Soft modes around QCD critical points

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Nonperturbative Aspects of QCD at Finite Temperature and Density
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Conjectured QCD phase diagram

- Di-quark fluctuation effects
- Chiral fluctuation effects
- Critical point $T_c$
- $2T_c$
- QGP
- Hadron
- Di-quark fluctuation effects
- CSC
Contents

1. Introduction
2. QCD critical points; alternatives of QCD phase diagram with vector int. and charge neutrality?
3. Density and energy fluctuations around QCD CP
4. Summary and concluding remarks
Phase diagram in NJL model

\[ m_{q0} = 0 \]

\[ m_{q0} = 5.5 \text{ MeV} \]

Asakawa, Yazaki, (1989)
Caution!
Effects of $G_V$ on Chiral Restoration

As $G_V$ is increased,

- Chiral restoration is shifted to higher densities.
- The phase transition is weakened.

Asakawa, Yazaki ’89 / Klimt, Luts, & Weise ’90 / Buballa, Oertel ’96

What would happen when the CSC joins the game?
Importance of Vector-type Interaction

- Vector interaction naturally appears in the effective theories.
  - Instanton-anti-instanton molecule model \( G.\text{Shaefer, Shuryak (‘98)} \)
    \[
    L = G \left\{ \frac{2}{N_C^2} \left[ (\bar{\psi} \tau^a \psi)^2 + (\bar{\psi} \tau^a i \gamma_5 \psi)^2 \right] - \frac{1}{2N_C^2} \left[ (\bar{\psi} \tau^a \gamma^\mu \psi)^2 + (\bar{\psi} \tau^a \gamma^\mu \gamma_5 \psi)^2 \right] \right\} + L_8
    \]
  - Renormalization-group analysis \( G.\text{N.Evans et al. (‘99)} \)
    \[
    L_{LL}^0 = G_{\parallel} \left\{ (\bar{\psi}_L \gamma^0 \psi_L)^2 - (\bar{\psi}_L \gamma^\mu \psi_L)^2 \right\}
    \]
    \[
    -G_V \left( \bar{\psi} \gamma^\mu \psi \right)^2 \overset{\leftrightarrow}{\text{density-density correlation}}
    \]
    \[
    -G_V \left( \bar{\psi} \gamma^0 \psi \right)^2 \rightarrow -G_V \left< \bar{\psi} \gamma^0 \psi \right>^2 = -G_V \rho^2 \quad \rho = \left< \bar{\psi} \gamma^0 \psi \right>
    \]

Chiral restoration is punished by the vector interaction!
Another end point appears from lower temperature, and hence there can exist two end points in some range of $G_V$!
Intuitive understanding of the effect of vector interaction on chiral restoration

Kitazawa, Koide, Kunihiro & Nemoto (’02)

Contour maps of thermal potential

The possible large density leading to CSC is `blamed’ by the vector interaction.

\[ \rho_F = \sqrt{\mu^2 - M^2} \]
Contour of $\omega$ with $G_V/G_S=0.35$

M. Kitazawa, et al ('02)

Very shallow or soft for creating diquark-chiral condensation!
Asymmetric homogenous CSC with charge neutrality

\[ n_d > n_u > n_s \]

Mismatch cooper paring

\[ \delta \mu, \delta m \rightarrow \delta p_F \]

Standard BSC paring, rare case

Mismatch paring or pair breaking, real case

For two flavor asymmetric homogenous CSC

\[ \delta \mu = \frac{\mu e}{2} \]

- Abnormal thermal behavior of diquark gap
- Chromomagnetic instability, imaginary meissner mass
Abnormal thermal behavior of diquark energy gap

Smearing by $T$ induces the pairing!

Double effects of $T$:

- Melting the condensate
- More and more components take part in cooper pairing

Competition between these two effects gives rise to abnormal thermal behavior of diquark condensate

Enhancing the competition between chiral condensate and diquark condensate for somewhat larger $T$, leading to a nontrivial impact on chiral phase transition

Shovkovy and Huang, PLB 564, (2003) 205
Effects of Charge neutrality on QCD phase diagram


- QCD phase diagram with chiral and CSC transitions with charge neutrality
- Pairing with mismatched Fermi surface
- Competition between chiral and CSC
- Charge neutrality play a role similar to the vector-vector(density-density) interaction and leads to proliferation of critical points.
Combined effect of Vector Interaction and Charge Neutrality constraint


\[ \mathcal{L} = \bar{\psi} (i \gamma^\mu \partial_\mu - \hat{m}) \psi + G_S \left[ (\bar{q}(x)q(x))^2 + (\bar{q}(x)i \gamma_5 \tau q(x))^2 \right] \]

\[ + G_D \sum_A [\bar{q}(x)\gamma_5 \tau_2 \lambda_A q_A(x)] [\bar{q}_A(x)\gamma_5 \tau_2 \lambda_A q(x)] \]

\[ - G_V \sum_{i=0}^{3} \left[ (\bar{q}(x)\gamma^\mu \tau_i q(x))^2 + (\bar{q}(x)i \gamma^\mu \gamma_5 \tau_i q(x))^2 \right] \]

\[ - K \left\{ \det_f [\bar{\psi}(1 + \gamma_5) \psi] + \det_f [\bar{\psi}(1 - \gamma_5) \psi] \right\} \]

Fierts tr.  

for 2+1 flavors

Kobayashi-Maskawa ('70); 't Hooft ('76)

diquark-chiral density coupl.

Kobayashi-Maskawa ('70); 't Hooft ('76)
Model set 2: $M(p=0) = 367.5$ MeV, $G_d/G_s = 0.75$

2-flavor case

Z. Zhang and T. K., PRD80 (2009)

Increasing $G_v/G_s$

4 critical points!

4 types of critical point structure

Order of critical-point number: 1, 2, 4, 2, 0
2+1 flavor case

\[ m_{u,d} = 5.5 \text{MeV} \quad m_s = 140 \text{MeV} \]

Similar to the two-flavor case, with multiple critical points.
Possible cause of the disappearance of QCD critical point at low density, suggested by a recent lattice stimulation?

Philippe de Forcrand and Owe Philipesen ('08)

\[
\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1}^{\infty} c_k \left( \frac{\mu}{\pi T_c} \right)^{2k}
\]

Fukushima, ('08), based on PNJL
3. Dynamical Critical Phenomena

Y. Minami and T.K., Prog. Theor. Phys. 122, (2009), 881
Chiral Transition and the collective modes

The low mass sigma in vacuum is now established:

- pi-pi scattering; Colangero, Gasser, Leutwyler ('06) and many others
- Full lattice QCD; SCALAR collaboration ('03)
- q-qbar, tetra quark, glue balls, or their mixed st’s?

c.f. The sigma as the Higgs particle in QCD

\[ \sigma = \sigma_0 + \tilde{\sigma} \]

\[ \phi \Rightarrow \langle \phi \rangle + \tilde{\phi} \]
Fluctuations of chiral order parameter around $T_c$ in Lattice QCD

\[ \chi_m = \frac{\partial}{\partial m} \langle \bar{\psi} \psi \rangle = \langle (\bar{\psi} \psi)^2 \rangle \]

Cf. Lattice Calculation of the generalized masses
F. Karsch, Lect. Note Phys. 583 (2002), 209. $N_f = 2, 8^3 \times 4$; Staggered fermion

the softening of the $\sigma$ with increasing $T$
The spectral function of the degenerate "para-pion" and the "para-sigma" at $T>T_c$ for the chiral transition: $T_c=164$ MeV

$$D_\sigma(x) = -2G_s \langle [\bar{\psi}\psi(x), \bar{\psi}\psi(0)] \rangle$$

$$= \langle \mathcal{O}_1 \rangle + \langle \mathcal{O}_2 \rangle + \cdots$$

$$\rho_\sigma(p, \omega) = -\frac{1}{\pi} \text{Im} D_\sigma(p, \omega)$$

(NJL model cal.)

Cf. C. De Tar ('85)
The same universality class; Z2

Fluctuations of conserved quantities such as the number and energy are the soft mode of QCD critical point! The sigma mode is a slaving mode of the density.

Dam. T. Son and M. A. Stephanov, PRD70 (’04) 056001
Sigma meson has still a non-zero mass at CP.
This is because the chiral symmetry is explicitly broken.

**What is the soft mode at CP?**

At finite density, scalar-vector mixing is present.

**Phonon mode in the space-like region softens at CP.**


See also, D. T. Son and M. Stephanov (2004)

does not affect particle creation in the time-like region.

It couples to hydrodynamical modes,
leading to interesting dynamical critical phenomena.

Spectral function of the chiral condensate

T-dependence ($\mu=\mu_{CP}$)

P=40 MeV

$\omega < p$

Space-like region $\omega < p$

(non-soft mode)

(3D) Space-like region (the soft modes)
Spectral function of conserved quantities

The density/energy fluctuation depends on the transport as well as thermodynamic quantities which show an anomalous behavior around the critical point.

For non-relativistic case with use of Navier-Stokes eq.

L.D. Landau and G. Placzek (1934),
L. P. Kadanoff and P. C. Martin (1963),
R. D. Mountain, Rev. Mod. Phys. 38 (1966), 38
H. E. Stanley, ‘Intro. To Phase transitions and critical phenomena’ (Clarendon, 1971)

We apply for the first time relativistic hydrodynamic equations to analyze the spectral properties of density and energy fluctuations, and examine possible critical phenomena.

Notice: The 1st-order can be valid for describing the hydrodynamic modes with a long wave-length without encountering the causality problem.
Linear approximation around the thermal equilibrium;

\[ u^\mu(\vec{r}, t) = u^\mu_0 + \delta u^\mu(\vec{r}, t) \quad n(\vec{r}, t) = n_0 + \delta n(\vec{r}, t) \quad \text{etc} \]

In the rest frame of the fluid,

\[ u^\mu_0 = (1, 0) \quad \delta u^\mu(\vec{r}, t) = (0, \vec{v}(\vec{r}, t)) \]

Inserting them into \( T^{\mu\nu}, N^\mu \), and taking the linear approx.

• Linearized Landau equation (Lin. Hydro in the energy frame);

\[
\frac{\partial \delta n}{\partial t} + n_0 \nabla \cdot \vec{v} - \kappa \frac{n_0}{w_0} \frac{T_0}{w_0} \nabla^2 (\delta P) - \nabla^2 (\delta T) = 0
\]

\[
w_0 \frac{\partial \vec{v}}{\partial t} - \eta \nabla^2 \vec{v} - \left( \frac{1}{3} \eta + \zeta \right) \nabla (\nabla \cdot \vec{v}) + \nabla (\delta P) = 0
\]

\[
n_0 \frac{\partial \delta s}{\partial t} - \frac{\kappa}{T_0} \nabla^2 (\delta T) + \frac{\kappa}{w_0} \nabla^2 (\delta P) = 0
\]

with

\[
\delta P(x) = \frac{w_0 c_s^2}{n_0 \gamma} \delta n(x) + \frac{w_0 c_s^2 \alpha P}{\gamma} \delta T(x)
\]

\[
\delta s(x) = -\frac{w_0 c_s^2 \alpha P}{n_0^2 \gamma} \delta n(x) + \frac{\dot{c}_n}{T_0} \delta T(x)
\]

Solving \( \ddot{\mathbf{x}} \) as an initial value problem using Laplace transformation, we obtain

\[ S_{nn}(\vec{k}, \omega) = \langle \delta n(\vec{k}, \omega) \delta n(\vec{k}, t = 0) \rangle \]

in terms of the initial correlation.
Spectral function of density fluctuations

\[
\frac{S_{nn}(\vec{k}, \omega)}{\langle \delta n(\vec{k}, t = 0) \rangle^2} = (1 - \frac{1}{\gamma}) \frac{2 \Gamma_R k^2}{\omega^2 + \Gamma_R^2 k^4} + \frac{1}{\gamma} \left( \frac{\Gamma_B k^2}{(\omega + c_s k)^2 + \Gamma_B^2 k^4} + \frac{\Gamma_B k^2}{(\omega - c_s k)^2 + \Gamma_B^2 k^4} \right)
\]

Rel. effects appear only in the width of the peaks.

\[
\Gamma_R = \chi \quad \Gamma_B = \Gamma + \frac{1}{2} c_s T_0 (\kappa / w_0 - 2 \chi \alpha_p)
\]

\[
\Gamma = \frac{1}{2} [\chi (\gamma - 1) + \nu_l]
\]

\[
\alpha_p \quad \text{rate of isothermal exp.}
\]

\[
\nu_l = (\zeta + \frac{4}{3} \eta) / w_0
\]

\[\gamma = c_p / c_n = t^{\gamma + \tilde{\alpha}} \rightarrow \infty\]

Notice: As approaching the critical point, the ratio of specific heats diverges! The strength of the sound modes vanishes out at the critical point.
Critical behavior of the density-fluctuation spectral functions

So, the divergence of and the viscosities therein can not be observed, unfortunately.

In the vicinity of CP, only the Rayleigh peak stays out, while the sound modes (Brillouin peaks) die out.

c.f. The dynamical critical exponent.

\[ z = 2 + (\gamma - a) / \nu \approx 3 \]

So, the divergence of \( \Gamma_B \) and the viscosities therein can not be observed, unfortunately.
Spectral function of density fluctuation at CP

The sound mode (Brillouin) disappears. Only an enhanced thermal mode remains. Furthermore, the Rayleigh peak is enhanced, meaning the large energy dissipation. Suggesting interesting critical phenomena related to sound mode.

The soft mode around QCD CP is thermally induced density fluctuations, but not the usual sound mode.

$S_{\delta n}(k,\omega) / \langle \delta n(kt=0)^2 \rangle$

$t = 0.1$

$S_{\delta n}(k,\omega) / \langle \delta n(kt=0)^2 \rangle$

$t = (T - T_c) / T_c = 0.4$

Y. Minami and T.K., (2009)
Why at all do sound modes die out at the Critical Point?

The correlation length
\[ \xi = \xi_0 t^{-\nu} \]

The wave length of sound mode
\[ \lambda_s \]

The hydrodynamic regime!
\[ \xi \ll \lambda_s \]

However, around the critical point
\[ \xi \to \infty \quad \text{as} \quad t \to 0 \]

\[ \lambda_s \ll \xi \]

The system behaves as an aggregate of clumps of matter with a diameter \( \xi \).

So the hydrodynamic sound modes can not be developed around CP!

\[ C_P = n^2 \int d^3 r \langle \delta s(r, t = 0) \delta s(0, 0) \rangle \]

Ornstein-Zernike;
\[ \langle \delta s(r, 0) \delta (0, 0) \rangle \sim \frac{e^{-r/\xi}}{r} \]

\[ C_P \propto \xi^2 \quad \leftrightarrow \quad \xi^{2-\eta} \quad \eta \sim 0.03 \]
Possible disappearance or strong suppression of Mach cone at the QCD critical point

Mach cone developed from sound mode as the density fluctuation

However, Around the CP:

(i) Attenuation of the sound mode; the dynamical density fluctuation is hardly developed.

(ii) The enhancement of the Rayleigh peak suggests that the energy dissipation is so large that the possible density fluctuation gets dissipated rapidly.

Possible disappearance or strong suppression of Mach cone at the QCD critical point!

Thus, if the identification of the Mach cone in the RHIC experiment is confirmed, possible disappearance or suppression along with the variation of the incident energy can be a signal of the existence of the critical point belonging to the same universality class as liq.-gas transition.
Cf. STAR, arXiv:0805/0622. 3-body correlations
Cf. the idea of Mach cone: E. Stoecker, E. Shuryak and many Others.

R. B. Neufeld, B. Muller, and J. Ruppert, arXiv:0802.2254
Comments

1. Mach cone physics with the critical dynamics incorporated is a new field which has not been explored so much.

- Critical Opalescence is a general phenomenon for the matter with 1st order transition.
There are still a room of other structure of the QCD phase diagram with multiple critical points when the color superconductivity and the vector interaction are incorporated. It seems that the QCD matter is very soft along the critical line when the color superconductivity is incorporated; there can be a good chance to see large fluctuations of various observables like diquark-density mixed fluctuations,

\[ aq_c q + bq q + c q^\dagger q. \]

Future problem: An explicit incorporation of the diquark-chiral density coupling due to the anomaly term, together with the chargeneutrality and The vector interaction.
Summary and concl. remarks (continued)

- The dynamical density fluctuations have been analyzed using rel. hydro,
  in which the entropy fluctuations are automatically incorporated.
- The sound modes due to density fluctuations are attenuated, and the Rayleigh peak due to the entropy fluctuation in turn gets enhanced around the QCD critical point.
  - The attenuation of the sound mode may lead to the suppression or even total disappearance of Mach cone at the CP.
  - If the identification of the Mach cone in the RHIC experiment is confirmed, possible disappearance or strong suppression along with the variation of the initial energy can be a signal of the existence of the critical point and the created matter went through the critical region of the CP.
  - Need further explicit calculation for confirmation.
Future problems

• Application of the rel. Langevin + dynamical RG theory a la Kawasaki, Onuki and others; Y. Minami and T.K. in preparation.

• Density fluctuations around the expanding background.

• Hydro. → Dual Magneto-hydro.? (Shuryak)
Back UPs
The vector interaction tends to suppress the mismatch of the Fermi spheres of the Cooper pairs.

Suppression of the Chromomagnetic instability!
Suppression of the Chromomagnetic instability due to the vector interaction!


(Partial) resolution of the chromomagnetic instability problem!
The coupling of the density fluctuation with the scalar mode and the importance of the vector interaction were discussed in, T.K. Phys. Lett. B271 (1991), 395:

1. The quark number susceptibility or density-density correlator $\langle q\gamma^0 q \rangle \langle q\gamma^0 q \rangle$ is coupled to the scalar susceptibility $\langle \bar{q}q \rangle \langle \bar{q}q \rangle$ at $\mu \neq 0$

Because $\langle \bar{q}q \rangle \neq 0$ for $\mu \neq 0$ Charge conjugation symm. Is violated.

2. The density-density correlator is a part of the vector correlator $\langle \bar{q}\gamma^\mu q\bar{q}\gamma^\nu q \rangle$ and hence has something to do with the vector interaction. $g_\gamma$


$\frac{\partial n(E_k)}{\partial \mu}$ involves $\frac{\partial M}{\partial \mu}$, which is divergent at the first order chiral transition because $M$ changes to $m$ with a gap.
Possible \((T, \mu)\) dependence of \(g_{V}\)

\[\kappa_T = \frac{\chi_q}{\rho^2}\]
\[\kappa_T \equiv -N_q^{-1}(\partial V/\partial \mu_q)_{T,N_q}\]

isothermal compressibility

\[\chi_q = \frac{\chi_q^{(0)}(T)}{1 + 2g_V\chi_q^{(0)}(T)}\]

\(g_{V}\) should decrease as \(T\) is raised.

\[\mu \neq 0\]

The vector-scalar mixing Arises:

\[\chi_q = \frac{D_s^{-1}(0)\chi_q^{(0)} + 2g_s(\chi_V^{(0)})^2}{D_s^{-1}(0)D_v^{-1}(0) + 4g_vg_s\chi_V^{(0)}\chi_q^{(0)}}\]

The diverging behavior of the density fluctuation around the CP ;
Relativistic Hydrodynamics

\[
\begin{align*}
\partial_{\mu} T^{\mu \nu} &= 0 \\
\partial_{\mu} N^{\mu} &= 0
\end{align*}
\]

\[T^{\mu \nu} = (\epsilon + P)u^\mu u^\nu - Pg^{\mu \nu} + \tau^{\mu \nu}\]

\[N^{\mu} = nu^{\mu} + \nu^{\mu}\]

\[\tau^{\mu \nu}, \nu^{\mu}\text{ dissipative terms}\]

(1) Energy-frame

\[\tau^{\mu \nu} = \eta \left[ \partial_{\perp}^{\mu} u^{\nu} + \partial_{\perp}^{\nu} u^{\mu} - \frac{2}{3} \Delta^{\mu \nu} (\partial_{\perp} \cdot u) \right] + \zeta \Delta^{\mu \nu} (\partial_{\perp} \cdot u)\]

\[\nu^{\mu} = \kappa \left( \frac{nT}{w} \right)^2 \partial_{\perp}^{\mu} \left( \frac{\mu}{T} \right)\]

\[u_{\mu} \nu^{\mu} = 0 \quad u_{\mu} \tau^{\mu \nu} = 0\]

(2) Particle frame; Eckart(1940), unstable

\[u_{\mu} \tau^{\mu \nu} u_{\nu} = 0\]


\[\nu^{\mu} = 0\]

\[\tau^{\mu \nu} = \eta \left[ \partial_{\perp}^{\mu} u^{\nu} + \partial_{\perp}^{\nu} u^{\mu} - \frac{2}{3} \Delta^{\mu \nu} (\partial_{\perp} \cdot u) \right] - \zeta' \left( 3u^{\mu} u^{\nu} - \Delta^{\mu \nu} \right)(\partial_{\perp} \cdot u) + \kappa (u^{\mu} \partial_{\perp}^{\nu} + u^{\nu} \partial_{\perp}^{\mu})T\]

\[\tau^{\mu}_{\mu} = 0\]

(3) Israel-Stewart
Particle frame; the new equation

\[
\frac{S_{nn}(\vec{k}, \omega)}{< (\delta n(\vec{k}, t = 0))^2 >} = \left(1 - \frac{1}{\gamma} \right) \frac{2 \chi k^2}{\omega^2 + \chi k^4} + \frac{1}{\gamma} \left( \frac{\Gamma_B k^2}{(\omega + c_s k)^2 + \Gamma_B^2 k^4} + \frac{\Gamma_B k^2}{(\omega - c_s k)^2 + \Gamma_B^2 k^4} \right)
\]

\[
\Gamma_B = \Gamma - \frac{\alpha P c_s^2}{n_0 \gamma} (\kappa T_0 + 3 \zeta')
\]

Rel. effects appear in the Brillouin peaks (sound mode) but not in The Rayleigh peak.


“Non-rel.’’
Spectral function from I-S eq.

\[ S_{nn}(\vec{k}, \omega) / \langle (\delta n(\vec{k}, t = 0))^2 \rangle \]

For \( \tau_\kappa > \frac{\kappa T_0}{w_0} \)

\[ \frac{2 \chi k^2}{\omega^2 + \chi^2 k^4} + \frac{w_0}{(\omega + c_s k)^2 + \Gamma_B k^2} + \frac{w_0}{(\omega - c_s k)^2 + \Gamma_B k^2} \]

\[ + O(k^2) \times \left[ \frac{1}{\omega^2 + (1/\tau_\zeta)^2} + \frac{1}{\omega^2 + (1/\tau_\eta)^2} + \frac{w_0/(\tau_\kappa w_0 - \kappa T_0)}{\omega^2 + [w_0/(\tau_\kappa w_0 - \kappa T_0)]^2} \right] \]

No contribution in the long-wave length limit \( k \to 0 \).

Conversely speaking, the first-order hydro. Equations have no problem to describe the hydrodynamic modes with long wave length, as it should.
How about the anomaly-induced new critical point?

a la Hatsuda-Tachibana-Yamamoto-Baym (2006)

(A) In flavor-symmetric limit; Abuki et al, PRD81 (2010)
(A’) Role of 2SC in 3-flavor quark matter

H. Basler and M. Buballa, arXiv:1007.5198

with equal bare quark masses of $(m_u = m_d = m_s = 5.5 \text{ MeV})$.
(B) Realistic case with massive strange quark;

\[ m_u = m_d = 5.5 \text{ MeV} \quad \ll \quad m_s = 140.7 \text{ MeV} \]

H. Basler and M. Buballa, arXiv:1007.5198

Notice!
Without charge neutrality nor vector interaction.

phases and new endpoints. On the other hand, the low-temperature critical endpoint, which was found earlier in the same model without 2SC pairing, is almost removed from the phase diagram and cannot be reached from the low-density chirally broken phase without crossing a preceding first-order phase boundary. For physical quark masses no additional critical endpoint is found.
4. Diquark fluctuations and pseudogap in the quark spectral function in hot and dense quark matter
As $T$ is lowered toward $T_C$, the peak of $\rho$ becomes sharp. (Soft mode) Pole behavior

The peak survives up to $\epsilon \sim 0.2$ electric SC $\epsilon \sim 0.005$
The mechanism of the pseudogap in High-TcSC is still controversial, but see, Y. Yanase et al, Phys. Rep. 387 (2003),1, where the essential role of pair fluc. is shown.

Anomalous depression of the density of states near the Fermi surface in the normal phase.

A typical non-Fermi liq. behavior!

The mechanism of the pseudogap in High-TcSC is still controversial, but see, Y. Yanase et al, Phys. Rep. 387 (2003),1, where the essential role of pair fluc. is shown.
T-matrix Approximation for Quark Propagator

\[
G(k, \omega_n) = \frac{1}{G^0(k, i\omega_n) - \Sigma(k, i\omega_n)}
\]

\[
G^0(k, i\omega_n) = \left[ (i\omega_n + \mu)\gamma^0 - k \cdot \vec{\gamma} \right]^{-1}
\]

\[
\Sigma(k, \omega_n) = \sum \text{diagram} = \sum_{\text{diagram}} + \sum_{\text{diagram}} + \cdots
\]

\[
\equiv T \sum \int \frac{d^3q}{(2\pi)^3} \Xi(k + q, \omega_n + \omega_m) G^0(q, \omega_m)
\]

Di-quark soft mode

Density of States \( N(\omega) \):

\[
N(\omega) = \int \frac{d^3k}{(2\pi)^3} \rho^0(k, \omega) \quad \rho^0(k, \omega) = \frac{1}{4} \text{Tr} \left[ \gamma^0 \text{Im} G^R(k, \omega) \right]
\]

\[
N = \int d^3x \langle \bar{\psi} \gamma^0 \psi \rangle
\]
Possible pseudogap formation in heated quark matter

\[ \mu = 400 \text{ MeV} \]

M. Kitazawa, T. Koide, T. K. and Y. Nemoto
Phys. Rev. D70, 956003(2004);
Prog. Theor. Phys. 114, 205(2005),

Pseudogap is formed above \( T_c \) of CSC in heated quark matter!

How?

\[ N(\omega) = \int \frac{d^3k}{(2\pi)^3} \rho(k, \omega) \]

\[ \varepsilon = \frac{T - T_c}{T_c} \]

\( \mu \) = 400 MeV
5. Chiral soft mode and anomalous quark spectral function
Digression: The poles of the S matrix in the complex mass plane for the sigma meson channel:


See also, I. Caprini, G. Colangelo and H. Leutwyler, PRL(2006);
H. Leutwyler, hep-ph/0608218; M_sigma = 441 – i 272 MeV
The spectral function of the degenerate hadronic ``para-pion'' and the ``para-sigma'' at $T>T_c$ for the chiral transition: $T_c=164$ MeV


- response function in RPA

$$D(k, \omega) = \langle \psi \phi \rangle + \langle \psi\langle \phi \rangle \langle \phi \rangle \psi \rangle + \langle \psi\langle \phi \rangle \langle \phi \rangle \langle \phi \rangle \psi \rangle + \ldots$$

- spectral function

$$A(k\omega) = -\frac{1}{\pi} \text{Im} D(k\omega)$$

$T \rightarrow T_c$, they become elementary modes with small width!

- sharp peak in time-like region

1. two γ decay
2. modified quark spectrum

M. Kitazawa, Y. Nemoto and T.K. (05)
Quarks coupled to chiral soft modes near $T_c$

We incorporate the fluctuation mode into a single particle Green function of a quark through a self-energy.

$$G(\omega_n, p) = \frac{1}{G_0(\omega_n, p) - \Sigma(\omega_n, p)} = + \quad \Sigma \quad + \quad \Sigma \quad \Sigma \quad + \cdots$$

Non self-consistent $T$-approximation (1-loop of the fluctuation mode)

$$\Sigma(\omega_n, p) = \Sigma \quad + \quad \Sigma \quad + \quad \Sigma \quad + \cdots$$

$$\equiv \quad k + q, i\omega_n + i\omega_m \quad = T \sum_m \int \frac{d^3q}{(2\pi)^3} D(\omega_n + \omega_m, p + q) G_0(\omega_m, q)$$

N.B. This is a complicated multiple integral owing to the compositeness of the para-sigma and para-pion modes.
Spectral Function of Quark

\[ \rho(p^0, p) = \rho_+(p^0, p) \Lambda_+ \gamma^0 + \rho_-(p^0, p) \Lambda_- \gamma^0 \]

\[ \epsilon = 0.05 \]

- Three-peak structure emerges.
- The peak around the origin is the sharpest.

Quasi-dispersion relation for eye-guide;

\[ \Sigma^\pm(p_0, p) = \Sigma^0(p_0, p) \mp \Sigma^0(p_0, p) \]

\[ \text{Re} \left[ S_+(\omega, p) \right]_1 = \omega - |p| - \text{Re} \Sigma_+(\omega, p) = 0 \]

Quark Spectrum in Yukawa models

Yukawa model with scalar/pseudoscalar or vector/axial vector boson.

\[ \Sigma(i\omega_n, \mathbf{p}) = \] (at one-loop)

Massie scalar/pseudoscalar boson

Massie vector/axial vector boson

The 3-peak structure emerges irrespective of the type of the boson at \( T \sim m_B \).

Remark: Bosonic excitations in QGP may include \( \sigma, \pi, \rho, J/\psi, \ldots \) / glue balls… How about the case of Vector manifestation (B-R, H-Y-S)
If a QCD phase transition is of a second order or close to that, there should exist specific soft modes, which may be easily thermally excited.

In the fermion-boson system with $m_F \ll m_B$, the fermion spectral function has a 3-peak structure at 1-loop approximation at $T \sim m_B$.

If the chiral transition is close to a second order, quarks may have a 3-peak structure in the QGP phase near $T_c$.

Any Yukawa model;
Kitazawa, Nemoto and T.K., Prog. Theor. Phys. 117, 103 (2007),
D. Sato, Y. Hidaka and T.K., Poster