Heavy quark physics with NRQCD bs and light dynamical quarks

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HPQCD and UKQCD collaborations

Key aim of HPQCD collabn: accurate calcs in lattice QCD, emphasising heavy q physics. Requires a whole range of lattice systematic errors to be simultaneously minimised - critical one has been inclusion of light dynamical quarks.

- Current results on heavyonium, α_s etc
- Developments for calculations for next 1-2 years - moving NRQCD, HISQ

Japan Dec 2004

People involved in various aspects of this work:

I. Allison, S. Collins, CD, A. Dougall, K. Foley, E. Follana, E. Gamiz, A. Gray, E. Gulez, A. Hart, P. Lepage, Q. Mason, M. Nobes, J. Shigemitsu, H. Trottier, M. Wingate
HPQCD/UKQCD

C. Aubin, C. Bernard, T. Burch, C. DeTar, S. Gottlieb, E. Gregory, U. Heller, J. Hetrick, J. Osborn, R. Sugar, D. Toussaint,
MILC

M. Di Pierro, A. El-Khadra, A. Kronfeld, P. Mackenzie, D. Menscher, M. Okamoto, J. Simone
HPQCD/Fermilab

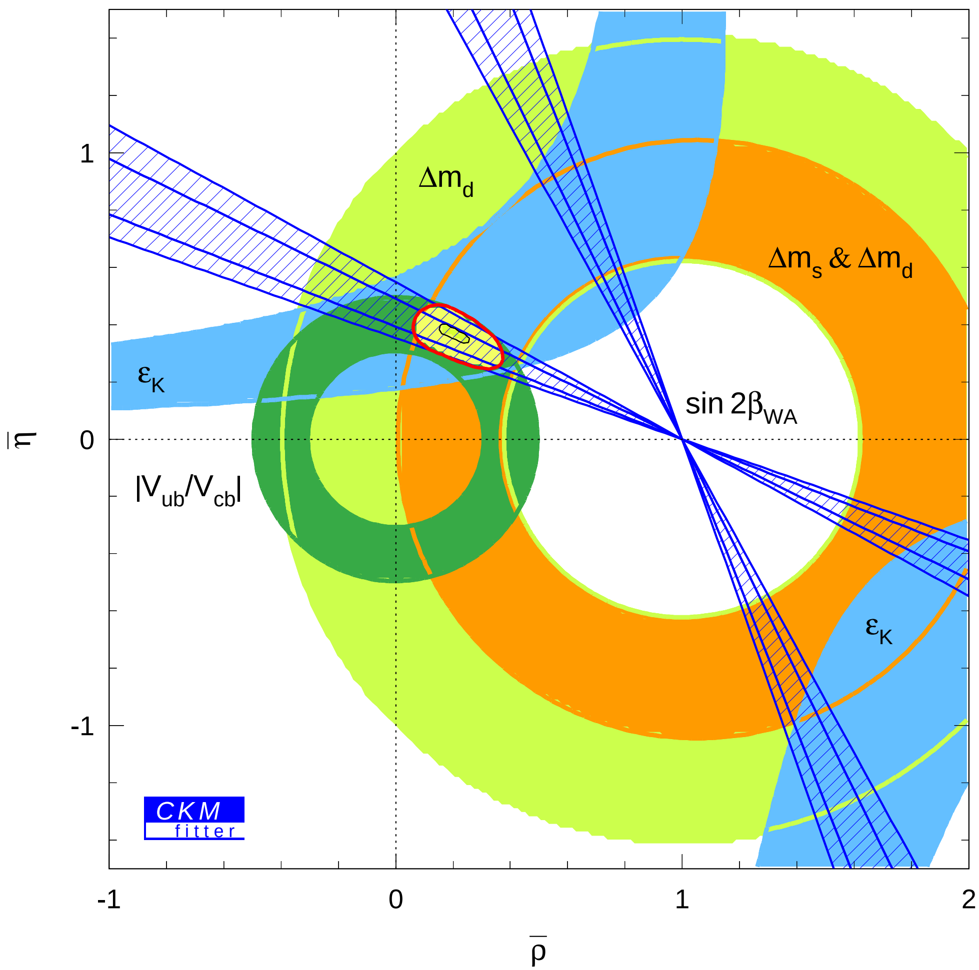
Heavy quark physics is an important part of the Standard Model and place where lattice QCD can make key calculations.

- Form many 'gold-plated', well-characterised heavy-heavy bound states whose masses can be calculated accurately in lattice QCD. New states being discovered there currently - *predictions* possible.
- Heavyonium states test the b and c quark actions for use in calculations for heavy-light mesons and baryons.
  Heavy-light bound states are critical to understanding CKM unitarity triangle.

Problem on lattice is $m_Q a$ not small $\rightarrow$ special techniques needed.

The Unitarity triangle

Important objective of current particle physics: accurate determination of elements of CKM matrix.



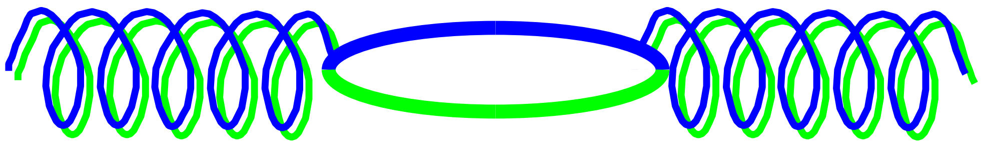
B factory prog. needs small 2-3% *reliable* lattice QCD errors for $B_{s/d}$ oscillations, $B \to D$ or π decay.

CLEO-c will test lattice predictions for D physics in next 2 years.

Requires *all* systematic errors to be small simultaneously. Precise quenched calcs are no good!

HPQCD/MILC spectrum results 2003

MILC collab. have used improved staggered quark formalism ($+$ highly improved gluon action) to generate ensembles of configurations which include 2$+$1 flavours of dynamical quarks.

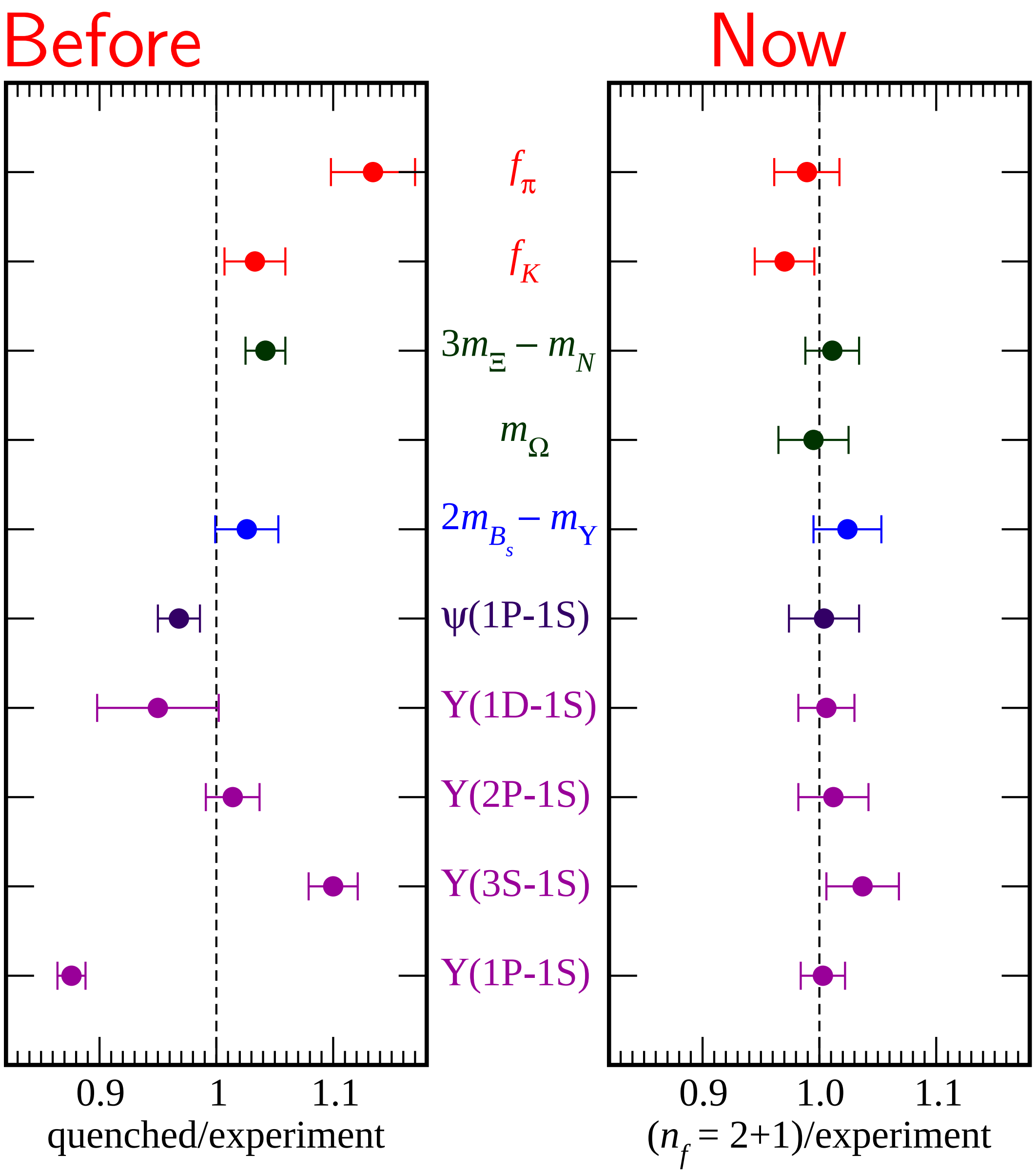


2 $= u, d$ degenerate with masses down to $m_s/8$.
1 $= s$ (can ignore heavy c, b, t dynamical qs.)
3 values of lattice spacing, $a \approx$ 0.087 fm and 0.12fm and 0.18fm.

Fix 5 free parameters of QCD (bare $m_u = m_d$, m_s, m_c, m_b, and $a \equiv \alpha_s$) using
$m_\pi, m_K, m_{D_s}, m_\Upsilon$ and $\Delta E_\Upsilon(2S - 1S)$. These are 'gold-plated' quantities (e.g. stable hadron masses).

Compute other 'gold-plated' quantities as a test of (lattice) QCD.

Lattice QCD/Experiment (no free parameters!):



Tests:

light mesons and baryons

heavy-light mesons

heavyonium

Find agreement with expt (at last!) when correct dyn. quark content is present.

Quenched approx. has syst. errors 10% and internal inconsistency.

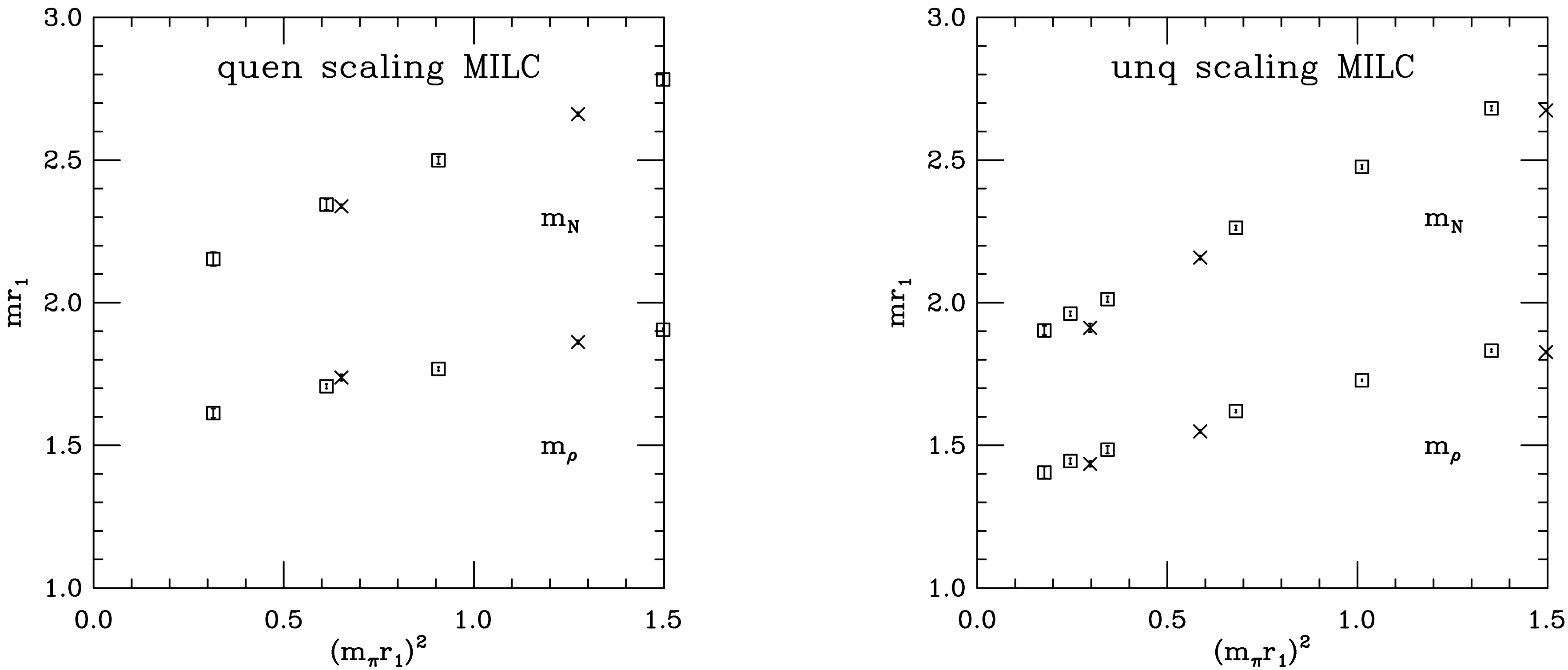
Davies *et al*, hep-lat/0304004 + Toussaint,Davies, LAT04

These results needed:

- Large ensembles to get good statistical errors. Long length in the time direction gives good π mass.
- Large physical volume.
- Very light u and d quark masses so chiral extrapolation is not far.
- Good control of discretisation errors with a highly improved gluon and quark action.

In fact discretisation errors are largest source of remaining uncertainty. Disc. errors are worse unquenched than they were quenched. (a few % vs zero)

Is this from glue or from a^2 errors in dynamical quarks (handled by staggered chiral pert. th.)?



Future calculations will improve further on this in two ways:

- Run at even finer lattice spacing values - $a = 0.06$fm with $m_l/m_s = 0.2$ costs 3 Tflopyrs. $(48^3 \times 144)$. This halves all discretisation errors compared to MILC fine lattice set. May be done by UKQCD+MILC.

- Run with more highly improved gluon and quark action. Gluon $N_f \alpha_s a^2$ corrections being calculated (Mason and Horgan). Highly Improved Staggered Quarks have half the taste-changing errors of asqtad, (Follana).

NonRelativistic QCD (NRQCD)

Discretisation errors are naively a worse problem for heavy quarks because $m_Q a$ is large. However, their non-relativistic nature saves us. NRQCD good for heavy quarks - can match order by order in v_Q and α_s to continuum full QCD.

L is $\psi^\dagger(D_t + H)\psi$, where ψ is a 2-spinor.

$$\begin{aligned} H_0 &= -\frac{\Delta^{(2)}}{2M} \\ \delta H &= -c_4 \frac{g}{2M}\vec{\sigma}\cdot\vec{B} + c_2\frac{ig}{8M^2}(\nabla\cdot\vec{E} - \vec{E}\cdot\nabla) \\ &- c_3\frac{g}{8M^2}\vec{\sigma}\cdot(\nabla\times\vec{E} - \vec{E}\times\nabla) \\ &- c_1\frac{(\Delta^{(2)})^2}{8M^3}(1 + \frac{Ma}{2n}) + c_5\frac{a^2\Delta^{(4)}}{24M} + \ldots \end{aligned}$$

Fast to solve on one pass thru lattice. All Us tadpole-improved.

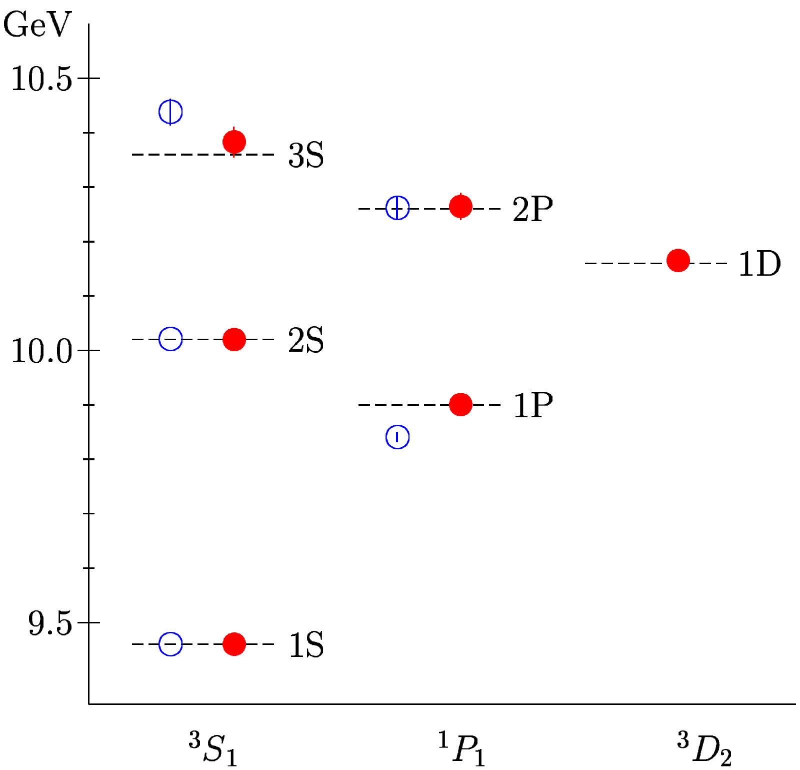
For b quarks this is an excellent action. For c quarks more problematic.

$\Upsilon(b\bar{b})$ spectrum

Lattice NRQCD for bs on MILC configs. Tests/tunes action for Bs.

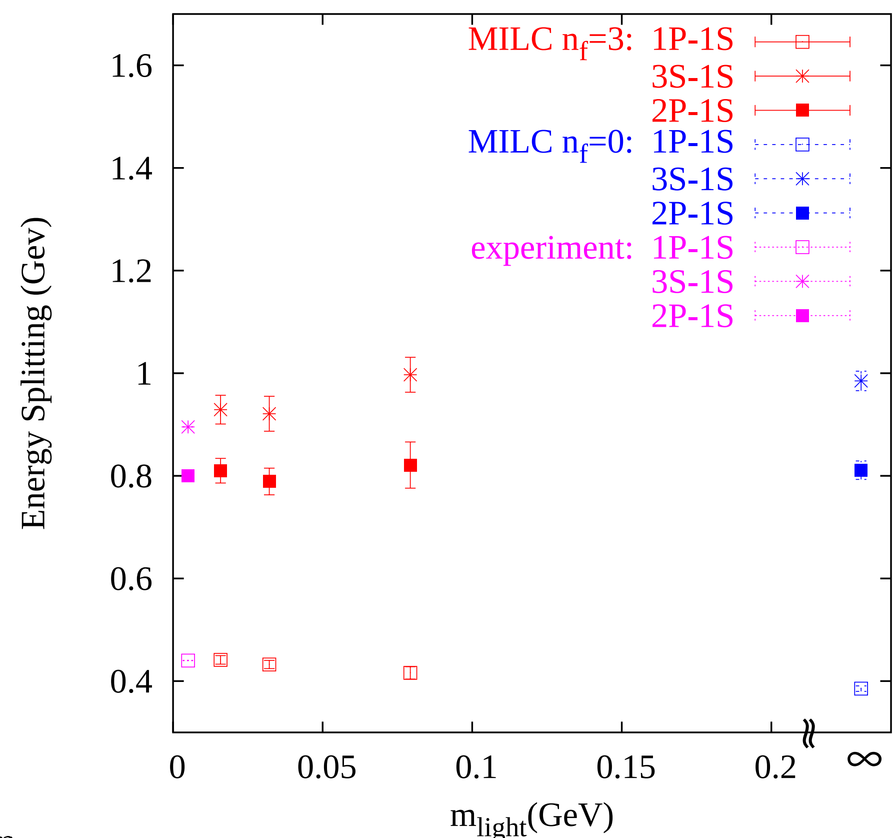
2S-1S fixes a and 1S fixes am_b.

1-loop matching gives $m_{b,\overline{MS}}(m_{b,\overline{MS}})$=4.3(3) GeV.



--- : Experiment ○ : Quenched MILC

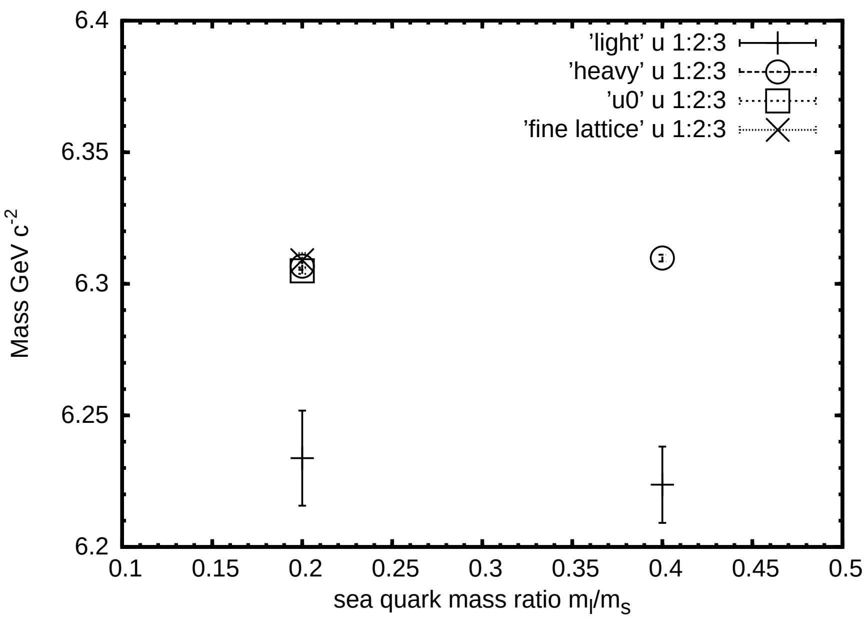
● : 2+1 flavors MILC, $m_{u,d} = m_s/4$, $a = 0.13$fm.

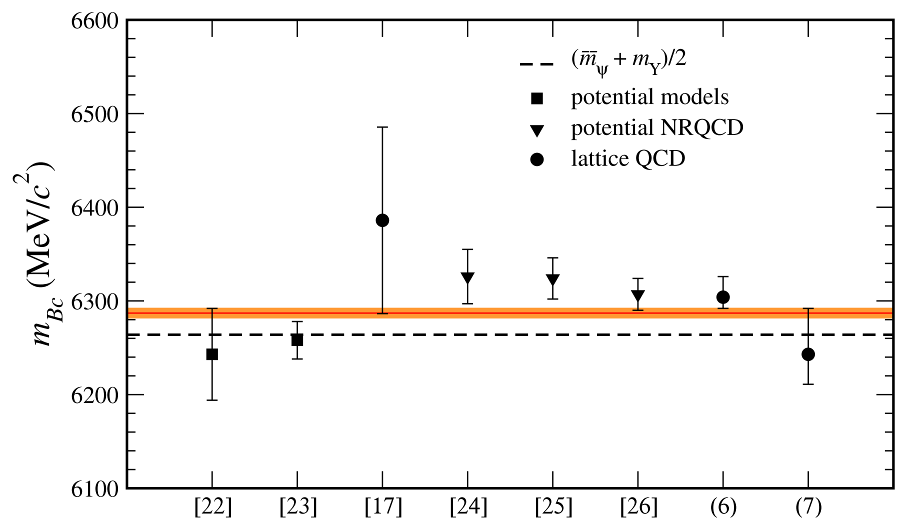


Gray, Davies et al, HPQCD, hep-lat/0310041, Gulez,Shigemitsu, hep-lat0312017.

Prediction of B_c mass.

From difference between mass of B_c (NRQCD b, Fermilab c) and average of Υ and J/ψ, get 6.304±12+18-0 GeV.





New experimental result from CDF (Glasgow, FNAL and Texas Tech) 6287(5) MeV.

Allison, Davies, Gray, Kronfeld, Mackenzie, Simone (HPQCD), LAT04

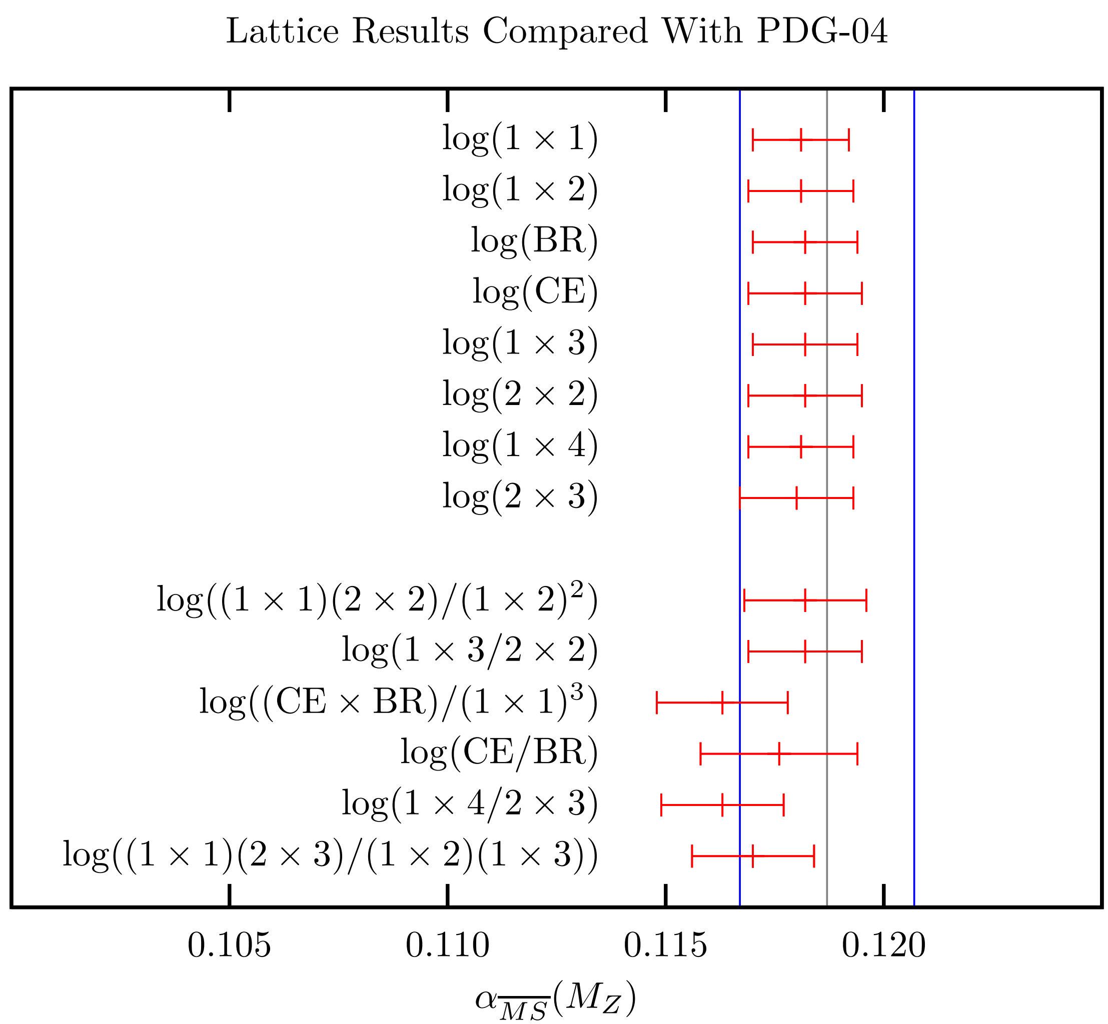
Precise determination of α_s.

Mean value of various Wilson loops and their ratios calculated to 3rd order in lattice pert. th. and on the lattice.

Results available at 3 values of a (inc. MILC super-coarse) allows higher order terms to be estimated.

Preliminary result: $\alpha_s(M_z) = 0.1181(15)$.

Improves on PDG. Will get better with even finer lattices.



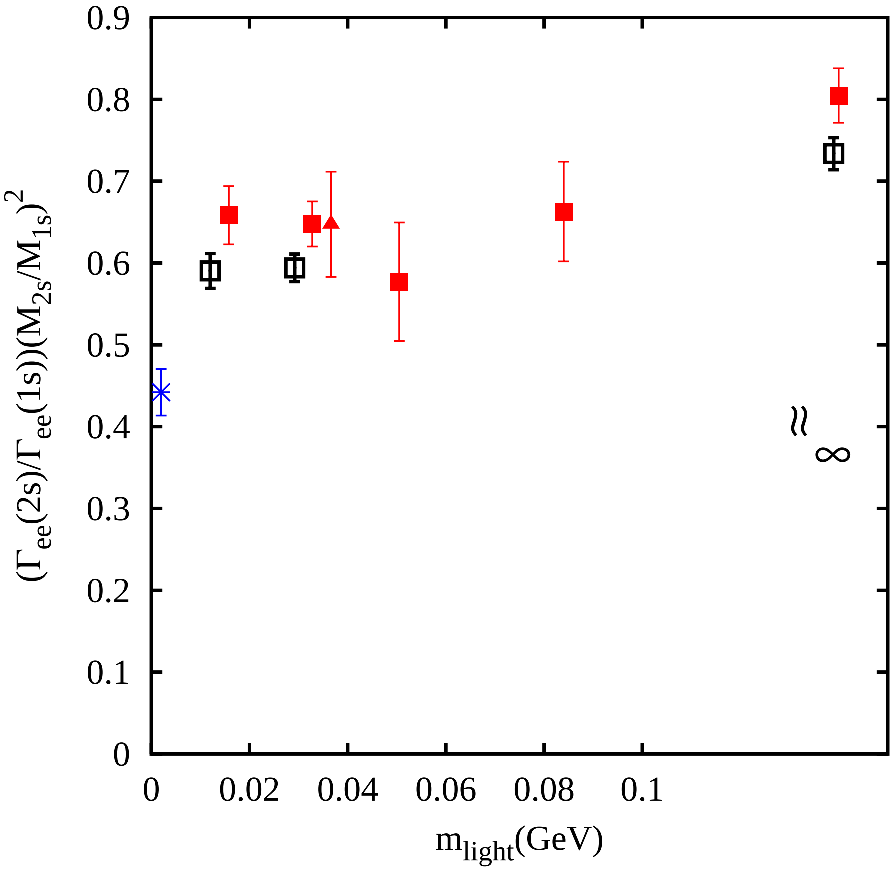
Mason, Trottier, Lepage *et al* (HPQCD), LAT04

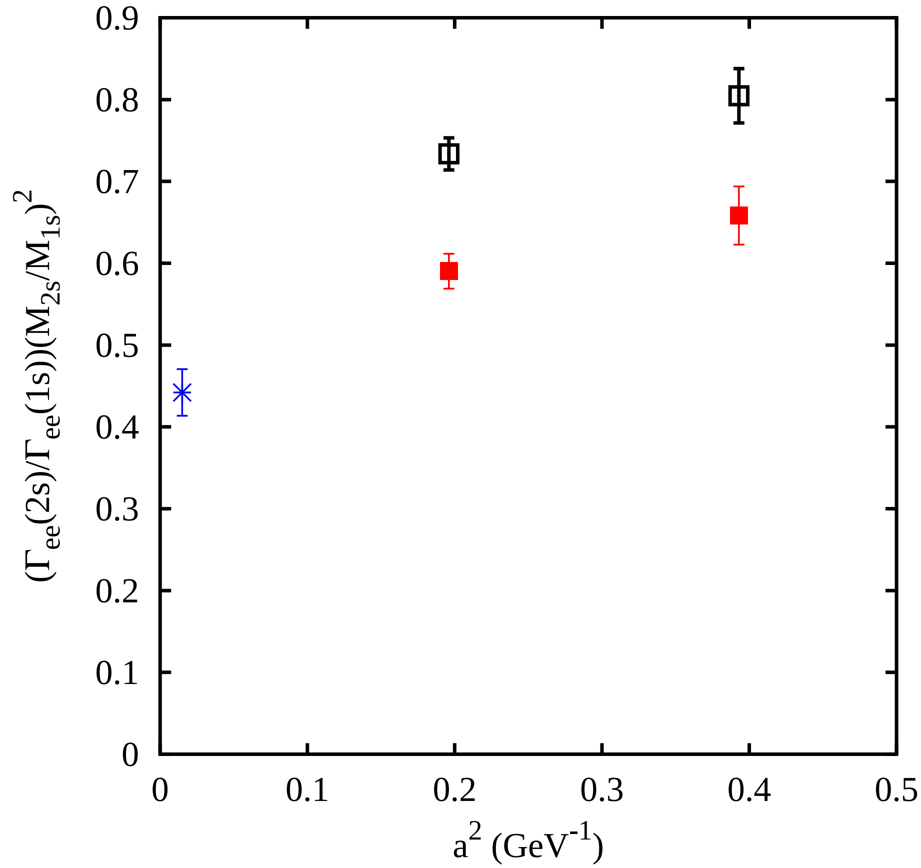
Matrix elements in heavyonium

Accurate calculation of Υ leptonic width is good check.

Renormln calc. by Horgan in progress. Meanwhile take ratio of $2S$ to $1S$ to cancel leading piece.

Clear that discretisation errors are main problem here - improve operators, action etc.





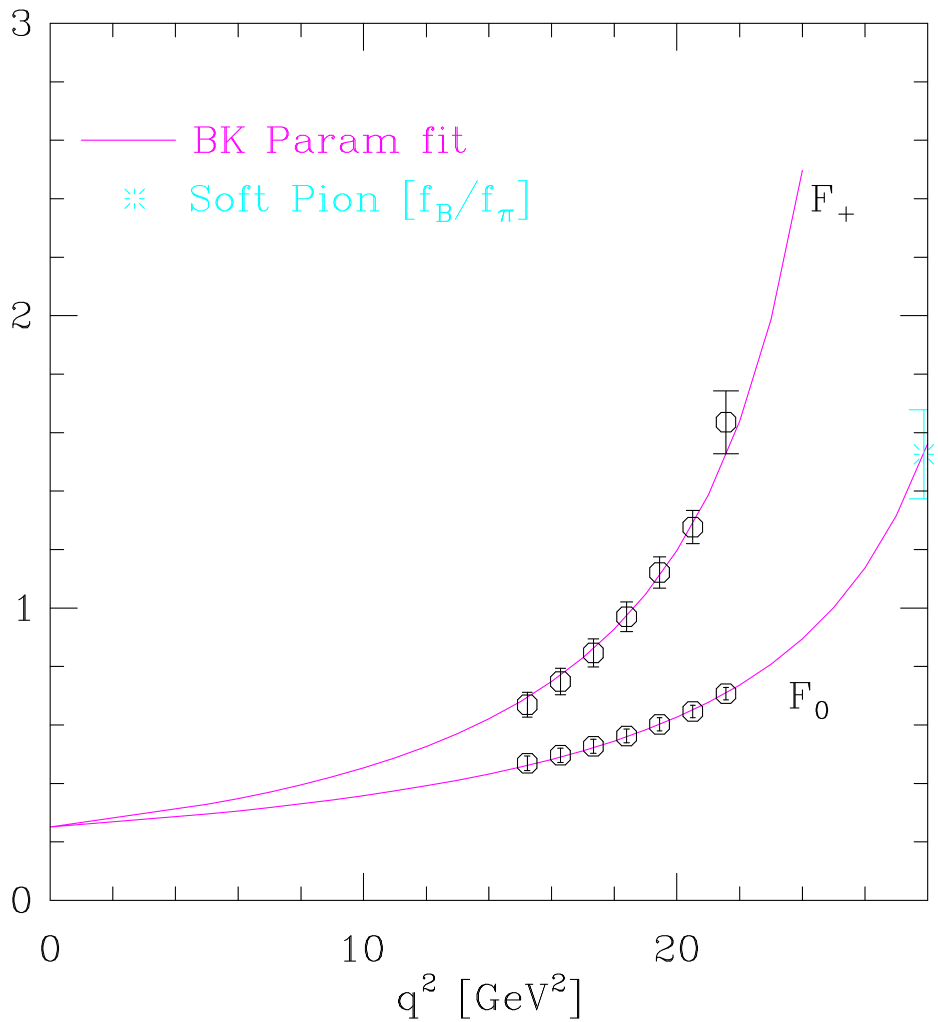
Gold-plated quantities for the CKM matrix

Gold-plated decays (i.e at most one hadron in final state) exist for almost every element (+ $K - \overline{K}$ mixing). Can now calculate these accurately in lattice QCD. Important for lattice calcs to have extensive cross-checks for error calibration: Υ, B, ψ, D, etc.

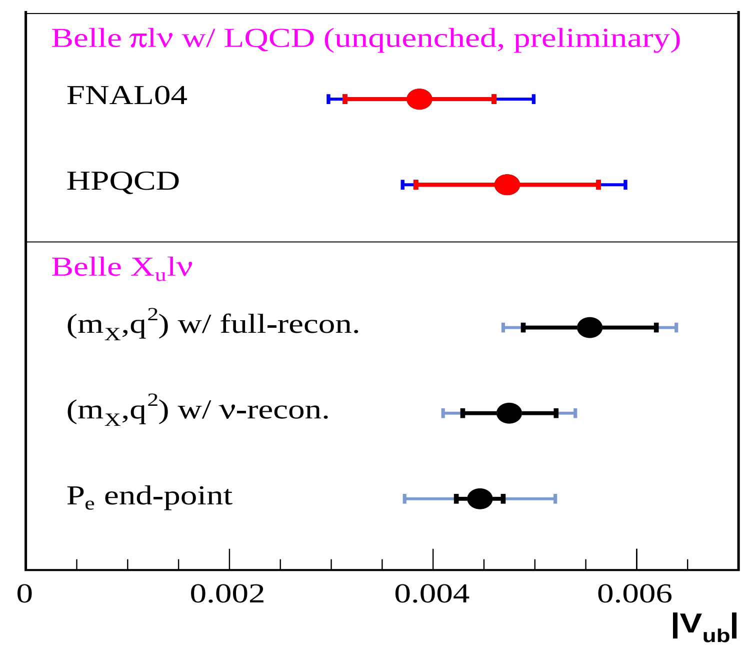
$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to l\nu & K \to l\nu & B \to \pi l\nu \\ & K \to \pi l\nu & \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to l\nu & D_s \to l\nu & B \to Dl\nu \\ D \to \pi l\nu & D \to Kl\nu & \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d|\overline{B}_d\rangle & \langle B_s|\overline{B}_s\rangle & \end{pmatrix}$$

Unquenched results for $B \to \pi$ form factors

Extrapoln to physical m_π is done at fixed E_π and is not far (lightest m_π = 260 MeV).



Shigemitsu+Gulez, HPQCD, LAT04



Used by Belle in new V_{ub} determn. (hep-ex/0408145)

These results are great but still suffer from one problem - they are unable to cover the full q^2 range.
$B \to \pi$ at small q^2 corresponds to π at large lattice momentum. This gives:

- increased statistical errors since signal/noise set by splitting to zero momentum π.

- increased systematic errors from discretisation errors at large $p_\pi a$.

Can solve these problems by moving B instead provided that we do not increase systematic errors in B system.
In fact we can treat large B momentum accurately since it is mostly b momentum and b momentum we can treat exactly with moving NRQCD.

Moving NRQCD

Write $P_b = m_b u + k$. $u = \gamma(1, \vec{v})$.

Now remove $m_b u$ from the action in the same way that m_b was removed for NRQCD.

For NRQCD, do FWT in rest frame of $b \equiv$ lattice frame.

For moving NRQCD rest frame of b boosted wrt lattice, and must boost back to get L in lattice frame.

Early work by Hashimoto and Sloan.

We have extended action, tested heavy-heavy and heavy-light and done $\mathcal{O}(\alpha_s)$ pert. th.

Foley, Lepage, Davies, Dougall, HPQCD, LAT04

Moving NRQCD

In b rest frame:

$$\begin{aligned} L &= \psi^\dagger(iD_t + \frac{D^2}{2m} + \frac{\sigma\cdot B}{2m} + \ldots)\psi \\ \Psi &= Te^{-im\gamma_0 t}\begin{pmatrix} \psi \\ \chi \end{pmatrix} \end{aligned}$$

In lattice frame:

$$\begin{aligned} L &= \psi^\dagger(iD_t + i\mathbf{v}\cdot\mathbf{D} + \frac{\mathbf{D}^2}{2\gamma m} + -\frac{(\mathbf{v}\cdot\mathbf{D})^2}{2\gamma m} + \frac{\sigma\cdot\tilde{B}}{2\gamma m}\ldots)\psi \\ \Psi &= \frac{\Lambda(\mathbf{v})}{\sqrt{\gamma}}Te^{-imu\cdot x}A_{D_t}\begin{pmatrix} \psi_v \\ \chi_v \end{pmatrix};\ \Lambda = \frac{1}{\sqrt{2(1+\gamma)}}\begin{pmatrix} 1+\gamma & \sigma\cdot\mathbf{v} \\ \sigma\cdot\mathbf{v} & 1+\gamma \end{pmatrix} \end{aligned}$$

Tests of Moving NRQCD

Use simplest action (with no spin-dependence).

Take quenched lattices at β=5.7 as cheap.

Do HH and HL. For L use clover propagators at κ_s.

Antiquark G is G_q^* with $v \rightarrow -v$

Check v dependence of e.g. kinetic mass for fixed ma. Extract this from:

$$E_v(k) + C(v) = \sqrt{(Z_p \mathbf{P_0} + \mathbf{k})^2 + M_{kin}^2}$$

$\mathbf{P_0} = \gamma m \mathbf{v}$ (twice this for HH).

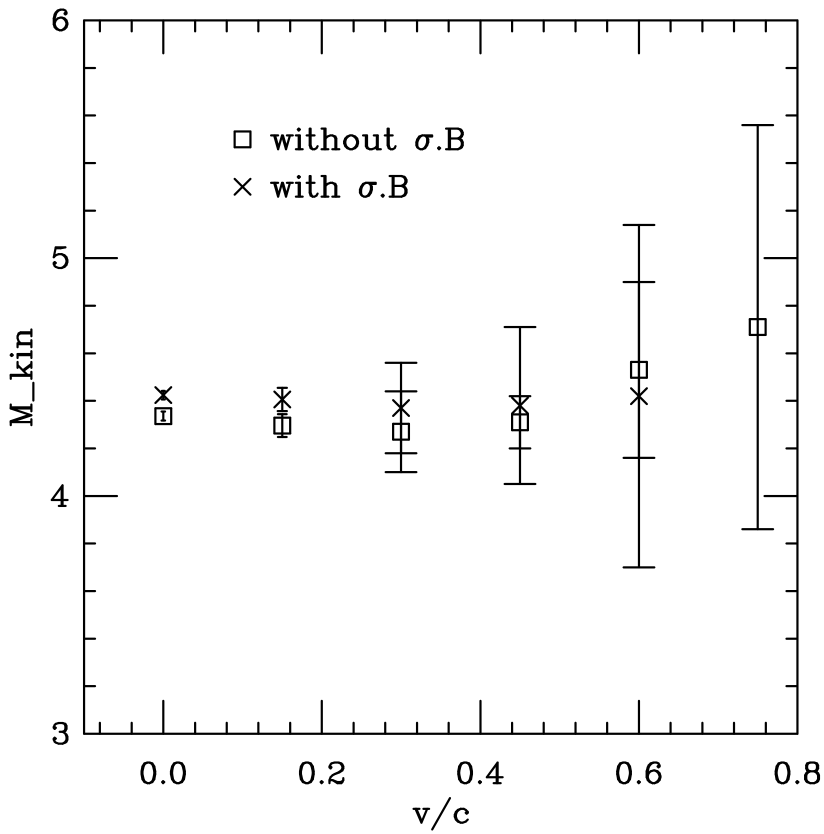
Working with different k dirns wrt to P_0 can extract Z_p and M_{kin}.

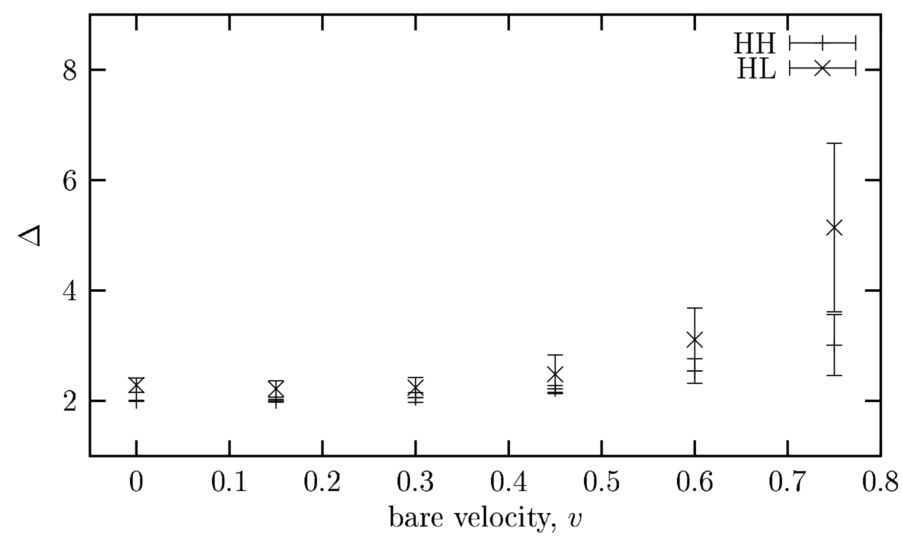
Noise grows as v and k grow as again set by zero total momentum states. Ameliorate with smearing.

Tests of Moving NRQCD

Find HH M_{kin} not strongly v-dependent for fixed ma i.e. will not need to change ma rapidly as a function of v to tune.

Find shift between χ and $E_v(k=0)$ (per quark) is same for HL and HH.





Tests of Moving NRQCD

Can calculate heavy quark self-energy perturbatively as done in NRQCD.

Working to $\mathcal{O}(\alpha_s)$:

$$\begin{aligned}\mathcal{G}^{-1} &= Q^{-1} - a\Sigma(k) \\ &= -ik_4 a - \alpha_s \Omega_0 + \alpha_s i k_4 a \Omega_1 + \mathbf{v}\cdot\mathbf{k}a - \alpha_s \mathbf{v}\cdot\mathbf{k}\Omega_v + \ldots \\ &= Z_\psi\left(-i\overline{k}_4 a + \frac{\mathbf{k}^2 a^2}{2\gamma_R m_R a} + \frac{\mathbf{P_R}\cdot\mathbf{k}}{\gamma_R m_R a} + \ldots\right)\end{aligned}$$

with $\Omega_v = \frac{1}{v_x}\frac{\partial\Sigma}{\partial k_x}$ etc.

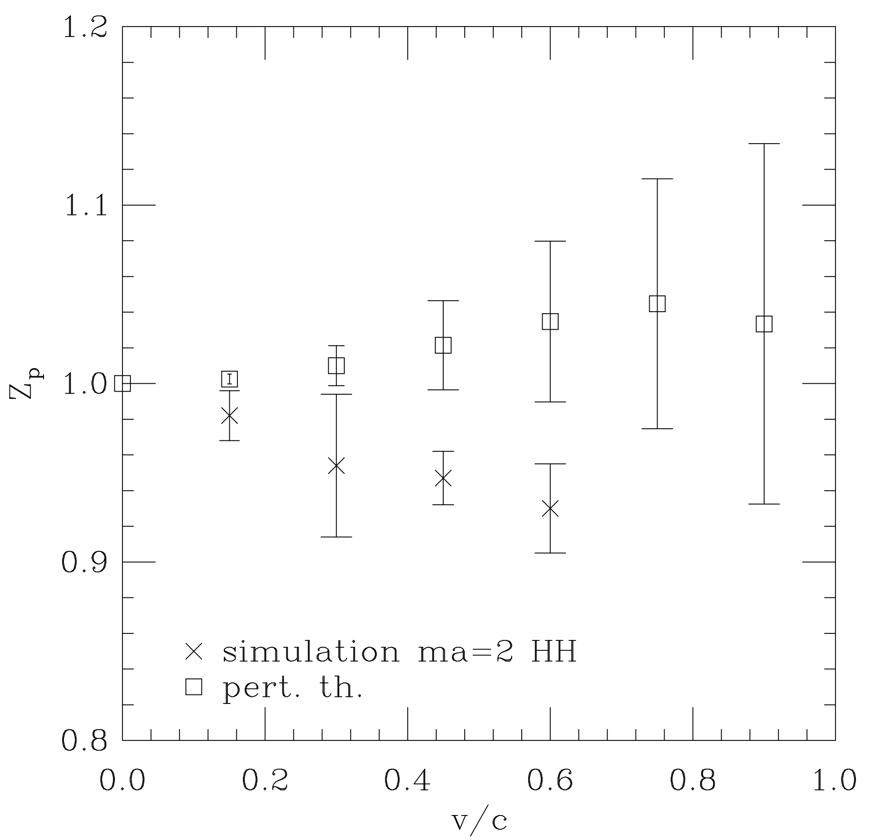
Remnant of reparameterisation invariance keeps renormln of P_0 small.

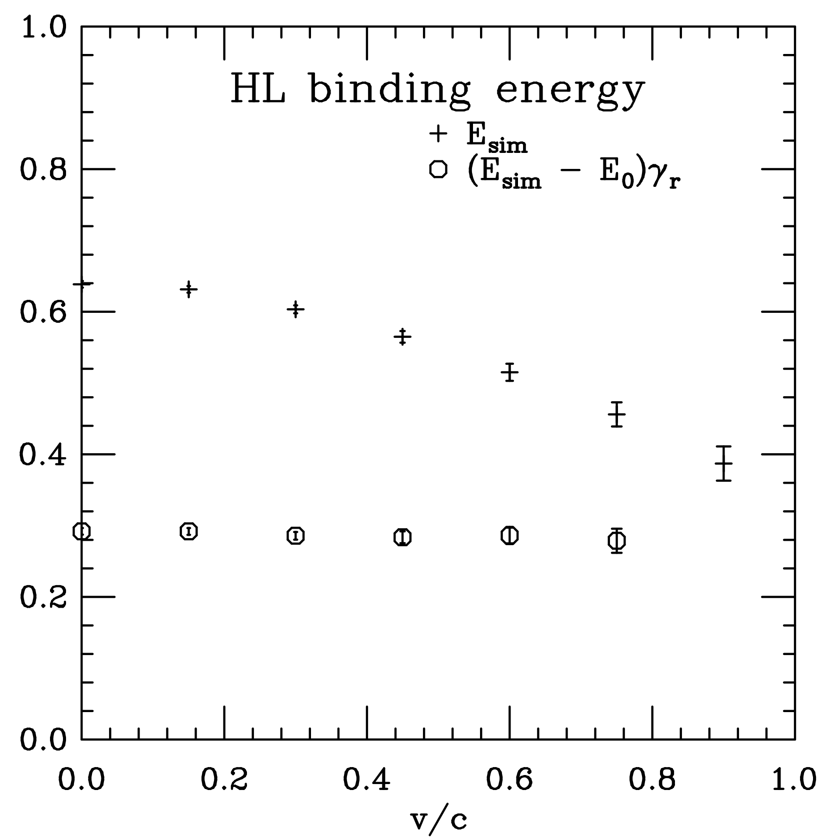
Physics is same if shift momentum between k and P_0.

Tests of Moving NRQCD

Test pert.th. against non-perturbative extraction of e.g. $Z_p =$ renormln of P_0.

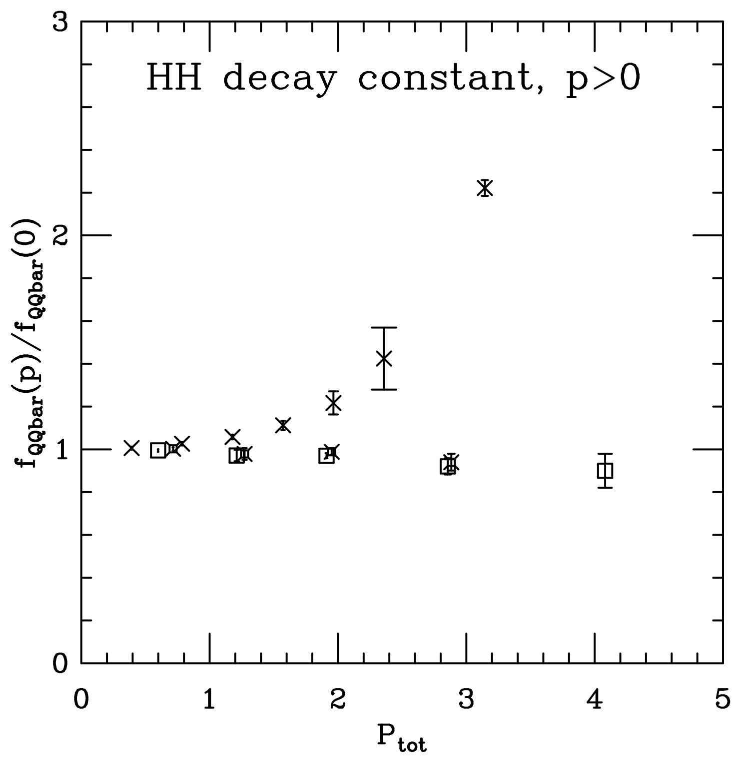
Also calculate HL binding energy as $\gamma_R(E_v(k=0) - E_0)$ and find v-independent.

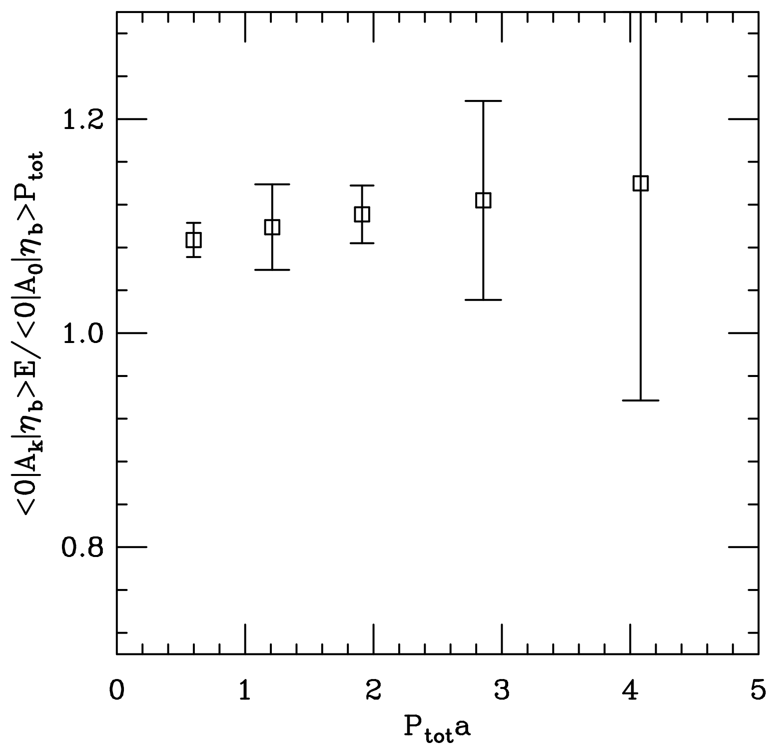




Tests of moving NRQCD - HH

Compare "decay constant" of η_b at rest and moving and from spatial and temporal axial vector current.





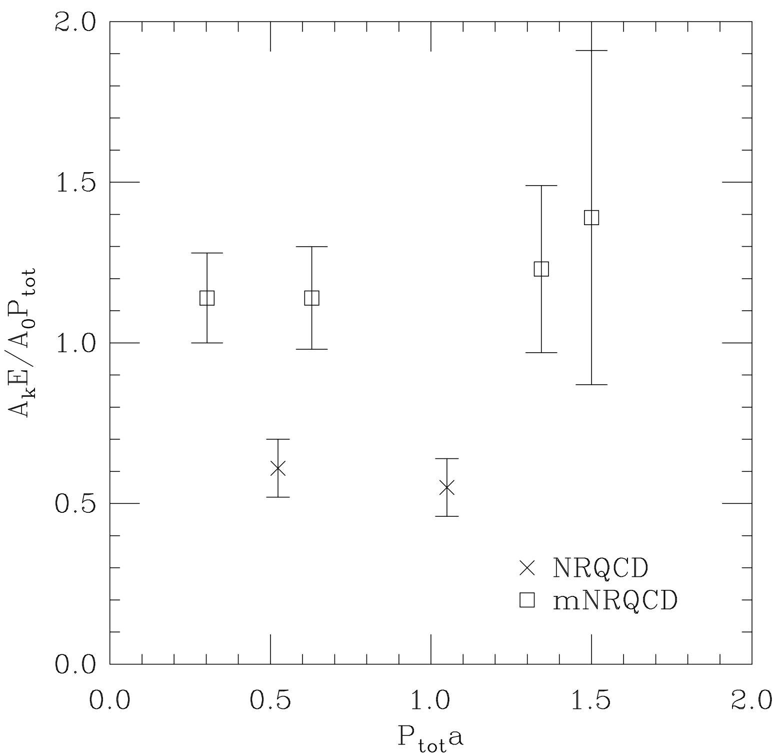
$J_0 = \chi_v^\dagger \psi_v$ and $J_k = \chi_v^\dagger v_k \psi_v$.

Relativistic corrns will fix A_k case to 1.

Tests of moving NRQCD - HL

Ditto for HL. Now have two pieces to currents *at leading order* coming from boost operator Λ.

e.g. $J_k \propto (1+\gamma)\chi^\dagger_{\frac{}{v}}\sigma_k q_{34} + \gamma\chi^\dagger_{\frac{}{v}}\sigma_k \sigma\cdot\mathbf{v}q_{12}$.



Now makes a big difference to getting f_B right from spatial axial current.

For non-moving case, sub-leading current is same size as leading current because $A_k \propto k$.

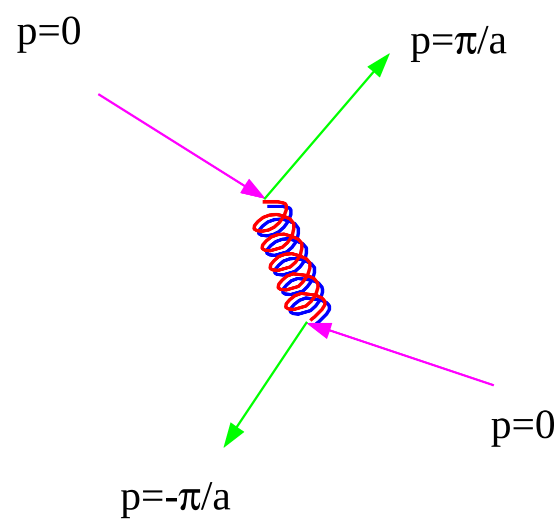
Renormln Z_v not yet calculated.

Moving NRQCD - future

- Need to start calculations on real MILC configs with staggered light quarks for appropriate 3-pt functions for $B \longrightarrow \pi$.
- Need to calculate renormalisation of current operators, preferably to 2-loops.

Further improving the staggered formalism

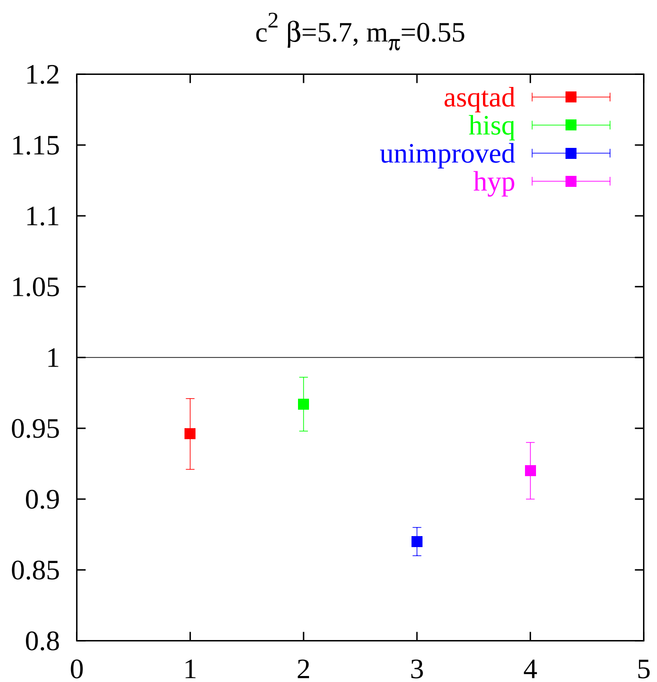
Limit to precision with asqtad improved staggered quarks is still taste-changing interactions associated with high-momentum gluon exchange.

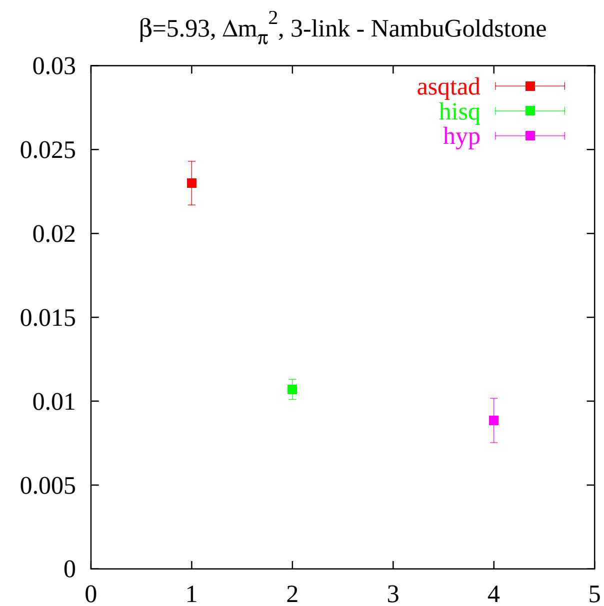


Improve action further by repeating the 'Fat7' smearing. Add Naik and Lepage terms (x2) as before to keep an action with $\alpha_s a^2$ errors *only*. This is the Highly Improved Staggered Quark action (HISQ).

Discretisation errors

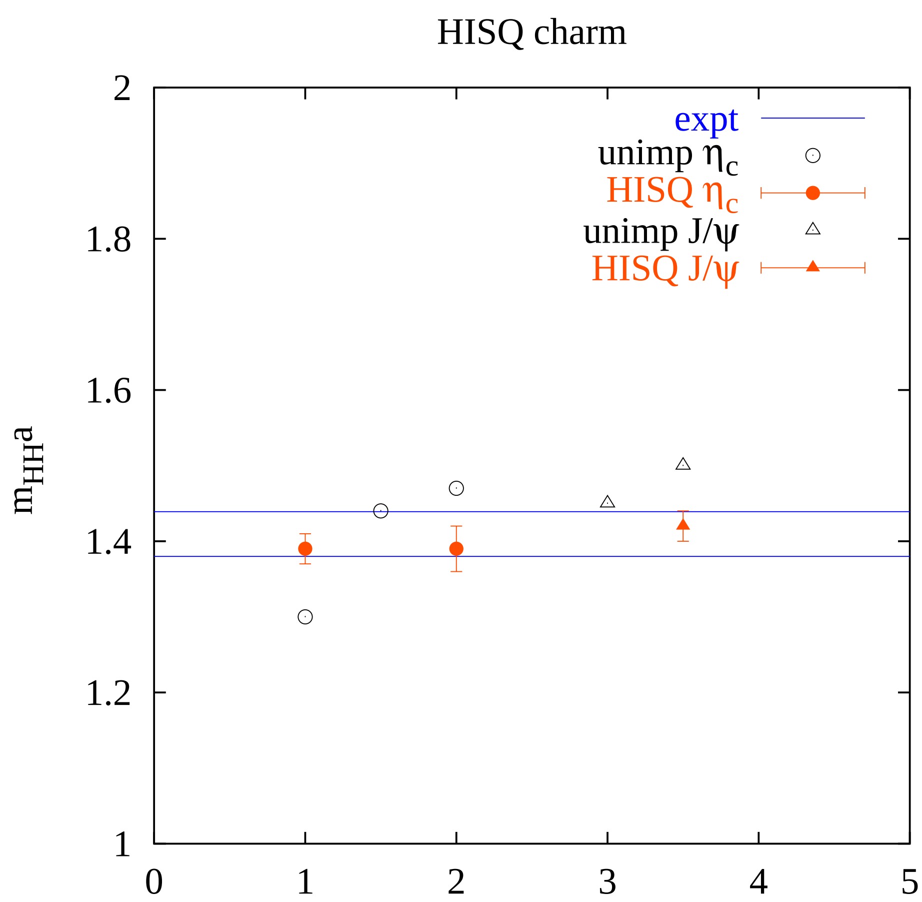
HISQ shows v. good behaviour on taste-changing and dispersion reln.





Follana, Mason, Davies, HPQCD, in preparation

Future: Use HISQ for charm



Unimproved calcs (JLQCD hep-lat/9411012) had problems with taste-changing in $\pi \equiv \eta_c$. This is much improved for HISQ.

Plan: try this on MILC fine (and planned super-fine) lattices where $\alpha_s(m_c a)^2$ = a few %.

Allison, Davies, Follana, Lepage, Mason HPQCD

Future First QCDOC machine being tested in Edinburgh.

Conclusions

- Calculations with 2+1 flavors of light dynamical quarks have made first high precision lattice *prediction* of a hadron mass, that of the mass of B_c meson.
- Gold-plated matrix elements for CKM determinations are in progress (more in Andreas and Junko's talks).
- Future work based on some new techniques which have lots of promise.