B Physics with NRQCD Heavy and AsqTad Light Quarks

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Good progress in recent years in Lattice simulations of heavy-light systems.

- More and more unquenched simulations using for instance the $N_f = 2 + 1$ MILC configurations

- Use of AsqTad improved staggered light quarks has led to considerable reduction in chiral extrapolation uncertainties.

Outline

- Formalism
- Perturbative Matching and Dim. 4 Current Corrections
- Decay Constants
- Semi-leptonic Form Factors
- Future Plans

Formalism

Symmetries of staggered or naive fermions are well documented in the literature, especially within the context of light quark physics.

The situation is simpler for heavy-light systems, if the heavy-quark action has no doublers as in NRQCD, or only heavy doublers as with Fermilab heavy quarks.

The free naive fermion action (unimproved for simplicity) is given by,

$$\mathcal{S}_0 = a^4 \sum_x \left\{ \overline{\Psi}(x) \left[\sum_\mu \gamma_\mu \frac{1}{a} \nabla_\mu \; + \; m \right] \Psi(x) \right\},$$

with

$$\nabla_\mu f(x) = \frac{1}{2} [f(x + a_\mu) - f(x - a_\mu)].$$

The naive action has a set of 16 discrete "doubling symmetries",

$$\begin{aligned}\Psi(x) &\rightarrow e^{ix\cdot\pi_g} M_g \Psi(x) \\ \overline{\Psi}(x) &\rightarrow e^{ix\cdot\pi_g} \overline{\Psi}(x) M_g^\dagger.\end{aligned}$$

"g" is an element of the set, G, of ordered sets of indices.

$$G = \{g : g = (\mu_1, \mu_2, ...),\ \mu_1 < \mu_2 < ...\},$$

and π_g is the 4-vector,

$$(\pi_g)_\mu = \begin{cases} \frac{\pi}{a} & \mu \in g, \\ \\ 0 & \text{otherwise.} \end{cases}$$

The M_g are transformation matrices,

$$M_g = M_{\mu_1} M_{\mu_2} ...., \qquad \mu_i \in g,$$

with

$$M_\mu = i\gamma_5\gamma_\mu.$$

Momentum Space Naive Fermions

$$\mathcal{S}_0 = \int_{k,D} \overline{\psi}(k) \left[\sum_\mu i\gamma_\mu \frac{1}{a} sin(k_\mu a) \; + \; m \right] \psi(k)$$

Using the 4-vectors π_g this can be written as,

$$\mathcal{S}_0 \;=\; \sum_g \int_{k,D_\emptyset} \overline{\psi}(k+\pi_g) \left[\sum_\mu i\gamma_\mu \frac{1}{a} sin([k+\pi_g]_\mu a) \right.$$

$$\left. + \; m \right] \psi(k+\pi_g)$$

D denotes the full Brillouin zone, $-\frac{\pi}{a} \le k_\mu < \frac{\pi}{a}$, and $D_\emptyset$ just the central region, $-\frac{\pi}{2a} \le k_\mu < \frac{\pi}{2a}$.

The next step is to define 16 new momentum space spinors $q^g(k)$ labeled by the elements g of the set G.

$$q^g(k) = M_g \psi(k+\pi_g), \;\; \overline{q}^g(k) = \overline{\psi}(k+\pi_g) M_g^\dagger.$$

Momentum Space Naive Fermions (cont'd)

Using,

$$M_g \gamma_\mu M_g^\dagger \, sin([k + \pi_g]_\mu a) = \gamma_\mu \, sin(k_\mu a),$$

the action $\mathcal{S}_0$ becomes

$$\mathcal{S}_0 = \sum_g \int_{k,D_\emptyset} \overline{q}^g(k) \left[\sum_\mu i\gamma_\mu \frac{1}{a} sin(k_\mu a) \; + \; m \right] q^g(k).$$

Heavy-Light Bilinears

Since there are 16 light tastes and one heavy flavor one has the possibility of forming 16 different B mesons labeled by the light taste index g, i.e. B_g. The obvious choice for an interpolating heavy-light operator has the general form

$$\mathcal{W}_{B_g}(x) = \overline{\Psi}_H(x)\gamma_5 M_g e^{i\pi_g \cdot x}\Psi(x)$$

The 16 B_g mesons are degenerate and do not mix. No information is lost by working with just one of them, e.g. with $g = \emptyset$.

Momentum Space Heavy-Light Bilinears

$$\sum_{\vec{x}} \mathcal{W}_B(\vec{x},t) = \sum_{g_s \in G_s} \int_{\vec{k},D_{s,\emptyset}} \int_{-\pi/2a}^{\pi/2a} \frac{dk_0}{2\pi} e^{ik_0 t}$$
$$\Big\{ \overline{\tilde{\psi}}_H(\vec{k}+\vec{\pi}_{g_s},t)\,\gamma_5 \Big[M^\dagger_{g_s} q^{g_s}(\vec{k},k_0)$$
$$+(-1)^t M^\dagger_{g_t g_s} q^{g_t g_s}(\vec{k},k_0) \Big] \Big\}$$

Use fact that $\overline{\tilde{\psi}}_H(\vec{k}+\vec{\pi}_{g_s},t)$, for $\vec{\pi}_{g_s} \neq \vec{\pi}_{\emptyset}$, represents a highly energetic heavy quark.

$$\sum_{\vec{x}} \mathcal{W}_B(\vec{x},t) \to \int_{\vec{k},D_{s,\emptyset}} \int_{-\pi/2a}^{\pi/2a} \frac{dk_0}{2\pi} e^{ik_0 t}$$
$$\Big\{ \overline{\tilde{\psi}}_H(\vec{k},t)\,\gamma_5 \Big[q(\vec{k},k_0) + (-1)^t M^\dagger_{g_t} q^{g_t}(\vec{k},k_0) \Big] \Big\}$$

$+$ highly energetic state contributions

Momentum Space Heavy-Light Bilinears (cont'd)

One can estimate the splitting between physical and lattice artifact levels.

$$\Delta E = E_{\tilde{H}} - E_H \approx \sqrt{M_b^2 + (\frac{\pi}{a})^2} - M_b$$

For the coarse MILC lattices, $a^{-1} \approx 1.6\text{GeV}$ and $\Delta E \sim 2.1\text{GeV}$.

Note: Wilson type fermions have heavy doublers with $\Delta E \sim a^{-1}$.

Note: one cannot go to $M_H \to \infty$

So,

- Effect of multiple light tastes simpler when studying heavy-light systems.

- The undoubled heavy quark picks out a unique light taste. It is sufficient to work with one of the B_g's.

- This is true in heavy-light meson decay constant (2-point), semi-leptonic form factor (3-point), and in B_B (four-fermion operator) calculations.

- Expressions for heavy-light currents and four-fermion operators are the same as in the continuum theory (no hypercubic constructions or point-splittings necessary).

Relation between Naive and Staggered Propagators

Simple but very useful relation between staggered and naive light propagators.

$$\Psi(x) = \Omega(x)\,\Phi(x), \qquad \overline{\Psi}(x) = \overline{\Phi}(x)\,\Omega(x)^{\dagger}$$

with

$$\Omega(x) = \prod_{\mu=0}^{3} (\gamma_\mu)^{x_\mu}$$

$$\mathcal{S}_0 \to \mathcal{S}_\Phi = a^4 \sum_x \left\{ \overline{\Phi}(x) \left[\sum_\mu \eta_\mu(x) \frac{1}{a} \nabla_\mu \, + \, m \right] \Phi(x) \right\}$$

$$\eta_\mu(x) = (-1)^{x_0 + \ldots x_{(\mu-1)}}.$$

So,

$$G_\Psi(x,y) = \Omega(x)\,G_\Phi(x,y)\Omega^{\dagger}(y)$$

$$G_\Phi(x,y) = \hat{I}_D\,G_\chi(x,y)$$

$$\boxed{G_\Psi(x,y) = \Omega(x)\,\Omega^{\dagger}(y)\,G_\chi(x,y)}$$

Perturbative Matching

(Emel Gulez, Matt Wingate, J.S.)

Matching has been carried out for AsqTad light, $\mathcal{O}(a^2)$ and $\mathcal{O}(1/M^2)$ improved NRQCD heavy, and Symanzik improved glue actions.

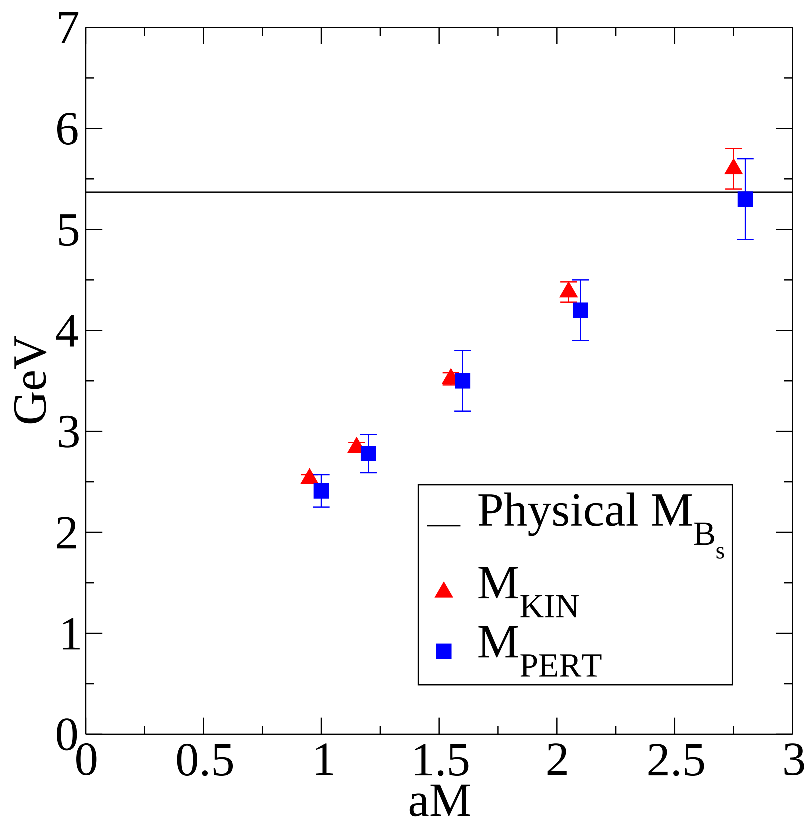
One-loop matching of V_0, A_0, V_k and A_k through $\mathcal{O}(\alpha_s)$, $\mathcal{O}(a\,\alpha_s)$, $\mathcal{O}(\alpha_s/(aM))$, and $\mathcal{O}(\alpha_s\,\Lambda_{QCD}/M)$
i.e. including all dimension 4 current corrections.

As part of the matching we reproduced H.Trottier's Z_q for massless AsqTad quarks using a gluon mass IR regulator and calculated the NRQCD heavy quark self energy (E_0, Z_m and Z_Q) at one-loop order (generalizing previous calculation by C.Morningstar to improved glue).

"Kinetic" and "Perturbative" B Mass

$$M_{pert} = Z_m M_0 - E_0 + E_{sim}(0)$$

$$M_{kin} = \frac{p^2 - \Delta E^2}{2\,\Delta E}, \qquad \Delta E \equiv E_{sim}(p) - E_{sim}(0)$$



1/M Current Corrections

For V_0, A_0 :

$$
\begin{aligned}
J_0^{(0)}(x) &= \bar{q}(x)\,\Gamma_0\,Q(x), \\
J_0^{(1)}(x) &= \frac{-1}{2M_0}\bar{q}(x)\,\Gamma_0\,\boldsymbol{\gamma}\cdot\boldsymbol{\nabla}\,Q(x), \\
J_0^{(2)}(x) &= \frac{-1}{2M_0}\bar{q}(x)\,\boldsymbol{\gamma}\cdot\overleftarrow{\boldsymbol{\nabla}}\,\gamma_0\,\Gamma_0\,Q(x).
\end{aligned}
$$

and for V_k, A_k :

$$
\begin{aligned}
J_k^{(0)}(x) &= \bar{q}(x)\,\Gamma_k\,Q(x), \\
J_k^{(1)}(x) &= \frac{-1}{2M_0}\bar{q}(x)\,\Gamma_k\,\boldsymbol{\gamma}\cdot\boldsymbol{\nabla}\,Q(x), \\
J_k^{(2)}(x) &= \frac{-1}{2M_0}\bar{q}(x)\,\boldsymbol{\gamma}\cdot\overleftarrow{\boldsymbol{\nabla}}\,\gamma_0\,\Gamma_k\,Q(x), \\
J_k^{(3)}(x) &= \frac{-1}{2M_0}\bar{q}(x)\,\nabla_k\,Q(x) \\
J_k^{(4)}(x) &= \frac{1}{2M_0}\bar{q}(x)\,\overleftarrow{\nabla}_k\,Q(x),
\end{aligned}
$$

Matching of A_0

We use :

$$\langle A_0 \rangle_{QCD} = (1 + \alpha_s \, \tilde{\rho}_0) \, \langle J_0^{(0)} \rangle + (1 + \alpha_s \, \rho_1) \, \langle J_0^{(1),sub} \rangle + \alpha_s \, \rho_2 \, \langle J_0^{(2),sub} \rangle$$

$$J^{(i),sub} = J^{(i)} - \alpha_s \, \zeta_{10} J^{(0)}$$

The second term subtracts power law contributions through $\mathcal{O}(\alpha/(aM))$.

Similar expressions for V_k involving, however, 5 currents.

Note : $Z(V_\mu) \equiv Z(A_\mu)$

$J^{(1)}$ **versus** $J^{(1),sub}$

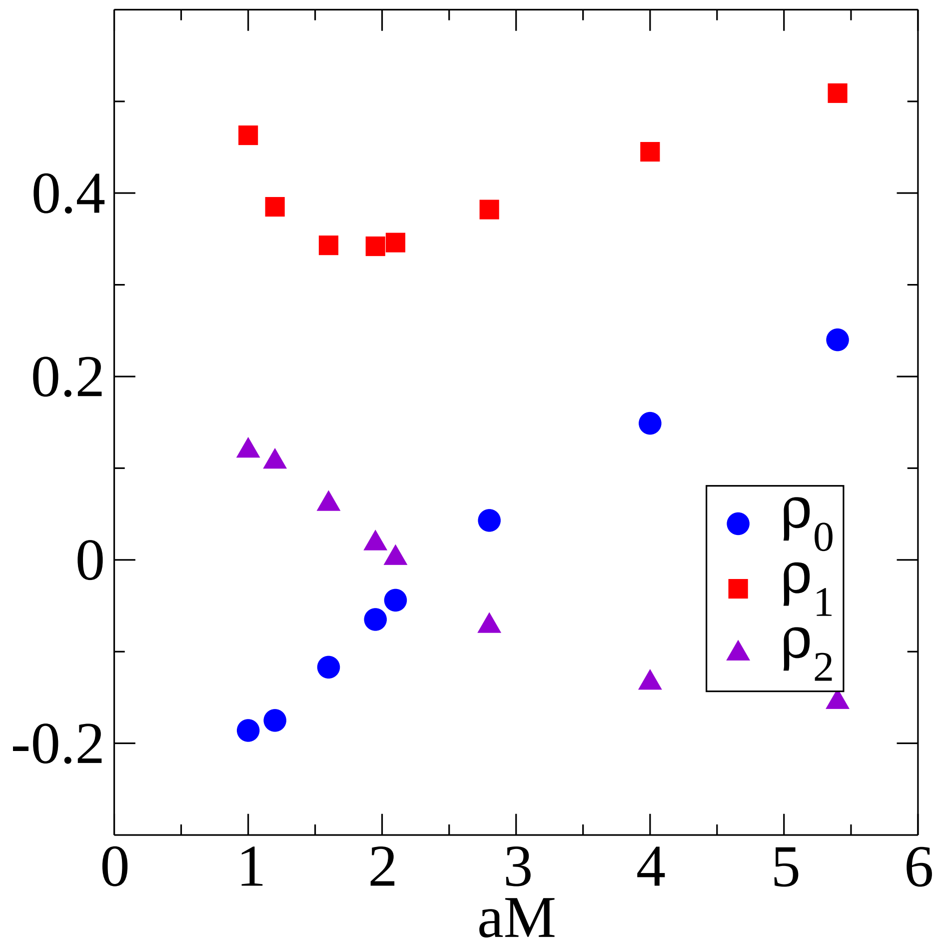
aM_0	$\lvert J^{(1)}\rvert/J^{(0)}$ [%]	$\lvert J^{(1),sub}\rvert/J^{(0)}$ [%]
2.8(B_s)	9.0(4)	3.7(4)
2.1	11.7(4)	5.0(4)
1.6	14.7(4)	6.4(4)
1.2	18.3(4)	7.8(4)
1.0	20.7(4)	8.6(4)

Quenched NRQCD/Clover results at the physical B_s

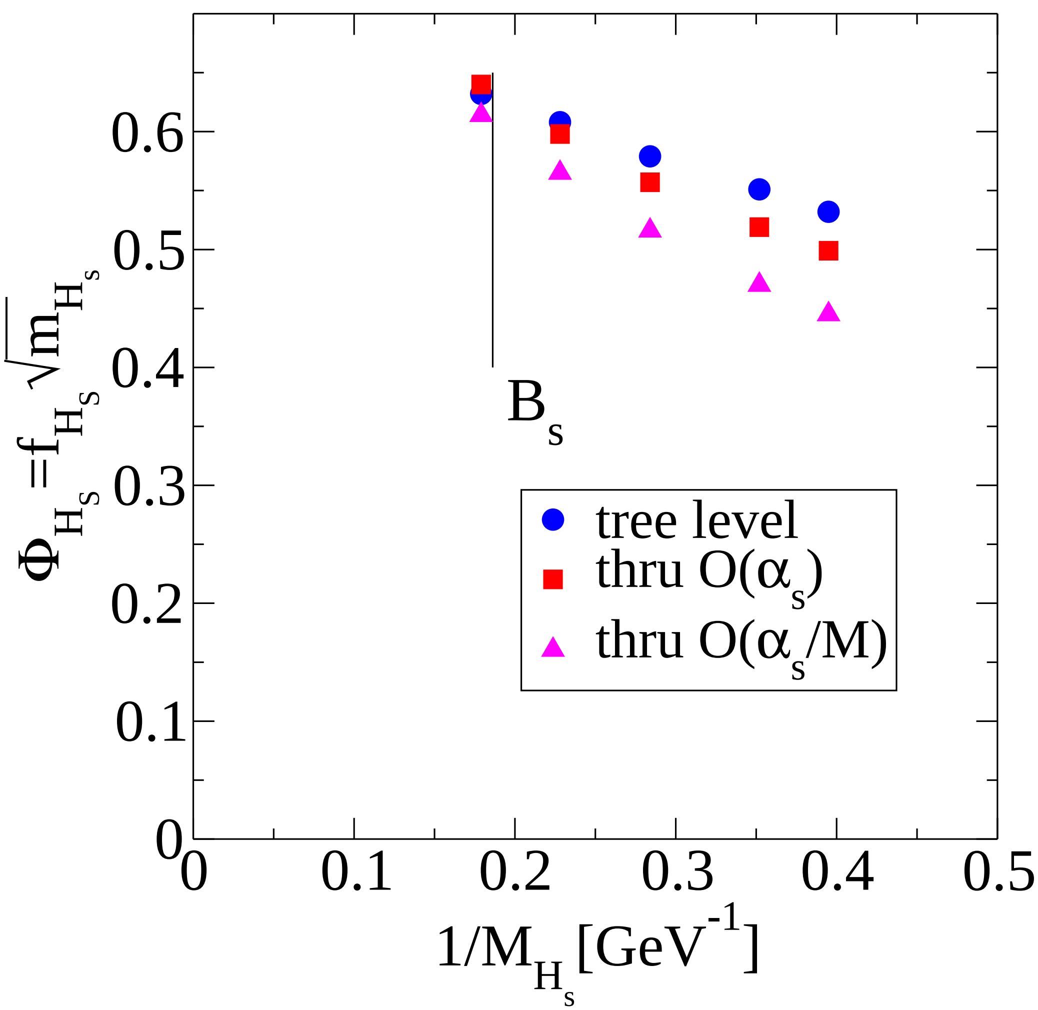
β	$\lvert J^{(1)}\rvert/J^{(0)}$ [%]	$\lvert J^{(1),sub}\rvert/J^{(0)}$ [%]
5.7	~ 8	~ 4
6.0	~ 10	~ 4
6.2	~ 13.5	~ 5

Much better scaling for $J^{(1),sub}$.

Matching Coefficients ρ_i



$\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha/M)$ Corrections to $\Phi = f_{H_s}\sqrt{M_{H_s}}$



$1/M$ current corrections to semi-leptonic form factors will be discussed later.

Heavy-Light Meson Decay Constants

(Alan Gray, Matt Wingate, et al.)

To date, all our results are from the coarser MILC configurations with $a^{-1} \sim 1.6$GeV.

aM_0	$u_0\, am_q(sea)$	$u_0\, am_q(valence)$
2.8	0.01	0.05, 0.04, 0.02, 0.01 0.005
2.1	0.01	0.04
1.9	0.01	0.04
1.6	0.01	0.04
1.2	0.01	0.04
1.0	0.01	0.04
2.8	0.02	0.04, 0.02
2.1	0.02	0.04
1.9	0.02	0.04
1.6	0.02	0.04
1.2	0.02	0.04
1.0	0.02	0.04

Results for f_{B_s} and f_{D_s}

(Matt Wingate, et al.; PRL 92, 2004)

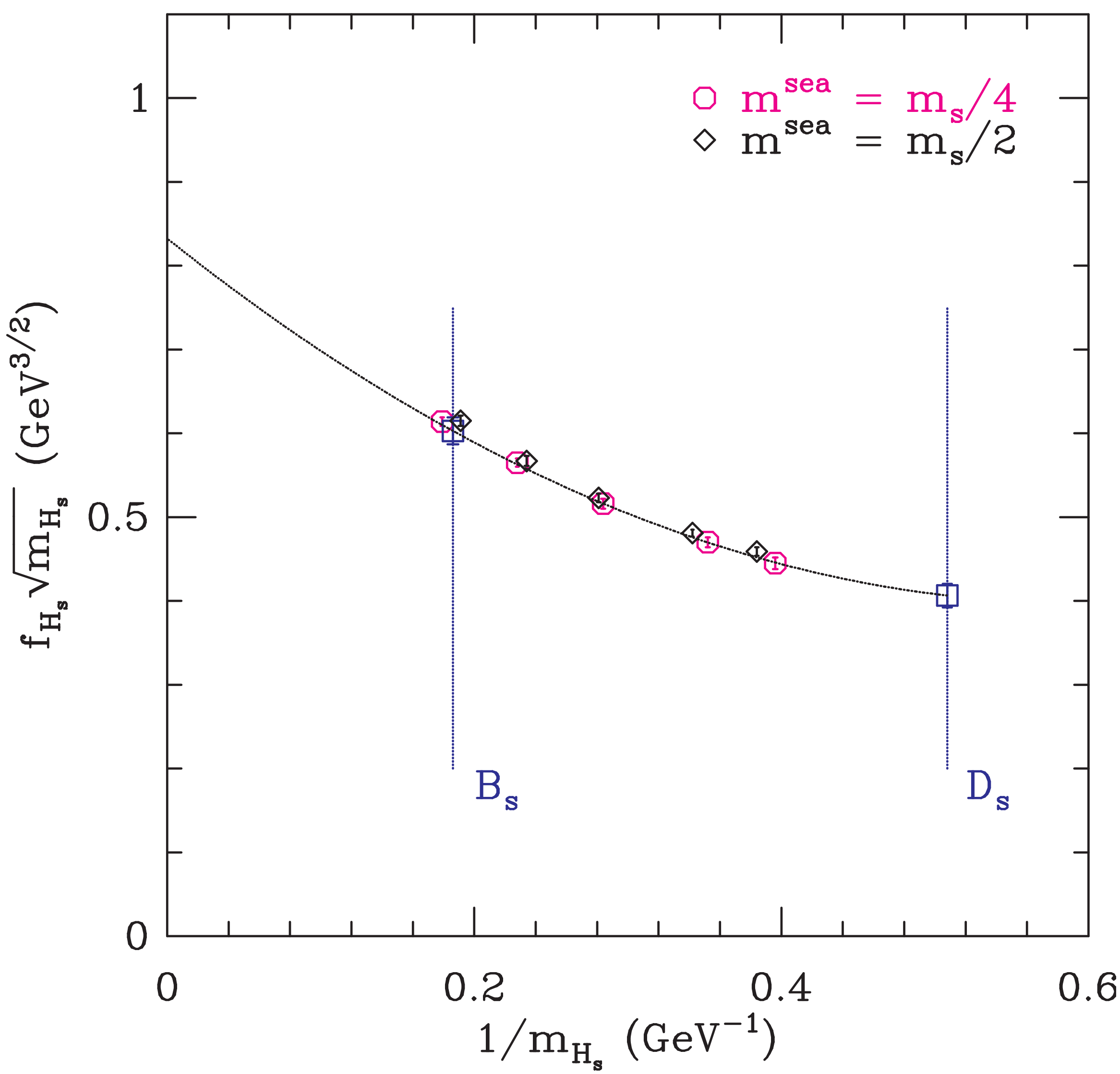
We find,

$$f_{B_s} = 260 \pm 7 \pm 26 \pm 8 \pm 5 \; MeV$$

$$f_{D_s} = 290 \pm 20 \pm 29 \pm 29 \pm 6 \; MeV$$

The dominant systematic error for f_{B_s} comes from uncertainties in higher order perturbative matching.

$f_{H_s}\sqrt{M_{H_s}}$ versus $1/M_{H_s}$



f_B and Chiral Extrapolation to Physical B

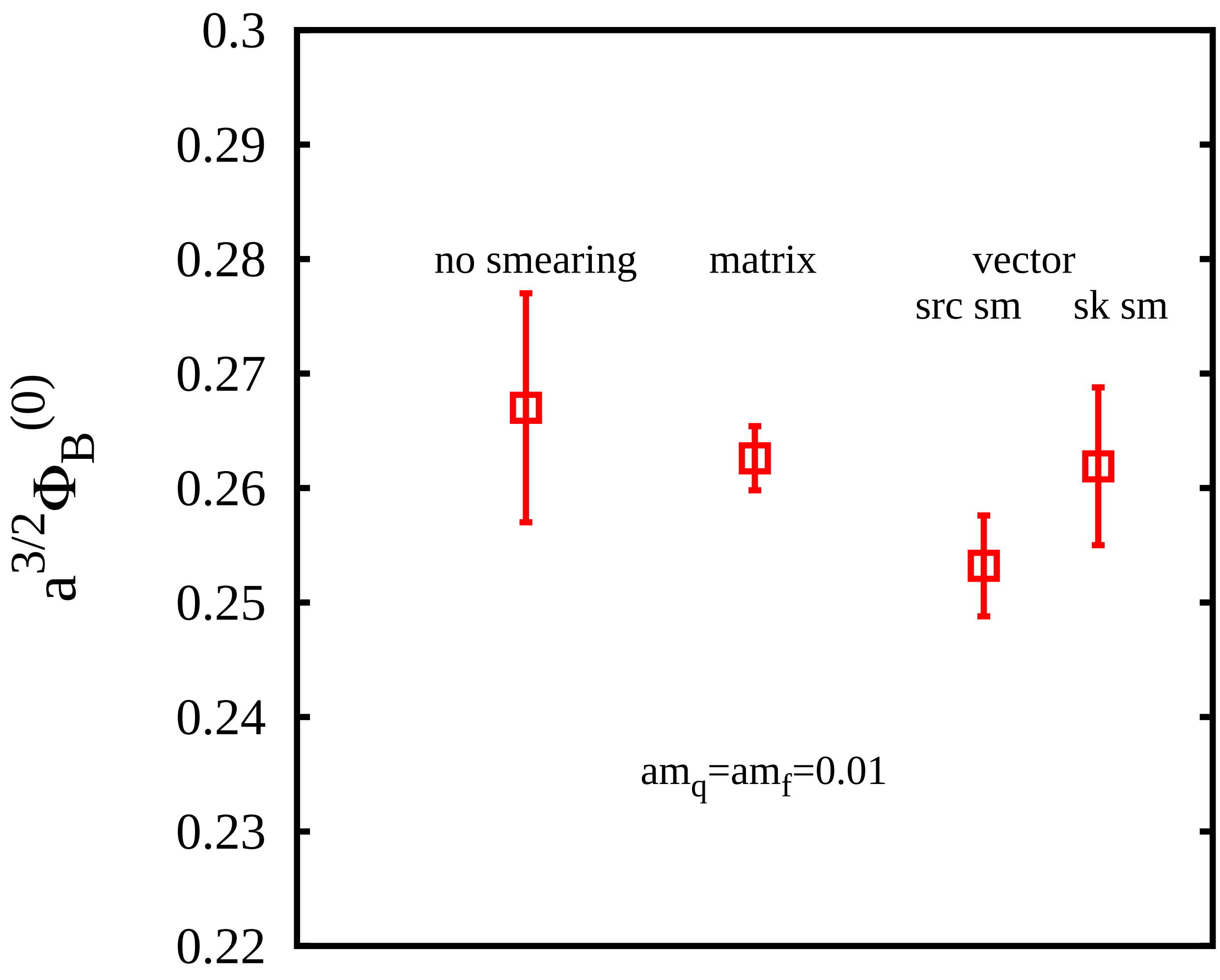
During the past year we have worked hard on reducing statistical errors in decay constant calculations, especially at lighter light quark masses.

We find that smearing the heavy quarks and employing a matrix of smeared correlators significantly reduces errors.

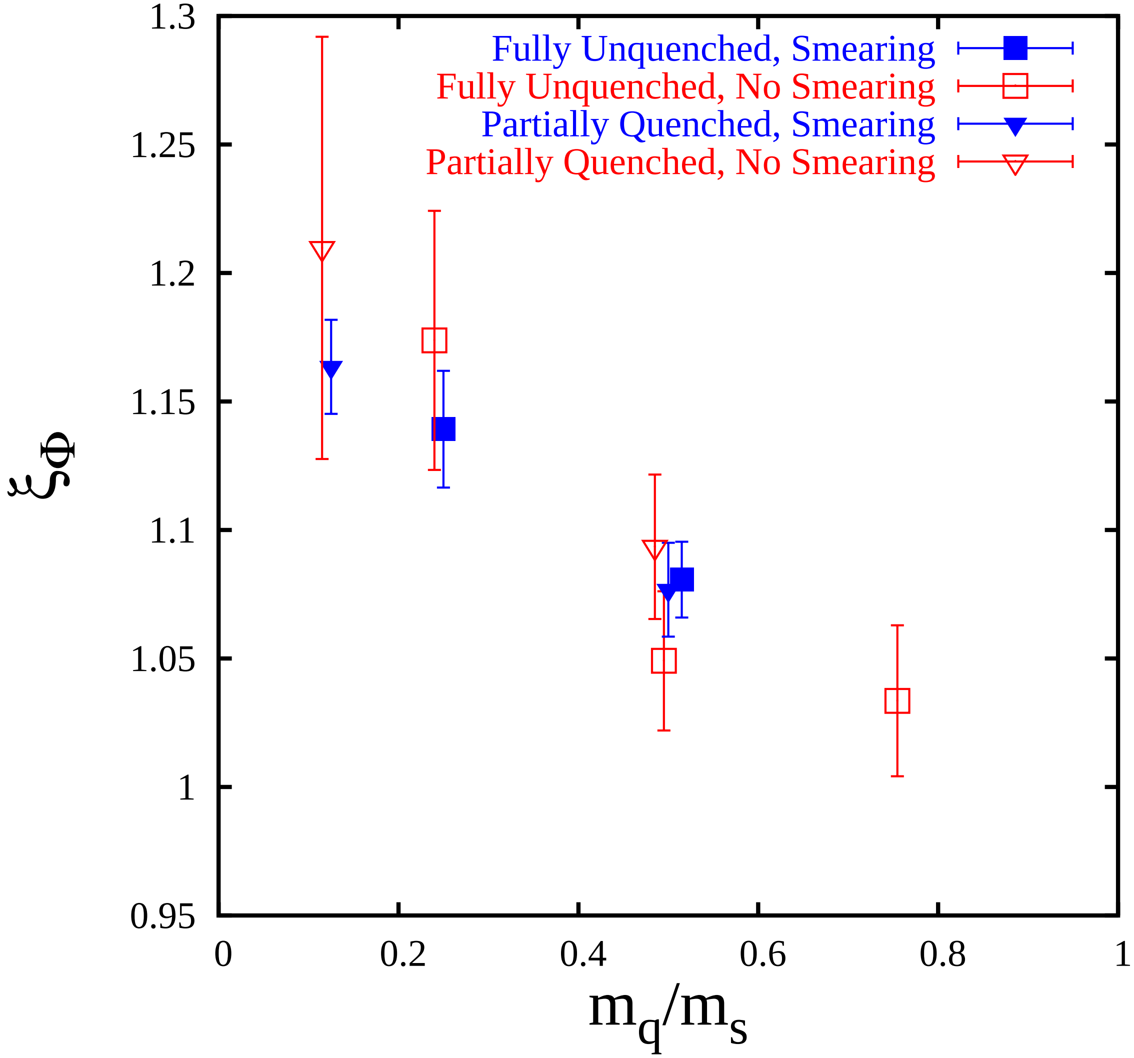
We have also started to implement the Staggered Chiral PT formulas of Aubin & Bernard for heavy-light physics.

Work is still underway to accumulate more fully unquenched data on the coarser and finer MILC lattices.

Effect of Smearing on $\Phi_B = f_B\sqrt{M_B}$

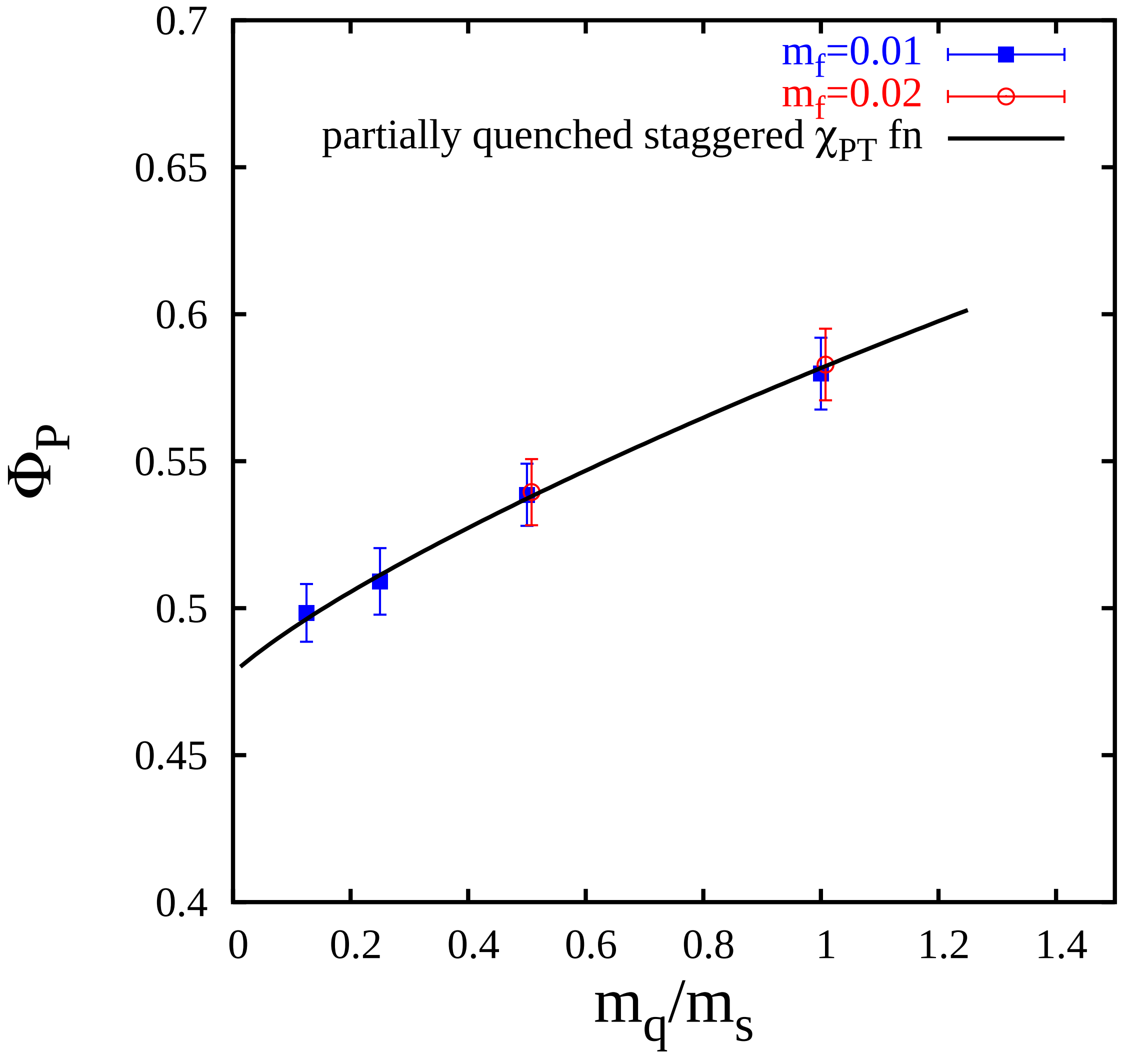


$\xi = \Phi_{B_s}/\Phi_B$ versus m_q



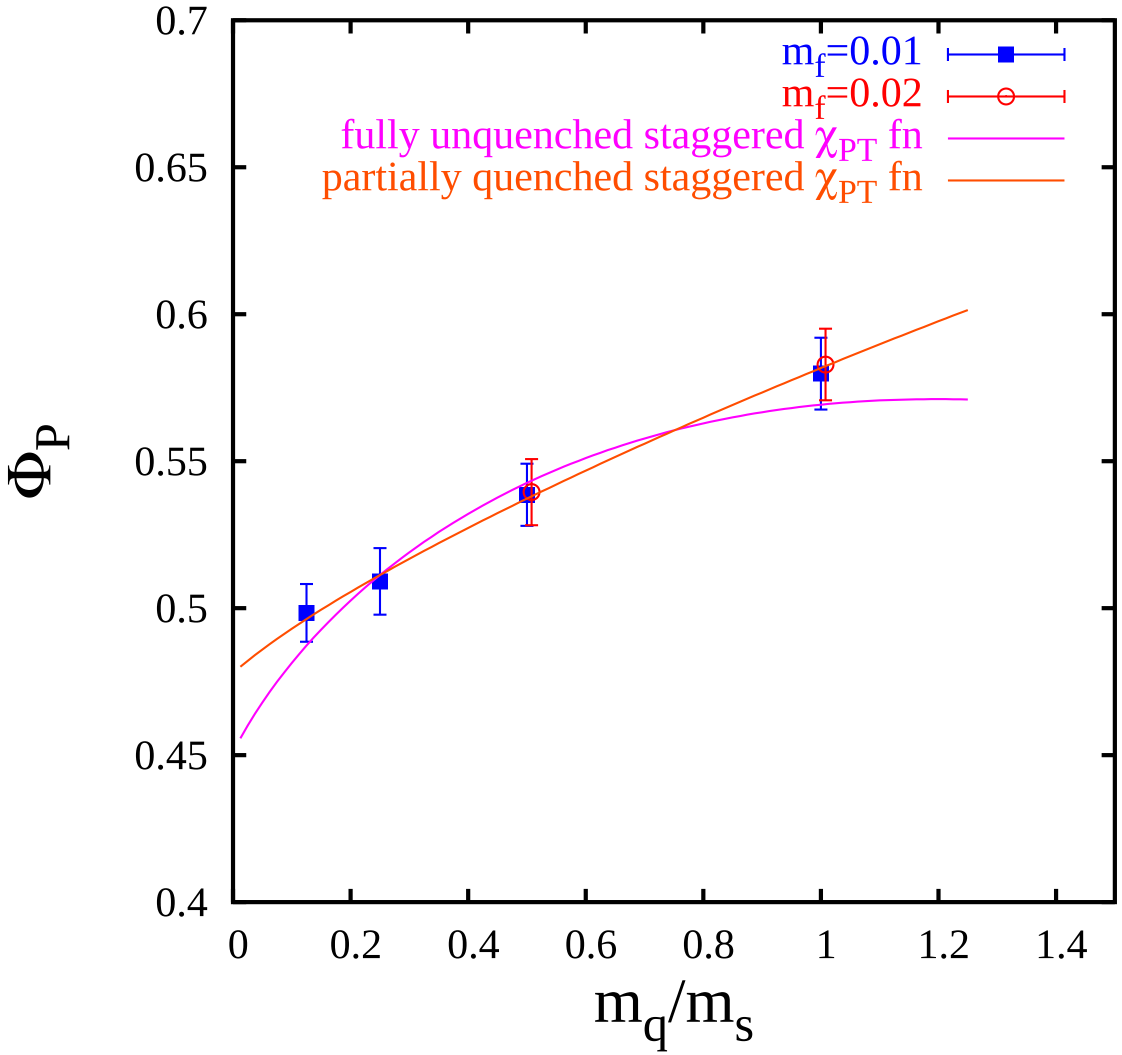
Φ_B versus m_q and $S\chi PT$

Uses $S\chi PT$ of Aubin & Bernard



Φ_B versus m_q and $S\chi PT$

Uses $S\chi PT$ of Aubin & Bernard



B Semileptonic Decay Form Factors

(Emel Gulez, J.S. et al.)

aM_0	$u_0\, am_q(sea)$	$u_0\, am_q(valence)$
2.8	0.01	0.04, 0.02, 0.01, 0.005
2.8	0.02	0.02

Simulations at other dynamical quark masses and on finer lattices are underway.

Since LAT'04 we are,

- accummulating more fully unquenched data
- analysing dimension four ($1/M$, α/M and $a\,\alpha$) current corrections to the form factors.
- starting to think about $S\chi PT$ chiral extrapolations

3-pnt Correlators

$$C^{(3)}(\vec{p}_\pi, \vec{p}_B, t, T_B) = \sum_{\vec{z}} \sum_{\vec{y}} \langle \Phi_\pi(0) J^\mu(\vec{z}, t) \Phi^\dagger_B(\vec{y}, T_B) \rangle \, e^{i\vec{p}_B \cdot \vec{y}} \, e^{i(\vec{p}_\pi - \vec{p}_B)\cdot \vec{z}}$$

$\vec{p}_B = 0$ throughout and $T_B = 16$ (also 20)

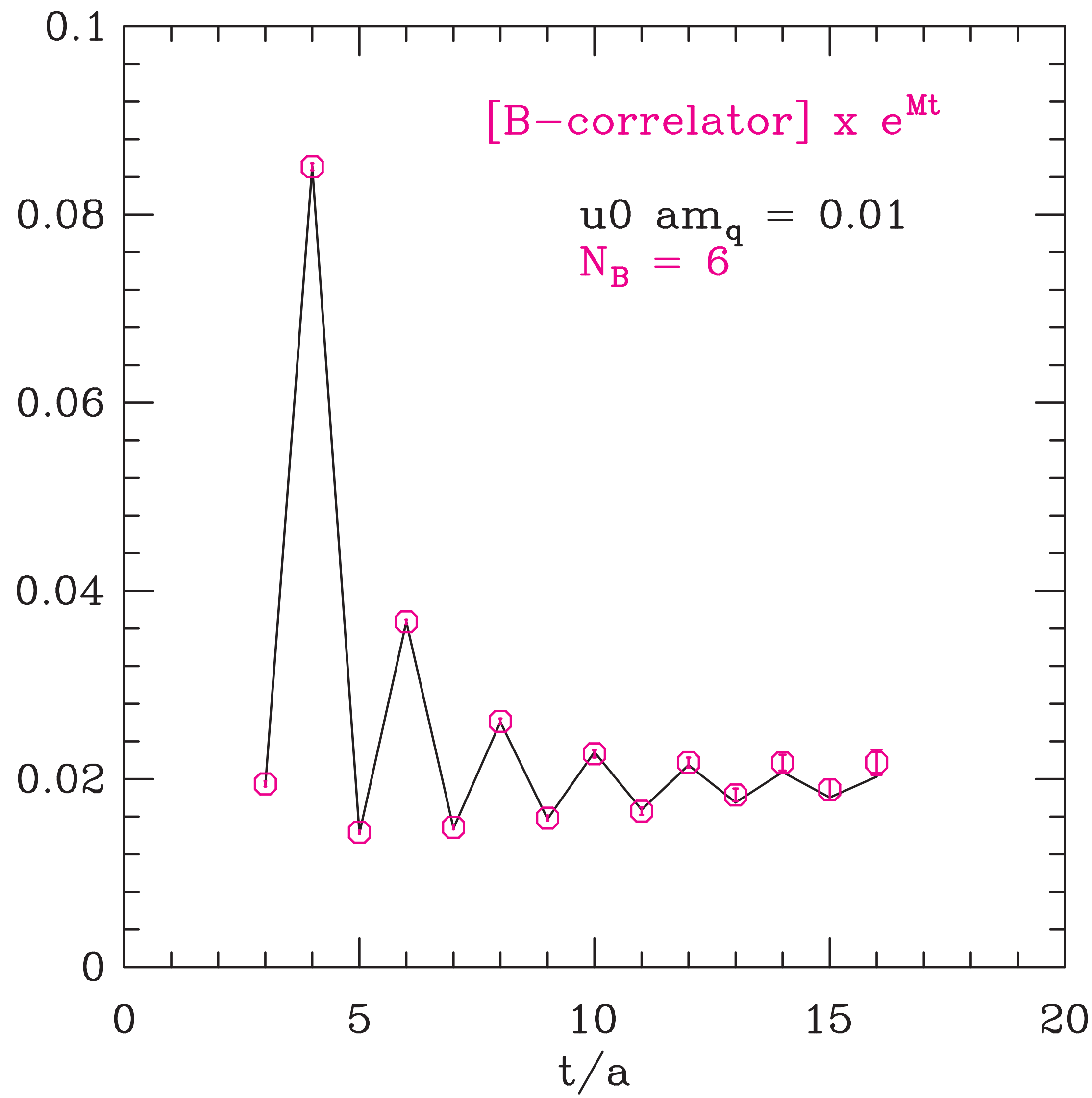
Fits :

$$C^{(3)}(\vec{p}_\pi, \vec{p}_B, t, T_B) \rightarrow \sum_{k=0}^{N_\pi - 1} \sum_{j=0}^{N_B - 1} (-1)^{k*t} (-1)^{j*(T_B - t)} \times A_{jk} \, e^{-E_\pi^{(k)} t} \, e^{-E_B^{(j)}(T_B - t)}$$

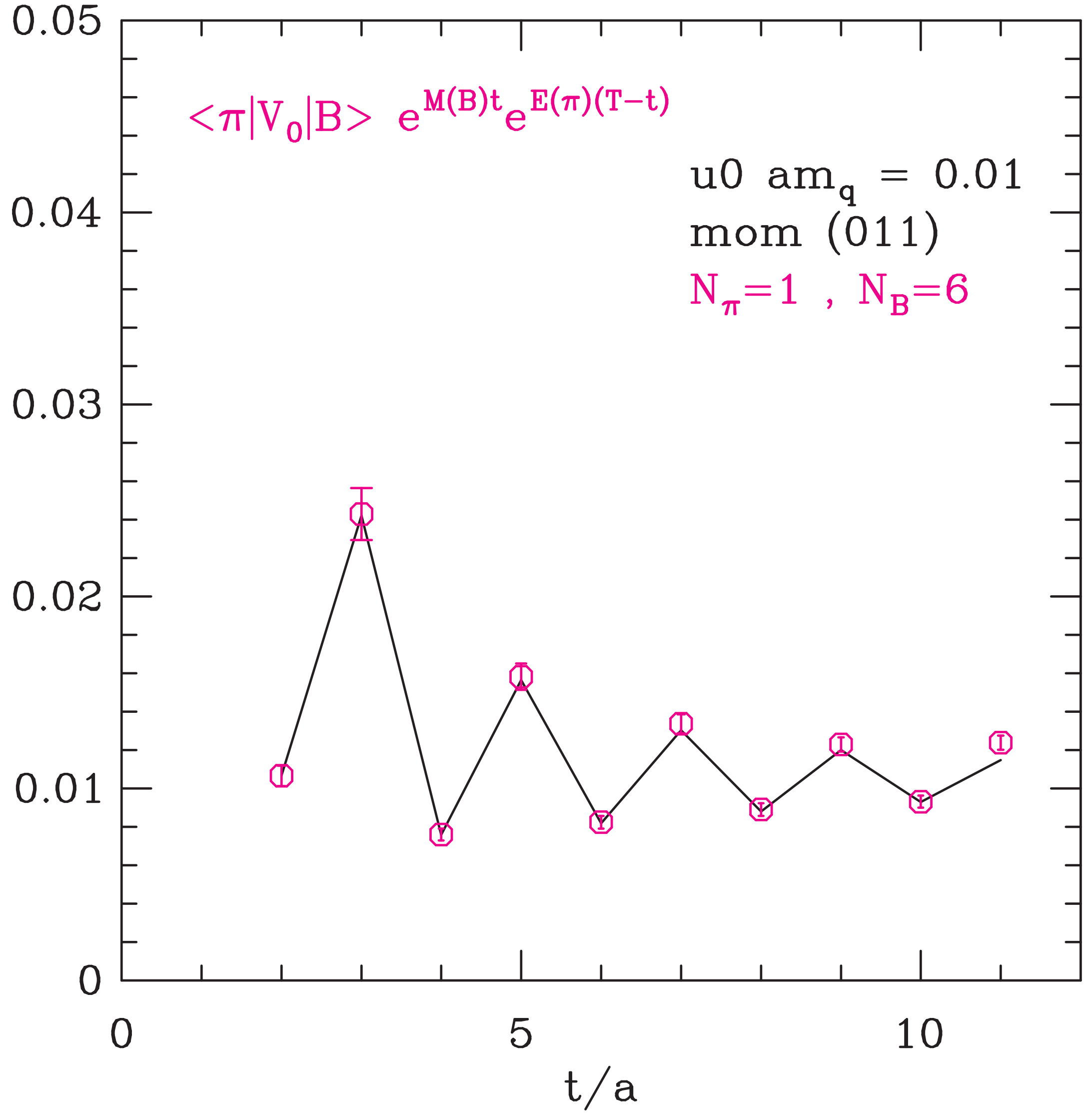
Most fits used $N_\pi = 1$ and $N_B = 3$ - 8 (Bayesian fits)

Goal is to extract $\Longrightarrow$ A_{00}

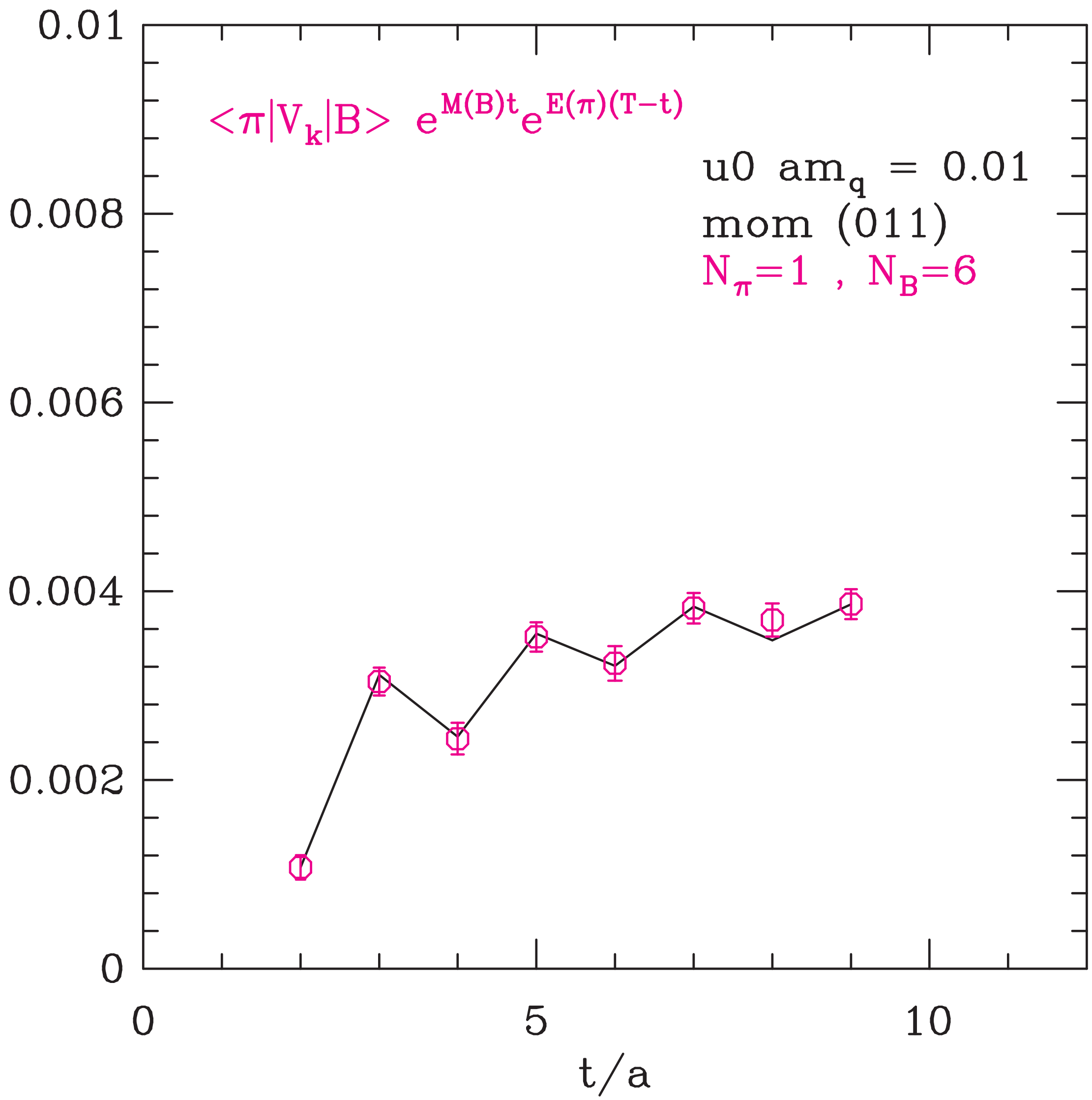
Fit to B- correlator



Fit to $\langle\pi|V_0|B\rangle$



Fit to $\langle \pi | V_k | B \rangle$



Form Factors

$$\langle \pi(p_\pi)|V^\mu|B(p_B)\rangle = f_+(q^2)\left[p_B^\mu + p_\pi^\mu - \frac{M_B^2 - m_\pi^2}{q^2}\,q^\mu\right] + f_0(q^2)\,\frac{M_B^2 - m_\pi^2}{q^2}\,q^\mu = \sqrt{2M_B}\,[v^\mu f_\parallel + p_\perp^\mu f_\perp]$$

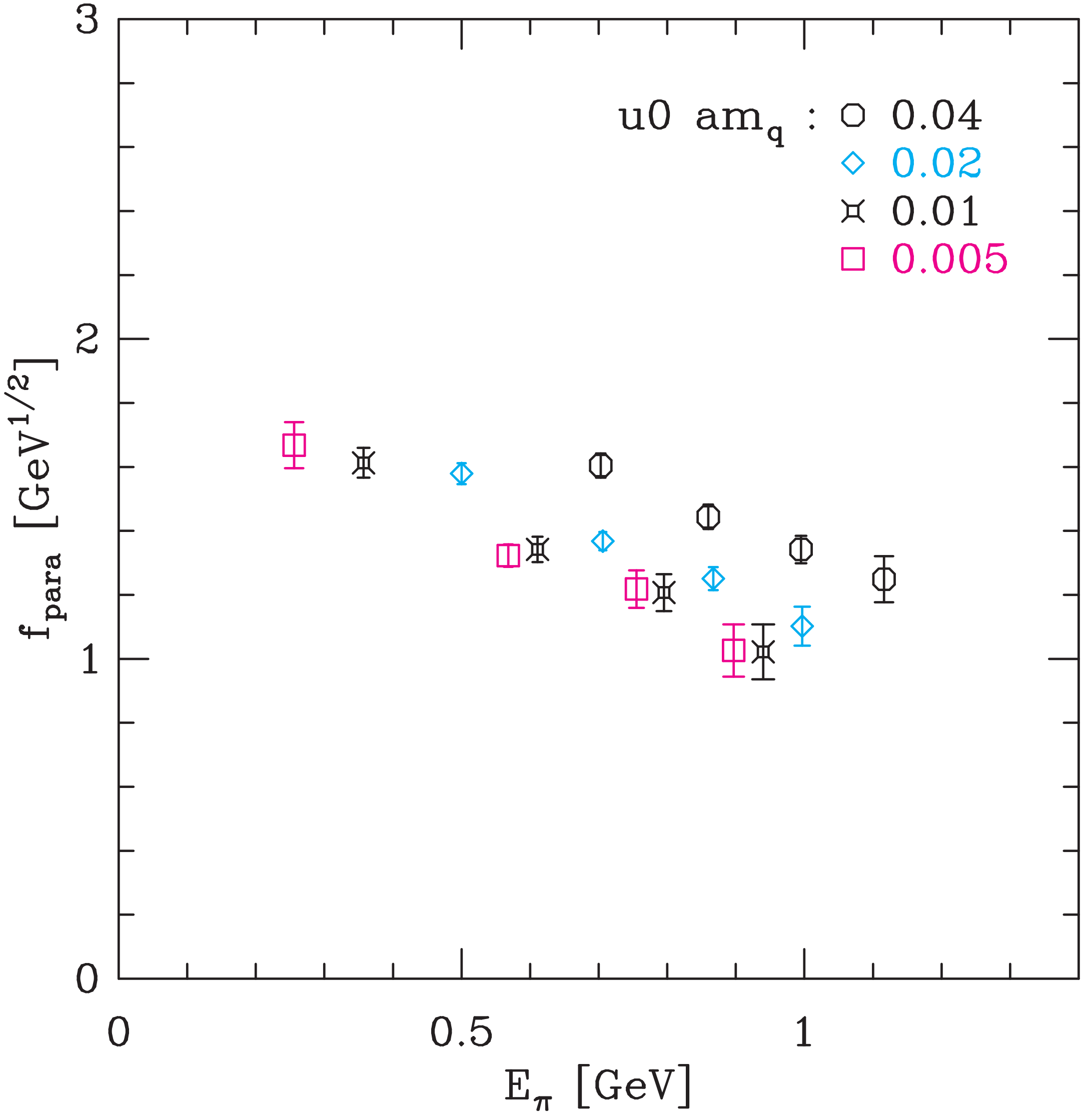
$v^\mu = \frac{p_B^\mu}{M_B}$, $p_\perp^\mu = p_\pi^\mu - (p_\pi \cdot v)\,v^\mu$, $q^\mu = p_B^\mu - p_\pi^\mu$

$$f_\parallel = \frac{A_{00}(V^0)}{\sqrt{\zeta_\pi^{(0)}\zeta_B^{(0)}}}\sqrt{2E_\pi}\,Z_{V_0}$$

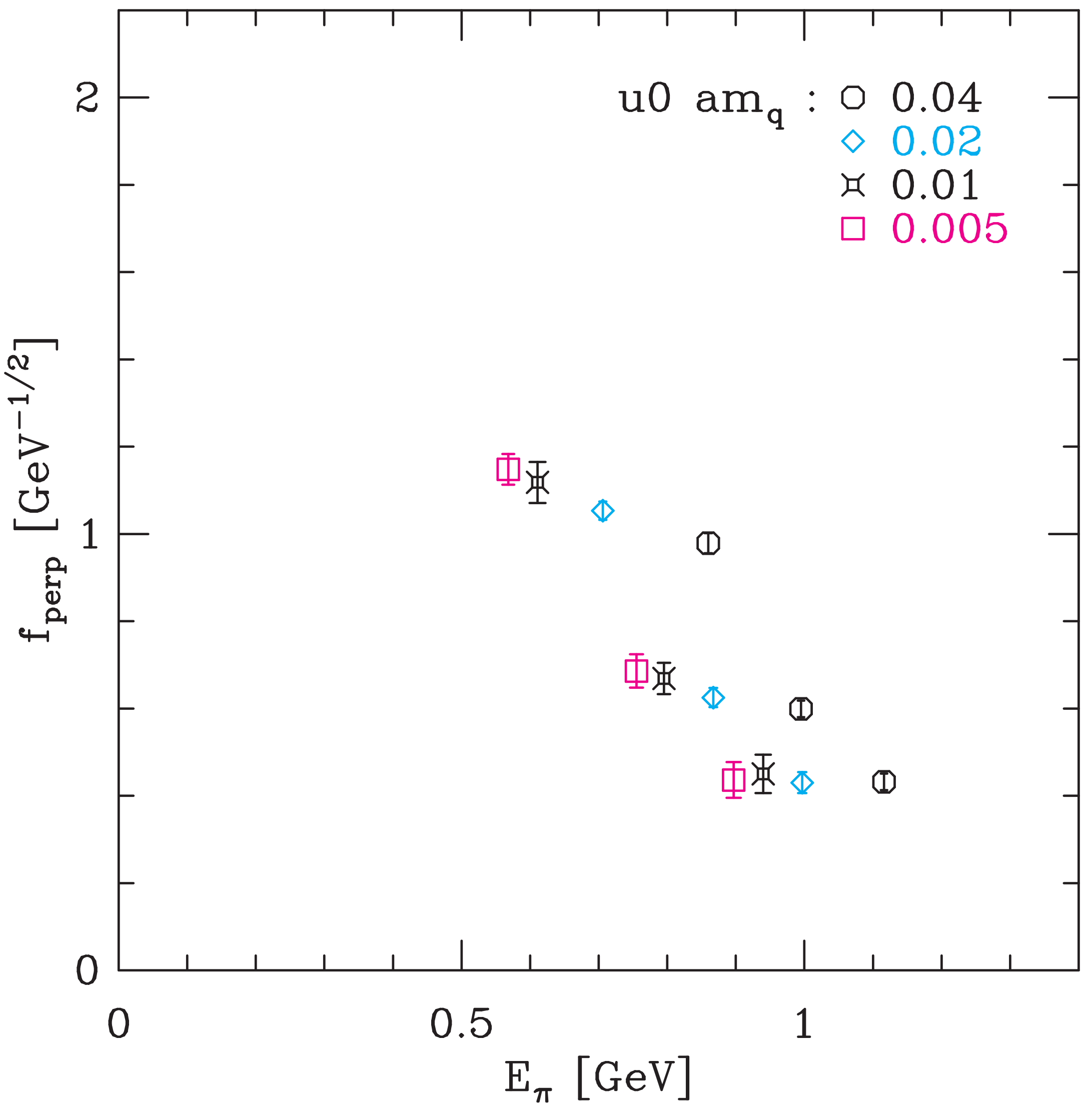
$$f_\perp = \frac{A_{00}(V^k)}{\sqrt{\zeta_\pi^{(0)}\zeta_B^{(0)}}\;p_\pi^k}\sqrt{2E_\pi}\,Z_{V_k}$$

Z_{V_0}, Z_{V_k} estimated via 1-loop pert. th.

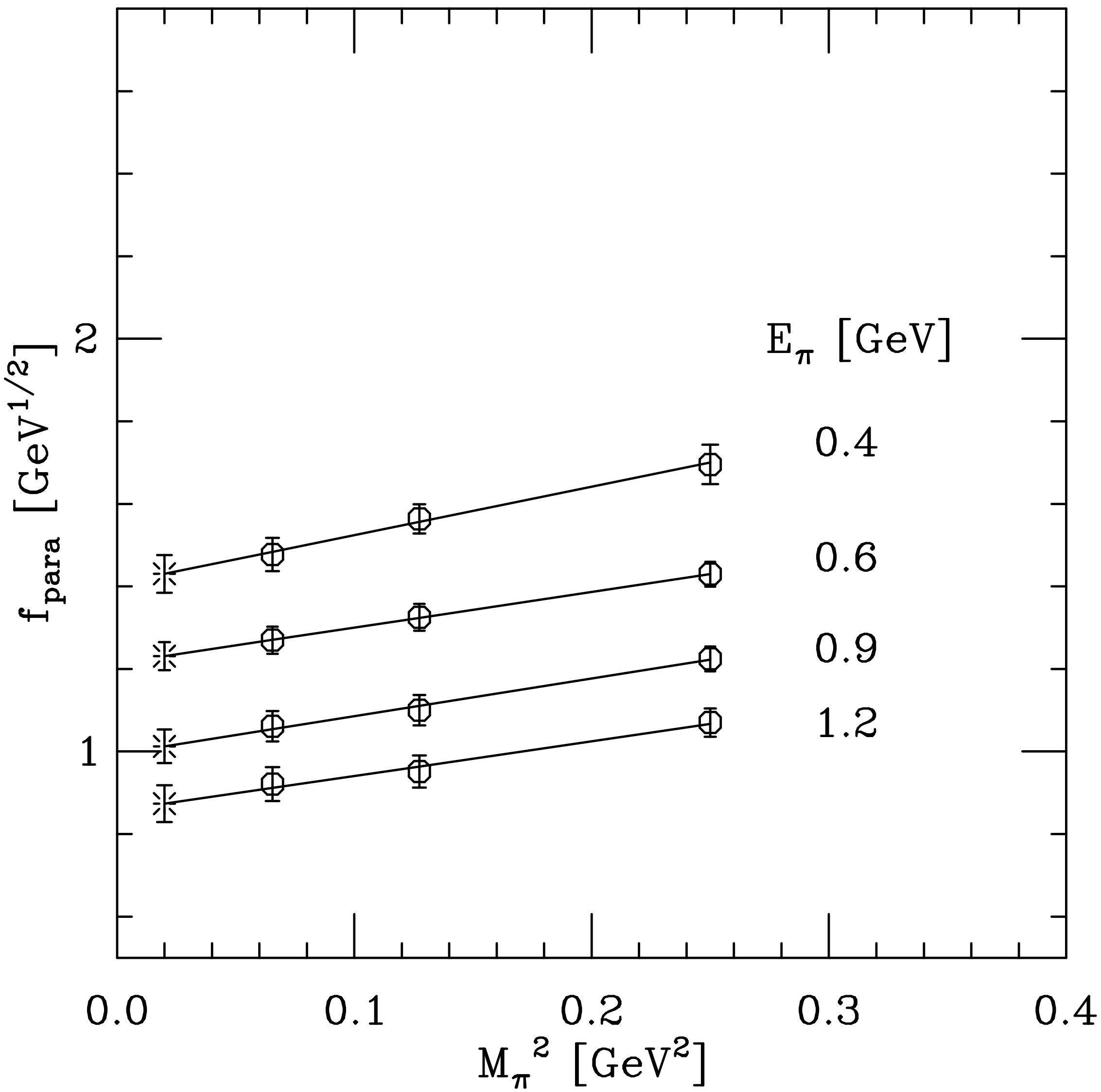
Results for $f_{\|}$



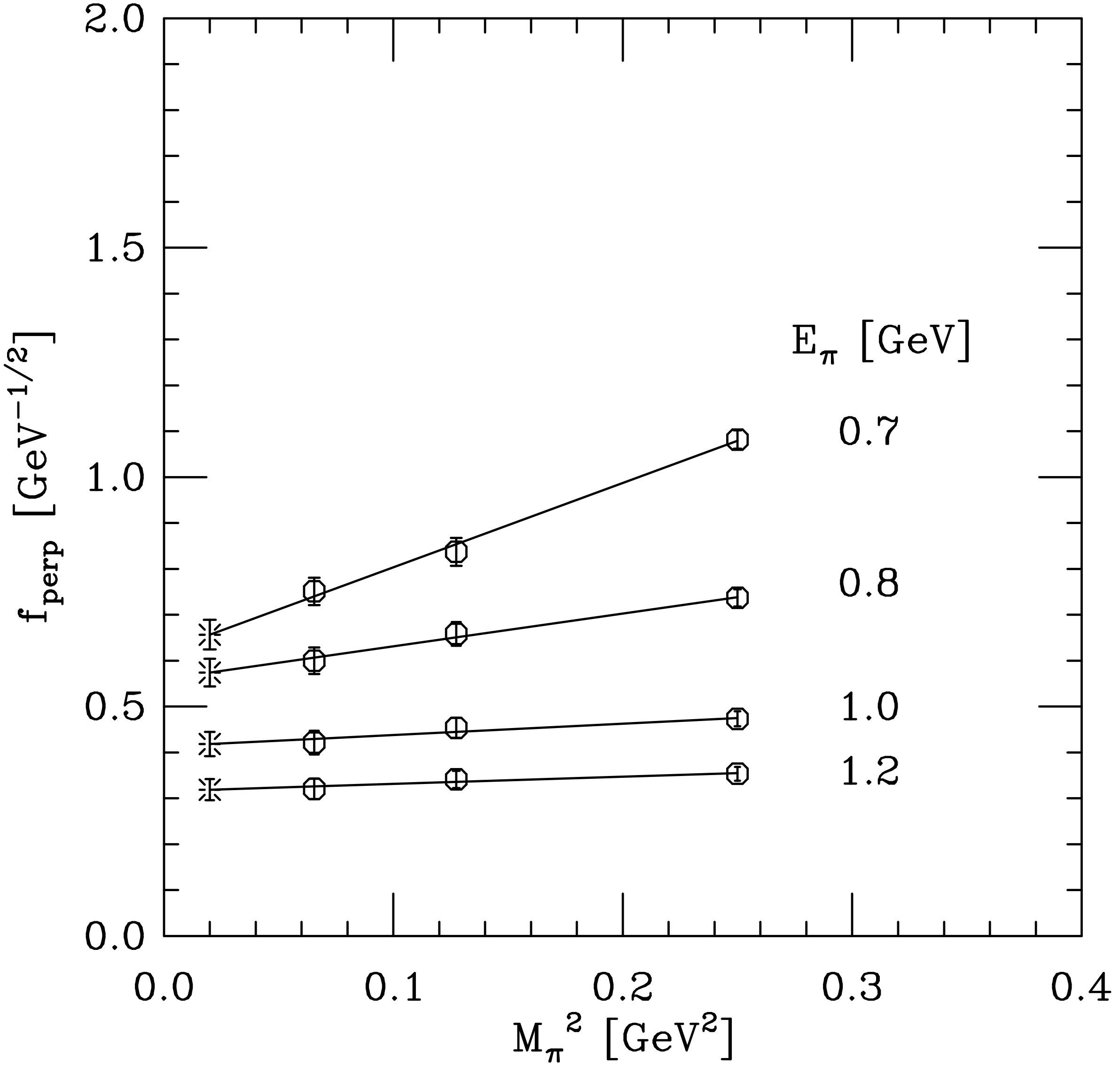
Results for $f_\perp$



Chiral Extrapolations for $f_{\parallel}$



Chiral Extrapolations for $f_\perp$



Becirevic-Kaidalov (BK) Parametrization

This ansatz satisfies :

— $f_+(0) = f_0(0)$

— HQET scaling laws

— position of pole at $q^2 = M_{B^*}^2$

$$f_+(q^2) = \frac{C_B\,(1-\alpha_B)}{(1-\tilde{q}^2)(1-\alpha_B\tilde{q}^2)} \qquad f_0(q^2) = \frac{C_B\,(1-\alpha_B)}{(1-\tilde{q}^2/\beta_B)}$$

$(\tilde{q}^2 \equiv q^2/M_{B^*}^2)$

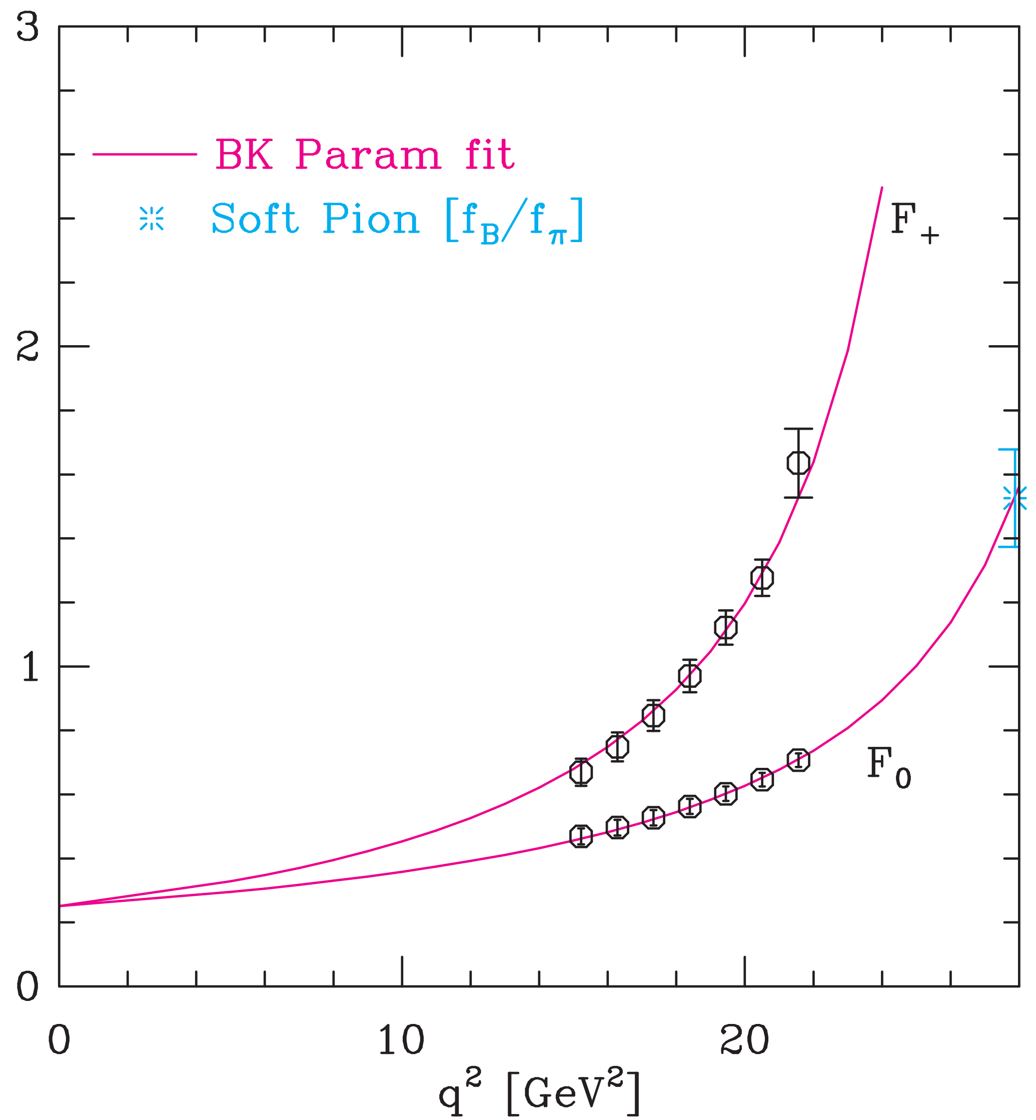
The chirally extrapolated f_0 & f_+ are fit very well by a BK ansatz using the physical M_{B^*} mass and

$C_B = 0.42(3) \qquad \alpha_B = 0.41(7) \qquad \beta_B = 1.18(5)$

which leads to $f_0(0) = f_+(0) = 0.25(2)$.

BK parametrization fit to f_0 and f_+

(at the physical pion)



Extracting $|V_{ub}|$, Lattice + CLEO

Several experimental groups are studying the process
$B \longrightarrow \pi^+, e^-\overline{\nu}$
CLEO, BaBar, Belle

Using lattice determination of $f_+(q^2)$ one can integrate

$$\frac{1}{|V_{ub}|^2}\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3} p_\pi^3 \, |f_+(q^2)|^2$$

to get $\frac{\Gamma}{|V_{ub}|^2}$ $\Longrightarrow$ $\boxed{|V_{ub}|}$

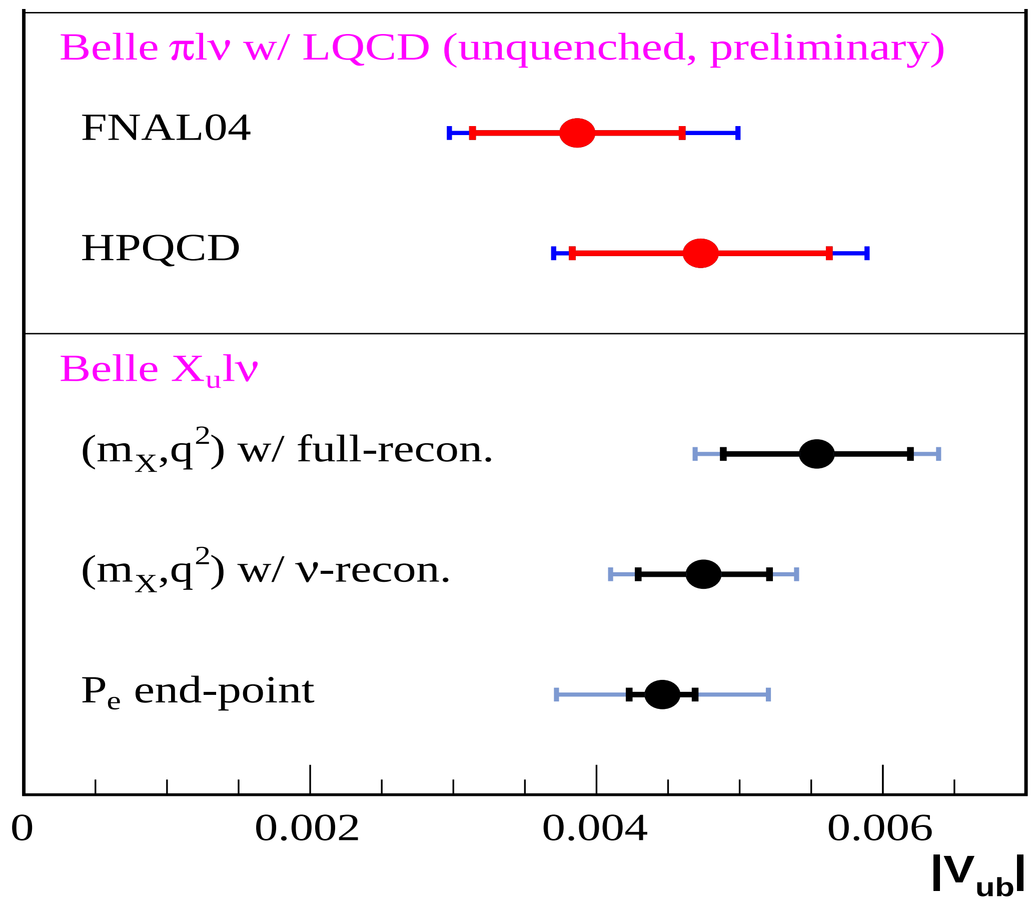
Using branching fractions Γ/Γ_{full} from CLEO [S.B.Athar et al.,PRD 68,072003 (2003)] we find,

(Preliminary)

$$|V_{ub}| = \begin{cases} 3.86(32)(58) \times 10^{-3} & 0 \le q^2 \le q^2_{max} \\ 3.52(73)(44) \times 10^{-3} & 16GeV^2 \le q^2 \end{cases}$$

Extracting $|V_{ub}|$, Lattice + Belle

[Belle collaboration contribution to ICHEP'04]
(K.Abe et al.,hep-ex/0408145)



Improvements

- other dynamical quark masses (more fully unquenched results)

- $1/M$ current corrections

- better chiral extrapolations based on $S\chi PT$ (Aubin & Bernard).

- use Moving NRQCD to get to lower q^2 (K.Foley, LAT'04)

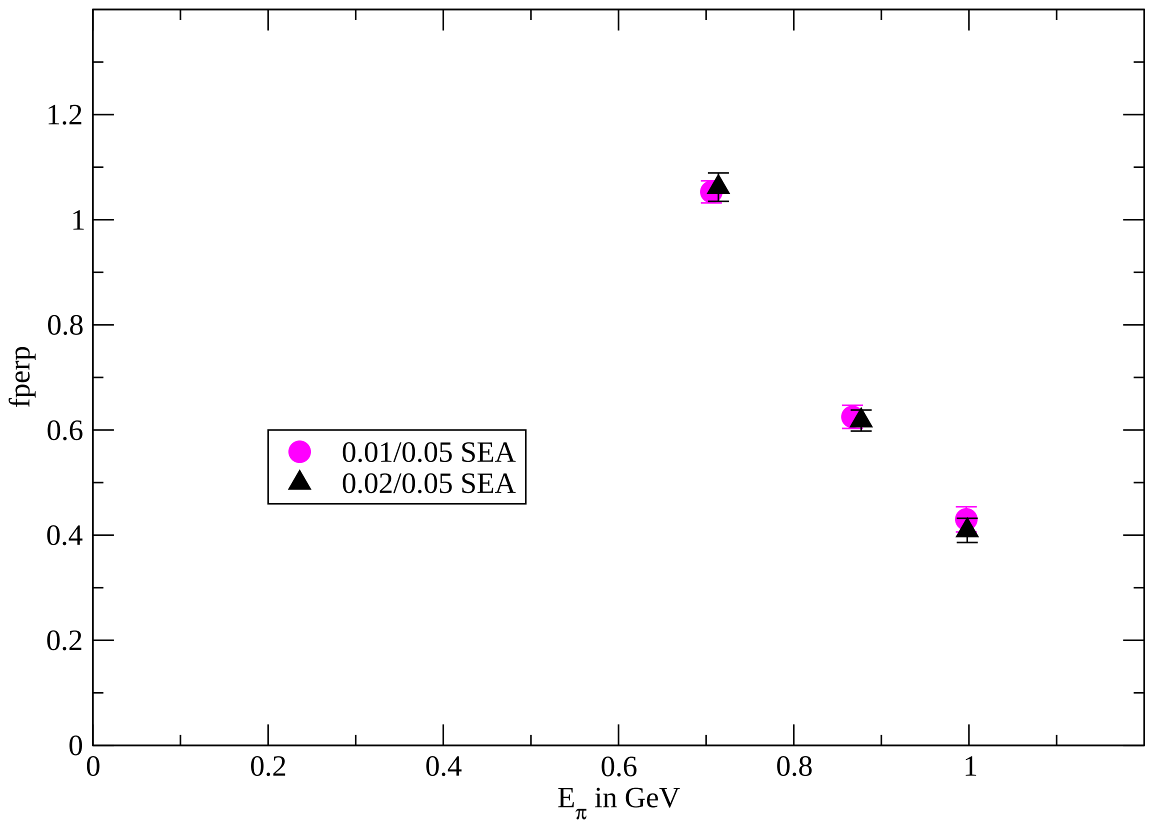
- work with finer MILC configurations

Systematic Errors

	order	error	how to improve	status
matching	α_s^2	9%	do 2-loop matching	about to embark
relativistic + finite a corrections	$\frac{\Lambda}{M}$, $\frac{\alpha_s}{(aM)}$ $\frac{\alpha_s \Lambda}{M}$, $a\alpha_s$	5%	include mixing with Dim.4 currents	done
chiral extrapolations		5%	use $S\chi PT$ check m_l^{sea} dep.	in progress
finite a error in action	$a^2\alpha_s$	2%	improve action finer lattices	in progress
Total		11%		

$f_\perp$ at Two Sea Quark Masses

$u_0\, am_q(valence)$ fixed at 0.02



1/M Current Corrections (revisited)

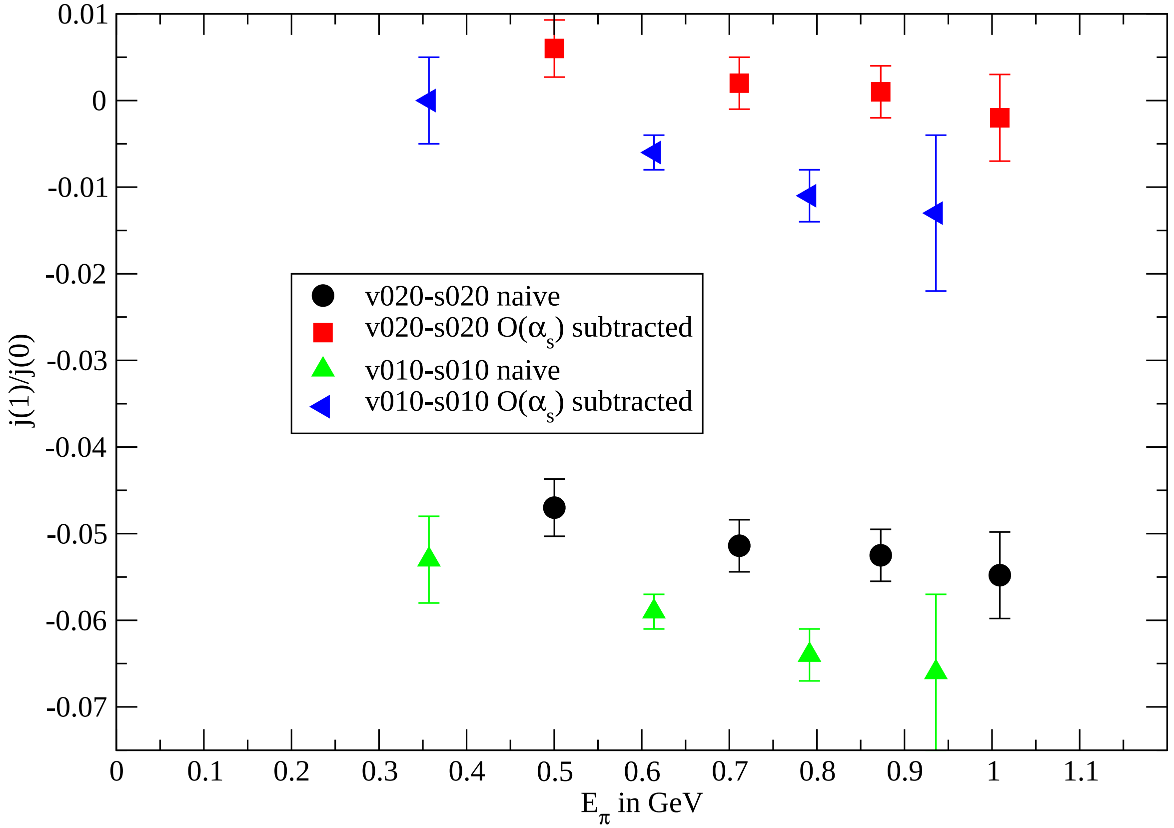
For V_0, A_0 :

$$\begin{aligned}
J_0^{(0)}(x) &= \bar{q}(x)\,\Gamma_0\,Q(x), \\
J_0^{(1)}(x) &= \frac{-1}{2M_0}\bar{q}(x)\,\Gamma_0\,\gamma\cdot\boldsymbol{\nabla}\,Q(x), \\
J_0^{(2)}(x) &= \frac{-1}{2M_0}\bar{q}(x)\,\gamma\cdot\overleftarrow{\boldsymbol{\nabla}}\,\gamma_0\,\Gamma_0\,Q(x).
\end{aligned}$$

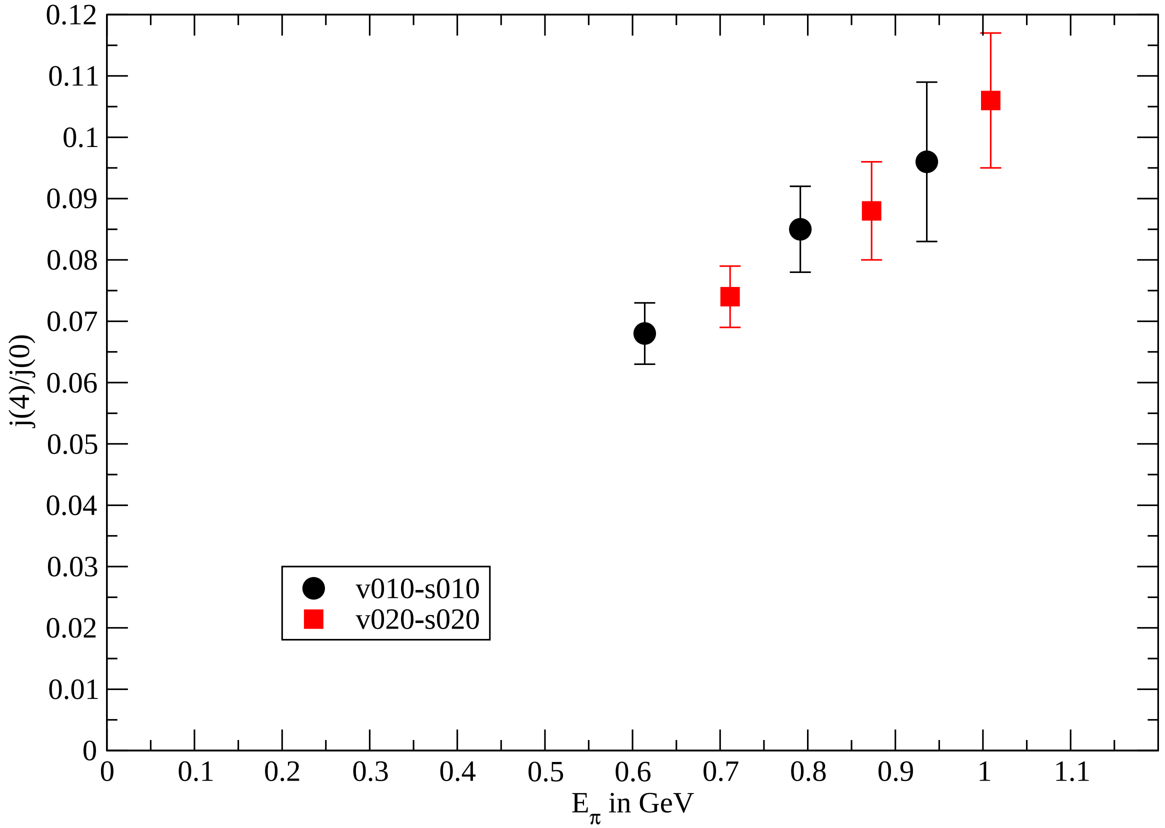
and for V_k, A_k :

$$\begin{aligned}
J_k^{(0)}(x) &= \bar{q}(x)\,\Gamma_k\,Q(x), \\
J_k^{(1)}(x) &= \frac{-1}{2M_0}\bar{q}(x)\,\Gamma_k\,\gamma\cdot\boldsymbol{\nabla}\,Q(x), \\
J_k^{(2)}(x) &= \frac{-1}{2M_0}\bar{q}(x)\,\gamma\cdot\overleftarrow{\boldsymbol{\nabla}}\,\gamma_0\,\Gamma_k\,Q(x), \\
J_k^{(3)}(x) &= \frac{-1}{2M_0}\bar{q}(x)\,\nabla_k\,Q(x) \\
J_k^{(4)}(x) &= \frac{1}{2M_0}\bar{q}(x)\,\overleftarrow{\nabla}_k\,Q(x),
\end{aligned}$$

$V_0^{(1)}/V_0^{(0)}$ versus Pion Energy

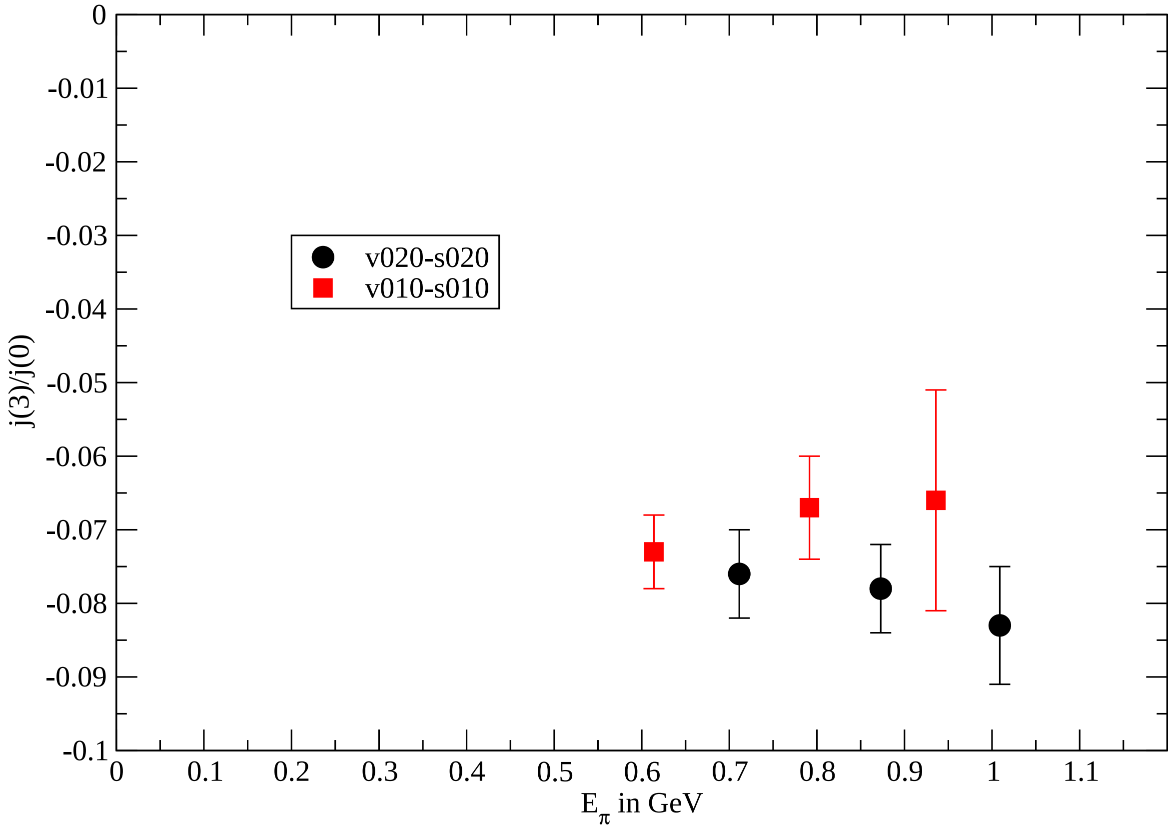


$V_k^{(4)}/V_k^{(0)}$ versus Pion Energy



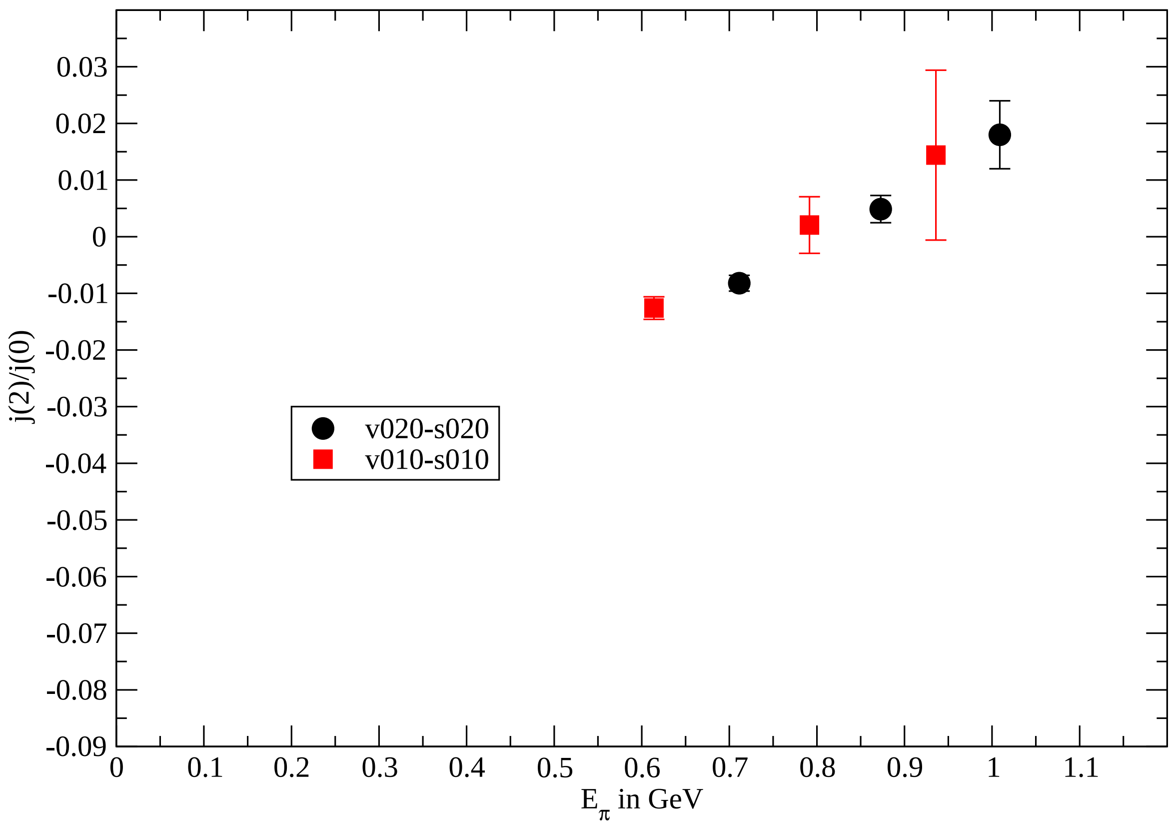
multiplied by $\rho_4\,\alpha_s = -0.029\,\alpha_s$

$V_k^{(3)}/V_k^{(0)}$ versus Pion Energy



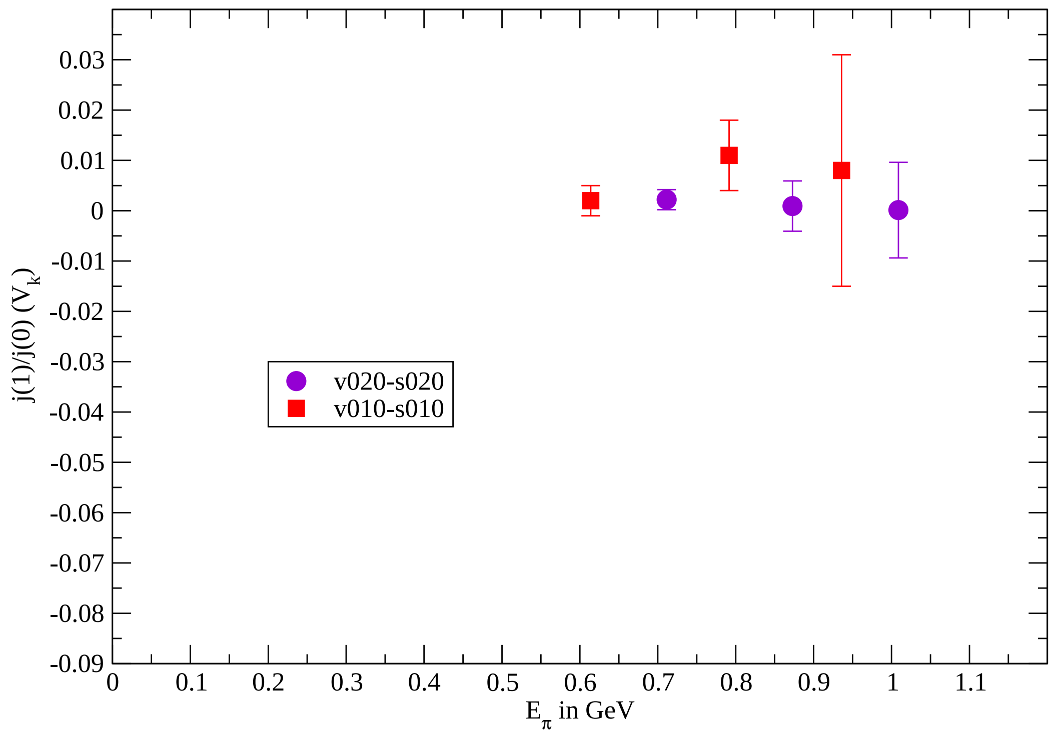
multiplied by $\rho_3\,\alpha_s = 0.218\,\alpha_s$

$V_k^{(2)}/V_k^{(0)}$ versus Pion Energy



multiplied by $\rho_2\,\alpha_s = 0.169\,\alpha_s$

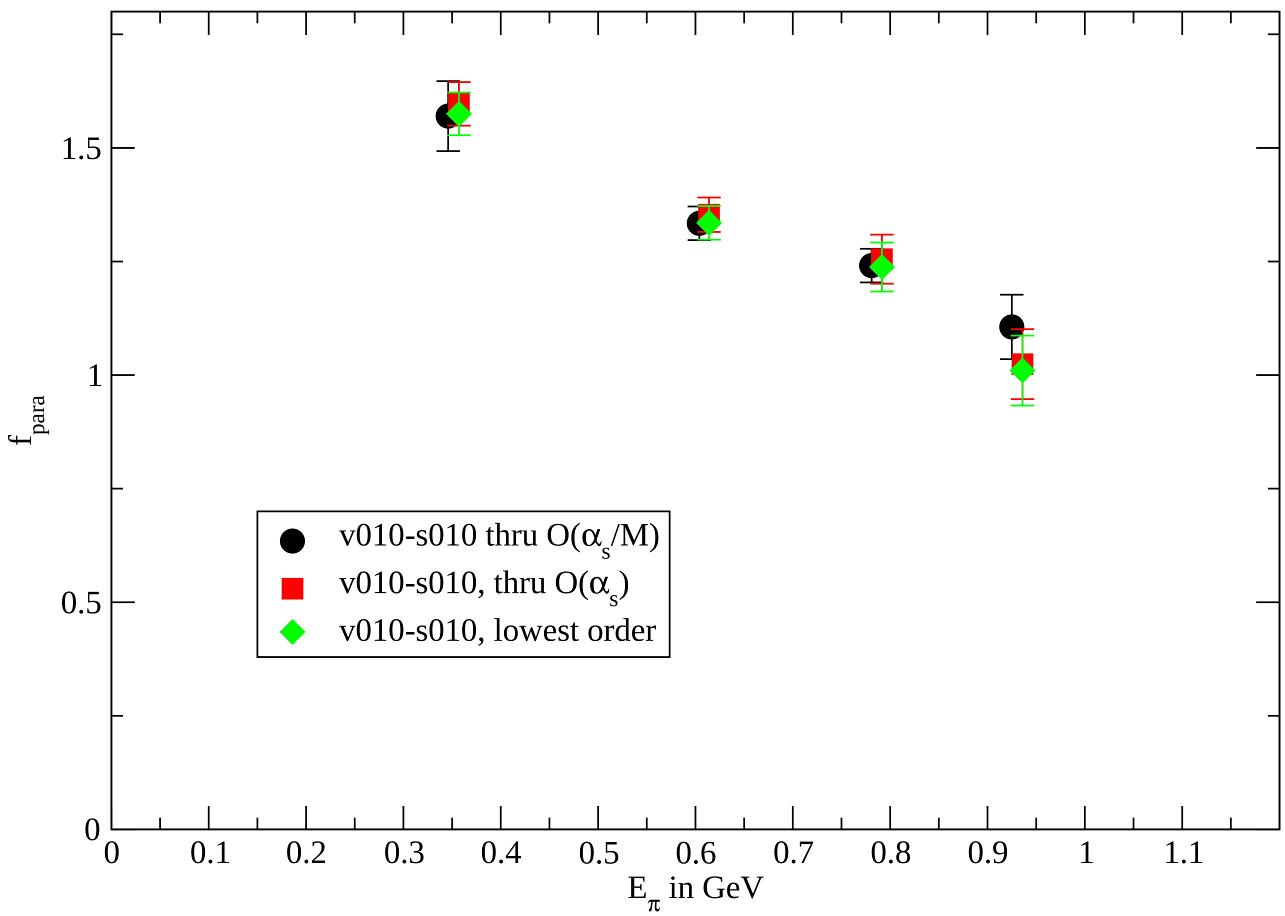
$V_k^{(1)}/V_k^{(0)}$ versus Pion Energy



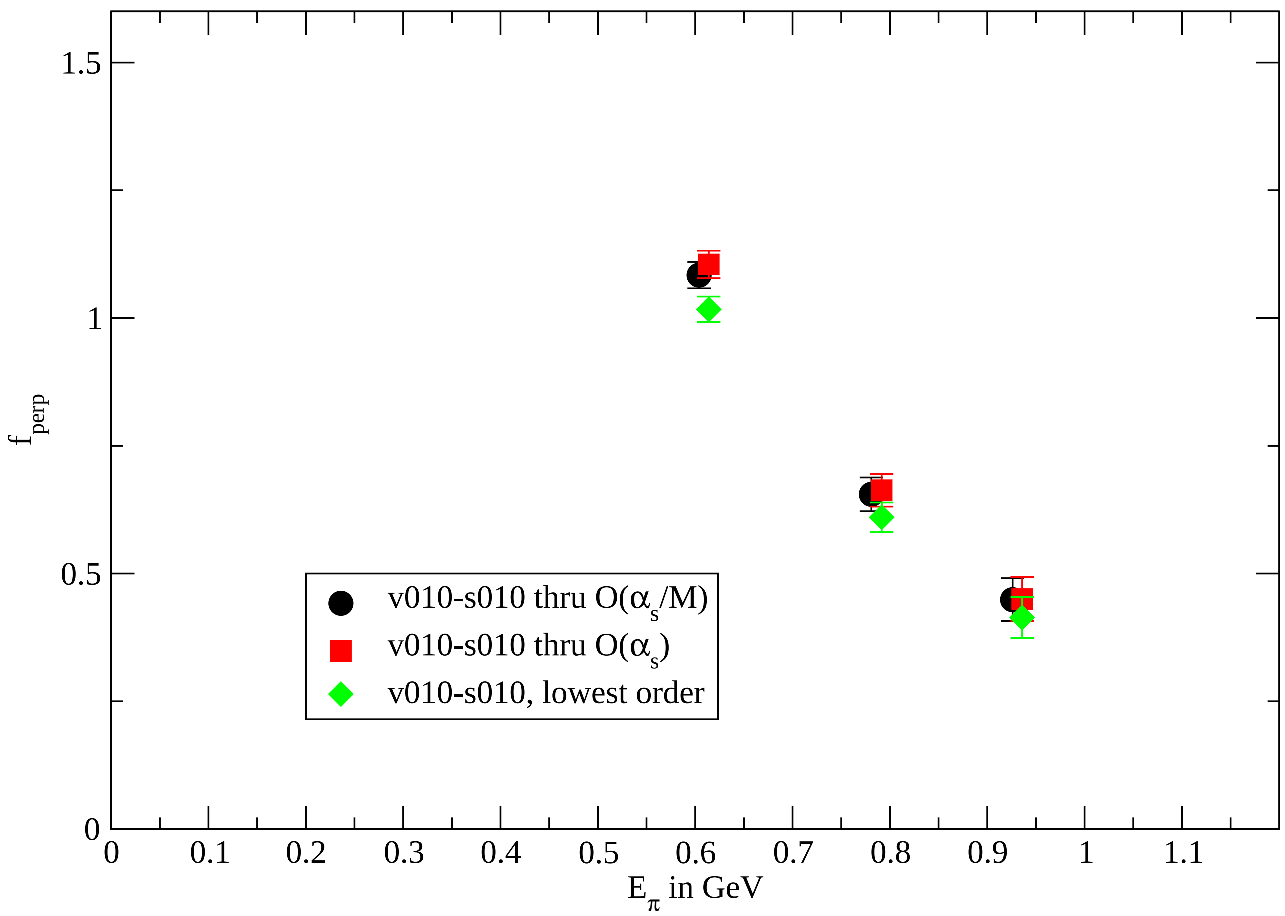
subtract $\zeta_{10}\,\alpha_s = 0.055\,\alpha_s$

multiply by $[1 + \rho_1\,\alpha_s] = [1 + 0.349\,\alpha_s]$

Effect of $1/M$ Current Corrections on $f_{\|}$



Effect of $1/M$ Current Corrections on $f_\perp$



Summary and Future Plans

The general availability of the MILC dynamical configurations and the use of improved staggered valence quarks in heavy-light simulations, have led to significant progress in heavy meson decay constant and semi-leptonic form factor determinations.

Much work remains to be done, however.

- more fully unquenched data and simulations on finer lattices

- further development of Moving NRQCD

- determination of B_B

- Higher order matching of lattice operators

Work on all these fronts is underway.