Heavy quark physics from unquenched clover calculations

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Collaborators

- hep-lat/0404010, HADRONIC DECAY OF A SCALAR B MESON FROM THE LATTICE, C. McNeile, C. Michael, G. Thompson.
- hep-lat/0402012, AN ESTIMATE OF THE FLAVOR SINGLET CONTRIBUTIONS TO THE HYPERFINE SPLITTING IN CHARMONIUM. C. McNeile, C. Michael,
- hep-lat/0312007, EXCITED B MESONS FROM THE LATTICE, A.M. Green, J. Koponen, C. McNeile, C. Michael, G. Thompson
- hep-lat/0408025, AN UNQUENCHED LATTICE QCD CALCULATION OF THE MASS OF THE BOTTOM QUARK. C. McNeile, C. Michael, G. Thompson

Motivation

We compute the masses of hadrons containing heavy quarks for a number of reasons.

- To validate the lattice QCD methods.
- To extract quark masses and the QCD coupling.
- Because the dynamics of QCD is hard and fascinating.
- Experimentalists worry when theory can't explain some part of their data.

I wouldn't claim that any of these calculations are "high precision", but we deal with QCD dynamics that requires techniques that are not fully mature.

Outline

In this project we use unquenched gauge configurations generated with the non-perturbatively improved clover action. We have not had a computer that could generate new gauge configurations.

Outline:

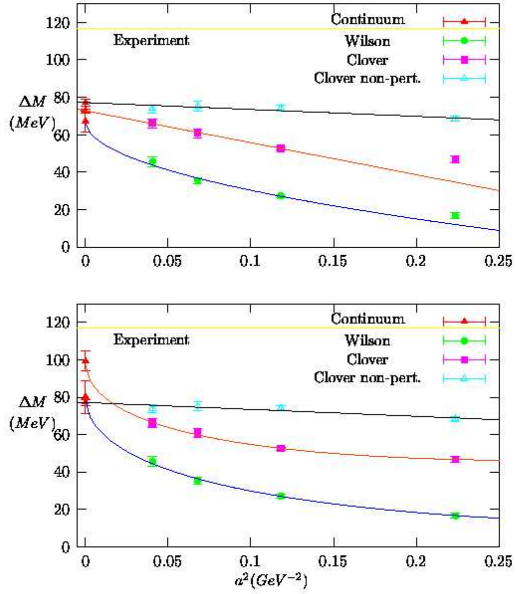
- OZI effects in Charmonium
- Mass of the bottom quark
- Searching for chiral logs in f_B.
- hadronic decay

Hyperfine splitting in charmonium

Experimentally the mass splitting between the J/ψ and the η_c is 117 MeV. It is clearly a critical task for lattice QCD to compute this. QCD-TARO (hep-lat/0307004) QCD get 77(2)(6) MeV from a large quenched study.
On the MILC lattices FNAL+MILC get $97 \pm 2 MeV$ (hep-lat/0310042). They suggest that the remaining discrepancy is due to only using the tree level tadpole estimate of the clover coefficient.

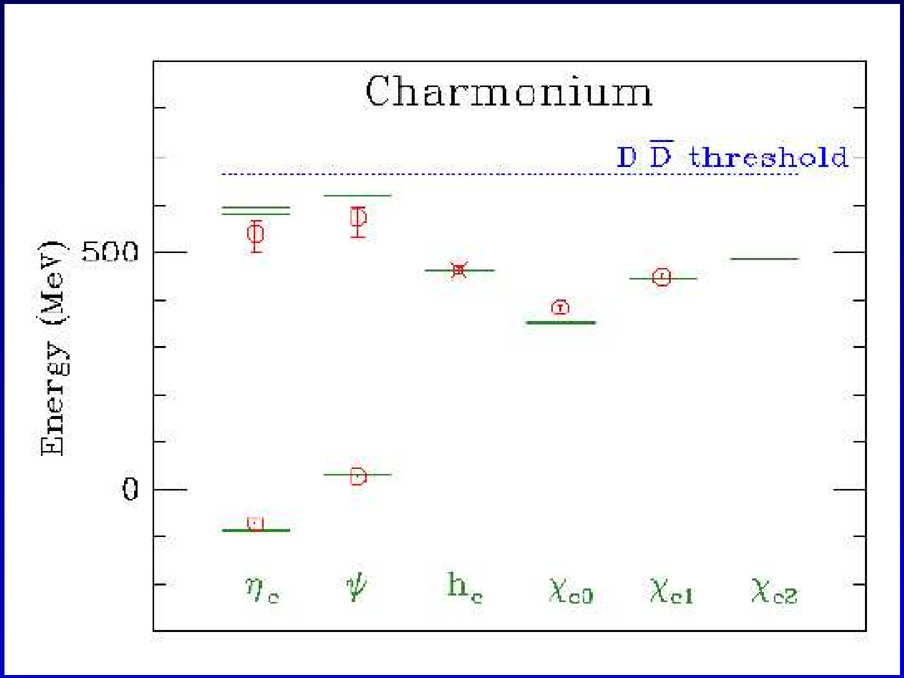
Charmonium spectroscopy is hard!

From QCD-TARO, hep-lat/0307004



Unquenched charmonium masses

Latest (work in progress) charmonium spectroscopy from unquenched lattice QCD hep-lat/0310042, FNAL lattice group



Lattice: $m_{J/\psi} - m_{\eta_c}$ = 97 $\pm$ 2 MeV (Expt. 116 MeV)

Singlet Charmonium

UKQCD, McNeile and Michael, hep-lat/0402012 looked at singlet charmonium interpolating operators for the first time.

$$\overline{c}\gamma_5 c$$

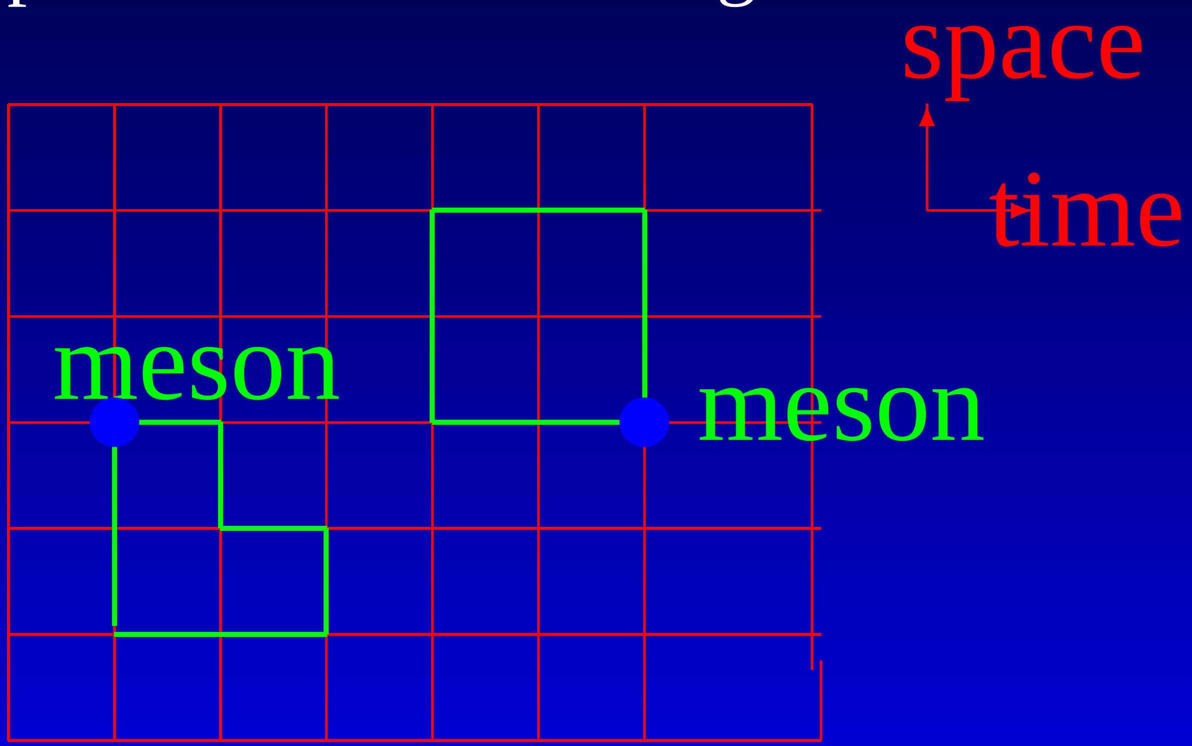
Hence the Wick contractions will have both connected and disconnected contributions. Perhaps, the disconnected contributions are small, because they are OZI suppressed, but ...
Worth investigating at least once.

What are singlet mesons?

Singlet interpolating operators such as:

$$O(x,t) = \overline{\psi}(x,t)\gamma_5\psi(x,t)$$

produce additional diagrams



Why might this be important?

The decay width of the $Jpsi$ is 87 kev, but the decay width of the η_c is 33 MeV (from BaBar). The decay width of the η_c is small relative to its mass, but not small relative to this splitting. The singlet diagrams “look like” the decay of the η_c. Somehow the bubbles are involved.

We extract masses from correlators using fit models of the form $c(t) \equiv e^{-mt}$. This fit model assumes that the hadron is stable

Remember King Midas!

Estimating OZI effects

One way to estimate decay widths (Kwong et al. Phys.Rev.D37:3210,1988)

(1) $$\Gamma_{tot}(\eta_c) \sim \alpha_s^2 \mid \psi(0) \mid^2 (1 + O(\alpha_s))$$

A more "modern way" of computing decay widths using NRQCD (Bodwin, Lepage, Braaten).

(2) $$\Gamma(H) = \sum_n \frac{F_n}{m^{d_n - 4}} \langle H \mid O_n \mid H \rangle$$

where O_n is a four fermion NRQCD operator. I am unclear about the reliablity of the above methods.

How to compute loop diagrams

This is essentially the same calculation that is done for the η.

$$E(\langle \chi_\alpha^\dagger \chi_\beta \rangle_S) = \delta_{\alpha\beta} \tag{3}$$

Solve the normal propagator equation.

$$M\phi = \chi \tag{4}$$

$$trace(M^{-1}) = E(\langle \chi^\dagger \phi \rangle_S) \tag{5}$$

Lattice details

Use UKQCD's unquenched (NP clover) ensemble β=5.2 κ= 0.1350, volume $16^3 32$ with sea quarks around the mass of the strange quark. Same parameters as D_s meson spectrum (hep-lat/0307001), used configurations separated by 10 sweeps with block sizes of 40. (The code was run on all configurations available).

The κ value for the charm quark was determined by a variety of different methods.

Stochastic errors

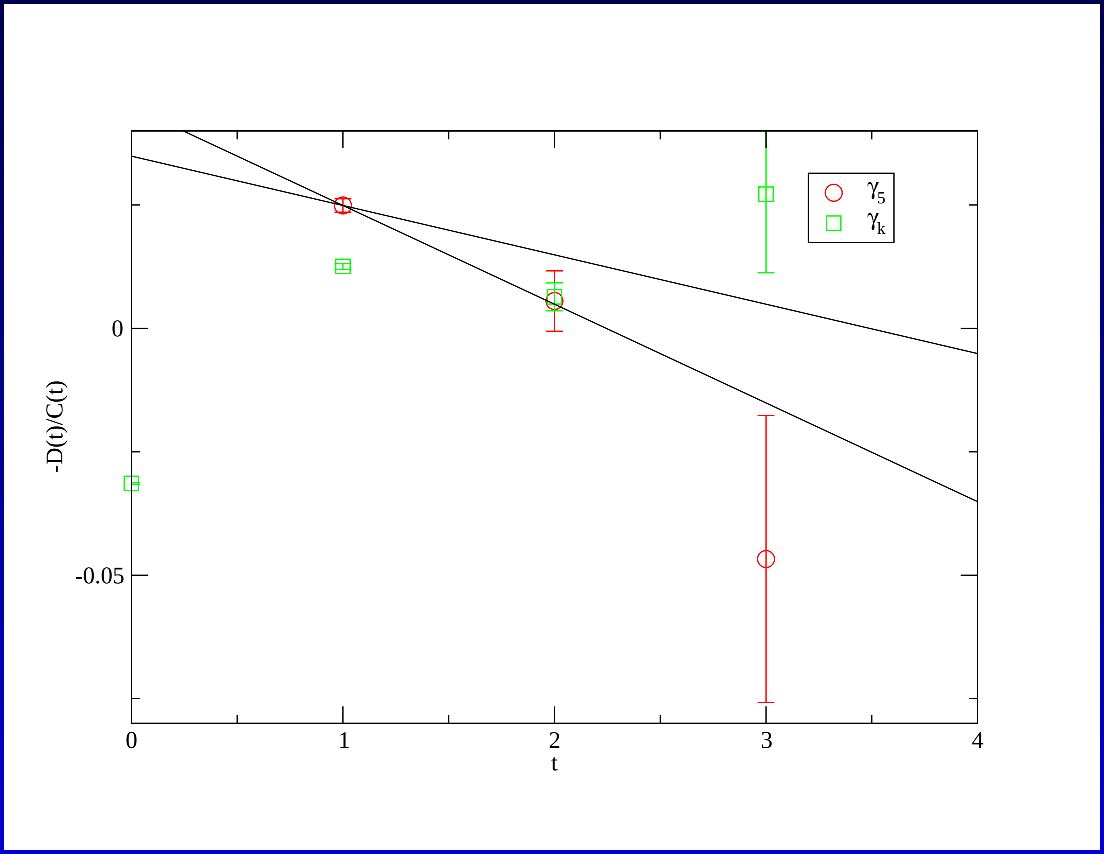
Used 2 x 100 Z_2 volume sources.

$$\sigma_{\rm gauge} = (\sigma^2_{\rm obs} - \sigma^2_{\rm stoch})^{1/2}$$

κ	Γ	J^{PC}	$\sigma_{\rm obs}$	$\sigma_{\rm stoch}$	$\sigma_{\rm gauge}$
0.135	γ_5	0^{-+}	33.6	13.91	30.6
0.135	γ_k	1^{--}	14.7	14.45	2.7
0.135	I	0^{++}	53.0	15.0	50.8
0.119	γ_5	0^{-+}	15.9	8.3	13.6
0.119	γ_k	1^{--}	9.1	9.003	1.2
0.119	I	0^{++}	23.6	10.9	20.9

Disconnected over connected

The slope is proportional to the mass splitting between the singlet and non-singlet.

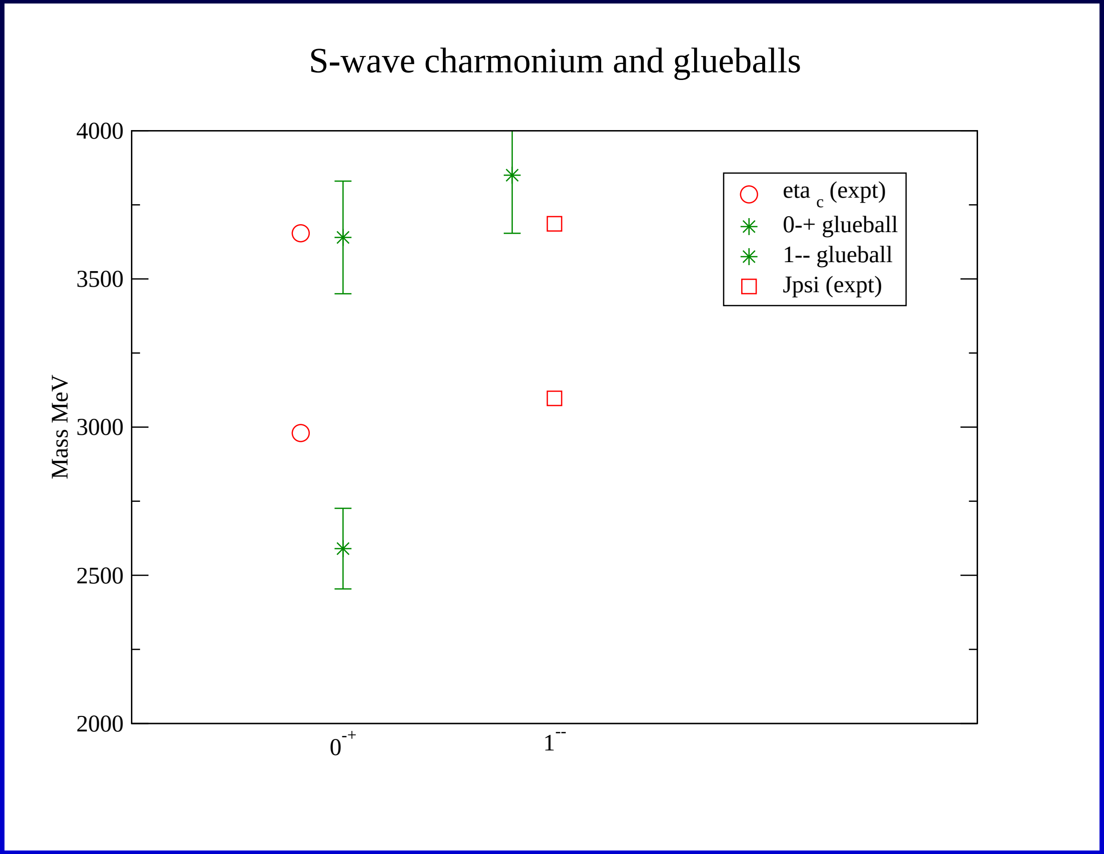


New Charmonium results

Our conclusion (hep-lat/0402012) was that, the singlet contribution could be as much as 20 MeV to the mass of the pseudo-scalar. The data was consistent with the vector channel having a nelgible contribution. Hence it is possible that the hyperfine splitting is increased by the singlet contribution decreasing the η_c mass by 20 MeV but not effecting the J/ψ mass. Paper by QCD-TARO (hep-lat/0404016) that is consistent with this analysis.

Glueballs and charmonium

McNeile & Michael (hep-lat/0402012). Also Bali (hep-lat/0308015)



2S hyperfine splitting

Group	Method	$M_{\psi(2S)} - M_{\eta_c(2S)}$ MeV
Columbia	lattice	75(44)
CP-PACS	lattice	26(17)
PDG	Belle expt	$32 \pm 6 \pm 8$
CLEO	Experiment	$43 \pm 4 \pm 4$
BABAR	Experiment	55 ± 4
Crystal Ball	Experiment	92 ± 5

Table 1: Excited hyperfine splittings between the $\psi(2S)$ and $\eta_c(2S)$.

Glueball mixing used get better agreement between $\psi(2S) \rightarrow \rho\pi$ theory and experiment.

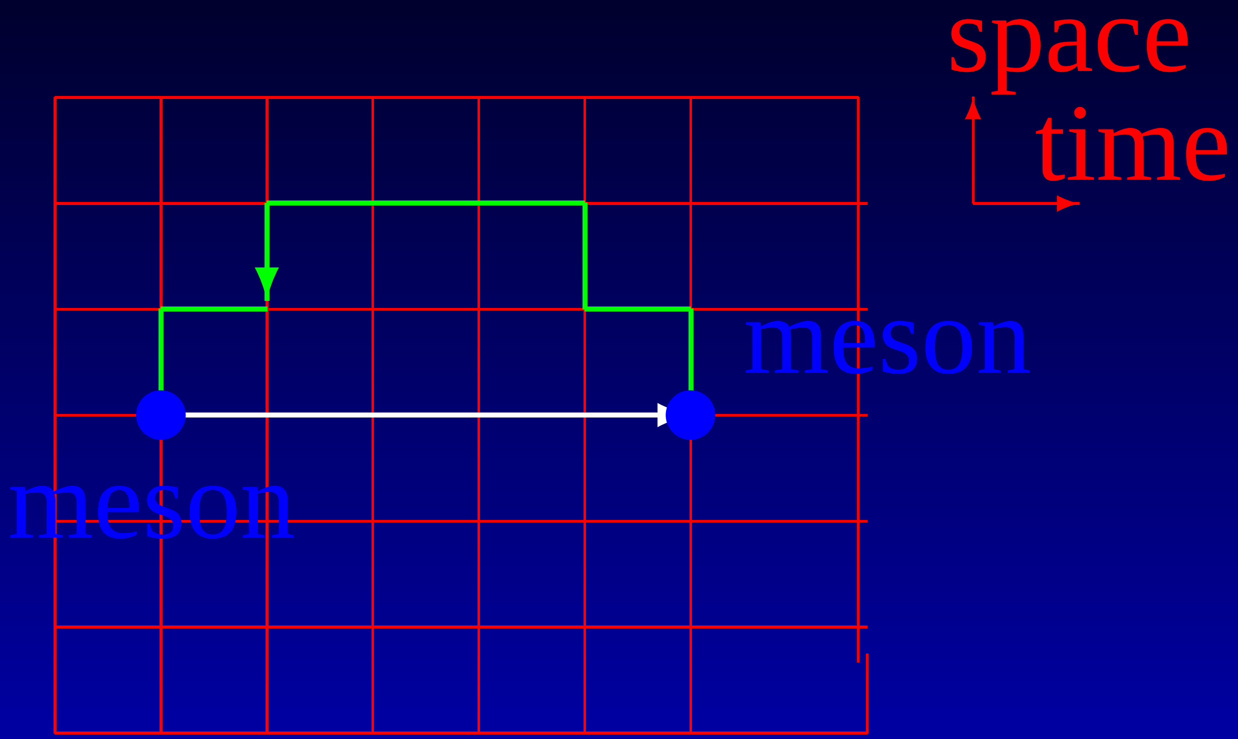
Static-light formalism

For $M_Q > \Lambda_{QCD}$, the heavy quark mass is no longer important for the dynamics.
The lattice static theory of Eichten and Hill,

$$S_{static} = ia^3 \sum_x b^\dagger(x)(b(x) - U_0(x-\widehat{0})b(x-\widehat{0})) \quad (6)$$

There are corrections to this result ordered by $1/m_Q$, but is still useful. It is not so easy to do good numerics with this formalism.

Static-light in pictures



All to all propagators 6 times more expensive than point inversion.

Quark masses

The masses of quarks are fundamental parameters of the standard model however they are quite poorly known.

Quark	Mass (PDG)	"references used"
up	1.5 to 4.5 MeV	0
strange	80 to 155 MeV	0
bottom	4 to 4.5 GeV	0
top	174.3 ± 5.1 GeV	5

Table 2: Quark masses from PDG

Compare to the Higgs mass bounds in the PDG (from precision electroweak data + direct searches)

$$115 \text{ GeV} < M_H < 196 \text{ GeV}$$

Why the bottom quark mass is important

It is important for extracting CKM matrix elements from inclusive measurements.

$$\Gamma(b \rightarrow X_u l \overline{\nu}_l) = \frac{G_F^2 \mid V_{ub} \mid^2 m_b^5}{192\pi^2}(1 - 2.41\frac{2.41\alpha_s}{\pi} + \ldots) \quad (7)$$

El-Khadra and Luke (hep-ph/0208114) note that a 100 MeV error on m_b corresponds to a 6 % error on the V_{ub} CKM matrix element (currently only known to 19 % accuracy PDG).

Lattice details

There have been very few published numbers for the mass of the bottom quark from unquenched lattice QCD.
We essentially following the method in Gimenez et al. (hep-lat/0002007), with some minor improvements.

$$C(t) = \sum_x \langle 0 \mid \Phi_B(x,t)\Phi_B^\dagger(x,0) \mid 0\rangle \tag{8}$$

$$\rightarrow Z^2 \exp(-a\mathcal{E}t) \tag{9}$$

Lattice details

No.	$r_0 m_{PS}$	κ_{sea}	κ_{val}	Volume	$a\mathcal{E}$
20	1.92(4)	0.1395	0.1395	12^3 24	0.87(1)
78	1.94(3)	0.1395	0.1395	16^3 24	0.842(5)
60	1.93(3)	0.1350	0.1350	16^3 32	0.772^{+7}_{-8}
60	1.48(3)	0.1355	0.1355	16^3 32	0.739^{+9}_{-8}
60	1.82(3)	0.1355	0.1350	16^3 32	0.748^{+9}_{-8}
55	1.06(3)	0.1358	0.1358	16^3 32	0.707^{+14}_{-12}

Table 3: Lattice binding energy ($a\mathcal{E}$) for each data set.

Basic idea

At leading order in HQET the pole mass of the bottom quark is related to the meson mass via

$$M_{B_s} = m_b^{pole} + (\mathcal{E} - \delta m) + O(1/m_b) \tag{10}$$

where $\mathcal{E}$ is the 'mass' extracted from the lattice calculation and δm is a lattice renormalization factor. To convert to $\overline{MS}$

$$\overline{m_b}_{\overline{MS}}(\mu) = m_b^{pole} Z_{pm}(\mu) \tag{11}$$

Complications

The two perturbative expressions for Z_{pm} and δm are a bit sick. For example the matching of the pole mass to $\overline{MS}$ has a perturbative expression.

$$
\begin{aligned}
Z_{pm} &= 1 - \frac{4}{3}\frac{\alpha_s(\overline{m_b})}{\pi} \\
&- (11.66 - 1.04 n_f)(\frac{\alpha_s(\overline{m_b})}{\pi})^2 \\
-(157 &-24 n_f + 0.65 n_f^2)(\frac{\alpha_s(\overline{m_b})}{\pi})^3
\end{aligned}
$$

The poor perturbative behaviour of δm (almost) cancels with that from Z_{pm}. (In this case two wrongs do make a right.)

Why the static limit can be used?

Mass of a heavy-light meson as a function of heavy quark mass m_b.

$$M_B = m_b + \Lambda_{static} - \frac{\lambda_1}{2m_b} + \frac{3\lambda_2}{2m_b} \tag{12}$$

λ_1 is the matrix element due to the insertion of the kinetic energy and λ_2

$\lambda_2 \sim 0.12\ GeV^2$ $\lambda_1 = -(0.45 \pm 0.12) GeV^2$ by Kronfeld and Simone

The corrections due $\frac{1}{m_b}$ are about 30 MeV Gimenez et al.

Mass of the bottom quark

Group	comment	$m_b(m_b) GeV$
UKQCD	unquenched	$4.25(2)(11)$
Collins	unquenched	$4.34(7)^{+0}_{-7}$
Gimenez et al.	unquenched	$4.26(9)$
Bali et al.	quenched!	$4.19(6)(15)$
Sommer et al.	quenched!	$4.12(8)$

Table 4: Lattice QCD results for $m_b(m_b)$.

Sommer and Heitger used fully non-perturbative techniques to compute $m_b(m_b)$.

How to do a better calculation?

Our analysis of the errors goes like:

$$\overline{m_b}(\overline{m_b}) = 4.25 \pm 0.02 \pm 0.03 \pm 0.03 \pm 0.08 \pm 0.06 \quad (13)$$

in GeV. The errors are (from left to right): statistical, perturbative, neglect of $1/m_b$ terms, ambiguities in the choice of lattice spacing, and error in the choice of the mass of the strange quark.

There are numerical techniques to estimate the α_s^3 terms to δm (Di Renzo et al). At the moment these are only known for quenched QCD and $n_f = 2$ unquenched calculations with Wilson fermions.

Chiral logs in f_{B_s}.

$$C(t) = \sum_x \langle 0 \mid A_4(x,t)\Phi_B^\dagger(x,0) \mid 0 \rangle \quad (14)$$

$$\rightarrow Z_L Z_{\Phi_B} \exp(-a\mathcal{E}t) \quad (15)$$

where Φ_B is the interpolating operator for static-light mesons. The static light A_4 axial current is (perturbatively) improved using the ALPHA scheme.

$$\langle 0 \mid A_\mu \mid B(p) \rangle = i p_\mu f_B \quad (16)$$

The decay constant is related to the amplitude:

$$f_B^{static} = Z_L \sqrt{\frac{2}{M_B}} Z_A^{static} \quad (17)$$

$\frac{f_{B_s}}{f_B}$ matters

The ratio of the decay constants of the B_s to B mesons ($\frac{f_{B_s}}{f_B}$) is a crucial QCD quantity for the unitarity checks of the CKM matrix. This combination of matrix elements places a constraint on $\mid V_{ts} \mid / \mid V_{td} \mid$. (It will become more important once B_s mixing is directly measured at run II of the Tevatron)

The JLQCD collaboration (hep-ph/0307039) quote $f_{B_s}/f_{B_d} = 1.13(3)(^{+13}_{-\ 2})$, where the errors are statistical and systematic (dominated by the chiral extrapolation).

Static-light chiral perturbation theory

The one loop expression for the static-light decay constant

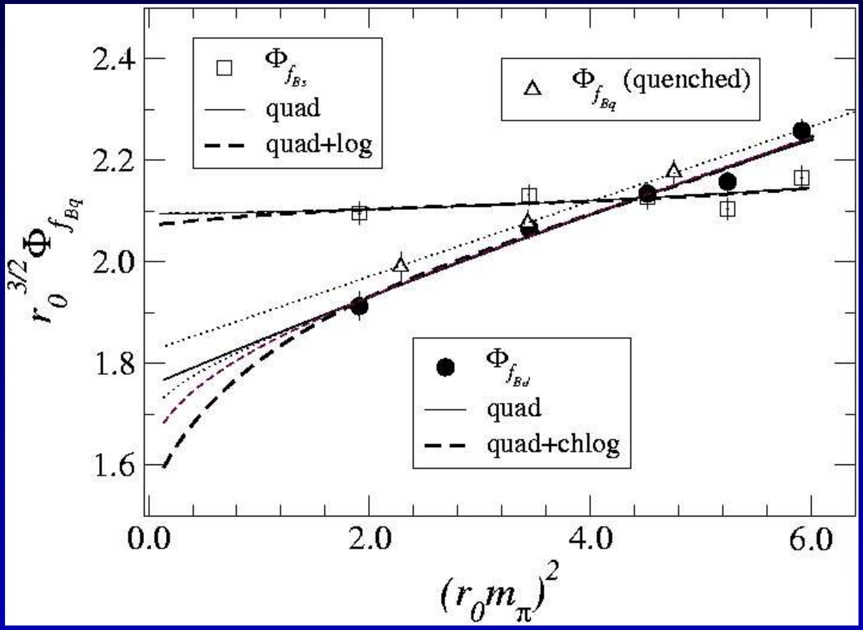
$$\frac{\Phi_{f_{B_d}}}{\Phi_{f^0_{B_d}}} = 1 - \frac{3(1+3g^2)}{4} \frac{m_\pi^2}{(4\pi f)^2} \log(\frac{m_\pi^2}{\mu^2}) \tag{18}$$

where $\Phi_{f_{B_d}} \equiv f_{B_d}\sqrt{M_{B_d}}$. and g is the $B^* B\pi$ coupling (g=0.6, $\mu = 1.5$ GeV). This has recently been "measured" at CLEO. It has also been determined from quenched lattice QCD.

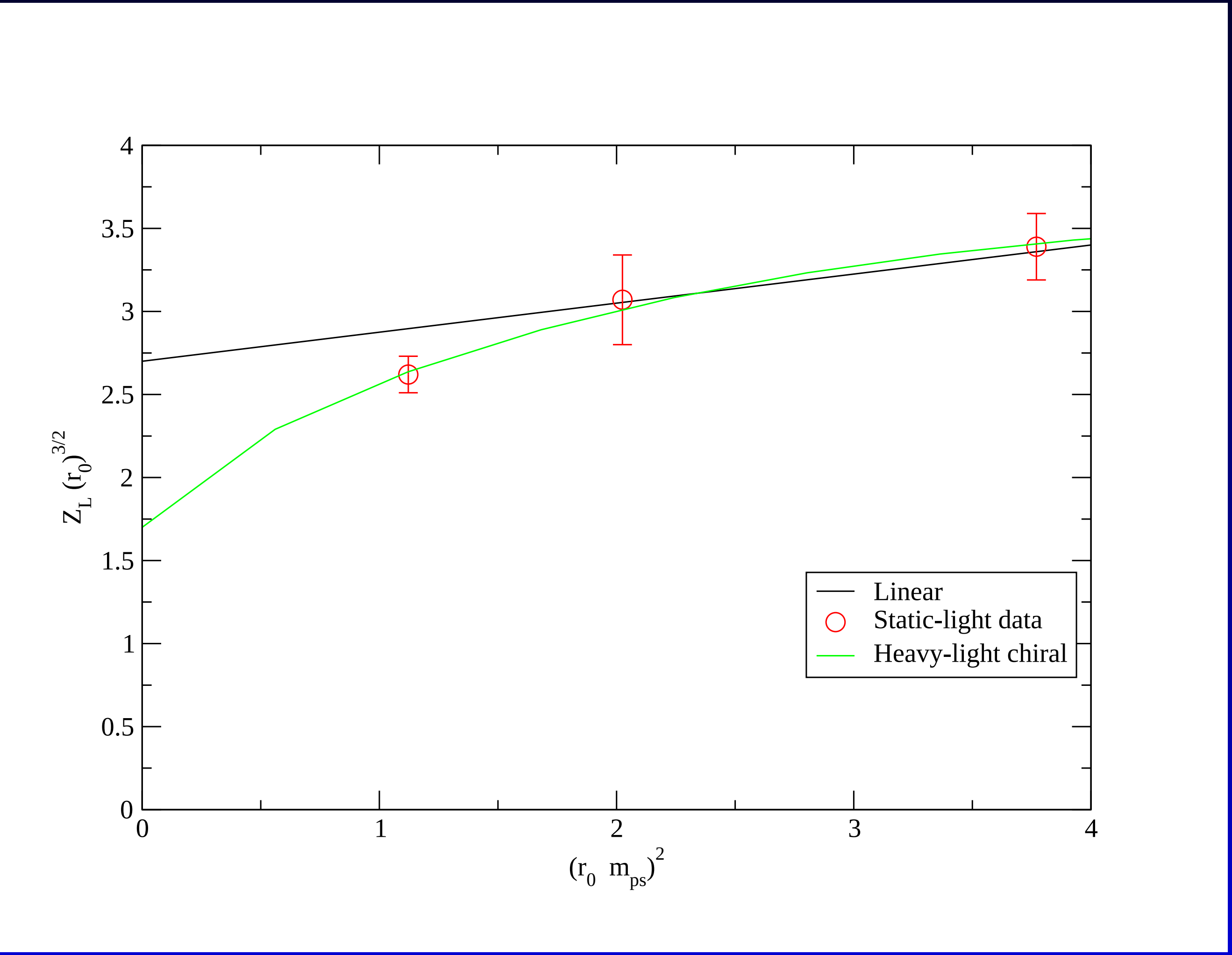
This has a similar structure to the one loop expression for the pion decay constant.

Do we expect to see anything?

From JLQCD, hep-ph/0307039. Our lightest data point $m_\pi r_0 = 1.059$.



Amplitude with quark mass



Finite volume effects

Using heavy-light chiral perturbation theory Arndt and Lin (hep-lat/0403012) estimated that for $m_\pi \sim 400\ MeV$, $L \sim 1.6$ fm, the finite size effects in $\frac{f_{B_s}}{f_B}$ were 0.006.

However, at κ_{sea} = 0.1358, UKQCD (hep-lat/0403007) claimed that the finite size effects in f_π were $\sim$ 8 %. The calculation of finite volume effects in f_π by Colangelo and Haefeli beyond leading order showed the corrections were sizeable.

Trying to decrease the statistical errors

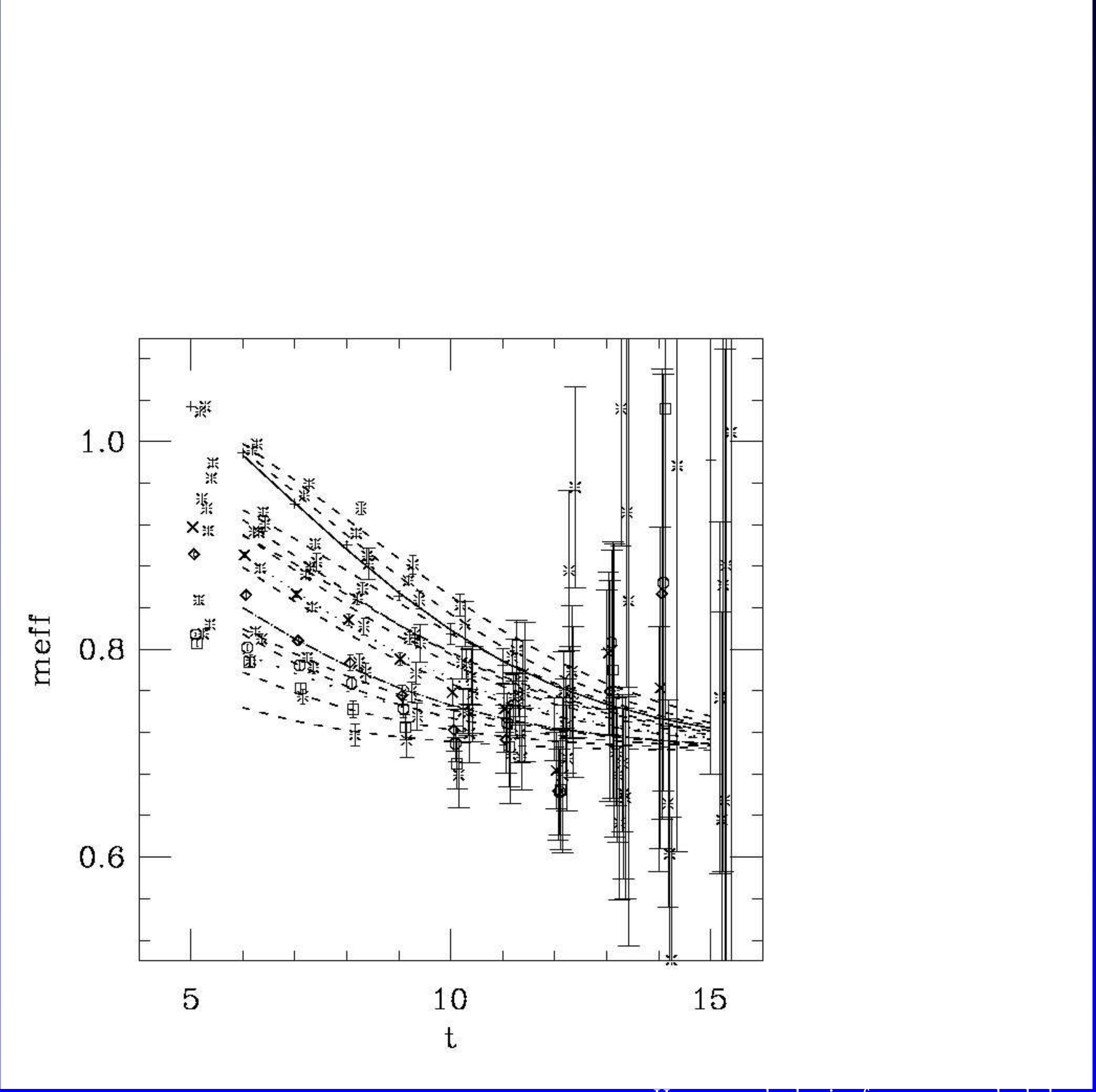
The ALPHA collaboration (hep-lat/0307021) have developed a new variant of the static formalism with a reduced $1/a$ mass dependence.

$$S_h = a^4 \sum_x \overline{\psi}_h(x) D_0 \psi_h(x) \tag{19}$$

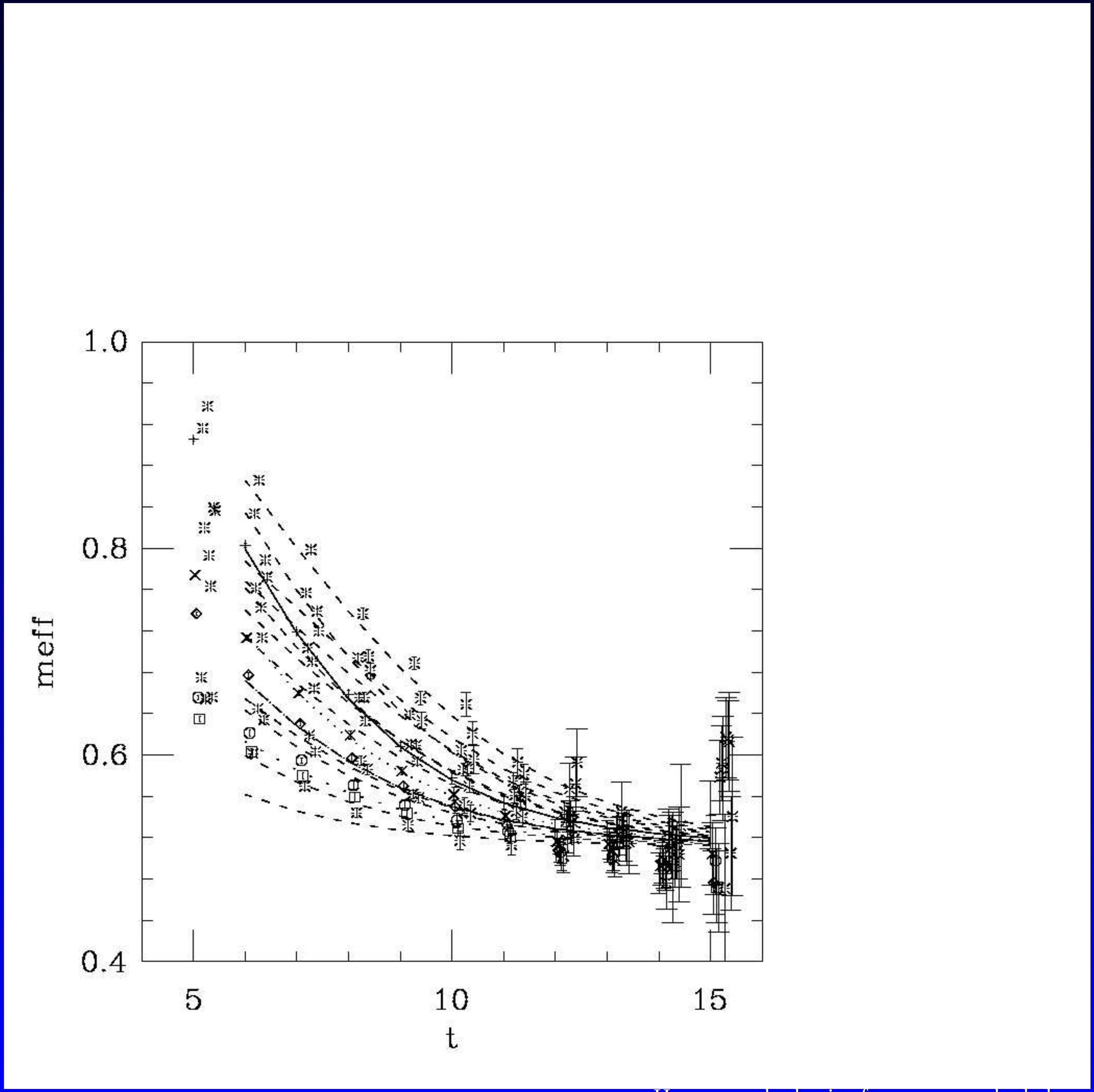
$$D_0 \psi_h(x) = \frac{1}{a}[\psi_h(x) - W^\dagger(x - a\hat{t})_t]\psi_h(x - \hat{t}) \tag{20}$$

use a “single fuzz” for W.

Effective mass for Eichten-Hill



Effective mass ALPHA action



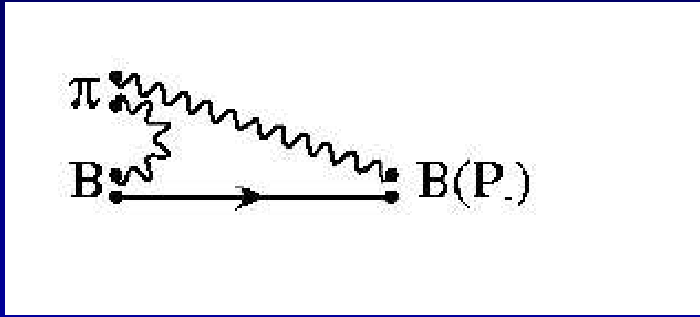
Is this any use?

It may be possible to study the chiral logs in f_{B_s}/f_B with static-clover quarks combined at coarse lattice spacing with Wilson ChPT (WChPT) that includes a^2 effects associated with explicit chiral symmetry breaking.

This would require careful validating, as the Wilson ChPT would be use to avoid having to take the continuum limit. The first step is seeing the effect of the chiral log.

Hadronic decay from the lattice

UKQCD, McNeile, Michael, and Thompson (hep-lat/0404010) recently looked at the hadronic decay of a “B”(P^-) meson to “B” (S) meson and a pion using lattice QCD.

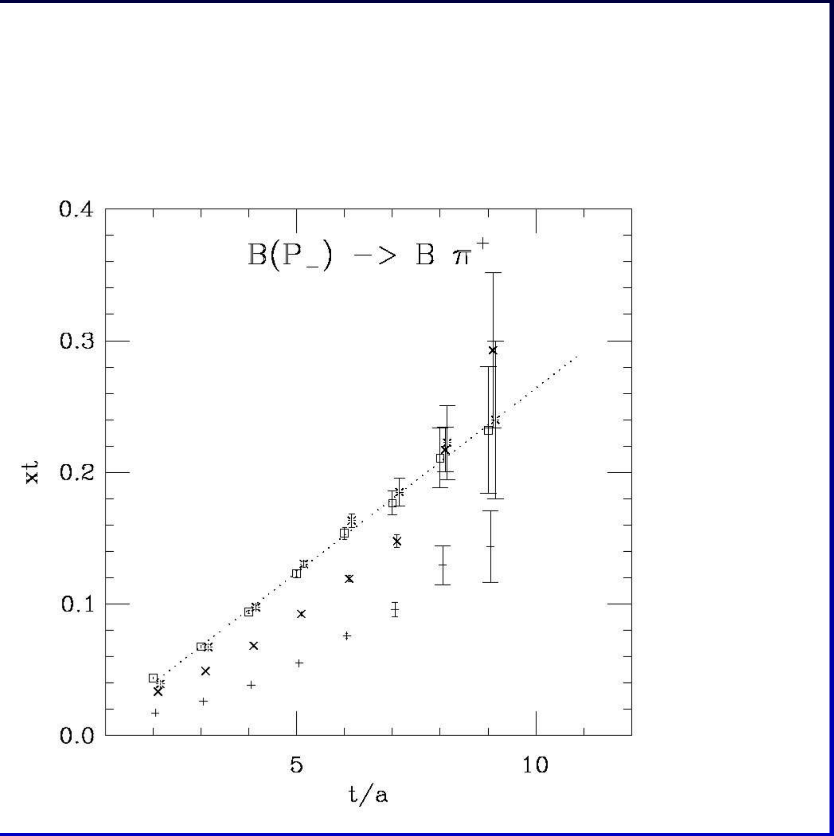


Picture of decay.

Estimate the hadronic coupling (ratio of three point function to two point functions) and plug that into Fermi’s golden rule to get the width.

Hadronic decay from the lattice

Some lattice data from UKQCD (hep-lat/0404010)



Hadronic decay width (hep-lat/0404010)

UKQCD gets Γ/k = $0.46(9)$ for $B(P^-) \longrightarrow B(S) + \pi$

Source	Decay	Γ/k
Expt	$D(0^+) \longrightarrow D(0^-) + \pi$	0.73^{+28}_{-24}
PDG	$B^{\star\star} \longrightarrow B + \pi$	$0.34(5)$
PDG	$K(1412) \longrightarrow K + \pi$	$0.48(5)$

Table 5: Reduced width from experiments

The width is 1/3 the width from the “chiral symmetry calculation” of Bardeen, Eichten, and Hill, (hep-ph/0305049)

Conclusions

- Getting the hyperfine splitting correct in charmonium is a key goal of lattice QCD. The singlet contribution may need to be computed.
- It may be possible to compute f_{B_s}/f_B reliably using clover fermions + lattice spacing enhanced chiral perturbation theory
- If I was going to do another mass of the bottom quark calculation, I would use Asqtad + static quarks (requires a perturbative calculation).