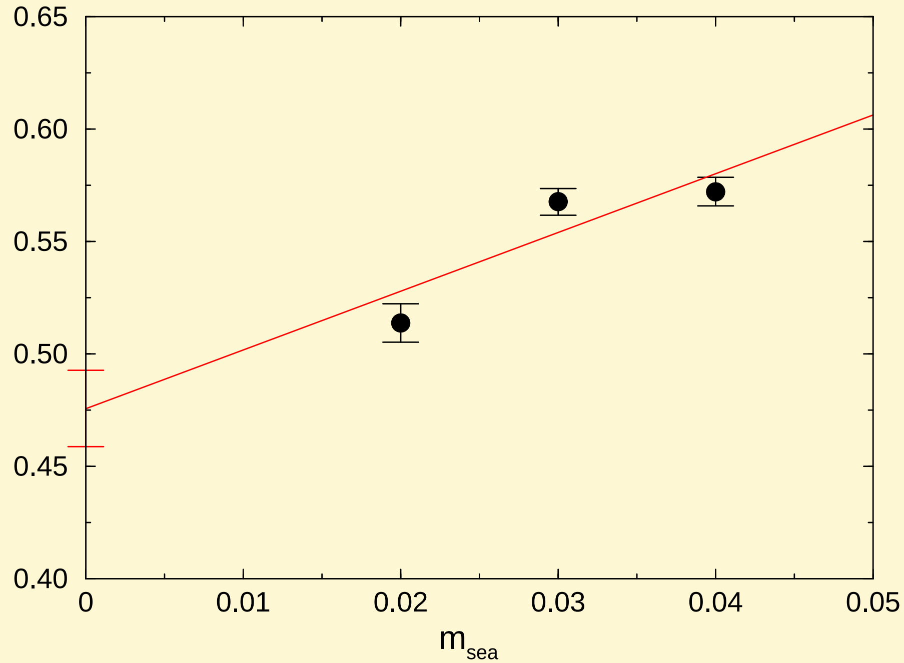
- **Kaon B-parameter**

PQChPT: $B_K = B\left[1 - \frac{6}{(4\pi f)^2} M_{\rm PS}^2 \ln\left(\frac{m_{\rm PS}^2}{\Lambda_\chi^2}\right)\right] + \mathcal{O}(m^2)$

$B_K^{\overline{\rm MS},\rm NDR}(\mu = 2\ \text{GeV})$ vs $m_{\rm sea}$

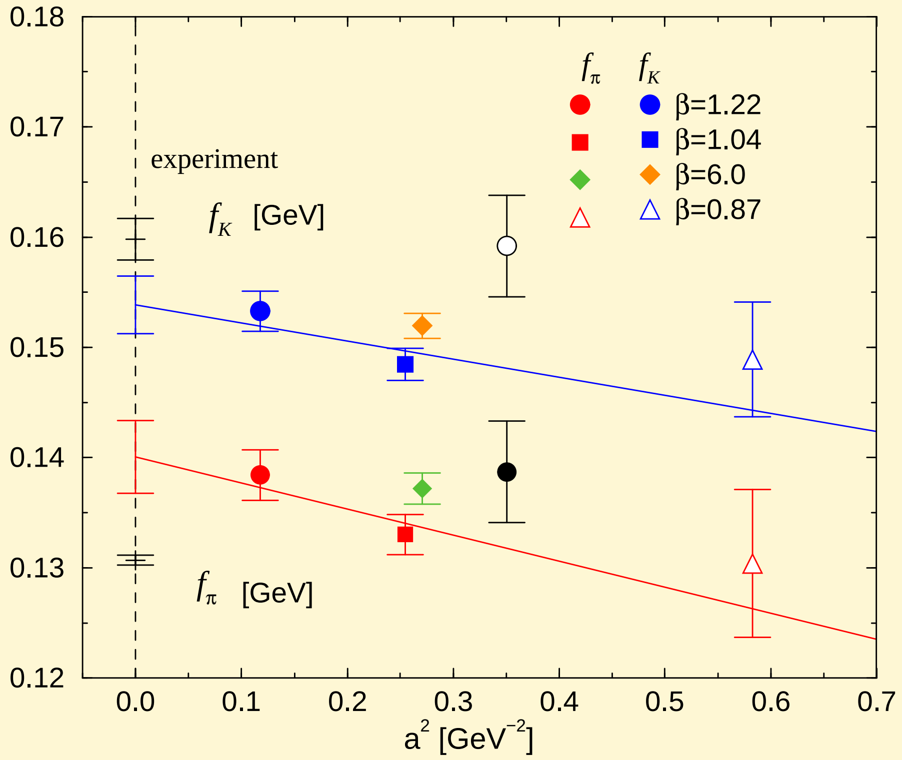


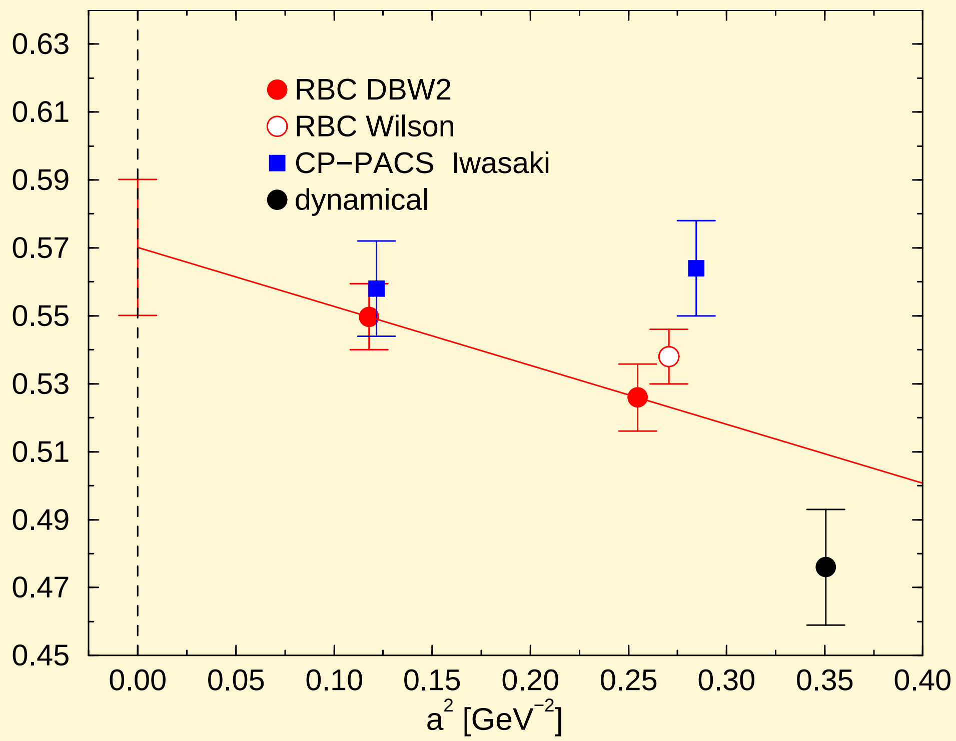
$m_{\rm valence} = m_s/2$



linear fit: $B_K^{\overline{\rm MS},\rm NDR}(\mu = 2\ \text{GeV}) = 0.476(17)$

- **Comparison with the quenched results**





Tends to be smaller : Dinamical effect?

f_π: consistent with quenched,

but not the experiment

f_K: consistent with experiment,

but not the quenched

UKQCD found same tendency

Flynn *et al.*, 2004

Conclusion

- B_K and f_K as an exact science
  - Quenched: Results from two Collabs. agree on a fine lattice

    Larger scale dependnce ?

    continuum value $\hat{B}_K = 0.762(27)$ $B_K^{\overline{\text{MS}}}(\mu = 2\text{GeV}) = 0.569(21)$
  - Dynamical quarks tend to lead smaller B_K
- Future Work (with QCDOC ?)
  - Scale dependence fully understood ? $\Rightarrow$ more data point(s)
  - Size effect?
  - $N_{\text{sea}} = 2 + 1$ to finalize B_K