$B \to D^{(*)} l\nu$ Form Factors: Status and Prospects and Introduction to Lattice QCD

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Belle-Lattice V_{xb} Mini-workshop

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Form Factors

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 \sqrt{w^2 - 1}\mathcal{G}(w)|V_{cb}|^2|\mathcal{F}_{B\to D^*}(w)|^2$$

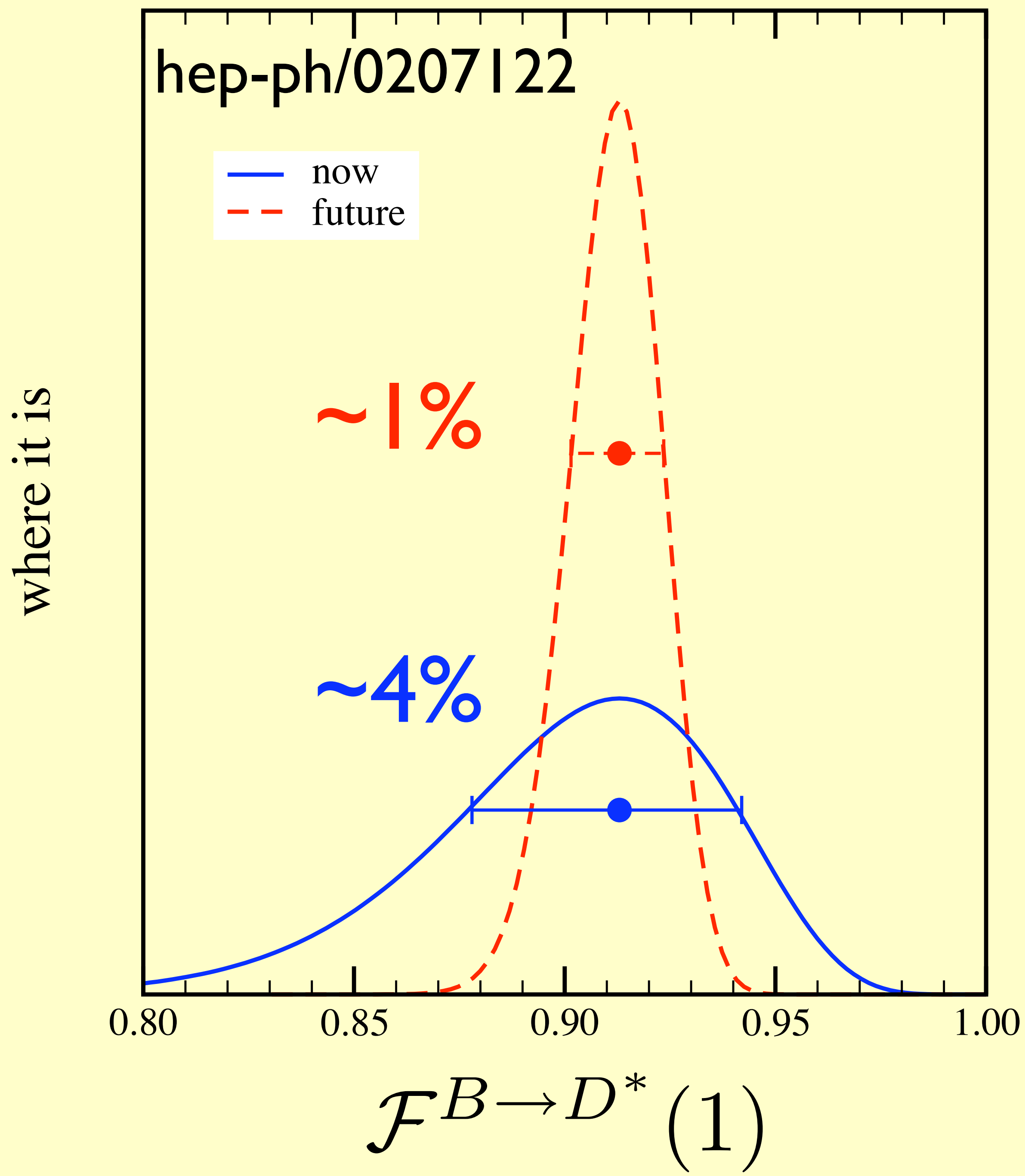
$$\mathcal{F}_{B\to D^*}(1) = h_{A_1}(1) \qquad \langle D^*(v)|\mathcal{A}^\mu|\bar{B}(v)\rangle = i\sqrt{2m_B\, 2m_{D^*}}\, \overline{\epsilon'}^\mu h_{A_1}(1)$$

$$\frac{d\Gamma(\bar{B}\to Dl\bar{\nu})}{dw} = \frac{G_F^2}{48\pi^3}(m_B + m_D)^2 m_D^3 (w^2 - 1)^{3/2}|V_{cb}|^2|\mathcal{F}_{B\to D}(w)|^2$$

$$\mathcal{F}_{B\to D}(w) = h_+^{B\to D}(w) - \frac{m_B - m_D}{m_B + m_D} h_-^{B\to D}(w)$$

$$\langle D(\mathbf{p}')|\mathcal{V}_\mu|\bar{B}(\mathbf{p})\rangle = \sqrt{m_B m_D}\left[h_+^{B\to D}(w)(v+v')_\mu + h_-^{B\to D}(w)(v-v')_\mu\right]$$

$\mathcal{F}^{B\to D^*}(1), \quad \mathcal{G}^{B\to D}(1)$



$$\mathcal{F}^{B\to D^*}(1) = 0.913 \; {}^{+0.024}_{-0.017} \pm 0.016 \; {}^{+0.003}_{-0.014} \; {}^{+0.000}_{-0.016} \; {}^{+0.006}_{-0.014}$$

hep-ph/0110253

stats **match**

a **chiral**

quench

$$\mathcal{G}^{B\to D}(1) = 1.058 \; \pm 0.016 \; {}^{+0.014}_{-0.006} \; \pm -- \; \pm 0.000 \; \pm --$$

hep-ph/9906376

Monte Carlo Method

hep-ph/0110253: Budget of statistical and systematic uncertainties for $h_{A_1}(1)$ and $1 - h_{A_1}(1)$. The row labeled "total systematic" does not include uncertainty from fitting, which is lumped with the statistical error. The statistical error is that after chiral extrapolation.

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$$h_{A_1}(1) = \mathcal{F}^{B \to D^*}(1)$$

Space-time Lattice

- quarks live on sites

$\psi(x), \quad \bar{\psi}(y)$

- gauge fields live on links

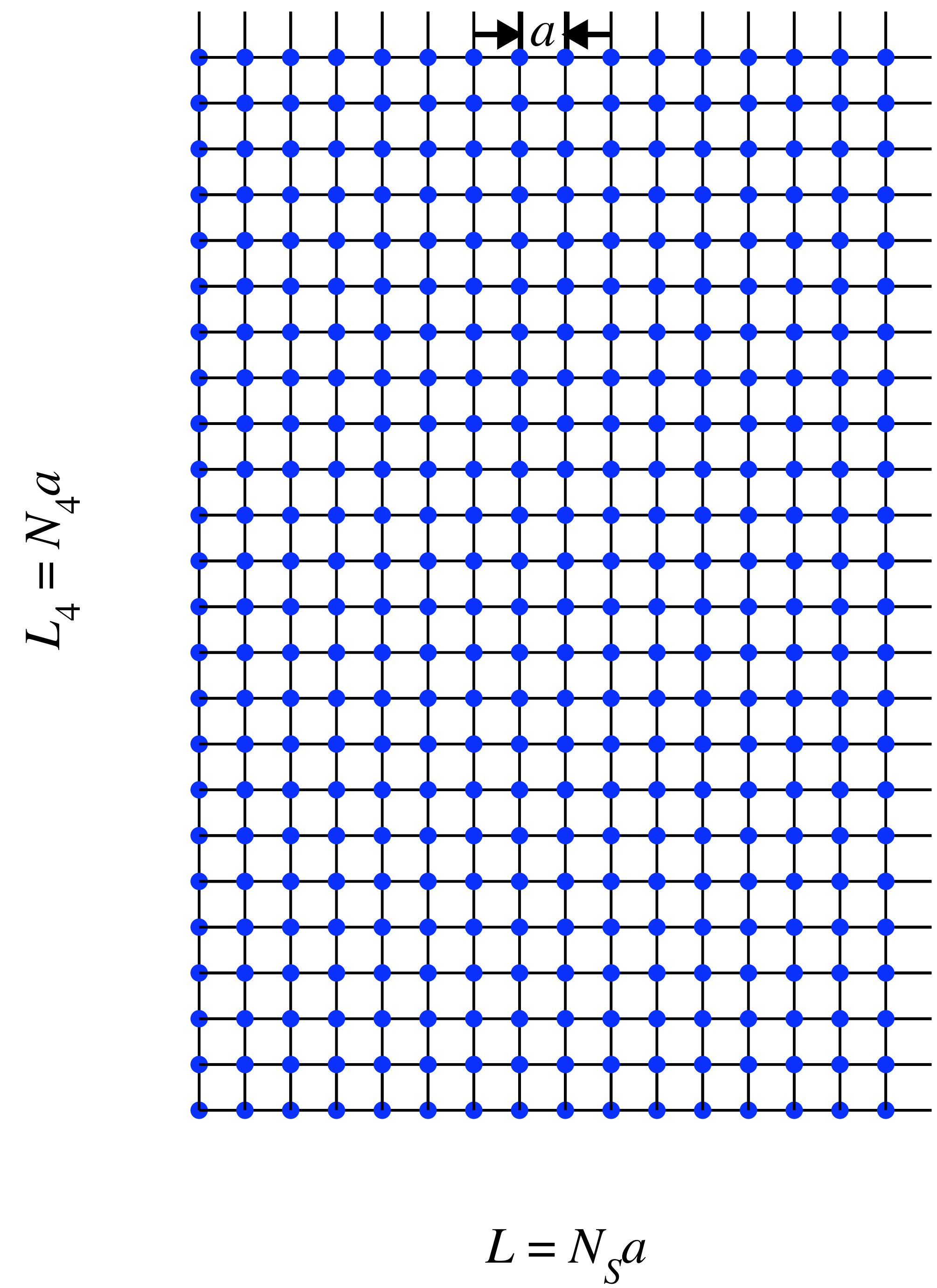
$U(x, x+ae_\mu) = \mathsf{P}e^{\int_0^a ds\, A_\mu(x+se_\mu)}$

- gauge invariance

$\bar{\psi}(x)U(x, x+ae_\mu)\psi(x+ae_\mu)$

- confinement emerges simply

K. Wilson, 1974



- Integrate the functional integral numerically (with finesse and brute force):

$$\int \mathcal{D}A\,\mathcal{D}\psi\mathcal{D}\bar{\psi}\;\bar{\psi}_u\gamma_5\psi_d(x)\bar{\psi}_d\gamma_5\psi_u(y)\;e^{-S_g-\bar{\psi}M\psi} = \int \mathcal{D}A\;\mathrm{tr}[G_d(x,y)\gamma_5 G_u(y,x)\gamma_5]\,\det M\,e^{-S_g}$$

$M = [\not{D} + m]_{\mathrm{lat}}$ $\qquad$ S_g = lattice gauge action

- $G = M^{-1}$ (quark propagators): expensive

- $\det M$ (sea quark loops): very expensive

- Only feasible integration method is Monte Carlo with importance sampling.
    - ensembles of a few hundred lattice gauge fields yield statistical errors of a few %.
    - active industry to devise better algorithms.
    - details not familiar to non-experts, but errors *usually* not underestimated by the experts.

Masses and Matrix Elements

mass

$$\langle B(t)B^\dagger(0)\rangle = \sum_n |\langle B_n|\hat{B}|0\rangle|^2 \exp(-m_{B_n}t)$$

decay constant

$$\langle \bar{b}\gamma_4\gamma_5 u(t)B^\dagger(0)\rangle = \sum_n \langle 0|\bar{b}\gamma_4\gamma_5 u|B_n\rangle\langle B_n|\hat{B}|0\rangle e^{-m_{B_n}t}$$

form factor

$$\langle D(t')\bar{b}\gamma_\mu c(t)B^\dagger(0)\rangle = \sum_{n,m} \langle D_m|\bar{b}\gamma_\mu c|B_n\rangle \times \langle 0|\hat{D}|D_m\rangle\langle B_n|\hat{B}|0\rangle e^{-m_{B_n}t-m_{D_m}(t'-t)}$$

Fitting Methods

- Key issue here is controlling contribution from excited states.
  - Larger *t* helps, but noise grows.
  - Matrix correlator $\langle B_i(t) B_j^\dagger(0) \rangle$
  - Fit to several exponentials, but constrain higher ones “to be sensible”.
- Like resolving lifetimes of several isotopes.

Double Ratios

$$\mathcal{R}_+ = \frac{\langle D|\bar{b}\gamma_4 c|B\rangle\langle B|\bar{c}\gamma_4 b|D\rangle}{\langle D|\bar{c}\gamma_4 c|D\rangle\langle B|\bar{b}\gamma_4 b|B\rangle} \quad \to \quad h_+(1)$$

$$\mathcal{R}_- = \frac{\langle D(\boldsymbol{p})|\bar{b}\gamma_i c|B\rangle\langle D(\boldsymbol{p})|\bar{b}\gamma_4 b|D\rangle}{\langle D(\boldsymbol{p})|\bar{b}\gamma_4 c|B\rangle\langle D(\boldsymbol{p})|\bar{c}\gamma_i b|D\rangle} \quad \to \quad h_-(1)$$

$$\mathcal{R}_1 = \frac{\langle D^*|\bar{b}\gamma_4 c|B^*\rangle\langle B|\bar{c}\gamma_4 b|D\rangle}{\langle D^*|\bar{c}\gamma_4 c|D^*\rangle\langle B^*|\bar{b}\gamma_4 b|B^*\rangle} \quad \to \quad h_1(1)$$

$$\mathcal{R}_{A_1} = \frac{\langle D^*|\bar{b}\gamma_i\gamma_5 c|B\rangle\langle B^*|\bar{c}\gamma_i\gamma_5 b|D\rangle}{\langle D^*|\bar{c}\gamma_i\gamma_5 c|D\rangle\langle B^*|\bar{b}\gamma_i\gamma_5 b|B\rangle} \quad \to \quad \check{h}_{A_1}(1)$$

$\mathcal{R}_+, \mathcal{R}_1, \mathcal{R}_{A_1}$ needed to obtain $h_{A_1}(1)$

Many uncertainties cancel: a key to $B \to D^{(*)}l\nu$

(also to $K \to \pi l\nu$)

Adjusting Masses I

hep-ph/0110253: Budget of statistical and systematic uncertainties for $h_{A_1}(1)$ and $1-h_{A_1}(1)$. The row labeled "total systematic" does not include uncertainty from fitting, which is lumped with the statistical error. The statistical error is that after chiral extrapolation.

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$$h_{A_1}(1) = \mathcal{F}^{B\to D^*}(1)$$

Adjusting Masses

- Lattice gauge theory gives a definition of QCD.
- Use 1 + n_f hadronic inputs to deduce a in fm and fix the (bare) quark masses.
- Subsequent calculations are predictions, *i.e.*, use same bare gauge coupling and quark masses.
- Have to propagate (statistical and systematic) uncertainties of the inputs to predictions.

Quenched Approximation

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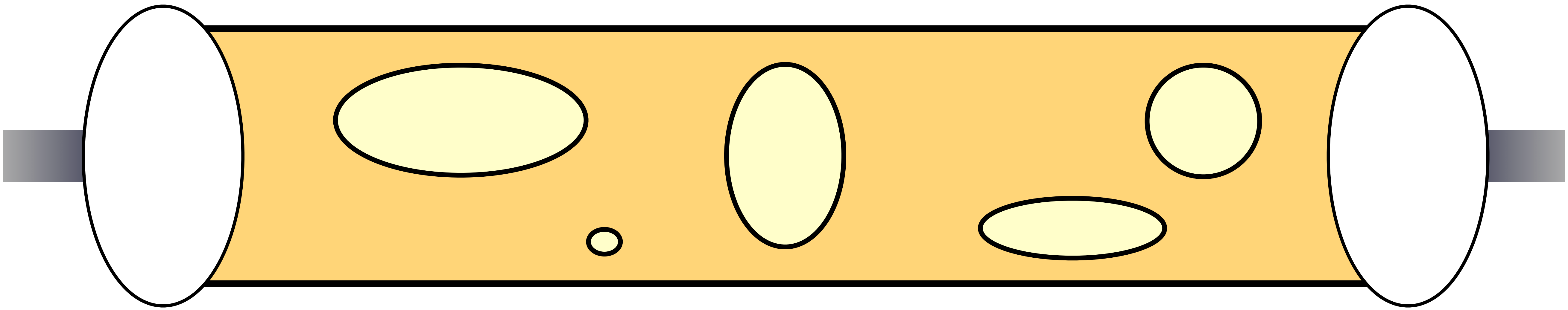
Quenched Approximation

Full QCD has (expensive) quark loops.

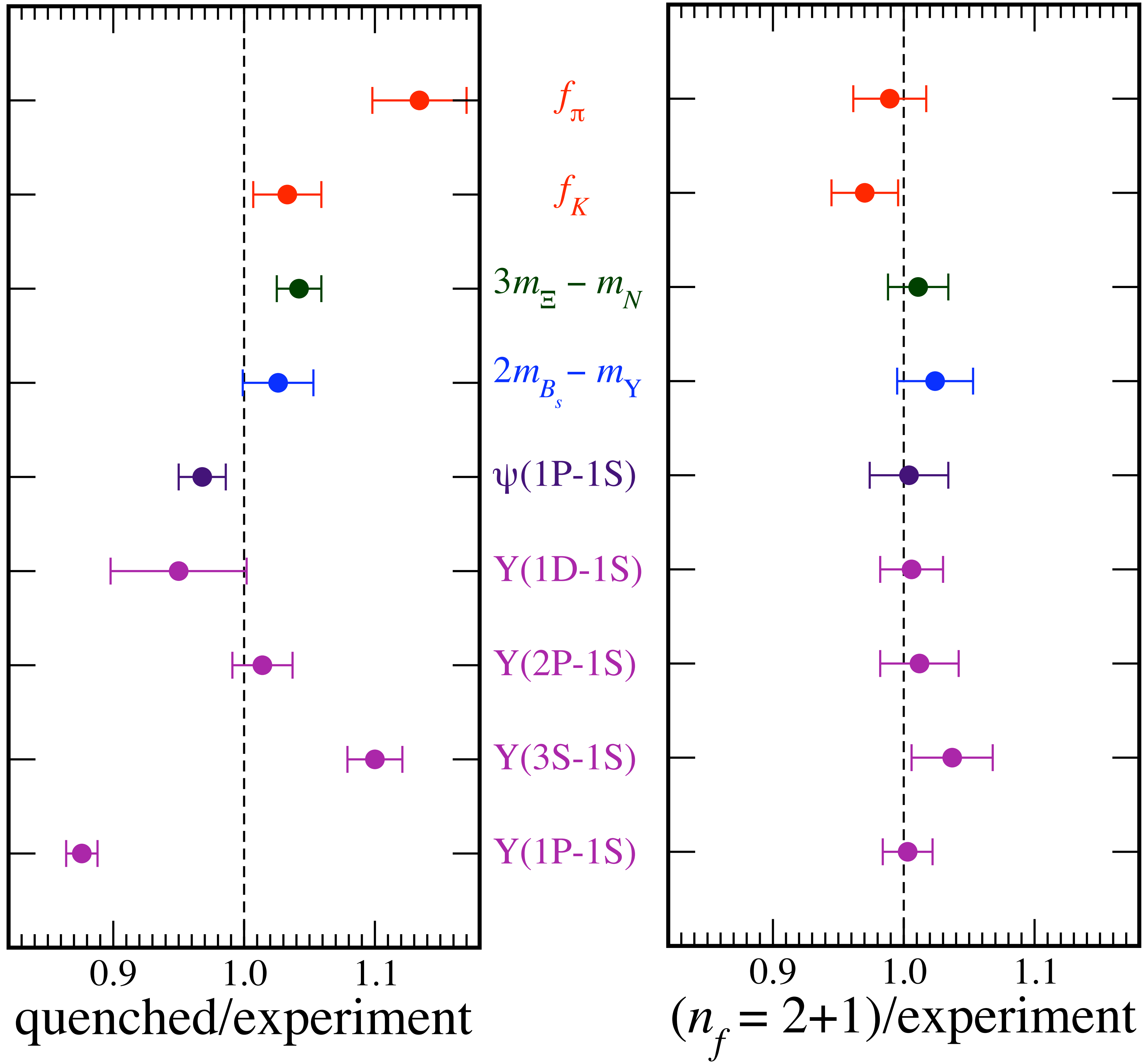


- Replace $\det M$ with 1, and compensate by shifting bare gauge coupling and bare masses. "Dielectric".
- Arguably OK if all light quarks had mass $m_q \sim \Lambda$.
- This error will go away in future calculations.

Unquenched QCD

- Only one method for $\det M$ is fast enough to generate a realistic sea.
  - "improved staggered quarks"
- Price for speed is an assumption that has not been proven rigorously.
  - also not disproven
  - for flavor physics, the stakes are high so we will have to settle this

Davies et al., hep-lat/0304004
[HPQCD, MILC, and Fermilab Lattice Col'ns]



Update: Ω^- works too

Tests

- $D \to Kl\nu$, $D \to \pi l\nu$:
  - $f_+^{D\to K}(0) = 0.73(3)(7)$, $f_+^{D\to\pi}(0) = 0.87(3)(9) f_+^{D\to K}$ [hep-ph/0408306]

    $f_+^{D\to K}(0) = 0.78(5)$, $f_+^{D\to\pi}(0) = 0.86(9) f_+^{D\to K}$ BES [hep-ex/0406028] CLEO [hep-ex/0407035]
- f_{D_s}, f_D:
  - $f_D = 225^{+11}_{-13} \pm 21$ MeV [hep-lat/0410030]

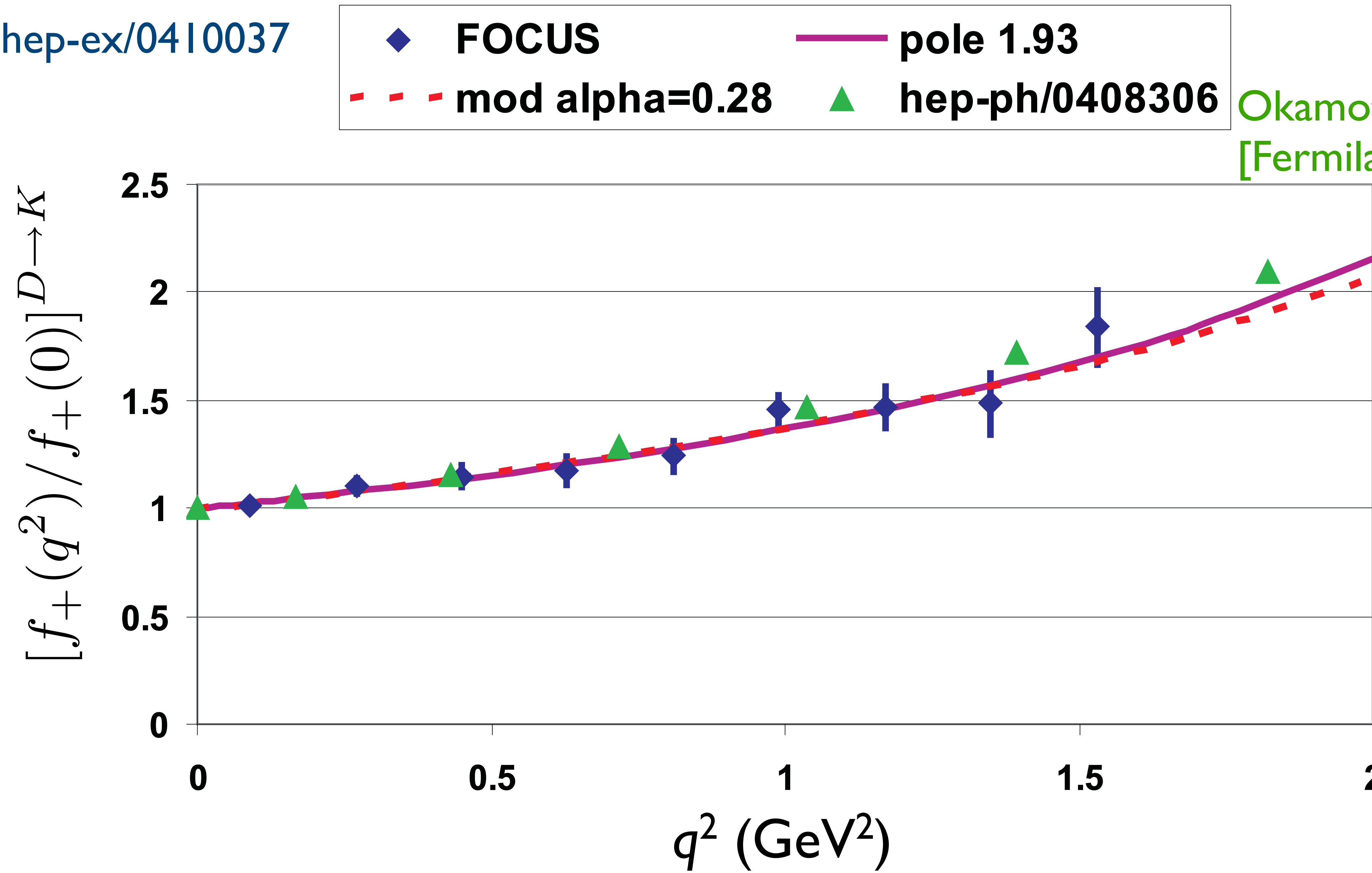
    $f_D = 202 \pm 41 \pm 17$ MeV CLEO[hep-lat/0411050]
- B_c mass:

  $m_{B_c} = 6304 \pm 12^{+18}_{-0}$ MeV [hep-lat/0411027]

  $m_{B_c} = 6287 \pm 5$ MeV CDF, W&C seminar, 12/03/04

$D \to Kl\nu$ vs. q^2

hep-ex/0410037



Okamoto *et al.* [Fermilab/MILC]

a Dependence

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Discretization Effects

- Putting field theory onto space-time lattice gives an non-perturbative ultraviolet cutoff.
- Creates discretization effects at non-zero a.
  - analogous to discrete approximations to partial differential equations
  - complicated by renormalization
  - in computer a is always non-zero

Symanzik LE£

- Symanzik proposed a continuum effective field theory to describe cutoff effects

details of lattice

$$\mathcal{L}_{\text{LGT}} \doteq \mathcal{L}_{\text{Sym}}$$

$$\mathcal{L}_{\text{Sym}} = \mathcal{L}_{\text{QCD}} + \sum_i a^{s_i} K_i(g^2, m_q a; \mu a)\, \mathcal{O}_i(\mu)$$

- Separates cutoff effects into short-distance coefficients and long-distance operators.

- Can be used to make back-of-the-envelope estimates of cutoff effects.
- Proven to all orders in perturbative QCD.
  - with mathematicians’ rigor for Wilson fermions
  - with physicists’ rigor for staggered fermions
- Hard to see what would go wrong non-perturbatively.

Symanzik Improvement

- The Symanzik effective theory provides a strategy for improving the discretization: reduce the K_i for any observable, and the LE£ shows that it is reduced everywhere.
- If *a* is only the short distance, it justifies a simple Ansatz—O(*a*), O(a^2)—for extrapolating to the continuum limit.

Heavy Quarks

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Heavy Quarks in LGT

- With heavy quarks there are two short distances, a and m_Q^{-1}.
- In practice $m_Q a$ is of order 1.
- Symanzik's split into "QCD + small corrections" breaks down.

Heavy Quark Theory

- In many aspects of heavy quark physics, including the bound-state problem, the scale m_Q decouples—heavy quark symmetry.
- Heavy quark Lagrangian—the Lagrangian for heavy-quark effective theory (HQET) and non-relativistic QCD (NRQCD).

HQET for $\bar{q}Q$

$$\mathcal{L}_{\mathrm{QCD}} \doteq \mathcal{L}_{\mathrm{HQ}}$$

$$\begin{aligned}
\mathcal{L}_{\mathrm{HQ}} &= \mathcal{L}_{\mathrm{light}} - \bar{h}_v(m_1 + iv\cdot D)h_v \\
&\quad + \frac{\bar{h}_v D_\perp^2 h_v}{2m_2} + z_B(\mu)\frac{\bar{h}_v s_{\mu\nu}B^{\mu\nu}h_v}{2m_2} \\
&\quad + z_{\mathrm{D}}(\mu)\frac{\bar{h}_v D_\perp\cdot E h_v}{4m_2^2} + z_{\mathrm{s.o.}}(\mu)\frac{\bar{h}_v s_{\mu\nu}D_\perp^\mu E^\nu h_v}{4m_2^2} \\
&\quad + \cdots \\
&= \sum_i \quad \mathcal{C}_i(m_Q, m_Q/\mu) \quad \mathcal{O}_i(\mu/\Lambda)
\end{aligned}$$

short distances: $1/m_Q$, a: lumped into coefficients

long distances: $1/\Lambda$, L: described by operators

NRQCD for $\bar{Q}Q$

$$
\begin{aligned}
\mathcal{L}_{\text{QCD}} &\doteq \mathcal{L}_{\text{HQ}} \\
\mathcal{L}_{\text{HQ}} &= \mathcal{L}_{\text{light}} - \bar{h}_v(m_1 + iv\cdot D)h_v + \frac{\bar{h}_v D_\perp^2 h_v}{2m_2} \\
&\quad + z_B(\mu)\frac{\bar{h}_v s_{\mu\nu}B^{\mu\nu}h_v}{2m_2} - z_R(\mu)\frac{\bar{h}_v(D_\perp^2)^2 h_v}{8m_2^3} \\
&\quad + z_{\text{D}}(\mu)\frac{\bar{h}_v D_\perp\cdot E h_v}{4m_2^2} + z_{\text{s.o.}}(\mu)\frac{\bar{h}_v s_{\mu\nu}D_\perp^\mu E^\nu h_v}{4m_2^2} \\
&+ \cdots \\
&\doteq \sum_i \quad \mathcal{C}_i(m_Q, m_Q/\mu) \quad \mathcal{O}_i(\mu/m_Q v^n)
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$$

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Lattice NRQCD

$$\begin{array}{c}\mathcal{L}_{\mathrm{QCD}} \\ \downarrow \\ \mathcal{L}_{\mathrm{HQ}} \\ \downarrow \\ \mathcal{L}_{\mathrm{LGT}} \\ \downarrow \\ \mathcal{L}_{\mathrm{HQ}(a)} \\ \downarrow \\ \mathcal{L}_{\mathrm{HQ}} \\ \downarrow \\ \mathcal{L}_{\mathrm{QCD}}\end{array}$$

Fermilab Method

$$\begin{array}{c}\mathcal{L}_{\mathrm{QCD}} \\ \downarrow \\ \mathcal{L}_{\mathrm{LGT}} \\ \downarrow \\ \mathcal{L}_{\mathrm{HQ}(a)} \\ \downarrow \\ \mathcal{L}_{\mathrm{HQ}} \\ \downarrow \\ \mathcal{L}_{\mathrm{QCD}}\end{array}$$

HQET & LGT

- The application of heavy-quark theory to understand the cutoff effects is essential in the calculations of $B \to D^{(*)}l\nu$ form factors.
- In particular, need HQET formalism to show that m_Q^{-1} corrections to $\mathcal{L}$ and J yield m_Q^{-2} corrections to $h_+, h_1, \check{h}_{A_1}$.

$$\check{h}_{A_1}(1) = \check{\eta}_A \left[1 - \ell_A \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2\right]$$

$$h_+(1) = \eta_V \left[1 - \ell_P \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2\right]$$

$$h_1(1) = \eta_V \left[1 - \ell_V \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2\right]$$

1. calculate lattice versions of the above;
2. remove (lattice) short distance factors;
3. fit mass dependence to obtain *l*s;
4. reconstitute

$$h_{A_1}(1) = \eta_A \left[1 - \frac{\ell_V}{(2m_c)^2} + \frac{2\ell_A}{2m_c\,2m_b} - \frac{\ell_P}{(2m_b)^2}\right]$$

Chiral Extrapolation

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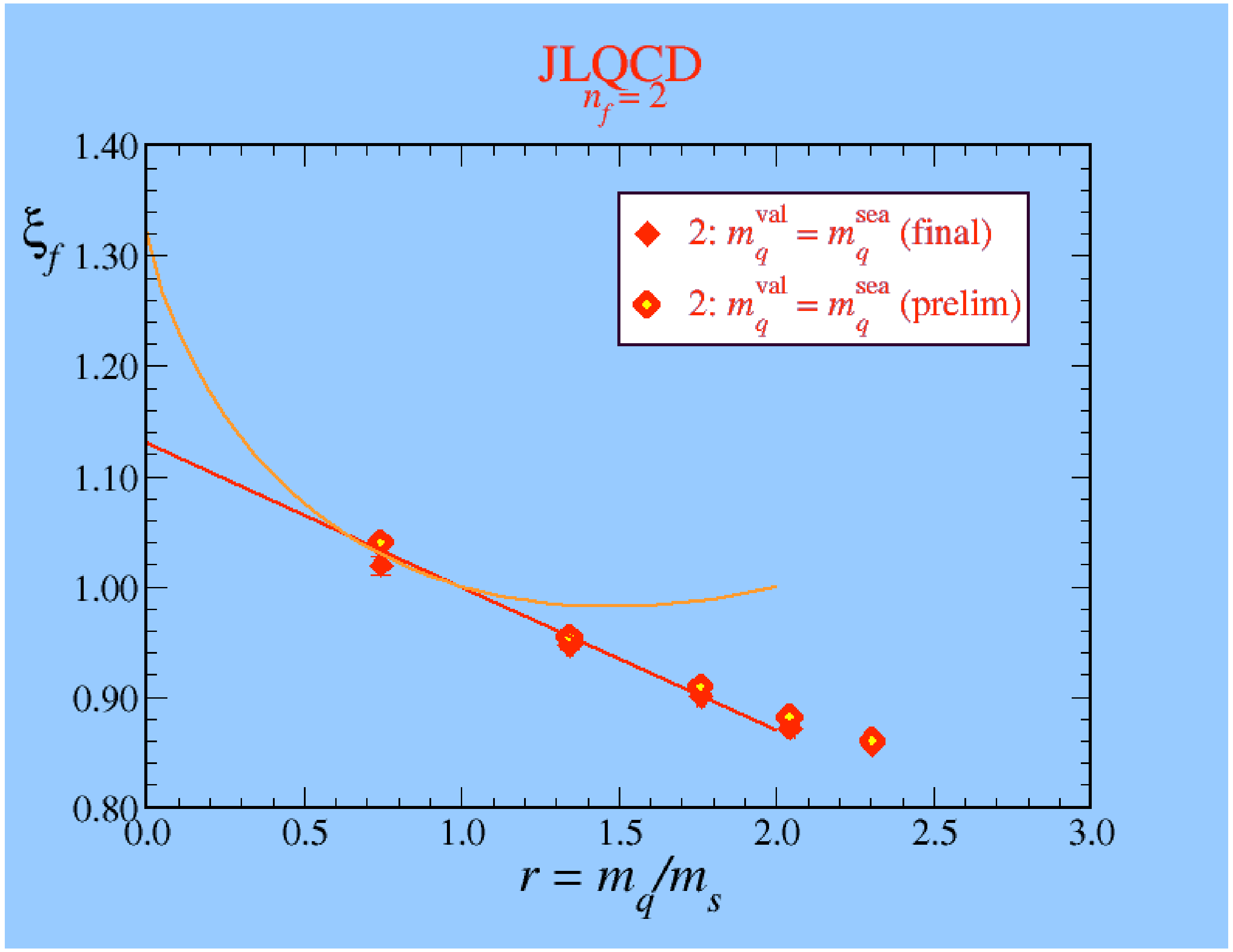
Chiral Extrapolation

- The algorithms for sea quarks (det *M*) and quark propagators are much too slow if the light quark mass is as small as down or up.
- Consequently, the pion cloud is not right.
- It can be corrected using chiral perturbation theory (ChPT) to guide an extrapolation...
  - ... if the light quark masses in the computer are small enough.

χlog vs linear

$$\xi_f = f_{B_s}/f_B$$

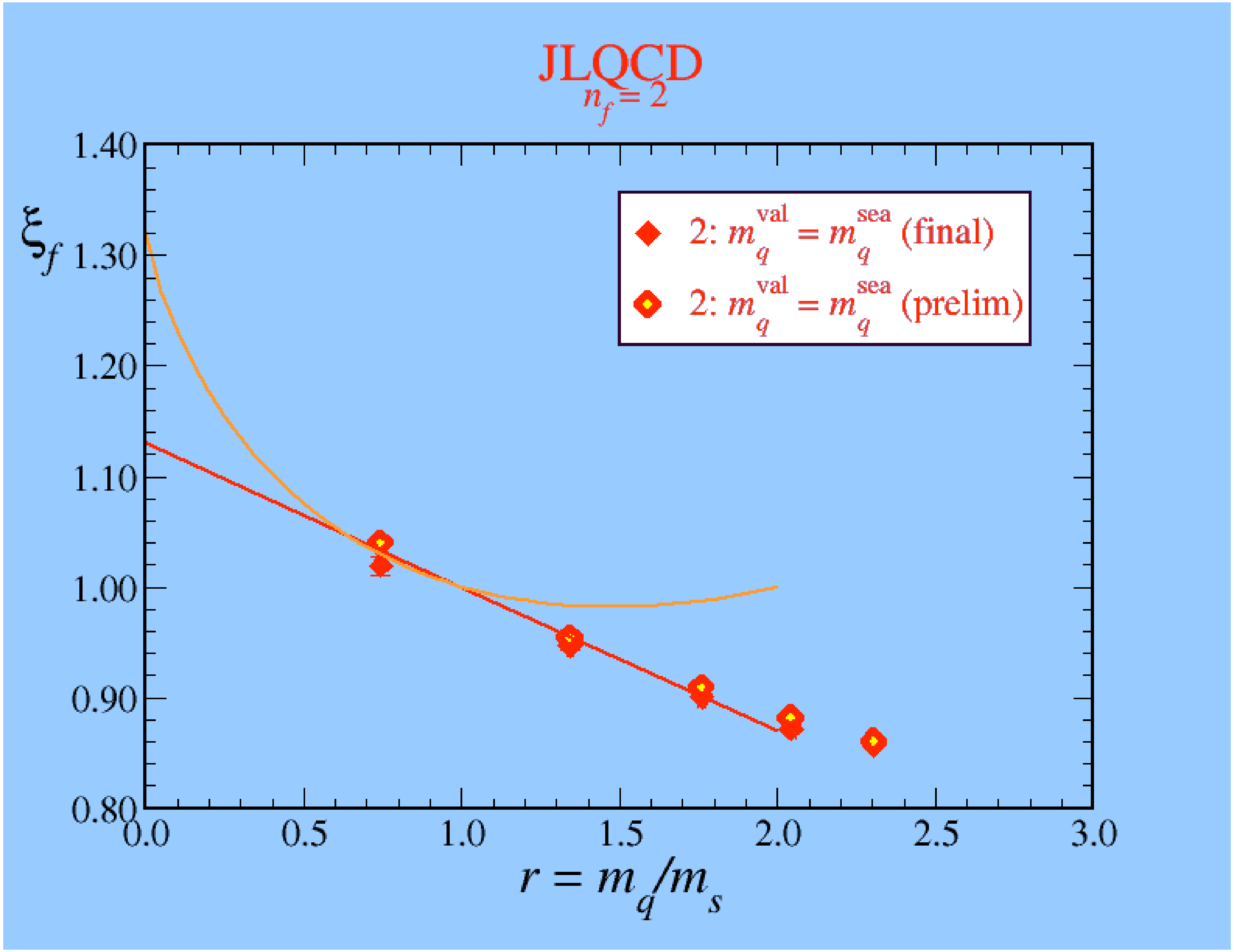


The plot compares JLQCD's linear fit with one that feeds their slope into the χlog expression.
ASK & Ryan, hep-ph/0206058

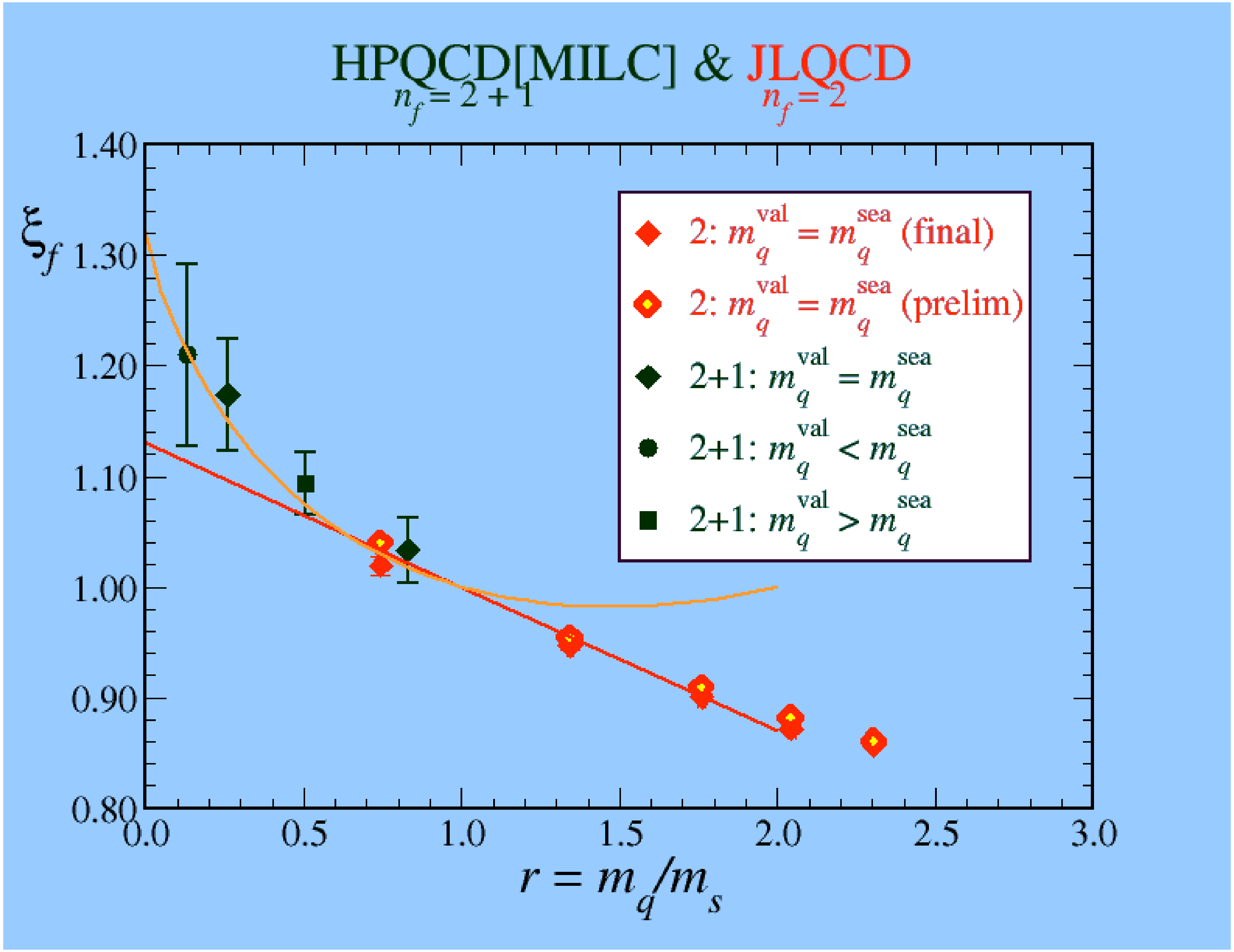
Other Ansätze lie between these two.

Thanks to N. Yamada, S. Hashimoto, and T. Onogi

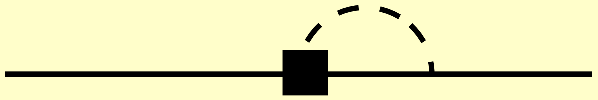
Now add 2+1 (MILC) results from Wingate (HPQCD)

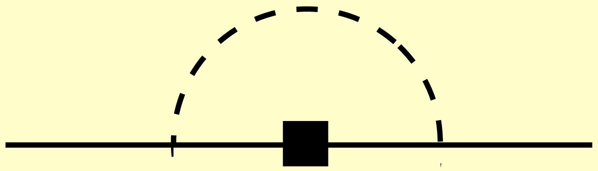


S. Aoki et al. [JlQCD], hep-lat/0307039 → PRL


HPQCD[MILC] & JLQCD
$n_f = 2+1$
$n_f = 2$
ξ_f
1.40
1.30
1.20
1.10
1.00
0.90
0.80
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$r = m_q/m_s$
2: $m_q^{val} = m_q^{sea}$ (final)
2: $m_q^{val} = m_q^{sea}$ (prelim)
2+1: $m_q^{val} = m_q^{sea}$
2+1: $m_q^{val} < m_q^{sea}$
2+1: $m_q^{val} > m_q^{sea}$

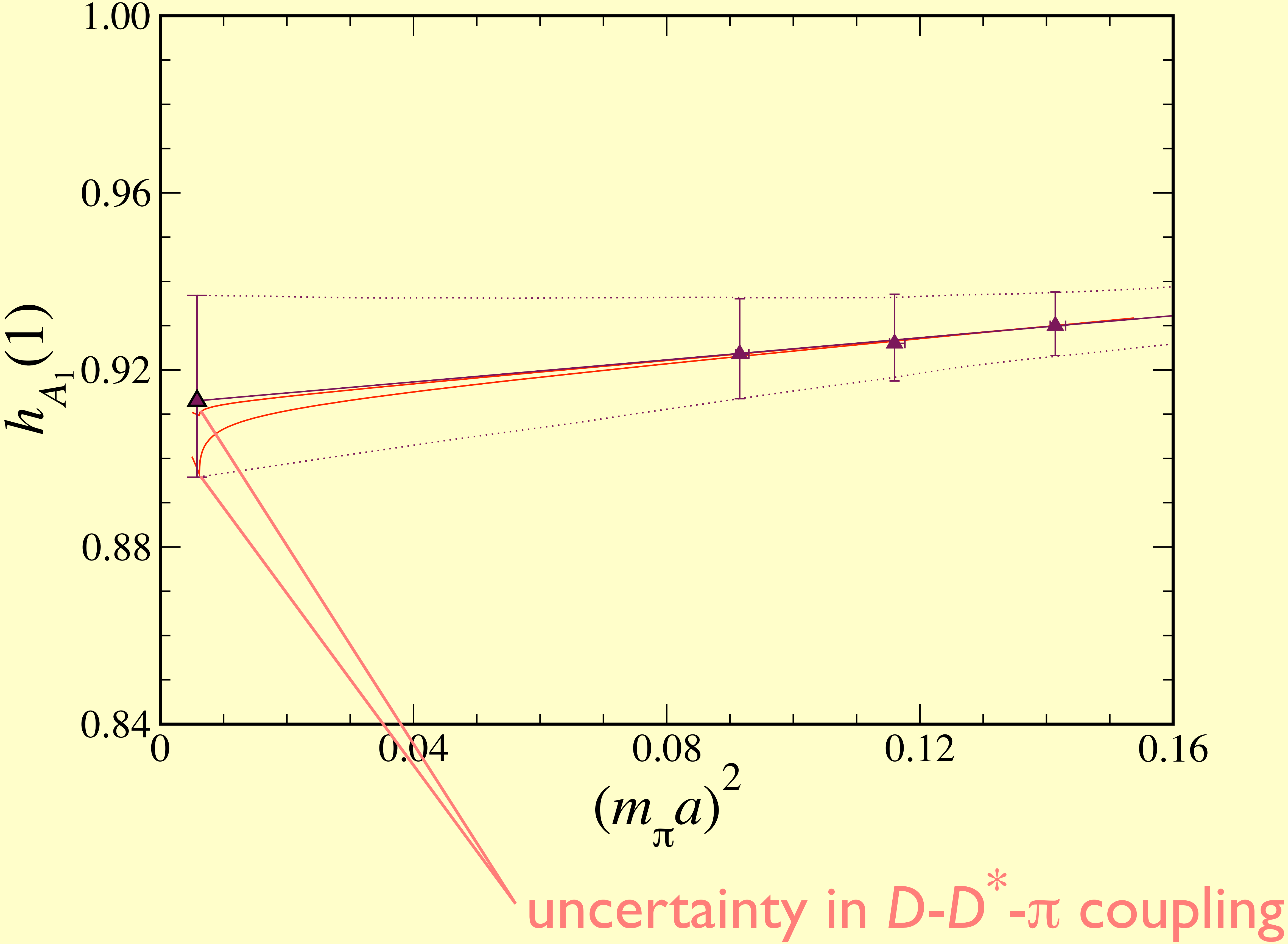
Form Factors





- In the case of $B \to Dl\nu$, the relevant internal state is $D^*\pi$; the m_π^2 dependence is flat.

- In the case of $B \to D^*l\nu$, the relevant internal state is $D\pi$; a cusp develops when $m_\pi + m_D < m_{D^*}$


1.00
0.96
0.92
0.88
0.84
0
0.04
0.08
0.12
0.16
$h_{A_1}(1)$
$(m_\pi a)^2$
uncertainty in D-D^*-π coupling

Prospects
(for $B \to D^{(*)} l \nu$)

Perturbative Matching

- The most important improvement needed (now that sea quarks are included) is better matching.
- Work in progress:
  - higher-dimension operators: Oktay, ASK, ...
  - perturbative part: El-Khadra, Nobes, Trottier; Aoki, Kayaba, Kuramashi, Yamada.

Chiral Extrapolation

- Will need to add cusp essentially by hand: there will be no sign whether the numerical data follow functional form of ChPT.
- Discretization effects (of light quarks) will have to be added to ChPT: Laiho.

New Error Budget(?)

hep-ph/0110253: Budget of statistical and systematic uncertainties for $h_{A_1}(1)$ and $1-h_{A_1}(1)$. The row labeled "total systematic" does not include uncertainty from fitting, which is lumped with the statistical error. The statistical error is that after chiral extrapolation.

uncertainty	h_{A_1}		$1-h_{A_1}$ (%)	
statistics and fitting	±0.005	+27		−20
adjusting m_c and m_b	±0.003	+ 8		− 8
α_s^2			± 9	
$\alpha_s(\bar{\Lambda}/2m_Q)^2$	±0.004		±13	
$(\bar{\Lambda})^3/(2m_Q)^3$			± 2	
a dependence	±0.003	+ 4		−16
chiral	±0.008	+ 0		−19
quenching	removed	+ 7		−16
total systematic	±0.011	+20		−35
total (stat ⊕ syst)		+34		−40

$$h_{A_1}(1) = \mathcal{F}^{B\to D^*}(1)$$

Prospects

- Lattice QCD should provide reliable results to help interpret experiments in flavor physics.
- My favorite paradigm
  - use trees to measure CKM
  - use loops to find new physics

Bibliography

- Reviews of lattice QCD
  - S. Hashimoto and T. Onogi, hep-ph/0407221
  - A. S. Kronfeld, hep-lat/0205021
- Numerical Results for $B \to D^{(*)}l\nu$
  - S. Hashimoto *et al.*, hep-ph/9906376 (for D)
  - S. Hashimoto *et al.*, hep-ph/0110253 (for D^*)

- Power Corrections
  - A. S. Kronfeld, hep-lat/0002008
- Radiative Corrections
  - J. Harada *et al.*, hep-lat/0112045
- Preliminary *unquenched* calculation
  - M. Okamoto *et al.*, hep-lat/0408306

    $\mathcal{G}^{B\to D}(1) = 1.074 \pm 0.018 \pm 0.016$