Light hadron spectrum in Nf=2+1 QCD

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Introduction

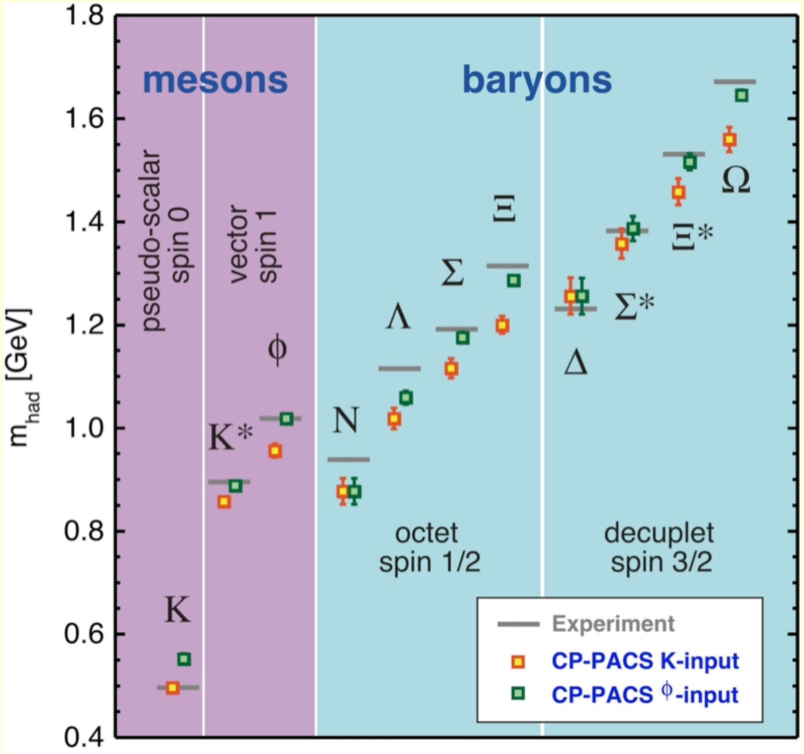
- **Light hadron spectrum**
  - Direct test of QCD at low energy scale
  - Determination of fundamental parameters

    quark masses, QCD coupling, ...
- **Systematic studies by CP-PACS and JLQCD**
  - quenched QCD (continuum limit)
    - RG-improved gauge
      + clover quark (tad.imp. c_{SW})
      (CP-PACS, 2001)
    - plaquette + Wilson
      (CP-PACS, 2001)

Systematic deviation from experiment (5-10%)

⇩

artifact due to the quenched approx.

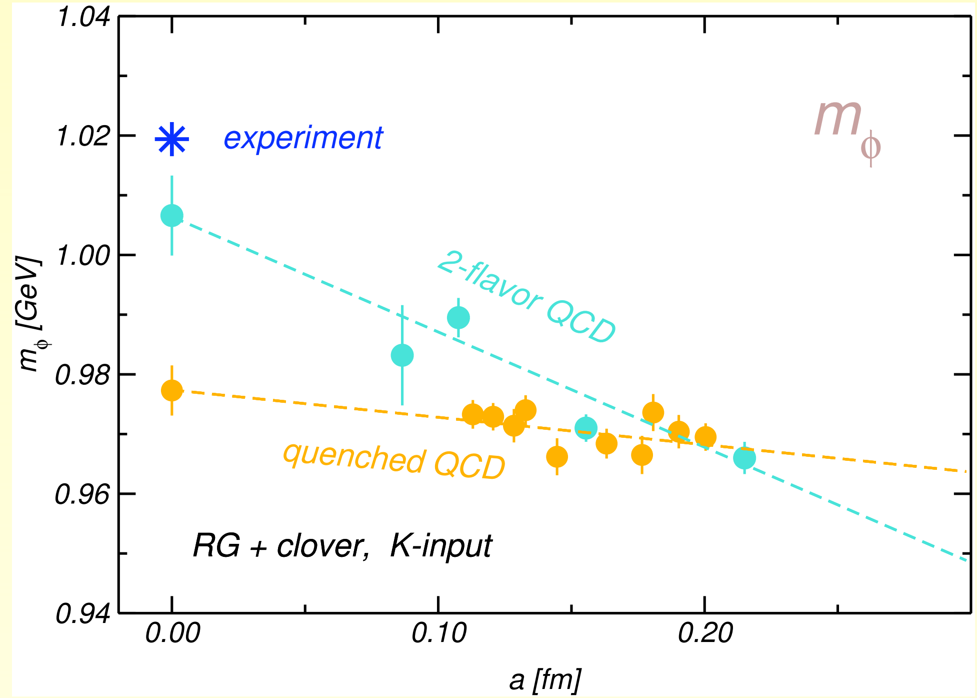


2-flavor QCD (continuum limit)

ud : dynamical
s : quenched

- RG-improved gauge
  + clover quark (tad.imp. c_{SW})
  (CP-PACS, 2001)

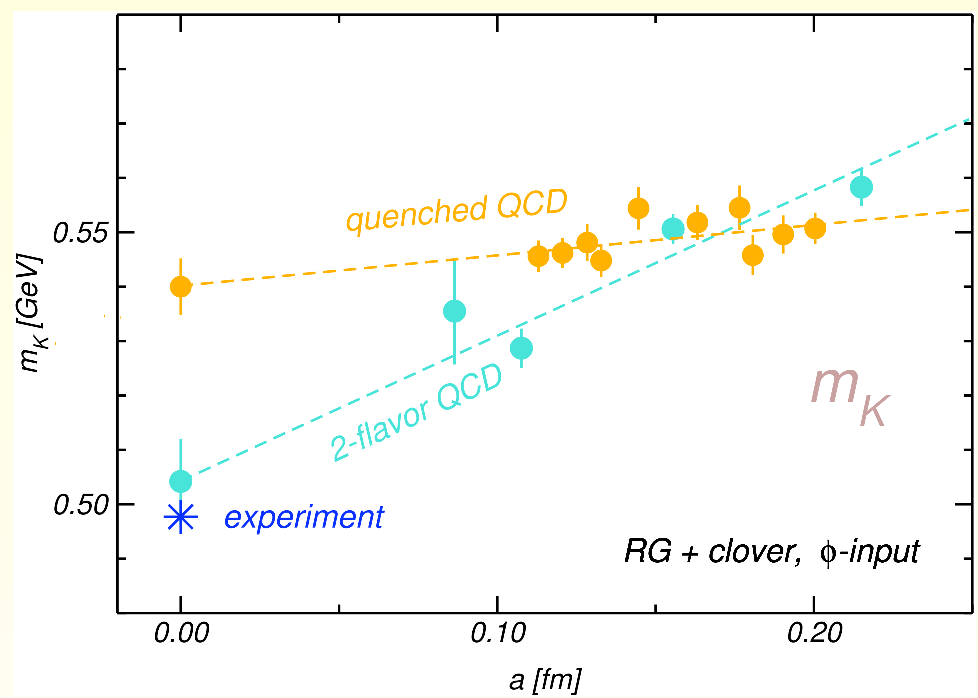
deviation is reduced



Next Step :

3-flavor full QCD

ud : dynamical
s : dynamical



Contents

Strategy of the project

Lattice action

- gauge : RG improved action (Iwasaki, '83)
- quark : non-perturbatively $\mathcal{O}(a)$ improved Wilson action

  $c_{\rm SW}$ is non-perturbatively determined. (K-I. Ishikawa, '03)

Algorithm

- with degenerate up and down quarks and strange quark

  We employ HMC algorithm for up and down quarks.

- Polynomial HMC (Forcrand and Takaishi, '97, K-I.Ishikawa et.al., '02)
  - Polynomial approx. of inv. of Dirac matrix

    $$D^{-1} \sim P_{2N_{poly}}[D] = \bar{T}[D]T[D]$$

  - Metropolis test for correction factor $\det\left[P_{2N_{poly}}[D]D\right]$

    ⇨ exact algorithm

We employ PHMC algorithm for strange quark.

Simulation points and statistics

production : on-going

$\beta = 2.05$
$(a \sim 0.07\ \mathrm{fm})$
$L^3 \times T = 28^3 \times 56$
$c_{\mathrm{sw}} = 1.628$
$\mathrm{stat.} = 2000\ \mathrm{traj.}$

production : almost finished

$\beta = 1.83$
$(a \sim 0.122\ \mathrm{fm})$
$L^3 \times T = 16^3 \times 32$
$c_{\mathrm{sw}} = 1.761$
$\mathrm{stat.} = 5000\ \mathrm{traj.}$

$0 \quad \frac{1}{2} \quad 1 \quad \frac{3}{2} \quad a^2$

fixed physical volume
$\sim (2.0\ \mathrm{fm})^3$

$\beta = 1.90$
$(a \sim 0.10\ \mathrm{fm})$
$L^3 \times T = 20^3 \times 40$
$c_{\mathrm{sw}} = 1.715$
$\mathrm{stat.} = 5000 - 8000\ \mathrm{traj.}$

production : already finished

Earth Simulator
@JAMSTEC

SR8000/F1
@KEK

CP-PACS
@Tsukuba

SR8000/G1
@Tsukuba

VPP5000
@Tsukuba

K parameters

- 5 ud

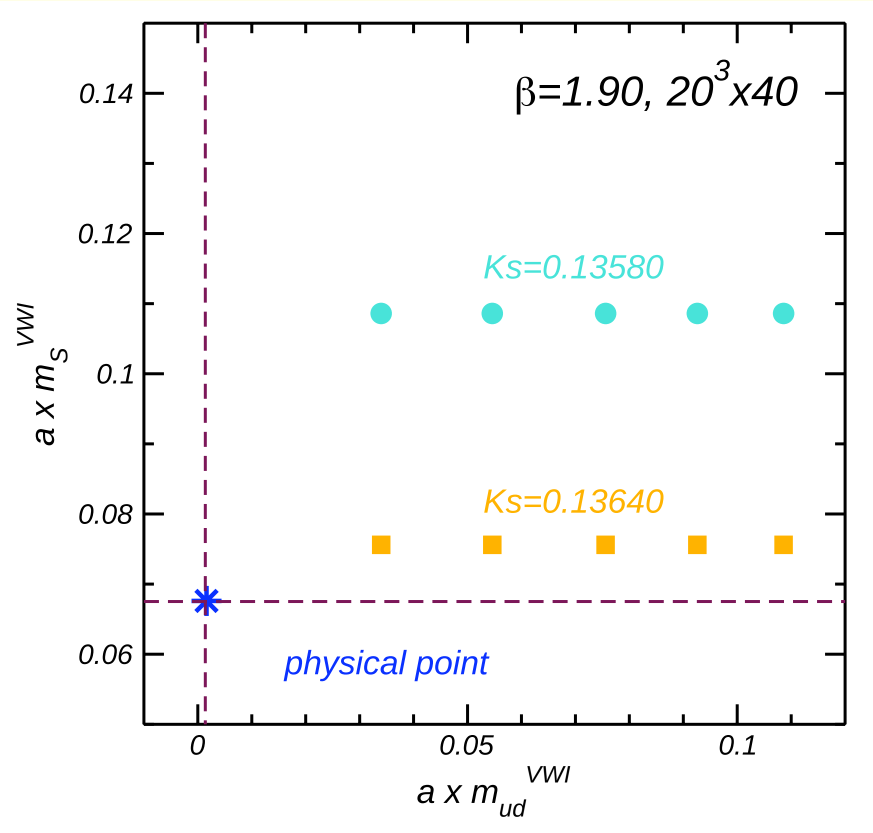
  $m_{PS}/m_V \sim 0.62 - 0.78$

  $(0.18 : \text{experiment})$

- 2 strange

  $m_{PS}/m_V \sim 0.7$

  $(0.68 : \text{1-loop ChPT})$



Measurement

- measure meson masses every 10 trajectories
- doubly smeared source - local sink is used.
- physical volume $\sim (2.0\ \text{fm})^3$ is not large to calculate baryon

⇒ We focus on the meson sector.

Simulation at β=1.90

Simulation parameters

- $\beta = 1.90 \ \ (a \sim 0.1 \text{ fm}), \ \ L^3 \times T = 20^3 \times 40, \ \ c_{SW} = 1.715$

Kud	Ks	$\delta\tau$	N_{poly}	traj.	$m_{PS}/m_V(LL)$	$m_{PS}/m_V(SS)$
0.13580		1/125		5000	0.7674(16)	0.7674(16)
0.13610		1/125		6000	0.7427(18)	0.7640(17)
0.13640	0.13580	1/140	110	7500	0.7206(19)	0.7687(14)
0.13680		1/160		8000	0.6710(28)	0.7677(17)
0.13700		1/180		8000	0.6384(22)	0.7687(15)
0.13580		1/125		5200	0.7666(17)	0.7210(22)
0.13610		1/125		8000	0.7448(15)	0.7187(17)
0.13640	0.13640	1/140	140	9000	0.7153(18)	0.7153(18)
0.13680		1/160		9000	0.6642(22)	0.7139(18)
0.13700		1/180		8000	0.6237(28)	0.7100(20)

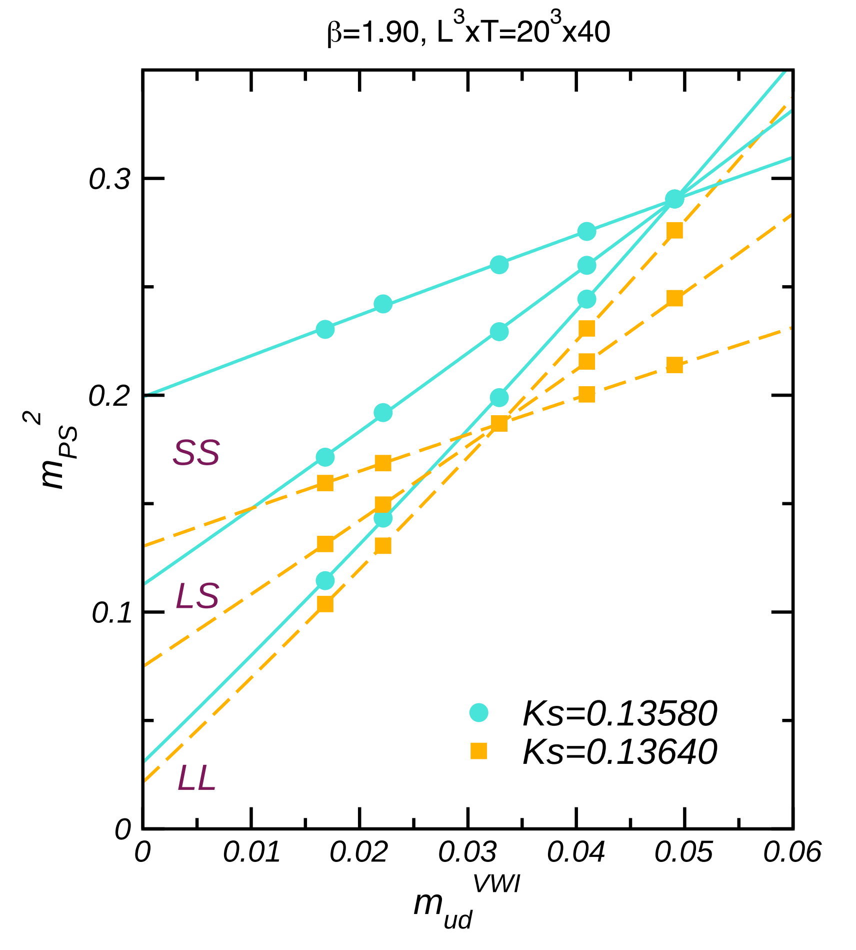
$\delta\tau, \ \ N_{poly} \ \Longleftarrow \ P_{HMC} \simeq 85\%, \ \ P_{GMP} \simeq 90\%$

Analysis at β=1.90

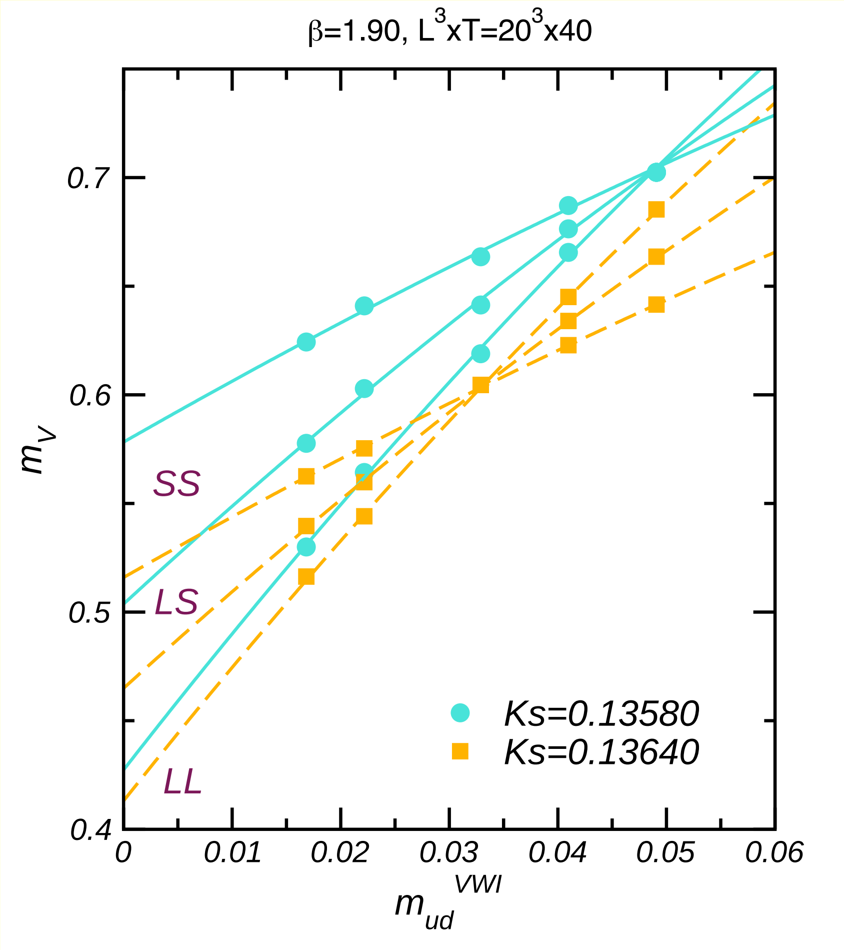
- **Chiral fit function**
  - polynomial in quark masses
  - Chiral fits are made to
    light-light (LL), light-strange (LS) and strange-strange (SS) simultaneously.
  - ignoring correlations among LL, LS and SS
  - include up to quadratic terms

$$
\begin{aligned}
&m_{PS}^2(m_{ud}, m_s; m_{val,1}, m_{val,2})\\
&= B_S^{PS}(2m_{ud} + m_s) + B_V^{PS}(m_{val,1} + m_{val,2})\\
&\quad + D_{SV}^{PS}(2m_{ud} + m_s)(m_{val,1} + m_{val,2})\\
&\quad + C_{S1}^{PS}(2m_{ud}^2 + m_s^2) + C_{S2}^{PS}(m_{ud} + 2m_{ud}m_s)\\
&\quad + C_{V1}^{PS}(m_{val,1}^2 + m_{val,2}^2) + C_{V2}^{PS} m_{val,1} m_{val,2}
\end{aligned}
$$

$$
\begin{aligned}
&m_V(m_{ud}, m_s; m_{val,1}, m_{val,2})\\
&= A^V + B_S^V(2m_{ud} + m_s) + B_V^V(m_{val,1} + m_{val,2})\\
&\quad + D_{SV}^V(2m_{ud} + m_s)(m_{val,1} + m_{val,2})\\
&\quad + C_{S1}^V(2m_{ud}^2 + m_s^2) + C_{S2}^V(m_{ud} + 2m_{ud}m_s)\\
&\quad + C_{V1}^V(m_{val,1}^2 + m_{val,2}^2) + C_{V2}^V m_{val,1} m_{val,2}
\end{aligned}
$$



$\chi^2/d.o.f. = 1.09$



$\chi^2/d.o.f. = 0.93$

LL : light-light meson
LS : light-strange meson
SS : strange-strange meson

Inputs

- Inputs to fix the quark masses

$$m_{ud} \longleftarrow \frac{m_{PS}(m_{ud},m_{ud})}{m_V(m_{ud},m_{ud})} = \frac{m_\pi}{m_\rho}$$

$$m_s(K\text{-input}) \longleftarrow \frac{m_{PS}(m_{ud},m_s)}{m_V(m_{ud},m_{ud})} = \frac{m_K}{m_\rho}$$

$$m_s(\phi\text{-input}) \longleftarrow \frac{m_V(m_s,m_s)}{m_V(m_{ud},m_{ud})} = \frac{m_\phi}{m_\rho}$$

- Input to fix the lattice spacing

$$a \leftarrow m_\rho$$

at β=1.90

$$a^{-1} = \begin{cases} 1.97(4)\ \text{GeV} & K\text{-input} \\ 1.97(4)\ \text{GeV} & \phi\text{-input} \end{cases}$$

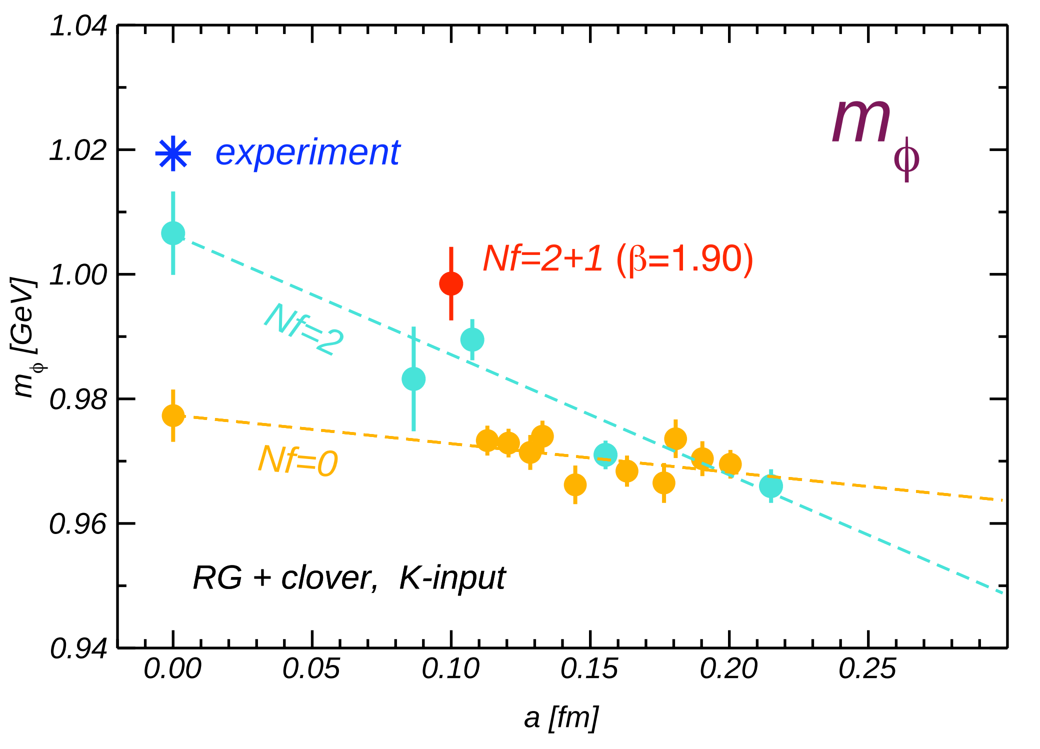
- independent of inputs
- consistent with the value determined from Sommer scale

$R_0 \simeq 0.5\text{fm}$

Light meson spectrum at β=1.90

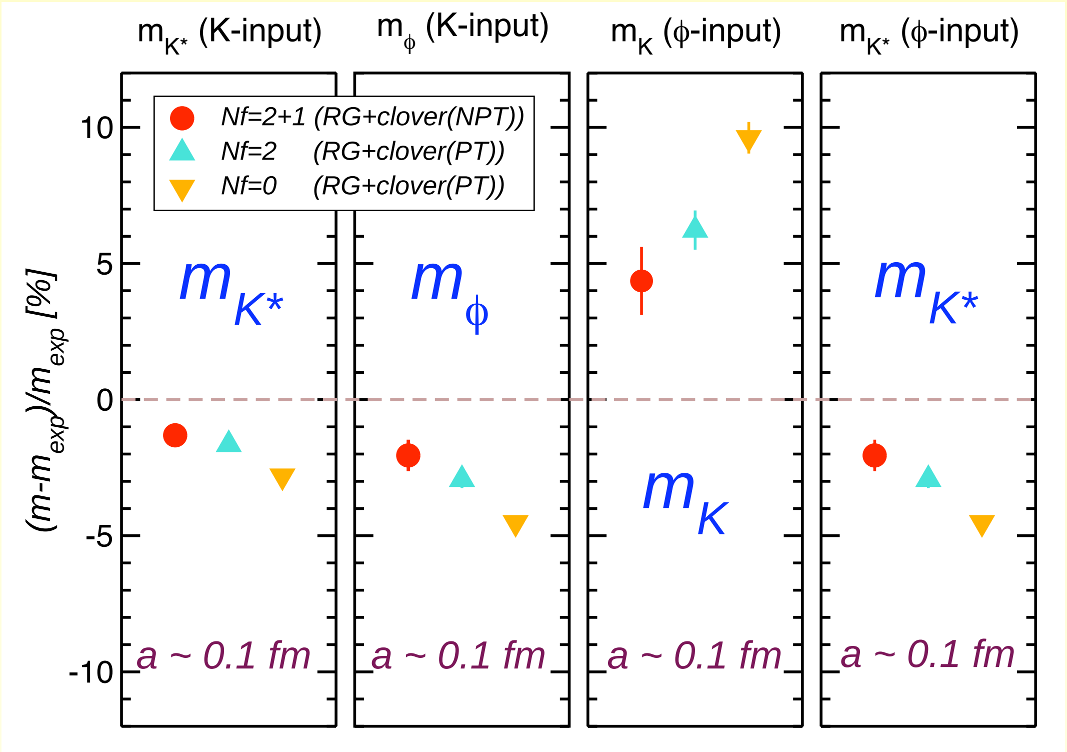
- **Results at β=1.90**

$$m_{K^*} = 884.4(3.0)\ \mathrm{MeV} \quad (\text{K-input})$$
$$m_\phi = 998.5(5.9)\ \mathrm{MeV} \quad (\text{K-input})$$
$$m_K = 519.4(6.2)\ \mathrm{MeV} \quad (\phi\text{-input})$$
$$m_{K^*} = 895.1(0.3)\ \mathrm{MeV} \quad (\phi\text{-input})$$



At a ~ 0.1 fm

We observe that meson spectrum in Nf=2+1 are closer to experiment than that in Nf=2 and Nf=0 at a ~ 0.1 fm.



Dynamical strange effect or non-perturbative $c_{\rm SW}$ effect

At this stage we cannot give a conclusion to this question.

Light quark masses at β=1.90

- **VWI quark mass**

$$m_q = \frac{1}{2}\left(\frac{1}{K} - \frac{1}{K_c}\right)$$

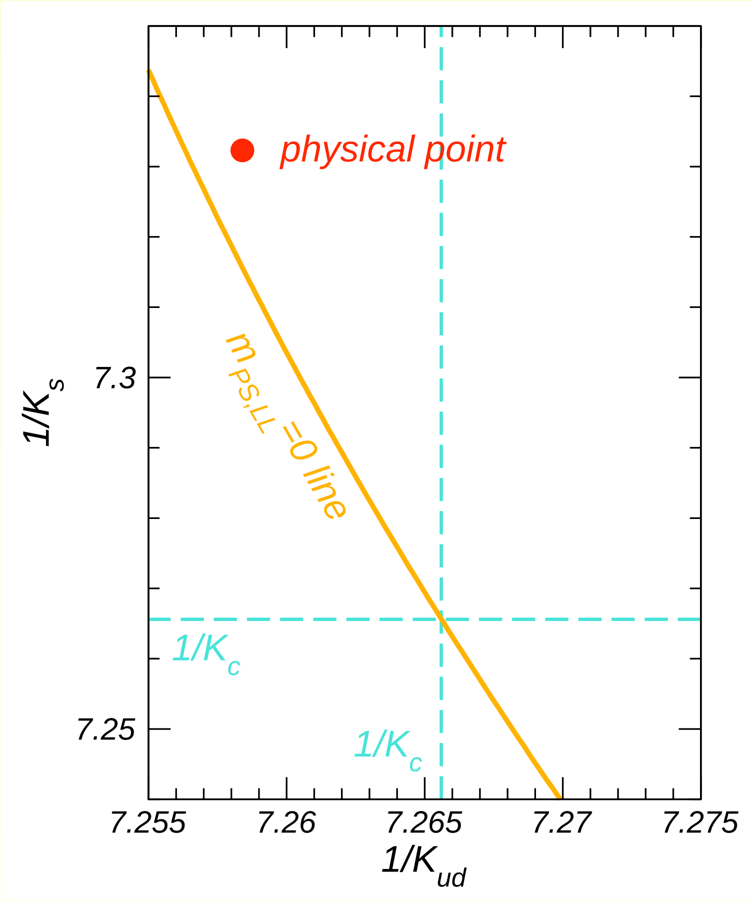
- VWI ud quark mass has negative value.
  - due to the chiral symmetry breaking of the Wilson quark
- define $K_{c,L}(K_s)$

⇧

$$m_{PS,LL}(K_{c,L}, K_s) = 0$$

- re-define

$$m_q^{VWI} = \frac{1}{2}\left(\frac{1}{K} - \frac{1}{K_{c,L}}\right) > 0$$



- **AWI quark mass** (We use this definition.)

$$m_q = \frac{\langle \Delta_4 A_4(t) P(0) \rangle}{2 \langle P(t) P(0) \rangle}$$

  - no such problem as in the VWI quark mass
  - The scaling violation is small in $N_f = 2$ case.

- **renormalization**
  - MF-improved 1-loop matching with $\overline{\text{MS}}$ at $\mu = a^{-1}$
  - 4-loop running to $\mu = 2\ GeV$

Results

Quark masses in Nf=2+1 are slightly smaller than those in Nf=2 and Nf=0 at a ~ 0.1 fm.

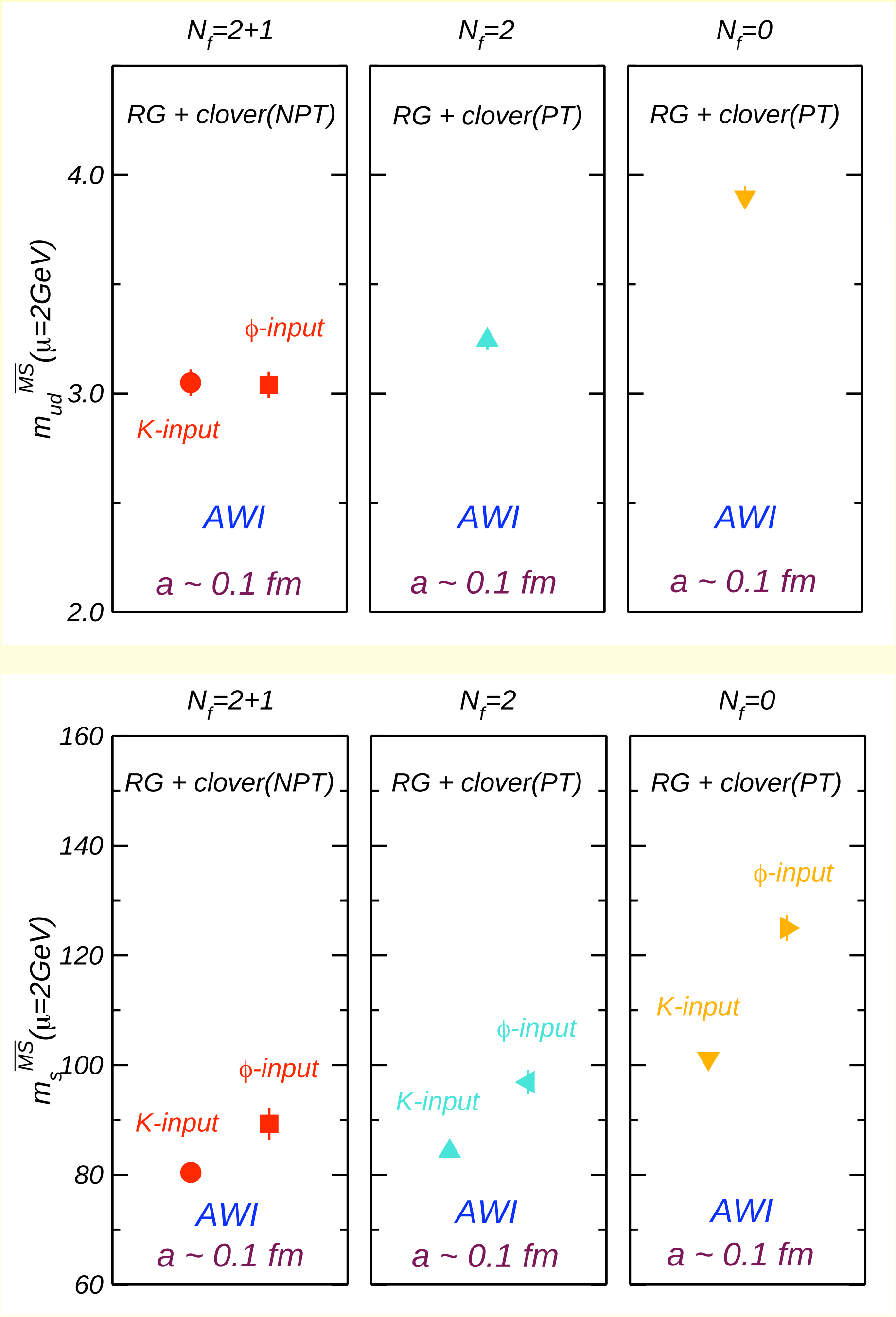
$\overline{\text{MS}}$ scheme ($\mu = 2GeV$) at β=1.90

$$\begin{cases} m_{ud} = 3.05(6)\ \text{MeV} \\ m_s = 80.4(1.9)\,\text{MeV} \end{cases}$$

(K-input)

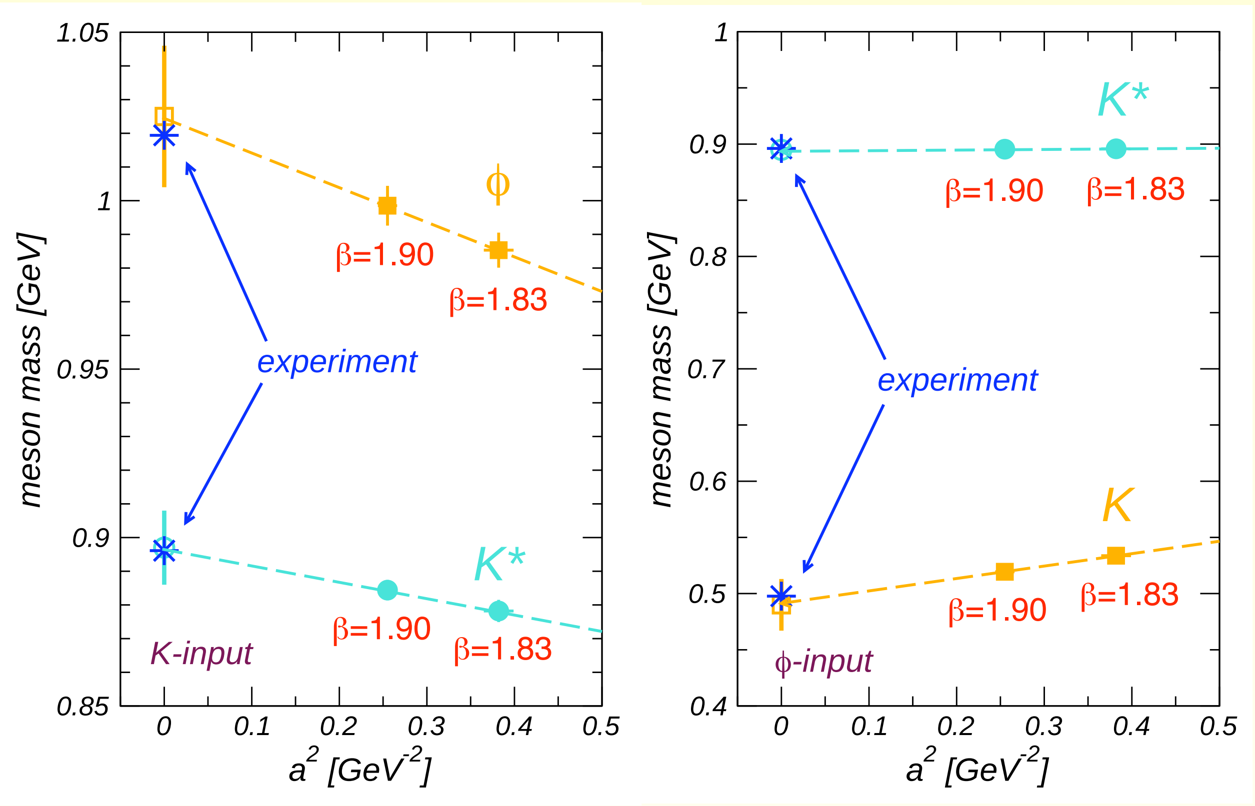
$$\begin{cases} m_{ud} = 3.04(6)\ \text{MeV} \\ m_s = 89.3(2.9)\,\text{MeV} \end{cases}$$

(ϕ-input)



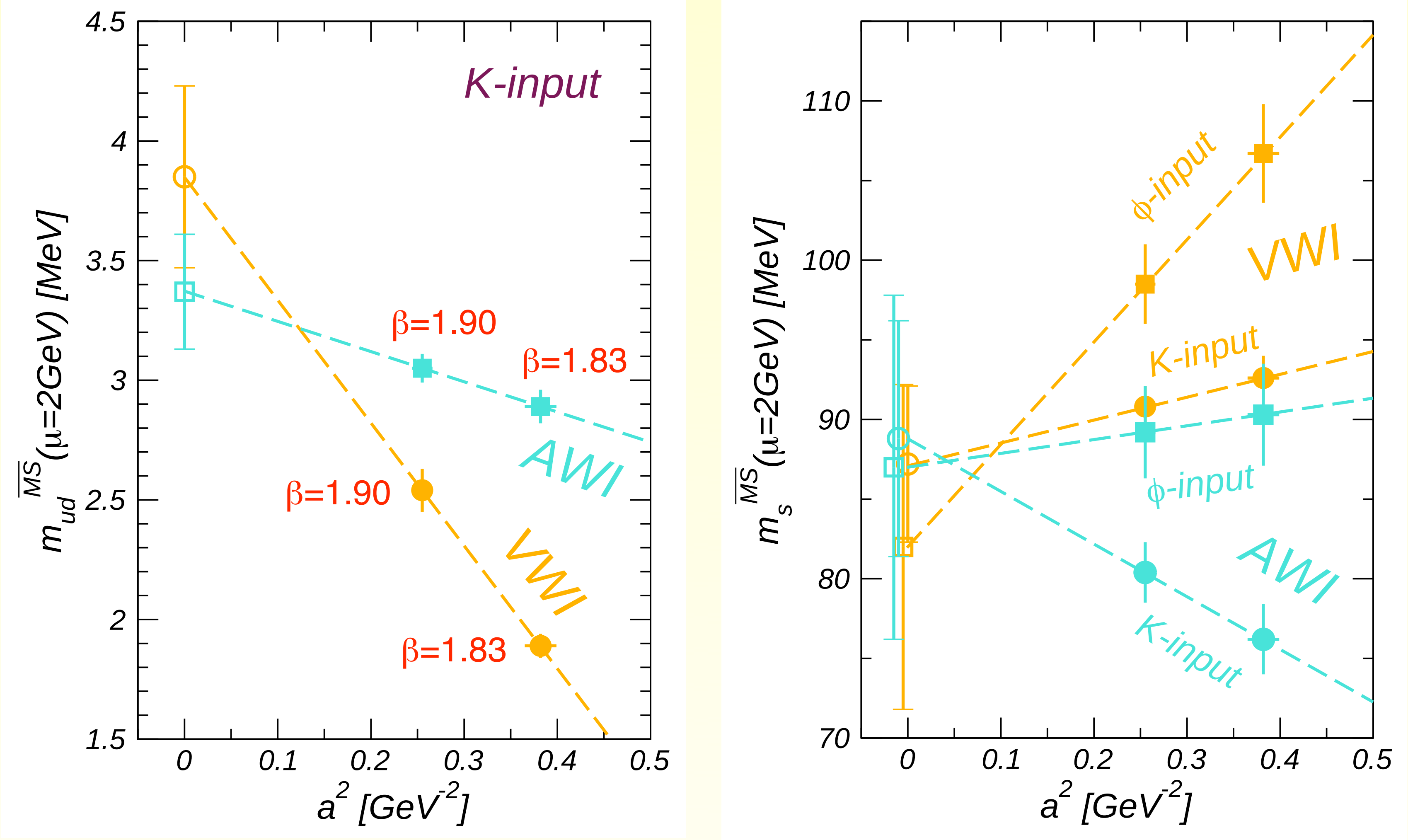
Scaling study (preliminary results)

- **Continuum extrapolation**
  - 2 point extrapolation (β=1.90 + β=1.83)
  - function : $m(a) = A + Ba^2$
- **Light meson spectrum**
  - In the continuum, meson spectrum is consistent with experiment.
  - Scaling violation is $\mathcal{O}(a^2)$.

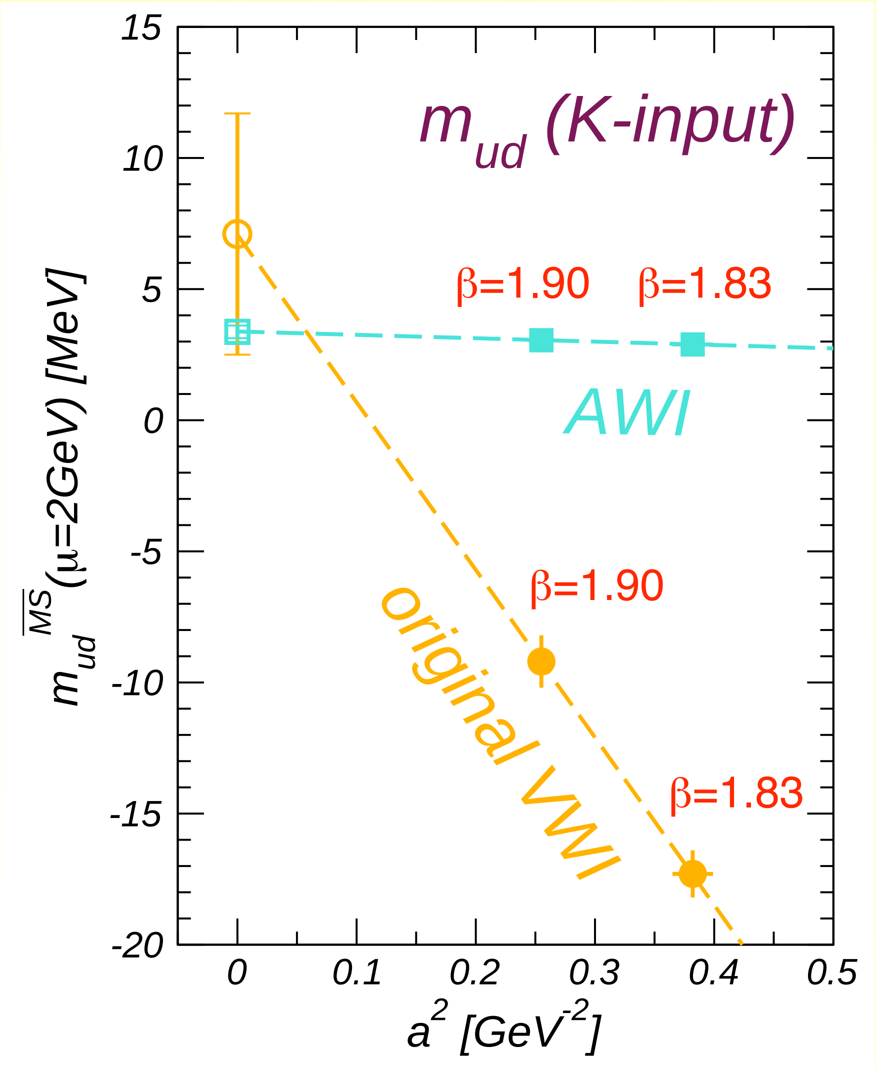


Quark masses

- AWI and VWI quark mass are consistent each other in the continuum limit.
- K-input and φ-input also give consistent results.

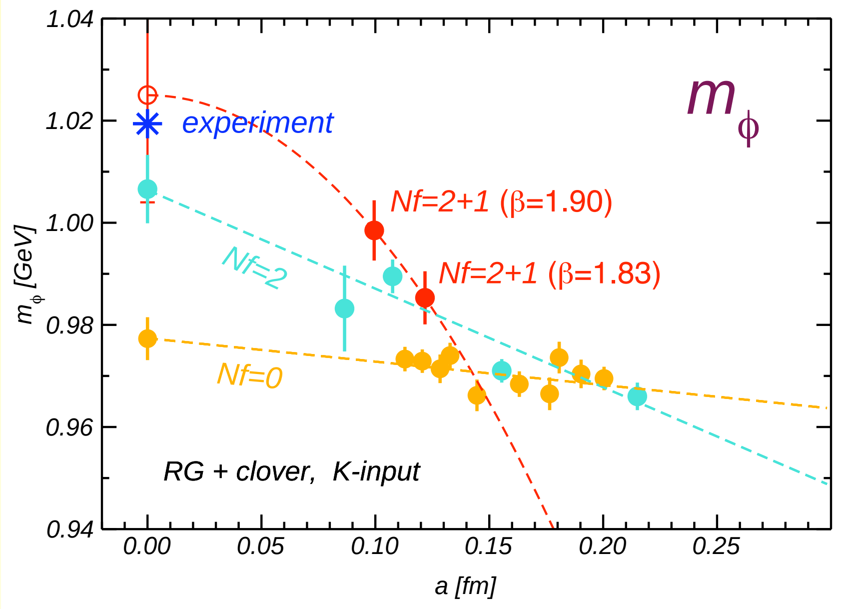


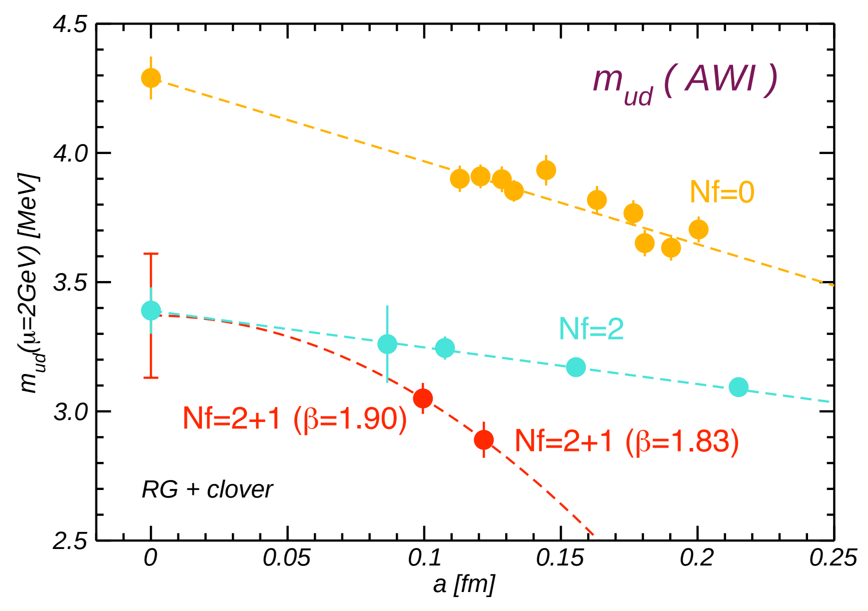
- ud quark mass in the original VWI definition has positive value and consistent with AWI quark mass in the continuum limit.
- But the error in the continuum limit is very large at this stage.

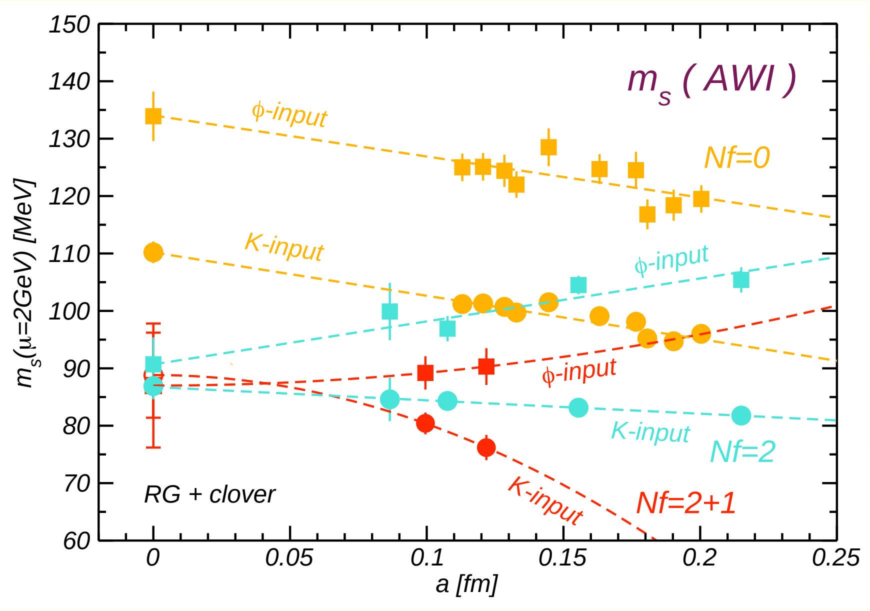


Comparison with Nf=2, 0

- Meson spectrum in Nf=2+1 seems closer to experiment than that in Nf=2.
- Quark masses in Nf=2+1 are almost same as in Nf=2.
- But these statement are unclear, because the error in the continuum limit is large at this stage.







Summary

- **Nf=2+1 project**
  - RG-improved gauge action
    + non-perturbatively $\mathcal{O}(a)$ improved Wilson quark action
  - Exact Nf=2+1 algorithm (PHMC)
  - Production @ β=1.90 has been already finished.
    Production @ β=1.83 has been almost finished.
    Production @ β=2.05 is on-going.
- **Analysis of meson spectrum and quark masses**
  - @ β=1.90 the analysis has been finished.
    @ β=1.83 preliminary results are obtained.
  - Encouranging results in the continuum limit are obtained.
    However a large error remains.
  - β=2.05 results are quite desired !!