On-shell improvement of the massive Wilson quark action

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CP-PACS collaboration

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§1. Introduction

$m_b a \sim 1 - 2$ in quenched QCD
$\qquad 2 - 3$ in full QCD

- static approximation ($m_Q \to 0$)
- NRQCD (absence of continuum limit)
- nonrelativistic interpretation of Wilosn/SW results
  (Fermilab interpretation)

relativistic approach

cutoff effects

$(m_Q a)^n$

$(a\Lambda_{\text{QCD}})^k$

$(m_Q a)^n \cdot (a\Lambda_{\text{QCD}})^k$

relativistic on-shell improvement in the massive case

$$S_{\text{eff}} = S_{\text{cont}} + \sum_{k\geq 1} a^k \int d^4 x C_{4+k,i}(g, m_Q a)\mathcal{O}_{4+k,i}(x)$$

$\longrightarrow$ remove $(m_Q a)^n a\Lambda_{\text{QCD}}$ errors

remaining cutoff effects are $O((a\Lambda_{\text{QCD}})^2)$

$a^{-1} \sim 3\text{Gev},\ \Lambda_{\text{QCD}} \sim 300\text{MeV} \longrightarrow (a\Lambda_{\text{QCD}})^2 \sim 1\%$

extend Symanzik's improvement program to the massive case

§**2.** Relativistic on-shell improvement

§**3.** Determination of improvement coefficients in the action

§**4.** Charmed hadron spectrum in $N_f = 2$ QCD

§**5.** Summary

§2. Relativistic on-shell improvement

cutoff effects

$$(m_Q a)^n, \ (a\Lambda_{\rm QCD})^k, \ (m_Q a)^n \cdot (a\Lambda_{\rm QCD})^k$$

we assume $m_Q a \ll a\Lambda_{\rm QCD} \ll 1$

$$a\Lambda_{\rm QCD} > (a\Lambda_{\rm QCD})^2 > \cdots$$

Symanzik's on-shell improvement program

$$S_{\rm eff} = S_{\rm cont} + \sum_{k,i\geq 1} a^k \int d^4x C_{4+k,i}(g)\mathcal{O}_{4+k,i}(x)$$

§2. Relativistic on-shell improvement

cutoff effects

$$(m_Q a)^n, \ \ (a\Lambda_{\rm QCD})^k, \ \ (m_Q a)^n \cdot (a\Lambda_{\rm QCD})^k$$

we assume $m_Q a \ll a\Lambda_{\rm QCD} \ll 1$

$$a\Lambda_{\rm QCD} > (a\Lambda_{\rm QCD})^2 > \cdots$$

Symanzik's on-shell improvement program

$$S_{\rm eff} = S_{\rm cont} + \sum_{k,i\geq 1} a^k \int d^4x C_{4+k,i}(g)\mathcal{O}_{4+k,i}(x)$$

later, we consider $m_Q a \gg a\Lambda_{\rm QCD}$ and $m_Q a \sim O(1)$

$$f_0(m_Q a) > f_1(m_Q a) a\Lambda_{\rm QCD} > f_2(m_Q a)(a\Lambda_{\rm QCD})^2 > \cdots$$

$f_i(m_Q a)$ have Taylor expansions around $m_Q a = 0$

allowed operators with axis interchange symmetry

$$
\begin{aligned}
\text{dim.3}: \quad & \mathcal{O}_3(x) = \bar{q}(x)q(x) \\
\text{dim.4}: \quad & \mathcal{O}_4(x) = \bar{q}(x)\not{D}q(x) \\
\text{dim.5}: \quad & \mathcal{O}_{5a}(x) = \bar{q}(x)D_\mu^2 q(x) \\
& \mathcal{O}_{5b}(x) = i\bar{q}(x)\sigma_{\mu\nu}F_{\mu\nu}q(x) \\
\text{dim.6}: \quad & \mathcal{O}_{6a}(x) = \bar{q}(x)\gamma_\mu D_\mu^3 q(x) \\
& \mathcal{O}_{6b}(x) = \bar{q}(x)D_\mu^2\not{D}q(x) \\
& \qquad\vdots
\end{aligned}
$$

allowed operators with axis interchange symmetry

$$
\begin{aligned}
\text{dim.3}: \quad & \mathcal{O}_3(x) = \bar{q}(x)q(x) \\
\text{dim.4}: \quad & \mathcal{O}_4(x) = \bar{q}(x)\not{D}q(x) \\
\text{dim.5}: \quad & \mathcal{O}_{5a}(x) = \bar{q}(x)D_\mu^2 q(x) \ \to \text{redundant by E.O.M.} \\
& \mathcal{O}_{5b}(x) = i\bar{q}(x)\sigma_{\mu\nu}F_{\mu\nu}q(x) \ \to O(a\Lambda_{\mathrm{QCD}}) \\
\text{dim.6}: \quad & \mathcal{O}_{6a}(x) = \bar{q}(x)\gamma_\mu D_\mu^3 q(x) \ \to O((a\Lambda_{\mathrm{QCD}})^2) \\
& \mathcal{O}_{6b}(x) = \bar{q}(x)D_\mu^2\not{D}q(x) \ \to \text{redundant by E.O.M.} \\
& \qquad \vdots
\end{aligned}
$$

generic form of $O(a)$ improved quark action

$$
S_q = \sum_x \Big[m_0\bar{q}(x)q(x) + \bar{q}(x)\not{D}q(x) - \frac{a}{2}r\sum_\mu \bar{q}(x)D_\mu^2 q(x) - \frac{iga}{4}c_{\mathrm{SW}}\sum_{\mu,\nu}\bar{q}(x)\sigma_{\mu\nu}F_{\mu\nu}q(x) \Big]
$$

what happens in case of $m_Q a \gg a\Lambda_{\rm QCD}$ and $m_Q a \sim O(1)$?

$$f_0(m_Q a) > f_1(m_Q a) a\Lambda_{\rm QCD} > f_2(m_Q a)(a\Lambda_{\rm QCD})^2 > \cdots$$

allowed operators with axis interchange symmetry

dim.3 : $\mathcal{O}_3(x) = \bar{q}(x) q(x)$

dim.4 : $\mathcal{O}_4(x) = \bar{q}(x) \not{D} q(x)$

dim.5 : $\mathcal{O}_{5a}(x) = \bar{q}(x) D_\mu^2 q(x)$ → redundant

$\mathcal{O}_{5b}(x) = i\bar{q}(x) \sigma_{\mu\nu} F_{\mu\nu} q(x)$ → $O(a\Lambda_{\rm QCD})$

dim.6 : $\mathcal{O}_{6a}(x) = \bar{q}(x) \gamma_\mu D_\mu^3 q(x)$

$\mathcal{O}_{6b}(x) = \bar{q}(x) D_\mu^2 \not{D} q(x)$

$\vdots$

what happens in case of $m_Q a \gg a\Lambda_{\rm QCD}$ and $m_Q a \sim O(1)$?

$$f_0(m_Q a) > f_1(m_Q a) a\Lambda_{\rm QCD} > f_2(m_Q a)(a\Lambda_{\rm QCD})^2 > \cdots$$

allowed operators with axis interchange symmetry

dim.3 : $\mathcal{O}_3(x) = \bar{q}(x)q(x)$

dim.4 : $\mathcal{O}_4(x) = \bar{q}(x)\not{D}q(x)$

dim.5 : $\mathcal{O}_{5a}(x) = \bar{q}(x)D_\mu^2 q(x)$ $\rightarrow$ redundant

$\mathcal{O}_{5b}(x) = i\bar{q}(x)\sigma_{\mu\nu}F_{\mu\nu}q(x)$ $\rightarrow O(a\Lambda_{\rm QCD})$

dim.6 : $\underline{\mathcal{O}_{6a}(x) = \bar{q}(x)\gamma_\mu D_\mu^3 q(x)}$ still $O((a\Lambda_{\rm QCD})^2)$ for $m_Q a \sim O(1)$?

$\mathcal{O}_{6b}(x) = \bar{q}(x)D_\mu^2\not{D}q(x)$ $\qquad \partial_0 \sim O(m_Q), \ \partial_i \sim O(\Lambda_{\rm QCD})$

$\vdots$

contribution of $\mathcal{O}_{6a}(x) = \bar{q}(x)\gamma_\mu D_\mu^3 q(x)$

$$\begin{aligned} a^2\bar{q}(x)\gamma_0 D_0^3 q(x) &= -\frac{1}{a}(m_Q a)^3\bar{q}(x)q(x) \\ &\quad -(m_Q a)^2\bar{q}(x)\gamma_i D_i q(x) \\ &\quad +a(m_Q a)\bar{q}(x)D_i^2 q(x) + O((\Lambda_{QCD} a)^2) \\ a^2\bar{q}(x)\gamma_i D_i^3 q(x) &\sim O((a\Lambda_{QCD})^2) \end{aligned}$$

with eq. of motion: $\gamma_0 D_0 + \gamma_i D_i + m_Q = 0$

can be absorbed in coefficients of lower dimensional operators

$\longrightarrow$ asymmetries with $(m_Q a)^n$ in coefficients

$$\begin{aligned} \bar{q}(x)\gamma_0 D_0 q(x) &\leftrightarrow (1-(m_Q a)^2)\bar{q}(x)\gamma_i D_i q(x) \\ r_t\bar{q}(x)\gamma_0 D_0^2 q(x) &\leftrightarrow (r_t + (m_Q a))\bar{q}(x)\gamma_i D_i^2 q(x) \end{aligned}$$

general form of quark action for $O(f_1(m_Q a)a\Lambda_{\mathrm{QCD}})$ improvement

$$
\begin{aligned}
S_q^{\mathrm{RHQ}} = \sum_x \Bigg[& m_0 \bar{q}(x)q(x) + \bar{q}(x)\gamma_0 D_0 q(x) + \nu \sum_i \bar{q}(x)\gamma_i D_i q(x) \\
& - \frac{r_t a}{2}\bar{q}(x) D_0^2 q(x) - \frac{r_s a}{2}\sum_i \bar{q}(x) D_i^2 q(x) \\
& - \frac{iga}{2} c_E \sum_i \bar{q}(x)\sigma_{0i}F_{0i}q(x) - \frac{iga}{4} c_B \sum_{i,j} \bar{q}(x)\sigma_{ij}F_{ij}q(x) \Bigg],
\end{aligned}
$$

general form of quark action for $O(f_1(m_Q a) a\Lambda_{\rm QCD})$ improvement

$$
\begin{aligned}
S_q^{\rm RHQ} = \sum_x \Big[& m_0 \bar{q}(x) q(x) + \bar{q}(x)\gamma_0 D_0 q(x) + \nu \sum_i \bar{q}(x)\gamma_i D_i q(x) \\
& -\frac{r_t a}{2} \bar{q}(x) D_0^2 q(x) - \frac{r_s a}{2} \sum_i \bar{q}(x) D_i^2 q(x) \\
& -\frac{iga}{2} c_E \sum_i \bar{q}(x)\sigma_{0i} F_{0i} q(x) - \frac{iga}{4} c_B \sum_{i,j} \bar{q}(x)\sigma_{ij} F_{ij} q(x) \Big],
\end{aligned}
$$

r_t is redundant

ν, r_s, c_E, c_B should be adjusted in a $m_Q a$ dependent way

$\nu - 1$, $r_s - r_t$, $c_B - c_E$ represent contributions from higher dimensional ops. without space-time rotational symmetry like $\mathcal{O}_{6a} = \bar{q}\gamma_\mu D_\mu^3 q$

cf. r_t and r_s are redundant in Fermilab approach another derivation

what happens if axis interchange symmetry is given up from the begining

allowed operators

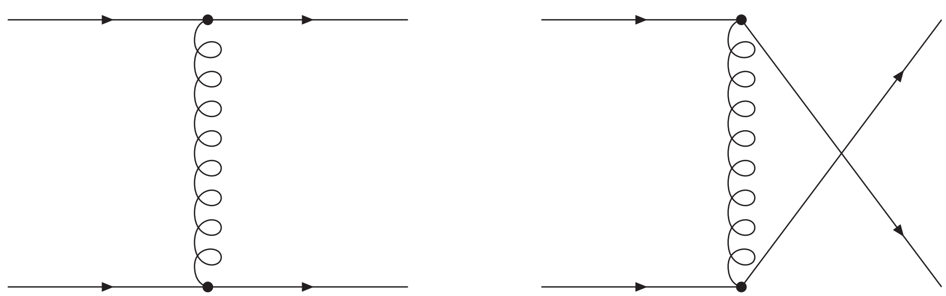
$$
\begin{aligned}
\text{dim.3}: \quad & \mathcal{O}_3(x) = \bar{q}(x)q(x) \\
\text{dim.4}: \quad & \mathcal{O}_{4a}(x) = \bar{q}(x)\gamma_0 D_0 q(x) \\
& \mathcal{O}_{4b}(x) = \bar{q}(x)\gamma_i D_i q(x) \\
\text{dim.5}: \quad & \mathcal{O}_{5a}(x) = \bar{q}(x)D_0^2 q(x) \ \rightarrow \text{redundant by E.O.M.} \\
& \mathcal{O}_{5b}(x) = \bar{q}(x)D_i^2 q(x) \\
& \mathcal{O}_{5c}(x) = i\bar{q}(x)\sigma_{0i}F_{0i}q(x) \\
& \mathcal{O}_{5d}(x) = i\bar{q}(x)\sigma_{ij}F_{ij}q(x) \\
& \mathcal{O}_{5e}(x) = i\bar{q}(x)[\gamma_0 D_0, \gamma_i D_i]q(x) \ \rightarrow \text{redundant by E.O.M.}
\end{aligned}
$$

the same form of quark action with ν, r_s, c_E, c_B to be tuned

§3. Determination of improvement coefficients
§3-1. $m_Q a$ corrections at tree level

"$\cdots$ on-shell quantities(particle energies, scattering amplitudes, normalized matrix elements of local composite fields between particle states etc.) $\cdots$" Lüscher-Sint-Sommer-Weisz, NPB478(1996)365

ν, r_s, c_E, c_B can be determined by on-shell quark-quark scattering



continuum scattering amplitude should be reproduced
massless case done by Wohlert to determine c_{SW}

the on-shell improvement condition yields

$$
\begin{aligned}
\nu^{(0)} &= \frac{\sinh(m_p^{(0)})}{m_p^{(0)}}, \\
r_s^{(0)} &= \frac{\cosh(m_p^{(0)}) + r_t \sinh(m_p^{(0)})}{m_p^{(0)}} - \frac{\sinh(m_p^{(0)})}{{m_p^{(0)}}^2}, \\
c_E^{(0)} &= r_t \nu^{(0)}, \\
c_B^{(0)} &= r_s^{(0)},
\end{aligned}
$$

points

- 4 parameters are uniquely determined
- r_t cannot be fixed $\longrightarrow$ consistent with redundancy
- $\nu \to 1$, $r_s \to r_t$, $c_B \to r_t$, $c_E \to r_t$ for $m_p^{(0)} \to 0$

§3-2. Universality of improvement coefficients

improvement coefficients in the action should be determined independent of improvement conditions

let us try to determine $\nu^{(0)}$, $r_s^{(0)}$ from quark propagator
same results as the quark-quark scattering?

inverting the Wilson-Dirac operator

$$S_q^{-1}(p) \;=\; i\gamma_0 \sin(p_0) + \nu i \sum_i \gamma_i \sin(p_i) + m_0$$
$$+ r_t(1-\cos(p_0)) + r_s \sum_i (1-\cos(p_i))$$

ν, r_s are determined by demanding

$$S_q(p) = \frac{1}{Z_q^{(0)}} \frac{-i\gamma_0 p_0 - i\sum_i \gamma_i p_i + m_p^{(0)}}{p_0^2 + \sum_i p_i^2 + {m_p^{(0)}}^2} + (\text{no pole terms}) + O((p_i a)^2)$$

around the pole

speed of light (coeffs. of $\gamma_0 p_0$ & $\gamma_i p_i$) $\to \nu^{(0)}$
dispersion relation ($E = {m_p^{(0)}}^2 + \sum_i p_i^2$) $\to r_s^{(0)}$

same results as the quark-quark scattering

$$\nu^{(0)} = \frac{\sinh(m_p^{(0)})}{m_p^{(0)}}$$

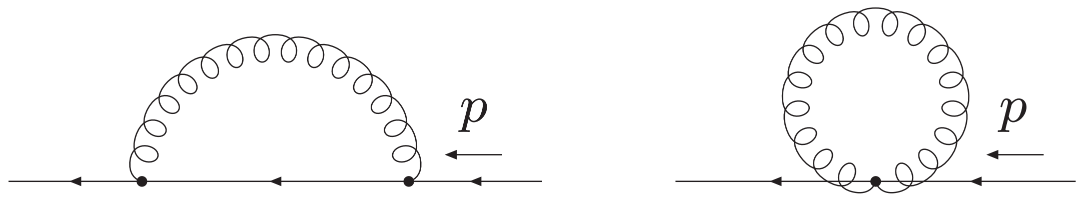
$$r_s^{(0)} = \frac{\cosh(m_p^{(0)}) + r_t \sinh(m_p^{(0)})}{m_p^{(0)}} - \frac{\sinh(m_p^{(0)})}{{m_p^{(0)}}^2}$$

$\longrightarrow$ universality of improvement coefficients

next step is to determine ν, r_s, c_E, c_B at one-loop level

§3-3. $\nu(m_Q a, g)$, $r_s(m_Q a, g)$ at one-loop level

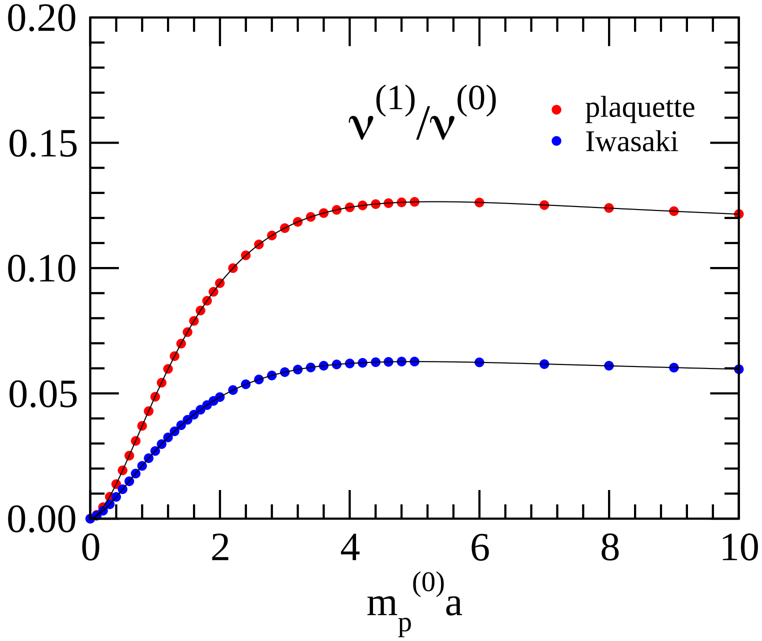
determined from quark propagator

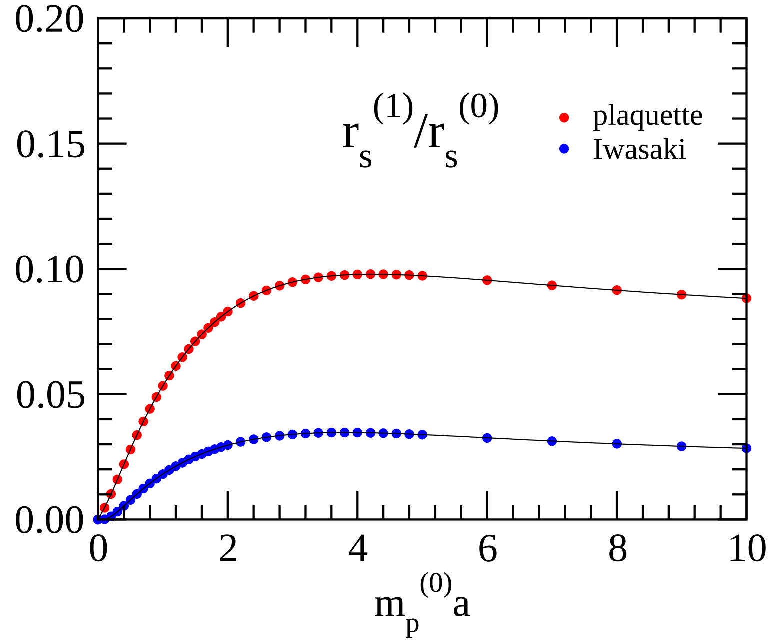


conditions for ν, r_s

- speed of light : adjust coefficients of $\gamma_0 p_0$ and $\gamma_i p_i$ in $S_q^{-1}(p)$

- dispersion relation : $E = m_p^2 + F(\nu, r_s) \sum_i p_i^2 + O(p_i^4)$
  $\longrightarrow F(\nu, r_s) = 1$



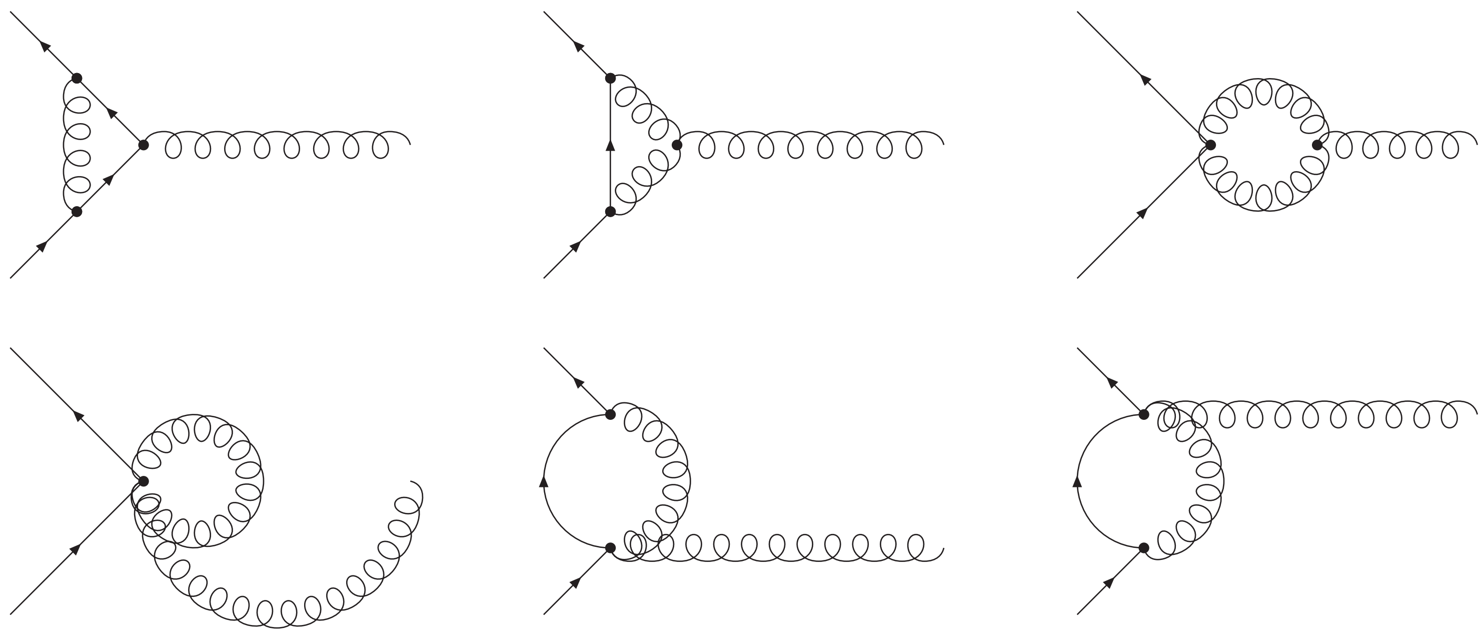


$\nu^{(1)}$, $r_s^{(1)}$ vanishes for $m_p^{(0)} \to 0$ as expected

§3-4. $c_B(m_Q a, g)$, $c_E(m_Q a, g)$ at one-loop level

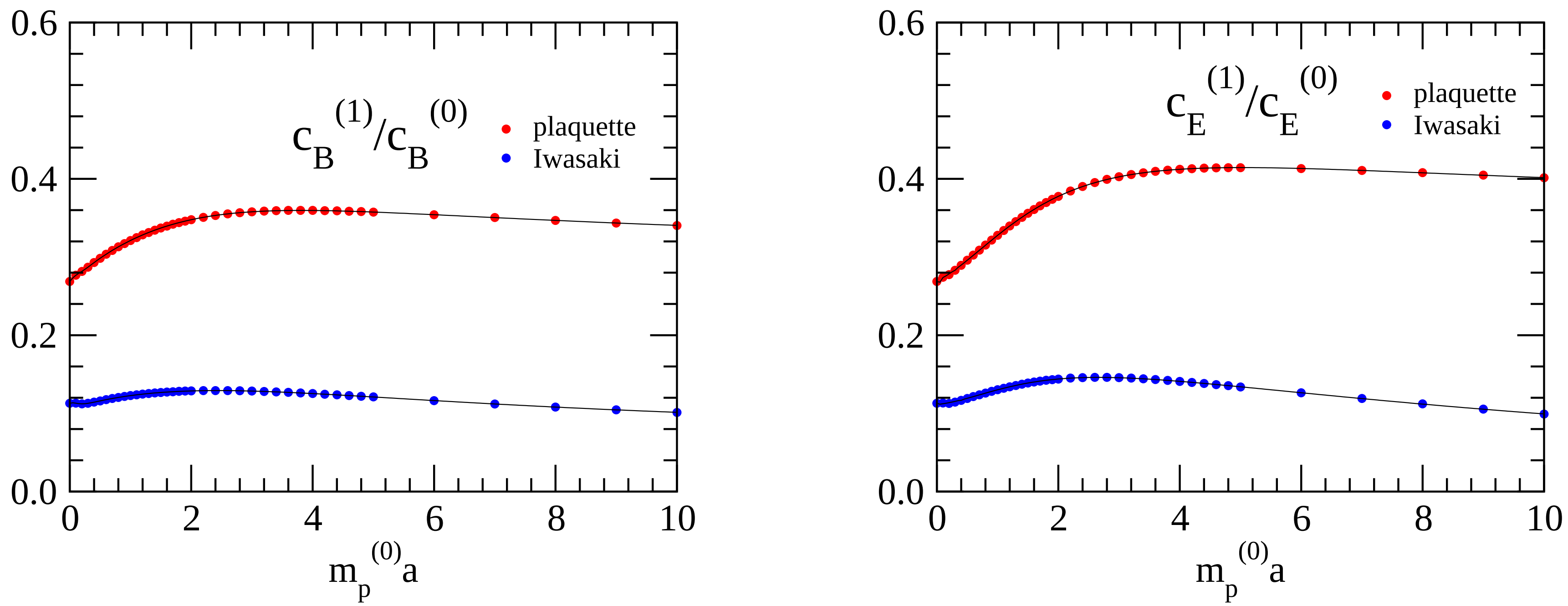
determined from on-shell quark-quark scattering

one-loop diagrams



$O(p_i a, q_i a)$ should vanish

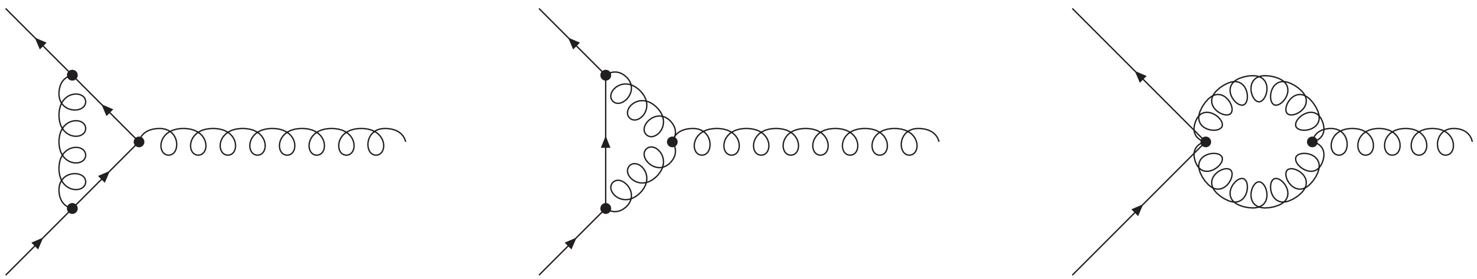
massless case was already done by Wohlert



$c_B^{(1)}$ is increased $\longrightarrow$ hyperfine splitting will be enhanced infrared divergence

c_B, c_E should be free from infrared divergence

three divergent diagrams



infrared divergences are cancelled out after summation
if and only if $\nu^{(0)}$, $r_s^{(0)}$, $c_B^{(0)}$, $c_E^{(0)}$ are properly tuned
$\longrightarrow$ another evidence for validity of our formulation
4 parameters in the action should be properly tuned

what we have achieved:

- a relativistic heavy quark action
  based on Symanzik improvement program

- perturbative determination of improvement coefficients
  ν, r_s, c_B, c_E in the action

- perturbative improvement of the currents

formulation is completed at least perturbatively
now time to embark on numerical simulations

$\S$4. Charmed hadron spectrum in $N_f = 2$ QCD

objective

1. check restoration of space-time symmetry

– dispersion relation of H-H and H-L mesons

$$E^2 = m^2 + {c_{\text{eff}}}^2 p_s^2$$

– PS meson decay constant from A_0 and A_k

$$R \equiv \frac{f_{PS}^{A_k}}{f_{PS}^{A_4}} = i\frac{\langle 0|A_k^R|PS\rangle}{\langle 0|A_4^R|PS\rangle} \cdot \frac{E}{|p_s|}$$

c_{eff} and R should be 1 in the continuum limit

2. investigate scaling property

– charmonium hyperfine splitting, $m_{J/\psi} - m_{\eta_c}$

pole mass $(m_{J/\psi}^{\text{pole}} - m_{\eta_c}^{\text{pole}}) \leftrightarrow$ kinetic mass $(m_{J/\psi}^{\text{kin}} - m_{\eta_c}^{\text{kin}})$

$$E = m_{\text{pole}} + \frac{1}{2m_{\text{kin}}} p_s^2 + O(p_s^4)$$

should have the same continuum limit

– D_s meson decay constant f_{D_s}

spatial component $A_k \leftrightarrow$ temporal component A_4

try to take the continuum limit

simulation parameters   CP-PACS, PRD65(2002)054505

Iwasaki gauge action

4 sea quark masses : $m_\pi/m_\rho = 0.5 - 0.7$

β	$L^3 \times T$	$a(r_0)$[fm]	#traj
1.8	$12^3 \times 24$	0.215	$\sim$ 5000
1.95	$16^3 \times 32$	0.156	$\sim$ 7000
2.1	$24^3 \times 48$	0.108	4000

light quarks (4 sea & 3 valence) : mean-field $O(a)$ improved action

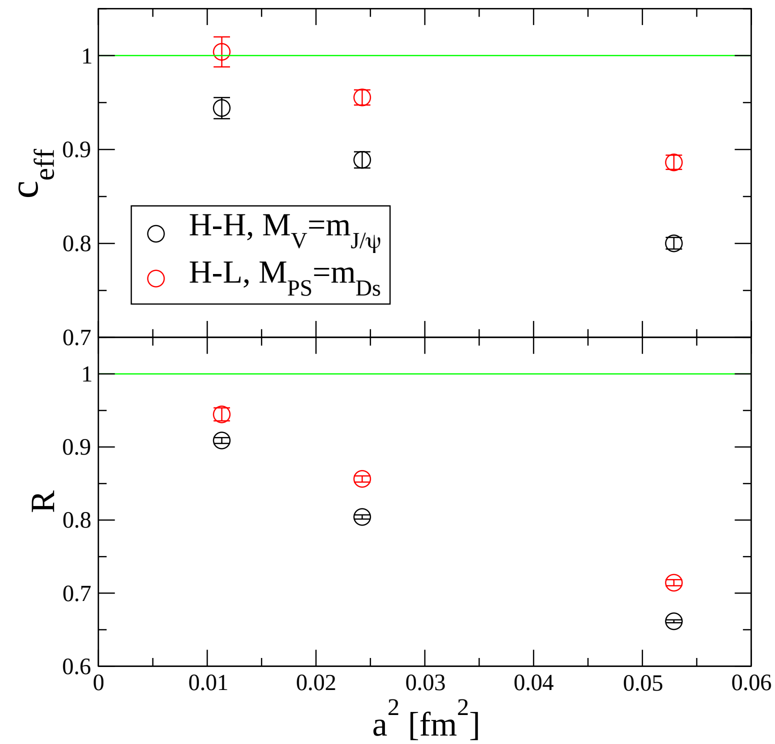
$\rightarrow$ $O(a)$ is not extinct

heavy quark action

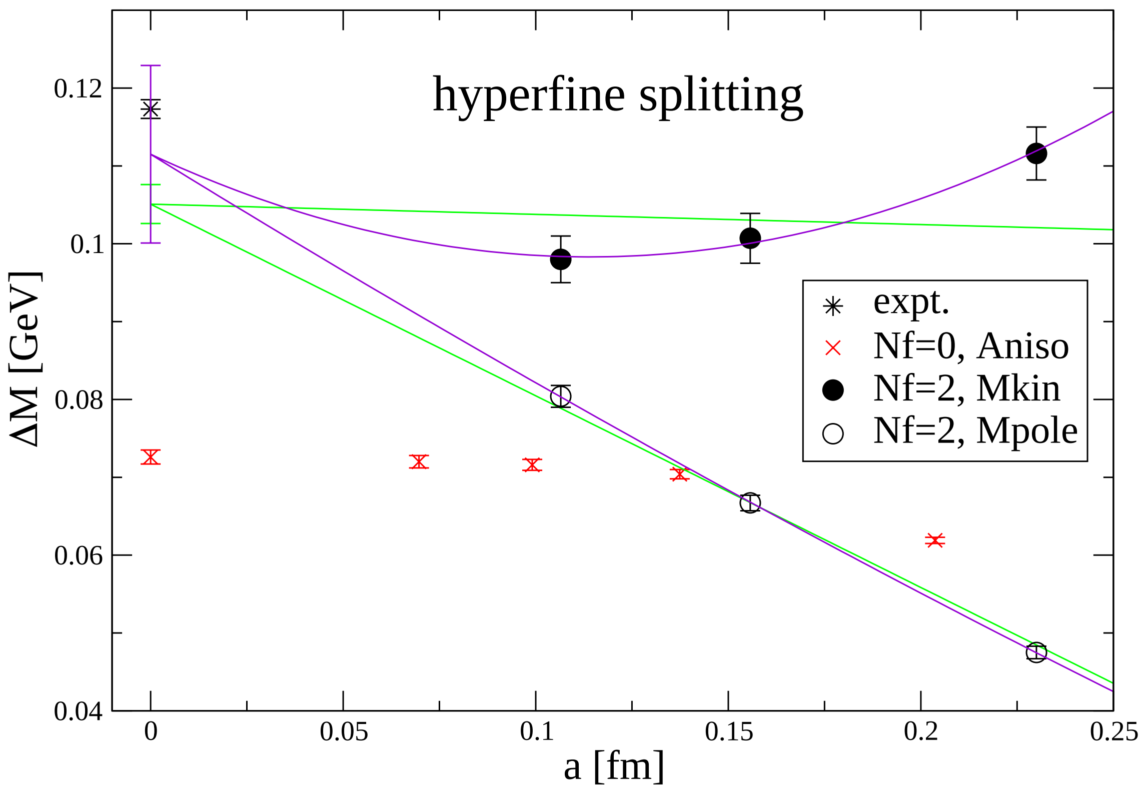
- ν, r_s, c_B, c_E : mean-field improved one-loop values
- $c_{B/E} = \{c^{\text{PT}}_{B/E}(m_Q a) - c^{\text{PT}}_{B/E}(0)\} + c^{\text{NP}}_{\text{SW}}$
  $\rightarrow$ nonperturbative contribution at $m_Q = 0$ is taken into account
- 4 heavy quark masses to cover m_{charm}

axial vector currents

- improvement coefficients : mean-field improved one-loop values



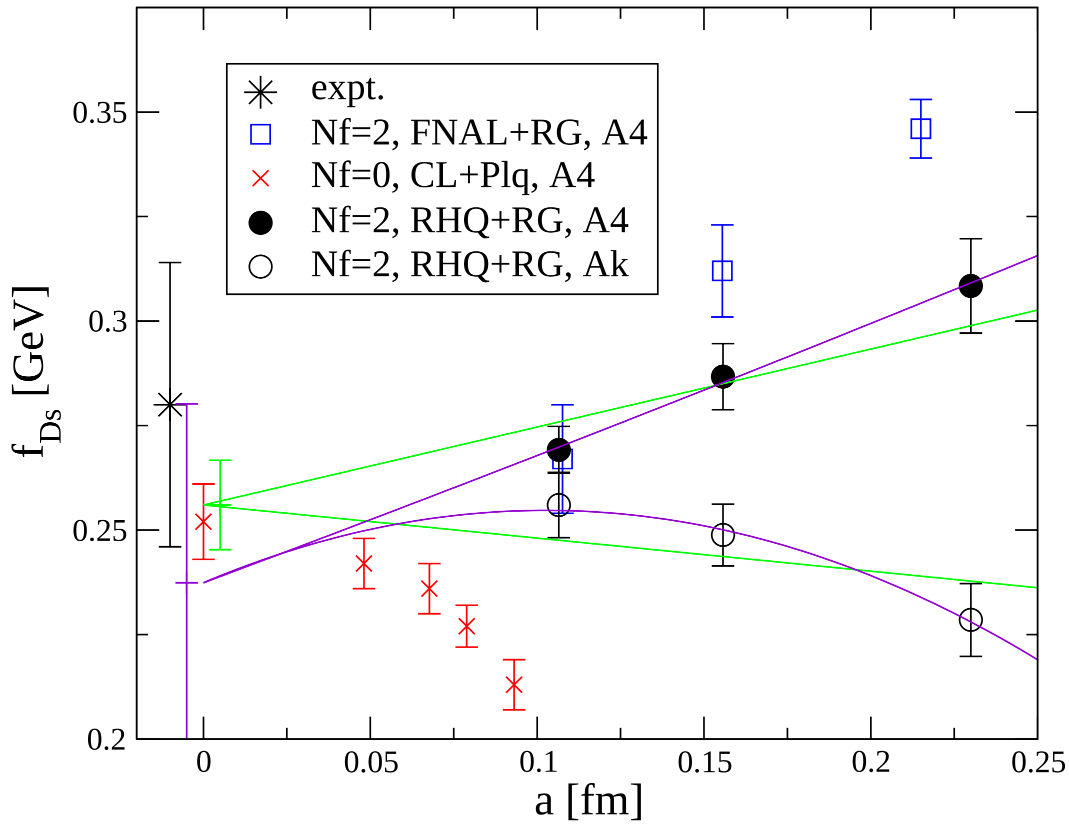
c_{eff} and R should be 1 in the continuum limit
almost $O(a^2)$ behavior
expected leading contribution is $O((a\Lambda_{\mathrm{QCD}})^2)$ or $O(g^4 a^2 m_c \Lambda_{\mathrm{QCD}})$
if $m_c a < 1$



dynamical quark effects are observed

already consistent with experiment

encouraged to explore $N_f = 3$ calculation



good scaling behavier

reliable continuum extrapolation with $f_{D_s}(A_4)$ and $f_{D_s}(A_k)$

§5. Summary

- general form of quark action

$$
\begin{aligned}
S_q^{\text{imp}} &= \sum_x \Bigg[m_0 \bar{q}(x) q(x) + \bar{q}(x)\gamma_0 D_0 q(x) + \nu \sum_i \bar{q}(x)\gamma_i D_i q(x) \\
&\quad - \frac{r_t a}{2} \bar{q}(x) D_0^2 q(x) - \frac{r_s a}{2} \sum_i \bar{q}(x) D_i^2 q(x) \\
&\quad - \frac{iga}{2} c_E \sum_i \bar{q}(x)\sigma_{0i} F_{0i} q(x) - \frac{iga}{4} c_B \sum_{i,j} \bar{q}(x)\sigma_{ij} F_{ij} q(x) \Bigg],
\end{aligned}
$$

r_t is redundant

ν, r_s, c_E, c_B should be adjusted

- 4 parameters in the action are determined up to one-loop level from on-shell quark-quark scattering

- $O(a)$ improvement of the vector and axial vector currents up to one-loop level

- numerical study shows encouraging results
  dynamical quark effects for HFS
  good scaling behavior for f_{D_s}

ongoing project

- renormalization and improvement of four-fermi ops.

- detailed scaling study in quenched QCD ranging from charm to bottom $\longrightarrow$ check the applicability to bottom

next plan

- nonperturbative determination of ν, r_s, c_E, c_B

- numerical calculation in $N_f = 3$ QCD