Quarks, Gluons, and Lattices

Michael Creutz
Brookhaven Lab

Why the lattice?

What drove us to it?

Where are we going?

LATTICE GAUGE THEORY

First principles ‘‘solution’’ of hadronic physics

- Basic formulation: Wilson 1974
- 1980’s: grew into a major industry
- Dominated by computer simulations
- Annual lattice conference: 300 participants
- New teraflop scale facilities coming

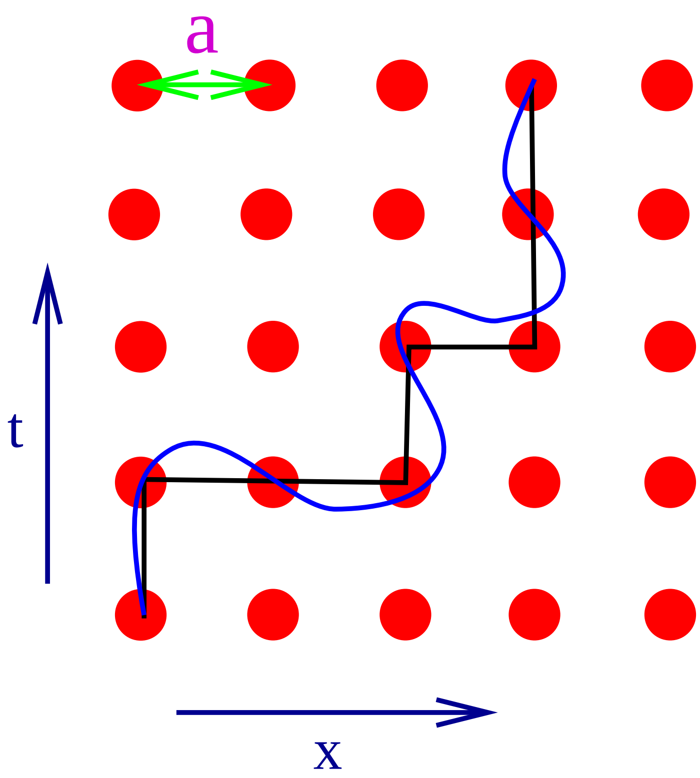
Grandiose goals -- some realized

- Hadronic spectra
- Weak matrix elements
- Quark gluon plasma
- Constraints on Higgs physics

Space-time Lattice

A mathematical trick

World lines $\longrightarrow$ discrete hops



Lattice spacing a

$a \to 0$ for physics

$a = \text{cutoff} = \pi/\Lambda$

Field theory has divergences

- bare charge, mass divergent
- must “regulate” for calculation
- Pauli Villars, dimensional regularization: perturbative
- based on Feynman diagrams

But important non-perturbative effects

- confinement
- chiral symmetry breaking

need a “non-perturbative” regulator

Wilson’s strong coupling lattice theory (1973)

- strong coupling limit confines
- only hadrons can move

space-time lattice $=$ non-perturbative cutoff

Lattice gauge theory

- A mathematical trick
- Minimum wavelength $=$ lattice spacing a
- Maximum momentum $= \pi/a$
- Allows computations
- Defines a field theory

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- Be discrete, do it on the lattice

- Be indiscreet, do it continuously

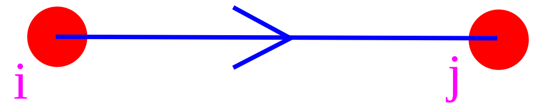
Wilson’s formulation

local symmetry + theory of phases

Variables:

- Gauge fields generalize “phases”

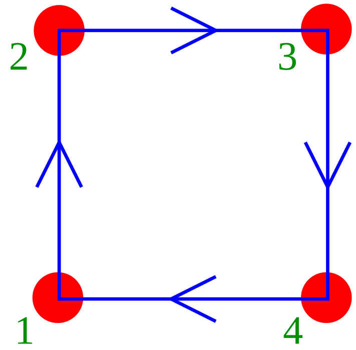
$$U_{i,j} \sim \exp(i \int_{x_i}^{x_j} A^\mu dx_\mu)$$



- On links connecting nearest neighbors
- $U_{ij} =$ 3 by 3 unitary matrix $\in$ SU(3)
- 3 quarks in a proton

Dynamics:

- Sum over elementary squares, “plaquettes”



$$U_p = U_{1,2}U_{2,3}U_{3,4}U_{4,1}$$

- like a “curl”
- flux through corresponding plaquette.

$$S = \int d^4x F^{\mu\nu}F_{\mu\nu} \longrightarrow \sum_p \left(1 - \frac{1}{3}\mathrm{ReTr}U_p\right)$$

Quantum mechanics:

- via path integral
- sum over paths $\longrightarrow$ sum over phases

$$Z = \int (dU) e^{-\beta S}$$

- invariant group measure
- β defines the ‘‘bare’’ charge

$$\beta = \frac{6}{g_0^2}$$

- must renormalize as $a \to 0$

Parameters

$$a \longrightarrow 0$$

Asymptotic freedom (2004 Nobel prize!):

$$g_0^2 \sim \frac{1}{\log(1/a\Lambda)} \longrightarrow 0$$

Overall scale Λ from “dimensional transmutation”

- Coleman and Weinberg
- depends on units: not a real parameter

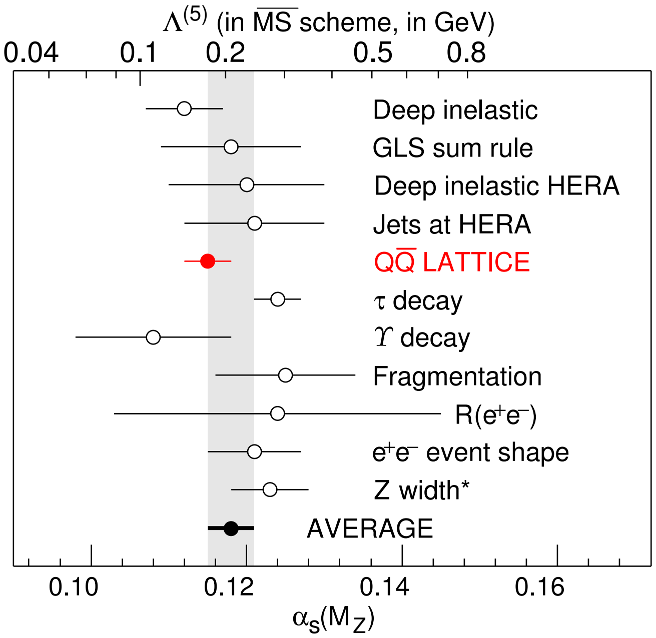
Only the quark masses!

$m_q = 0$: parameter free theory

- $m_\pi = 0$
- m_ρ / m_p determined
- close to reality

Example: strong coupling determined

$$\alpha_s(M_Z) = 0.115 \pm 0.003$$



(PDG, 1999)

(charmonium spectrum for input)

Numerical Simulation

$$Z = \int dU e^{-\beta S}$$

10^4 lattice ⇒

- $10^4 \times 4 \times 8 = 320,000$ dimensional integral
- 2 points/dimension ⇒

$$2^{320,000} = 3.8 \times 10^{96,329} \qquad \text{terms}$$

- age of universe $\sim 10^{27}$ nanoseconds

Use statistical methods

- $Z \longleftrightarrow$ partition function
- $\frac{1}{\beta} \longleftrightarrow$ temperature

Find “typical” equilibrium configurations C

$$P(C) \sim e^{-\beta S(C)}$$

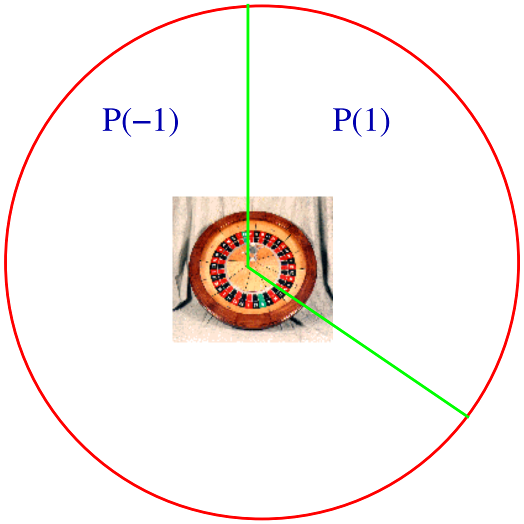
Use a Markov process

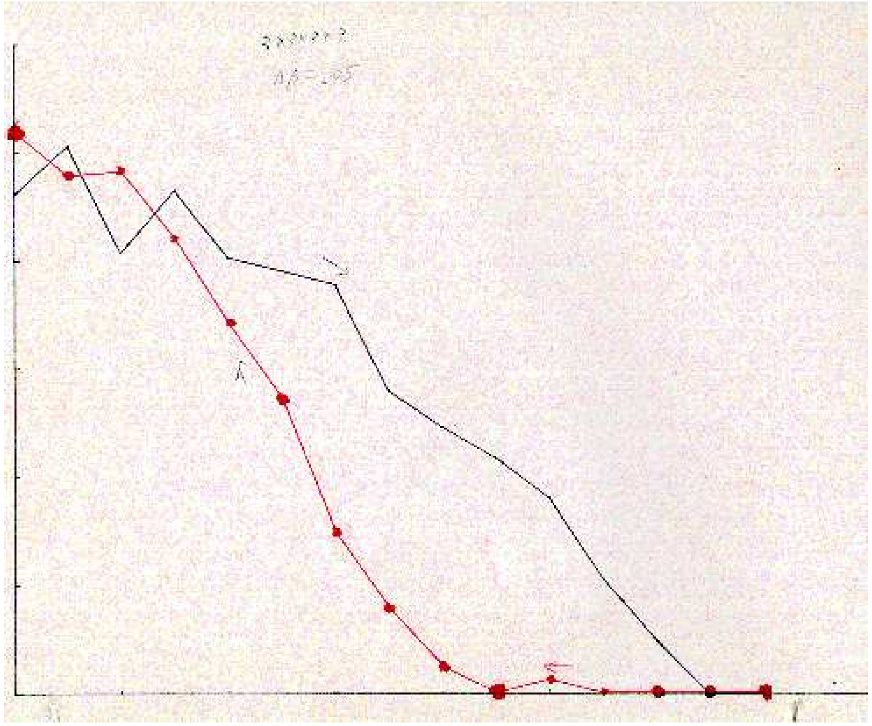
$$C \rightarrow C' \rightarrow \ldots$$

Z_2 example: (L. Jacobs, C. Rebbi, MC)

$$U = \pm 1$$

$$P(1) = \frac{e^{-\beta S(1)}}{e^{-\beta S(1)} + e^{-\beta S(-1)}}$$



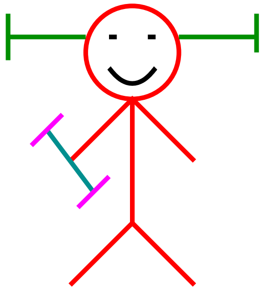


Monte Carlo methods

Make random field changes biased by Boltzmann weight.
Converge towards configurations in “thermal equilibrium.”

$$P(C) \sim e^{\beta S}$$

In principle can measure anything.
Fluctuations → theorists have error bars!

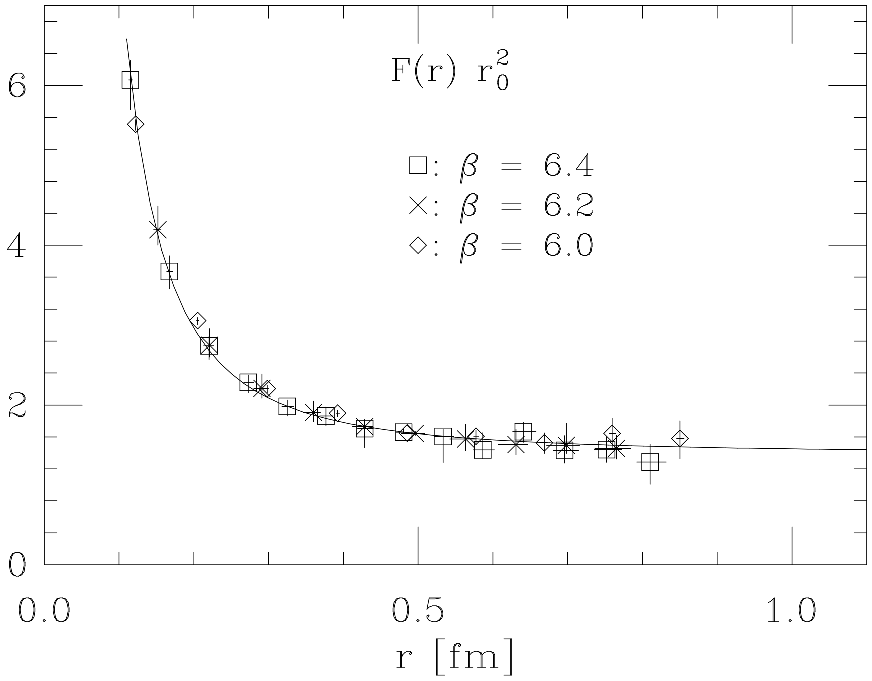


Systematic errors:

- finite volume
- finite lattice spacing
- quark mass extrapolations
- valence approximation for quarks

Interquark force

- constant at large distance
- confinement



C. Michael, hep-lat/9509090

Quarks: serious unsolved problems

Anticommuting fields

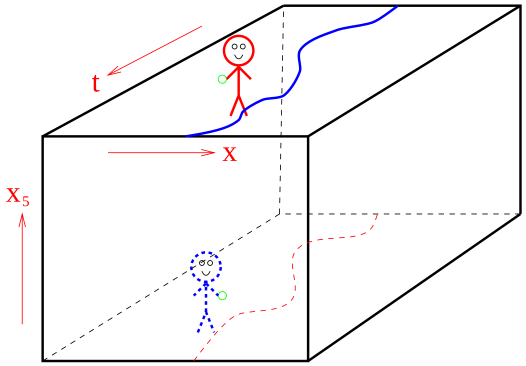
- $\not\Rightarrow$ classical statistical mechanics
- Integrate out as a determinant
- Tedious to simulate.

Chemical potential background baryon density

- Non-positive weight.
- No viable algorithms known!

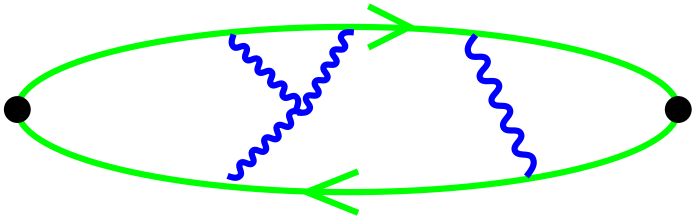
Chiral fermions and the “standard model”

- Unsolved difficulties tied with anomalies.
- Lots of recent activity.
- My favorite: 4d world an interface in 5d

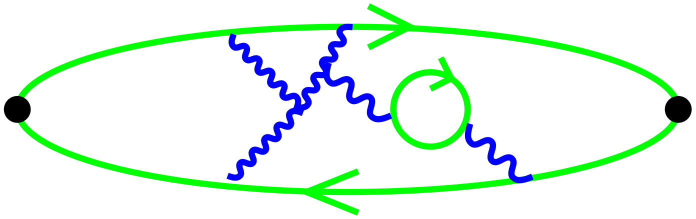


Valence or Quenched approximation:

- Simulate gauge fields ignoring $|\not{D} + m|$
- Propagate quarks in background gauge field
- include:



- neglect:



Saves orders of magnitude in computer time

Singular in the light quark limit

Hadronic spectra: as $t \to \infty$

$$\langle \phi(t)\phi(0) \rangle \longrightarrow e^{-mt}$$

- $m =$ mass of lightest hadron created by ϕ
- Bare quark mass is a parameter

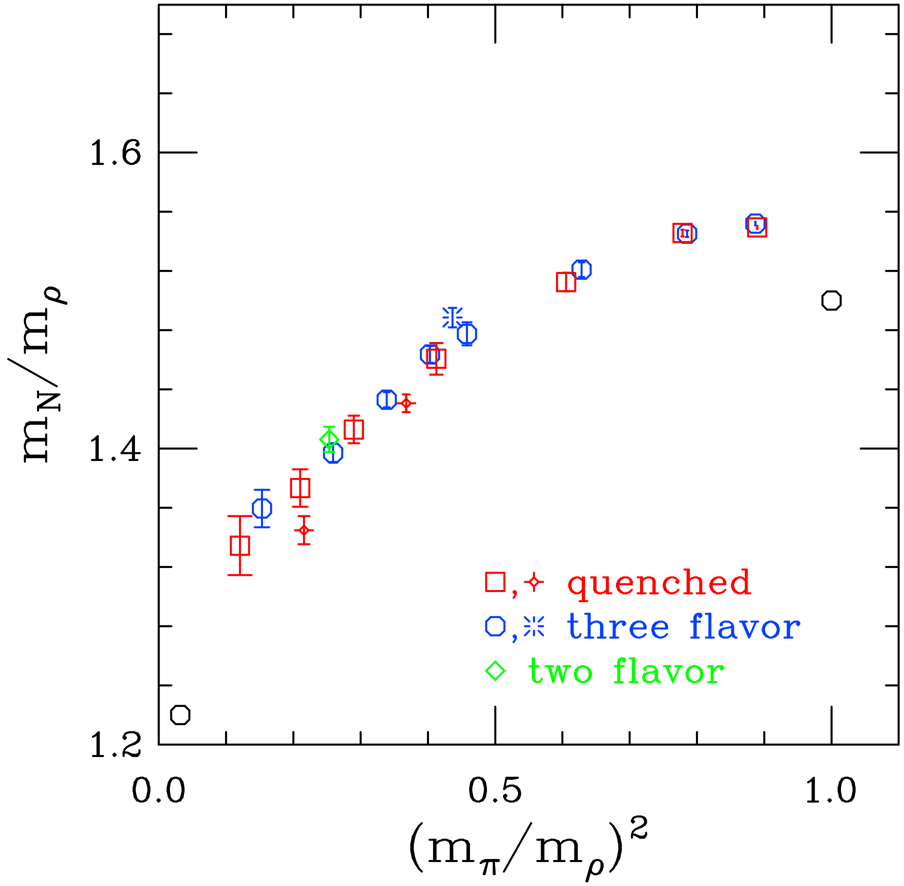
Chiral symmetry:

$$m_\pi^2 \sim m_q$$

Adjust M_q to get m_π / m_ρ (M_s for the kaon)

all other mass ratios determined

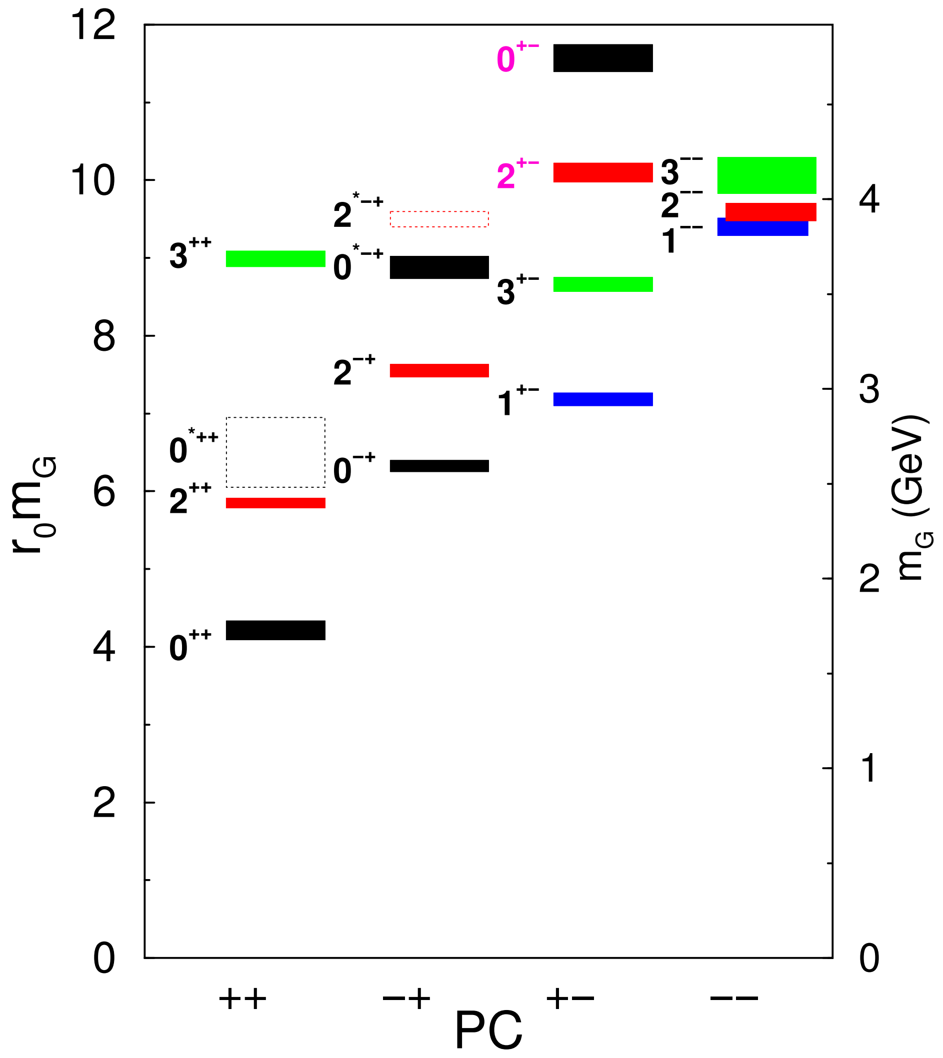
"APE" or "Edinburgh" plot:



- improved Kogut-Susskind quarks, $16^3 \times 48$ lattice
- MILC collaboration, Phys. Rev. D $\mathbf{64}$, 054506 (2001)

Glueballs

- gluonic excitations
- no quarks



- Morningstar and Peardon, Phys. Rev. D **60**, 034509 (1999)
- quenched, anisotropic lattice

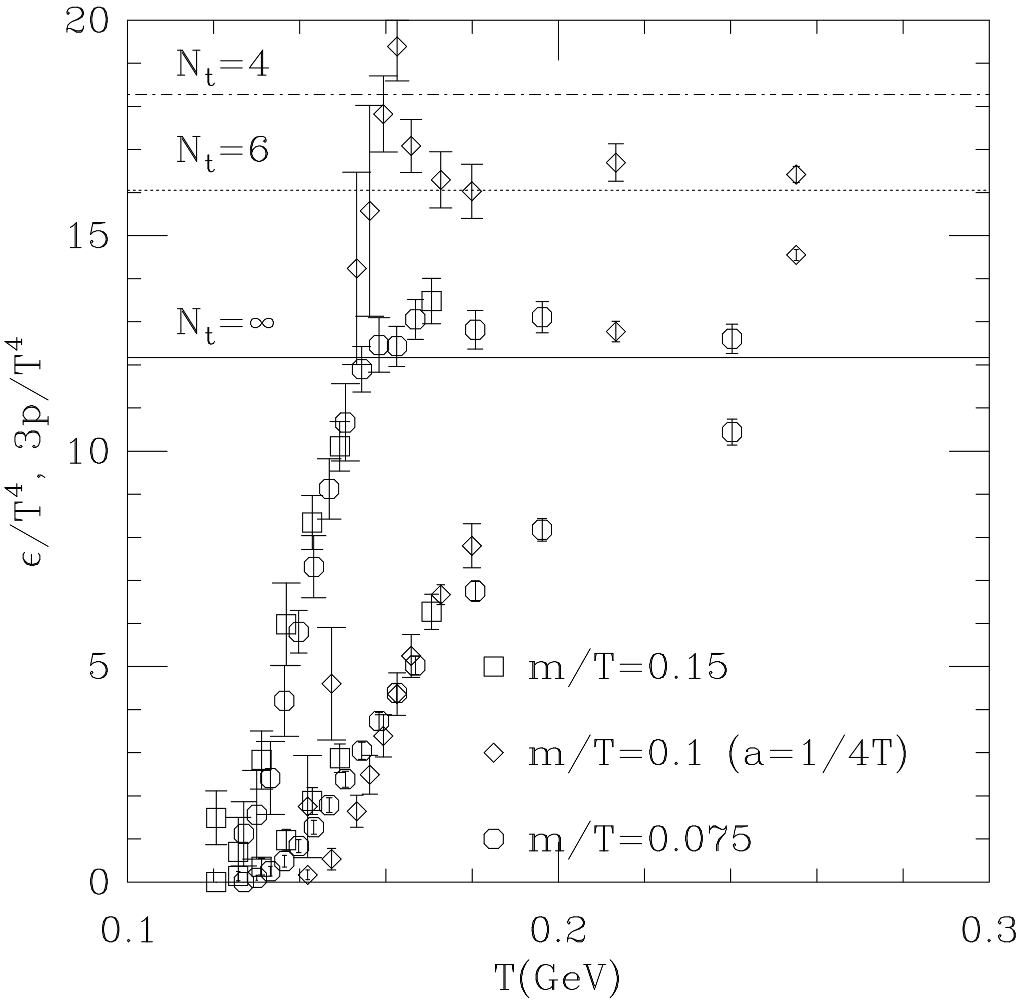
Quark Gluon Plasma

Finite temporal box of length t

$$Z \sim \mathrm{Tr}\ e^{-Ht}$$

- $1/t \leftrightarrow$ temperature
- confinement lost at high temperature
- chiral symmetry manifestly restored
- $T_c \sim 235$ MeV, 0 flavors (quenched)
- $T_c \sim 160$ MeV, 2 flavors

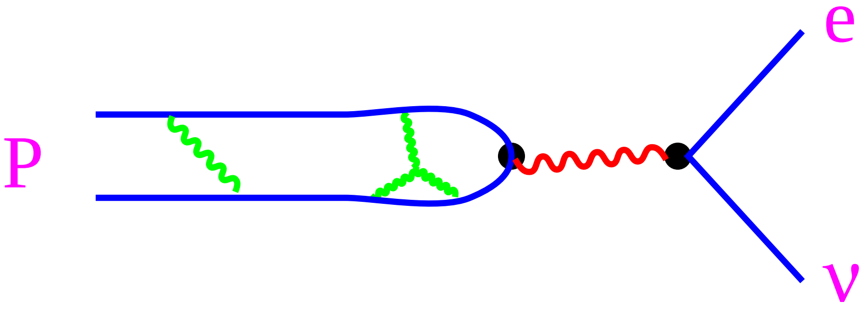
Energy ϵ and pressure p versus temperature.



Bernard *et al.*, MILC collaboration, Dec. 1996

Matrix elements

To test standard model predictions for weak decays, need strong interaction corrections.

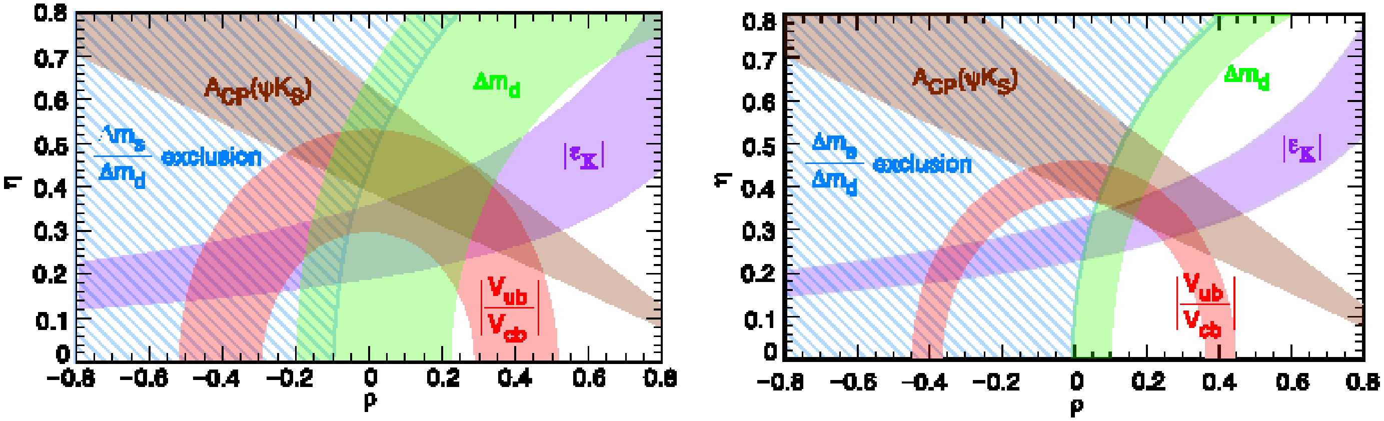


- $\Delta I = 1/2$ rule verified
- ϵ'/ϵ large quenching errors, heavy use of chiral perturbation theory
- dynamical simulations necessary: QCDOC

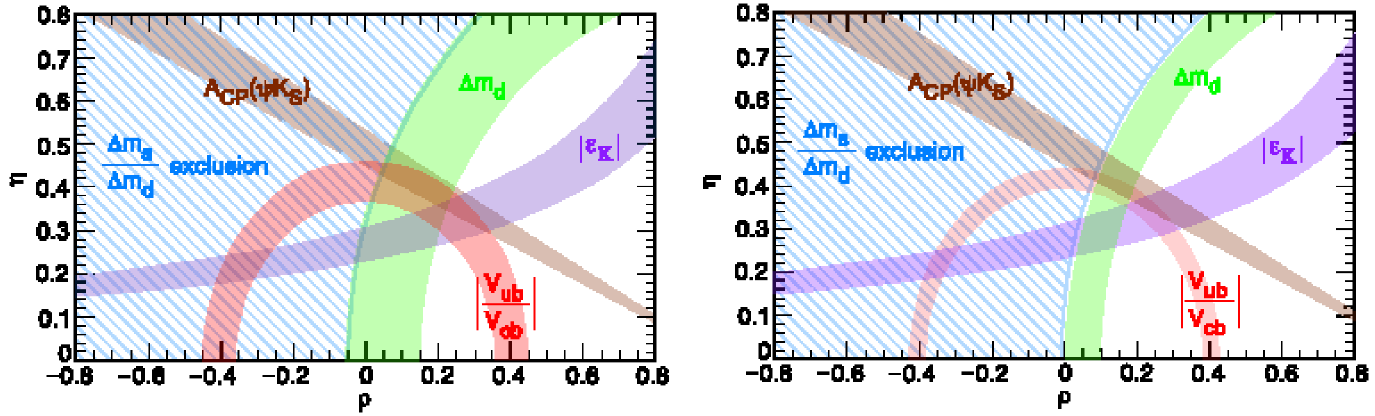
Impact of Reduced Lattice Errors

CKM today . . .

. . . and with 2–3% theory errors.



And with B Factories . . .



The impact of the B factories and improvements in lattice calculations on parameters of the CKM matrix. CLEO-c Collaboration (2001).

HEPAP, February 9, 2004 – p.7/24

The Lattice SciDAC Project

66 US lattice theorists; 9 member executive committee:

R. Brower, (Boston U.) N. Christ (Columbia U.), M. Creutz (BNL), P. Mackenzie (Fermilab), J. Negele (MIT), C. Rebbi (Boston U.), S. Sharpe (U. Washington), R. Sugar (UCSB) and W. Watson, III (JLab)

Two prong approach

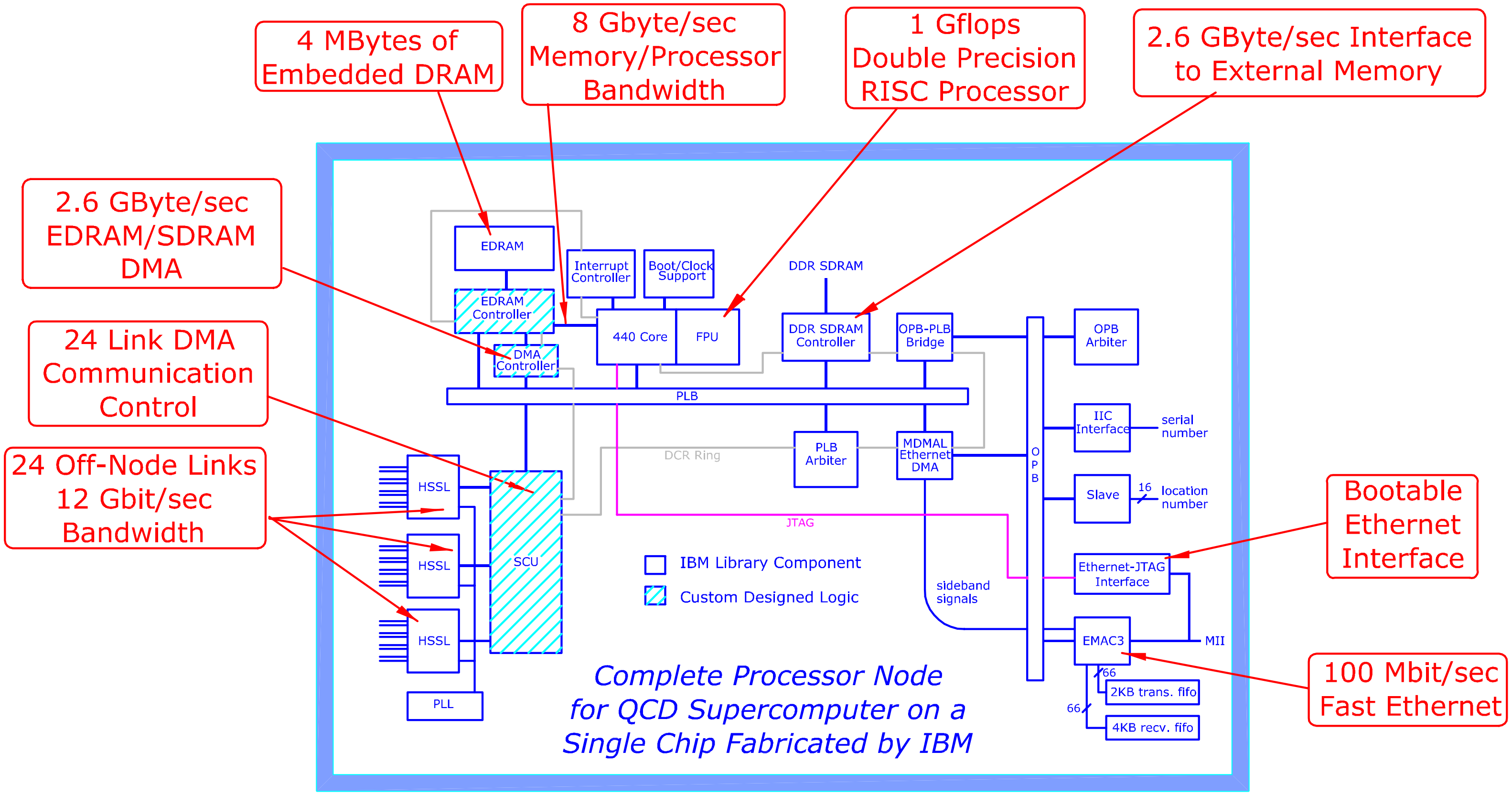
- QCDOC at BNL
- commodity clusters at Fermi Lab and Jefferson Lab
- $\sim 3 \times 10$ Teraflops distributed computing facility

QCDOC

- next generation after QCDSP
- designed by Columbia University with IBM
- on design path to IBM Blue Gene
- Power PC nodes connected in a 6 dimensional torus
- processor/memory/communication on a single chip

QCDOC places entire node on a single custom chip

QCDOC ASIC DESIGN



☐ IBM Library Component

▨ Custom Designed Logic

Complete Processor Node for QCD Supercomputer on a Single Chip Fabricated by IBM

4 MBytes of Embedded DRAM

8 Gbyte/sec Memory/Processor Bandwidth

1 Gflops Double Precision RISC Processor

2.6 GByte/sec Interface to External Memory

2.6 GByte/sec EDRAM/SDRAM DMA

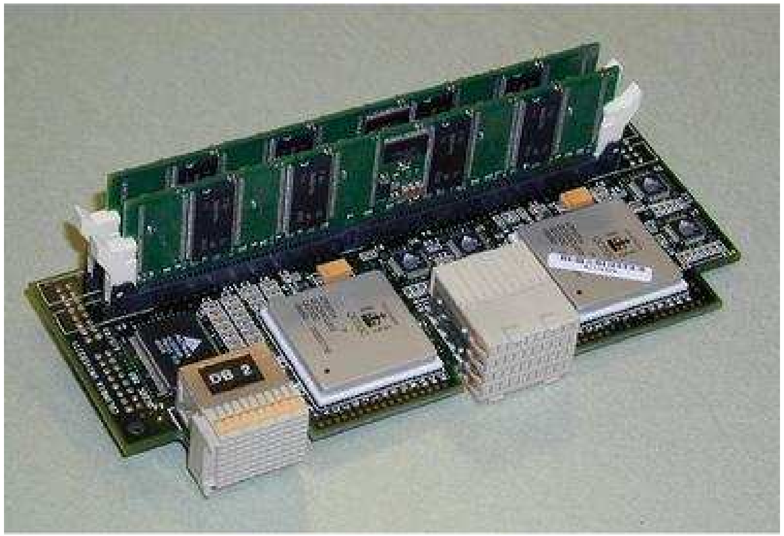
24 Link DMA Communication Control

24 Off-Node Links 12 Gbit/sec Bandwidth

Bootable Ethernet Interface

100 Mbit/sec Fast Ethernet

Misson-critical, custom logic (hatched) for high-performance memory access and fast, low-latency off-node communications is combined with standards-based, highly integrated commercial library components.



Two node daughterboard

64 node motherboard

128 node prototype

UKQCD QCDOC machine

128 node dual 2.4GHz P4 Myrinet cluster, commissioned at FNAL in January 2003

256 node single 2.66 GHz P4 Gigabit Ethernet cluster, commissioned at JLab in September 2003

DOE panel review, Feb. 2003

Frank Wilczek (MIT) - chair
Roy Briere (CMU)
David Ceperley (NCSA-UIUC)
Candy Culhane (NSA)
Lynn Kissel (LLNL)
Michael Ogilvie (Washington Univ)
Robert Swendsen (CMU)
Peter Varman (NSF)

‘‘In short, we feel the scientific merit of [the] suggested program is very clearly outstanding.’’

HEPAP, February 2004:

- strong endorsement of overall plan

Schedule

- first chips delivered beginning of June 2003
- 2x1000 node prototypes at Columbia now running
- three 5 teraflop sustained machines under construction
- UKQCD machine: recently shipped to Edinburgh
- RIKEN/Brookhaven Research Center machine by end of 2004
- 5 teraflop US community QCDOC ready by March 2005
- 5-8 teraflop clusters at JLAB and FNAL: end of 2005
- QCDOC-II? in early planning stages

My Pet Problems

Chiral gauge theories

- parity conserving theories in good shape
- chiral theories (neutrinos) remain enigmatic
- non-perturbative definition of the weak interactions?
- related problem: supersymmetry

Fermion algorithms

- very awkward
- background ‘‘sign problem’’ unsolved
- why treat fermions and bosons so differently?

We need new ideas!