The Pion Form-Factor on the Lattice

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Lattice Hadron Physics Collaboration

Research interests:

- Quark and gluon structure of hadrons
  - Nucleon & meson structure funcs, form-factors, GPD's…
- Spectroscopy of conventional and exotic states of hadrons
- Interactions between hadrons
- Fundamental aspects of QCD including confinement and chiral symmetry breaking
- Members:
  - R. Brower, C. Rebbi (Boston U.), C. Morningstar (CMU), S. Chandrasekharan (Duke), R. Fiebig, R. Narayanan (FIU)
  - R. Edwards, D. Richards, C. Watson (JLab),
  - X. Ji, I. Sato, S. Basak, S. Wallace (Maryland),
  - P. Dreher, J. Negele, K.Orginos, A. Pochinsky, D.Renner, W.Schroers (MIT)
  - M. Burkhardt (NMSU), E. Swanson (Pittsburg), H. Thacker (Virginia), F.X. Lee (GWU)

Pion Electromagnetic Form Factor $F_\pi(Q^2)$

- Considered a good observable for studying the interplay between perturbative and non-perturbative descriptions of QCD
  - Large Q^2 scaling as predicted by Brodsky-Farrar

$$\mathbf{F_\pi(Q^2) = \frac{8\pi\alpha_s(Q^2)f_\pi^2}{Q^2}} \quad \text{as} \quad \mathbf{Q^2 \to \infty}$$

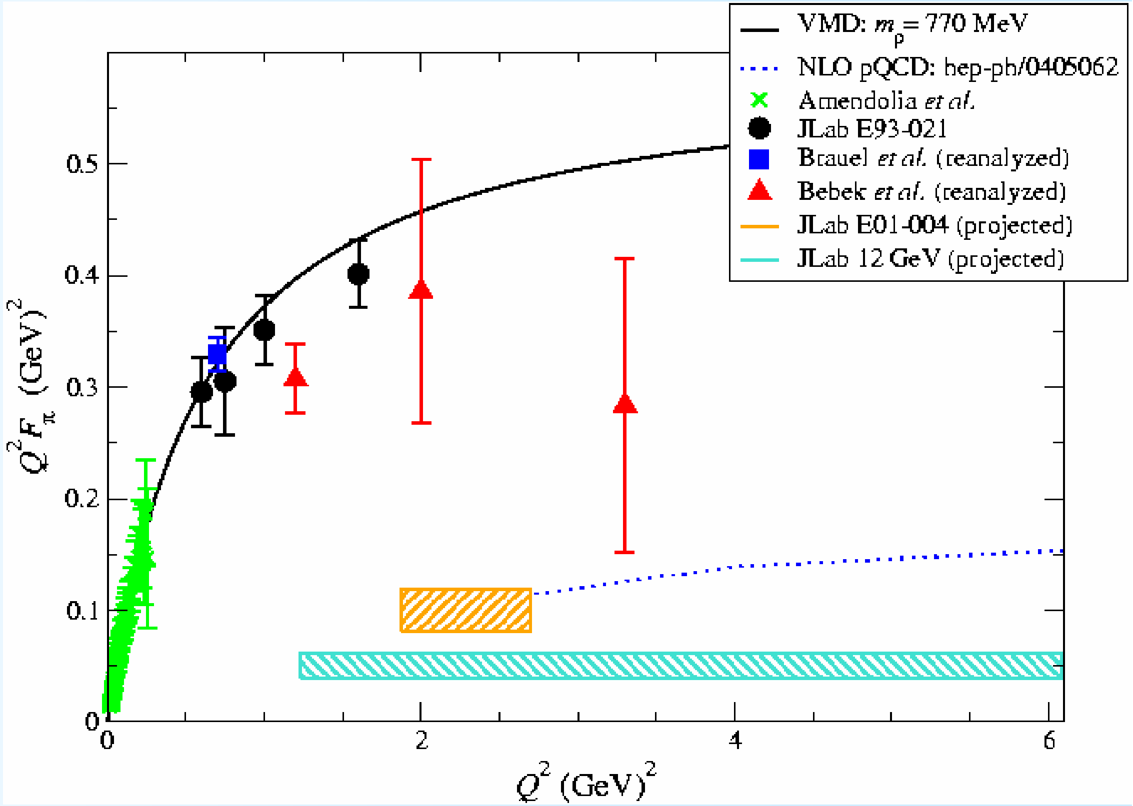
  - For small Q^2 , vector meson dominance gives an accurate description, with normalization $F_\pi(0) = 1$ given by charge conservation

$$\mathbf{F_\pi(Q^2) \sim \frac{1}{1 + Q^2/m_\rho^2}} \quad \text{for} \quad \mathbf{Q^2 \ll m_\rho^2}$$

  - Not too hard to calculate on the lattice – no disconnected diagrams
  - Experimental results are coming for Q^2 ¸ 1 GeV^2

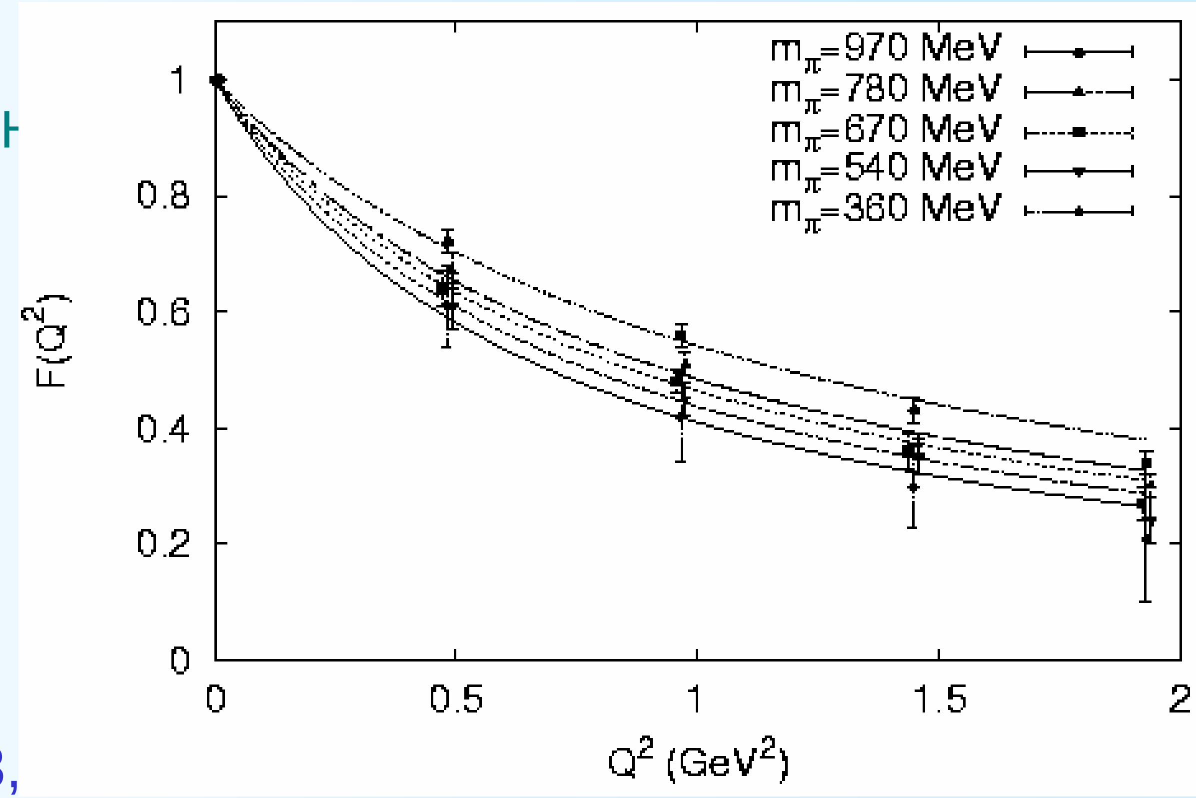
Experimental Results

- Existing data fit VMD monopole formulae too well. Where's perturbative QCD?
- Because of simple valence structure, argued that PQCD should apply to lower Q^2 than other hadrons
- PQCD calc. far off.
- Experimental interpretation problematic – must extrapolate to the pion pole
- Use lattice QCD!

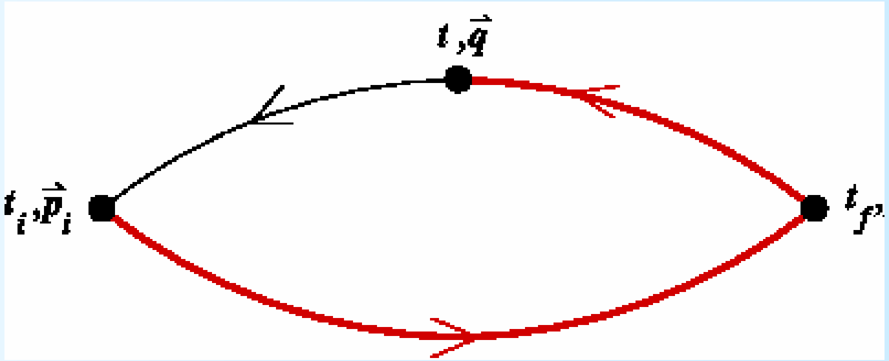


Latest published quenched lattice results

- Quenched clover results:
  - J. van der Eide, M. Lutterot, J.H. Kock, and E. Laermann, PLB566, 131 (2003)
  - J. van der Heide, J.H. Koch, and E. Laermann, hep-lat/0312023
- The results still consistent with VMD up to 2 GeV^2
- Preliminary results by RBC Collaboration, hep-lat/0309173, with quenched DBW2+DWF, $Q^2 < 0.5$ GeV^2



Lattice techniques for extracting $F_\pi(Q^2)$



- Form factor definition

$$\langle \pi(\tilde{\mathbf{p}}_\mathbf{f}) \, |\mathbf{V}_\mu(\mathbf{0})| \, \pi(\tilde{\mathbf{p}}_\mathbf{i}) \rangle_{\text{cont}}$$

$$= Z_V \left\langle \pi(\vec{p}_f) \, |V_\mu(0)| \, \pi(\vec{p}_i) \right\rangle$$

$$= F(Q^2)(p_i + p_f)_\mu$$

- Use the sequential-sink method:
- Disadvantage: momentum p_f and pion quantum numbers are fixed at the sink
- Advantages: variety of form-factors, e.g. ρ!γπ computable from same set of propagators
- Largest Q^2 available in Breit frame (p_f = - p_i). Need sequential propagators for several p_f .
- $F_\pi(Q^2)$ extracted either from simultaneous fit to several 2 and 3-point functions or by constructing ratios.

Three point functions, smearings and ratios

- The three-point correlator with APE smeared source A and sink B

$$\Gamma^{AB}_{\pi 4\pi}(t_i, t, t_f, \vec{p}, \vec{p}_f) = e^{-i(\vec{x}_f - \vec{x})\cdot\vec{p}_f} \left\langle \pi^B(\vec{x}_f, t_f) | V_4(\vec{x}, t) | \pi^A(\vec{x}_i, t_i) \right\rangle e^{-i(\vec{x} - \vec{x}_i)\cdot\vec{p}_i}$$

- Ratio gives $F_\pi(Q^2,t)$ independent of Z's and exponentials (C = smeared or Local)

$$F(Q^2, t) = \frac{\Gamma^{AB}_{\pi 4\pi}(t_i, t, t_f, \vec{p}_i, \vec{p}_f)\Gamma^{CL}_{\pi\pi}(t_i, t, \vec{p}_f)}{\Gamma^{AL}_{\pi\pi}(t_i, t, \vec{p}_i)\Gamma^{CB}_{\pi\pi}(t_i, t_f, \vec{p}_f)} \left(\frac{2 Z_V E_\pi(\vec{p}_f)}{E_\pi(\vec{p}_i) + E_\pi(\vec{p}_f)} \right)$$

- Compare ratio with explicit fits

Computational Strategies

- Quenched
  - Here Wilson gauge and Wilson fermion actions
  - Difficulty getting to chiral limit, m_π=1350MeV ! 290MeV
  - Use quenched chiral perturbation theory – not QCD!
  - Low energy Gasser-Leutwyler constants not those of QCD
  - Pathologies in quenched approx

- Dynamical (full QCD)
  - $N_f = 2 + 1$
  - Asqtad staggered sea quarks
  - Domain Wall valence quarks, m_π=758MeV ! 320MeV
  - Use partially quenched chiral perturbation theory – have lattice corrections
  - Low energy Gasser-Leutwyler constants are those of QCD!

Calculation Details

- Quenched unimproved Wilson fermions
  - Pion masses smaller than clover, but O(a) scaling violations
  - β=6.0, a=0.1fm
  - Conserved current Z_V=1

κ	lattice	m_π MeV	m_ρ MeV
0.1480	$16^3 \times 32$	1350	1420
0.1520	$16^3 \times 32$	954	1100
0.1540	$16^3 \times 32$	728	936
0.1555	$24^3 \times 32$	518	796
0.1563	$24^3 \times 32$	358	716
0.1566	$24^3 \times 32$	290	686
0.1566	$32^3 \times 48$	290	686

- Full QCD
  - N_f=2+1 Asqtad, a=0.12 fm, 20^3£64 & 28^3£64 am=0.01+0.05, m_π=356MeV, m_K=790MeV
  - N_f=3, Asqtad, a=0.125fm, 20^3£ 64, am=0.05, m_{PS}=790MeV
  - DWF valence, HYP smeared, 20^3£32 using Dirichlet BC, same a*m values -> 320MeV, 607MeV

5D Domain Wall

- Domain wall action: $\mathbf{S_{DW} = -\bar{\Psi} D^{(5)}_{DW} \Psi}$

$$\mathbf{S}_{\mathsf{DW}} = -\sum_{\mathbf{s=1}}^{\mathbf{L_s}} \bar{\mathbf{\Psi}}_{\mathbf{s}} \left[(\mathbf{D_w}(-\mathbf{M}) + \mathbf{1})\, \mathbf{\Psi_s} - \mathbf{P_-} \mathbf{\Psi_{s+1}} - \mathbf{P_+} \mathbf{\Psi_{s-1}} \right]$$

- 5D Domain wall kernel:

$$D^{(5)}_{DW}(\mu) = \begin{pmatrix} D_+ & -P_- & 0 & \cdots & 0 & \mu P_+ \\ -P_+ & D_+ & -P_- & \cdots & 0 & 0 \\ 0 & -P_+ & D_+ & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & D_+ & -P_- \\ \mu P_- & 0 & 0 & \cdots & -P_+ & D_+ \end{pmatrix}$$

with quark mass μ, and $\mathbf{D_\pm = a_5 D_w(-M) \pm 1}$

- Integrate out L_s-1 extra fields to obtain

$$\begin{aligned} D_{tov}(\mu; H) &= \left\{ \mathcal{P}^{-1} \left[D^{(5)}(1) \right]^{-1} D^{(5)}(\mu) \mathcal{P} \right\}_{11} \\ &= \tfrac{1}{2}\left[1 + \mu + (1-\mu)\gamma_5 \varepsilon_{L_s/2}(a_5 H) \right] \end{aligned}$$

- Here **P** is such that $(P^{-1}\,\Psi)_1 = q$ is the light fermion

Induced 4D action – truncated overlap

- Core piece of induced kernel:

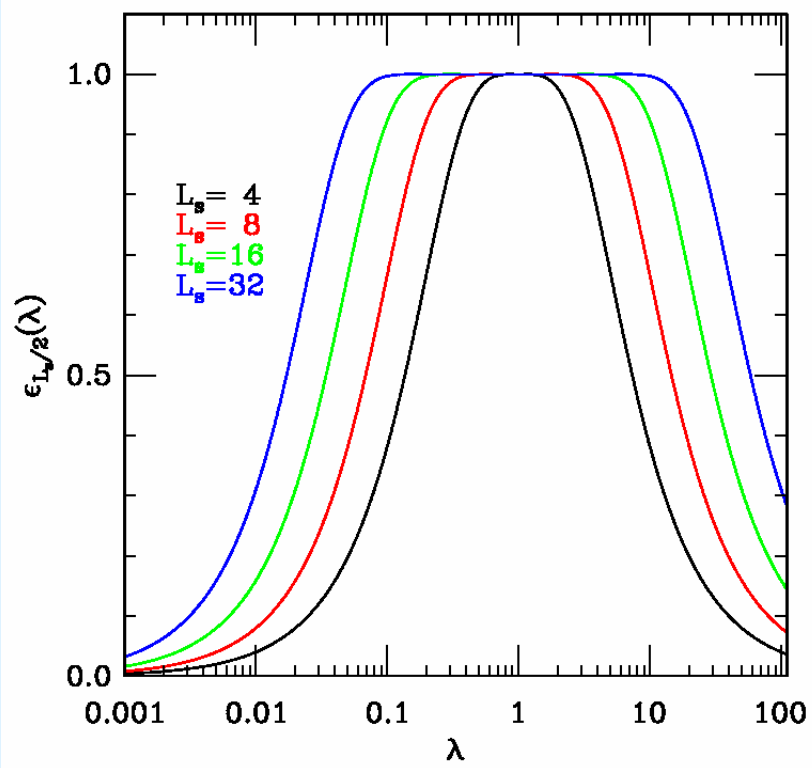
$$D_{tov}(\mu; H) = \frac{1}{2}\Big[1 + \mu + (1-\mu)\gamma_5 \varepsilon_{L_s/2}(a_5 H)\Big]$$

$$\varepsilon_{L_s/2}(a_5 H) = \frac{(1+a_5 H)^{L_s} - (1-a_5 H)^{L_s}}{(1+a_5 H)^{L_s} + (1-a_5 H)^{L_s}}$$

$$\sim \tanh(\tfrac{1}{2} a_5 L_s H) \stackrel{L_s \to \infty}{\longrightarrow} \epsilon(a_5 H)$$

Two variants:

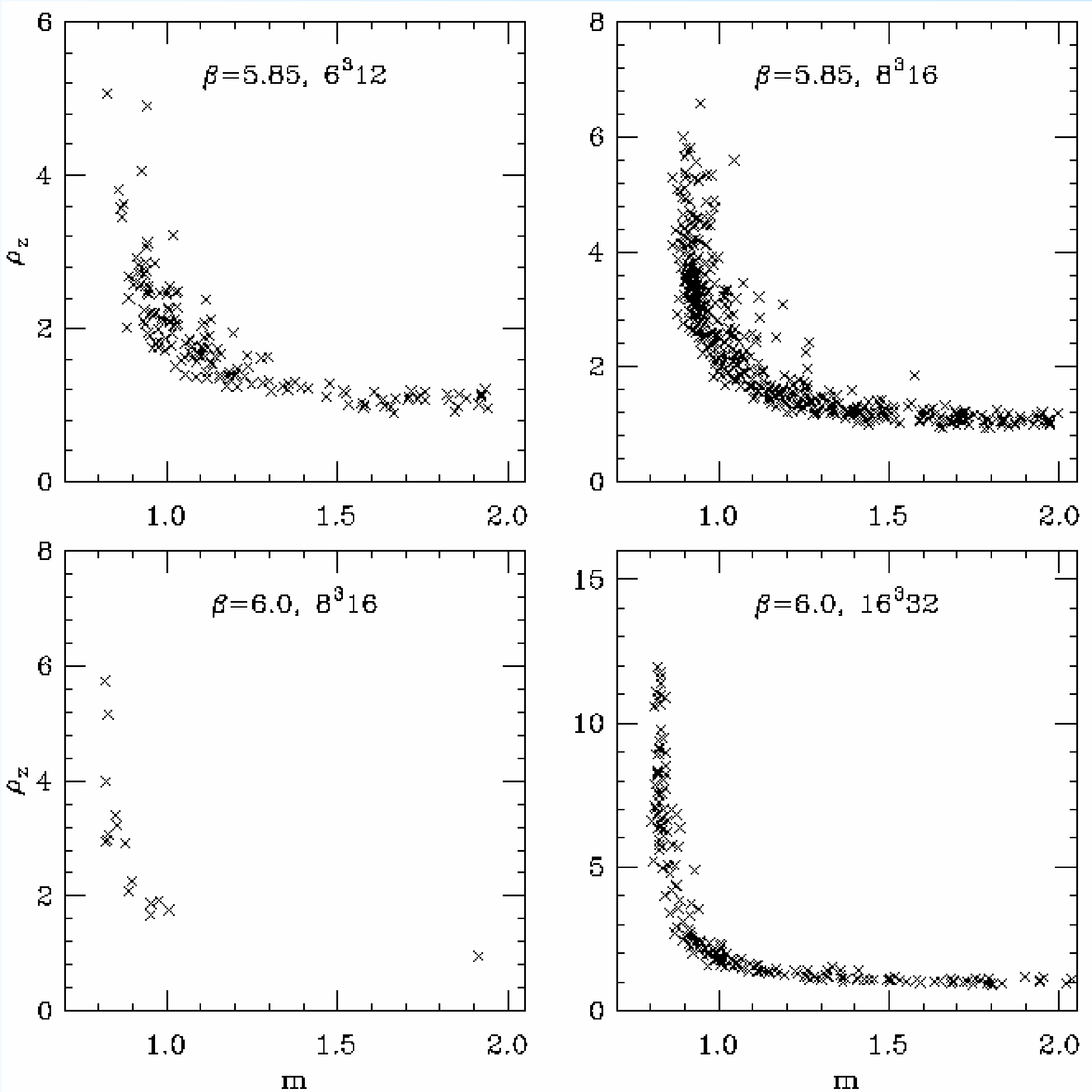
- Domain wall: $H = H_T = \gamma_5 D_w / (2 + a_5 D_w)$
- Overlap: $\lim a_5 ! \, 0$: $H = H_w = \gamma_5 D_w$



Zero mode size distribution

- Size of zero modes at each crossing.
- Modes become small

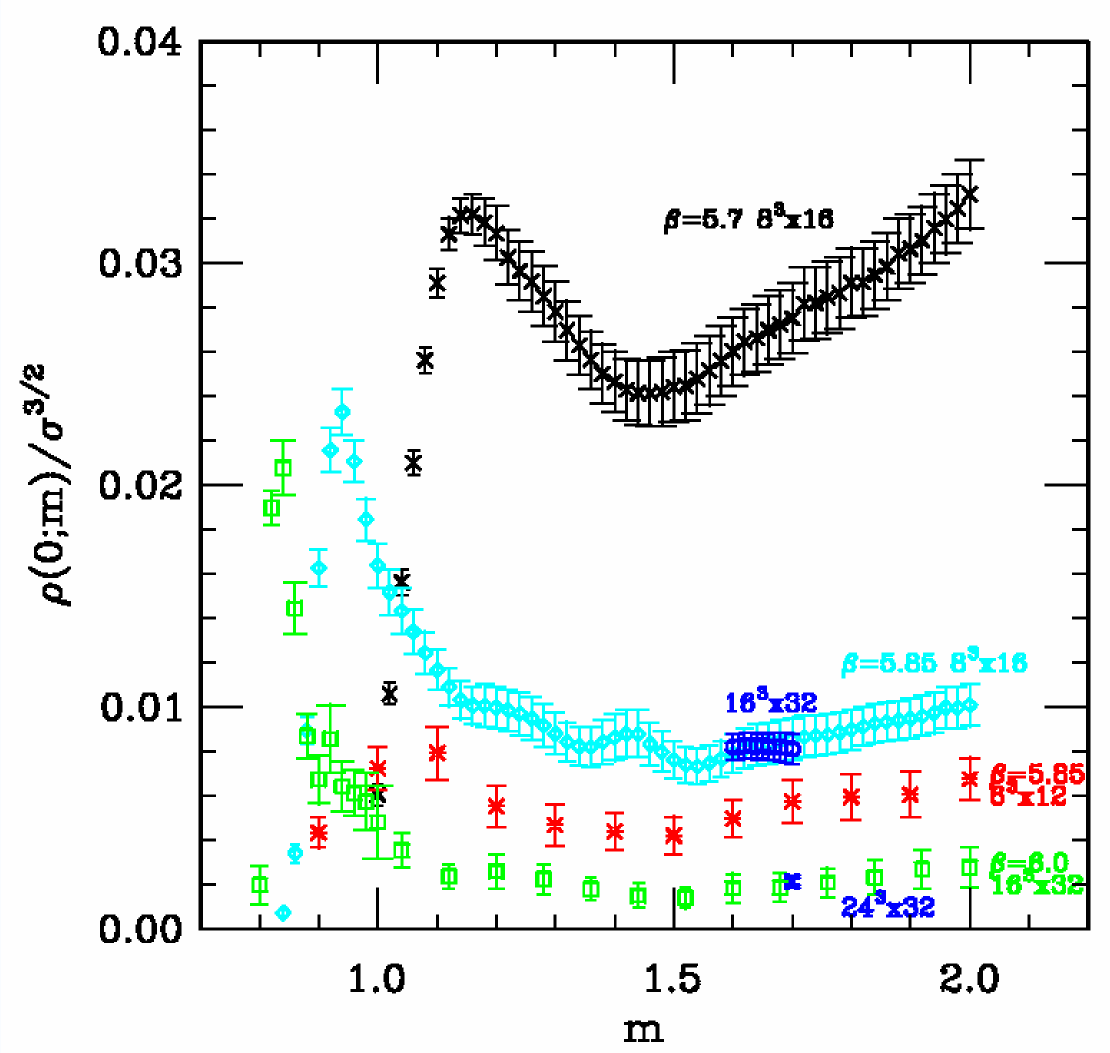
- Upshot: large contamination from small quantum fluctuations



Main problem for chiral fermions

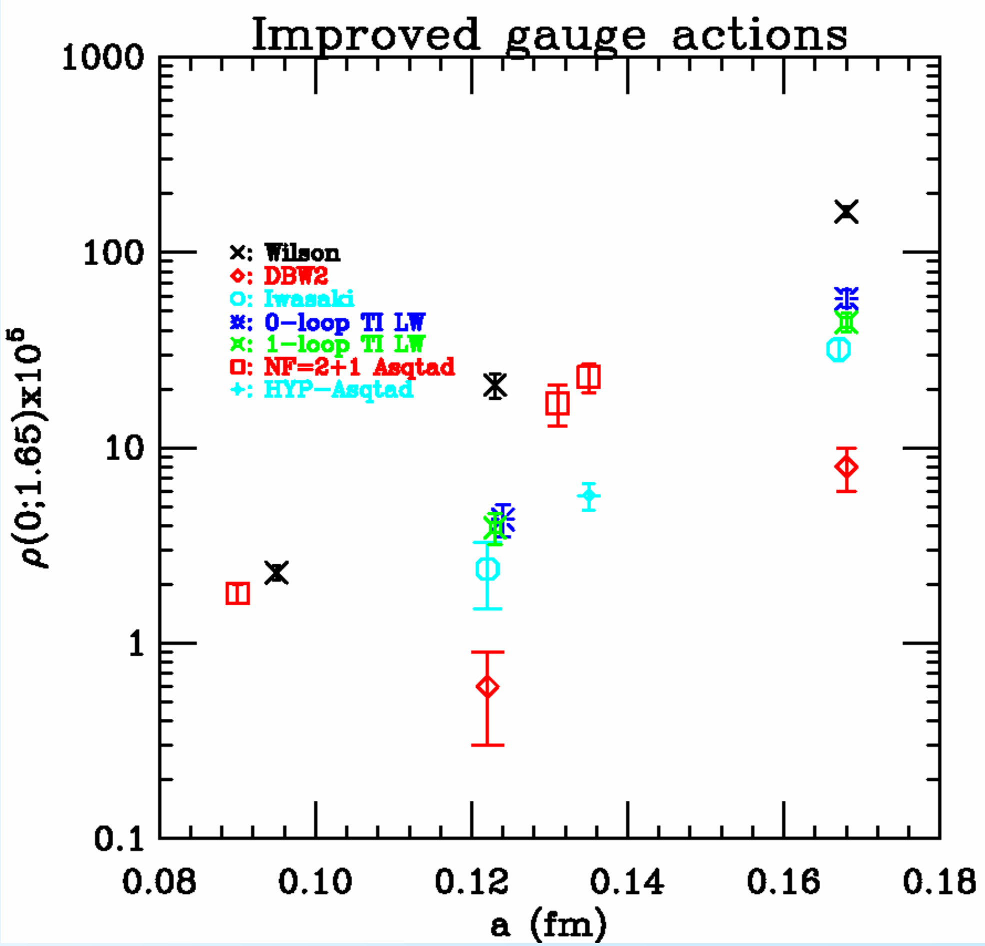
- The density of (near)zero eigenvalues $\rho(0^+;m)$ is computed by fitting the integrated density

$$\int_{0^+}^{\lambda} \rho(\lambda')\,d\lambda' = \rho(0^+)\lambda + \frac{1}{2}\rho_1\lambda^2 + \ldots$$



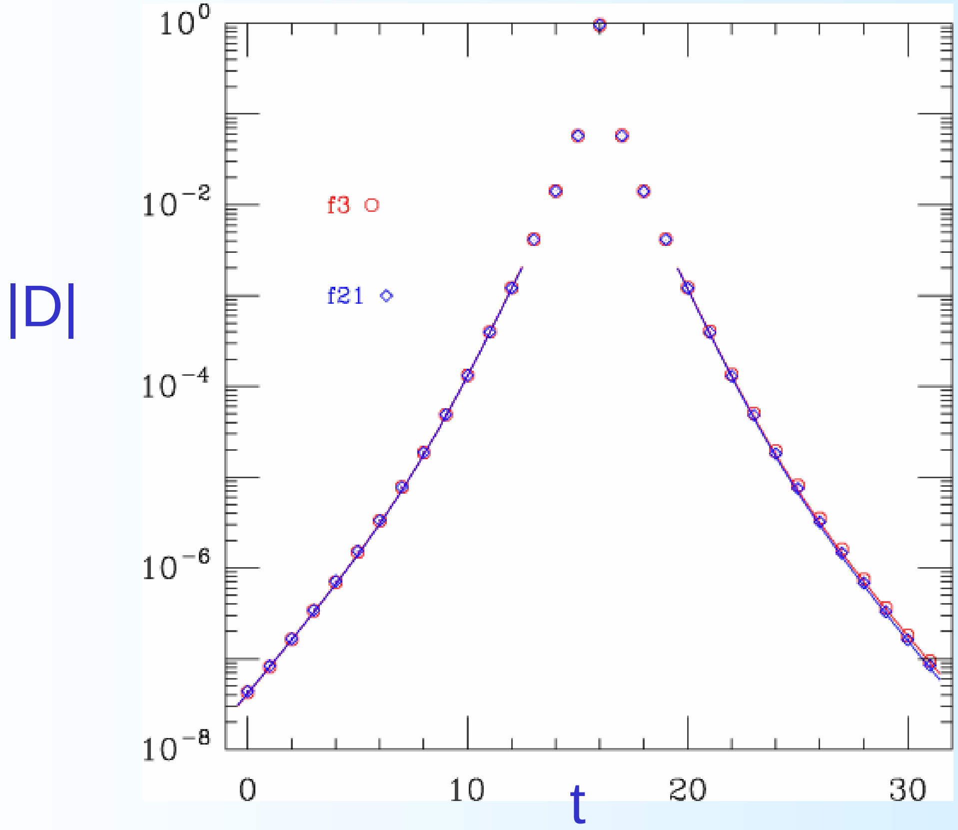
Improving the gauge action

- Gauge action improvements – reduce fluctuations
- Comparison of density of small (near-zero) evs. $\rho(0^+)$ from H_w for various gauge actions
- DBW2 (renorm. group) smallest
- Asqtad noisy!
- Surprise! dyn. fermions *induce* fluctuations!
- Fermions screen β-func., hence gauge coupling runs more slowly to short distance
- Accuracy problem worse for dynamical chiral fermions!



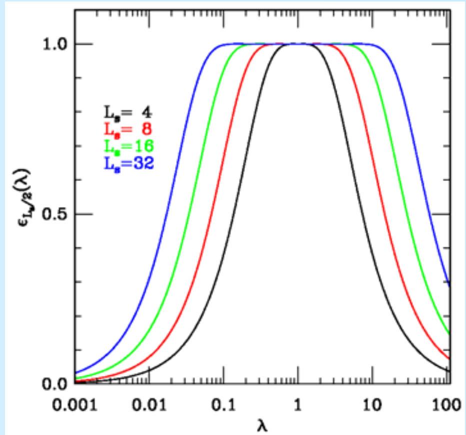
Operator Locality

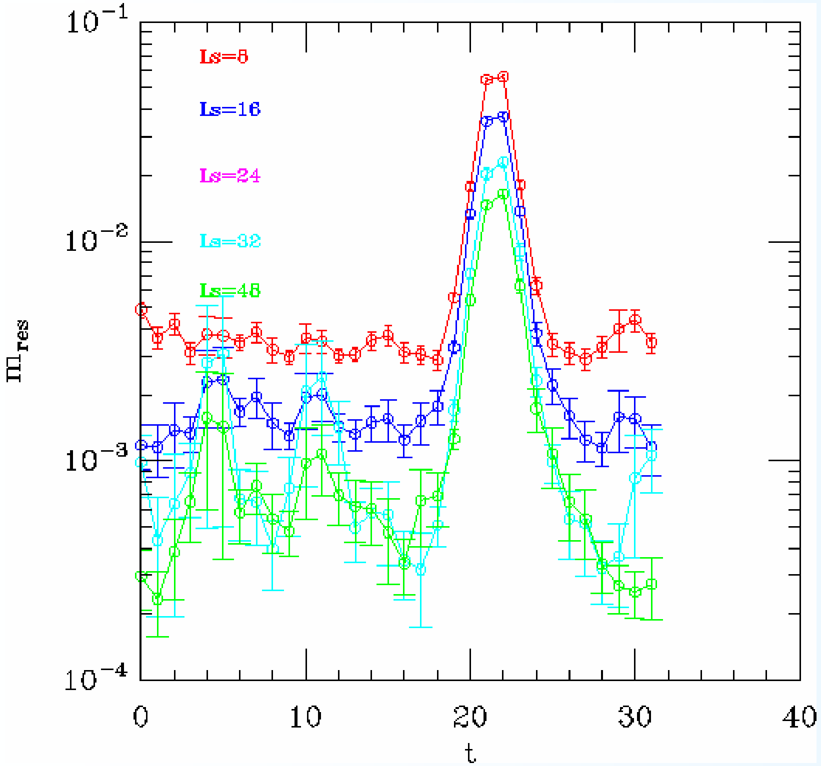
- Another sanity check:
  - Compute norm |D(x-y)|
  - A valid quantum field theory requires exp. falloff – locality
  - Tests on 20^3£32 lattice Asqtad lattice
  - Observe locality – this addresses a concern about Asqtad locality



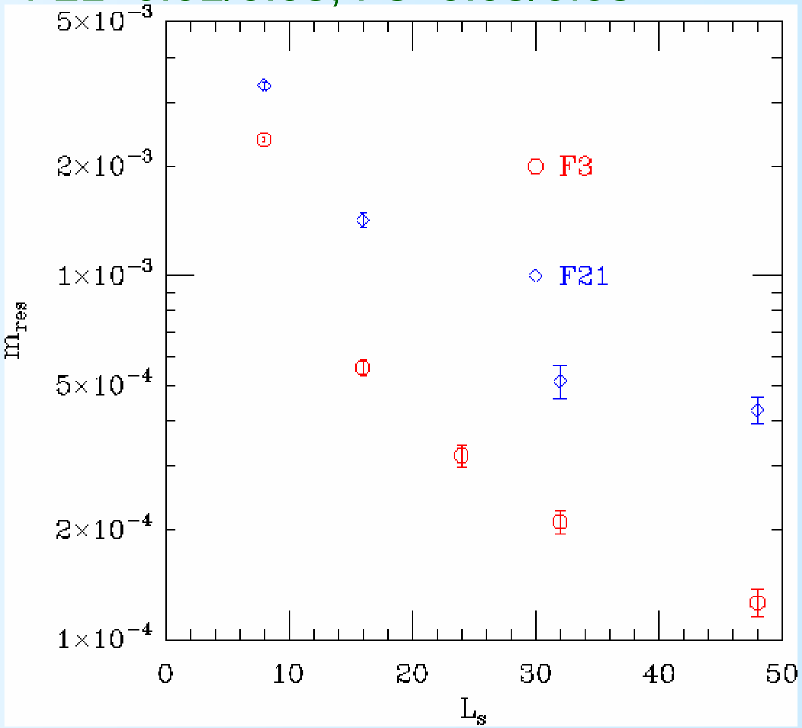
Induced quark mass dependence

- From 5D axial Ward identity, define an induced quark mass m_{res}
- Small modes in sign function induce chiral symmetry breaking
- We used Dirichlet BC – see mode on boundary
- Consistency in m_{res} » exp(-const*L_s)





•F21=0.01/0.05, F3=0.05/0.05

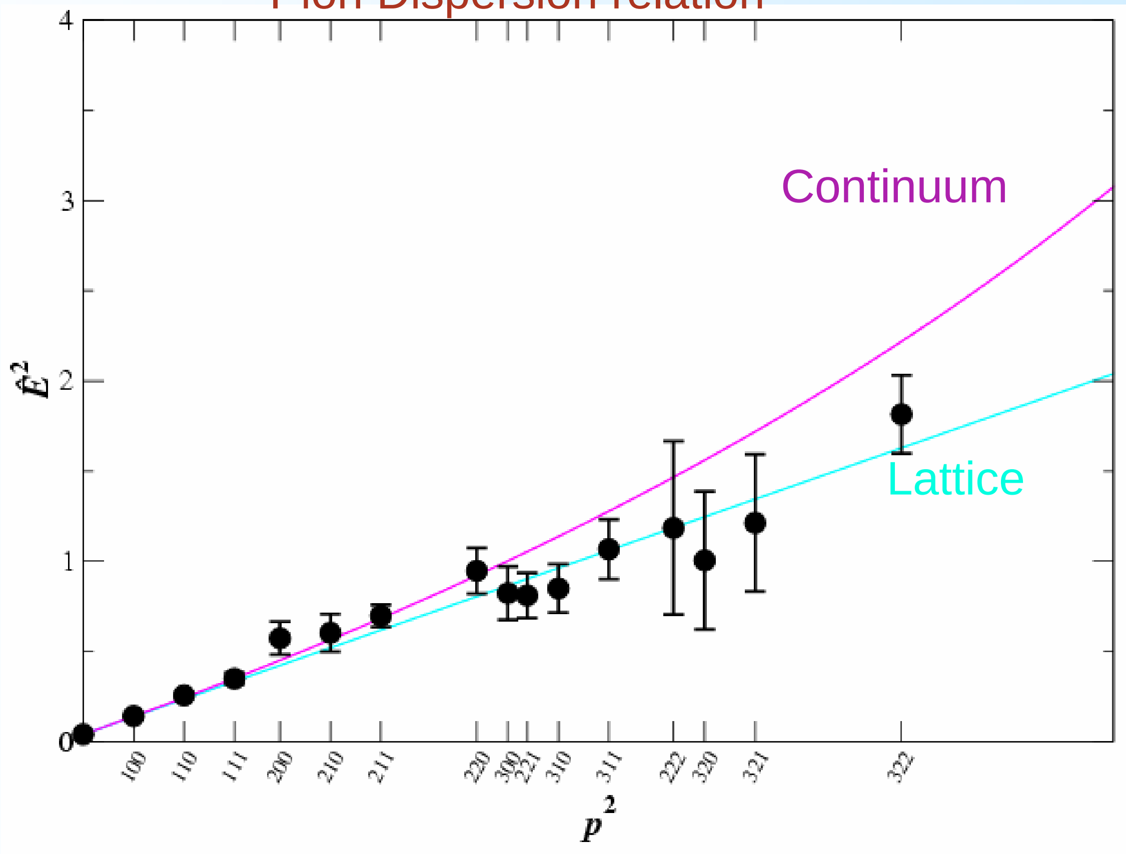


Dispersion Relation

- Pion computed at large momenta
- Test against both continuum and lattice dispersion relation
- Follows lattice dispersion relation

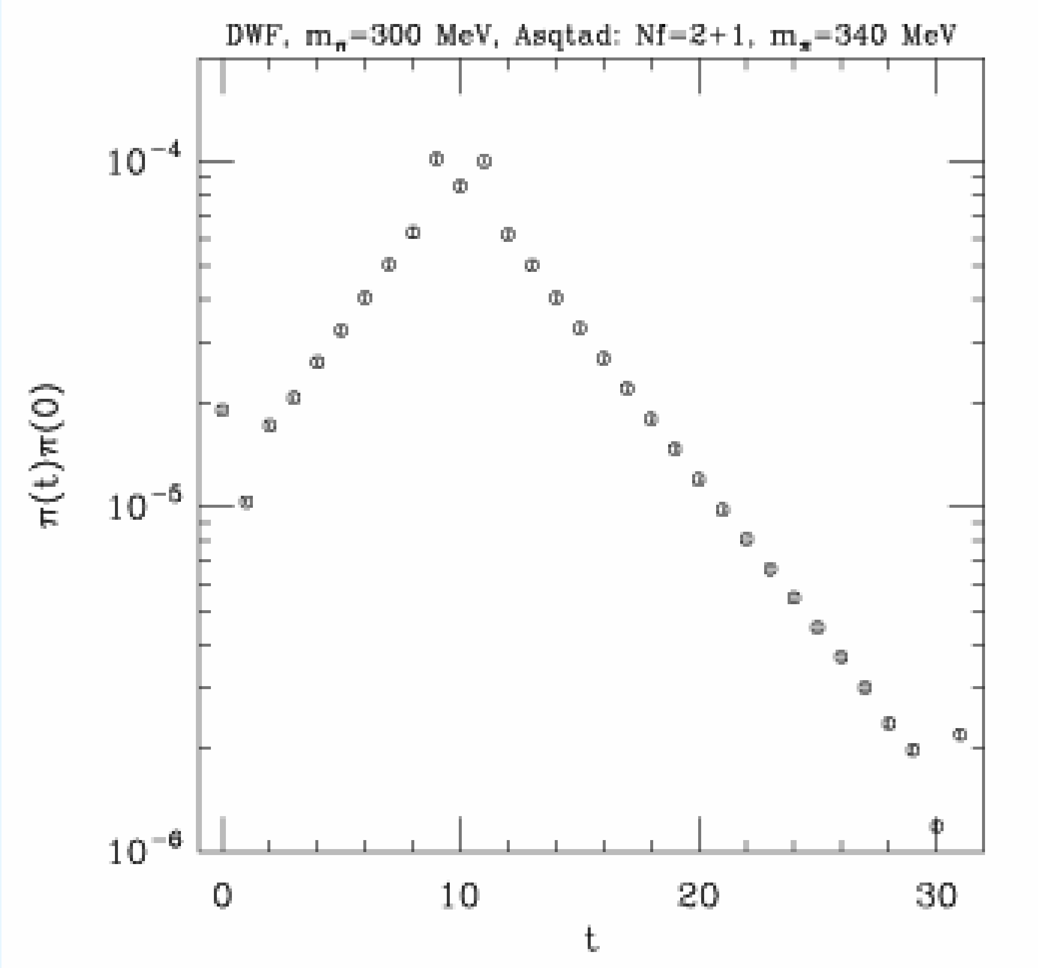
$$\frac{1}{4}\sinh^2\left(\frac{E_\pi(\vec{p})}{2}\right) = \frac{1}{4}\sin^2\left(\frac{\vec{p}}{2}\right) + \frac{1}{4}\sinh^2\left(\frac{E_\pi(0)}{2}\right)$$

Pion Dispersion relation



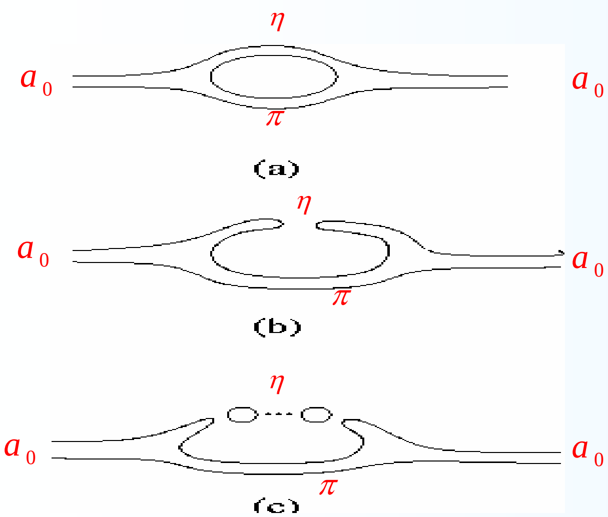
Pion Correlation function

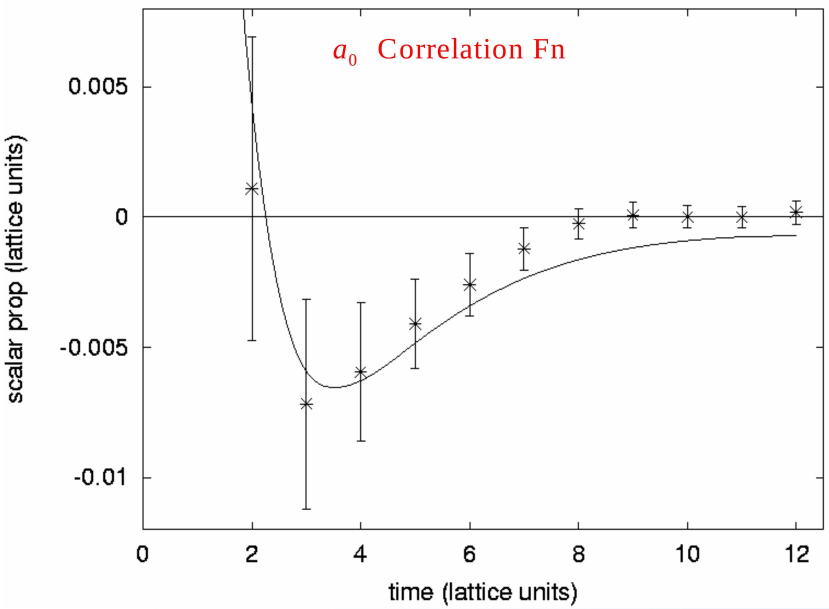
- Dirichlet BC: Source at $t_i=10$ (sink at $t_f=21$)
- DWF pion corr. shows odd behavior at $t=10$
- Originates from action – not positive definite transfer matrix



“Decay” in Quenched Approximation

Quark lines



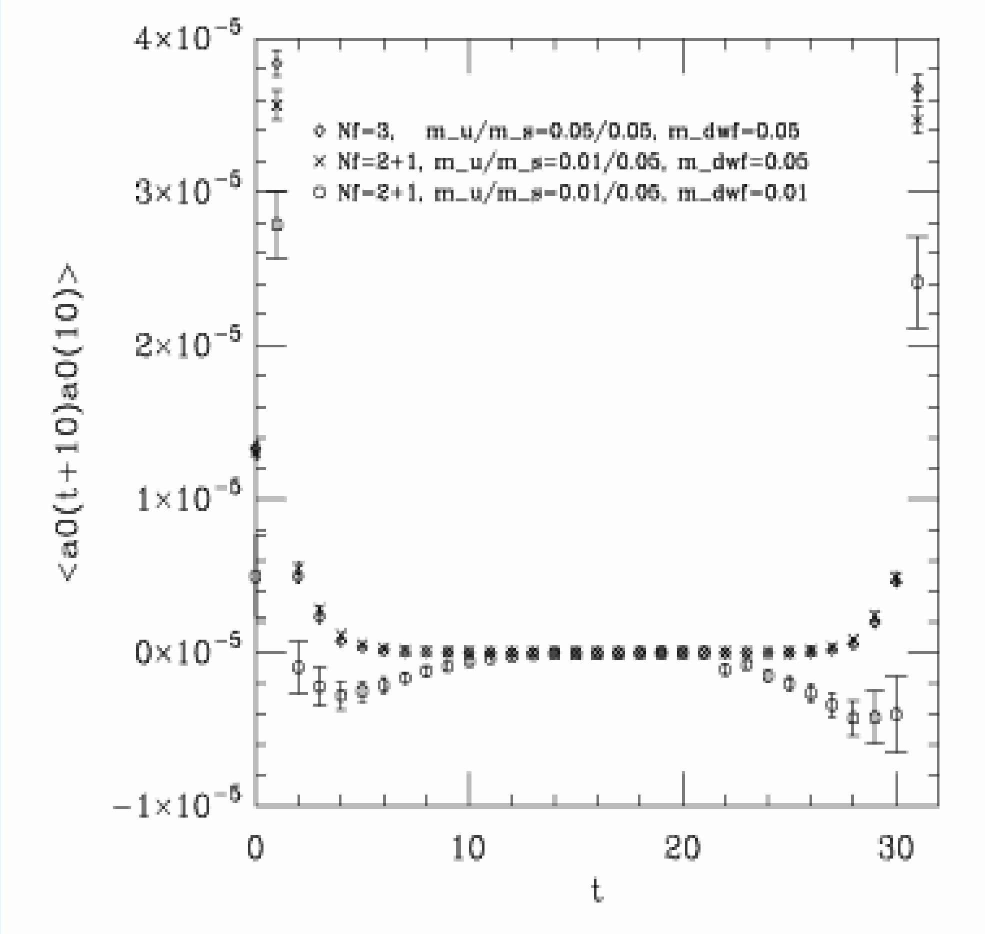


Bardeen, Duncan, Eichten, Thacker, 2000

- Dramatic behavior in Isotriplet scalar particle a_0!$\eta\pi$ intermediate state
- Loss of positivity of a_0 propagator from missing bubble insertions

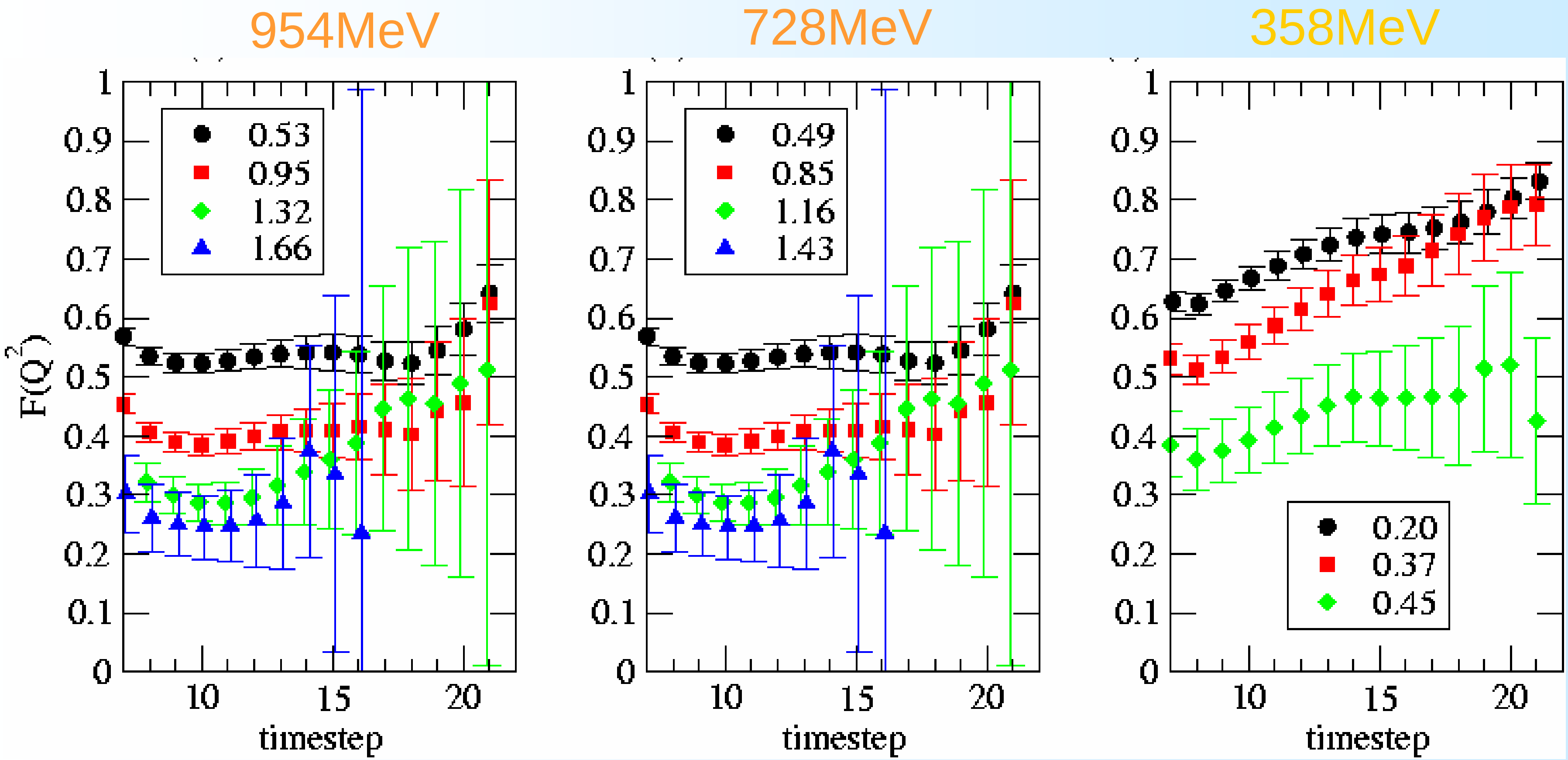
Partially Quenched Singularity

- Non-positivity of a_0 correlator
- (Partially) Quenched singularity (still) present at $m_{\pi,\text{ valence}}a = m_{\pi,\text{ sea}}a$.
- Suggests not single *staggered* pion in chiral loops – taste breaking not neglible
- Need complete partial χPT
  - Vary valence and sea masses
  - Theory under development…



Quenched Wilson Results

- $F_\pi(Q^2,t)$ ratio plots
  - For heavy pions, there are nice wide plateaus
  - For lighter pions, plateaus start further from source
  - Best/Better interpolating field at high momenta - $\bar{\psi}\gamma_4\gamma_5\psi$



Quenched Wilson Form Factor

- Quenched Wilson $F_\pi(Q^2,t)$
  - Data goes below experimental values
  - Can be understood from O(a) scaling violations (Wilson vector mass too small)

κ	m_π MeV	m_ρ MeV
0.1480	1350	1420
0.1520	954	1100
0.1540	728	936
0.1555	518	796
0.1563	358	716
0.1566	290	686
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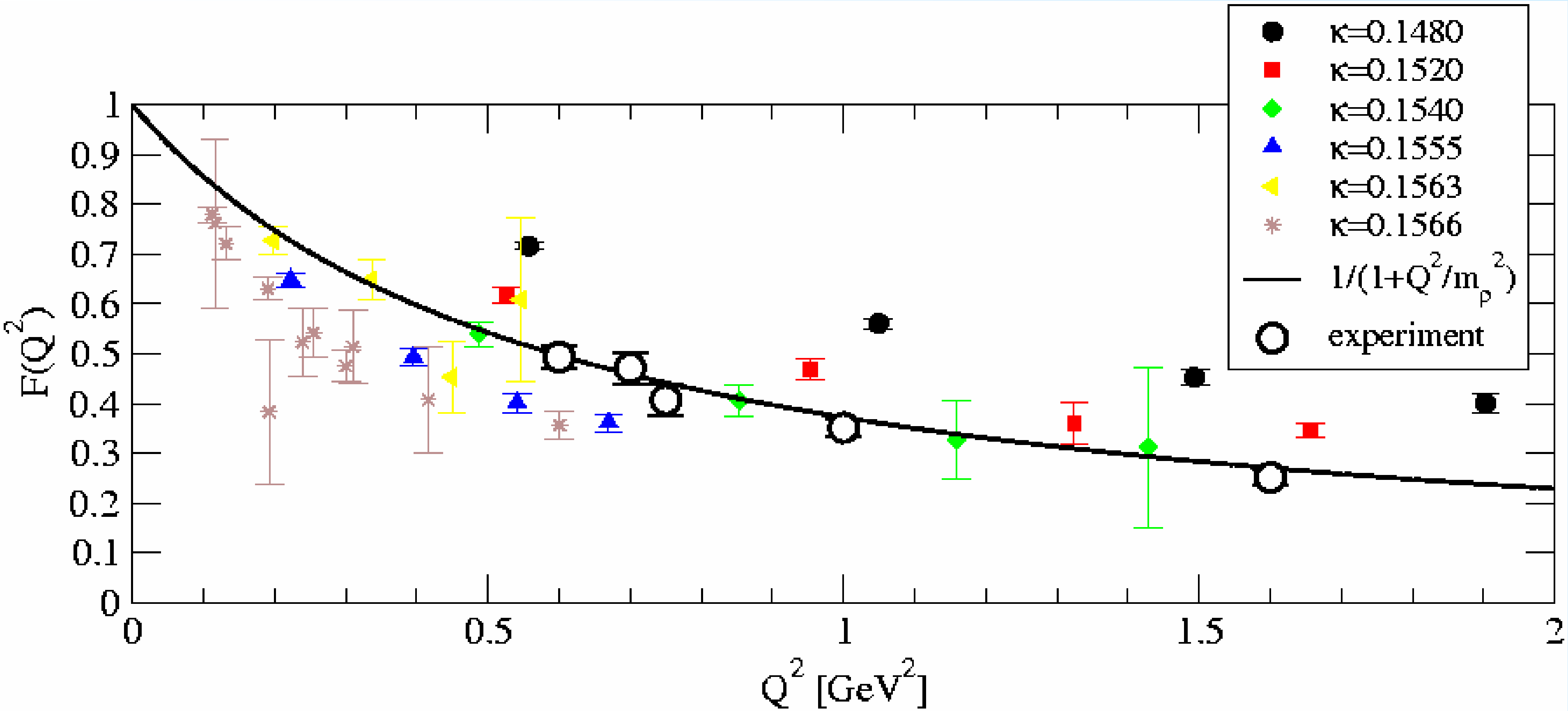
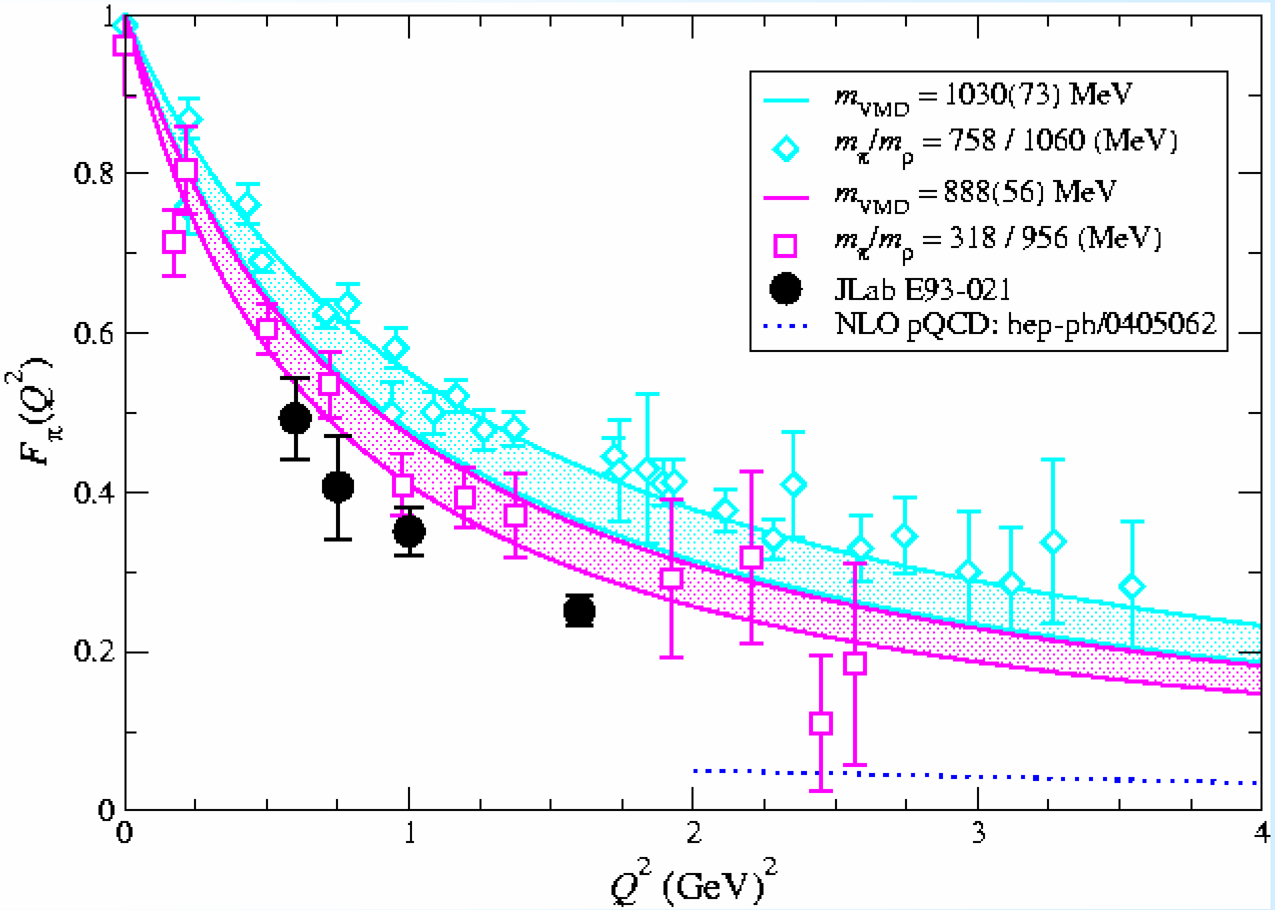


Partially Quenched DWF Form Factor

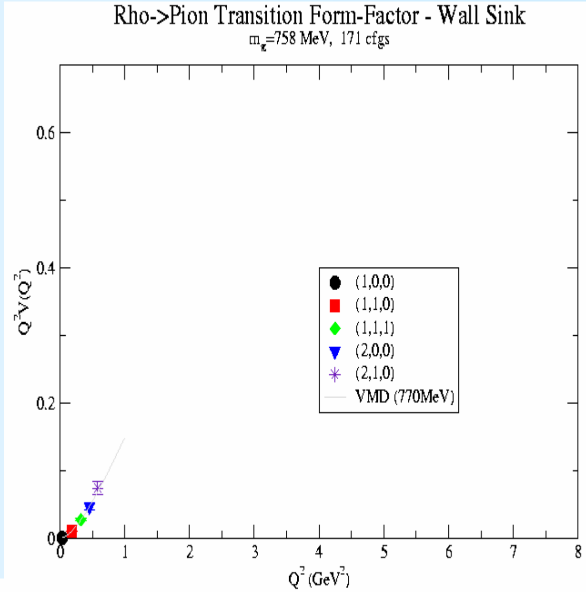
- DWF $F_\pi(Q^2,t)$
  - Smaller mass close to experimental VMD.
- Charge radius (crude analysis): $$\left.\frac{\partial \mathbf{F}(\mathbf{Q^2})}{\partial \mathbf{Q^2}}\right|_{\mathbf{Q^2=0}} = \frac{\mathbf{1}}{\mathbf{6}}\langle \mathbf{r^2}\rangle \quad \to \langle \mathbf{r^2}\rangle = \frac{\mathbf{6}}{\mathbf{m_V^2}}$$
  - Exp. h r^{2}i = 0.439(8)fm^2 , VMD ! 0.405fm^2
  - Statistical: 0.226(15)fm^2, 0.304(18)fm^2 strong mass dependence

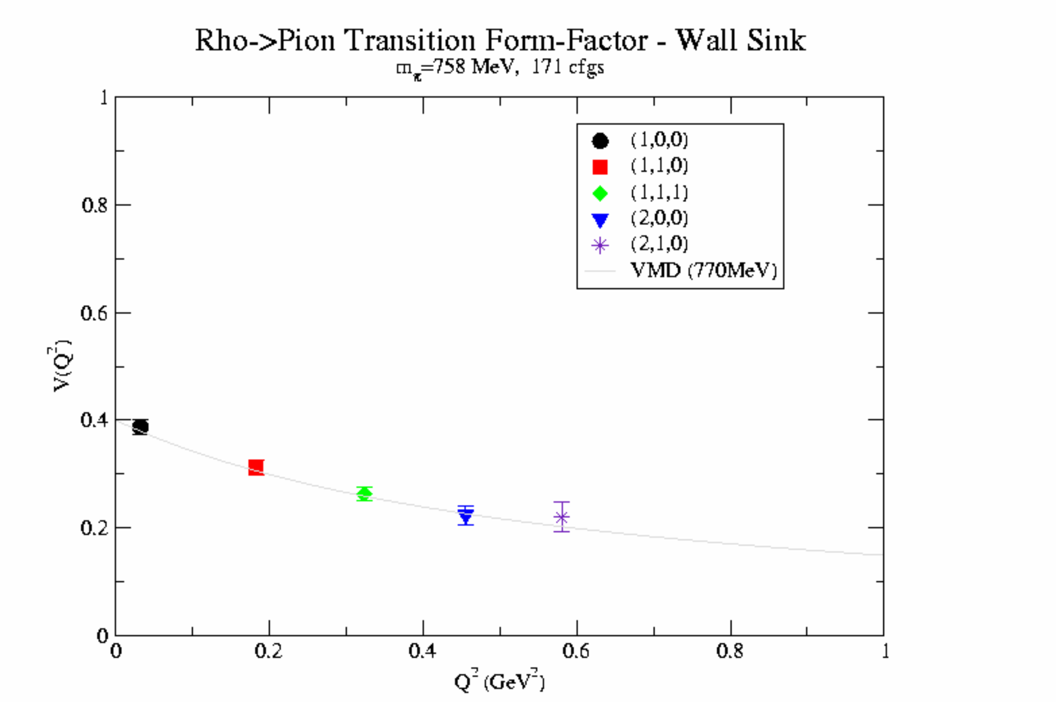


Rho!Pion Transition Form-Factor

- Electro-disintegration of deuteron intensively studied
  - Isovector exchange currents identified
  - Isoscalar exchange currents not clear
- h $\pi(p_f)|J_\mu|\rho_k(p_i)$i ! $V(Q^2)$

- Extracted coupling close to expt
- Need higher Q^2 to discern VMD





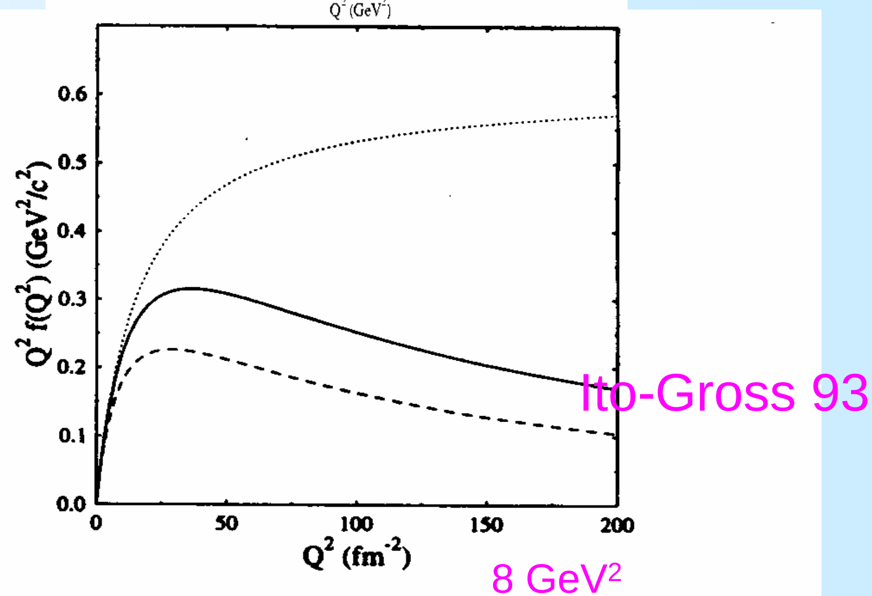
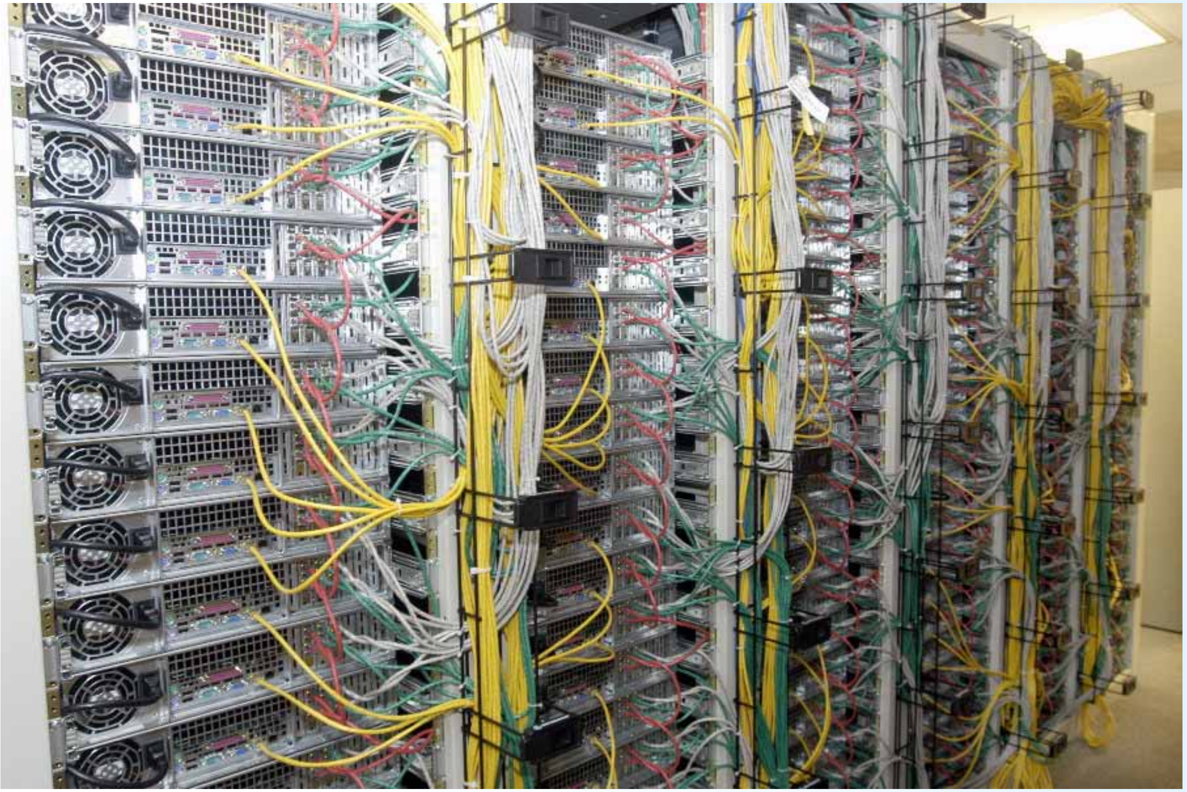
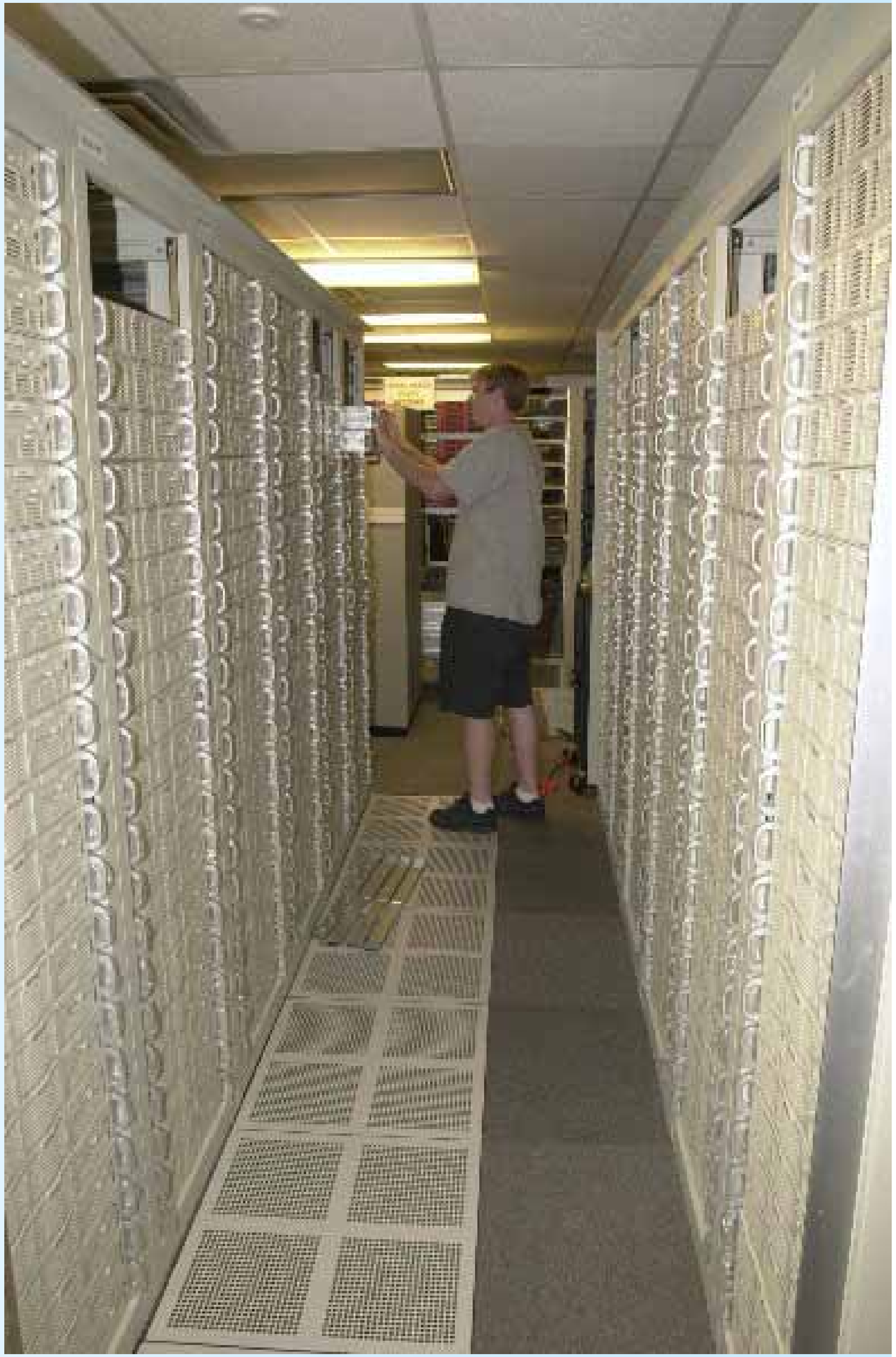


FIG. 3. The QL $\rho\pi\gamma$ (solid line) and $\omega\sigma\gamma$ (dashed line) form factors multiplied by Q^2. The VDM form factor (dotted line) is shown for comparison.

Current Hardware

- 128-node myrinet, 01 SciDAC, ~ 100Gflops
  - P4, 2.0Ghz, 400Mhz FSB
  - MPI and QMP
- 256-node, 3D mesh gigE, 02 SciDAC ~ 100Gflops
  - P4, 2.66Ghz, 533Mhz FSB
  - ONLY QMP – uses VIA drivers – lowers latency (15μ sec)
  - Config: 8-node (1D), 64-node (2D), 256node (3D)
- 20-node *spare* cluster, 100Mbit ethernet
  - P4, 2.66GHz: 8-node 3D gigE test cluster
- http://www.jlab.org/hpc

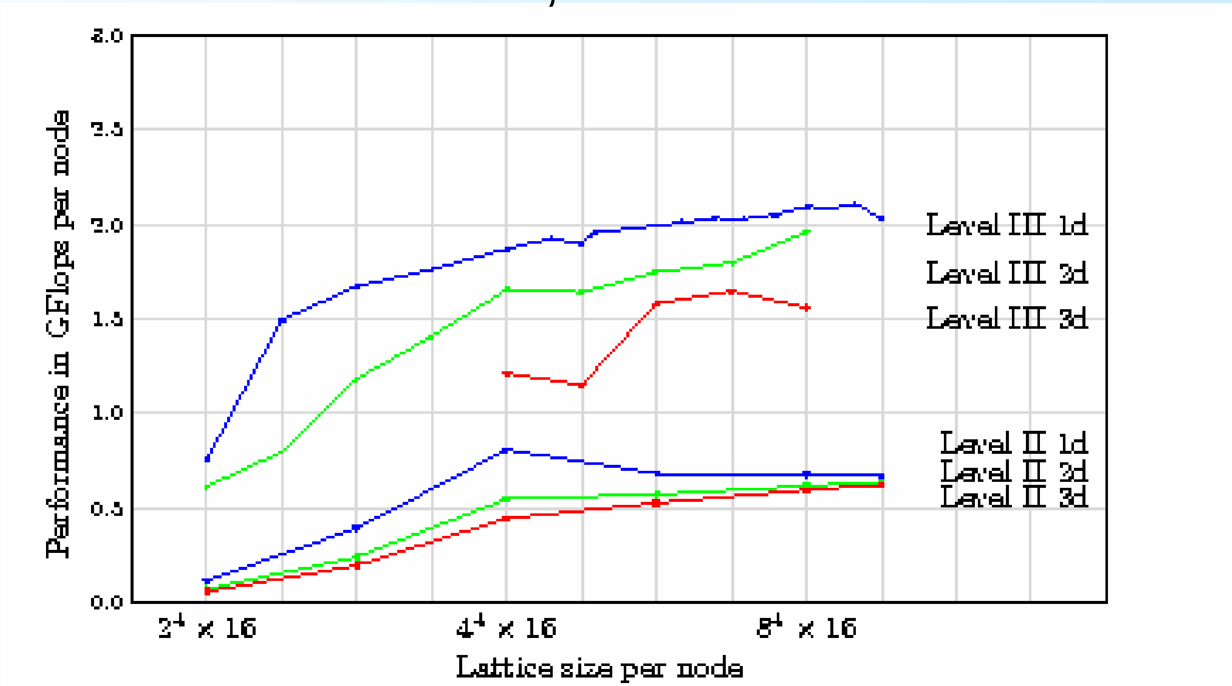




Near Term Hardware

- 512-node, 3d gigE mesh
  - P4, 3.0Ghz, 800Mhz FSB (memory bus)
  - New SciDAC DWF inverter (A. Pochinsky) – minimizes mem bus usage
  - Based on observed scaling from 533Mhz to 800Mhz mem bus, expect 1 Tflop sustained

Current 256-node 3d gigE
2.66Ghz P4, 533Mhz mem bus -> 384 Gflops



Conclusions

- Quenched Wilson $F_\pi(Q^2)$ decreases with pion mass, but undershoots.
  - Understood from O(a) scaling violations
- Partially quenched DWF $F_\pi(Q^2)$ also decreases with pion mass. Lowest mass agrees well with experiment up to 2 GeV^2
- Where is perturbative QCD?
- Have nearly completed a $p_f=(1,1,0)$ for the lightest mass. Will get higher in Q^2.
- Still need partial quenching control
- In future might use a=0.09fm Asqtad lattices – big boost in Q^2. Similarly for proton form-factor.