The QCD phase diagram for small values of the chemical potential

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OUTLINE:

- QCD phase diagram close to the chiral limit
- QCD close to the chiral limit, O(N) scaling
- Curvature of the critical line in the chiral limit
Phase diagram for $\mu_B \geq 0$, $m_q \geq 0$

- critical line at $m_q=0$
- physics on crossover line controlled by universal scaling relations?
- Is it related to the experimentally determined freeze-out curve?

Phase diagram for $\mu_B \geq 0$, $m_q = 0$

- critical line at $m_q=0$

$$\frac{T_c(\mu_q)}{T_c} = 1 - \kappa_q \left(\frac{\mu_q}{T}\right)^2 - \mathcal{O}(\mu_q^4)$$

- physics on crossover line controlled by universal scaling relations?

- Is it related to the experimentally determined freeze-out curve?

\textbf{Phase diagram for } \mu_B \geq 0, \ m_q > 0

\begin{itemize}
  \item critical line at \(m_q=0\)
  \[ \frac{T_c(\mu_q)}{T_c} = 1 - \kappa_q \left(\frac{\mu_q}{T}\right)^2 - \mathcal{O}(\mu_q^4) \]
  \item physics on crossover line controlled by universal scaling relations?
  \item Is it related to the experimentally determined freeze-out curve?
\end{itemize}

Phase diagram for $\mu_B \geq 0, \, m_q > 0$

Critical line at $m_q=0$

$$\frac{T_c(\mu_q)}{T_c} = 1 - \kappa_q \left( \frac{\mu_q}{T} \right)^2 - \mathcal{O}(\mu_q^4)$$

Physics on crossover line controlled by universal scaling relations?

Is it related to the experimentally determined freeze-out curve?

The RHIC low energy runs

Moments of charge fluctuations

charge fluctuations at freeze-out agree with HRG model predictions

freeze-out line and chiral phase transition

BNL-Bielefeld-GSI, in preparation
Critical behavior in hot and dense matter

QCD phase diagram & chiral limit

already $\mu_B = 0$
is not fully explored

possible

conventional

not excluded
Phase diagram for $\mu_B = 0$

Is physics at the physical quark mass point sensitive to (universal) properties of the chiral phase transition?

$\mathbf{n_f = 3}$: $m_s^{crit} \lesssim 70$ MeV

first order region starts below physical pion mass value
O(N) scaling and chiral transition

Thermodynamics in the vicinity of a critical point:

\[ \frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V, T) = h^{1+1/\beta} f_s(t/h^{1/\beta}) + f_r(V, T) \]

Critical behavior controlled by two relevant fields: \( t \), \( h \)

All couplings that do not explicitly break chiral symmetry contribute in leading order only to 't'

\[ t = \frac{1}{t_0} \left( \left( \frac{T}{T_c} - 1 \right) - \kappa_q \left( \frac{\mu_q}{T} \right)^2 - \kappa_s \left( \frac{\mu_s}{T} \right)^2 - \kappa_{qs} \left( \frac{\mu_q \mu_s}{T} \right)^2 \right) \]

\[ h = \frac{1}{h_0} \frac{m_l}{m_s} \]

3 unique scales at vanishing chemical potential
3 additional, unique scales at non-vanishing chemical potential: curvature of the critical surface
In the vicinity of \((t,h) = (0,0)\) the chiral order parameter and its susceptibility are given in terms of scaling functions

\[
M = h^{1/\delta} f_G(z), \quad \chi_M = \partial M / \partial h = h^{1/\delta - 1} f_\chi(z)
\]

\[
\chi_t = \partial M / \partial T = \frac{1}{t_0 T_c} h^{(\beta - 1)/\delta \beta} f'(z)
\]

\[
f_\chi(z) = \frac{1}{\delta} \left( f_G(z) - \frac{z}{\beta} f'_G(z) \right)
\]

known from 3d O(N) spin model

J. Engels et al., 2001/2003
Scaling analysis in (2+1)-flavor QCD

RBC-Bielefeld-GSI arXiv:0909.5122; hotQCD in preparation

QCD with 2 light and a “physical” strange quark mass:

- improved staggered fermions; most detailed: p4-action
  extended to asqtad and hisq

- calculations have been performed on $N^3_\sigma \times N_\tau$ lattices

\[ N_\sigma = 32, 48, N_\tau = 4, 6, 8 \]

- calculations with p4-action cover a wide quark mass range:

\[ N_\tau = 4 : \quad 1/80 \leq m_l/m_s \leq 2/5 \]

\[ \Rightarrow \quad 75 \text{ MeV} \leq m_\pi \leq 320 \text{ MeV} \]

- evidence for O(N) scaling
  expect O(2) rather than O(4) scaling with staggered fermions at non-zero lattice spacing
Goldstone modes at finite T

In the symmetry broken phase **Goldstone modes** give the dominant contribution to the scaling functions and lead to a **divergent susceptibility** in the chiral limit.

\[ T < T_c \iff z < 0 : \]

\[ \langle \bar{\psi} \psi \rangle \sim f_G(z) : \quad f_G^\infty(z) = (-z)^\beta (1 + \tilde{c}_2 \beta (-z)^{-\beta \delta/2}) \]

\[ \Rightarrow \quad M \sim (-t)^\beta + \tilde{c}_2 \beta |t|^{\beta(1-\delta/2)} h^{1/2} \]

\[ \chi_m \sim f_\chi(z) : \quad (-z)^{\beta(1-\delta/2)} \]

\[ \Rightarrow \quad \chi_M \sim \frac{1}{2} \tilde{c}_2 \beta |t|^{\beta(1-\delta/2)} h^{-1/2} \]
Goldstone modes dominate quark mass dependence of the chiral order parameter for \( T \lesssim T_c \)

analog of chiral logs at \( T=0 \)

\[
<\bar{\psi}\psi>_I
\]

\[
\frac{m}{m_s}
\]

\[
0.96<T/T_c<1.06
\]

\[
(p4\text{-action}) \quad N_\sigma^3 \times 4, \quad N_\sigma = 8 - 32
\]

Disconnected chiral susceptibility
(2+1)-flavor QCD

\[ \chi_{m,l} = \frac{\partial \langle \bar{\psi} \psi \rangle_l}{\partial m_l} = \chi_{l, disc} + 2\chi_{l, con} \]

\[ \frac{\chi_{l, disc}}{T^2} \]

asqtad-action: \(48^3 \times 8\)  hotQCD preliminary
Magnetic Equation of State
(2+1)-flavor QCD

chiral order parameter:

\[ m_b \equiv \frac{m_s \langle \overline{\psi}\psi \rangle}{T^4} \quad z \equiv \frac{t}{h^{1/\beta \delta}} \]

O(2) vs. O(4)

\[ z \rightarrow 1.2z \]

\[ \beta = 3.285, 3.29, 3.295, 3.30, 3.3025, 3.305, 3.3075, 3.31 \]

\[ \frac{m}{m_s} \leq 1/20 \quad |T/T_c - 1| < 0.03 \]

\[ \frac{t}{h^{1/\beta \delta}} \]

\[ \frac{M_b}{h^{1/\delta}} \]

\[ \frac{m}{m_s} = 2/5 \quad \frac{1}{5} \quad \frac{1}{10} \quad \frac{1}{20} \quad \frac{1}{40} \quad \frac{1}{80} \]

\[ t/h^{1/\beta \delta} \]

\[ t/h^{1/\beta \delta} \]

\[ N_\sigma^3 \times 4 \quad N_\sigma = 16, 32 \]

Magnetic Equation of State
\((2+1)-\text{flavor QCD}\)

scaling violations:

\[
M_b = h^{1/\delta} f_G(z) + \frac{m_l}{m_s} (a_t (T - T_c) + b_1) + \ldots
\]

\(p4\)-action: \(N_\sigma^3 \times 4\), \(N_\sigma = 16, 32\)

Scaling analysis in (2+1)-flavor QCD

$N_T = 8$

$M_b \equiv \frac{m_s \langle \bar{\psi} \psi \rangle}{T^4}$

$z \equiv \frac{t}{h^{1/\beta \delta}}$

$\Rightarrow T_c, t_0, h_0 \rightarrow z_0 = h_0^{1/\beta \delta} / t_0$

asqtad

hotQCD preliminary
Scaling analysis in (2+1)-flavor QCD

$O(2)$ vs $O(4)$ fits

$N_T = 8$

$M_b \equiv \frac{m_s \langle \bar{\psi} \psi \rangle}{T^4}$

$z \equiv \frac{t}{h^{1/\beta \delta}}$

$\beta_c = 3.4986(15)$

$h_0 = 0.00053(10)$

$t_0 = 0.00286(23)$

$z_0 = 3.8(5)$

$b_1 = 3.6(4)$

$a_1 = -11.2(17)$

$\beta_c = 3.4949(15)$

$h_0 = 0.00034(7)$

$t_0 = 0.00378(29)$

$z_0 = 3.4(4)$

$b_1 = 2.6(5)$

$a_1 = -6.4(17)$

$N_t=8$: $m_l/m_s$

1/5

1/10

1/20

$O(2) f_G(z)$

$\chi^2 = 11.19$

$O(4) f_G(z)$

$\chi^2 = 10.18$

$\Rightarrow T_c, t_0, h_0 \rightarrow z_0 = h_0^{1/\beta \delta}/t_0$

chiral phase transition controls $T_c(m_l/m_s)$

W. Unger, PhD thesis, Bielefeld, September 2010
The pseudo-critical line

\[ \Rightarrow T_c, \ t_0, \ h_0 \rightarrow z_0 = h_0^{1/\beta \delta} / t_0 \]

controls \( T_c (m_l / m_s) \)

chiral phase transition

- pseudo-critical line defined by location of maximum in \( f_\chi(z) \)

\[ z = z_p \quad \Rightarrow \quad t = z_p h^{1/\beta \delta} \]

\[ \frac{T_{pc}(m_l / m_s)}{T_c} = 1 - \frac{z_p}{z_0} \left( \frac{m_l}{m_s} \right)^{1/\beta \delta} + \text{scaling violating terms} \]

hotQCD preliminary
The curvature of the critical line

QCD, chiral limit (u,d quarks only)

\[ \mu_u = \mu_d > 0, \quad \mu_Q = \mu_s = 0 \]

\[ \frac{T}{T_c} = 1 - \kappa_q \left( \frac{\mu_q}{T} \right)^2 \]

\[ t \equiv \frac{1}{t_0} \left( \left( \frac{T}{T_c} - 1 \right) - \kappa_q \left( \frac{\mu_q}{T} \right)^2 \right) \]

scaling laws control curvature of chiral transition line for small \( \mu_q/T \)
"thermal" fluctuations of the order parameter

\[ t \equiv \frac{1}{t_0} \left( \left( \frac{T}{T_c} - 1 \right) - \kappa_q \left( \frac{\mu_q}{T} \right)^2 \right), \quad z = t/h^{1/\beta \delta} \]

\[ M_b \equiv \frac{m_s \langle \overline{\psi} \psi \rangle}{T^4} = h^{1/\delta} f_G(z) \text{ fixes } T_c, t_0, h_0 \]

\[ \chi_t = \frac{\partial^2 M_b}{\partial (\mu_q/T)^2} = \frac{m_s \partial \langle \overline{\psi} \psi \rangle / T^3}{T \partial (\mu_q/T)^2} = \frac{2 \kappa_q}{t_0 T_c} h^{(\beta-1)/\delta \beta} f'(z) \]
"thermal" fluctuations of the order parameter

\[ \chi_t = \partial^2 M_b / \partial (\mu_q / T)^2 = \frac{2\kappa_q}{t_0 T_c} h^{(\beta-1)} / \delta\beta \ f'(z) \]

analysis for 2 values of the cut-off and 4 different quark masses

\[ \kappa_q = 0.059 \pm 0.006 \]

compare to freeze-out curve:
Chiral Transition and Freeze-out

chiral phase transition curve:

\[
\frac{T(\mu_B)}{T_c} = 1 - 0.0066(7) \left( \frac{\mu_B}{T} \right)^2 + O(\mu_B^4)
\]

freeze-out curve in heavy ion collisions:

\[
\frac{T(\mu_B)}{T_c} = 1 - 0.023 \left( \frac{\mu_B}{T} \right)^2 - c \left( \frac{\mu_B}{T} \right)^4
\]

\[
\mu_B(\sqrt{s_{NN}}) = \frac{d}{1 + e \sqrt{s_{NN}}}
\]


open issues:
- continuum limit
- strangeness conservation
- non zero charge

F. Karsch, Tsukuba workshop, 2010
Chiral Transition and Freeze-out

Chiral phase transition curve at non-zero strangeness chemical potential:

$$\frac{T(\mu_B)}{T_c} = 1 - \kappa_B \left( \frac{\mu_B}{T} \right)^2 - \kappa_S \left( \frac{\mu_S}{T} \right)^2 - \kappa_{BS} \frac{\mu_B \mu_S}{T^2}$$

Freeze-out curve in heavy ion collisions:

$$\frac{T(\mu_B)}{T_c} = 1 - 0.023 \left( \frac{\mu_B}{T} \right)^2 - c \left( \frac{\mu_B}{T} \right)^4$$

$$\mu_B(\sqrt{s_{NN}}) = \frac{d}{1 + e \sqrt{s_{NN}}}$$

Conclusions

QCD with physical quark masses is sensitive to universal scaling properties in the chiral limit.

The curvature of the chiral phase transition line can be determined from O(N) scaling functions;

Current results suggest that the curvature of the chiral transition line is smaller than that of the freeze-out curve.

The freeze-out curve will be even further away from the crossover in QCD with physical quark masses.

An estimate:

\[ \frac{\mu_q}{T} = 1 : \ T_{cross} - T_{freeze} \approx 0.15T_c \]
Disconnected chiral susceptibility
(2+1)-flavor QCD

\[ \langle \bar{\psi} \psi \rangle_l = \frac{T}{V} \frac{\partial \ln Z}{\partial m_l} = \chi_{l,\text{disc}} + 2\chi_{l,\text{con}} \]

\[ \chi_{l,\text{disc}} = \begin{cases} 0.0125, N_\sigma = 32 \\ 0.025, N_\sigma = 32 \\ 0.025, N_\sigma = 16 \\ 0.05, N_\sigma = 16 \\ 0.1, N_\sigma = 16 \\ 0.2, N_\sigma = 16 \\ 0.4, N_\sigma = 8 \end{cases} \]

p4-action: \( N^3_\sigma \times 4 \), \( N_\sigma = 8 \rightarrow 32 \)

chiral susceptibilities:

\[ \chi_M = \partial M / \partial h = h^{1/\delta-1} f_\chi(z) \quad z \equiv t / h^{1/\beta\delta} \]

p4-action: \( N^3_\sigma \times 4 \), \( N_\sigma = 16, 32 \)