On-shell improvement of the massive Wilson quark action

Y.Kuramashi (U.Tsukuba)
in collaboration with
S.Aoki (U.Tsukuba)
Y.Kayaba (U.Tsukuba)
N.Yamada (RIKEN-BNL)

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§1. Introduction

$m_b a \sim 1 - 2$ in quenched QCD
$\qquad 2 - 3$ in full QCD

- static approximation ($m_Q \to 0$)
- NRQCD ($1/m_Q$ expansion)
- nonrelativistic interpretation of Wilosn/SW results (Fermilab interpretation)

relativistic approach

cutoff effects

$(m_Q a)^n$

$(a\Lambda_{\mathrm{QCD}})^k$

$(m_Q a)^n \cdot (a\Lambda_{\mathrm{QCD}})^k$

relativistic on-shell improvement in the massive case

$$S_{\mathrm{eff}} = S_{\mathrm{cont}} + \sum_{k\geq 1} a^k \int d^4x C_{4+k,i}(g, m_Q a)\mathcal{O}_{4+k,i}(x)$$

$\longrightarrow$ remove $(m_Q a)^n a\Lambda_{\mathrm{QCD}}$ errors

remaining cutoff effects are $O((a\Lambda_{\mathrm{QCD}})^2)$

$a^{-1} \sim 3\mathrm{Gev}$, $\Lambda_{\mathrm{QCD}} \sim 300\mathrm{MeV} \longrightarrow (a\Lambda_{\mathrm{QCD}})^2 \sim 1\%$

extend Symanzik's improvement program to the massive case

§**2.** Relativistic on-shell improvement

§**3.** Determination of improvement coefficients in the action

§**4.** Improvement of the axial current

§**5.** Numerical simulations

§**6.** Summary

§2. Relativistic on-shell improvement

cutoff effects

$$(m_Q a)^n, \ \ (a\Lambda_{\mathrm{QCD}})^k, \ \ (m_Q a)^n \cdot (a\Lambda_{\mathrm{QCD}})^k$$

we assume

$$f_0(m_Q a) > f_1(m_Q a)a\Lambda_{\mathrm{QCD}} > f_2(m_Q a)(a\Lambda_{\mathrm{QCD}})^2 > \cdots$$

$f_i(m_Q a)$ have Taylor expansions around $m_Q a = 0$

Symanzik's improvement program

$$S_{\mathrm{eff}} = S_{\mathrm{cont}} + \sum_{k,i\geq 1} a^k \int d^4x C_{4+k,i}(g)\mathcal{O}_{4+k,i}(x)$$

allowed operators with axis interchange symmetry

$$
\begin{aligned}
\text{dim.3}: \quad & \mathcal{O}_3(x) = \bar{q}(x)q(x) \\
\text{dim.4}: \quad & \mathcal{O}_4(x) = \bar{q}(x)\not{D}q(x) \\
\text{dim.5}: \quad & \mathcal{O}_{5a}(x) = \bar{q}(x)D_\mu^2 q(x) \\
& \mathcal{O}_{5b}(x) = i\bar{q}(x)\sigma_{\mu\nu}F_{\mu\nu}q(x) \\
\text{dim.6}: \quad & \mathcal{O}_{6a}(x) = \bar{q}(x)\gamma_\mu D_\mu^3 q(x) \\
& \mathcal{O}_{6b}(x) = \bar{q}(x)D_\mu^2\not{D}q(x) \\
& \qquad \vdots
\end{aligned}
$$

generic form of the improved quark action

$$
\begin{aligned}
S_q^{\mathrm{imp}} = \sum_x \big[& c_3\mathcal{O}_3(x) + c_4\mathcal{O}_4(x) + c_{5a}\mathcal{O}_{5a}(x) + c_{5b}\mathcal{O}_{5b}(x) \\
& + c_{6a}\mathcal{O}_{6a}(x) + c_{6b}\mathcal{O}_{6b}(x) + \cdots \big],
\end{aligned}
$$

allowed operators with axis interchange symmetry

dim.3 : $\mathcal{O}_3(x) = \bar{q}(x)q(x)$

dim.4 : $\mathcal{O}_4(x) = \bar{q}(x)\not{D}q(x)$

dim.5 : $\mathcal{O}_{5a}(x) = \bar{q}(x)D_\mu^2 q(x)$ $\rightarrow$ redundant

$\mathcal{O}_{5b}(x) = i\bar{q}(x)\sigma_{\mu\nu}F_{\mu\nu}q(x)$ $\rightarrow O(a\Lambda_{\mathrm{QCD}})$

dim.6 : $\mathcal{O}_{6a}(x) = \bar{q}(x)\gamma_\mu D_\mu^3 q(x)$ still $O((a\Lambda_{\mathrm{QCD}})^2)$ for $m_Q a \sim O(1)$?

$\mathcal{O}_{6b}(x) = \bar{q}(x)D_\mu^2\not{D}q(x)$ $\partial_0 \sim O(m_Q),\ \ \partial_i \sim O(\Lambda_{\mathrm{QCD}})$

$\vdots$

generic form of the improved quark action

$$S_q^{\mathrm{imp}} = \sum_x [c_3\mathcal{O}_3(x) + c_4\mathcal{O}_4(x) + c_{5a}\mathcal{O}_{5a}(x) + c_{5b}\mathcal{O}_{5b}(x) + c_{6a}\mathcal{O}_{6a}(x) + c_{6b}\mathcal{O}_{6b}(x) + \cdots],$$

contribution of $\mathcal{O}_{6a}(x) = \bar{q}(x)\gamma_\mu D_\mu^3 q(x)$

$$
\begin{aligned}
a^2\bar{q}(x)\gamma_0 D_0^3 q(x) &= -\frac{1}{a}(m_Q a)^3\bar{q}(x)q(x) \\
&\quad -(m_Q a)^2\bar{q}(x)\gamma_i D_i q(x) \\
&\quad +a(m_Q a)\bar{q}(x)D_i^2 q(x) + O((\Lambda_{\mathrm{QCD}}a)^2). \\
a^2\bar{q}(x)\gamma_0 D_0^3 q(x) &\sim O((a\Lambda_{\mathrm{QCD}})^2)
\end{aligned}
$$

with eq. of motion: $\gamma_0 D_0 + \gamma_i D_i + m_Q = 0$

expressed in terms of lower dimensional operators
$\longrightarrow$ asymmetries with $(m_Q a)^n$ in coefficients

$$
\begin{aligned}
\bar{q}(x)\gamma_0 D_0 q(x) &\leftrightarrow \bar{q}(x)\gamma_i D_i q(x) \\
\bar{q}(x)\gamma_0 D_0^2 q(x) &\leftrightarrow \bar{q}(x)\gamma_i D_i^2 q(x)
\end{aligned}
$$

axis interchange symmetry is not retained any more general form of quark action for $O(a\Lambda_{\mathrm{QCD}})$ improvement

$$
\begin{aligned}
S_q^{\mathrm{RHQ}} = \sum_x \Bigg[& m_0 \bar{q}(x) q(x) + \bar{q}(x)\gamma_0 D_0 q(x) + \nu \sum_i \bar{q}(x)\gamma_i D_i q(x) \\
& - \frac{r_t a}{2} \bar{q}(x) D_0^2 q(x) - \frac{r_s a}{2} \sum_i \bar{q}(x) D_i^2 q(x) \\
& - \frac{iga}{2} c_E \sum_i \bar{q}(x)\sigma_{0i} F_{0i} q(x) - \frac{iga}{4} c_B \sum_{i,j} \bar{q}(x)\sigma_{ij} F_{ij} q(x) \Bigg],
\end{aligned}
$$

r_t is redundant

ν, r_s, c_E, c_B should be adjusted in a $m_Q a$ dependent way

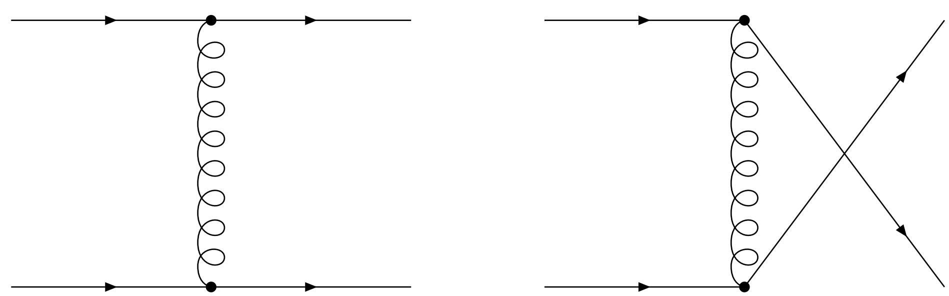
$\nu - 1$, $r_s - r_t$, $c_B - c_E$ represent contributions from higher dimensional ops. without space-time rotational symmetry

cf. r_t and r_s are redundant in Fermilab approach

§3. Determination of improvement coefficients
§3-1. $m_Q a$ corrections at tree level

"$\cdots$ on-shell quantities(particle energies, scattering amplitudes, normalized matrix elements of local composite fields between particle states etc.) $\cdots$" Lüscher-Sint-Sommer-Weisz, NPB478(1996)365

ν, r_s, c_E, c_B can be determined by on-shell quark-quark scattering



continuum scattering amplitude should be reproduced
massless case done by Wohlert to determine c_{SW}

the on-shell improvement condition yields

$$
\begin{aligned}
\nu^{(0)} &= \frac{\sinh(m_p^{(0)})}{m_p^{(0)}}, \\
r_s^{(0)} &= \frac{\cosh(m_p^{(0)}) + r_t \sinh(m_p^{(0)})}{m_p^{(0)}} - \frac{\sinh(m_p^{(0)})}{m_p^{(0)2}}, \\
c_E^{(0)} &= r_t \nu^{(0)}, \\
c_B^{(0)} &= r_s^{(0)},
\end{aligned}
$$

<u>points</u>

– 4 parameters are uniquely determined

– r_t is not fixed $\longrightarrow$ consistent with redundancy

– $\nu \to 1$, $r_s \to r_t$, $c_B \to r_t$, $c_E \to r_t$ for $m_p^{(0)} \to 0$

inverting the Wilson-Dirac operator

$$
\begin{aligned}
S_q^{-1}(p) &= i\gamma_0 \sin(p_0) + \nu i \sum_i \gamma_i \sin(p_i) + m_0 \\
&\quad + r_t(1-\cos(p_0)) + r_s \sum_i (1-\cos(p_i))
\end{aligned}
$$

ν, r_s are determined by demanding

$$
S_q(p) = \frac{1}{Z_q^{(0)}} \frac{-i\gamma_0 p_0 - i\sum_i \gamma_i p_i + m_p^{(0)}}{p_0^2 + \sum_i p_i^2 + {m_p^{(0)}}^2} + (\text{no pole terms}) + O((p_i a)^2)
$$

around the pole

speed of light (coeffs. of $\gamma_0 p_0$ & $\gamma_i p_i$) $\to \nu^{(0)}$

dispersion relation ($E = {m_p^{(0)}}^2 + \sum_i p_i^2$) $\to r_s^{(0)}$

same results as the quark-quark scattering

$$\nu^{(0)} = \frac{\sinh(m_p^{(0)})}{m_p^{(0)}}$$

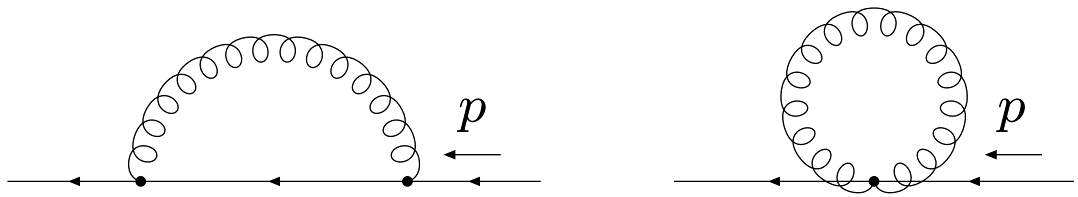
$$r_s^{(0)} = \frac{\cosh(m_p^{(0)}) + r_t \sinh(m_p^{(0)})}{m_p^{(0)}} - \frac{\sinh(m_p^{(0)})}{m_p^{(0)2}}$$

Dirac spinor has a correct continuum form up to $O((p_i a)^2)$
necessary for correct on-shell matrix elements

next step is to determine ν, r_s, c_E, c_B at one-loop level

§3-2. $\nu(m_Q a, g)$, $r_s(m_Q a, g)$ at one-loop level

determined from quark propagator



up to one-loop level

$$
\begin{aligned}
S_q^{-1}(p,m) &= i\gamma_0 \sin p_0 [1 - g^2 B_0(p,m)] + \nu i \sum_i \gamma_i \sin p_i [1 - g^2 B_i(p,m)] \\
&\quad + m + 2 r_t \sin^2\left(\frac{p_0}{2}\right) + 2 r_s \sum_i \sin^2\left(\frac{p_i}{2}\right) - g^2 \widehat{C}(p,m)
\end{aligned}
$$

with

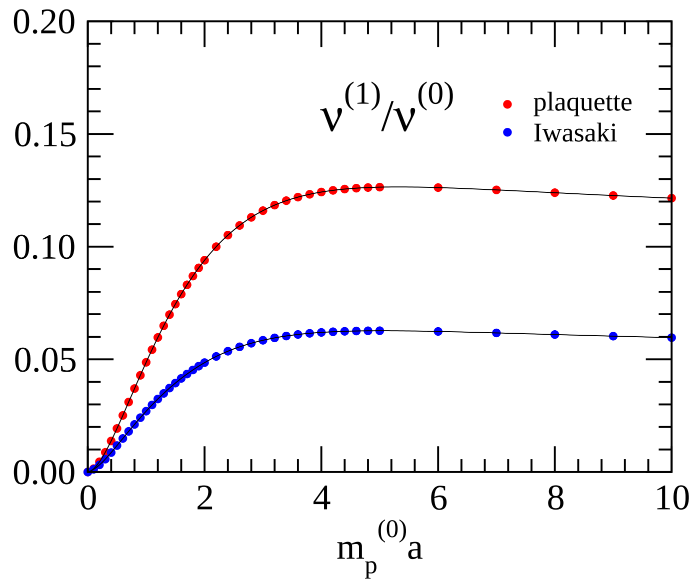
$$
\begin{aligned}
m &= m_0 - g^2 C(p=0, m=0) \\
\widehat{C}(p,m) &= C(p,m) - C(p=0, m=0)
\end{aligned}
$$

parameter $\nu = \nu^{(0)} + g^2\nu^{(1)}$

adjust speed of light

$$\nu[1 - g^2 B_i(p^*, m)] = \frac{\sinh(m_p)}{m_p}[1 - g^2 B_0(p^*, m)]$$

$$p^* \equiv (p_0 = im_p, p_i = 0)$$

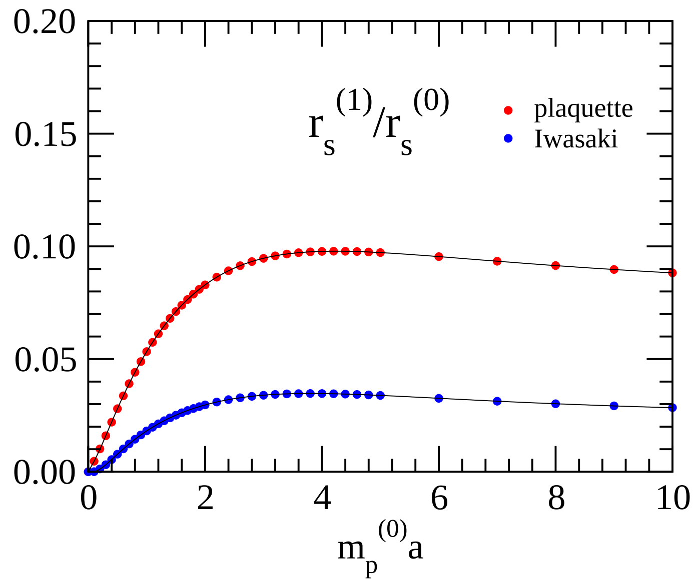


parameter $r_s = r_s^{(0)} + g^2 r_s^{(1)}$

dispersion relation

$$E = m_p^2 + F(\nu, r_s) \sum_i p_i^2 + O(p_i^4)$$

$$\rightarrow F(\nu, r_s) = 1$$



§3-3. $c_B(m_Q a, g)$, $c_E(m_Q a, g)$ at one-loop level

c_B, c_E from on-shell quark-quark scattering at tree level

$\longrightarrow$ extend to one-loop level

<u>one possible strategy</u>

Schrödinger functional with large quark mass?

unwilling to try

§3-3. $c_B(m_Q a, g)$, $c_E(m_Q a, g)$ at one-loop level

c_B, c_E from on-shell quark-quark scattering at tree level

$\longrightarrow$ extend to one-loop level

<u>one possible strategy</u>

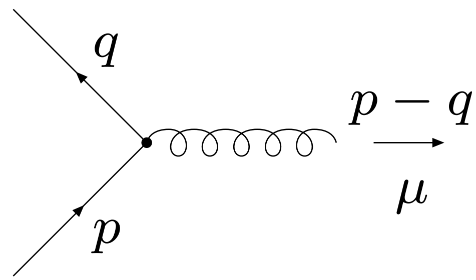
Schrödinger functional with large quark mass?

unwilling to try

<u>another strategy</u>

consider ordinary PT with fictitious gluon mass as infrared regulator

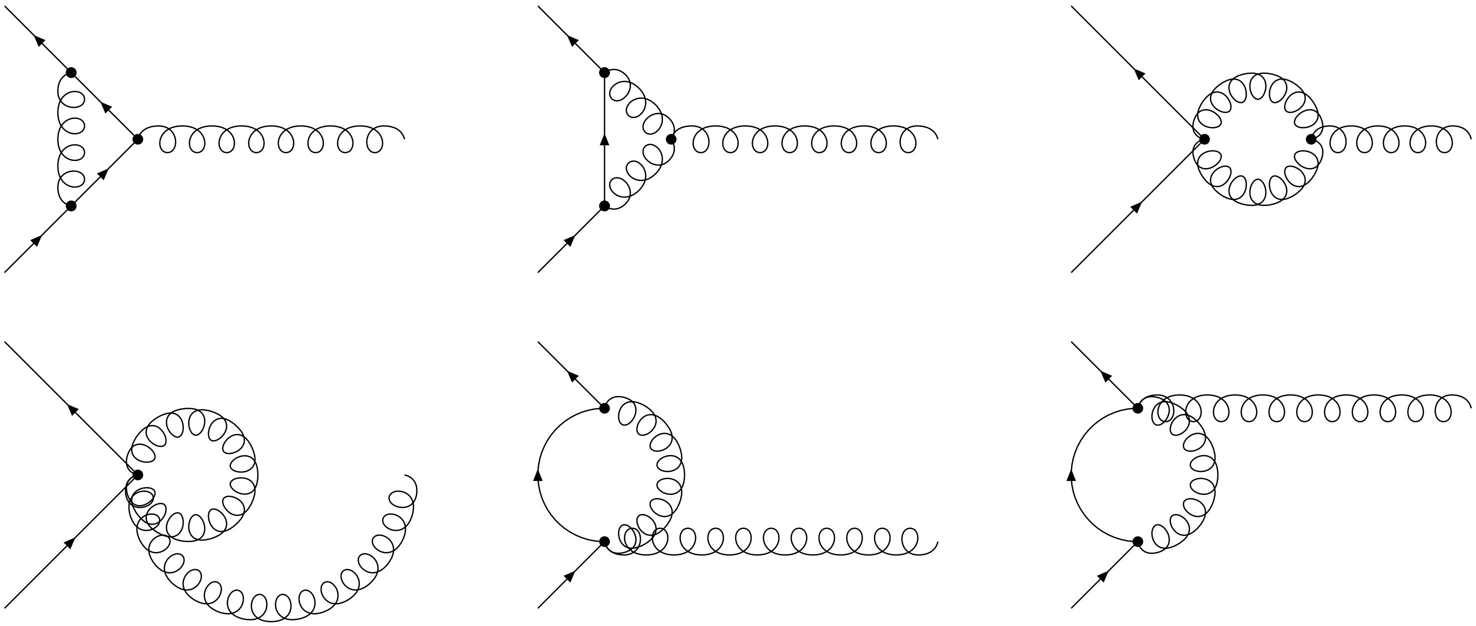
we first test this method in massless case determination of c_{SW} at tree level



$$
\begin{aligned}
V_\mu(p,q) \;&=\; -gT^A\{i\gamma_\mu + r\left(\frac{p_\mu a + q_\mu a}{2}\right)\} \\
&\quad -gr\frac{c_{\mathsf{SW}}}{2}T^A\sum_\nu \sigma_{\mu\nu}(p_\nu - q_\nu)a + O(a^2)
\end{aligned}
$$

$$
\bar{u}(q)V_\mu(p,q)u(p) = -gT^A\bar{u}(q)\{i\gamma_\mu + \frac{r}{2}(1-c_{\mathsf{SW}})(p_\mu + q_\mu)a\}u(p) + O(a^2)
$$

$\longrightarrow c_{\mathsf{SW}} = 1$ at tree level one-loop diagrams



$\frac{11}{3}(1 - c_{\mathrm{SW}}{}^{(0)})\frac{1}{16\pi^2}\ln\left|\frac{\pi^2}{\lambda^2 a^2}\right| + 0.26858825(1)$

consistent with previous work (Wohlert, Lüscher-Weisz)

cancellation of infrared divergences is not trivial determination of $c_B^{(1)}$

general form of off-shell vertex function at one-loop level
without space-time rotational symmetry

$$\begin{aligned}\Lambda_k^{(1)}(p,q,m) \;=\; & \gamma_k F_1^k + \gamma_k\{\not{p} F_2^k + \not{p}_0 F_3^k\} + \{\not{q} F_4^k + \not{q}_0 F_5^k\}\gamma_k \\ & + \not{q}\gamma_k\not{p} F_6^k + \not{q}\gamma_k\not{p}_0 F_7^k + \not{q}_0\gamma_k\not{p} F_8^k + \gamma_k\not{p}_0\not{p} F_9 + \not{q}\not{q}_0\gamma_k F_{10}^k \\ & + (p_k+q_k)\left[H_1^k + \not{p} H_2^k + \not{q} H_3^k + \not{q}\not{p} H_4^k\right] \\ & + (p_k-q_k)\left[G_1^k + \not{p} G_2^k + \not{q} G_3^k + \not{q}\not{p} G_4^k\right] + O(a^2)\end{aligned}$$

with $\not{p}_0 = p_0\gamma_0$, $\not{q}_0 = q_0\gamma_0$
charge conjugation symmetry demands

$$\begin{aligned}& F_2^k = F_4^k,\;\; F_3^k = F_5^k,\;\; F_7^k = F_8^k,\;\; F_9^k = F_{10}^k \\ & H_2^k = H_3^k \\ & G_1^k = G_4^k = 0,\;\; G_2^k = -G_3^k\end{aligned}$$

Sandwiching $\Lambda_k^{(1)}(p,q,m)$ by the on-shell quark states

$$
\begin{aligned}
& \bar{u}(q)\Lambda_k^{(1)}(p,q,m)u(p)|^{\mathrm{latt}} \\
= \; & \bar{u}(q)\gamma_k u(p)\left\{F_1^k + im_p(F_2^k + F_4^k) - m_p^2 F_6^k\right\}^{\mathrm{latt}} \\
& +(p_k+q_k)\bar{u}(q)u(p)\left\{H_1^k + im_p(H_2^k + H_3^k) - m_p^2 H_4^k\right\}^{\mathrm{latt}} \\
& +(p_k-q_k)\bar{u}(q)u(p)\left\{G_1^k + im_p(G_2^k + G_3^k) - m_p^2 G_4^k\right\}^{\mathrm{latt}} + O(a^2)
\end{aligned}
$$

withn $\not{p}u(p) = im_p u(p)$, $\bar{u}(q)\not{q} = im_p\bar{u}(q)$

Sandwiching $\Lambda_k^{(1)}(p,q,m)$ by the on-shell quark states

$$
\begin{aligned}
& \bar{u}(q)\Lambda_k^{(1)}(p,q,m)u(p)|^{\mathrm{latt}} \\
= \ & \bar{u}(q)\gamma_k u(p)\left\{\underbrace{F_1^k + im_p(F_2^k + F_4^k) - m_p^2 F_6^k}_{Z_q^{(1)}}\right\}^{\mathrm{latt}} \\
& +(p_k+q_k)\bar{u}(q)u(p)\left\{\underbrace{H_1^k + im_p(H_2^k + H_3^k) - m_p^2 H_4^k}_{c_B^{(1)}}\right\}^{\mathrm{latt}} \\
& +(p_k-q_k)\bar{u}(q)u(p)\left\{\underbrace{G_1^k + im_p(G_2^k + G_3^k) - m_p^2 G_4^k}_{=0 \text{ by C.C.}}\right\}^{\mathrm{latt}} + O(a^2)
\end{aligned}
$$

withn $\not{p}u(p) = im_p u(p)$, $\bar{u}(q)\not{q} = im_p\bar{u}(q)$

$\left\{H_1^k + \cdots\right\}^{\mathrm{latt}}$ contains both $O(a)$ and $O(1/m)$

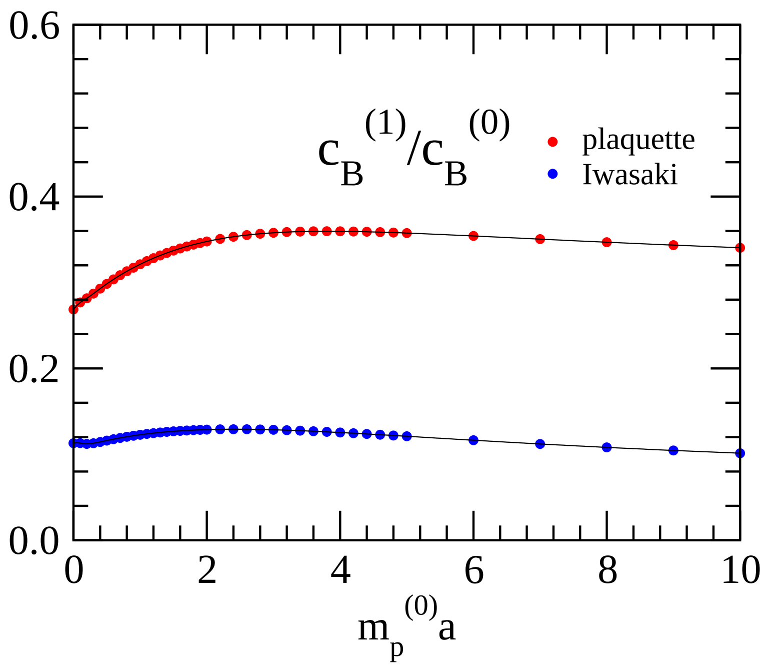
do the same calculation in the continuum

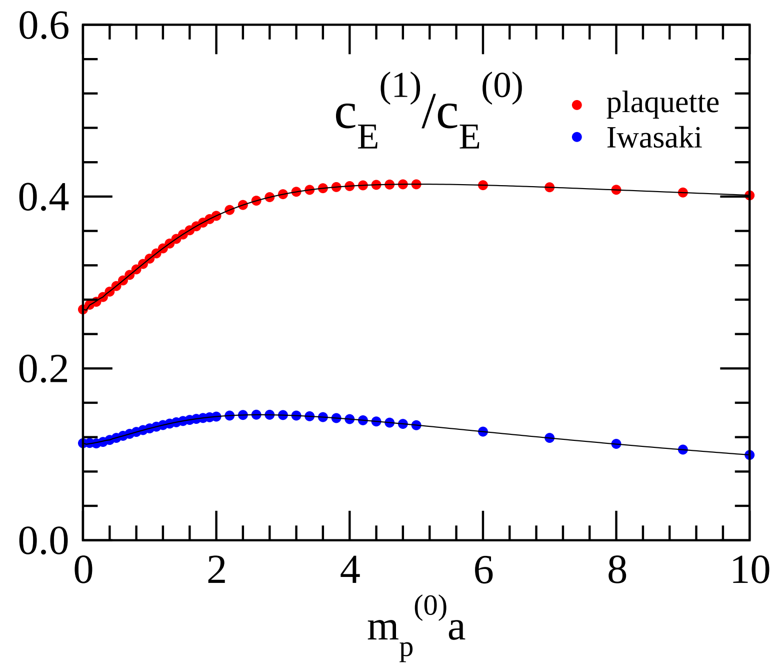
$$
\begin{aligned}
& \bar{u}(q)\Lambda_k^{(1)}(p,q,m)u(p)|^{\text{cont}} \\
= \; & \bar{u}(q)\gamma_k u(p)\left\{F_1^k + im_p(F_2^k + F_4^k) - m_p^2 F_6^k\right\}^{\text{cont}} \\
& +(p_k + q_k)\bar{u}(q)u(p)\left\{H_1^k + im_p(H_2^k + H_3^k) - m_p^2 H_4^k\right\}^{\text{cont}}
\end{aligned}
$$

subtract the physical contribution

$$
\begin{aligned}
\frac{c_B^{(1)} - r_s^{(1)}}{2} \; = \; & \left[H_1^k + im_p(H_2^k + H_3^k) - m_p^2 H_4^k\right]_{p=p^*,q=q^*}^{\text{latt}} \\
& -Z_q^{(0)}\left[H_1^k + im_p(H_2^k + H_3^k) - m_p^2 H_4^k\right]_{p=p^*,q=q^*}^{\text{cont}}
\end{aligned}
$$

with $p^* \equiv (p_0 = im_p, p_i = 0)$, $q^* \equiv (q_0 = im_p, q_i = 0)$

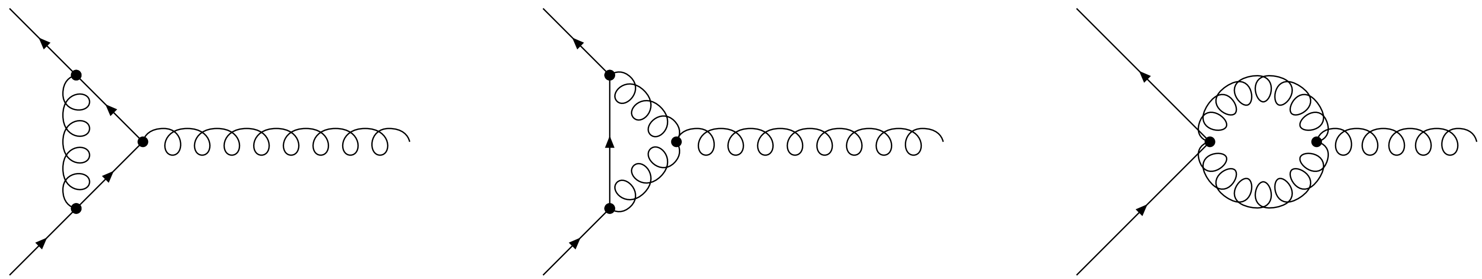




$c_B^{(1)}$ is increased $\longrightarrow$ hyperfine splitting will be enhanced infrared divergence

c_B, c_E should be free from infrared divergence

three divergent diagrams



infrared divergences are cancelled out after summation

if and only if $\nu^{(0)}$, $r_s^{(0)}$, $c_B^{(0)}$, $c_E^{(0)}$ are properly tuned

$\longrightarrow$ another evidence for validity of our formulation

4 parameters in the action should be properly tuned

§4. Improvement of the axial current

focus on the time component

form of improved operators

$$
\begin{aligned}
A_0^{\mathrm{latt},R}(x) &= Z_{A_0}^{\mathrm{latt}}\Big[\bar{q}(x)\gamma_0\gamma_5 Q(x) - g^2 c_{A_0}^{+}\partial_0^{+}\{\bar{q}(x)\gamma_5 Q(x)\} \\
&\quad - g^2 c_{A_0}^{-}\partial_0^{-}\{\bar{q}(x)\gamma_5 Q(x)\} + O(g^4)\Big]
\end{aligned}
$$

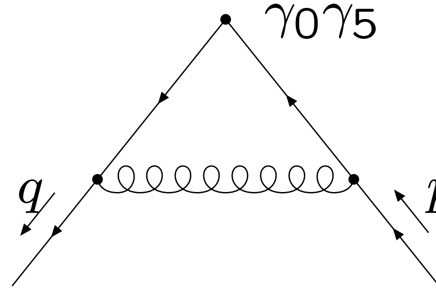
$$
\begin{aligned}
\partial_0^{+}\{\bar{q}(x)\gamma_5 Q(x)\} &= \bar{q}(x)\gamma_5\{\partial_0 Q(x)\} + \{\partial_0\bar{q}(x)\}\gamma_5 Q(x) \\
\partial_0^{-}\{\bar{q}(x)\gamma_5 Q(x)\} &= \bar{q}(x)\gamma_5\{\partial_0 Q(x)\} - \{\partial_0\bar{q}(x)\}\gamma_5 Q(x)
\end{aligned}
$$

in case of $m_Q = m_q$, $c_{A_0}^{-} = 0$ from charge conjugation symmetry

determination of $c_{A_0}^{\pm}$ is quite similar to that of c_B, c_E

general form of off-shell vertex function at one-loop level

$$\begin{aligned}
& \Lambda^{(1)}_{05}(p, q, m_{p1}, m_{p2}) \\
= \; & \gamma_0\gamma_5 F^{05}_1 + \gamma_0\gamma_5 \not{p} F^{05}_2 + \not{q}\gamma_0\gamma_5 F^{05}_3 + \not{q}\gamma_0\gamma_5\not{p} F^{05}_4 \\
& +(p_0 - q_0)\left[\gamma_5 G^{05}_1 + \gamma_5\not{p} G^{05}_2 + \not{q}\gamma_5 G^{05}_3 + \not{q}\gamma_5\not{p} G^{05}_4\right] \\
& +(p_0 + q_0)\left[\gamma_5 H^{05}_1 + \gamma_5\not{p} H^{05}_2 + \not{q}\gamma_5 H^{05}_3 + \not{q}\gamma_5\not{p} H^{05}_4\right] + O(a^2),
\end{aligned}$$



Sandwiching $\Lambda_k^{(1)}(p,q,m)$ by the on-shell quark states

$$
\begin{aligned}
& \bar{u}(q)\Lambda_{05}^{(1)}(p,q,m_{p1},m_{p2})u(p) \\
= \ & \bar{u}(q)\gamma_0\gamma_5 u(p)\left\{F_1^{05}+im_{p1}F_2^{05}+im_{p2}F_3^{05}-m_{p1}m_{p2}F_4^{05}\right\} \\
& +(p_0-q_0)\bar{u}(q)u(p)\left\{G_1^{05}+im_{p1}G_2^{05}+im_{p2}G_3^{05}-m_{p1}m_{p2}G_4^{05}\right\} \\
& +(p_0+q_0)\bar{u}(q)\gamma_5 u(p)\left\{H_1^{05}+im_{p1}H_2^{05}+im_{p2}H_3^{05}-m_{p1}m_{p2}H_4^{05}\right\} \\
& +O(a^2)
\end{aligned}
$$

with $\not{p}u(p)=im_{p1}u(p)$, $\bar{u}(q)\not{q}=im_{p2}\bar{u}(q)$

Sandwiching $\Lambda_k^{(1)}(p,q,m)$ by the on-shell quark states

$$
\begin{aligned}
& \bar{u}(q)\Lambda_{05}^{(1)}(p,q,m_{p1},m_{p2})u(p) \\
= \; & \bar{u}(q)\gamma_0\gamma_5 u(p)\left\{\underbrace{F_1^{05}+im_{p1}F_2^{05}+im_{p2}F_3^{05}-m_{p1}m_{p2}F_4^{05}}_{\Delta_{\gamma_0\gamma_5}\ \text{in}\ Z_{A_0}}\right\} \\
& +(p_0-q_0)\bar{u}(q)u(p)\left\{\underbrace{G_1^{05}+im_{p1}G_2^{05}+im_{p2}G_3^{05}-m_{p1}m_{p2}G_4^{05}}_{c_{A_0}^{+}\partial_0^{+}\{\bar{q}(x)\gamma_5 Q(x)\}}\right\} \\
& +(p_0+q_0)\bar{u}(q)\gamma_5 u(p)\left\{\underbrace{H_1^{05}+im_{p1}H_2^{05}+im_{p2}H_3^{05}-m_{p1}m_{p2}H_4^{05}}_{c_{A_0}^{-}\partial_0^{-}\{\bar{q}(x)\gamma_5 Q(x)\}}\right\} \\
& +O(a^2)
\end{aligned}
$$

with $\not{p}u(p)=im_{p1}u(p)$, $\bar{u}(q)\not{q}=im_{p2}\bar{u}(q)$

$\{F_1^{05} + \cdots\}$, $\{G_1^{05} + \cdots\}$, $\{H_1^{05} + \cdots\}$ contain both $O(a)$ and $O(1/m)$

subtract the continuum counter part

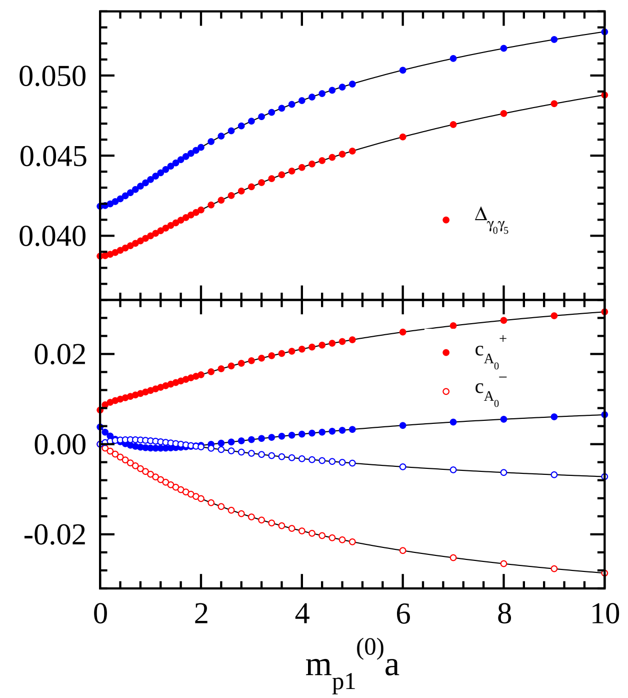
$$
\begin{aligned}
\Delta_{\gamma_0\gamma_5} &= \{F_1^{05} + \cdots\}^{\text{latt}} - \{F_1^{05} + \cdots\}^{\text{cont}} \\
ic_{A_0}^{+} &= \{G_1^{05} + \cdots\}^{\text{latt}} - \{G_1^{05} + \cdots\}^{\text{cont}} \\
ic_{A_0}^{-} &= \{H_1^{05} + \cdots\}^{\text{latt}} - \{H_1^{05} + \cdots\}^{\text{cont}}
\end{aligned}
$$

should be independent of infrared divergences heavy-light case

red : plaquette

blue : Iwasaki

small values for $\Delta_{\gamma_0\gamma_5}$, $c^{\pm}_{A_0}$



$O(a)$ improvement of V_0, V_k, A_0, A_k is done for H-H and H-L cases
see hep-lat/0401030

§5. Numerical simulations

§5-1. test of the action in quenched QCD

check restoration of space-time symmetry

– dispersion relation of H-H and H-L mesons

$$c_{\text{eff}} = \sqrt{\frac{E(p_s)^2 - E(0)^2}{p_s^2}}$$

– H-L PS meson decay constant from A_0 and A_k

$$R \equiv i \frac{\langle 0|A_k^R|PS\rangle}{\langle 0|A_4^R|PS\rangle} \cdot \frac{E}{|p_s|}$$

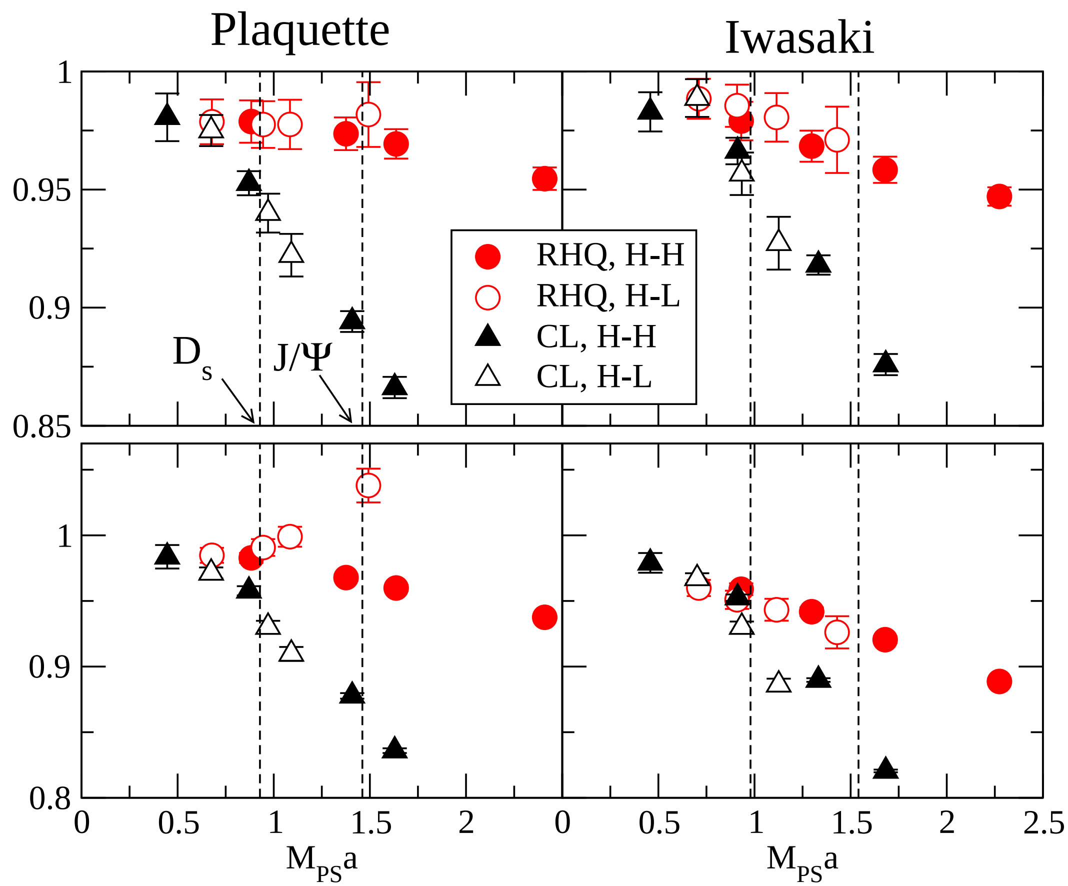
c_{eff} and R should be 1 in the continuum limit simulation parameters

quenched, $24^3 \times 48$, #conf.=300
plaquette gauge action @ $\beta = 5.9$, $a(r_0) \sim 0.1$fm
Iwasaki gauge action @ $\beta = 2.6$, $a(r_0) \sim 0.1$fm

light quarks : SW with nonperturbative c_{SW}
heavy quarks (A) : ν, r_s, c_B, c_E up to one-loop level

$$c_{B/E} = \{c_{B/E}^{\mathrm{PT}}(m_Q a) - c_{B/E}^{\mathrm{PT}}(0)\} + c_{\mathrm{SW}}^{\mathrm{NP}}$$

cover charm quark mass
heavy quarks (B) : SW with nonperturbative c_{SW} for comparison
axial currents : mass dependently improved up to one-loop level



clear improvement over the clover heavy quark action

§5-2. Preliminary results of $N_f = 2$ QCD

simulation parameters   PRD65(2002)054505

Iwasaki gauge action

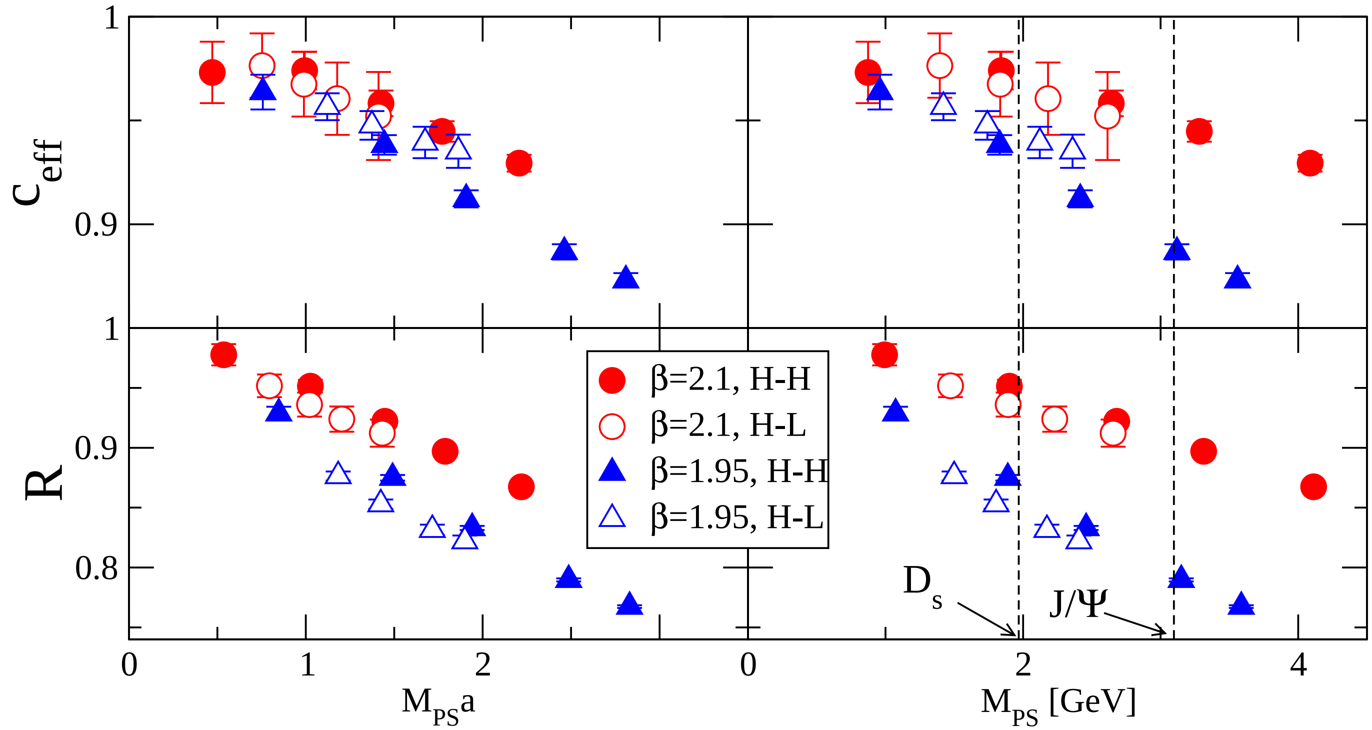
4 sea quark masses with mean-field improved SW

$\beta = 1.95$ $16^3 \times 32$ $a(r_0) \sim 0.15$fm   #conf.=400

$\beta = 2.1$ $24^3 \times 48$ $a(r_0) \sim 0.11$fm   #conf.=200

light quarks : mean-field improved SW

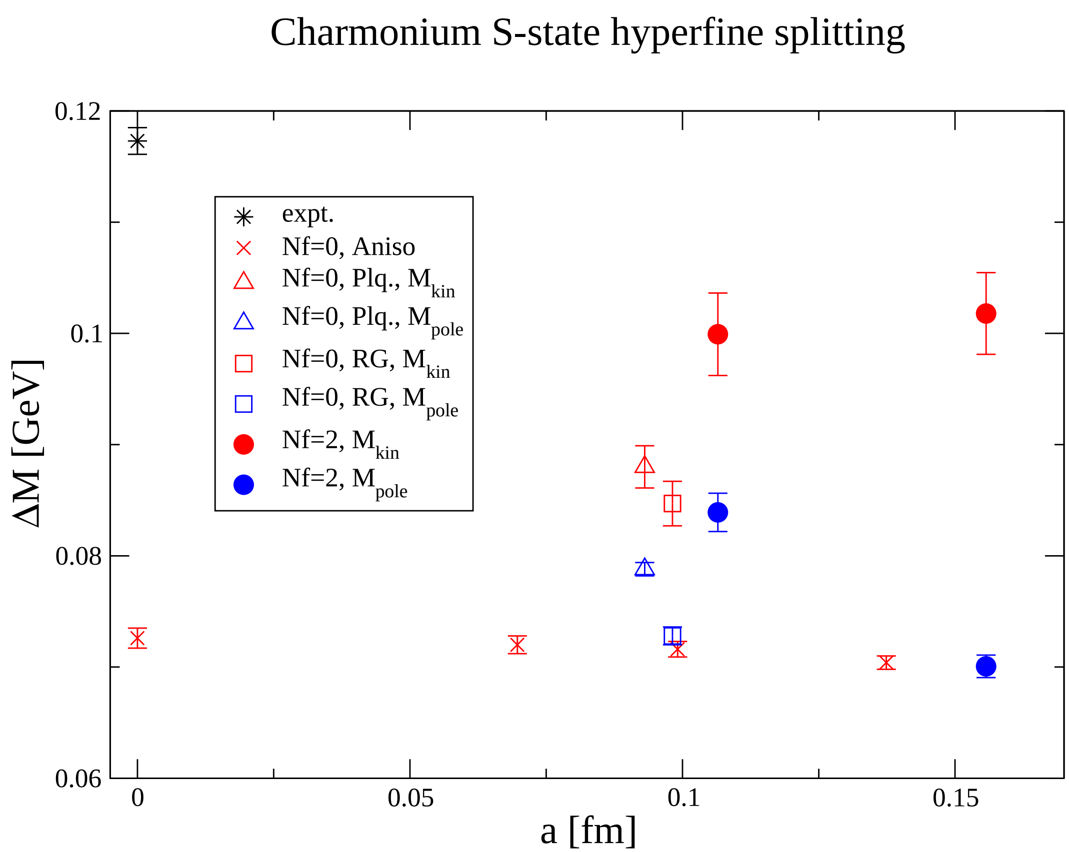
heavy quarks : same as quenched calculation



expected leading scaling violation:

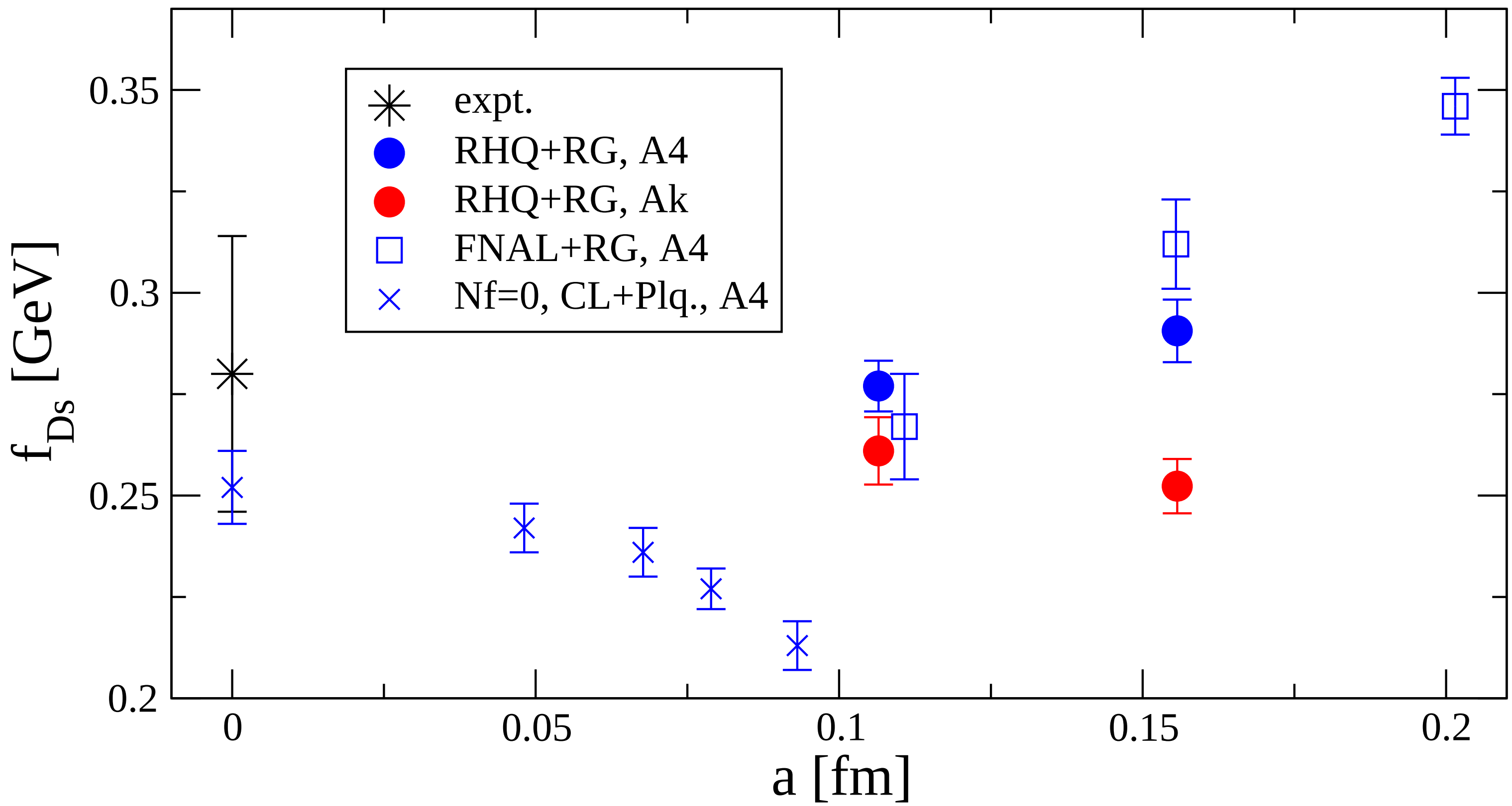
$$O(f_2(m_Q a)(a\Lambda_{\mathrm{QCD}})^2) \text{ or } O(f_1(m_Q a) g^4 a \Lambda_{\mathrm{QCD}})$$

a bit surprising is $f_{1,2}$ look linear beyond $m_Q a = 1$



dynamical quark effects?

$\longrightarrow$ need more scaling study for $N_f = 0$ and 2



possibility of reliable continuum extrapolation with $f_{D_s}(A_4)$ and $f_{D_s}(A_k)$

§5. Summary

- general form of quark action

$$
\begin{aligned}
S_q^{\mathrm{imp}} \;=\; \sum_x \Big[& m_0 \bar{q}(x) q(x) + \bar{q}(x)\gamma_0 D_0 q(x) + \nu \sum_i \bar{q}(x)\gamma_i D_i q(x) \\
& -\frac{r_t a}{2}\bar{q}(x) D_0^2 q(x) - \frac{r_s a}{2}\sum_i \bar{q}(x) D_i^2 q(x) \\
& -\frac{iga}{2} c_E \sum_i \bar{q}(x)\sigma_{0i}F_{0i}q(x) - \frac{iga}{4} c_B \sum_{i,j} \bar{q}(x)\sigma_{ij}F_{ij}q(x) \Big],
\end{aligned}
$$

r_t is redundant

ν, r_s, c_E, c_B should be adjusted

- 4 parameters in the action are determined up to one-loop level massless c_{SW} using the ordinary PT with fictitious gluon mass $\longrightarrow$ extend to c_B, c_E in the massive case

- $O(a)$ improvement of the vector and axial vector currents up to one-loop level

- numerical study shows encouraging results dynamical quark effects?$\longrightarrow$ need more scaling study ongoing project

- renormalization and improvement of four-fermi ops.
- numerical calculation of H-H and H-L meson spectrum, decay constants, form factors in $N_f = 2$ QCD

next plan

- detailed scaling study in quenched QCD
- nonperturbative determination of ν, r_s, c_E, c_B
- numerical calculation in $N_f = 3$ QCD