$\boxed{B \longrightarrow \pi \text{ DECAYS}}$

FROM THE LATTICE

ILFTNetwork Tsukuba Workshop

Lattice QCD and Particle Phenomenology

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presented by :

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$B \longrightarrow \pi$ Decays from the Lattice

[Current Status and Future Prospects]

Current Status

- Good news : huge progress in last couple of years

- Sobering fact : lattice errors in $|V_{ub}|$ still $10 \sim 14\%$

Future Prospects

- What does it take to reduce errors to $\leq 5\%$?

Recent Results for B Semi-leptonic Decays

Progress has come about mainly for two reasons.

- Unquenching

- Better control over chiral extrapolations ($m_l \to m_{u,d}$)

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The MILC collaboration has created and made available gauge field configurations that include effects of $N_f = 2+1$ dynamical quarks. They use a highly improved light quark action (AsqTad Action) which allows for simulations at much smaller quark masses than in the past (now well tested).

People are now using this same AsqTad action for the light valence quark inside the B meson.

Unquenched Results from Two Groups

Fermilab/MILC

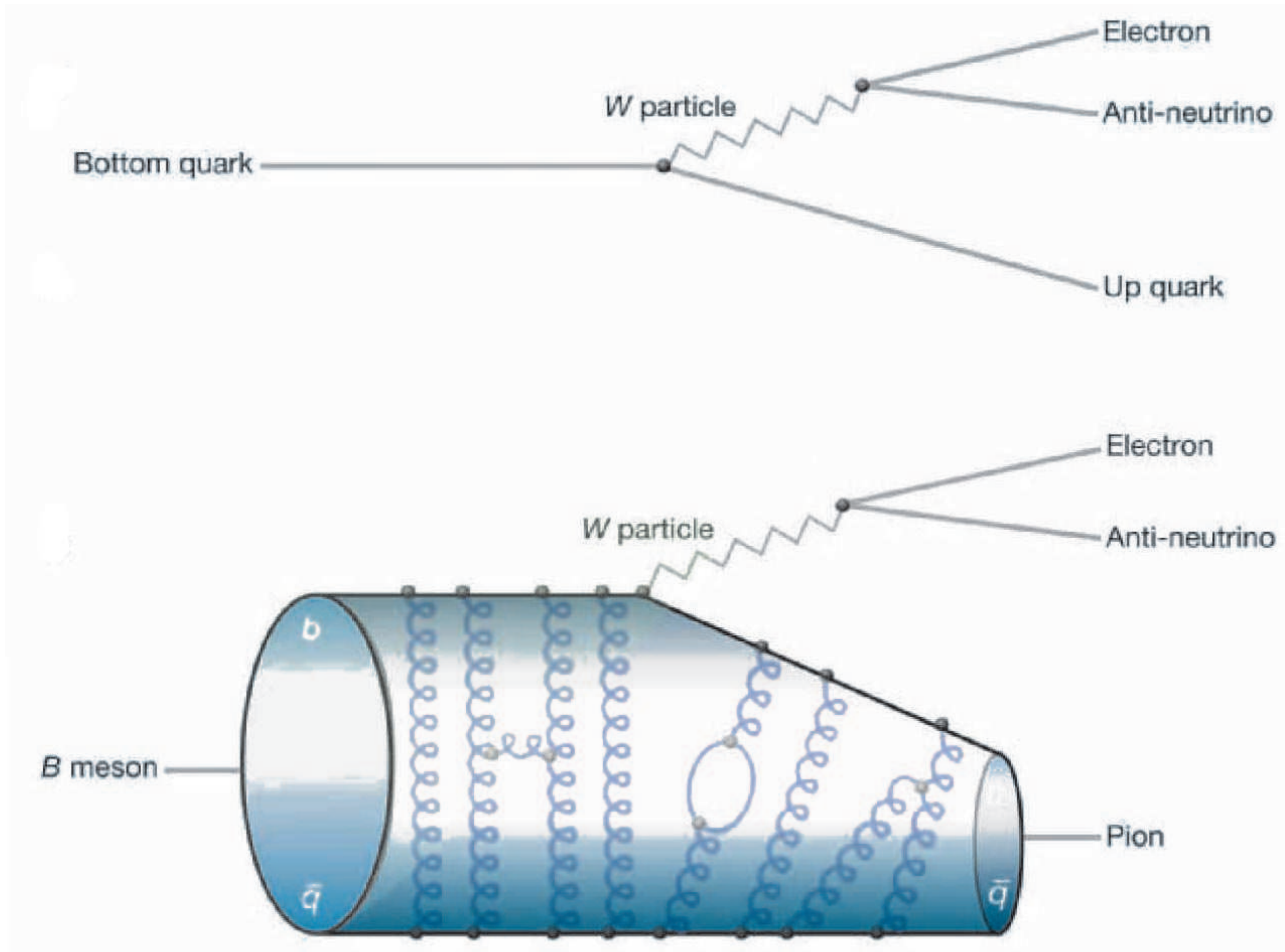
Fermilab, St.Louis, Utah, DePaul, Urbana, Bloomington, Arizona, APS, U. of the Pacific, Santa Barbara.

HPQCD

Cambridge, Cornell, Glasgow, Ohio State, Seattle, Simon Fraser.

Results from both groups are still preliminary.

The Process



$$\langle \pi | (V_\mu - A_\mu) | B \rangle$$

Some Lattice Details

So, the task for the Lattice is to evaluate the matrix element

$$\langle \pi(p_\pi)|V_\mu|B(p_B)\rangle$$

nonperturbatively.

- Both Fermilab/MILC and HPQCD use the MILC dynamical gauge configurations and the improved staggered, AsqTad, light quark action.

- Fermilab/MILC uses "Fermilab heavy quarks" (b-quark mass fixed by B_s).

- HPQCD uses nonrelativistic, NRQCD, heavy quarks (b-quark mass fixed by Υ).

- Both groups work with light quark masses ranging between $\frac{m_s}{8} \leq m_l \leq m_s$.

Form Factors

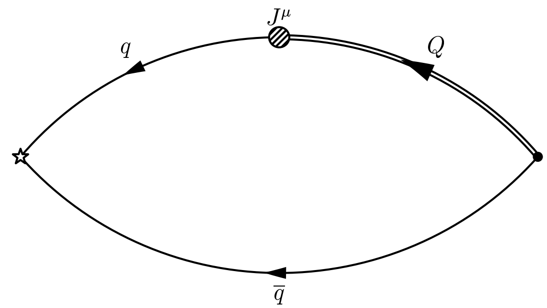
$$
\begin{aligned}
\langle \pi(p_\pi)|V^\mu|B(p_B)\rangle &= f_+(q^2)\left[p_B^\mu + p_\pi^\mu - \frac{M_B^2 - m_\pi^2}{q^2}\,q^\mu\right] \\
&+ f_0(q^2)\,\frac{M_B^2 - m_\pi^2}{q^2}\,q^\mu \\
&= \sqrt{2M_B}\,[v^\mu f_\parallel + p_\perp^\mu f_\perp]
\end{aligned}
$$

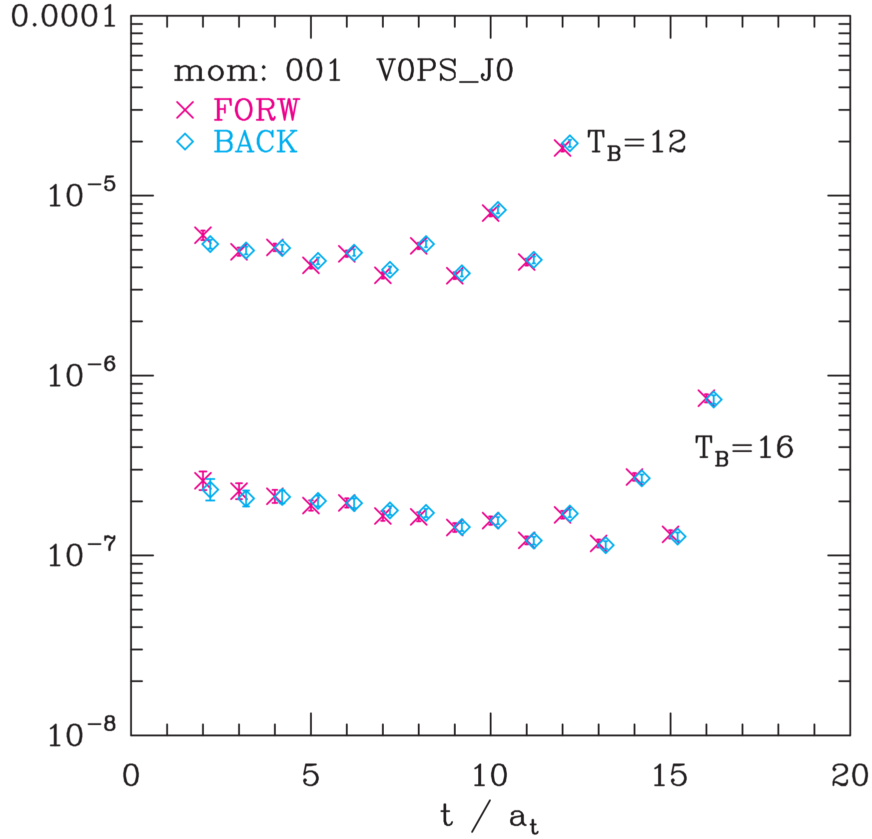
$v^\mu = \frac{p_B^\mu}{M_B}$, $p_\perp^\mu = p_\pi^\mu - (p_\pi \cdot v)\, v^\mu$, $q^\mu = p_B^\mu - p_\pi^\mu$

$f_\parallel$ and $f_\perp$ are more convenient for chiral extrapolations and have simpler HQET scaling properties.

In the B restframe ($\vec{p}_B = 0$), $f_\parallel$ is determined by $\langle V^0\rangle$ and $f_\perp$ by $\langle V^k\rangle$.

Sample Raw Data





3-pnt Correlators

$$C^{(3)}(\vec{p}_\pi, \vec{p}_B, t, T_B) =$$
$$\sum_{\vec{z}} \sum_{\vec{y}} \langle \Phi_\pi(0) J^\mu(\vec{z}, t) \Phi_B^\dagger(\vec{y}, T_B) \rangle \, e^{i\vec{p}_B \cdot \vec{y}} \, e^{i(\vec{p}_\pi - \vec{p}_B)\cdot \vec{z}}$$

$\vec{p}_B = 0$ throughout and $T_B = 16$

__Fits__ :

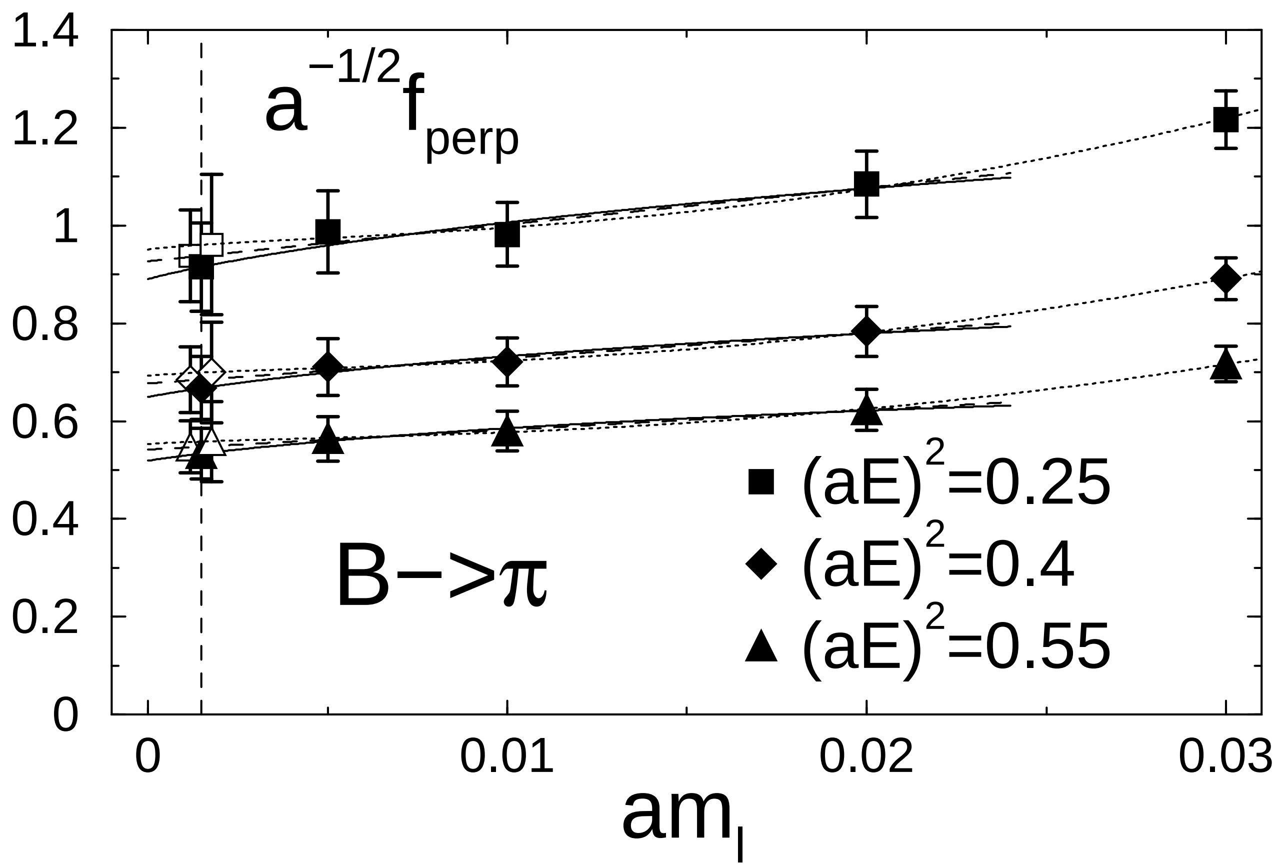
$$C^{(3)}(\vec{p}_\pi, \vec{p}_B, t, T_B) \rightarrow$$
$$\sum_{k=0}^{N_\pi - 1} \sum_{j=0}^{N_B - 1} (-1)^{k*t} (-1)^{j*(T_B - t)}$$
$$\times A_{jk} \, e^{-E_\pi^{(k)} t} \, e^{-E_B^{(j)} (T_B - t)}$$

Most fits used $N_\pi = 1$ and $N_B = 3$ - 8 (Bayesian fits)

Goal is to extract $\Longrightarrow$ $\boxed{A_{00}}$ $\Longrightarrow$ $\boxed{f_\perp, \, f_\|}$

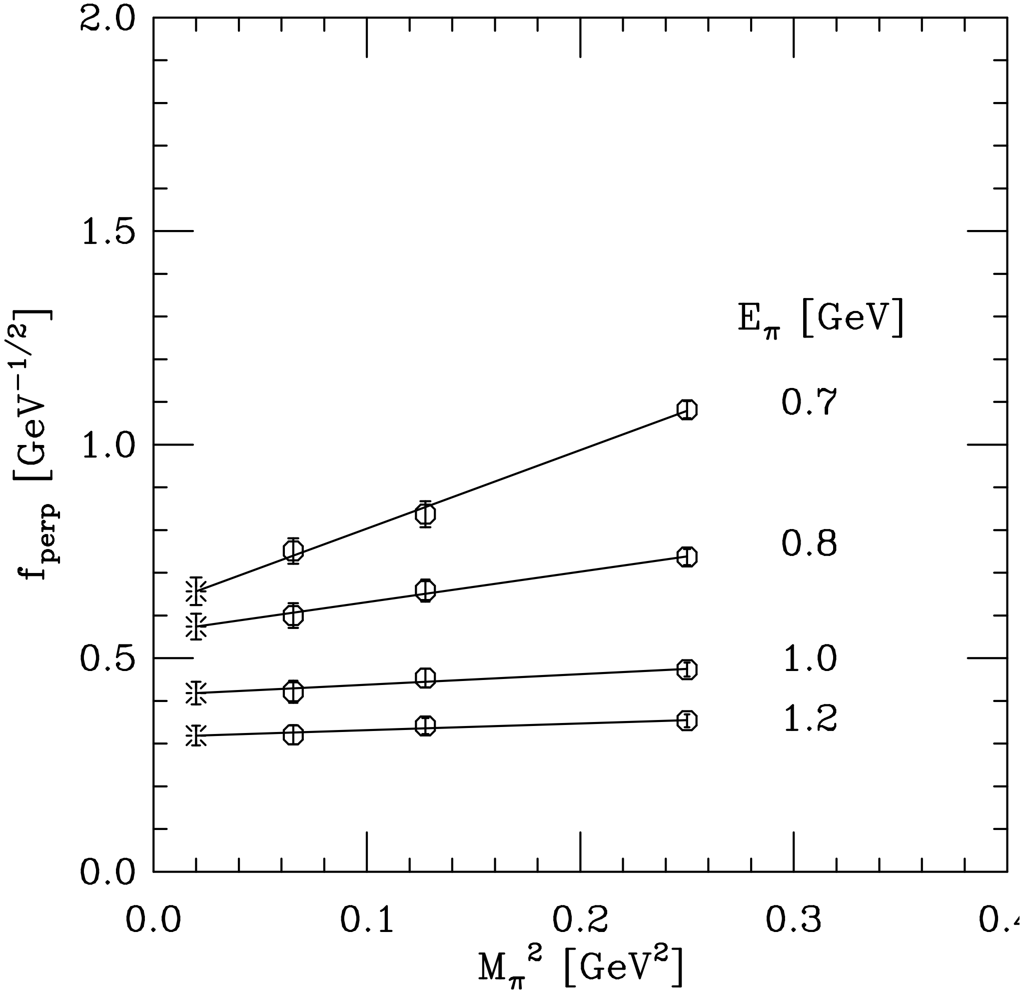
Chiral Extrapolations

Fermilab/MILC



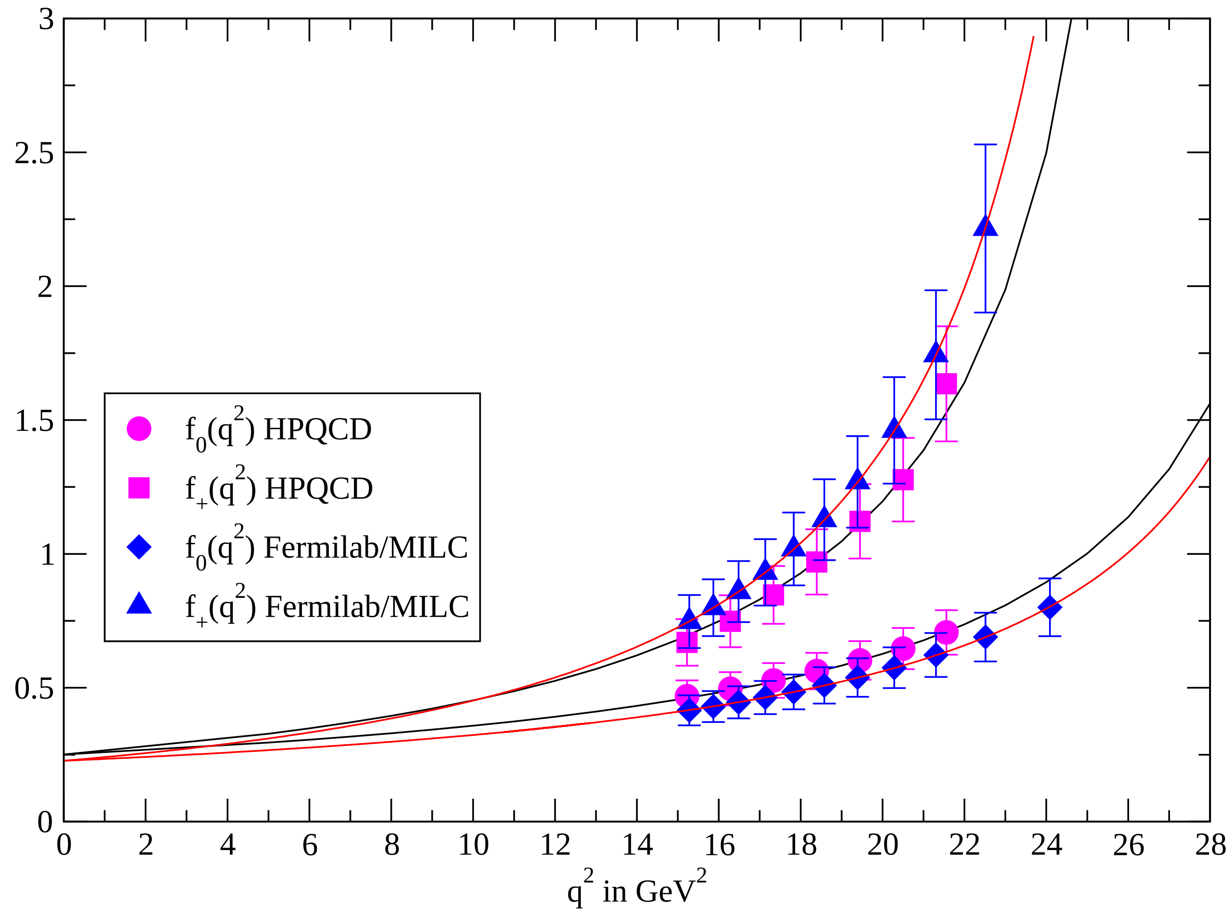
Chiral Extrapolations (cont'd)

HPQCD



Results for Form Factors

11% systematic + 4 ∼ 8% statistical errors added in quadrature



Becirevic-Kaidalov (BK) Parametrization

This ansatz satisfies :

— $f_+(0) = f_0(0)$

— HQET scaling laws

— position of pole at $q^2 = M^2_{B^*}$

$$f_+(q^2)=\frac{C_B\,(1-\alpha_B)}{(1-\tilde{q}^2)(1-\alpha_B\tilde{q}^2)} \qquad f_0(q^2) = \frac{C_B\,(1-\alpha_B)}{(1-\tilde{q}^2/\beta_B)}$$

$(\tilde{q}^2 \equiv q^2/M^2_{B^*})$

	Fermilab/MILC	HPQCD
α_B	0.63(5)	0.41(7)
β_B	1.18(5)	1.18(5)
$f_{0/+}(0)$	0.23(2)	0.25(2)

Extracting $|V_{ub}|$

Lattice results for $f_+(q^2)$ can be integrated

$$\frac{1}{|V_{ub}|^2}\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3} p_\pi^3 \, |f_+(q^2)|^2$$

to yield $\frac{\Gamma}{|V_{ub}|^2} \quad \Longrightarrow \quad \boxed{|V_{ub}|}$

Using branching fractions Γ/Γ_{full} from CLEO [S.B.Athar et al.,PRD 68,072003 (2003)] one finds

	$\vert V_{ub}\vert$ Fermilab/MILC	$\vert V_{ub}\vert$ HPQCD
$16 GeV^2 \leq q^2$	$3.0(4)(6) \times 10^{-3}$	$3.52(44)(73) \times 10^{-3}$
$0 \leq q^2 \leq q^2_{max}$		$3.86(58)(32) \times 10^{-3}$

The first error is from the lattice and second from experiment.

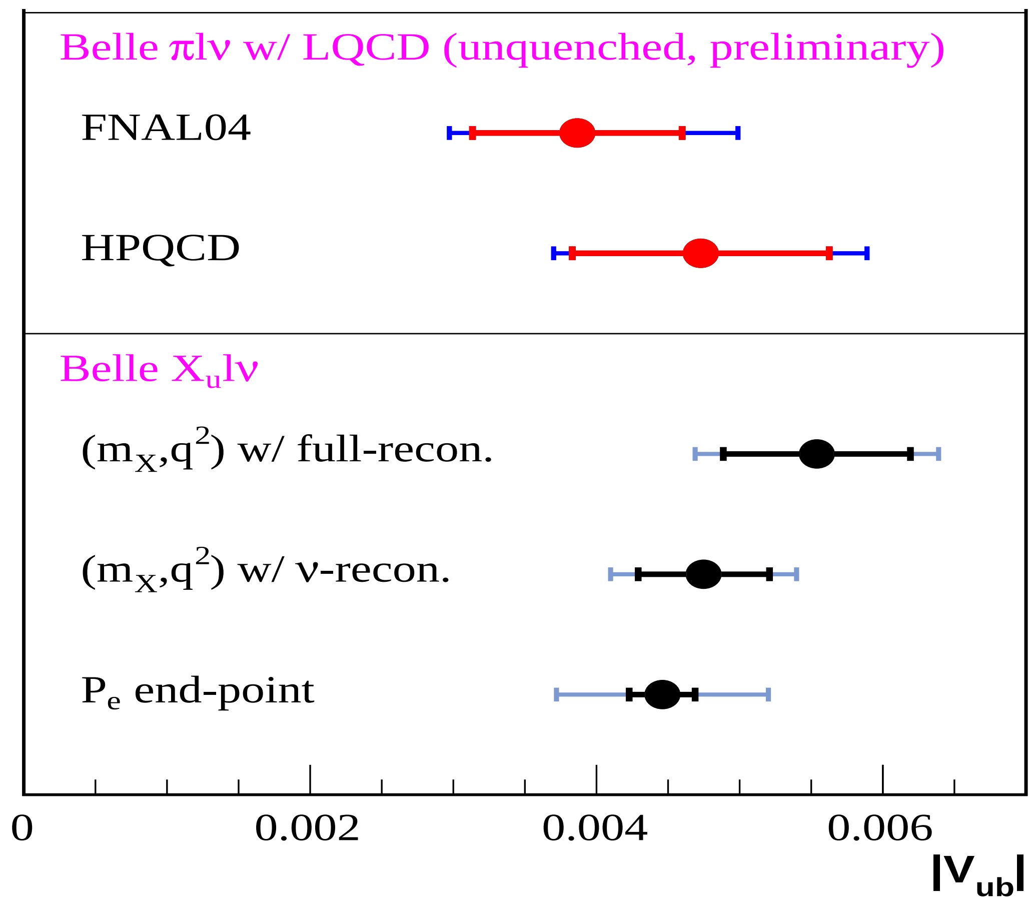
[L.Gibbons, private comm.:

$$|V_{ub}| = (3.84 \pm 0.25(stat) \pm 0.17(sys)) \times 10^{-3} \text{]}$$

(experimental errors only)

Extracting $|V_{ub}|$, Lattice + Belle

[Belle collaboration contribution to ICHEP'04]
(K.Abe et al.,hep-ex/0408145)



Errors in Present Lattice Calculations

Statistical errors are at the $4 \sim 8\%$ level.

Systematic Errors (Fermilab/MILC)

(from M.Okamoto et al.,hep-lat/0409116)

3-pt function	3%
BK fit	4%
chiral extrapolation	4%
matching	1%
α_s uncertainty	1%
finite a error	9%
Total	11%

Errors (cont'd)

Systematic Errors (HPQCD)

	order	error	how to improve	status
matching	α_s^2	9%	do 2-loop matching	about to embark
relativistic + finite a corrections	$\frac{\Lambda}{M}$, $\frac{\alpha_s}{(aM)}$ $\frac{\alpha_s\Lambda}{M}$, $a\alpha_s$	5%	include mixing with Dim.4 currents	done
chiral extrapolations		5%	use $S\chi PT$ check m_l^{sea} dep.	in progress
finite a error in action	$a^2\alpha_s$	2%	improve action finer lattices	in progress
Total		11%		

Future Prospects for Reducing Errors

For Fermilab/MILC, the dominant systematic error appears to be finite lattice spacing errors.

Need to

- improve the heavy quark action
- do the one-loop operator matching including dimension 4 current corrections

This will remove the dominating $\mathcal{O}(a\,\alpha_s)$ error from their calculations.

Just going to finer lattices will also help.

Reducing Errors (cont'd)

For HPQCD, the dominant systematic error comes from uncertainties in higher order operator matching .

Two-loop calculations with our highly improved lattice actions is a daunting task.

However, some members of HPQCD (Q.Mason, H.Trottier) have taken up the challenge. During the past couple of years they have developed "automated lattice perturbation theory". They now plan to apply their methods to two-loop heavy-light current matching calculations.

$$7\% \sim 9\% \quad \longrightarrow \quad 2\% \sim 3\%$$

Reducing Errors (cont'd)

Corrections due to dimension 4 current operators

For V_0, A_0 :

$$
\begin{aligned}
J_0^{(0)}(x) &= \bar{q}(x)\,\Gamma_0\,Q(x), \\
J_0^{(1)}(x) &= \frac{-1}{2M_0}\bar{q}(x)\,\Gamma_0\,\boldsymbol{\gamma}\cdot\boldsymbol{\nabla}\,Q(x), \\
J_0^{(2)}(x) &= \frac{-1}{2M_0}\bar{q}(x)\,\boldsymbol{\gamma}\cdot\overleftarrow{\boldsymbol{\nabla}}\,\gamma_0\,\Gamma_0\,Q(x).
\end{aligned}
$$

and for V_k, A_k :

$$
\begin{aligned}
J_k^{(0)}(x) &= \bar{q}(x)\,\Gamma_k\,Q(x), \\
J_k^{(1)}(x) &= \frac{-1}{2M_0}\bar{q}(x)\,\Gamma_k\,\boldsymbol{\gamma}\cdot\boldsymbol{\nabla}\,Q(x), \\
J_k^{(2)}(x) &= \frac{-1}{2M_0}\bar{q}(x)\,\boldsymbol{\gamma}\cdot\overleftarrow{\boldsymbol{\nabla}}\,\gamma_0\,\Gamma_k\,Q(x), \\
J_k^{(3)}(x) &= \frac{-1}{2M_0}\bar{q}(x)\,\nabla_k\,Q(x) \\
J_k^{(4)}(x) &= \frac{1}{2M_0}\bar{q}(x)\,\overleftarrow{\nabla}_k\,Q(x),
\end{aligned}
$$

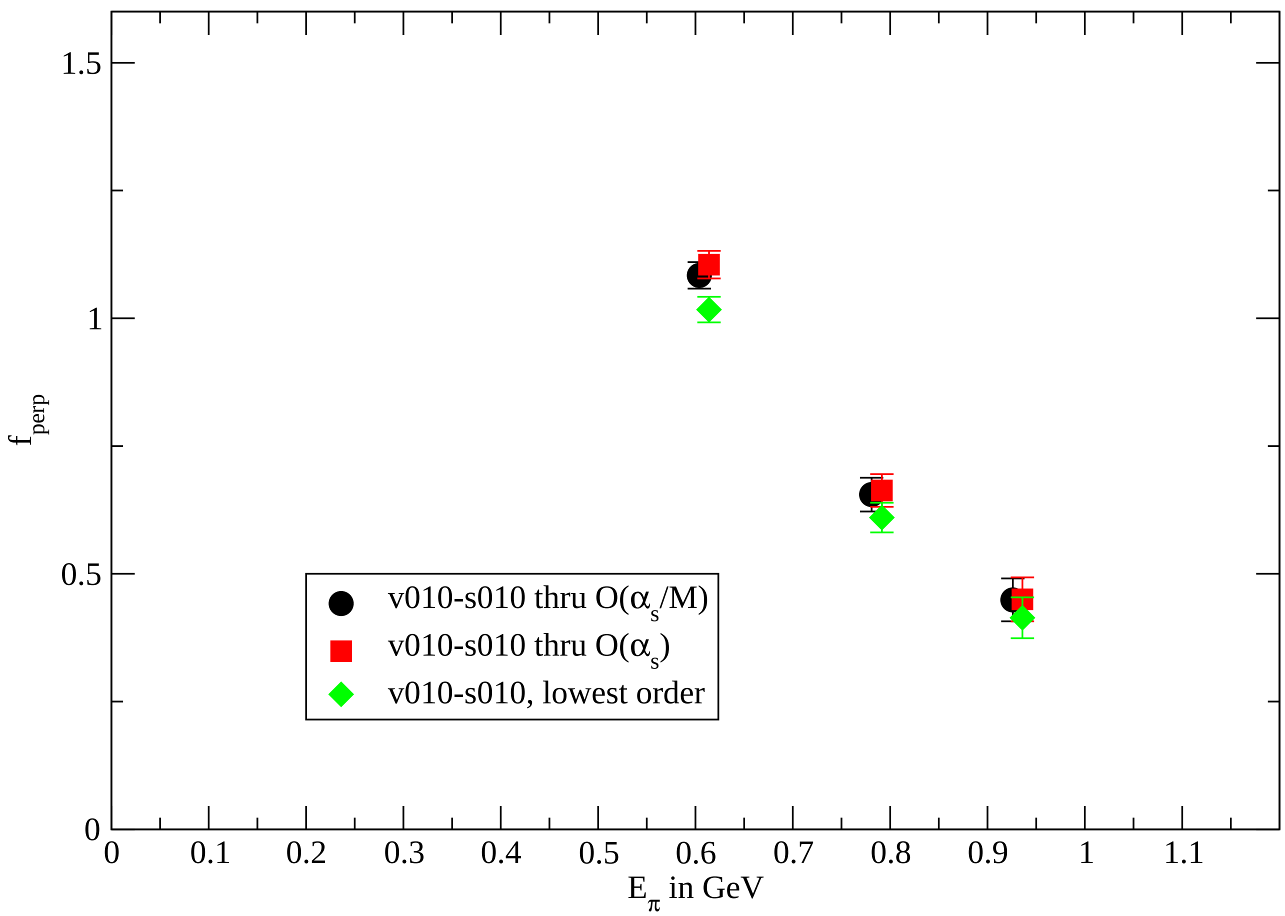
Corrections due to dimension 4 current operators

(HPQCD: Emel Gulez, Matt Wingate, J.S.)

By completing a one-loop calculation we have reduced the "relativistic" and "finite lattice spacing" errors by removing $\mathcal{O}(a\alpha_s)$, $\mathcal{O}(\frac{\Lambda}{M})$, $\mathcal{O}(\frac{\alpha_s}{(aM)}$, $\mathcal{O}(\frac{\alpha_s\Lambda}{M})$ errors.

$$5\% \quad \longrightarrow \quad 3\%$$

Effect of $1/M$ Current Corrections on $f_\perp$ (Emel Gulez)



So, in the Not Too Distant Future

Assume statistical errors of 4% or better.

Then with just one-loop matching one has
7∼9% (pert.) + 3% (disc.) + 3% (chiral) $\longrightarrow$ 8∼10% (syst.)
$\Longrightarrow$ 9∼11% total error

and with two-loop matching one has
2∼3% (pert.) + 3% (disc.) + 3% (chiral) $\longrightarrow$ 4.7∼5.2% (syst.)
$\Longrightarrow$ 6.2∼6.6% total error

Other Issues

Total error in $|V_{ub}|$ comes from both lattice & experimental uncertainties.

All lattice calculations to date are restricted to the region
$q^2 \geq 16 GeV^2$.

- How do experimental errors depend on q^2 ?

- How can we minimize both experimental & lattice errors ?

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Promising approach to carrying out lattice simulations at low q^2.
$\Longrightarrow$ Moving NRQCD (talk by C.Davies)

Table from Recent SciDAC Meeting

(B.Sugar, C.Bernard)

Measurement	CKM Matrix Element	Hadronic Matrix Element	Lattice Errors 0.6 TF-Yr MILC0	Lattice Errors 6.0 TF-Yr MILC1	Lattice Errors 60. TF-Yr MILC2/ DWF1
ε_K ($\bar{K}K$ mixing)	$\mathrm{Im}\, V_{td}^2$	$\hat{B}_K$	12%	5%	3%
ΔM_d ($\bar{B}B$ mixing)	$\lvert V_{td}\rvert^2$	$f_{B_d}^2 B_{B_d}$	16%–26%	8%–10%	6%–8%
$\Delta M_d/\Delta M_s$	$\lvert V_{td}/V_{ts}\rvert^2$	ξ^2	8%	6%	3%–4%
$B \to \pi \ell \nu$	$\lvert V_{ub}\rvert^2$	$\langle \pi \rvert (V-A)_\mu \lvert B \rangle$	10%–13%	5.5%–6.5%	4%–5%
$B \to \binom{D^*}{D} \ell \nu$	$\lvert V_{cb}\rvert^2$	$\lvert \mathcal{F}_{B \to \binom{D^*}{D} \ell \nu}\rvert^2$	3%–4%	1.8%–2%	1%–1.4%

Summary

- "realistic" unquenching and better control over the chiral limit have led to significant improvements in lattice calculations of B semi-leptonic decay form factors.

- lattice errors are currently at the $10 \sim 14\%$ level.

- with a lot of hard work it should be possible to reduce these errors down to $5 \sim 6\%$.

- the lattice community needs to stay in close contact with experimental and phenomenology colleagues.