Shuzenji, Izu
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Parity-broken phase: a new perspective on the past experience

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3. *Old studies since early-middle 90's*
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What this talk is about

One day in July at the Physics Faculty Meeting, I sat next to Sinya Aoki. He was reading a hep-lat article,

"Twisted mass quarks and the phase structure of lattice QCD"

F. Farchioni et al, hep-lat/0406039

I remembered that Karl Jansen told me at the reception at the Lattice 2004 Meeting that they had found a 1st order phase transition in their study of twised mass QCD.

In fact, Frezotti reported it and showed a metastability graph in his plenary talk, of which I was an audience.

I was slow (too much administrative work, perhaps....) and it took me some time before I realized that all this bears on something that Sinya and I, and many others, had worked on in early to middle 90's.

Trying to understand the transition, we also realized that Junichi Noaki, Taku Izubuchi and I had encountered a 2-dim analog in 1998.

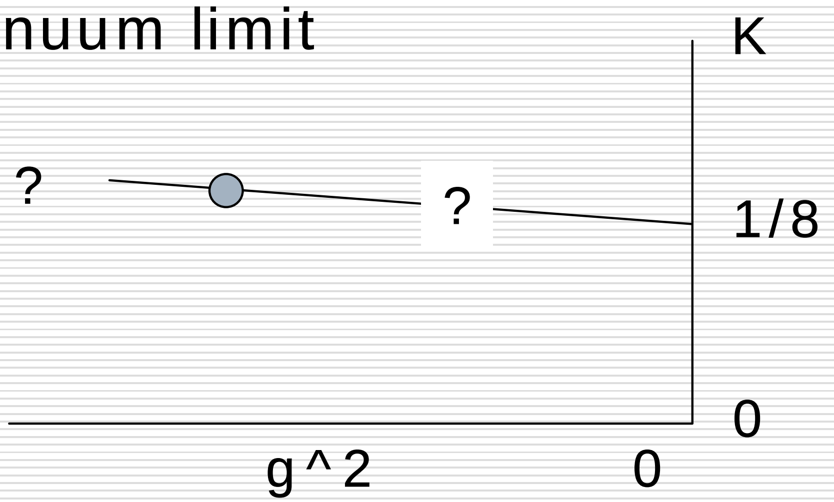
We realized further on that the suggestion of Farchioni et al may shed light on a puzzling 1st order transition MILC encountered in 1994, which was much talked about at the time.

It may also be relevant to the 1st order transition JLQCD encountered in Nf=3 QCD in 2001. → Shoji Hashimoto

I will try to tell you about the "old" studies in the light of "modern" analyses.

F. Farchioni et al, hep-lat/0406039

- 1st order transition in Nf=2 QCD
  - Plaquette + naïve Wilson quark
  - Beta=5.2, K=0.17150 on 12^3x24 (for mu=0)
  - m_pi is non-zero at the transition
- Their interpretation and suggestion
  - c_2<0 in the language of Sharpe-Singleton analysis of the Aoki phase
  - Will continue to the continuum limit



Sharpe-Singleton analysis of Aoki Phase

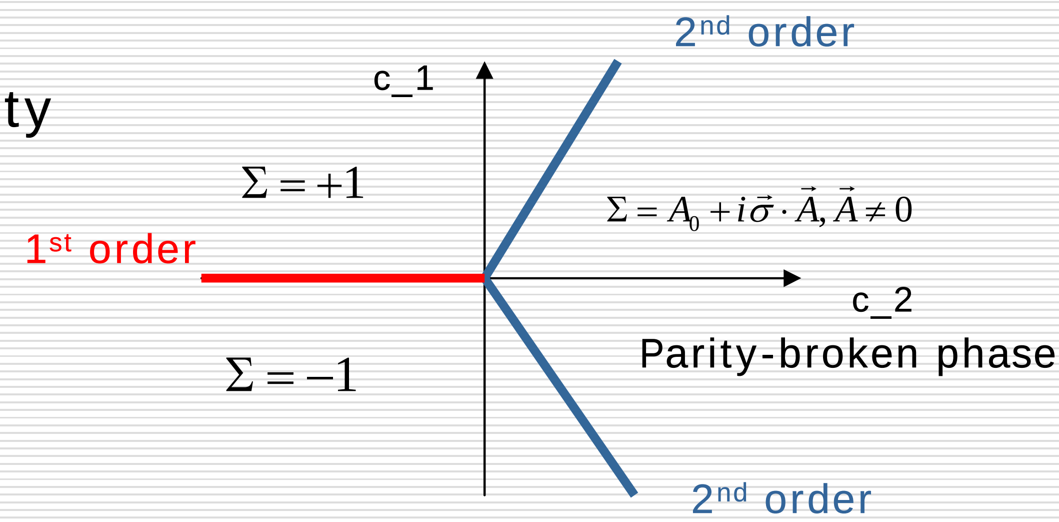
Nf=2

Sharpe and Singleton, Hep-lat/9804028 PRD 58,074501('98)

- Chiral lagrangian for Wilson quark action for Nf=2 flavors

$$L = \frac{f^2}{4} Tr\left(\partial_\mu \Sigma \partial_\mu \Sigma^+\right) - \frac{c_1}{4} Tr\left(\Sigma + \Sigma^+\right) + \frac{c_2}{16}\left(Tr\left(\Sigma + \Sigma^+\right)\right)^2$$

- If c_2>0, there is a region of parity-broken phase sandwiched by symmetric phase
- If c_2<0, there is no parity broken phase, but a 1st order transition



$$L = \frac{f^2}{4} Tr\left(\partial_\mu \Sigma \partial_\mu \Sigma^+\right) - \frac{c_1}{4} Tr\left(\Sigma + \Sigma^+\right) + \frac{c_2}{16}\left(Tr\left(\Sigma + \Sigma^+\right)\right)^2$$

$$+ \frac{c_3}{16}\left(Tr\left(\Sigma - \Sigma^+\right)\right)^2 + \frac{c_4}{16} Tr\left(\left(\Sigma + \Sigma^+\right)^2\right)$$

Extension to Nf=3 straightforward, but conclusions far less definite because 4 couplings are allowed

An old work from 1995-1996

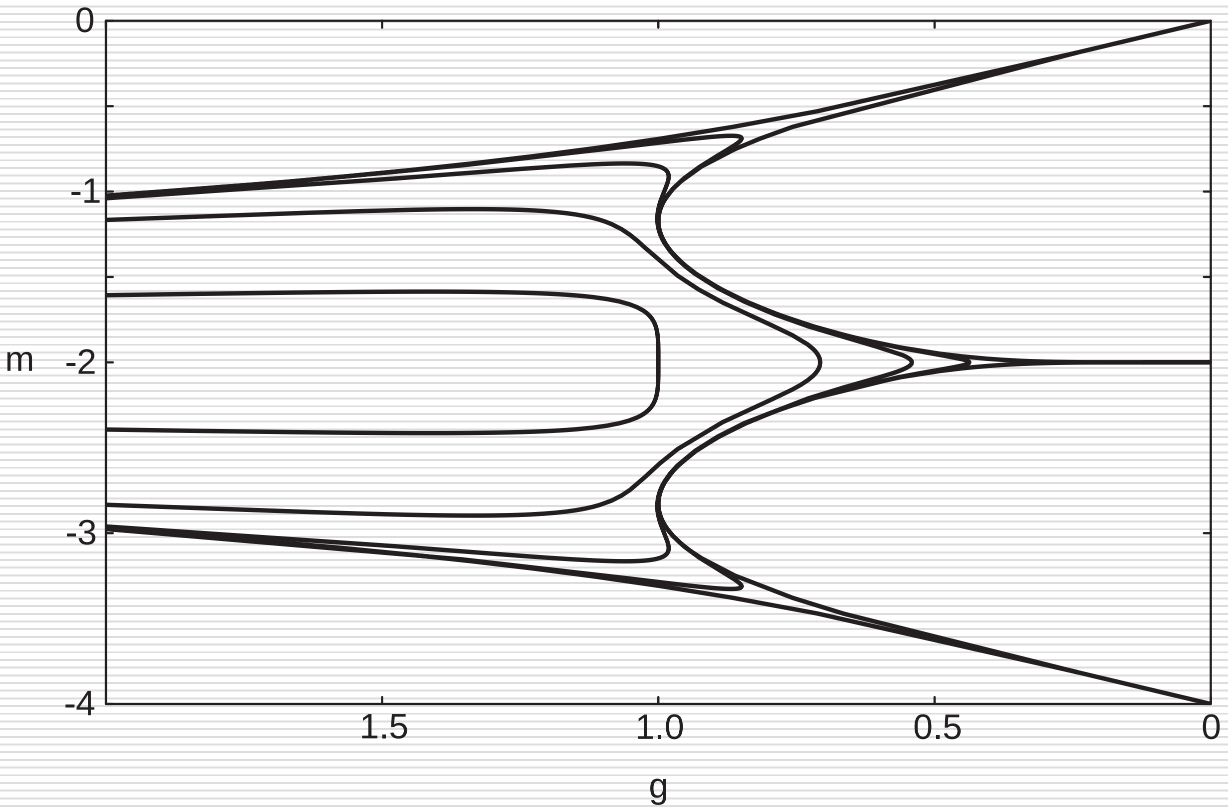
Aoki-Ukawa-Umemura, hep-lat/9508008 PRL 76 ('96) 873

Aoki-Kaneda-Ukawa-Umemura, hep-lat/9612010 Nucl.Phys.(Proc. Suppl.) 53('96)438

- The phase diagram on the (beta,K) plane for finite N_t
  - Gross-Neveu example
  - Existence of the Aoki phase at strong coupling
    - beta=3.5, K=0.225-0.245
  - Pion mass measurement with "twisted mass"
- Dependence on N_t
  - The "tip" moves very slowly with N_t

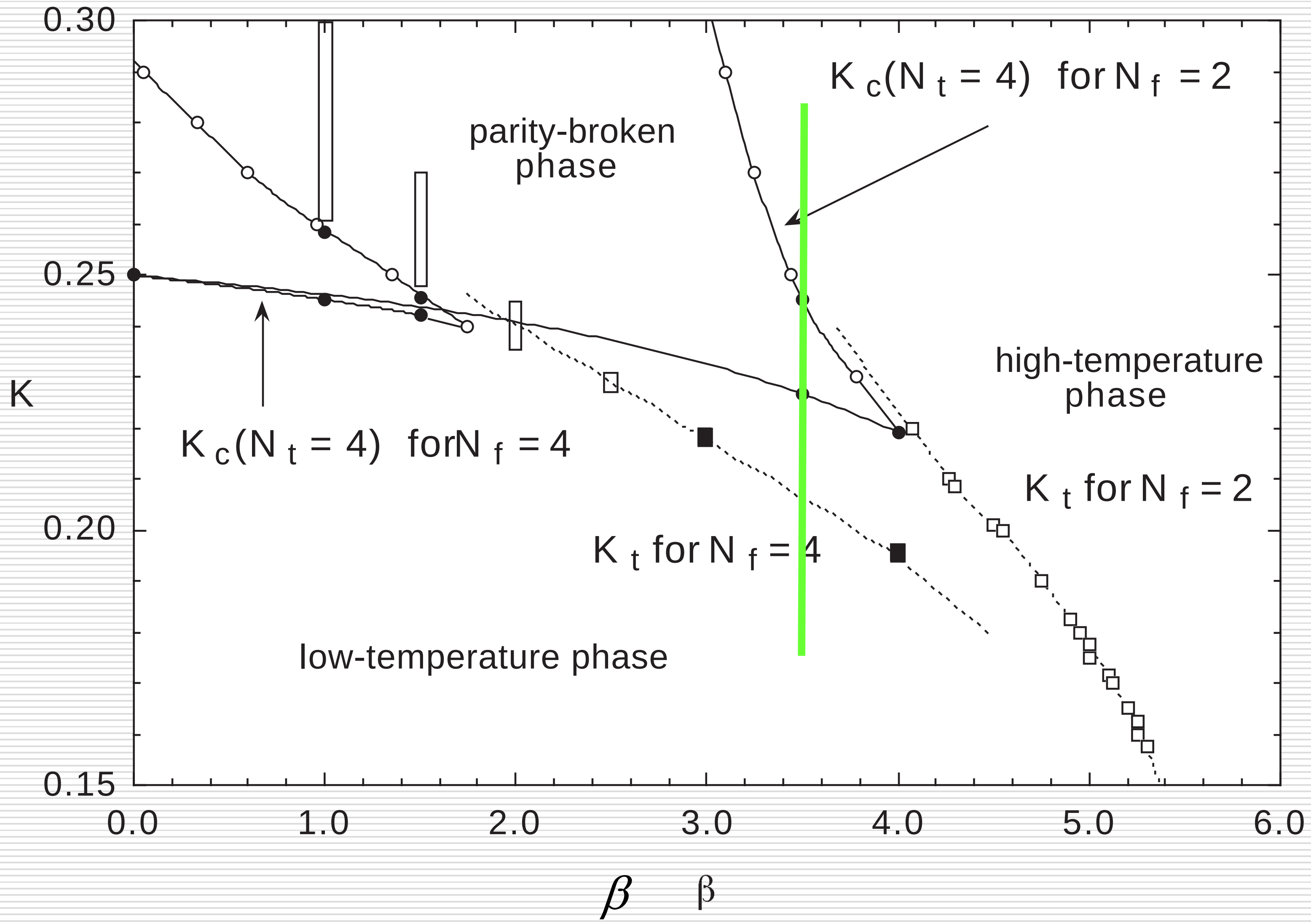
2d Gross-Neveu model

g=g_sigma=g_pi plane



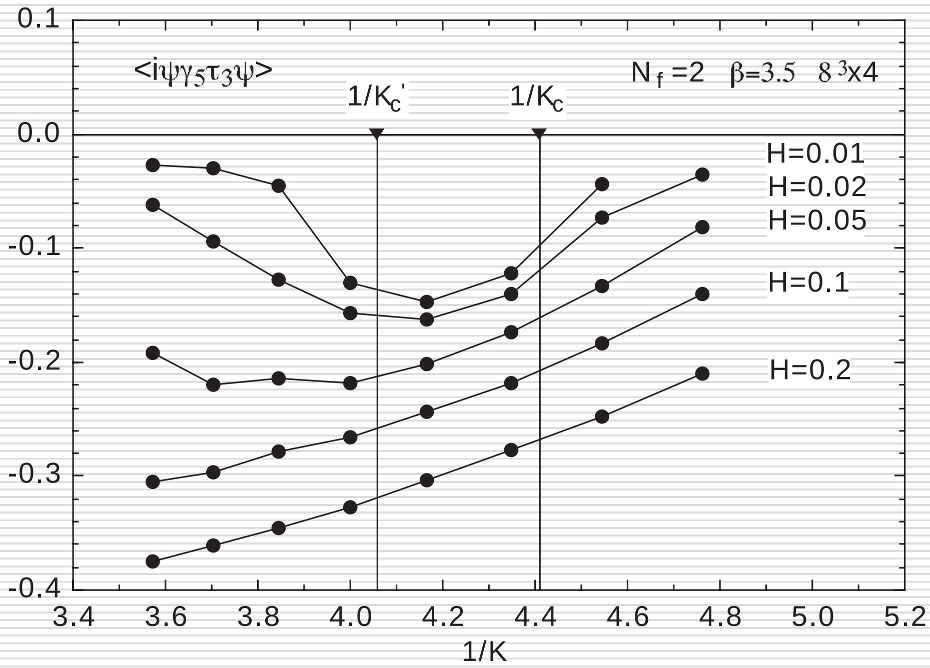
∞xN_t lattice , N_t=2,4, 8,16, ∞ from inside to outside

Phase daigram for Nf=2 QCD



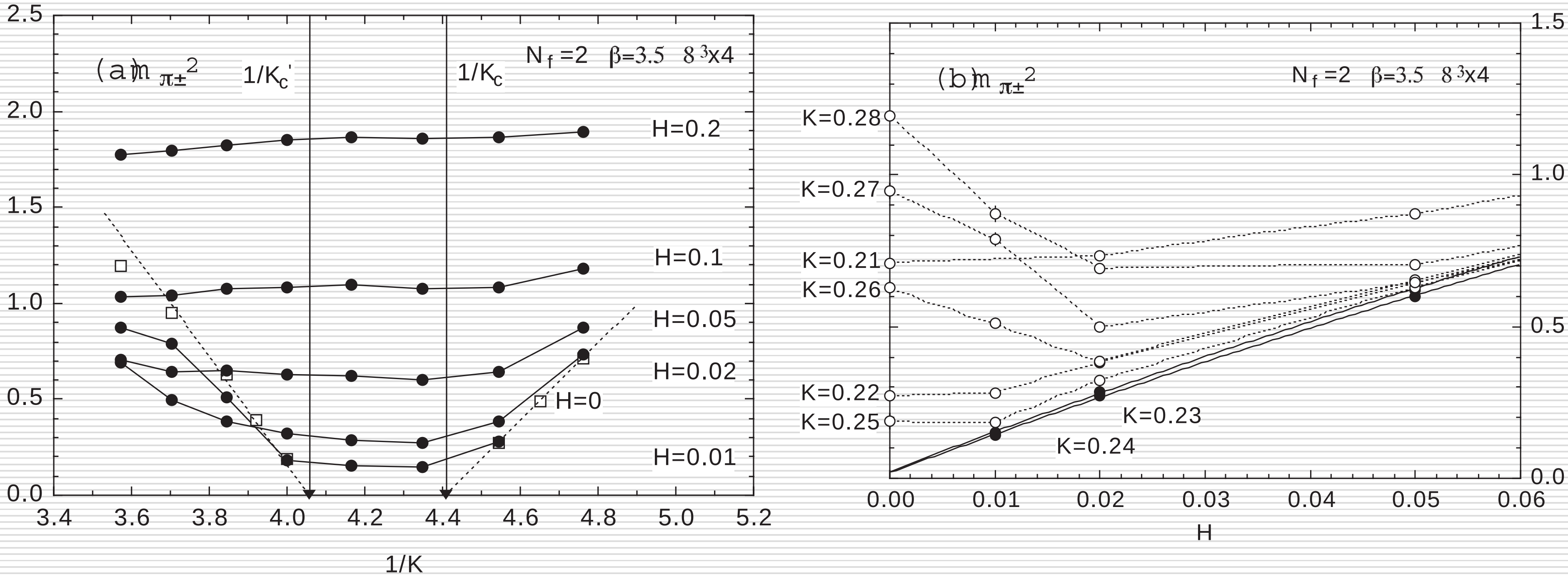
Evidence for parity-broken phase at beta=3.5 (I)

$$\langle \overline{\psi} i \gamma_5 \tau_3 \psi \rangle \quad 8^3 \times 4 \quad H \neq 0$$

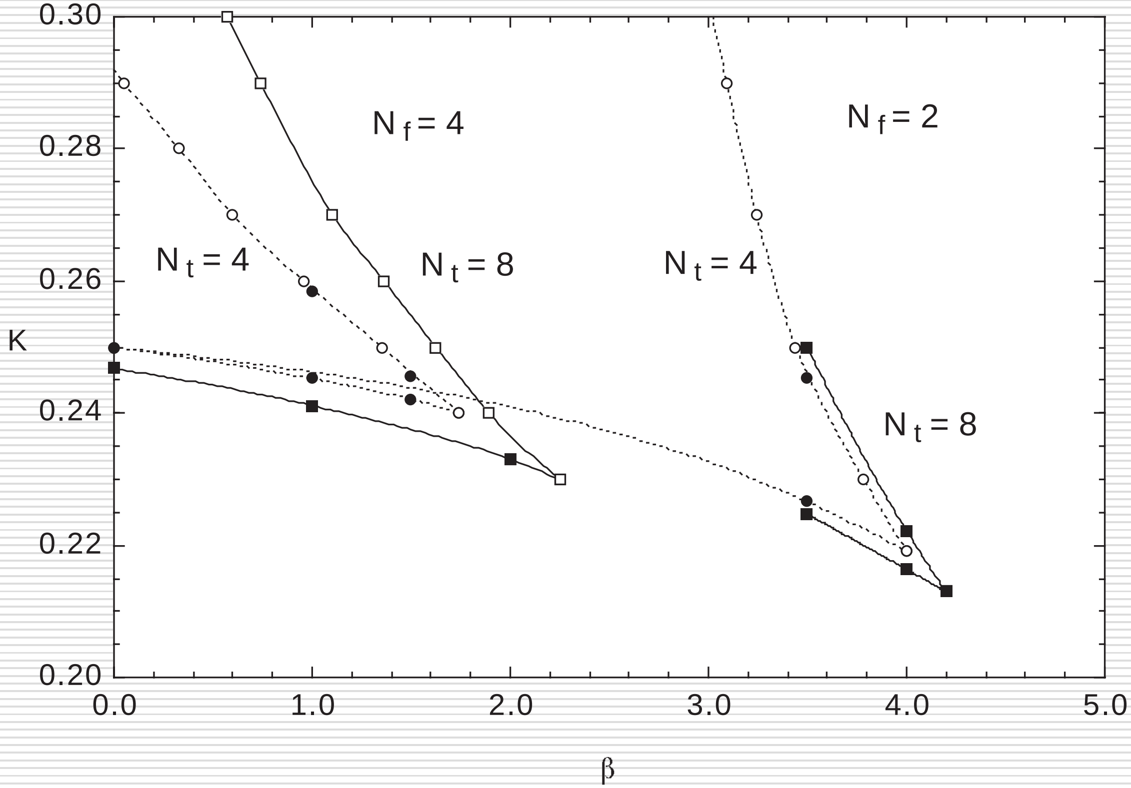


Evidence for parity-broken phase at beta=3.5 (II)

Screening pion mass^2 as a function of H
8^3x4 ->(8x2)x8x8x4



Dependence on N_t



The “tip” is supposed to move to g^2=0 as N_t goes to infinity.

In practice, the tip moved very little.

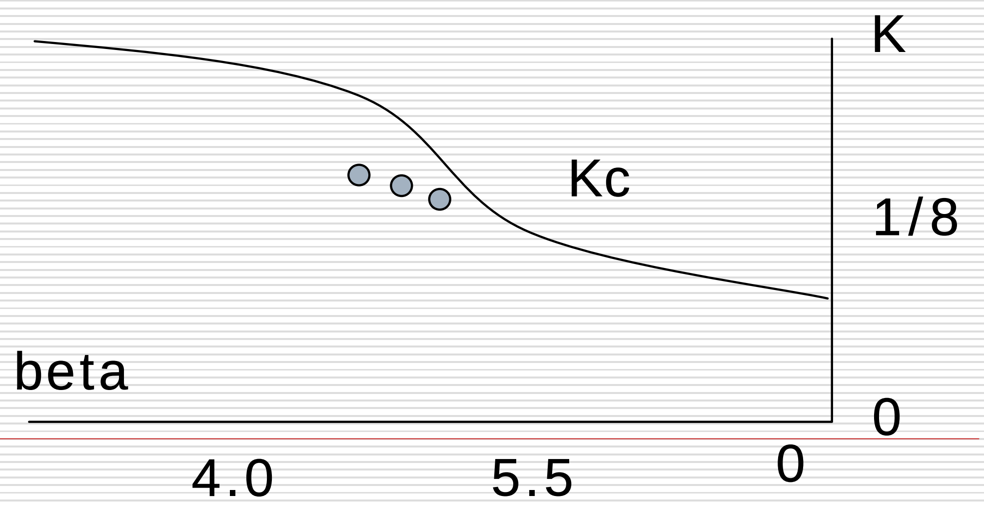
At the time, we thought that we simply had to take a large N_t for the tip to go into the scaling region, say beta>5.5……

MILC finding in 1994

Blum et al PRD 50 3377('94)

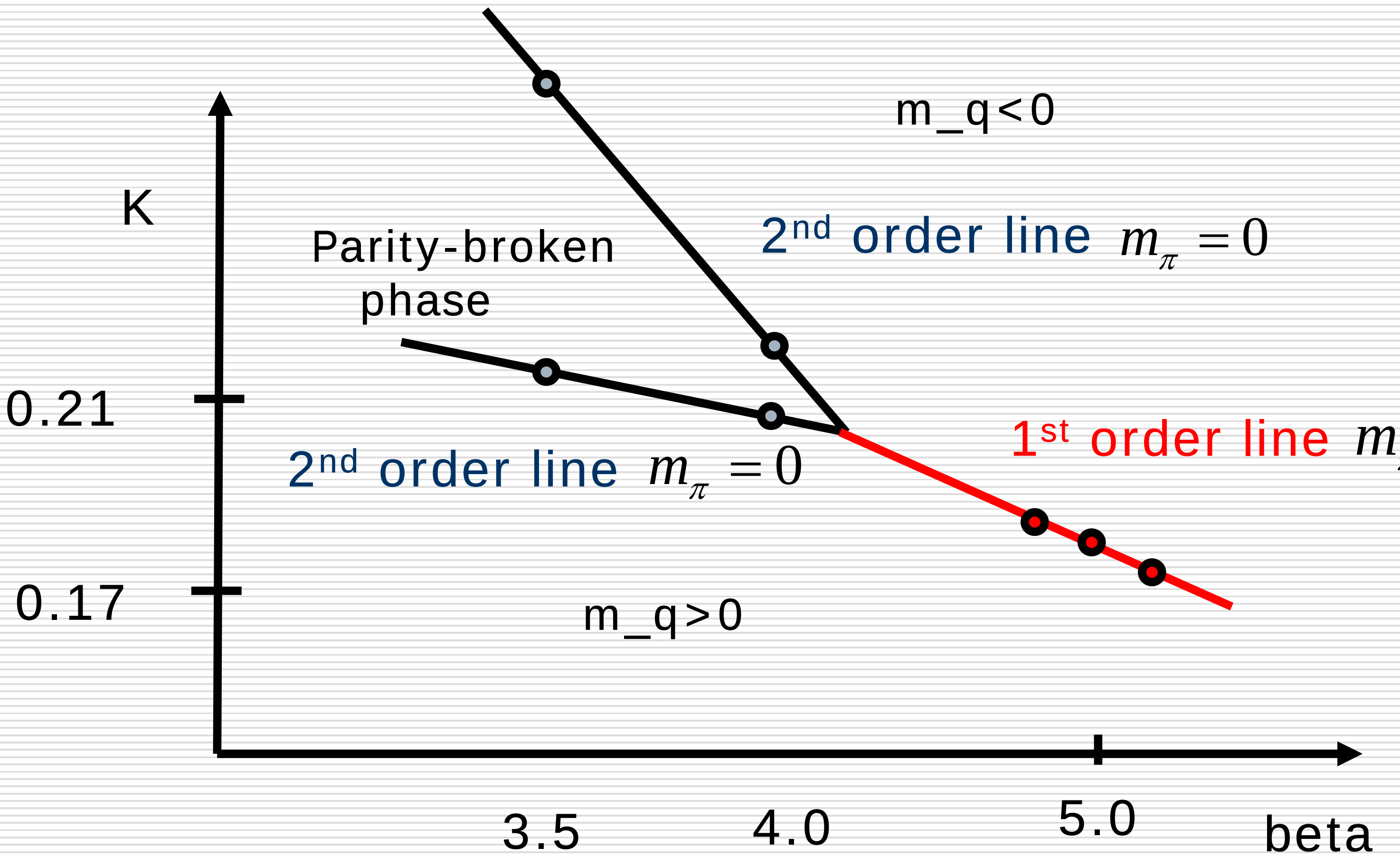
- Mostly 12^3x6 runs for finite-temperature study
- Metastability at
  - Beta=4.8, K=0.19
  - Beta=5.01, K=0.18 (Farchioni et al)
  - Beta=5.22, K=0.17 ← cf beta=5.2, K=0.1715
- Jump in plaquette → bulk transition suggested
- QCDPAX Collaboration (Iwasaki et al) suggested that this is a lattice artifact due to wiggling of K_c

Y. Iwasaki, hep-lat/9412103



Putting things together……

Perhaps, the tip simply stops moving and turns into a line of 1st order transition at some vakue of beta near 4.0…..



Cf M. Creutz, hep-lat/9608024

Other studies

- K.Bitar, hep-lat/9602027
  - $\langle \overline{\psi} i \gamma_5 \tau_3 \psi \rangle$ in the presence of ext. field h
  - L^4, L=6,8,10,
  - Consistent with zero at
    - Beta=5.0, K=0.15, 0.1810,0.1820,0.1850,0.1875
    - Beta=5.5, K=0.1300,0.1350,0.1425,0.1500,0.1550,0.1610,0.1620,0.1650
    - Beta=8.0, K=0.1200,0.1300,0.1400,0.1460,0.1500,0.1550,0.1600,0.1800
- Ilgenfritz et al hep-lat/0309057
  - Similar study
  - L=4,6,8,10
  - (Beta,K)=(4.0,0.22),(4.3,0.21) non-zero value at h->0
  - (Beta,K)=(4.6,0.1981),(5.0,0.18)

    consistent with zero at h->0

questions

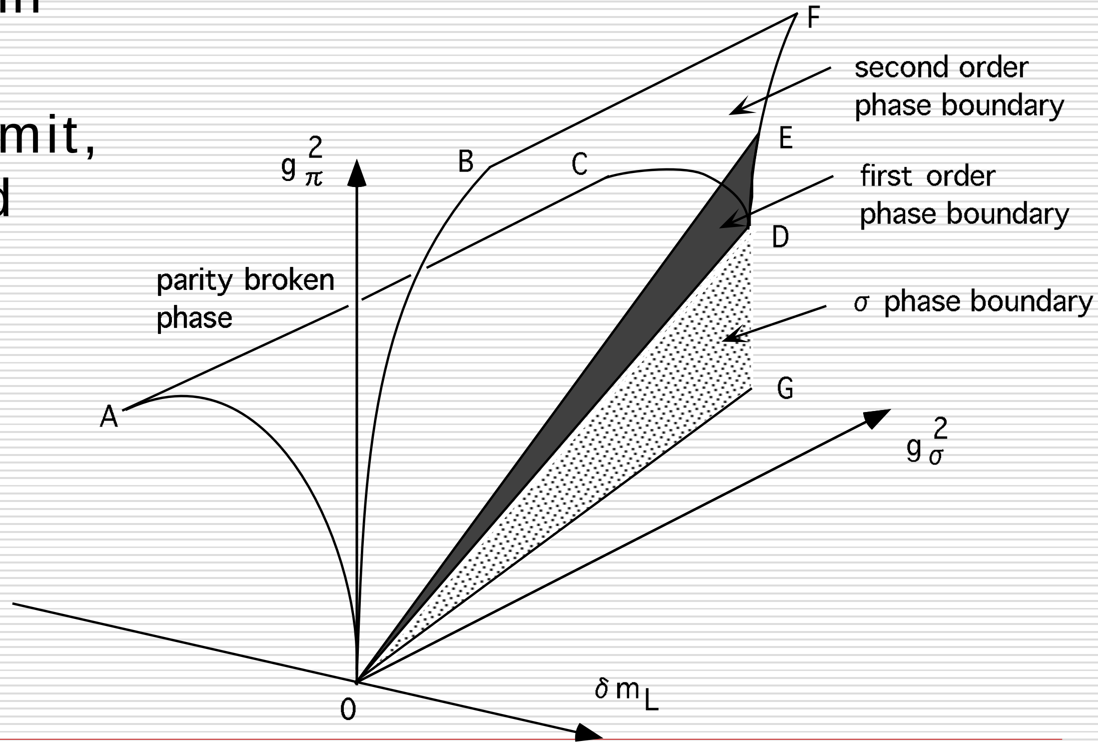
- What happens to the 1st order line toward weaker coupling?
- Does it affect the continuum limit?
- How universal is the 1st order transition?

An interlude:
2D Gross-Neveu model in the large N limit

Noaki-Izubuchi-Ukawa, PRD

$$L = \bar{\psi}\gamma_\mu \cdot \left(\nabla_\mu + \nabla_{-\mu} - \frac{1}{2}\nabla_\mu \nabla_{-\mu} \right)\psi + \frac{g_\sigma}{N}(\bar{\psi}\psi)^2 + \frac{g_\pi}{N}(\bar{\psi} i\gamma_5 \psi)^2$$

- Need two couplings to restore chiral symmetry in the continuum limit
- Solvable in the large N limit, but a rather complicated phase diagram



comments $O\left(a\sigma^3\right)$

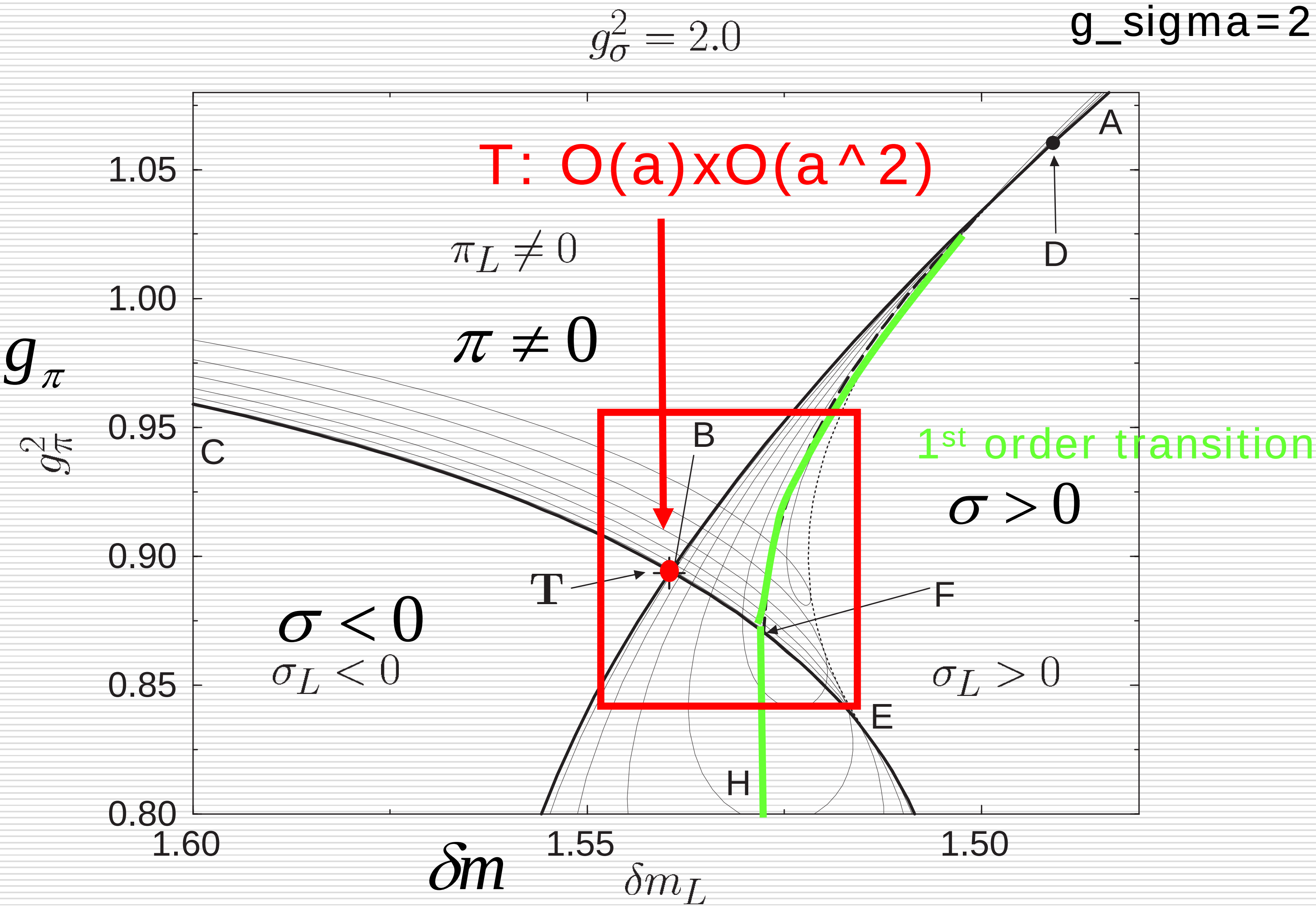
$$V(\sigma) = -\left(\frac{\delta m}{g_\sigma} + 2C_1\right)\sigma + \left(\frac{1}{2g_\pi} - C_0\right)\pi^2 + \left(\frac{1}{2g_\sigma} - C_0 + 2C_2\right)\pi^2 + + \frac{1}{4\pi}\left(\sigma^2 + \pi^2\right)\ln\frac{\sigma^2 + \pi^2}{e}$$
$$-\frac{8}{3}C_3\sigma^3 + 2\left[C_1' - \frac{1}{8\pi}\ln\left(\sigma^2 + \pi^2\right)\right]\sigma\left(\sigma^2 + \pi^2\right) + \cdots$$

- Origin of 1st order transition:negative O(a) cubic term in the effective potetial
- Tuning of couplings necessary, but a chirally symmetric continuum limit constructible:

$$T : \begin{cases} \dfrac{\delta m}{g_\sigma} = 2C_1 + O\left(a^2\right) \\ \dfrac{1}{2g_\pi} = \dfrac{1}{2g_\sigma} + 2C_2 + O(a) \end{cases}$$

Continuum extrapolation

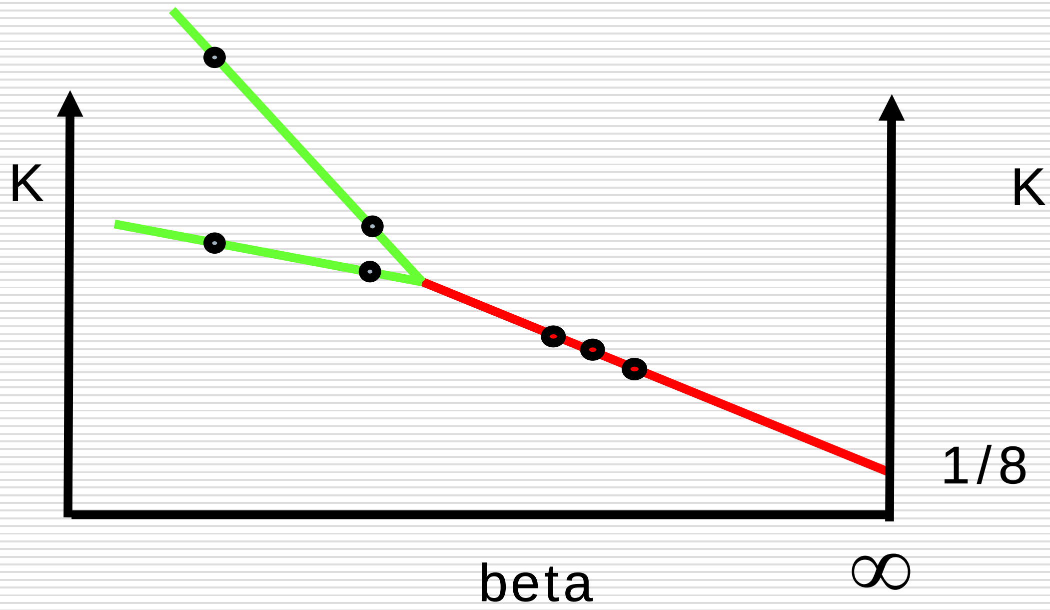
g_sigma=2.0



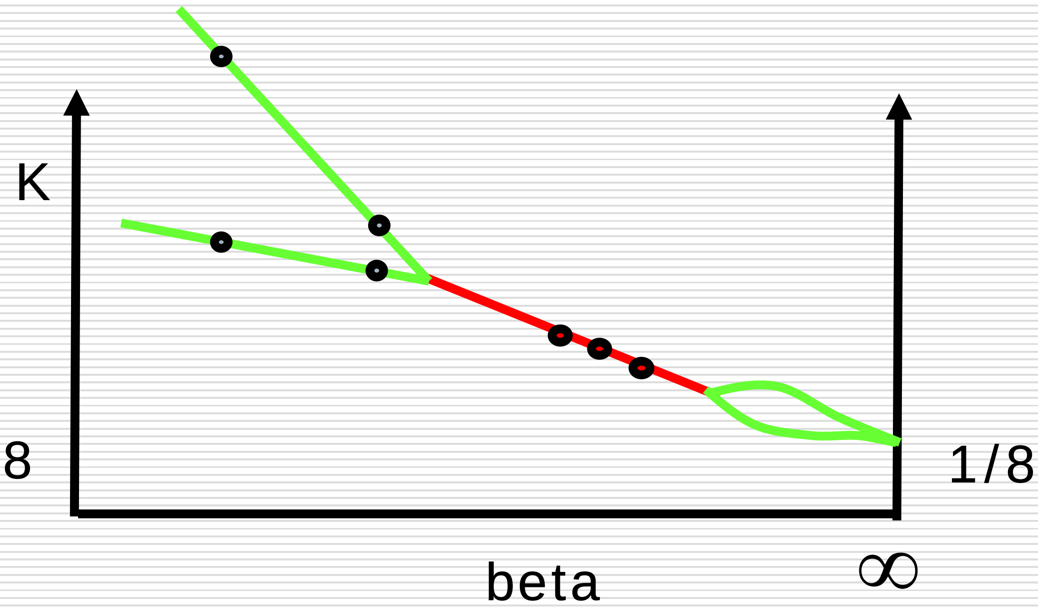
Possible scenarios for QCD ......

Plaquette + naïve Wilson

(I)

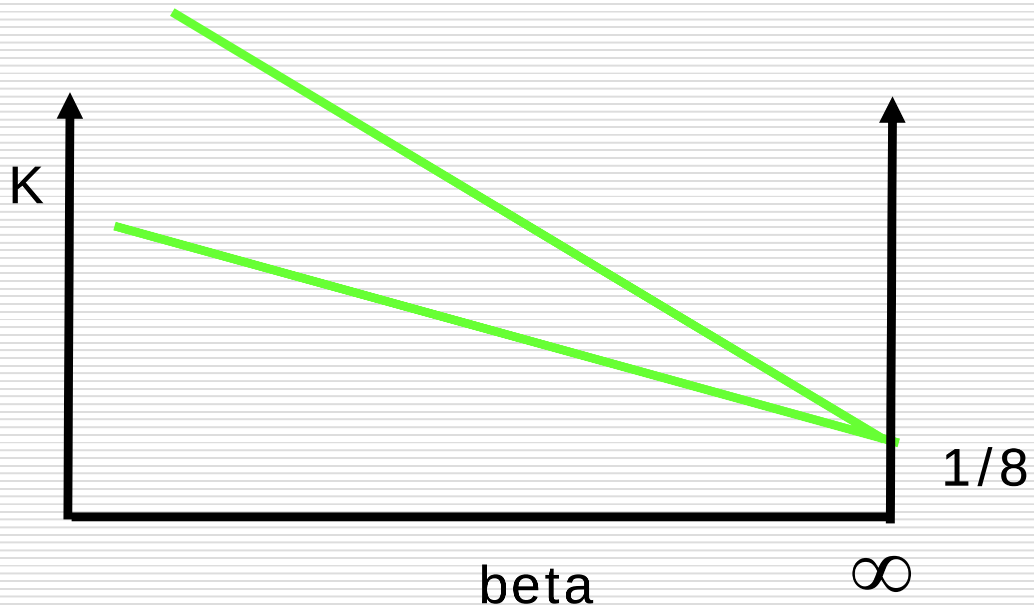


(II)



Possible scenarios for QCD ....

Improved gauge and/or improved quark



At the end:

- Quite a surprise
- If 1st order transtition is there and continues toward weak coupling,
  - No problem of principle, but
  - Quite a nuisance (I think) for phenomenology