Calculating ϵ'/ϵ using staggered fermions

Weonjong Lee

School of Physics
Seoul National University

Collaboration

- S. Sharpe (University of Washington, Seattle)
- W. Lee (Seoul National University)
- G. Fleming (Jefferson Lab)
- T. Bhattacharya (Los Alamos National Lab)
  R. Gupta (Los Alamos National Lab)
  G. Kilcup (Ohio State University)
- QCDSP support: N. Christ, G. Fleming, G. Liu, R. Mawhinney, L. Wu.

$Re(A_2)$, $Re(A_0)$, and ϵ'/ϵ (Exp)

- $\langle(\pi\pi)_I|H_W|K^0\rangle = A_I e^{i\delta_I}$
- $\omega = \frac{ReA_2}{ReA_0} = 0.045$
- $ReA_2 = 1.50 \times 10^{-8} GeV$
- $ReA_0 = 33.3 \times 10^{-8} GeV$
- $\epsilon'/\epsilon = -\frac{\omega}{\sqrt{2}|\epsilon|}\left[\frac{ImA_0}{ReA_0} - \frac{ImA_2}{ReA_2}\right]$

  KTeV $\rightarrow Re(\epsilon'/\epsilon) = (20.7 \pm 2.8) \times 10^{-4}$

  NA48 $\rightarrow Re(\epsilon'/\epsilon) = (15.3 \pm 2.6) \times 10^{-4}$

Main goals of the staggered ϵ'/ϵ project.

- Check the results obtained using the quenched Domain-Wall fermions: $\rightarrow$ negative ϵ'/ϵ (CP-PACS, RBC).
- Calculate ϵ'/ϵ in the full QCD.
  - Quenching $\rightarrow$ main systematic errors.
  - Using the improved actions $\rightarrow$ HYP or $\overline{\mathrm{Fat7}}$.
- Comparison with experiment $\rightarrow$ new physics (?)

Why staggered fermions?

- Advantage:
  1. Conserved $U(1)_V \otimes U(1)_A$.
  2. Quark mass is protected (No Residual Mass).
  3. Computationally cheap ($< 50 \times 4$).
  4. Discretization error = $\mathcal{O}(a^2)$.
  5. Easy to improve (no extra cost).
- Disadvantage
  1. Broken $SU(4)$ Flavor (Taste) Symmetry.
  2. Operator mixing matrix of 65536×65536.
  3. NPR is impractical.

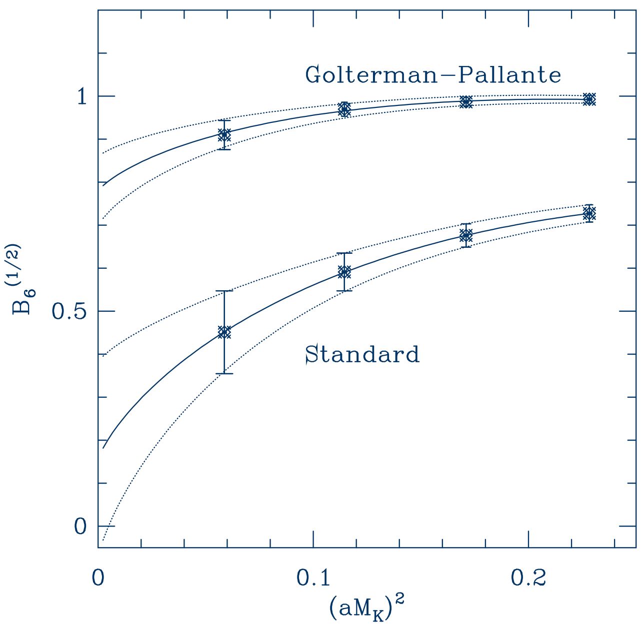
The First Stage

Perturbative matching for unimproved staggered operators

- Penguin diagrams (Sharpe, Patel, 1993)
$\longrightarrow$ small ($\approx 2\%$).
- Current-current diagrams (W. Lee, 2001)
$\longrightarrow$ large ($\approx 100\%$ for $[P \times P][P \times P]$).
- Hence, we must improve the staggered fermion action and operators to reduce the perturbative correction.

The Second Stage:

- Numerical study with unimproved staggered
- Check the previous work (Kilcup, Pekurovsky).
- Use fully one-loop matched operators.
- Nucl. Phys. B (Proc. Suppl.) 106, 311; hep-lat/0111004.
- Large quenching uncertainty.



The Third Stage:

Improving staggered fermions

- AsqTad: Fat7 + Lepage + Naik
- Fat7
- Fat7 + Lepage
- Hypercubic (HYP)
- Other possibilities: $\overline{\mathrm{Fat7}}$, APE

Main goal of improvement:

Reduce perturbative corrections

- **Key idea**:

  Suppress the taste changing interaction of high momemtum gluons with quarks, using the Symanzik improvement program and smearing.

- **Numerical Studies find**:
  - Taste symmetry breaking substantially reduced. (Hasenfratz, Knechtli)
  - Largest reduction in HYP and APE.

Bilinear Operator Renormalization

- W. Lee and S. Sharpe, PRD 66, (2002), 114501.
- Fat7 and HYP are the best in reducing the size of one-loop corrections. ($\leq 10\%$ level)
- Comparable to those for Wilson and Domain Wall Fermions.
- This leads us to choosing HYP for our numerical study.

The Fourth Stage: Study on HYP

- W. Lee, PRD 66 (2002) 114504.
- Properties:
  - Theorem 1 SU(3) Projections.
  - Theorem 2 Triviality of renormalization.
  - Theorem 3 Multiple SU(3) Projection.
  - Theorem 4 Uniqueness
  - Theorem 5 Equivalence (HYP $\leftrightarrow \overline{\mathrm{Fat7}}$)
- $|C_{\mathrm{HYP}}| \ll |C_{\mathrm{thin}}|, \quad SU(3)$ Projection $\leftrightarrow$ T.I. for doublers.
- Renormalization simplified by $\langle A_\mu A_\nu \rangle \rightarrow \langle B_\mu B_\nu \rangle$

Current-current Diagrams for HYP/$\overline{\mathrm{Fat7}}$

- W. Lee and S. Sharpe, PRD68, (2003) 054510; hep-lat/0306016.
- HYP/$\overline{\mathrm{Fat7}} \approx 10\%$.
- Tadpole improvement:
  $C_N = 13.159$
  $C_H = 1.4051$

Finite correction to $(\mathcal{O}_3)_{II}$

Operators	$[S \times P][S \times P]_{II}$
NAIVE	$2 \times (95.6 - 6C_N)$
HYP/$\overline{\mathrm{Fat7}}$	$2 \times (19.1 - 6C_H)$

Operators	$[P \times P][P \times P]_{II}$
NAIVE	$2 \times (111.3 - 2C_N)$
HYP/$\overline{\mathrm{Fat7}}$	$2 \times (6.9 - 2C_H)$

Penguin Diagrams for HYP

- K. Choi and W. Lee: hep-lat/0309070.
- Theorem 1 (Equivalence):

  At the one loop level, diagonal mixing coefficients are identical between unimproved and improved staggered operators of HYP (I), HYP (II), Fat7, Fat7+Lepage, and $\overline{\mathrm{Fat7}}$.
- Note that AsqTad is NOT included in the list.
- Off-diagonal mixing vanishes for Fat7, HYP (II), and $\overline{\mathrm{Fat7}}$.

The Fifth Stage:

Numerical Study with HYP staggered

- W. Lee NPB (P.S.) 128 (2004) 125, hep-lat/0310047; T. Bhattacharya, *et al.*, hep-lat/0309105.
- $\beta = 6.0$, $16^3 \times 64$ lattice, 218 confs.
- Bare quark mass shifted by ≈ 2.5.

  $Z_m \cong 2.5 \to Z_m \cong 1$.
- Small perturbative correction ($\approx 10\%$).
- Reduced taste symmetry breaking.

$Re(A_2)$, $Re(A_0)$, and ϵ'/ϵ (Exp)

- $\langle (\pi\pi)_I | H_W | K^0 \rangle = A_I e^{i\delta_I}$
- $\omega = \frac{ReA_2}{ReA_0} = 0.045$
- $ReA_2 = 1.50 \times 10^{-8} GeV$
- $ReA_0 = 33.3 \times 10^{-8} GeV$
- $\epsilon'/\epsilon = -\frac{\omega}{\sqrt{2}|\epsilon|} \left[\frac{ImA_0}{ReA_0} - \frac{ImA_2}{ReA_2} \right]$

  KTeV $\rightarrow Re(\epsilon'/\epsilon) = (20.7 \pm 2.8) \times 10^{-4}$

  NA48 $\rightarrow Re(\epsilon'/\epsilon) = (15.3 \pm 2.6) \times 10^{-4}$

Standard model H_W (Theory)

- $H_W = \frac{G_F}{\sqrt{2}} V_{us} V_{ud} \sum_i \Big(z_i(\mu) + \tau y_i(\mu) \Big) Q_i(\mu)$
- Current-current: Q_1, $Q_2 = (\bar{s}u)_L (\bar{u}d)_L$
- QCD Penguin: Q_3, Q_4, Q_5, $Q_6 = (\bar{s}d) \sum_q (\bar{q}q)$
- EW Penguin: Q_7, Q_8, Q_9, $Q_{10} = (\bar{s}d) \sum_q e_q (\bar{q}q)$
- Group representation:

Q_1, Q_2, Q_9, $Q_{10} \in (27_L, 1_R) + (8_L, 1_R)$

Q_3, Q_4, Q_5, $Q_6 \in (8_L, 1_R)$

Q_7, $Q_8 \in (8_L, 8_R)$

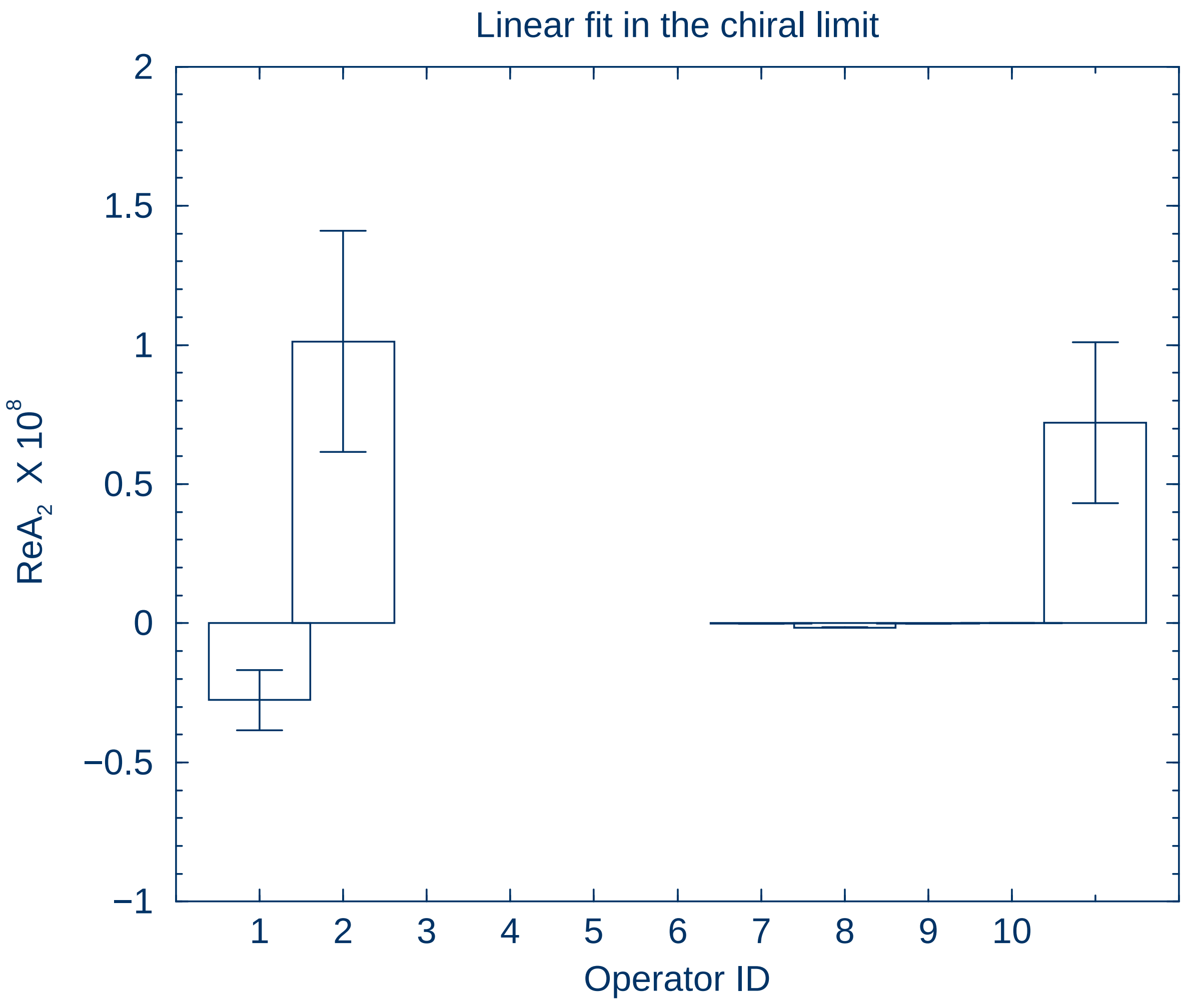
$Re(A_0)$ and $Re(A_2)$ (Theory)

- Standard model:

$$ReA_I \ = \ \frac{G_F}{\sqrt{2}}|V_{ud}V_{us}|\Big[\sum_{i=1,2} z_i\langle Q_i\rangle_I + Re(\tau)\sum_{i=3}^{10} y_i\langle Q_i\rangle_I\Big]$$

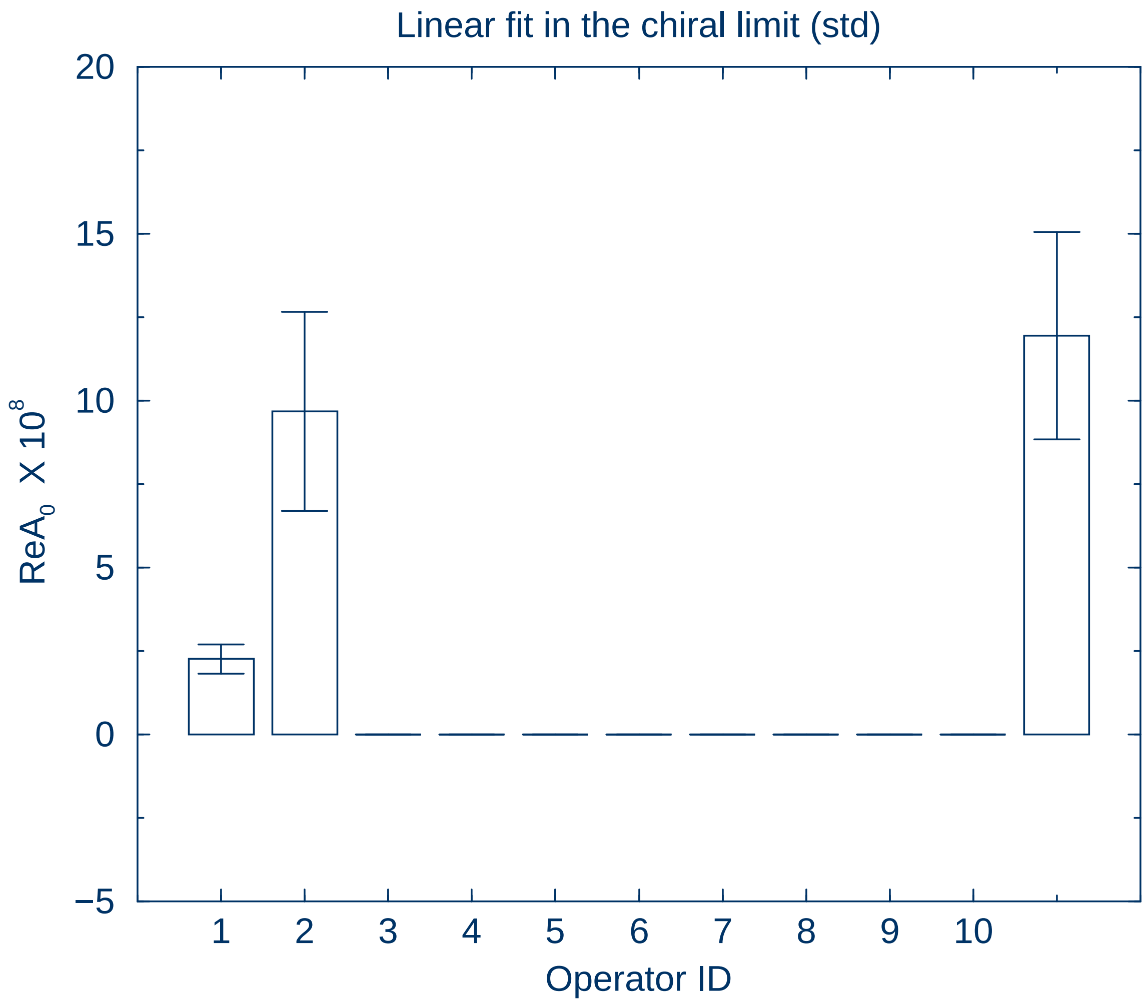
- $\langle Q_i\rangle_I \equiv \langle(\pi\pi)_I|Q_i|K^0\rangle$
- $z_i,\ y_i \approx 1$ or $\alpha = 1/129$
- $Re(\tau) = -Re(\lambda_t/\lambda_u) = 0.002$
- Hence, only $\langle Q_1\rangle_I$ and $\langle Q_2\rangle_I$ are important in ReA_I.

ReA_2 X 10^8



$ReA_2 = 1.50 \times 10^{-8} GeV$ (Experiment), $\mu = 1/a$

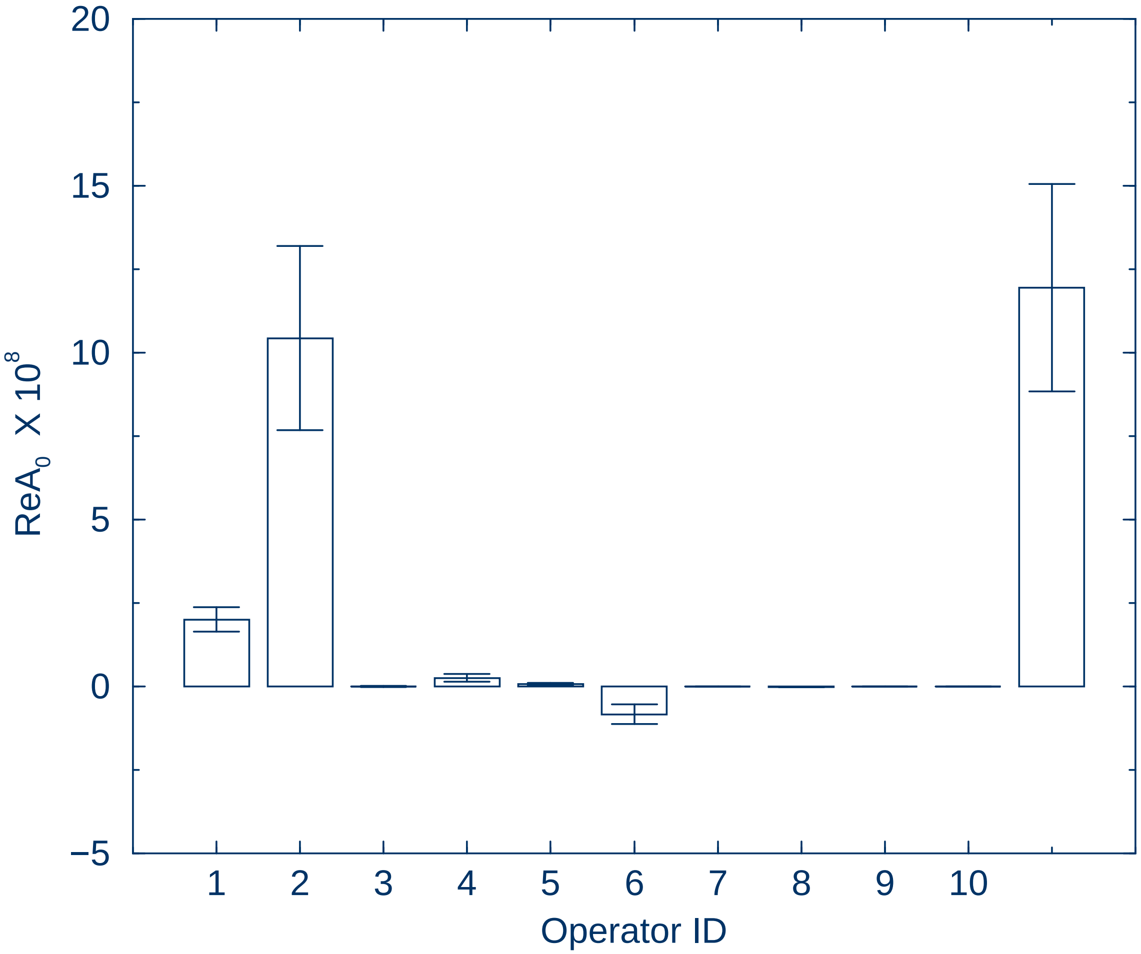
ReA_0 X 10^8



$$ReA_0 = 33.3 \times 10^{-8} GeV \text{ (Experiment)}, \mu = m_c$$

ReA_0 X 10^8

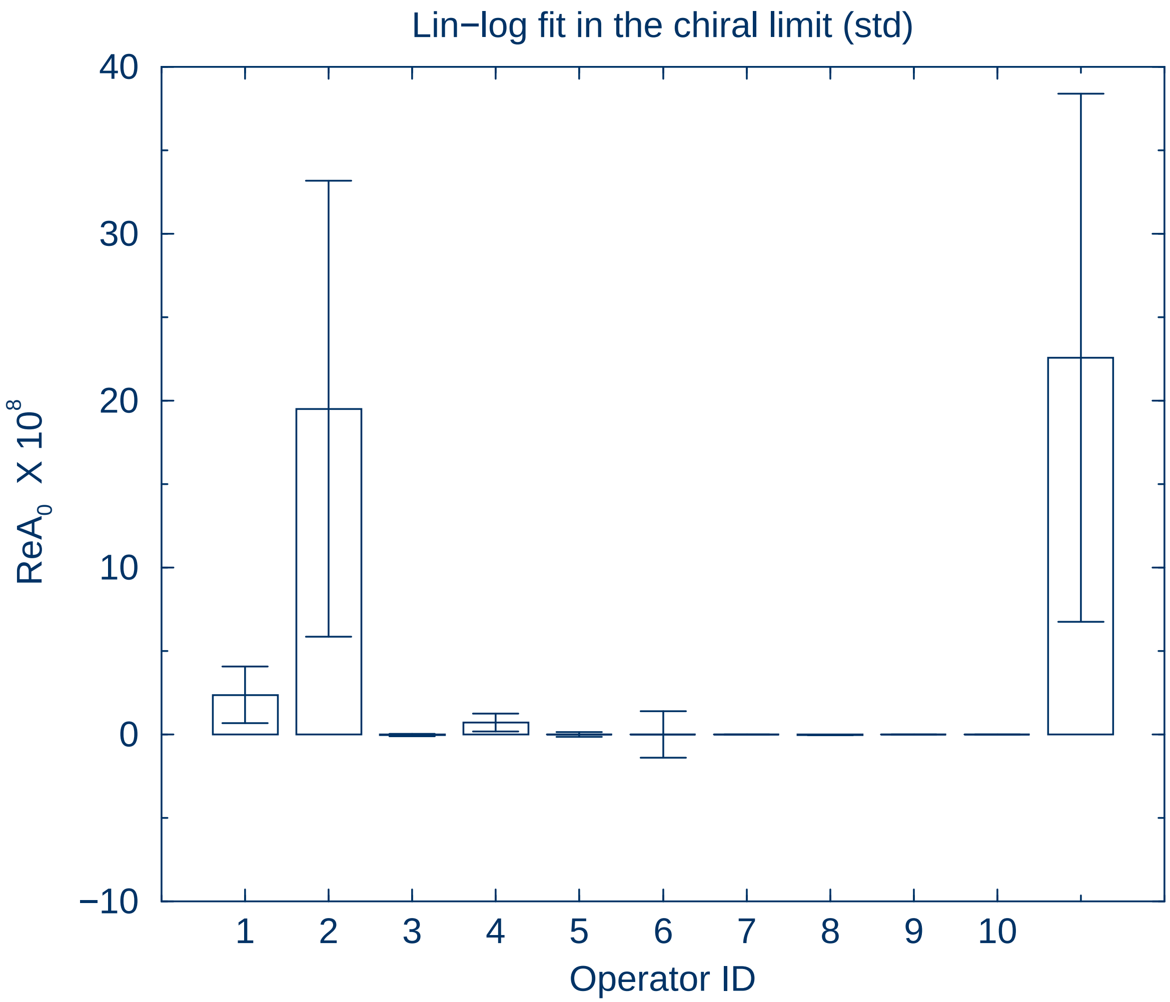
Linear fit in the chiral limit (std)



$ReA_0 = 33.3 \times 10^{-8} GeV$ (Experiment), $\mu = 1/a$

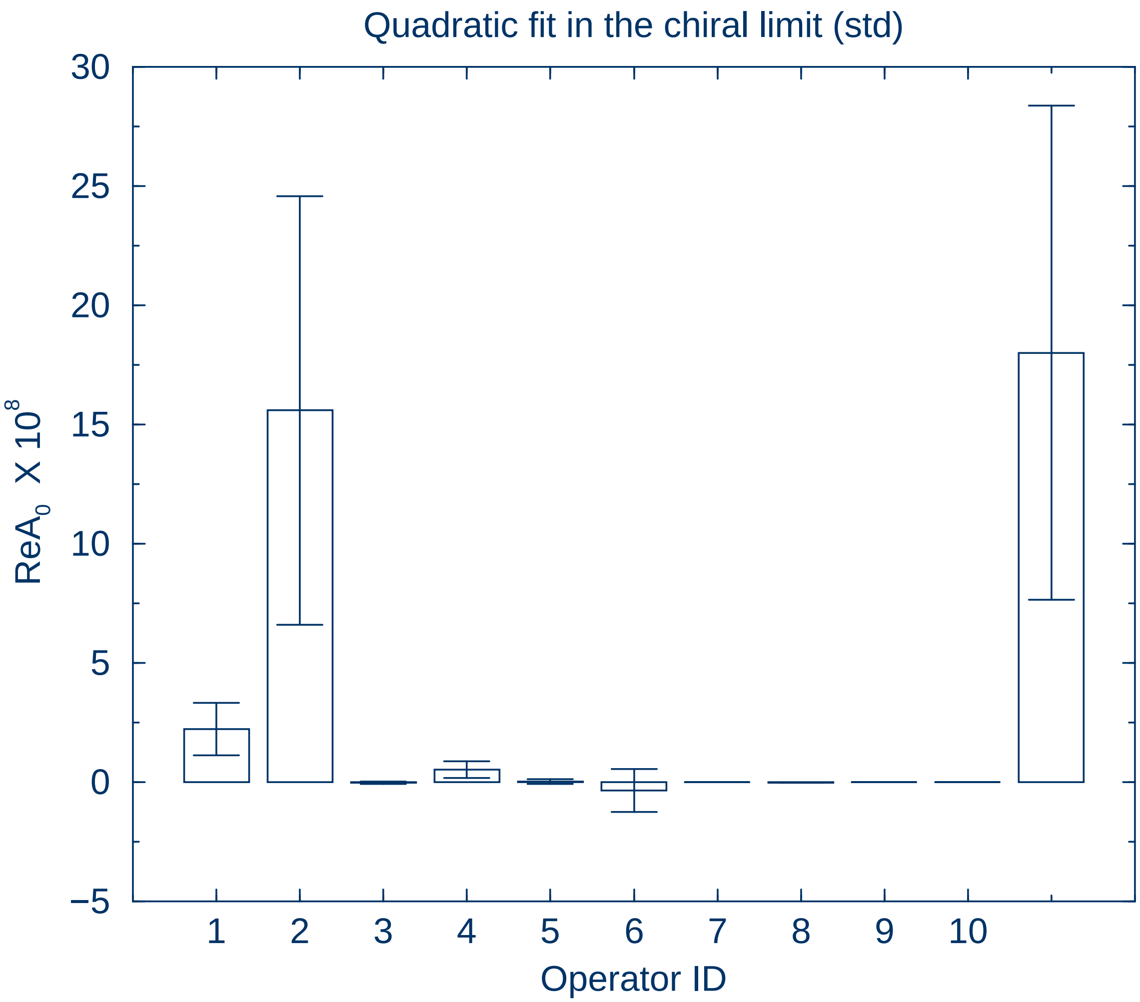
School of Physics
Seoul National University

$ReA_0 \times 10^8$



$$ReA_0 = 33.3 \times 10^{-8} GeV \text{ (Experiment)}, \ \mu = 1/a$$

ReA_0 X 10^8



$ReA_0 = 33.3 \times 10^{-8} GeV$ (Experiment), $\mu = 1/a$

ϵ'/ϵ (Theory)

- Standard model:

$$\begin{aligned}
\epsilon'/\epsilon &= Im(V_{ts}^* V_{td}) \Big[P^{(1/2)} - P^{(3/2)} \Big] \\
P^{(1/2)} &= r \sum_{i=3}^{10} y_i(\mu) < Q_i >_0 (\mu)(1 - \Omega_{\eta+\eta'}) \\
P^{(3/2)} &= \frac{r}{\omega} \sum_{i=3}^{10} y_i(\mu) < Q_i >_2 (\mu) \\
r &= \frac{G_F \omega}{2|\epsilon| Re A_0}
\end{aligned}$$

ϵ'/ϵ (Theory)

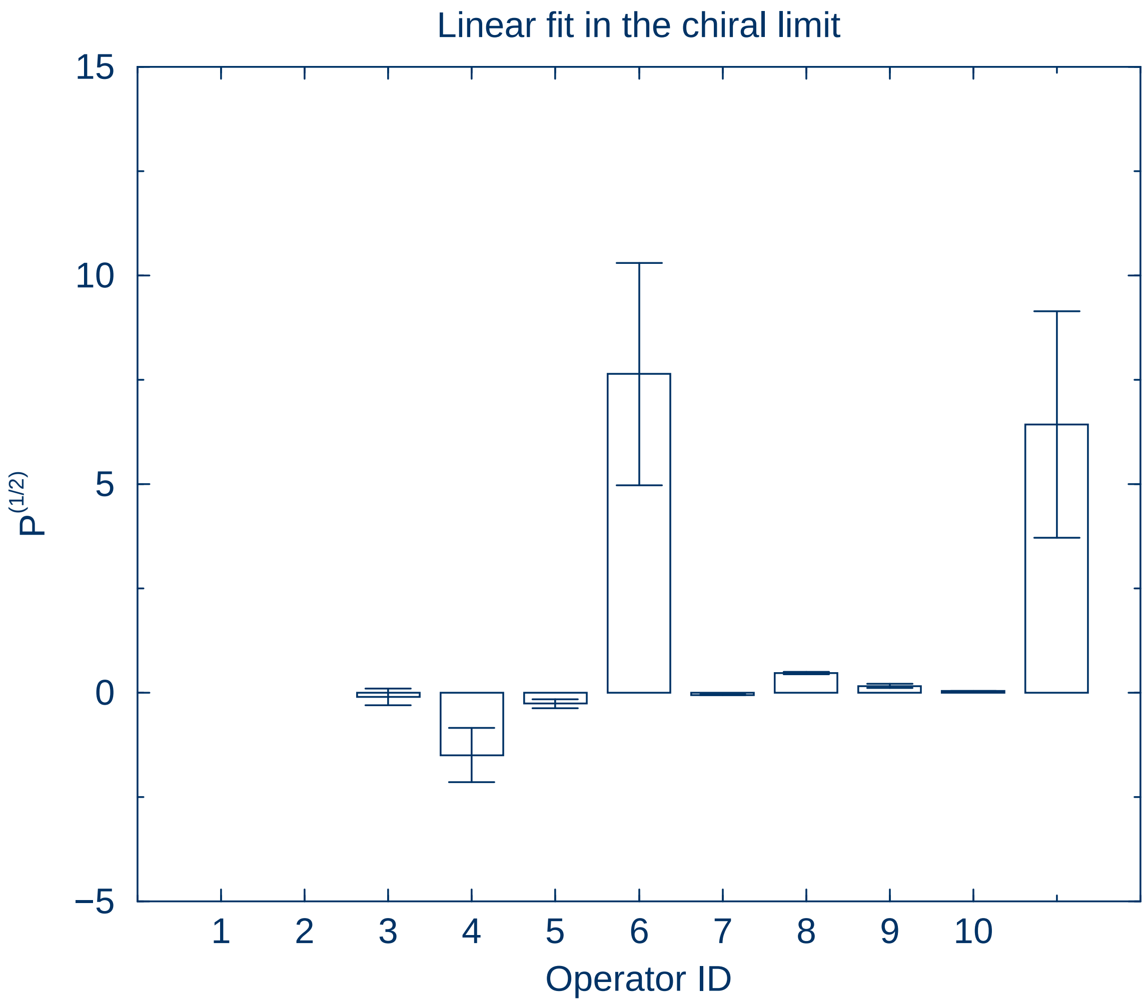
- NO contribution from $\langle Q_1 \rangle_I$ and $\langle Q_2 \rangle_I$.
- $P^{(1/2)}$ is dominated by $\langle Q_6 \rangle$.
- $P^{(3/2)}$ is dominated by $\langle Q_8 \rangle$.

Lattice version of Q_6 and Q_5

- Flavor symmetry of quenched QCD = $SU(3|3)$
- Flavor symmetry of full QCD = $SU(3)$
- Hence, one may choose a singlet in $SU(3)_R$ or a singlet in $SU(3|3)_R$ in quenched QCD.
- The standard operator $\in SU(3)_R$.
- Golterman-Pallante operator $\in SU(3|3)_R$.

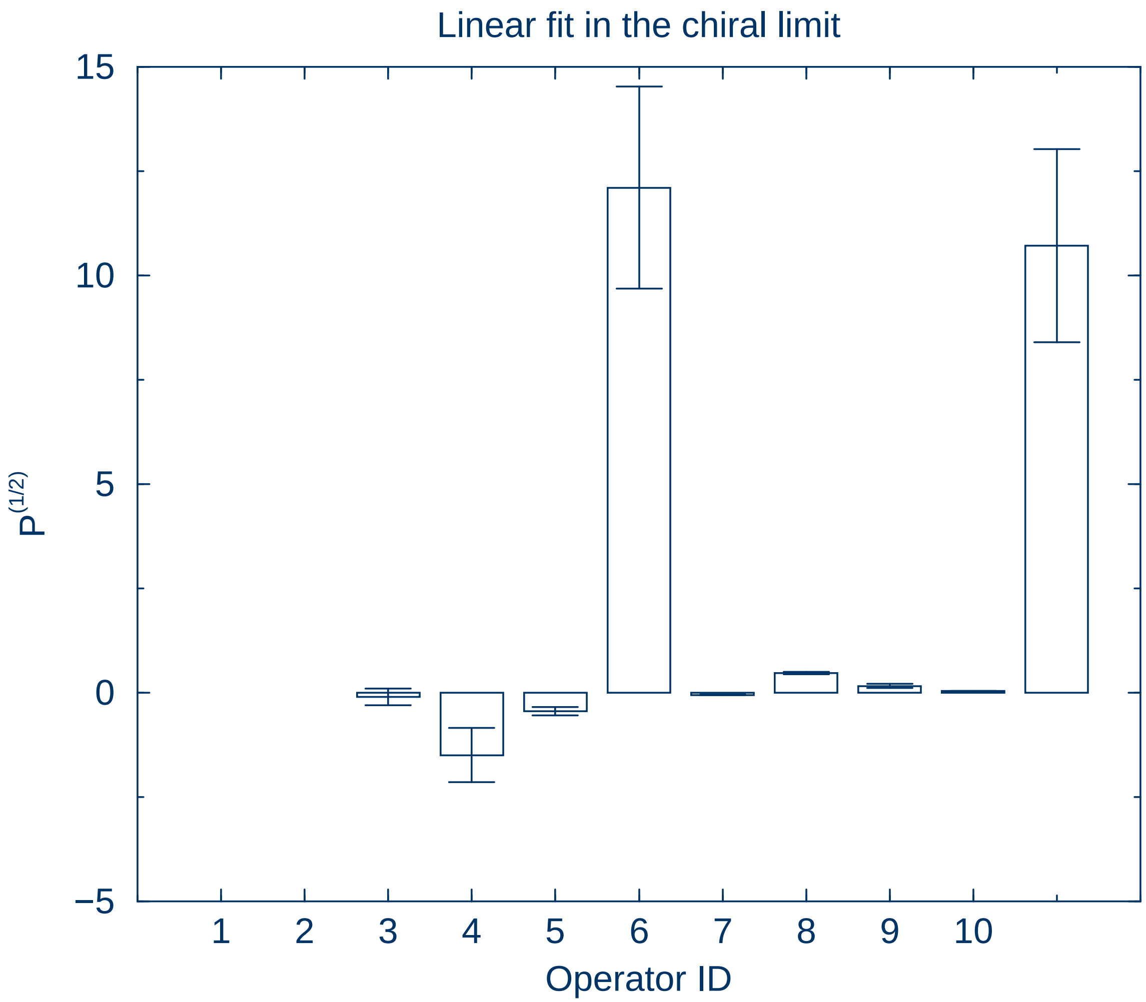
School of Physics
Seoul National University

$P^{(1/2)}$ (Standard op)



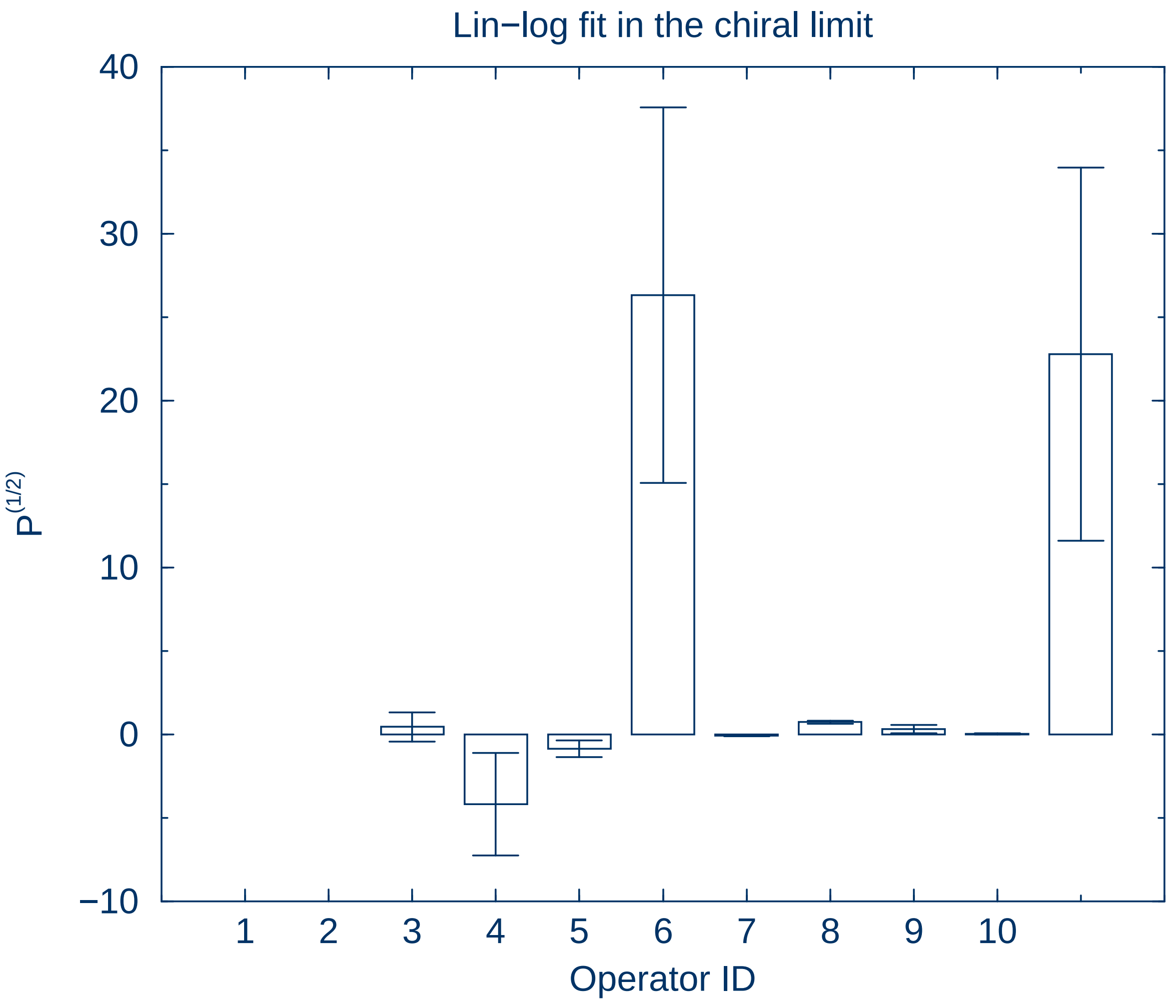
$$\mu = 1/a$$

$P^{(1/2)}$ (Golterman−Pallante op)



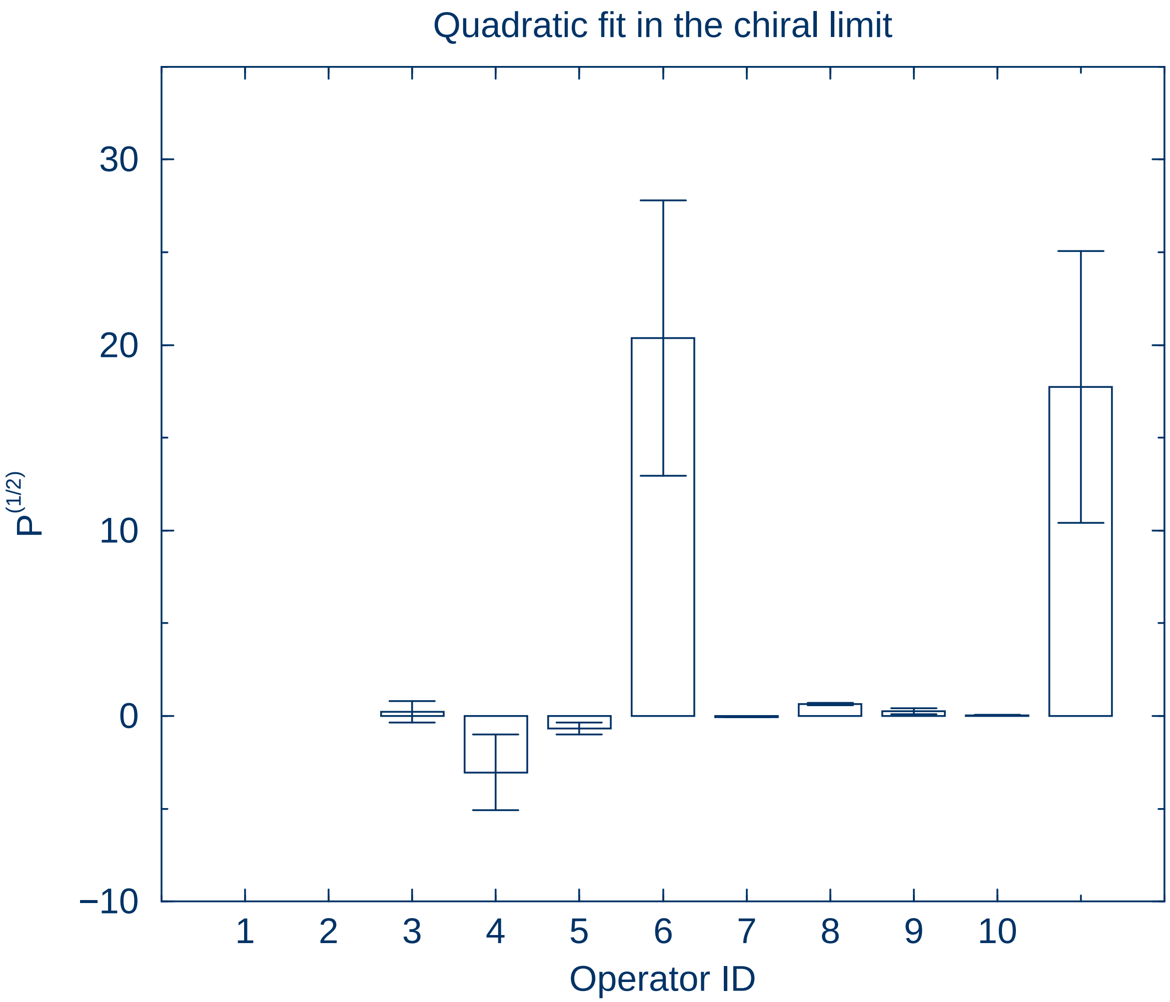
$$\mu = 1/a$$

$P^{(1/2)}$ (Golterman−Pallante op)



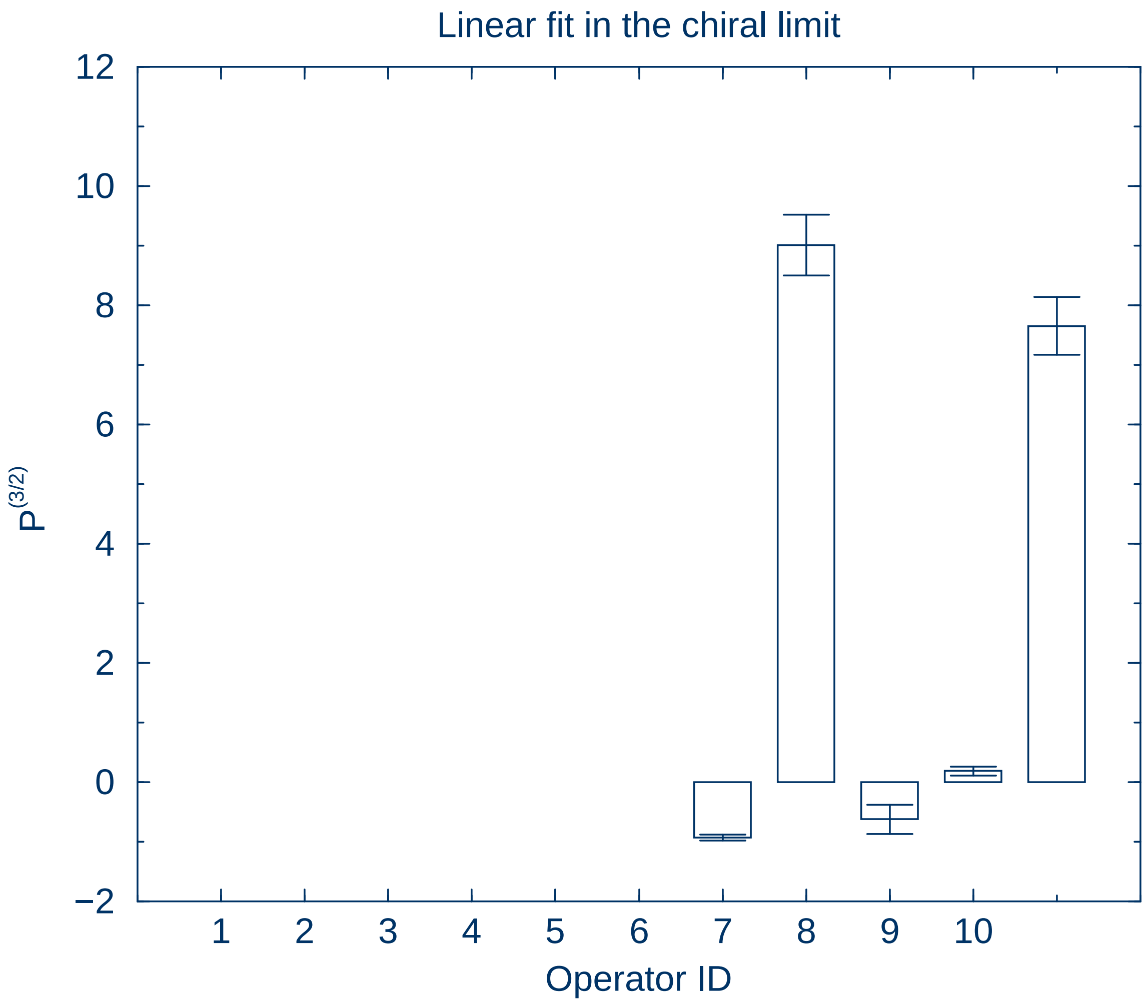
$$\mu = 1/a$$

$P^{(1/2)}$ (Golterman−Pallante op)



$$\mu = 1/a$$

$P^{(3/2)}$



Linear fit in the chiral limit
12
10
8
6
4
2
0
−2
1
2
3
4
5
6
7
8
9
10
Operator ID
P(3/2)

$$\mu = 1/a$$

ϵ'/ϵ ($\mu = 1/a$, preliminary results)

- Standard Operator:

$$\epsilon'/\epsilon = \begin{cases} -2.4(34) \times 10^{-4} & \text{(linear fit)} \\ -12.8(114) \times 10^{-4} & \text{(quadratic fit)} \\ -20.1(175) \times 10^{-4} & (\chi\text{-log fit}) \end{cases}$$

- Golterman-Pallante Operator:

$$\epsilon'/\epsilon = \begin{cases} +3.2(29) \times 10^{-4} & \text{(linear fit)} \\ +8.8(93) \times 10^{-4} & \text{(quadratic fit)} \\ +13.1(142) \times 10^{-4} & (\chi\text{-log fit}) \end{cases}$$

Calculating $K \to \pi\pi$ on the lattice

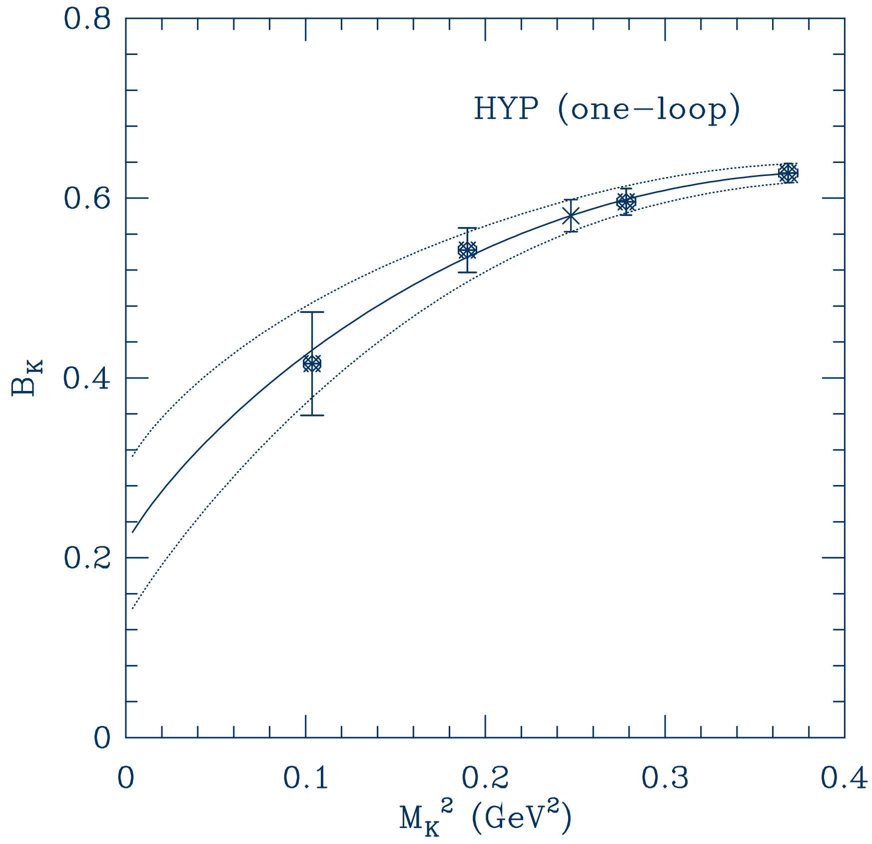
- Calculate $\langle \pi^+ | Q_i | K^+ \rangle$ and $\langle 0 | Q_i | K^0 \rangle$ on the lattice
- Use χPT at leading order to obtain $\langle \pi^+ \pi^- | Q_i | K^0 \rangle$

$$\left\{ \begin{array}{c} K^+ \to \pi^+ \\ K^0 \to 0 \end{array} \right\} \Longrightarrow \chi PT \Longrightarrow (K^+ \to \pi^+ \pi^-)$$

- Numerical study in quenched QCD.

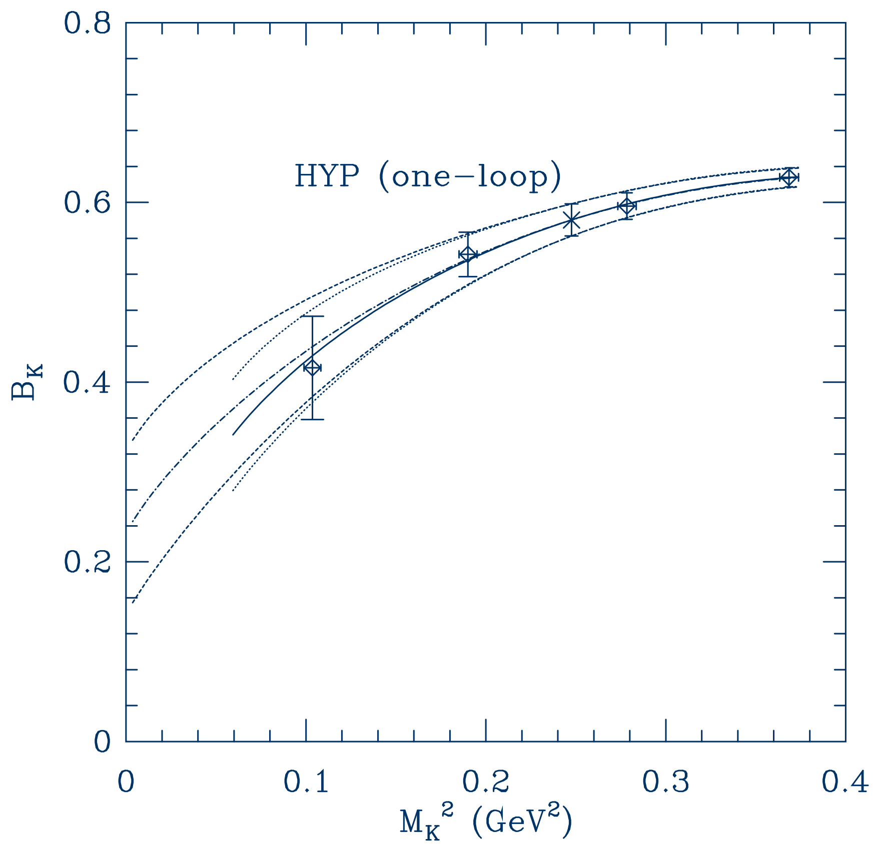
B_K

- Fit to quenched χPT.
- $B_K(m_K, \mu = 2GeV) = 0.580(18)$
- Consistent with JLQCD results at $a = 0$.
- $B_K(0, RGI) = 0.298(117)$
- Consistent with $1/N_c$ results.



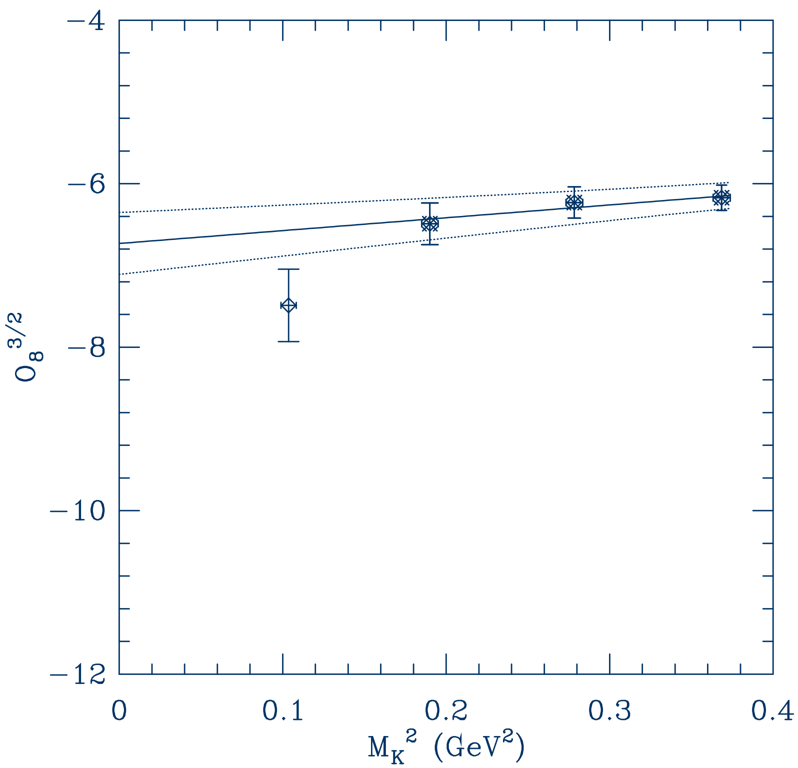
Finite volume effect on B_K

- Fit to quenched χPT with finite volume corrections.
- $B_K(m_K, 2GeV) = 0.580(18)(41)$
- $B_K(0, RGI) = 0.321(125)(175)$
- Consistent with $1/N_c$ results.

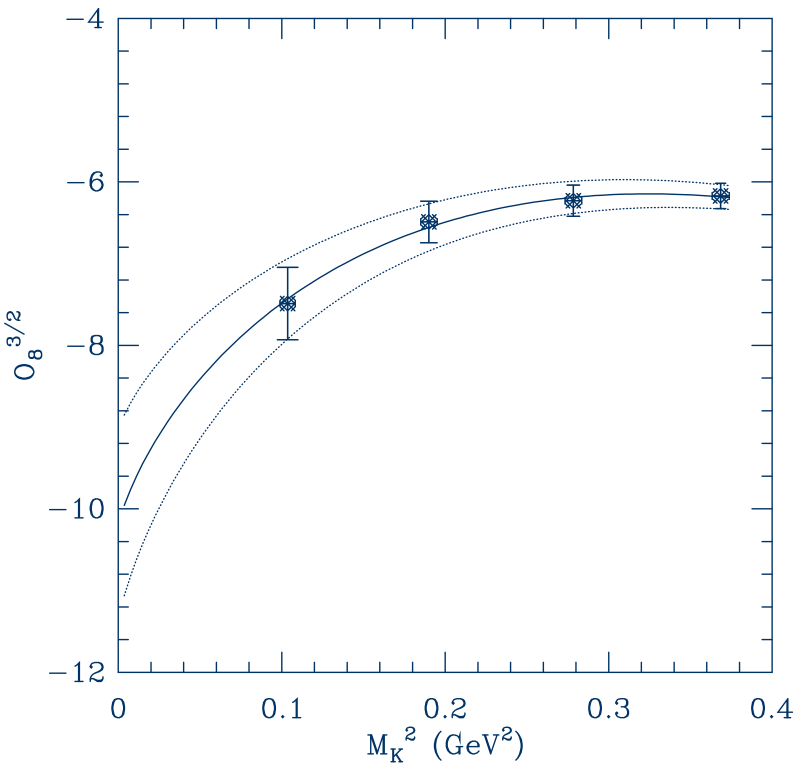


School of Physics
Seoul National University

$\langle Q_8^{\Delta I=3/2} \rangle$



Linear fit

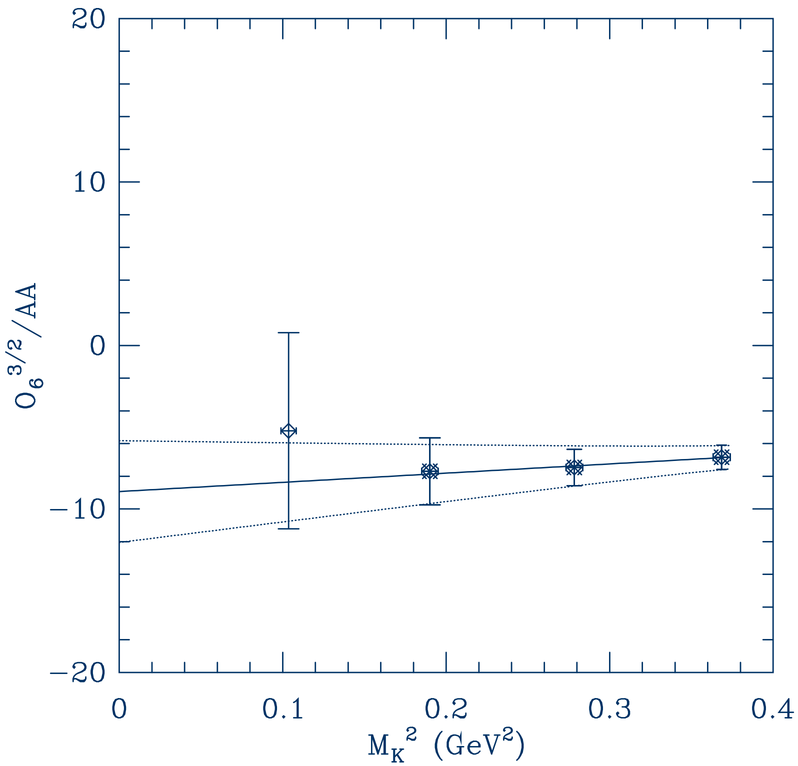


Chiral Log fit

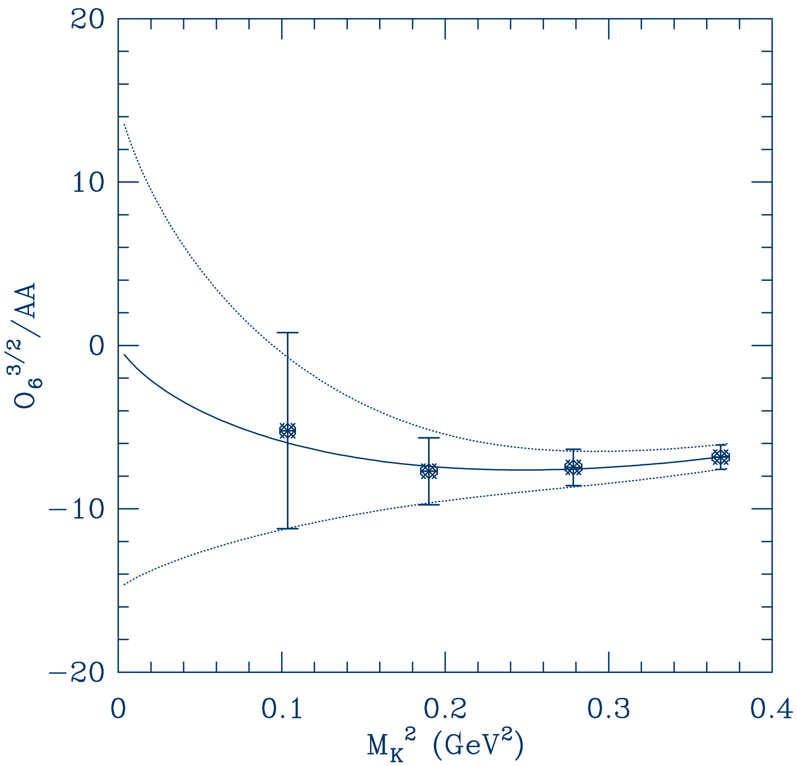
Lattice version of Q_6

- Flavor symmetry of quenched QCD = $SU(3|3)$
- Flavor symmetry of full QCD = $SU(3)$
- Hence, one may choose a singlet in $SU(3)_R$ or a singlet in $SU(3|3)_R$ in quenched QCD.
- The standard operator $\in SU(3)_R$.
- Golterman-Pallante operator $\in SU(3|3)_R$.

$\frac{\langle Q_6^{\Delta I=1/2} \rangle}{m_K^2 f^2}$ (standard op)

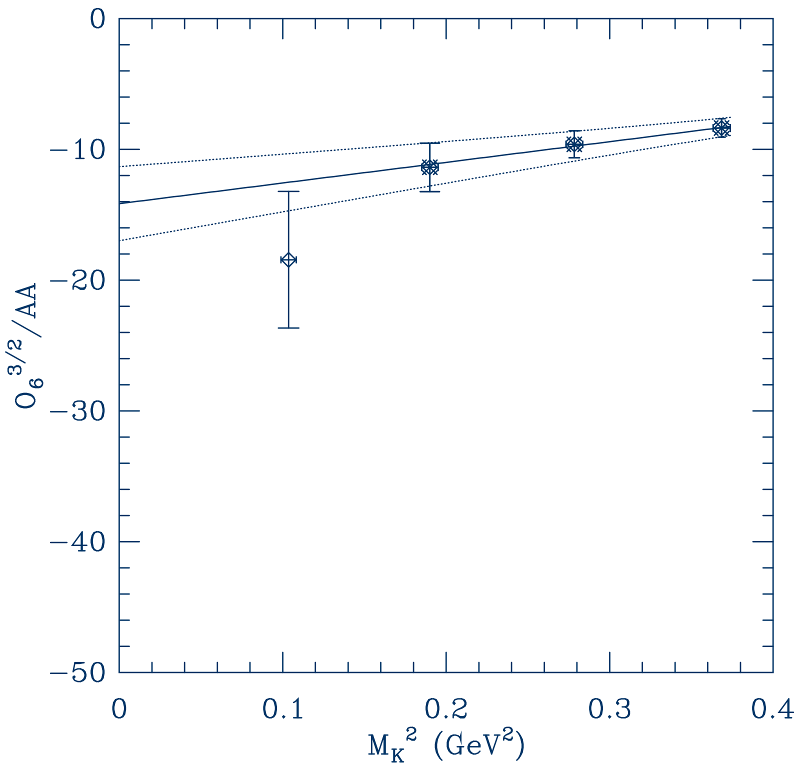


Linear fit

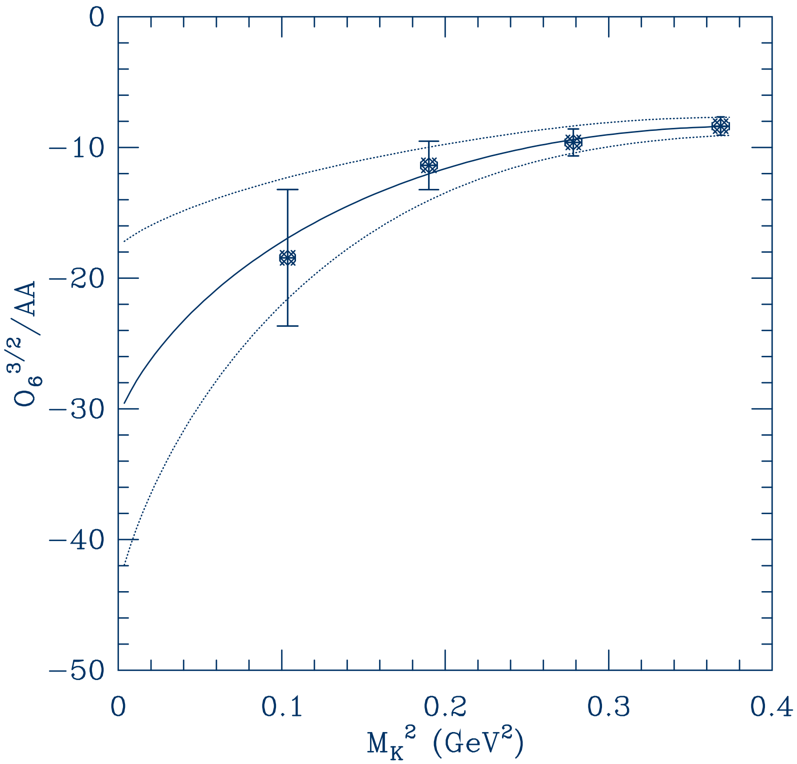


Chiral Log fit

$\frac{\langle Q_6^{\Delta I=1/2} \rangle}{m_K^2 f^2}$ (Golterman-Pallante op)



Linear fit



Chiral Log fit

Summary and Conclusion

- HYP/$\overline{\mathrm{Fat7}}$ resolves the long-standing problem of large perturbative corrections.
- The perturbative calculation for HYP/$\overline{\mathrm{Fat7}}$ is extended to current-current diagrams and penguin diagrams.
- This allows full matching between lattice and continuum matrix elements.
- The Golterman-Pallante method prefers positive ϵ'/ϵ.
- The standard method prefer negative ϵ'/ϵ.
- This tells us that uncertainty from the quenched approximation is large for ϵ'/ϵ.

- We are extending the current calculation to include more quark masses and to accumulate higher statistics.
- In order to constrain the form of the fitting functions, we need results of the staggered chiral perturbation.
- We need to check the finite volume effect (1.6 fm $\longrightarrow$ 2.4 fm).
- We need to extend it to weaker gauge couplings to check the scaling violation, which is expected to be quite small.
- We need to calculate ϵ'/ϵ in partially quenched QCD and full QCD.
- We need to include the next leading order in χPT to

check the size of NLO correction.

RG evolution for $N_f = 3$

- For $N_f = 3$, there is a removable singularity.
- $2\beta_0 + \gamma_8^{(0)} - \gamma_7^{(0)} = 0 \rightarrow$ singularity.
- Both the denomenator and numerator vanish.
- Final analytical form contains

$$\ln\left(\frac{\alpha_s(m_2)}{\alpha_s(m_1)}\right) \;, \quad \left[\ln\left(\frac{\alpha_s(m_2)}{\alpha_s(m_1)}\right)\right]^2$$