Taylor expansions of hadron screening masses in chemical potential

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Current results: hep-lat/0410017

Plan of the talk

☞ Introduction
- Properties of hadrons at finite T and μ

☞ Definitions
- Introduction of chemical potential into a theory
- Formulation on the lattice

☞ The prospects of the developed framework
- What can we do?
- Two scenarios: finite T and $\mu=0$, finite T and μ

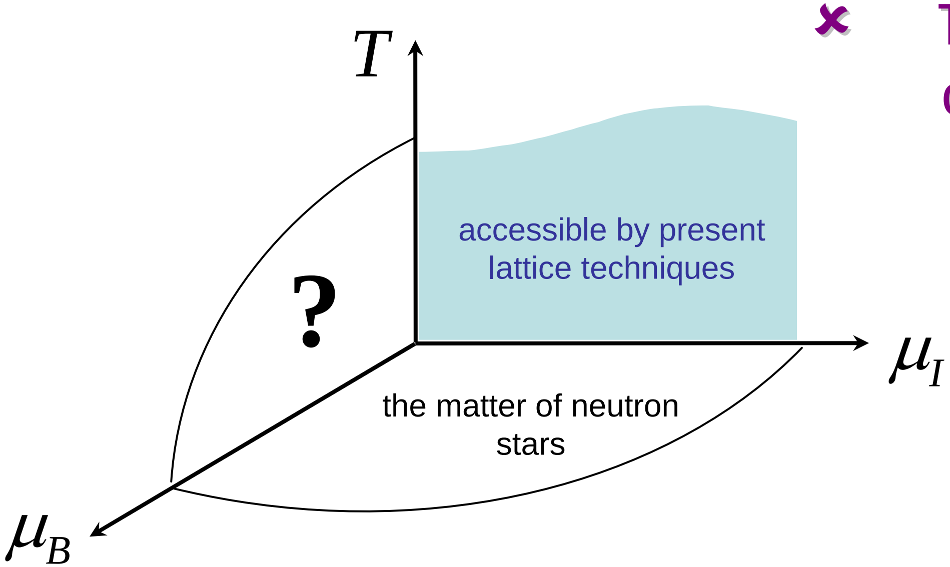
☞ Numerical simulations
- Results and discussions
- Change of hadron properties in the planes (T, μ_I), (T, μ_B) and (T, μ_I, μ_B)

☞ Conclusion

Introduction

Properties of hadrons at finite T

- ✓ Phenomenological models
  (the vector dominance model, QCD sum rules etc.)
- ✓ Lattice QCD simulations
  (QCD Plasma, hadron correlators, charmonium etc.)



Properties of hadrons at finite density

- ✓ Calculations in SU(2)
- ✘ There is no lattice QCD study of hadron pole mass

☞ Our study: the screening masses of various hadrons in the vicinity of zero chemical potential in SU(3)

Introduction

Application of a Taylor series expansion method

✓ The singlet and non-singlet quark number susceptibilities

- *S.Gottlieb et al., Phys.Rev.D55 (1997) 6852)*

✓ The susceptibilities for quenched and 2 flavors QCD (*Phys.Rev.D65 (2002) 054506).* The susceptibilities for 3 flavors with improved staggered fermions (*MILC, hep-lat/0209079*).

✓ The responses of the chiral condensate with respect to the chemical potential at m=0

- *QCD-TARO Collaboration, hep-lat/0110223*

Definitions

A thermodynamical system is described by the partition function

$$Z = Tr e^{-\frac{1}{\kappa T}(H-\mu N)} = \int [DU]\,[D\overline{\psi}]\,[D\psi]\, e^{-\frac{1}{\kappa T}S_G - \overline{\psi}\Delta\psi}$$

$$= \int [DU]\, e^{-\frac{1}{\kappa T}S_G} \det D(m_u,\mu_u) \det D(m_d,\mu_d) \cancelto{1}{\det D(m_s,\mu_s)}$$

For staggered fermions, the fermion matrix

$$D(x,y) = am_q\delta_{x,y} + \frac{1}{2}\sum_{i=1}^{3}\eta_i(x)\left\{U_i(x)\delta_{x+\hat{i},y} - U_i^{+}(y)\delta_{x-\hat{i},y}\right\}$$

$$+\frac{1}{2}\eta_4(x)\left\{\boxed{e^{+a\mu}}\, U_4(x)\delta_{x+\hat{4},y} - \boxed{e^{-a\mu}}\, U_4^{+}(y)\delta_{x-\hat{4},y}\right\}$$

The chemical potential is introduced as,

$$U_t(x) \to e^{a\mu}U_t(x)$$

$$U_t^{+}(x) \to e^{-a\mu}U_t^{+}(x)$$

Taylor series expansion method

A Taylor series expansion of the screening mass *M*

$$\frac{M(\mu)}{T} = \left.\frac{M}{T}\right|_{\mu=0} + \left(\frac{\mu}{T}\right)\left.\frac{\partial M}{\partial \mu}\right|_{\mu=0} + \frac{1}{2}\left(\frac{\mu}{T}\right)^2 \frac{T}{2}\left.\frac{\partial^2 M}{\partial \mu^2}\right|_{\mu=0} + O\left[\left(\frac{\mu}{T}\right)^3\right]$$

For the successful analysis we need

- ✓ to calculate the derivatives of hadron correlators with respect to chemical potential using the lattice QCD
- ✓ to extract the responses of hadron screening masses using two-exponential fitting

$$M(\mu=0),\ dM/d\mu(\mu=0),\ d^2M/d\mu^2(\mu=0)$$

- ✓ to take the chiral limit ($m_q \to 0$) $\Longrightarrow$ physical results

What can we do?

We can …

- ✓ explore a region which currently runs relativistic heavy-ion collision experiments at RHIC
- ✓ Enrich our theoretical understanding of QCD at finite isospin and baryonic density
- ✓ Investigate baryon mass at finite baryonic density in SU(3) by lattice QCD

Our targets:

Mesons
- pseudoscalar and vector

Baryons
- neutron (udd), proton (duu) and Δ

Lattice simulations

Problem: How to get the derivatives of the hadron correlator from lattice simulations?

$$\langle G \rangle = \frac{\int [dU]\, G\, \Delta\, e^{-S_G}}{\int [dU]\, \Delta\, e^{-S_G}}$$

the fermion determinant

$$\Delta = \det\left(D(U;\, \hat{\mu}_u)\right)^{N_f/4} \det\left(D(U;\, \hat{\mu}_d)\right)^{N_f/4}$$

the meson propagator part

$$G = Tr\left[P(\hat{\mu}_u)_{n:0}\Gamma P(\hat{\mu}_d)_{0:n}\Gamma^{+}\right]$$

the baryon correlator

$$\boxed{\sum_{\vec{x}} \varepsilon^{abc}\varepsilon^{a'b'c'} P_{\alpha\alpha'}^{aa'}(x;0) P_{\beta\beta'}^{bb'}(x;0) P_{\gamma\gamma'}^{cc'}(x;0)}$$

the quark propagator

$$P(\hat{\mu}) = D(U;\, \hat{\mu})^{-1}$$

Run parameters

2-flavor SU(3) gauge theory

- Lattice size: $12^2 \times 24 \times 6$ with $N_f=2$ KS fermions
- Quark masses: $ma=0.10$, 0.05 and 0.025
  lattice spacing $a=0.09\sim0.27fm$, temperature $0.5\sim1.6T_c$
- Setting the scale (Tamhankar, Nucl.Phys.(Proc.Suppl.)83(2000)212)
- "corner" wall source with Coulomb gauge fixing
- Stochastic method with 200 complex vectors
  (estimation of the traces of various fermionic operators)

Statistics:

- R-algorithm, 1000 confs./10 unit length trajs.
- MD step size, 0.02

Chemical potential for two flavor system:

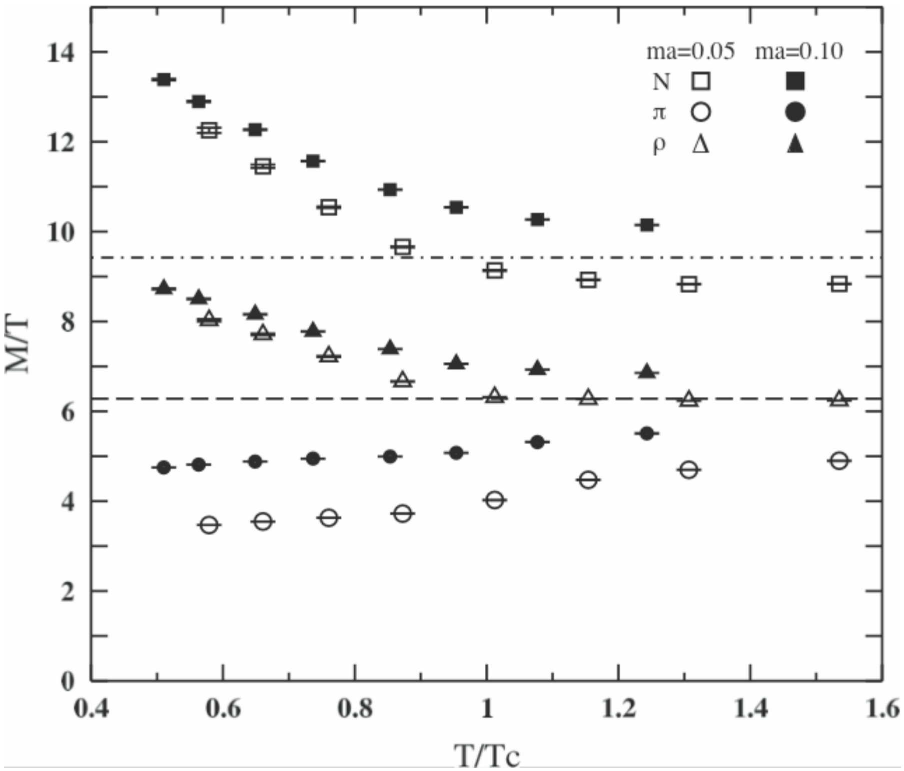
☞ isoscalar (baryon) μ_s, $\mu_s=\mu_u=\mu_d$
☞ isovector (isospin) μ_v, $\mu_v=\mu_u=-\mu_d$

M(μ=0) at finite T

$$C_\pi(z) = C_1\left(e^{-\hat{m}_1\hat{z}} + e^{-\hat{m}_1(N_z-\hat{z})}\right) + C_2\left(e^{-\hat{m}_2\hat{z}} + e^{-\hat{m}_2(N_z-\hat{z})}\right)$$

$$C_\rho(z) = C_1'\left(e^{-\hat{m}_1'\hat{z}} + e^{-\hat{m}_1'(N_z-\hat{z})}\right) + C_2' \boxed{(-1)^{\hat{z}}} \left(e^{-\hat{m}_2'\hat{z}} + e^{-\hat{m}_2'(N_z-\hat{z})}\right)$$

$$C_N(z) = C_1''\left(e^{-\hat{m}_1''\hat{z}} + \boxed{(-1)^{\hat{z}}}\, e^{-\hat{m}_1''(N_z-\hat{z})}\right) + C_2''\left(\boxed{(-1)^{\hat{z}}}\, e^{-\hat{m}_2''\hat{z}} + e^{-\hat{m}_2''(N_z-\hat{z})}\right)$$



$dM(\mu=0)/d\mu$ at finite T

Pseudoscalar and vector mesons

☞ The first order response of mesons to the chemical potential is equal to 0!

Baryons

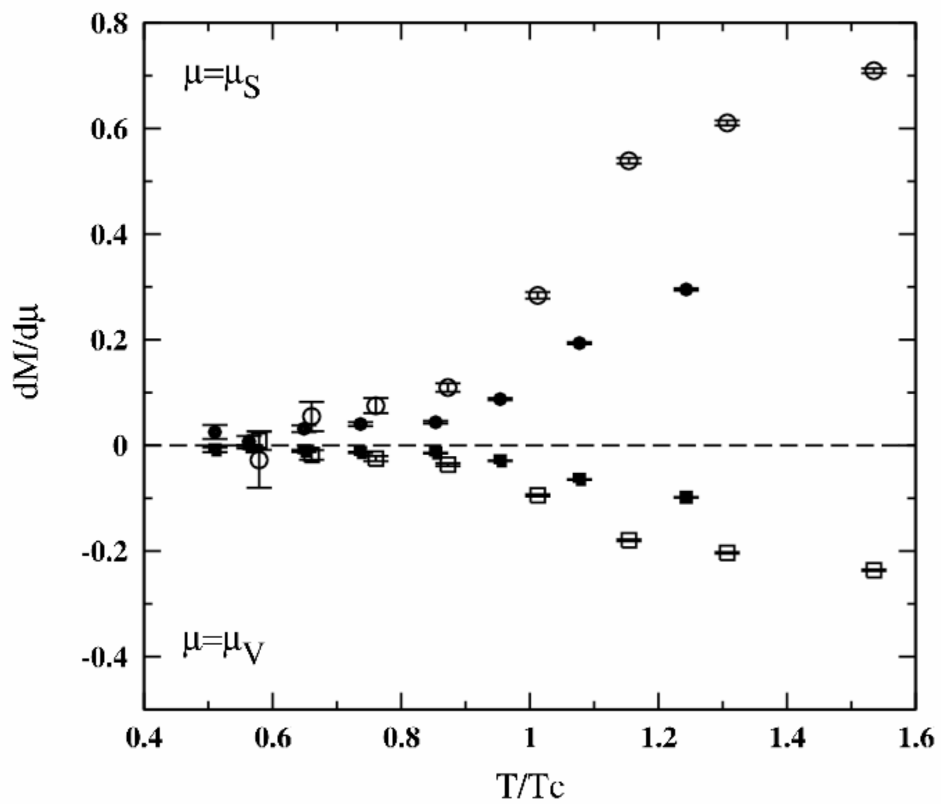
$$\left.\frac{\partial M_{neutron}}{\partial \mu_S}\right|_{\mu_S=0} = \left.\frac{\partial M_{proton}}{\partial \mu_S}\right|_{\mu_S=0}$$

$$\left.\frac{\partial M_{neutron}}{\partial \mu_V}\right|_{\mu_V=0} = -\left.\frac{\partial M_{proton}}{\partial \mu_V}\right|_{\mu_V=0}$$

ma=0.05 - open symbols

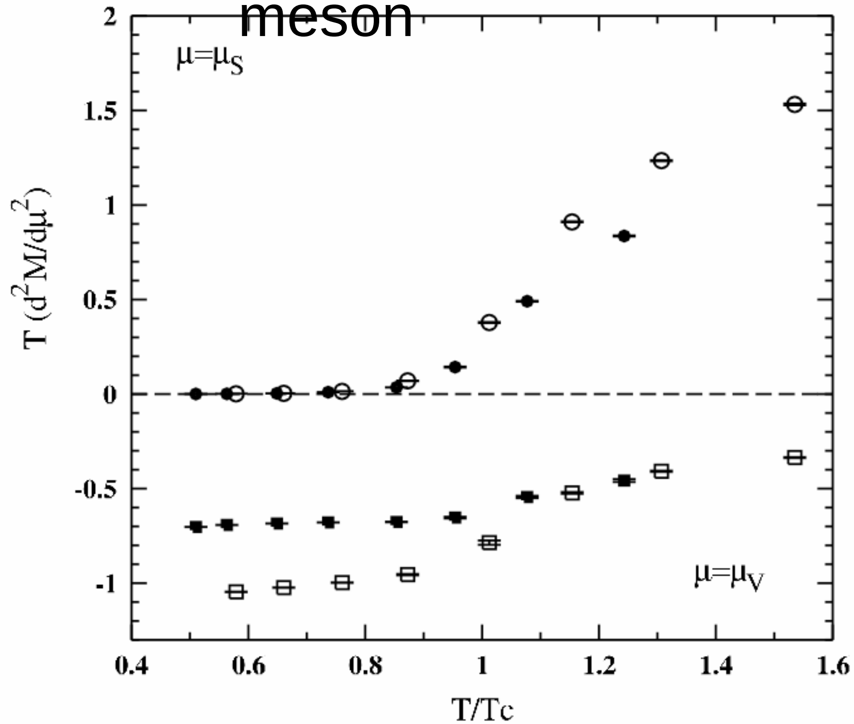
ma=0.10 - filled symbols

Nucleon

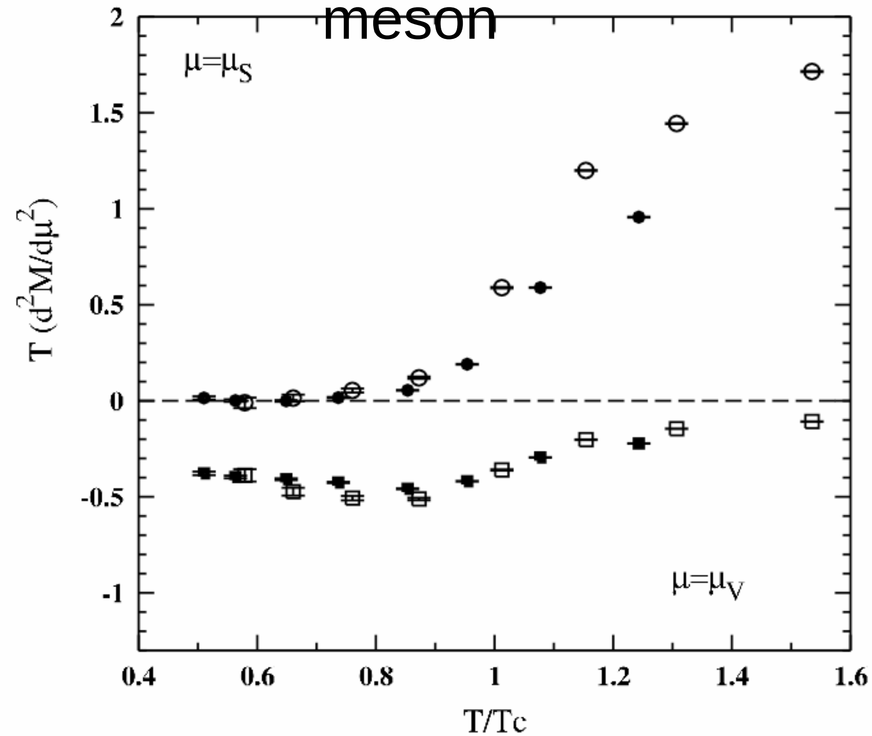


$d^2M(\mu=0)/d\mu^2$ at finite T

Pseudoscalar meson



Vector meson



☞ the confined mesons feel baryon density effect little

☞ the mesons become heavier with T

☞ The response to μ_V is negative and becomes smaller at large T

$\Rightarrow$ the mass becomes zero around $\mu_V \sim M$

$d^2M(\mu=0)/d\mu^2$ at finite T

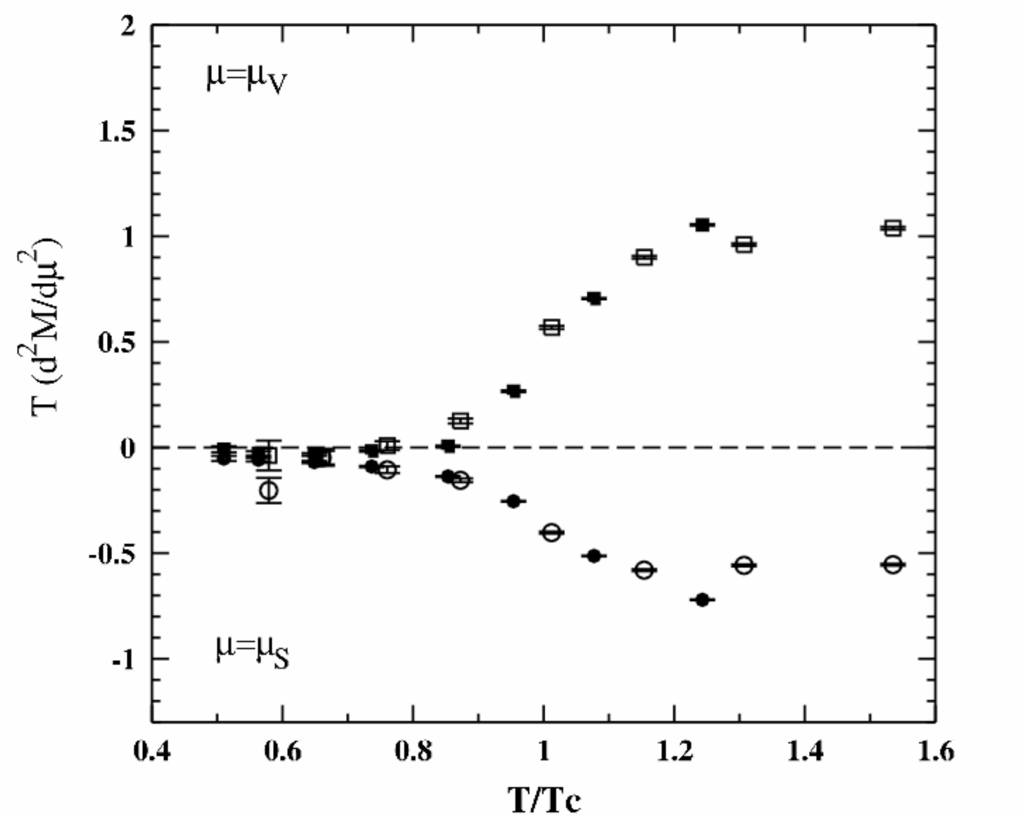
The second order term is of the same order as $dM/d\mu$

$\Rightarrow$ nucleon mass has less chemical potential effect

The difference between two quark mass cases is very small till $1.1T_c$

ma=0.05 - open symbols

ma=0.10 - filled symbols



Hadrons in the plane (T, μ_B)

Motivation

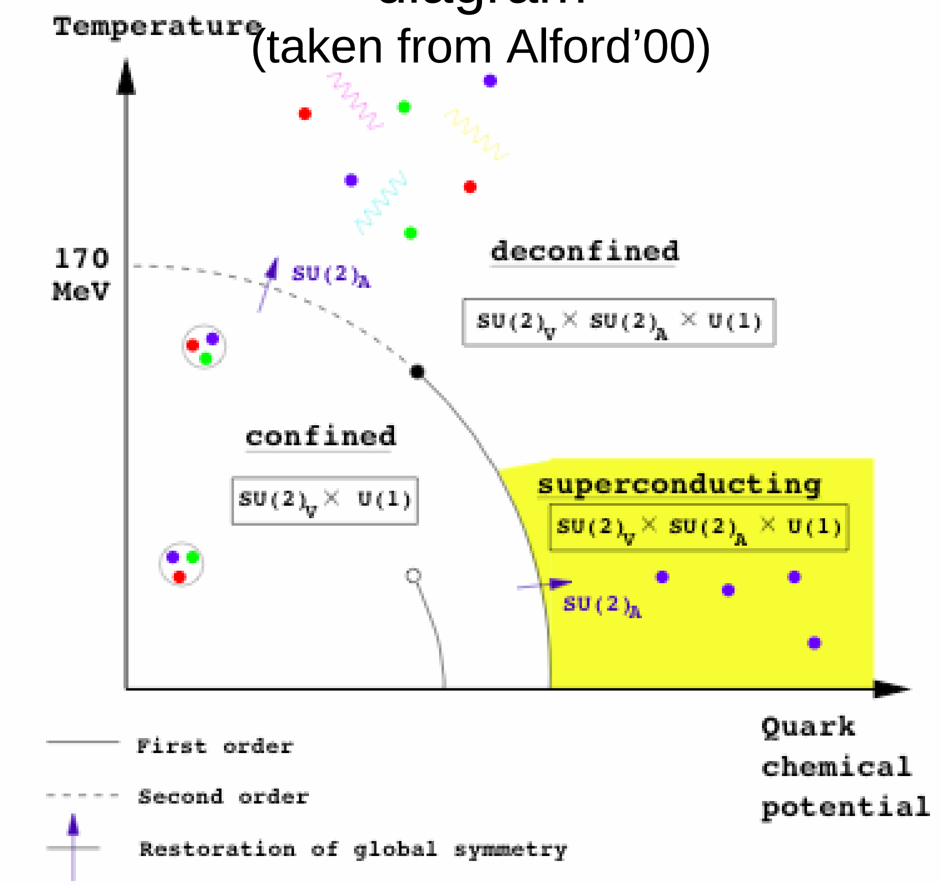
☞ Nature provides us with isospin-asymmetric matter

$$\mu_I << \mu_B$$

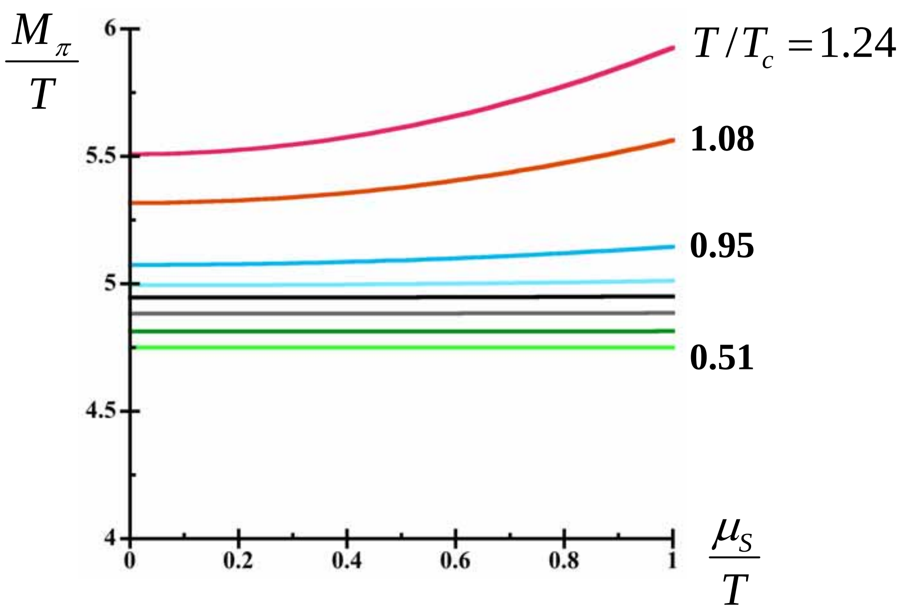
☞ A prime goal of heavy ion collision experiments at SPS, LHC (CERN) and RHIC (Brookhaven) is to probe the transition from hadronic matter to a quark gluon plasma at high T and small baryon density

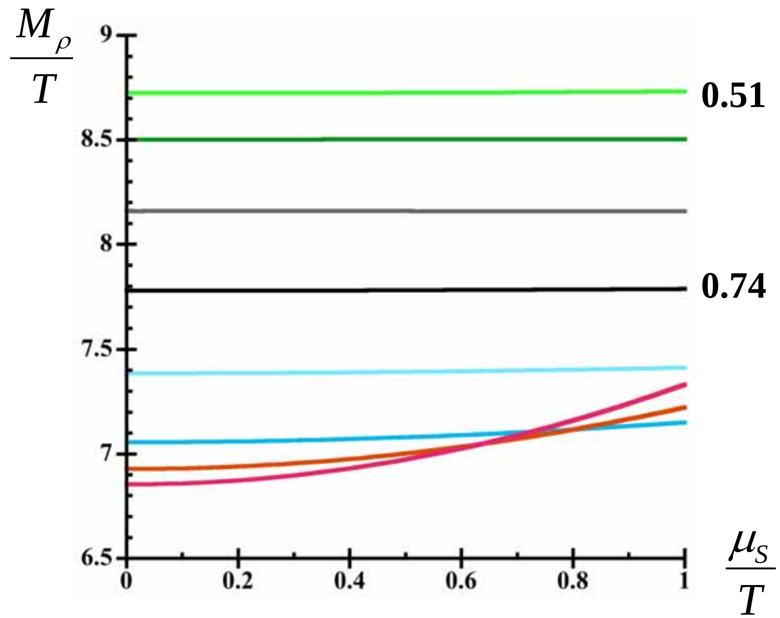
Two massless flavor phase diagram

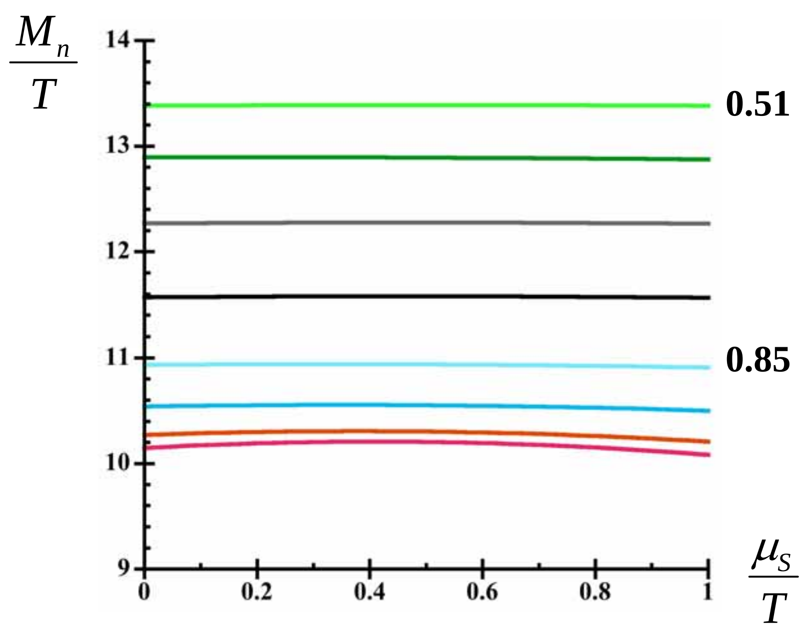
(taken from Alford'00)



Hadrons in the plane (T, μ_S)







$ma = 0.10$

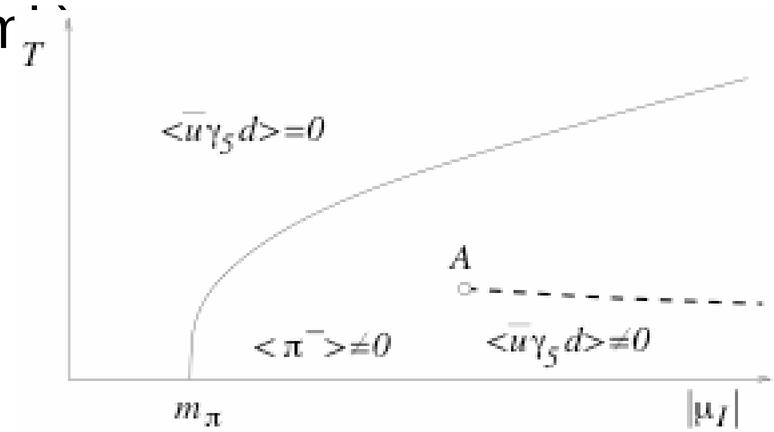
Hadrons in the plane (T, μ_I)

Main features

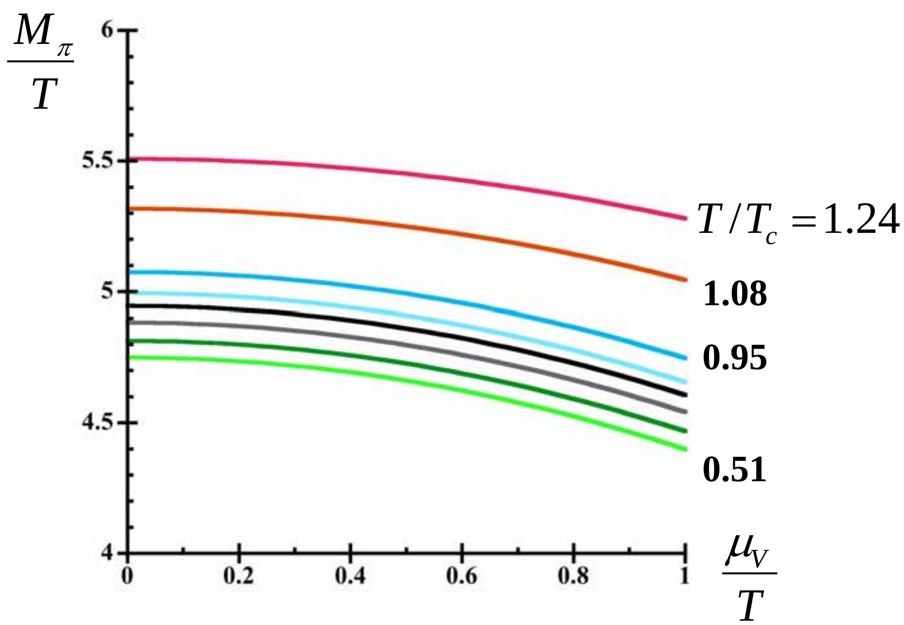
- ✓ The theory has no fermion sign problem
- ✓ Low μ_I, chiral perturbation theory is applicable
- ✓ Assimptotically high μ_I, perturbative QCD is at work

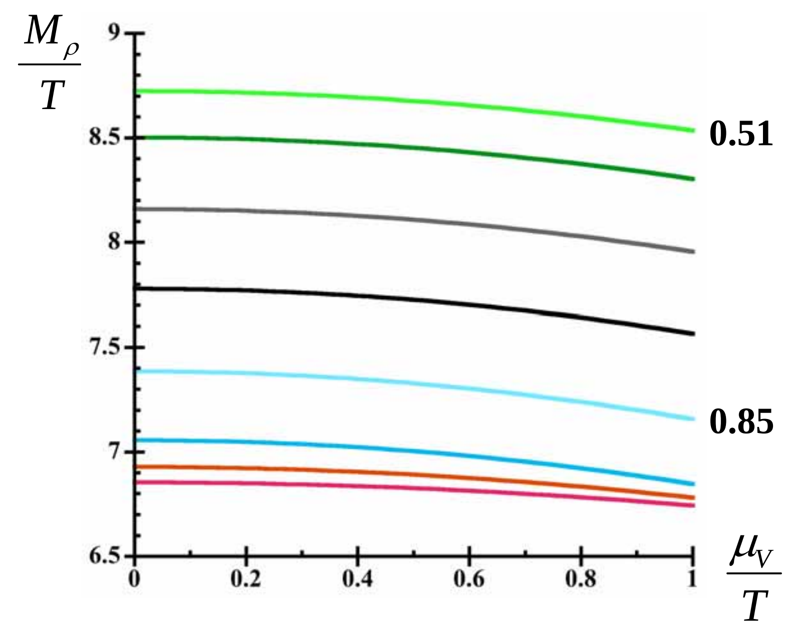
Drawbacks

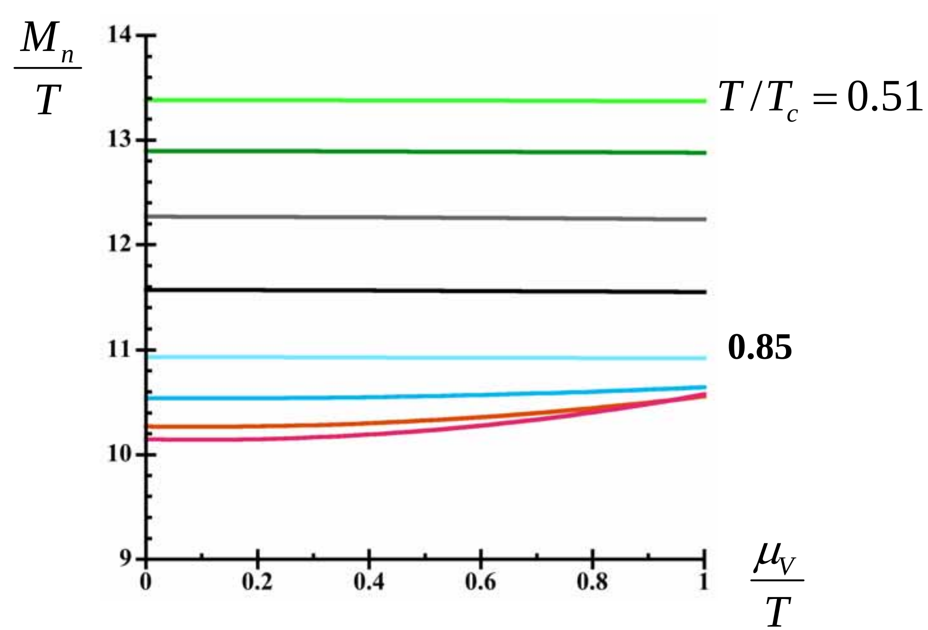
- ✓ A system is unstable with respect to weak decays (no thermodynamic lim



Hadrons in the plane (T, μ_V)







$ma = 0.10$

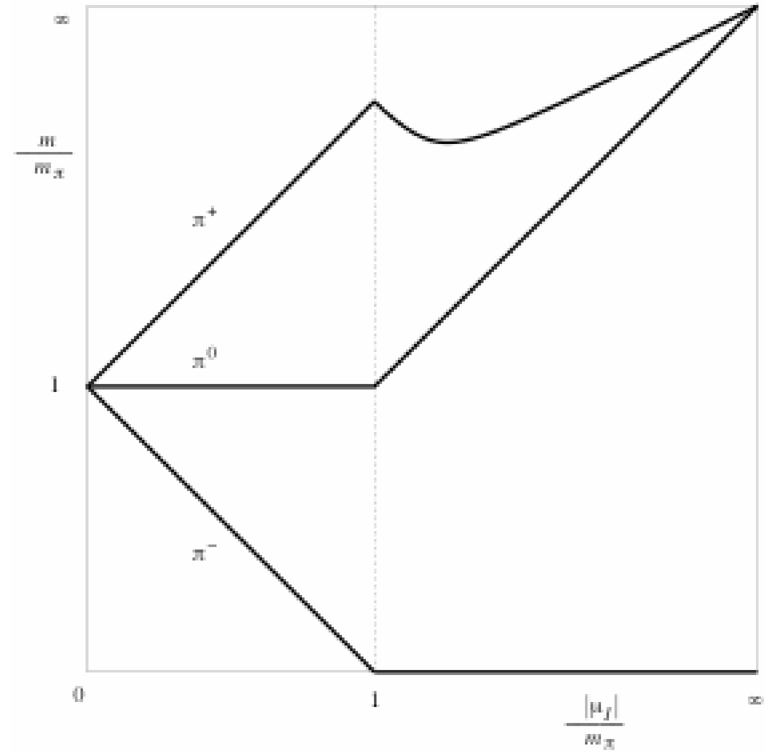
Chiral perturbation theory $\mu_I << m_\rho$, $T=0$

The low-energy dynamics is governed by the chiral Lagrangian, with the matrix pion field Σ

$$L_{eff} = \frac{f_\pi^2}{4} Tr \nabla_\nu \Sigma \nabla_\nu \Sigma^+ - \frac{m_\pi^2 f_\pi^2}{2} \mathrm{Re}\, Tr\Sigma$$

The chemical potential is included in the dervative

$$\nabla_0 \Sigma = \partial_0 \Sigma - \frac{\mu_I}{2}(\tau_3 \Sigma - \Sigma \tau_3), \qquad \nabla_i \Sigma = \partial_i \Sigma$$



Effects of μ_I on the baryon masses

☞ the π^-'s in the condensate tend to repel the baryons, lifting up their masses

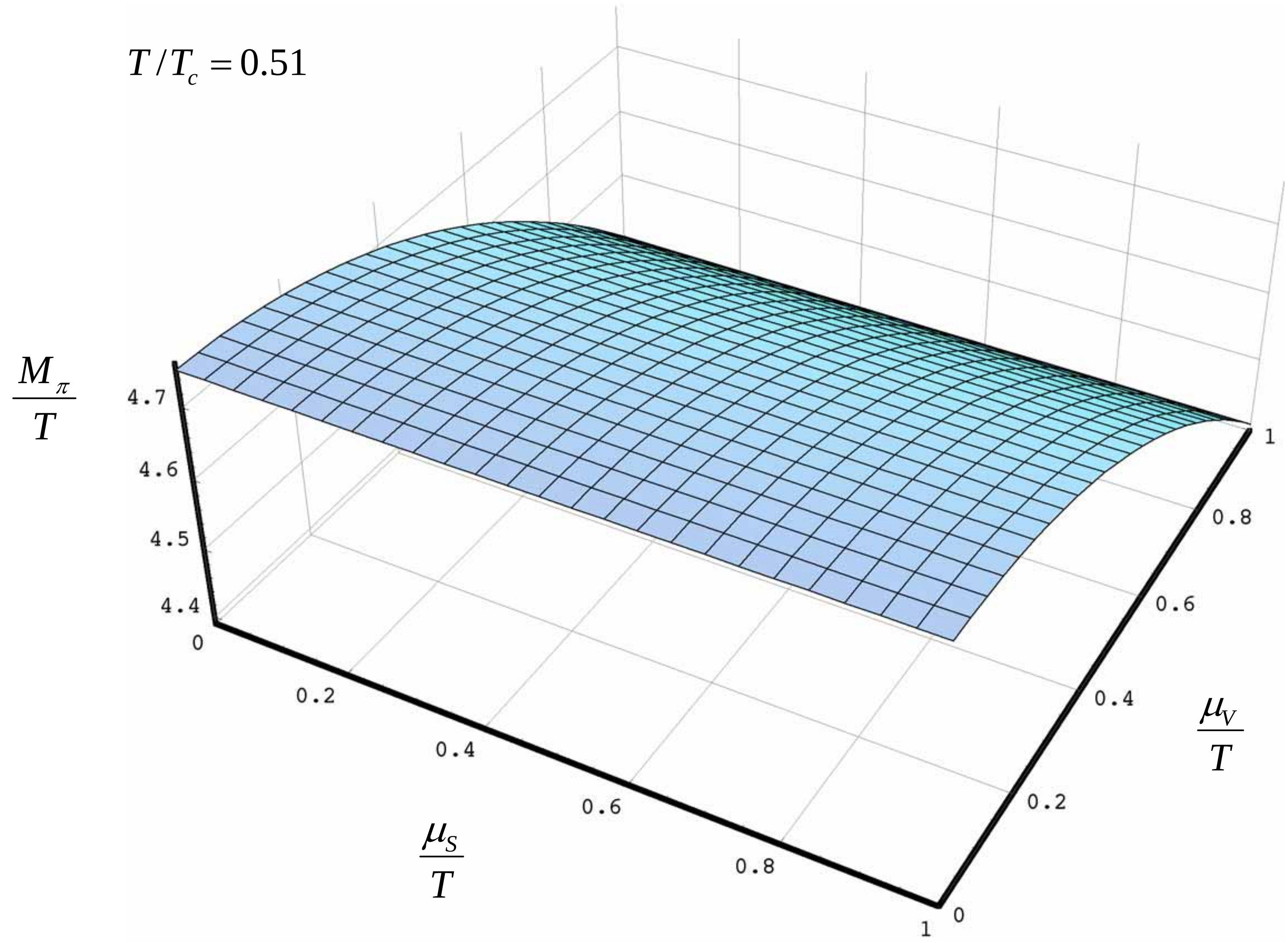
☞ the nucleon mass eigenstate becomes a superposition of vacuum n and p states

☞ the baryon mass never drops to zero!

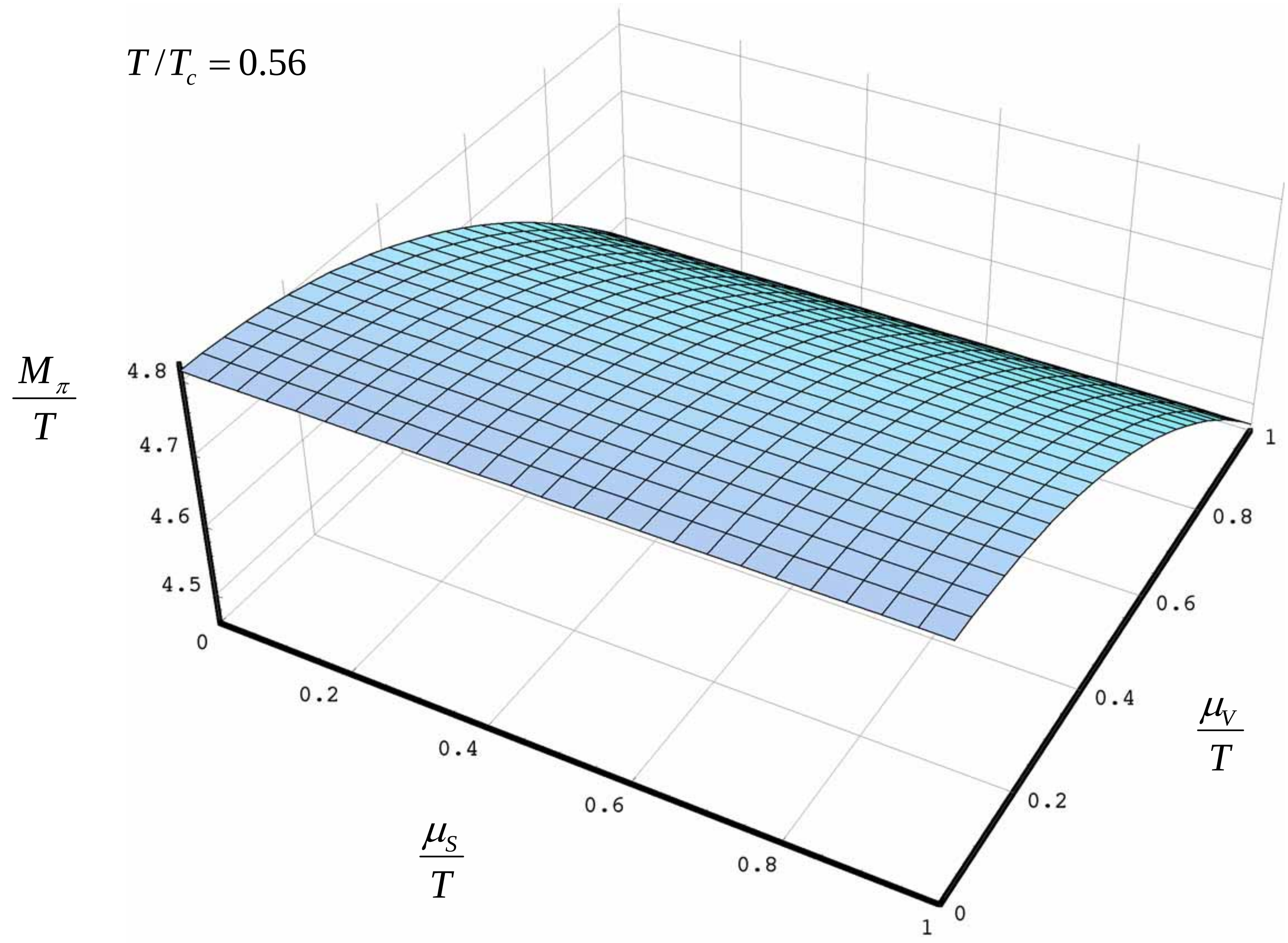
$$M_n = M_n(\mu_I = 0) - \frac{|\mu_I|}{2} \quad \text{if } |\mu_I| < m_\pi$$

Son & Stephanov

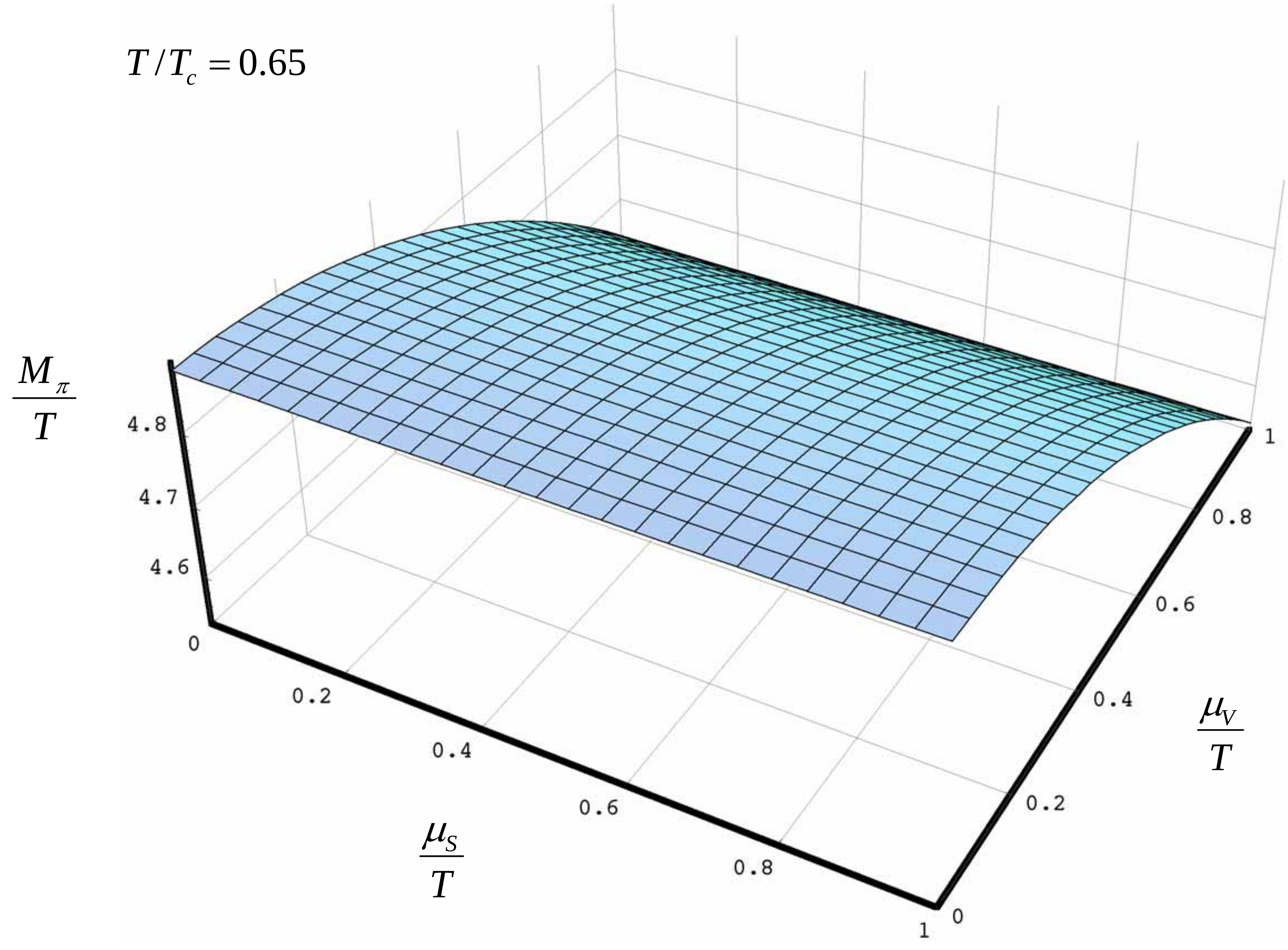
PS-meson in the plane (μ_I, μ_B)



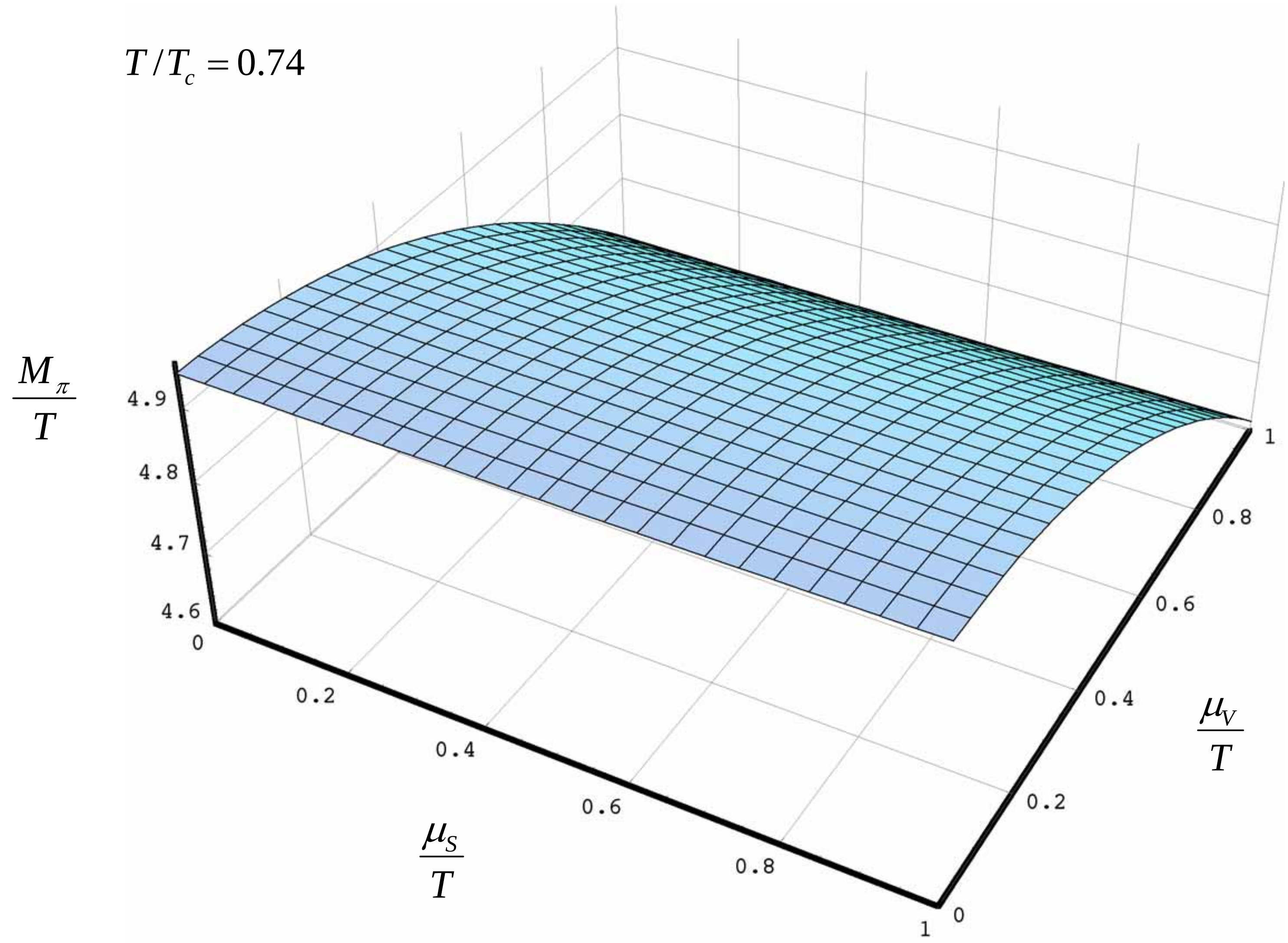
PS-meson in the plane (μ_I, μ_B)



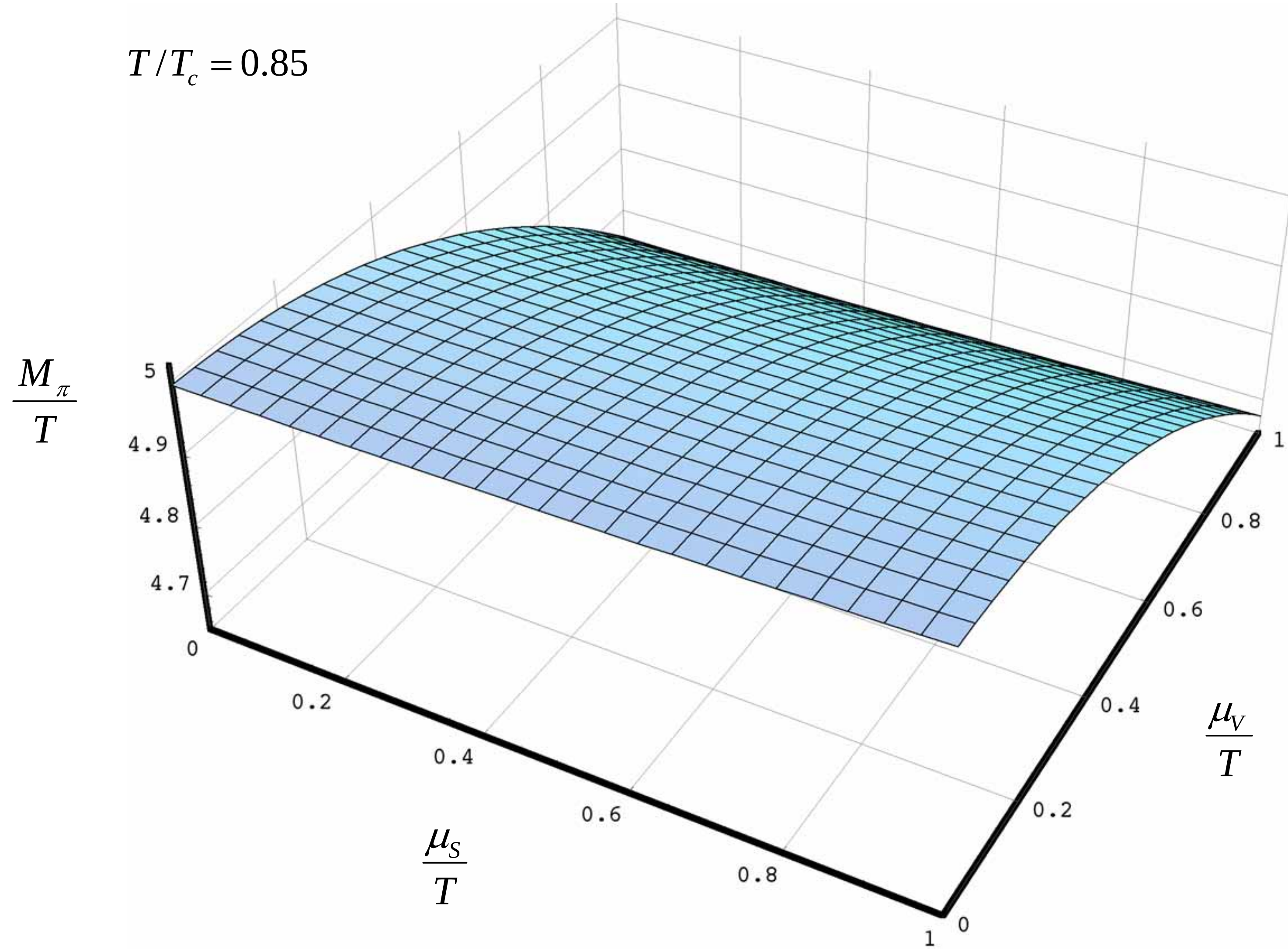
PS-meson in the plane (μ_I, μ_B)



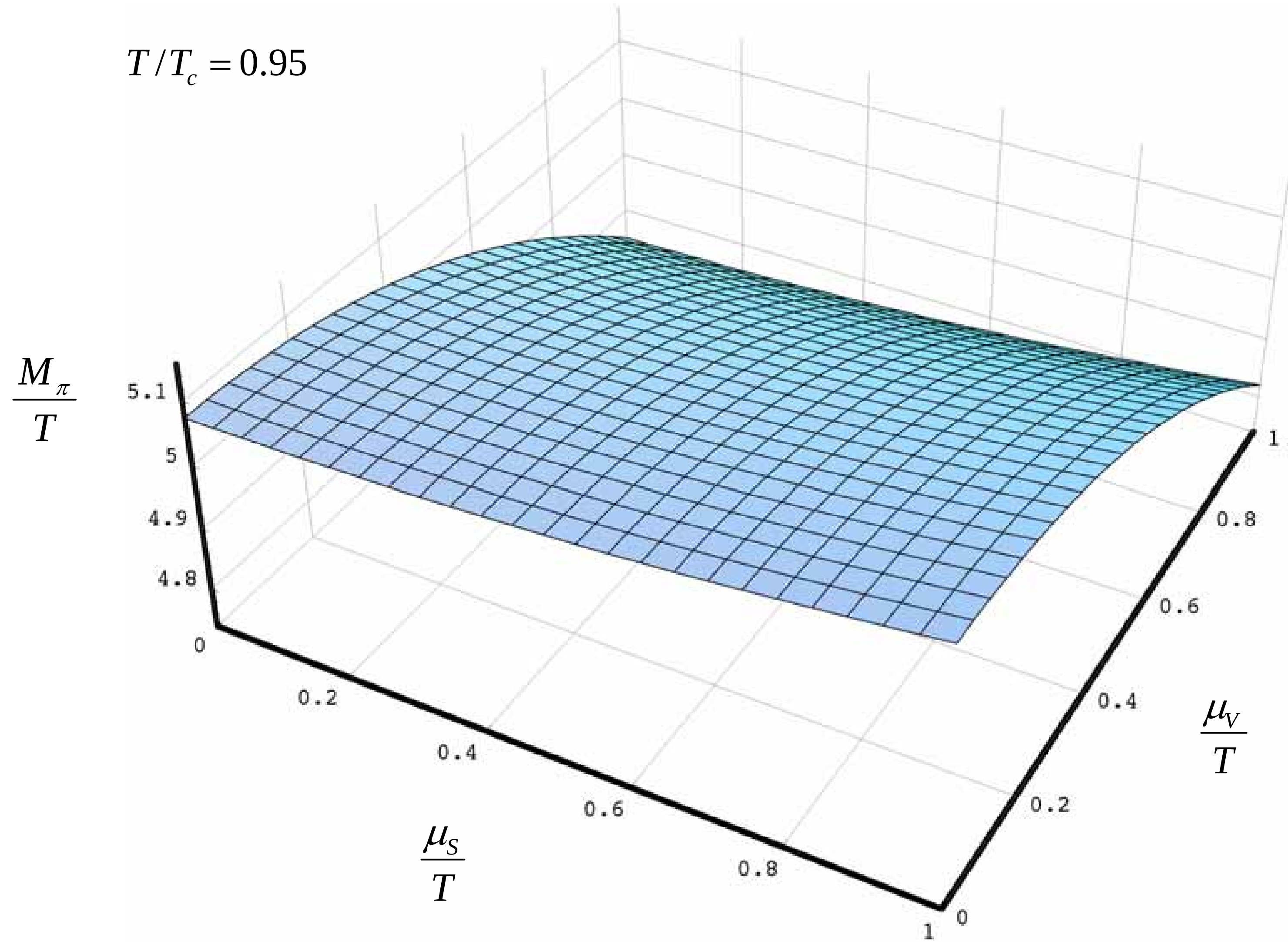
PS-meson in the plane (μ_I, μ_B)



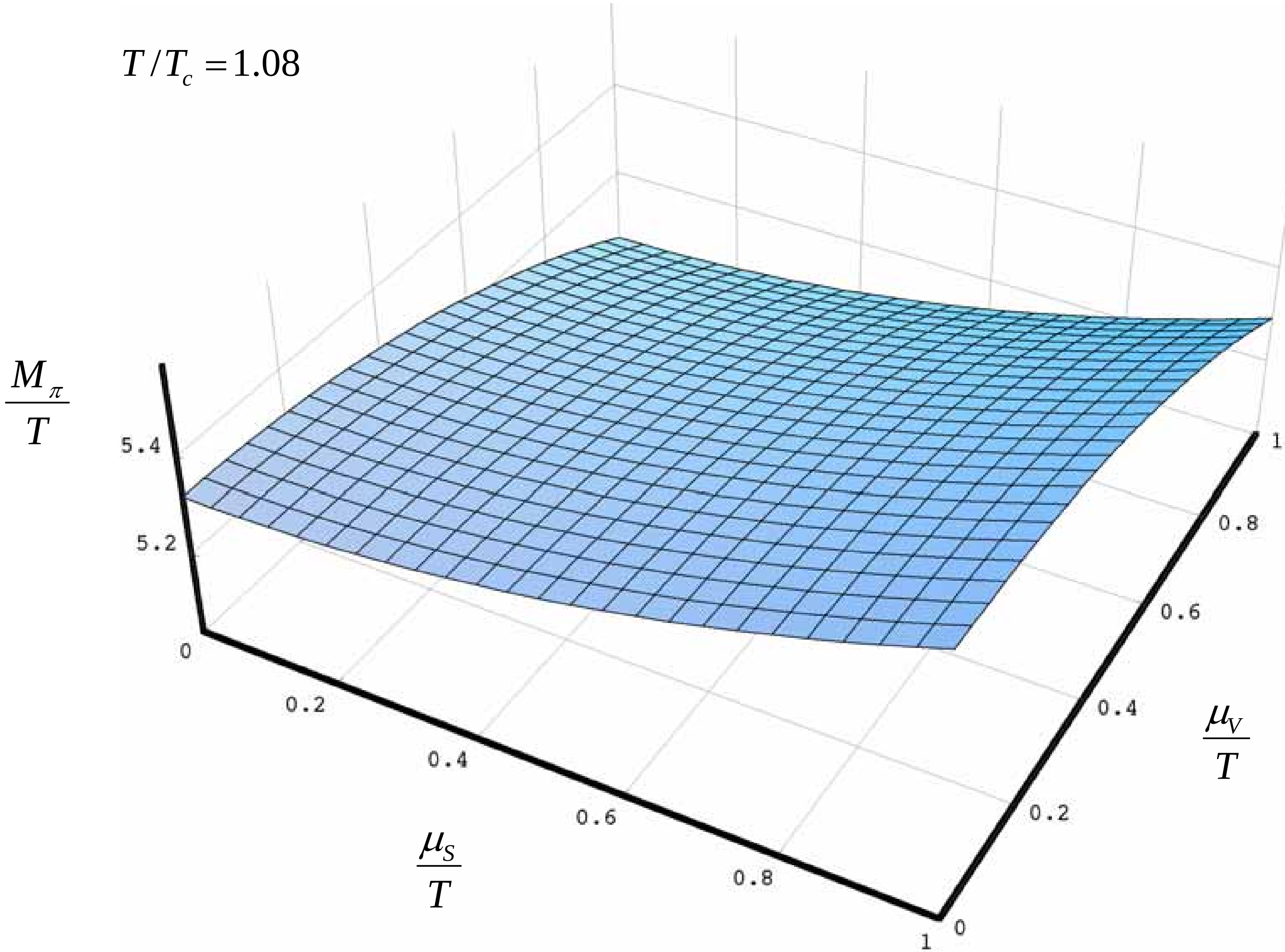
PS-meson in the plane (μ_I, μ_B)



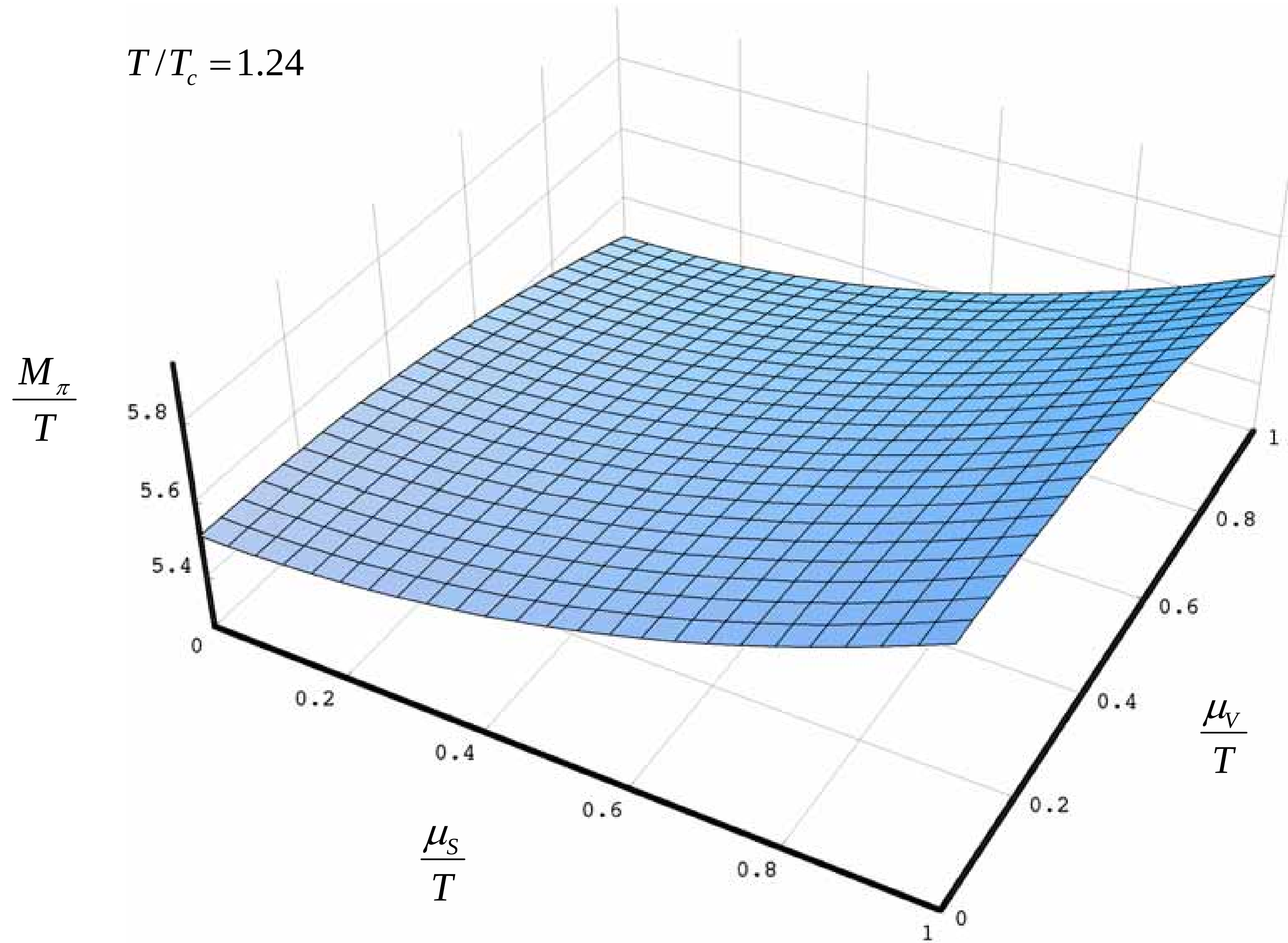
PS-meson in the plane (μ_I, μ_B)



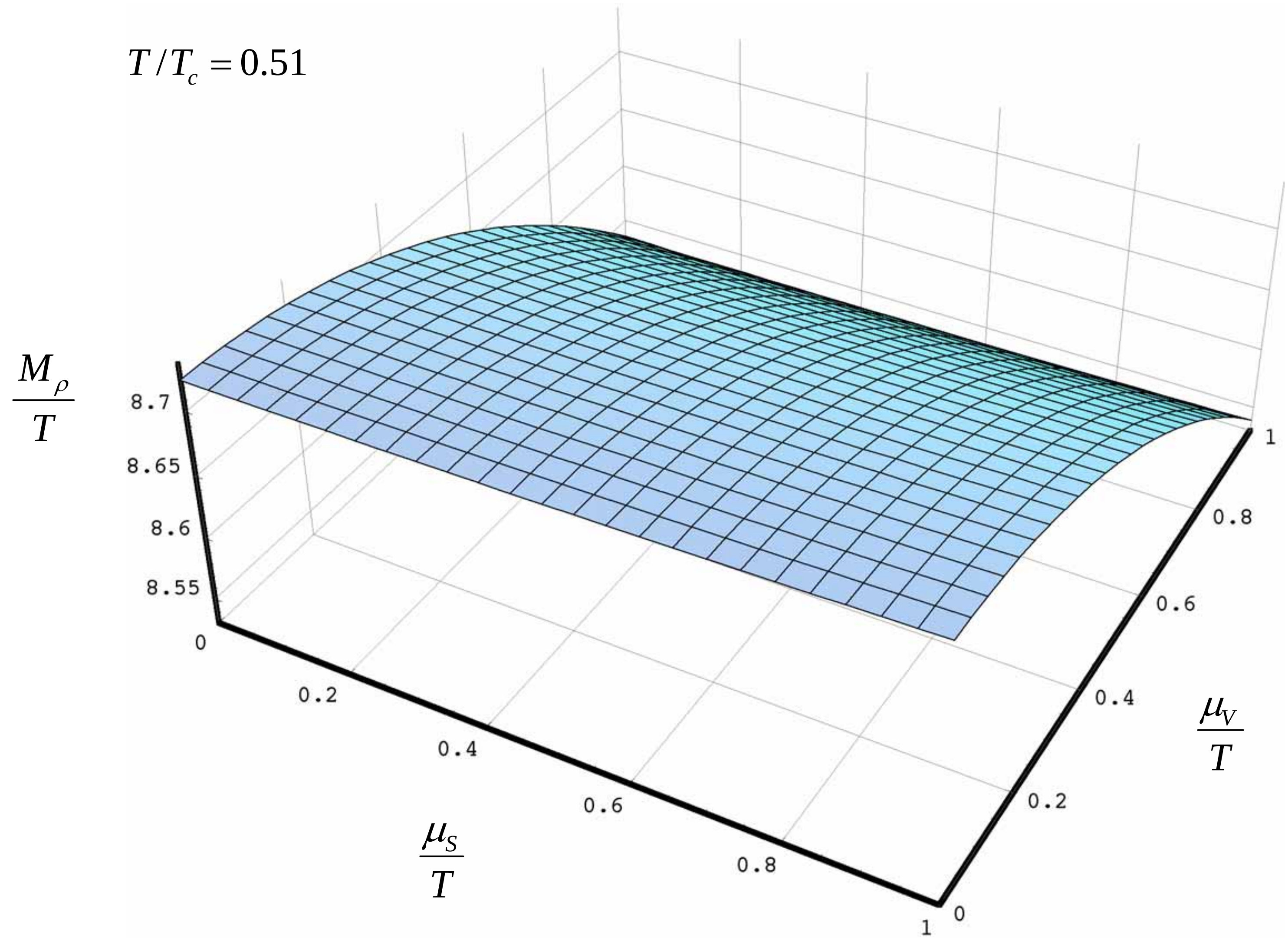
PS-meson in the plane (μ_I, μ_B)



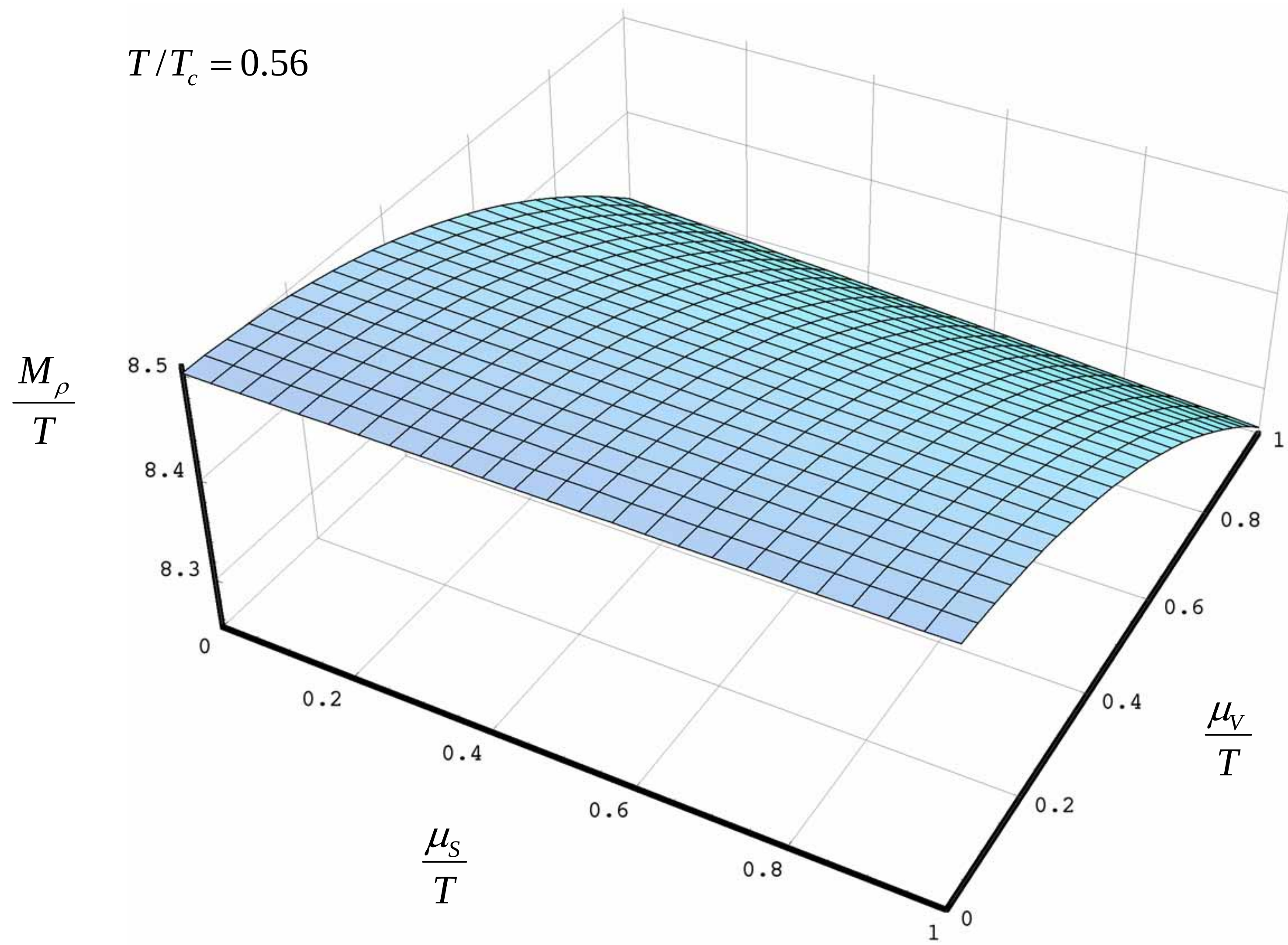
PS-meson in the plane (μ_I, μ_B)



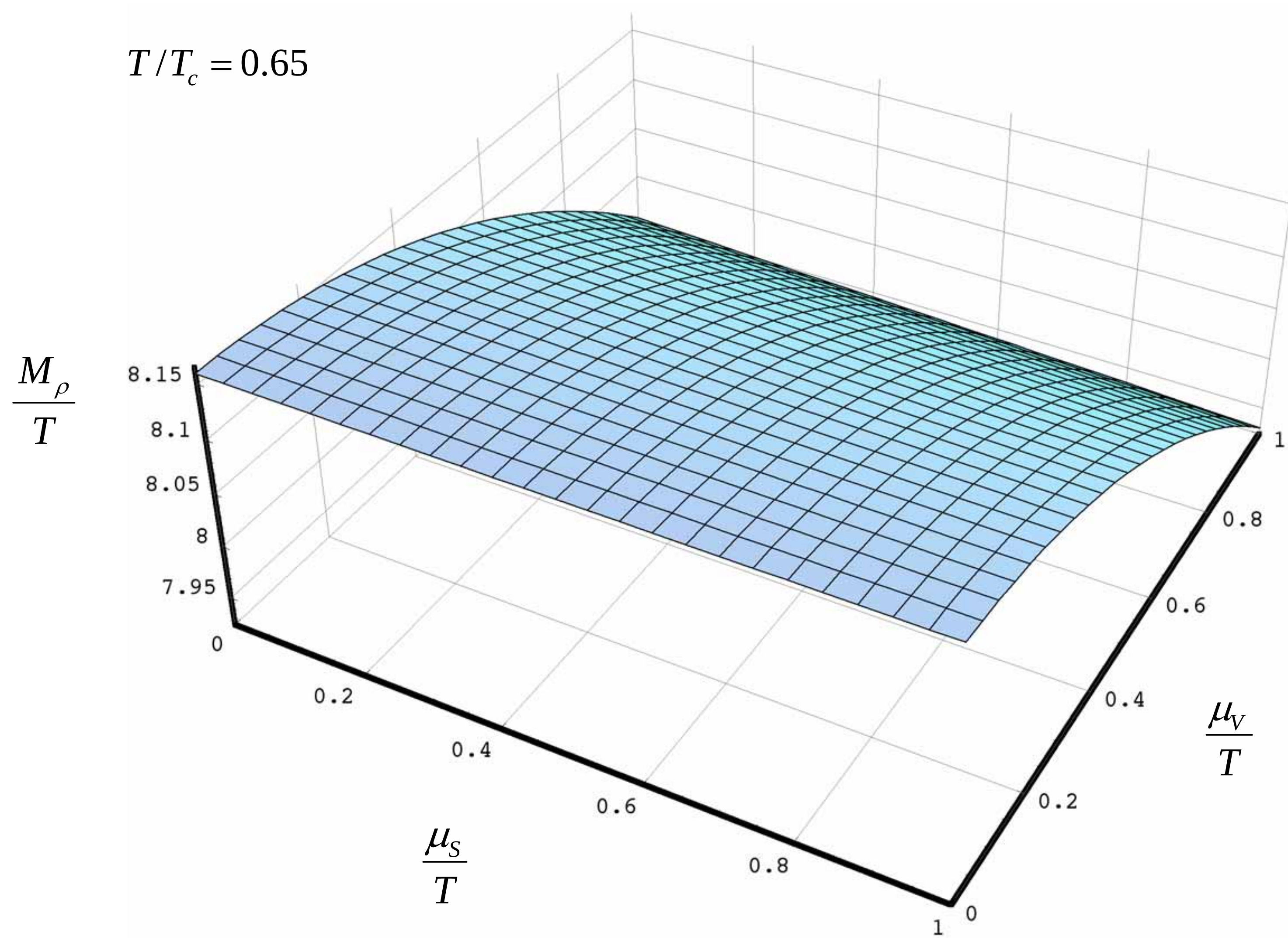
V-meson in the plane (μ_I, μ_B)



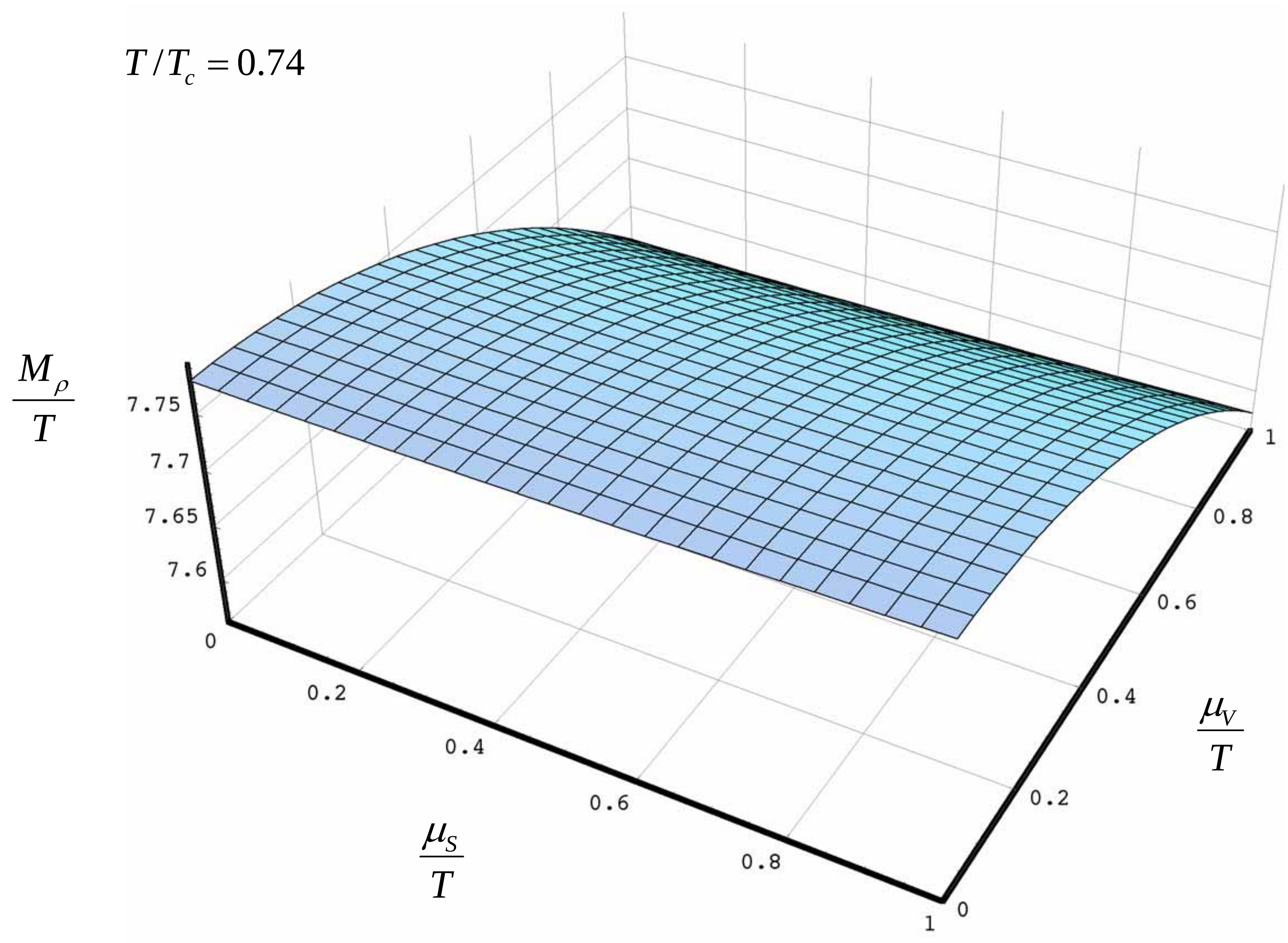
V-meson in the plane (μ_I, μ_B)



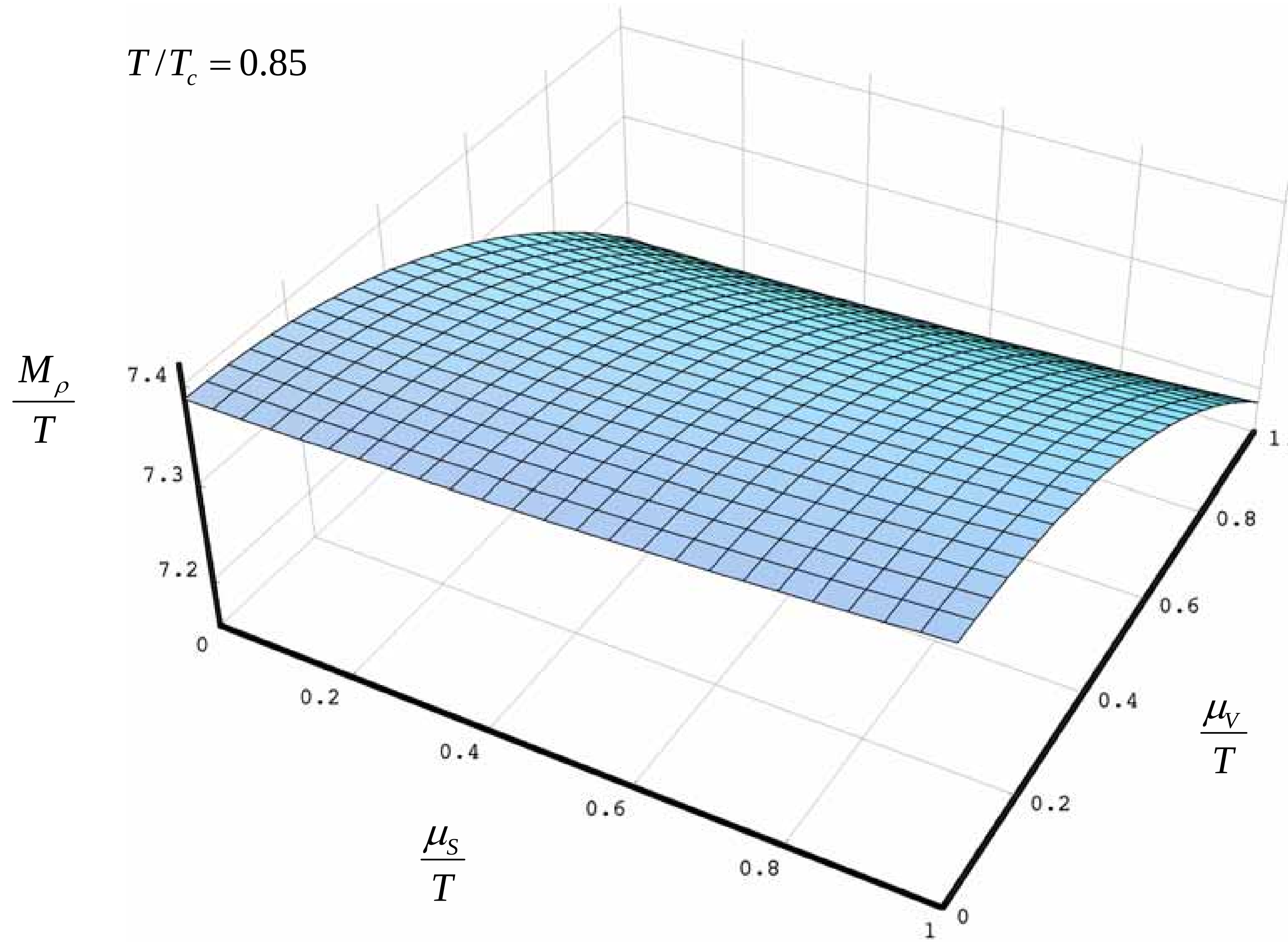
V-meson in the plane (μ_I, μ_B)



V-meson in the plane (μ_I, μ_B)



V-meson in the plane (μ_I, μ_B)



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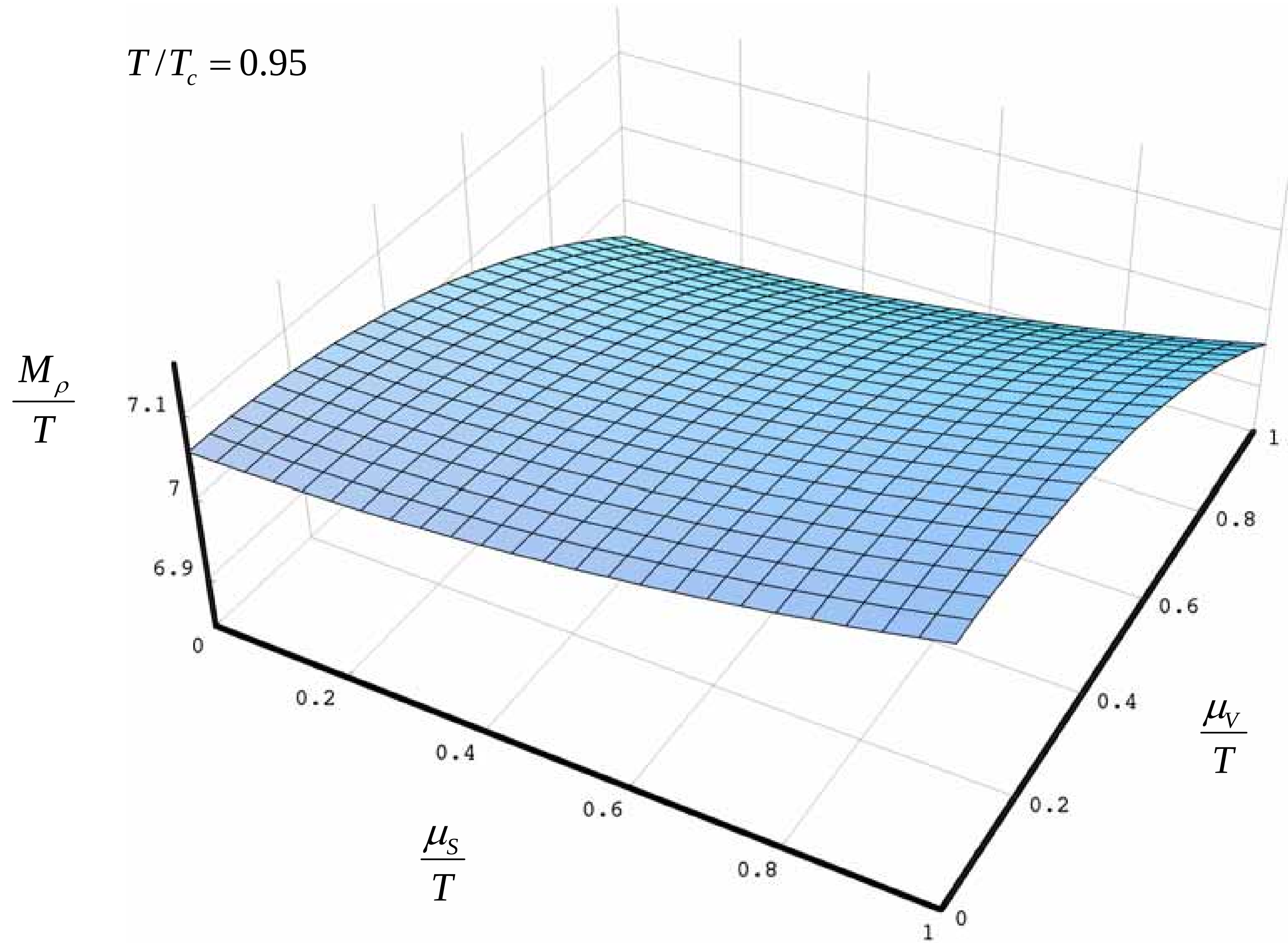
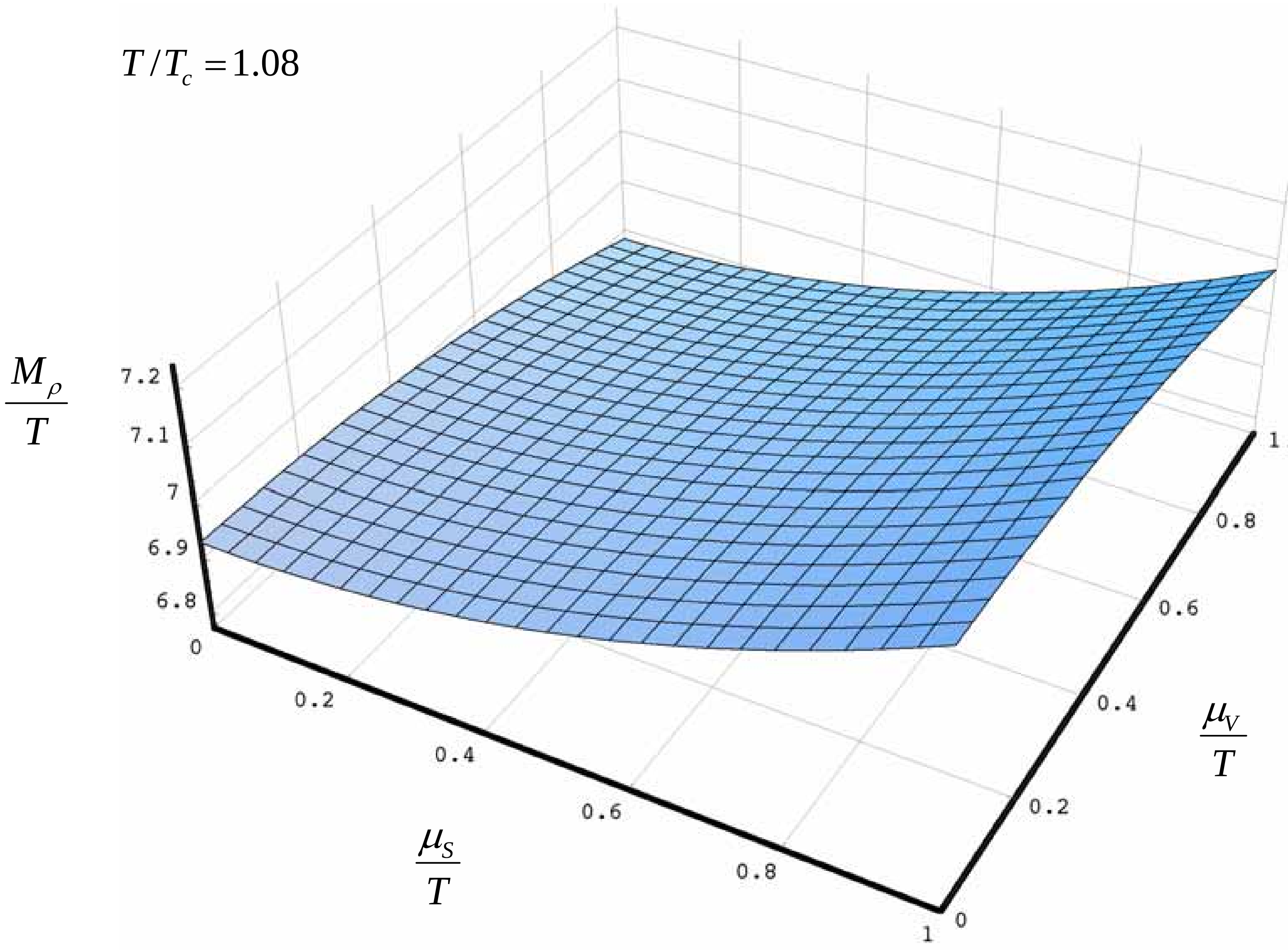
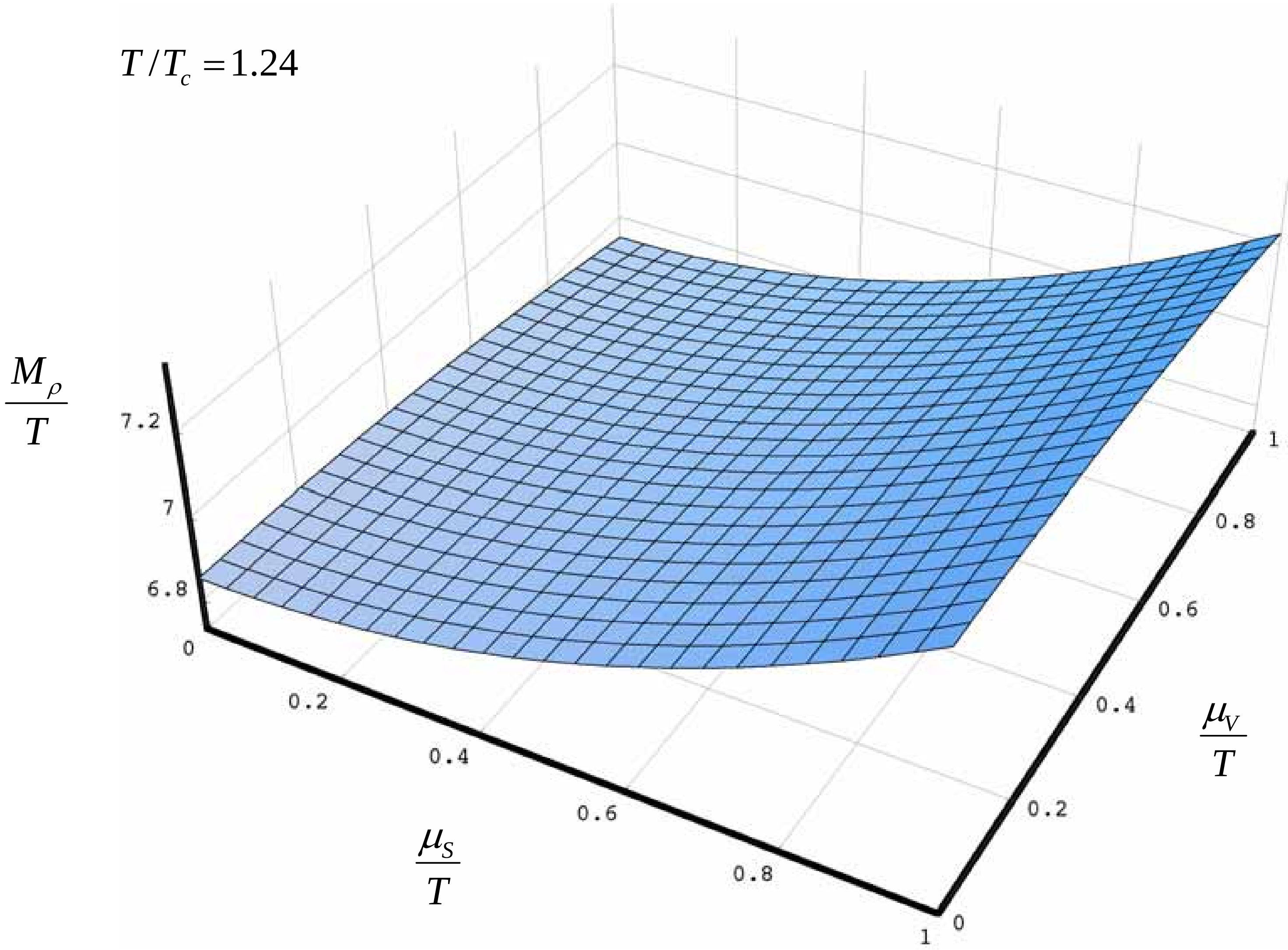


V-meson in the plane (μ_I, μ_B)



V-meson in the plane (μ_I, μ_B)



Conclusion

☞ We have analyzed the behavior of the 1st and 2nd order derivatives of the meson and baryon screening masses with respect to the chemical potential

(the hadron screening mass shows different behaivour as a function of both T and chemical potential in the vicinity of $\mu=0$)

☞ Results are very encouraging

The calculations for Δ baryon are in progress

Next steps:

$\Rightarrow$ extrapolation to the chiral limit $ma \to 0$

$\Rightarrow$ comparison with experimental data

$\Rightarrow$ investigation of other hadrons

Outlook