Hadronic Transitions

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Why?

- scattering phase shifts $\Rightarrow$ weak decays
- exploration of exotic states
  - hybrids
  - glueballs
  - $q\bar{q}q\bar{q}$
  - string breaking
  - $c\bar{s}$, $b\bar{s}$ mesons
- χPT

How?

- Lattice
  Non-perturbative QCD
- Euclidean time
  Finite spatial volume

Unstable Particle

Minkowski

$$\psi(t) \sim e^{imt}\, e^{-\frac{\Gamma}{2} t}$$

Euclidian

$$-it \to t$$

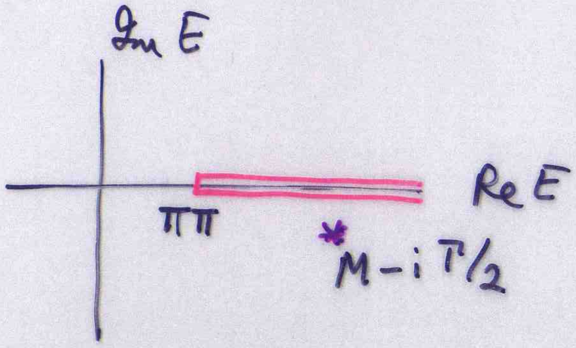
$$\psi(t) \sim e^{-mt}\, e^{-\frac{\Gamma}{2} i t}$$

$$\mathrm{Re} \qquad \cos \frac{\Gamma}{2} t \qquad \text{not} >$$

Resolve this apparent paradox

Euclidian time

Ex $\rho \to \pi\pi$



"ρ propagator"

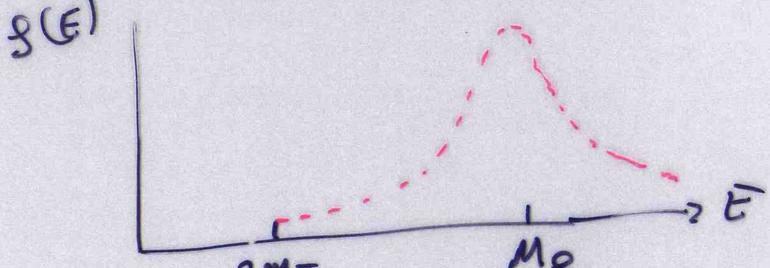
$$G_E(t) = \frac{1}{\pi}\int_{2m_\pi}^{\infty} dE\, e^{-Et}\, \rho(E)$$

Measurable on lattice

cut discontinuity spectral fn.

Typical Example:

$$\rho(E) = \frac{1}{2M}\,\frac{\Gamma(E)/2}{(M-E)^2 + (\Gamma(E)/2)^2}$$



• As $t \to$ large $G_E(t)$ is dominated by threshold $E \approx 2m_\pi$

$\frac{\Gamma}{2}t \ll (M-2m_\pi)t \ll 1$

CM89

$$2MG_E(t) \simeq e^{-Mt}\cos\frac{\Gamma}{2}t + e^{-2m_\pi t}\frac{\Gamma/2}{\pi(M-2m_\pi)^2 t}$$

SIGN!

$$2M\,G_M(t) = e^{iMt}e^{-\Gamma/2\, t} - i e^{i2m_\pi t}\frac{\Gamma/2}{\pi(M-2m_\pi)^2 t}$$

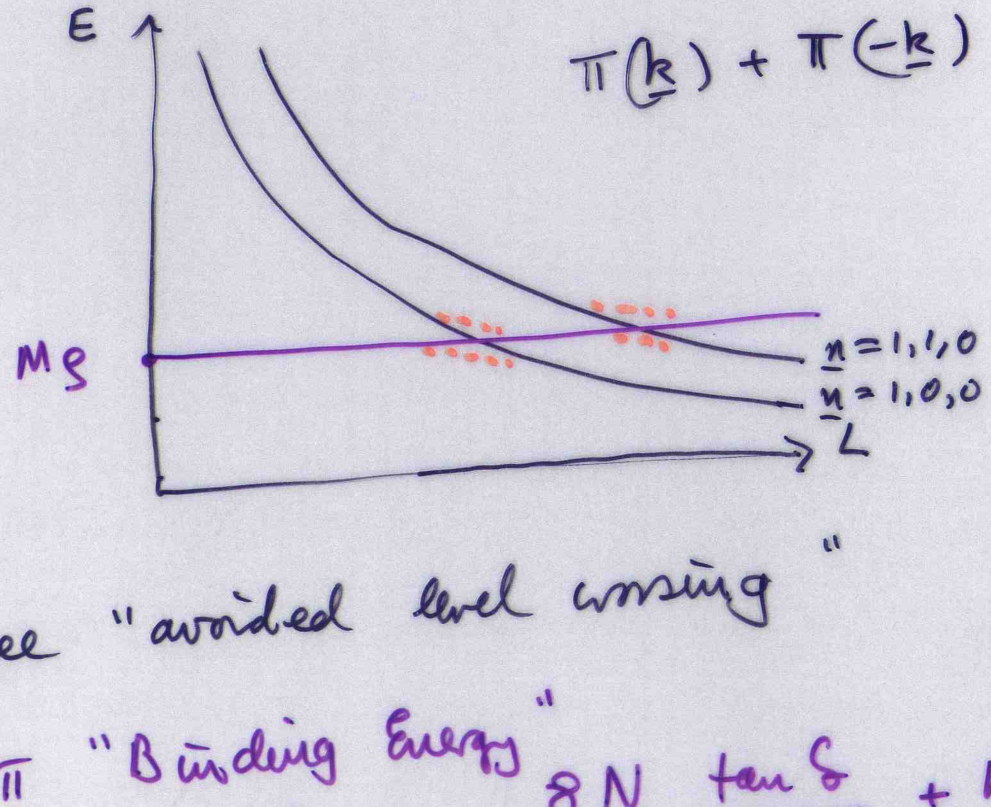
Furthermore extracting $\rho(E)$ from $G_E(t)$ is ill-posed numerically. [inverse Laplace transform]

This is basis of "no-go" theorems.

Finite Volume

Lüscher et al: 2 body states are discrete and their energy versus L $\Rightarrow$ phase shifts δ.
[Lu91

$\pi(\underline{k}) + \pi(-\underline{k})$



$\underline{k} = \frac{2\pi}{L}\underline{n}$

[N integers $|\underline{n}| = |\underline{n}'|$

See "avoided level crossing"

$\pi\pi$ "Binding Energy"

$$\Delta E = \frac{8N \tan\delta}{E \, \underline{n} \, L^2} + \text{known correction terms } O\left(\frac{1}{L^3}\right) \text{ etc}$$

We need dynamical quark simulations to measure ΔE — and it is small - see later.

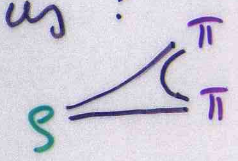
Has been used for $\pi^+\pi^+$ scattering, ...

A Direct Approach ?

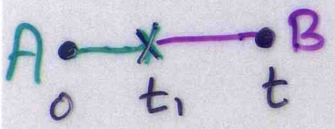
To measure the $\pi\pi$ energy precisely we need to use a basis ρ: $(\bar{q}\gamma_\mu q)_x$

and $\pi\pi$: $(\bar{q}\gamma_5 q)_y\,(\bar{q}\gamma_5 q)_z$

What do the transition amplitudes tell us ?

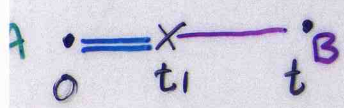


$$C_{AB}(t) = \langle 0 | \mathcal{O}_A \cdots\cdots \mathcal{O}_B^\dagger | 0 \rangle$$



$$\approx \sum_{t_1=0}^{t} c_A\, e^{-M_A t_1}\, x\, e^{-M_B(t-t_1)}\, c_B$$

$$\left(\approx c_1\, e^{-M_A t} + c_2\, e^{-M_B t} \right)$$



$$+ \sum_{t_1=0}^{t} c_{A'}\, e^{-M_{A'} t_1}\, x'\, e^{-M_B(t-t_1)}$$

$$\left(\approx c'\, e^{-M_B t} \right)$$

$$+ \cdots$$

So when t_1 small excited A states contribute
when $t-t_1$ small excited B states contribute

We can't choose t_1 (unlike weak or EM transitions)

all terms behave as $e^{-M_A t}$; $e^{-M_B t}$

so how to separate ?

One way forward:

if $M_A = M_B = M$

$$C_{AB}(t) = c_A c_B \propto \underline{t}\, e^{-Mt} \quad \text{dominates}$$

so excited state contributions less by $1/t$

ON-SHELL (ENERGY CONSERVING) HADRONIC TRANSITIONS ACCESSIBLE

more precisely

$(M_A - M_B)t \ll 5$ "ON SHELL"

$x\, t \ll 1$ "WEAK"

$(m' - m)\, t \gg 1$ "EXCITED STATE SUPPRESSION"

REMINDER:

More rigorous (but MUCH more computing)
Lüscher approach: only uses energies on a lattice.

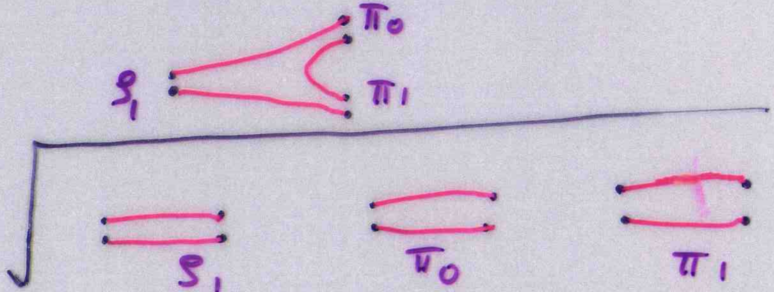
$A \text{ — } \underbrace{\begin{matrix} C \\ D \end{matrix}}$

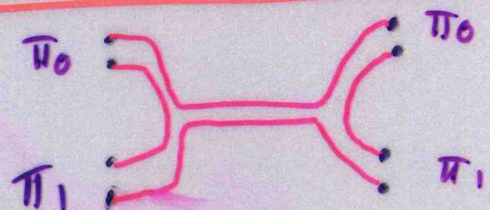
explore the CD spectrum for specific momentum for different lattice size L^3 : $\Rightarrow$ phase shift CD-CD

Large L  small L 

ILLUSTRATION $\rho \to \pi\pi$

$\rho_1 \to \pi_1 \pi_0 \qquad p = \frac{2\pi n}{L} \qquad n = 0, 1$

$xt =$ 

Cross checks • 

$\approx \frac{1}{2} x^2 t^2$

• different momenta

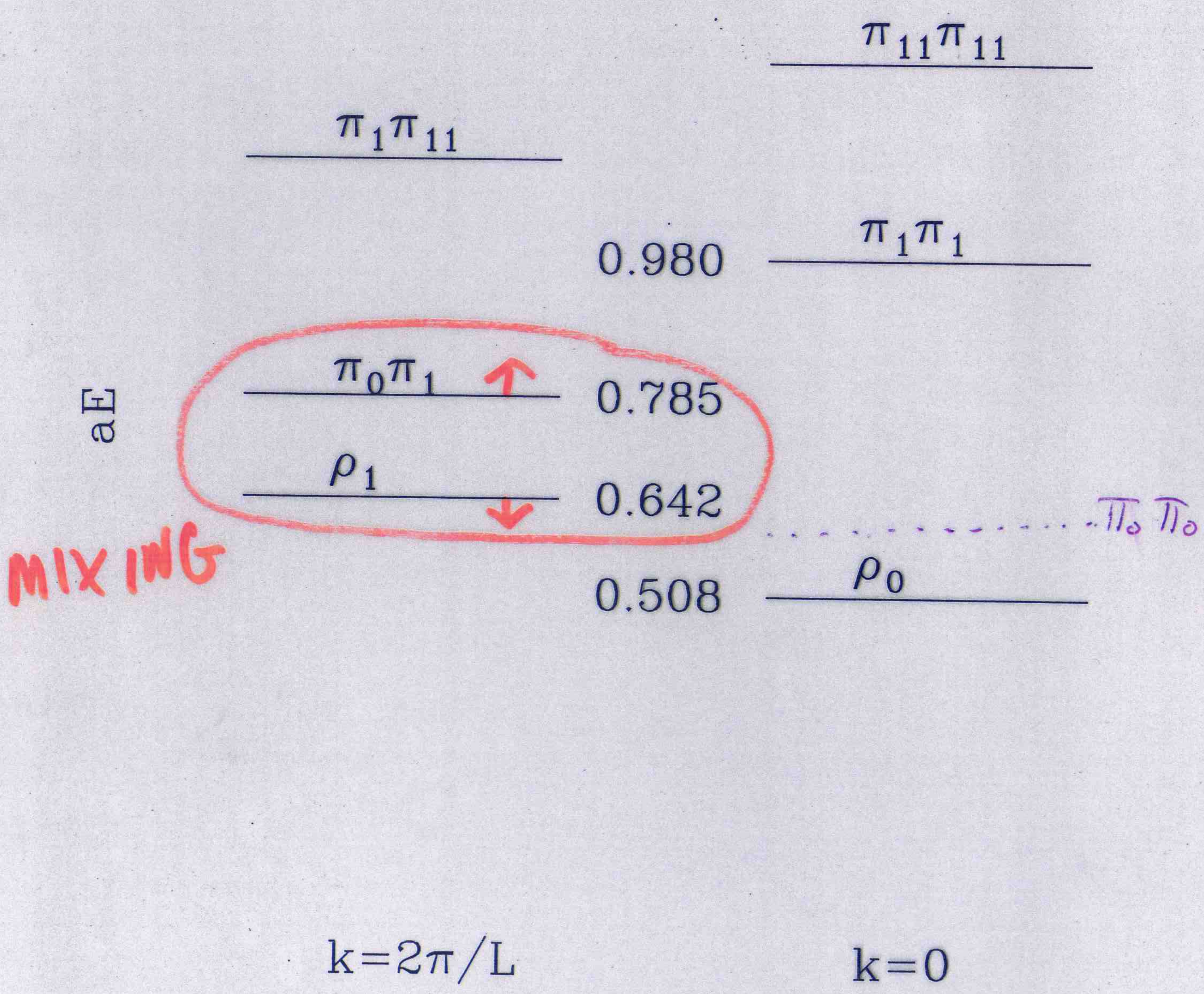
• Level mixing pushes down

$\rho \quad J_z$; P_z pushes down; $P_{x,y}$ no effect.

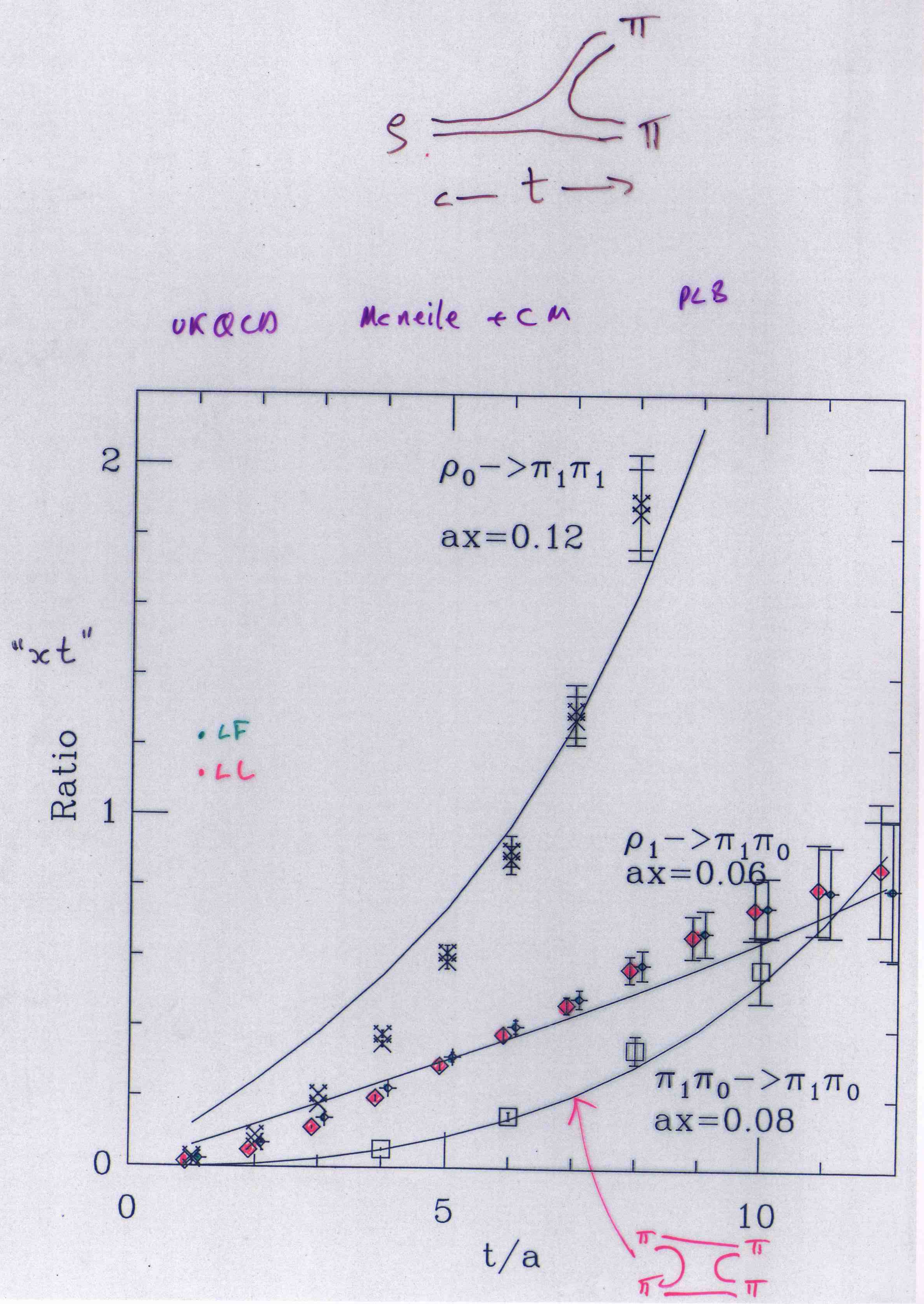
• [more data needed] observe $\pi_0 \pi_1$ energy shift versus L ($\sim 0.02(2)$ seen) shift in Eq

Lattice Spectrum

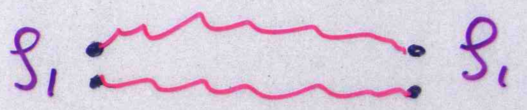
UKQCD
$N_f = 2$
$a \approx 0.11$ fm
$m_q \approx \frac{2}{3} m_s$
CM + Craig McNeile



All 7.2



ρ π π
t
UKQCD
Mcneile + CM
PLB
"xt"
Ratio
• LF
• LL
$\rho_0 \to \pi_1\pi_1$
ax=0.12
$\rho_1 \to \pi_1\pi_0$
ax=0.06
$\pi_1\pi_0 \to \pi_1\pi_0$
ax=0.08
0
1
2
5
10
t/a
π π π π

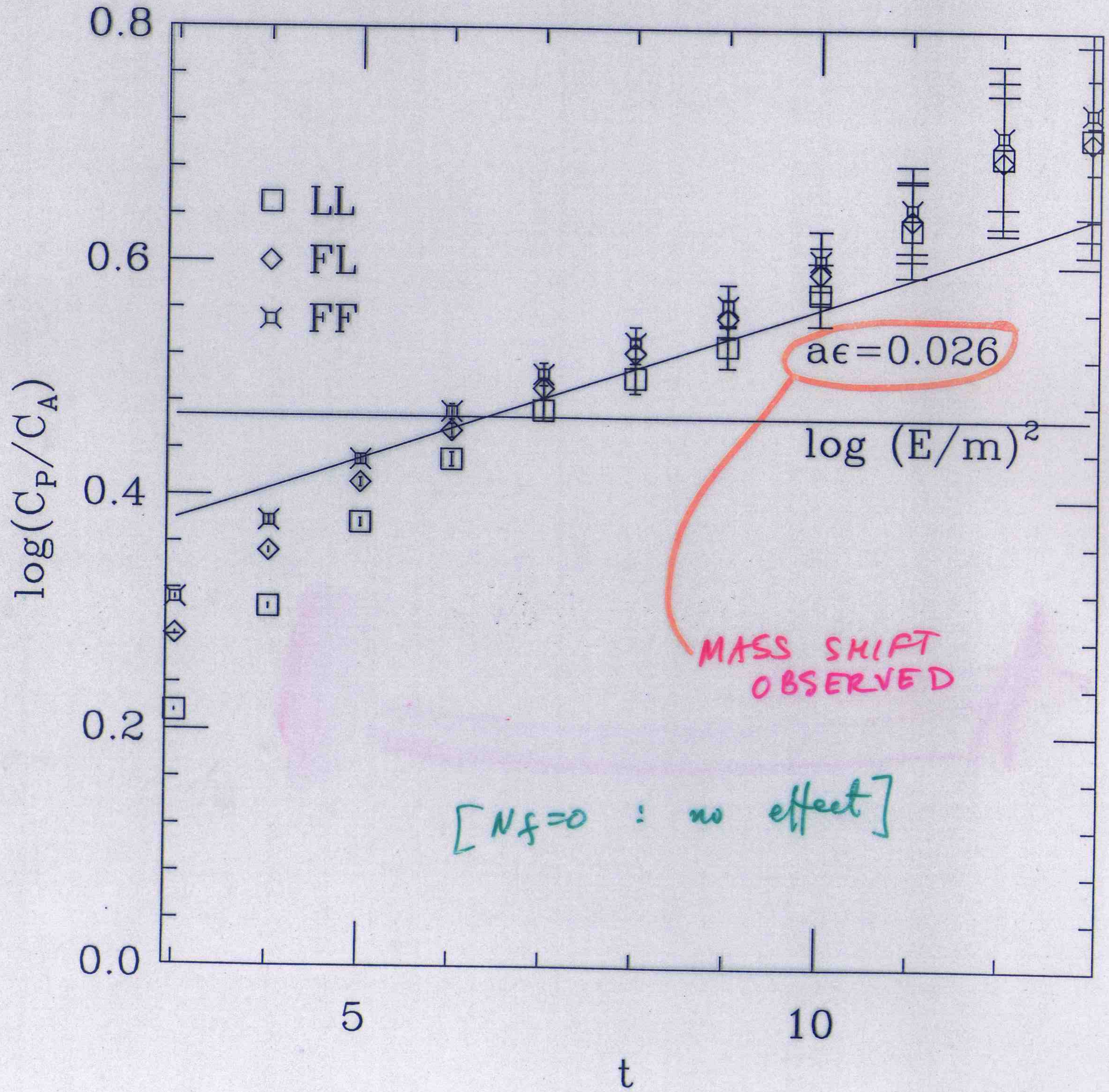


$\underline{P} = \frac{2\pi n}{L}$

P : spin $\parallel^{el}$ momentum : MIXES $\pi_0 \pi_1$
A : spin $\perp r$ momentum

$N_f = 2$

UKQCD McNeile + CM



Summary $\langle \rho | \pi\pi \rangle$

$(\bar{g})^2 = \frac{TME}{k^3}$ $\bar{g}$ 1.40^{47}_{23} $\rho - \pi\pi$ "xt"

1.56^{21}_{13} $\rho_{\parallel}$ vs $\rho_{\perp}$

1.5 expt $\phi \to K\bar{K}$

No decay allowed on lattice! but
on shell transition $\rho_1 - \pi_1 \pi_0$ gives $\bar{g}$.

[checked $\rho_0 \to \pi_1 \pi_{-1}$ but bigger energy gap so less "safe"]

Conclude:

when "on shell" can measure hadronic transitions directly

checks
- indirect (Lüscher L-dependence)
- ρ shift
- mom = $1/2$; 1
- $2 \to 2$ amplitude

Heavy light Mesons

B : $b\bar{u}$ $b\bar{d}$ $b\bar{s}$

HQET : Static b- quark. (corrections $1/m_b$)

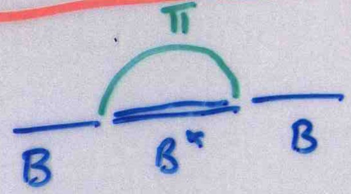
- Is $J^P = 0^+$ $b\bar{s}$ meson Stable? (Isospin violating decay only)

Is $B_s^*(0^+)$ lighter than BK

$D_s^*(0^+)$ is lighter than DK (Expt & some lattice results).

- How does f_B behave as $m_q \to m_{u,d}$

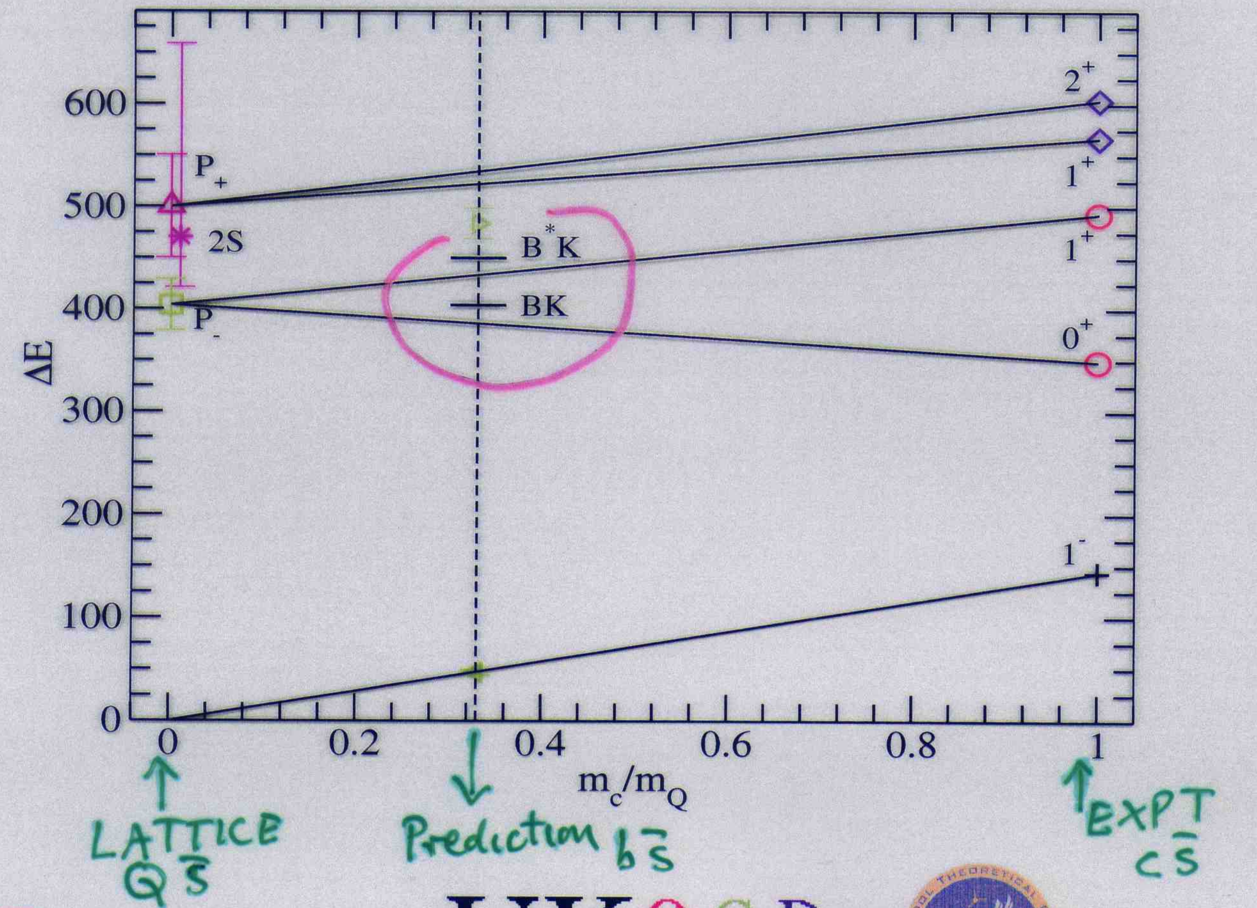
χPT



- $N_f = 2$ clover UKQCD
- stochastic all-to-all light quarks

HL 2.1

Heavy-light mesons $(b\bar{s})$



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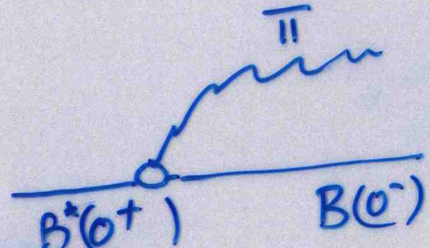
UK QCD collaboration

$b\bar{s}$ spectrum

F: $m_{B_s^*(0^+)} < m_B + m_K$ so stable

assuming: $1/m_Q$ corrections to HQET from c to b

decays



F) $\frac{\Gamma}{k} = 0.5(1)$

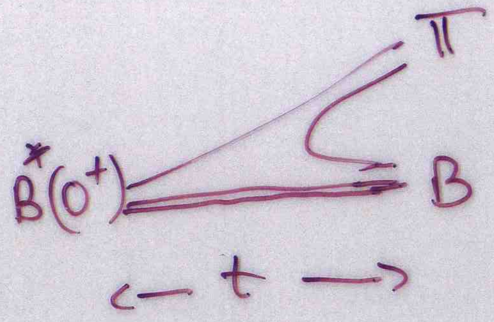
- predict $B^*(0^+) \to B\pi$
  $\Gamma = 160(30)$ MeV

- note $\frac{\Gamma}{k} \approx 0.5$ for $K^*(0^+) \to K\pi$
  ≈ 0.7 for $D^*(0^+) \to D\pi$

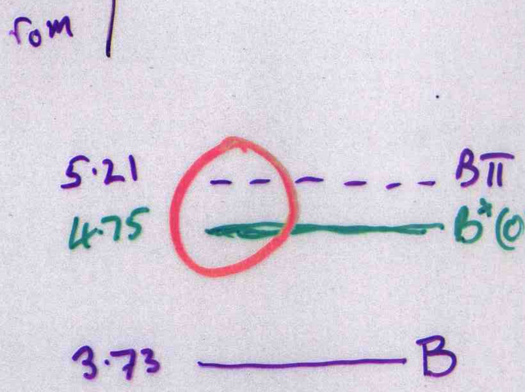
Is this $b\bar{s}$ 0^+ state a BK molecule
or a $b\bar{s}$ state?

Lattice evidence favours $b\bar{s}$.

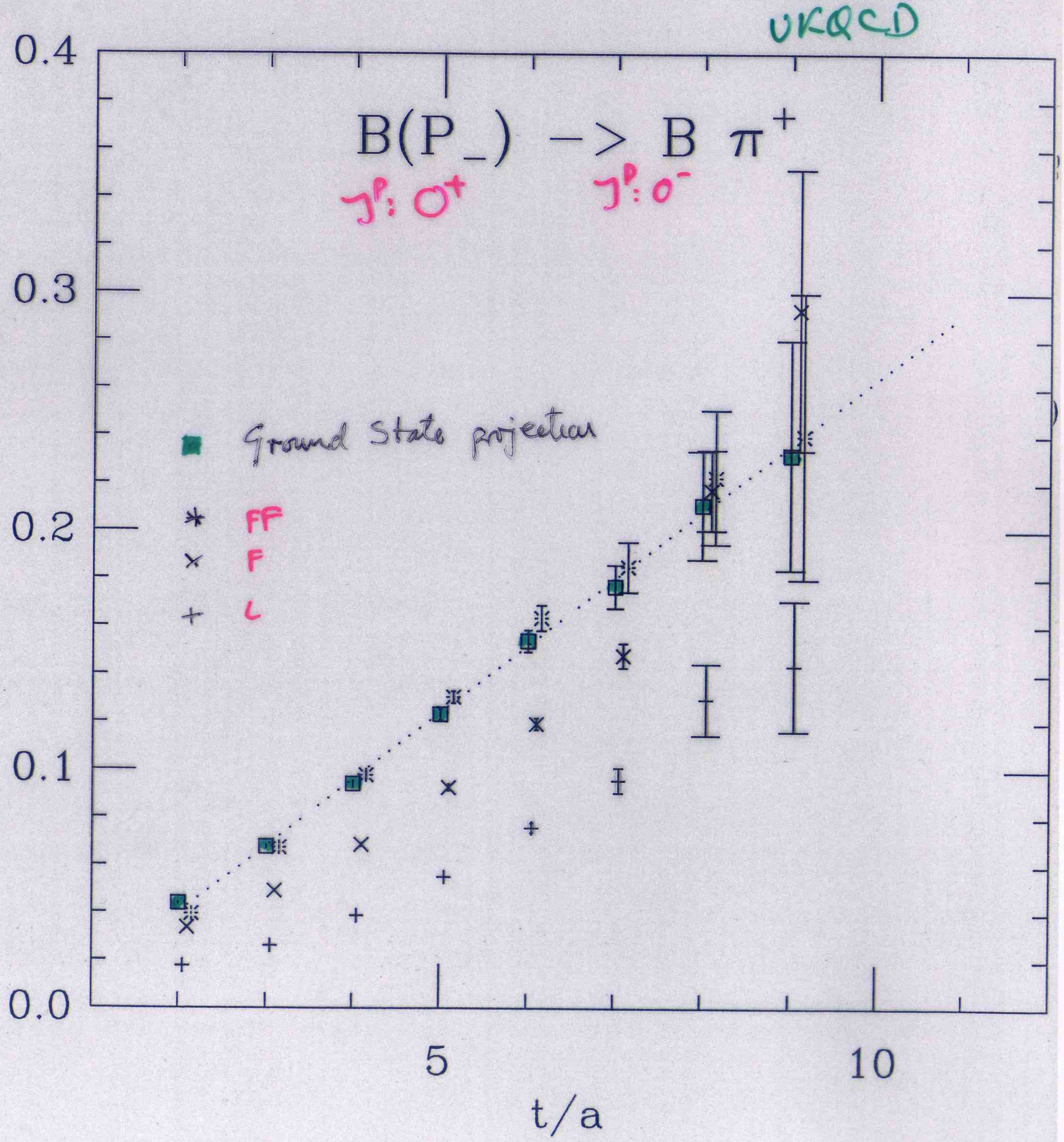
2.8



$\overset{*}{B}(0^+)$
π
B
<— t —>



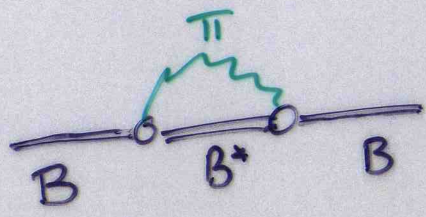
rom ↑
5.21
$B\pi$
4.75
$B^*(0^+$
3.73
B



UKQCD
$B(P\_) \; -> \; B \; \pi^+$
$J^P: 0^+$
$J^P: 0^-$
Ground State projection
FF
F
L
xt
0.0
0.1
0.2
0.3
0.4
5
10
t/a

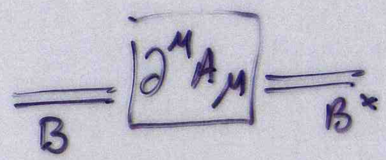
f_B and χPT

$\boxed{f_B}$ static b, all-to-all stochastic method, 5×5 matrix of observables, $N_f = 2$ sea = valence (F



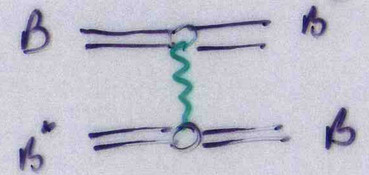
need $g^2_{B^* B\pi}$

$\boxed{g^2_{B^* B\pi}}$

- 

  and π dominance of $\partial^\mu A_\mu$

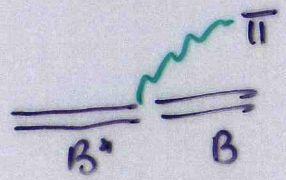
  $g = \begin{array}{l}.42(8) \\ .48(7)\end{array}$ [.67(8) @ charm]

- 

  R

  π exchange tail in B-B potential

  consistent $g = .42$

- 

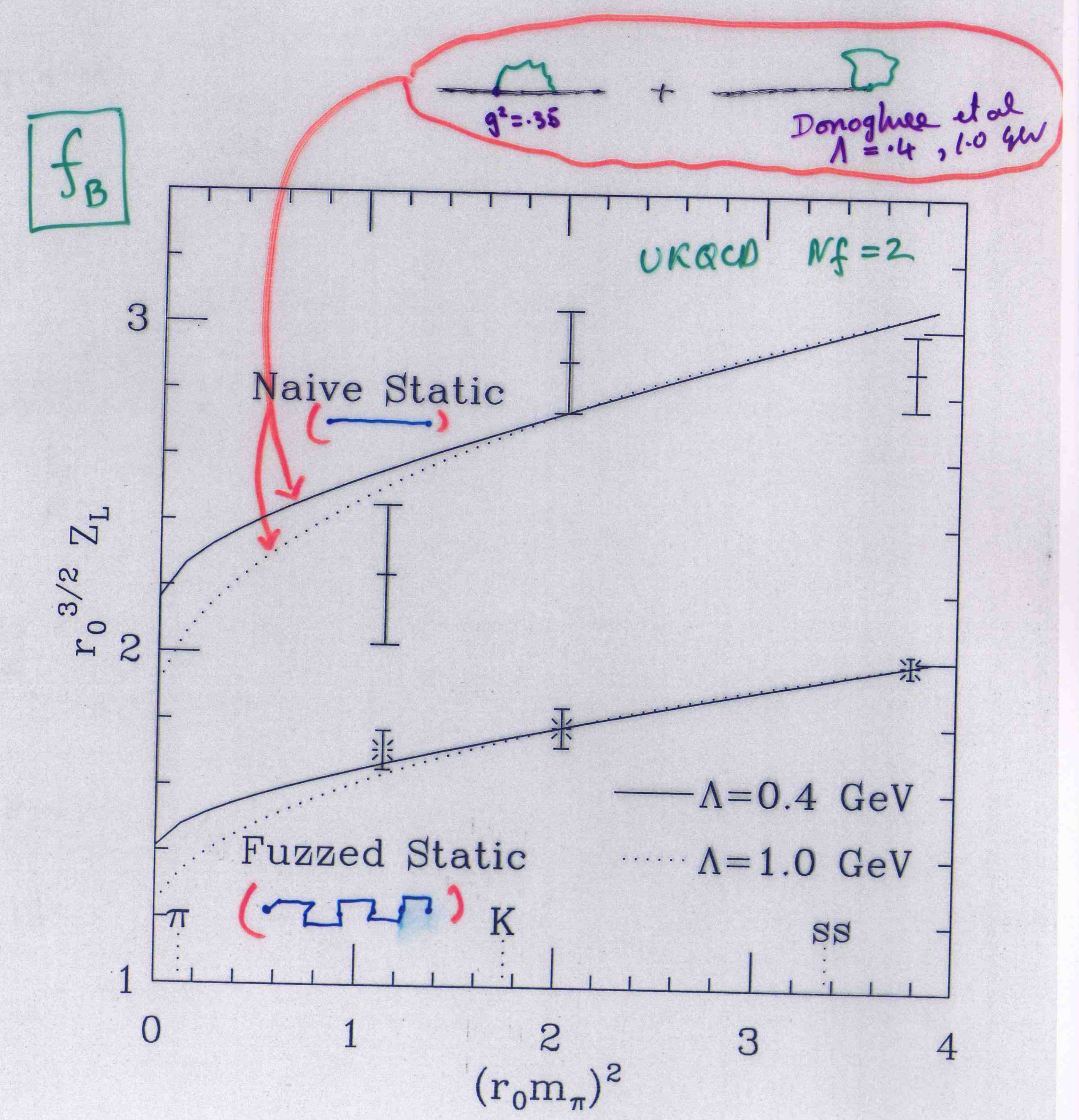
  hadronic transition.

  $g = .39(6)$ (F

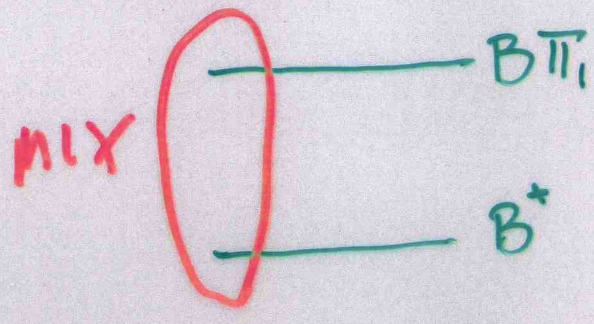
- $D^* \to D\pi$ (expt) CLEO $g = .61(6)$

- - - - - - -

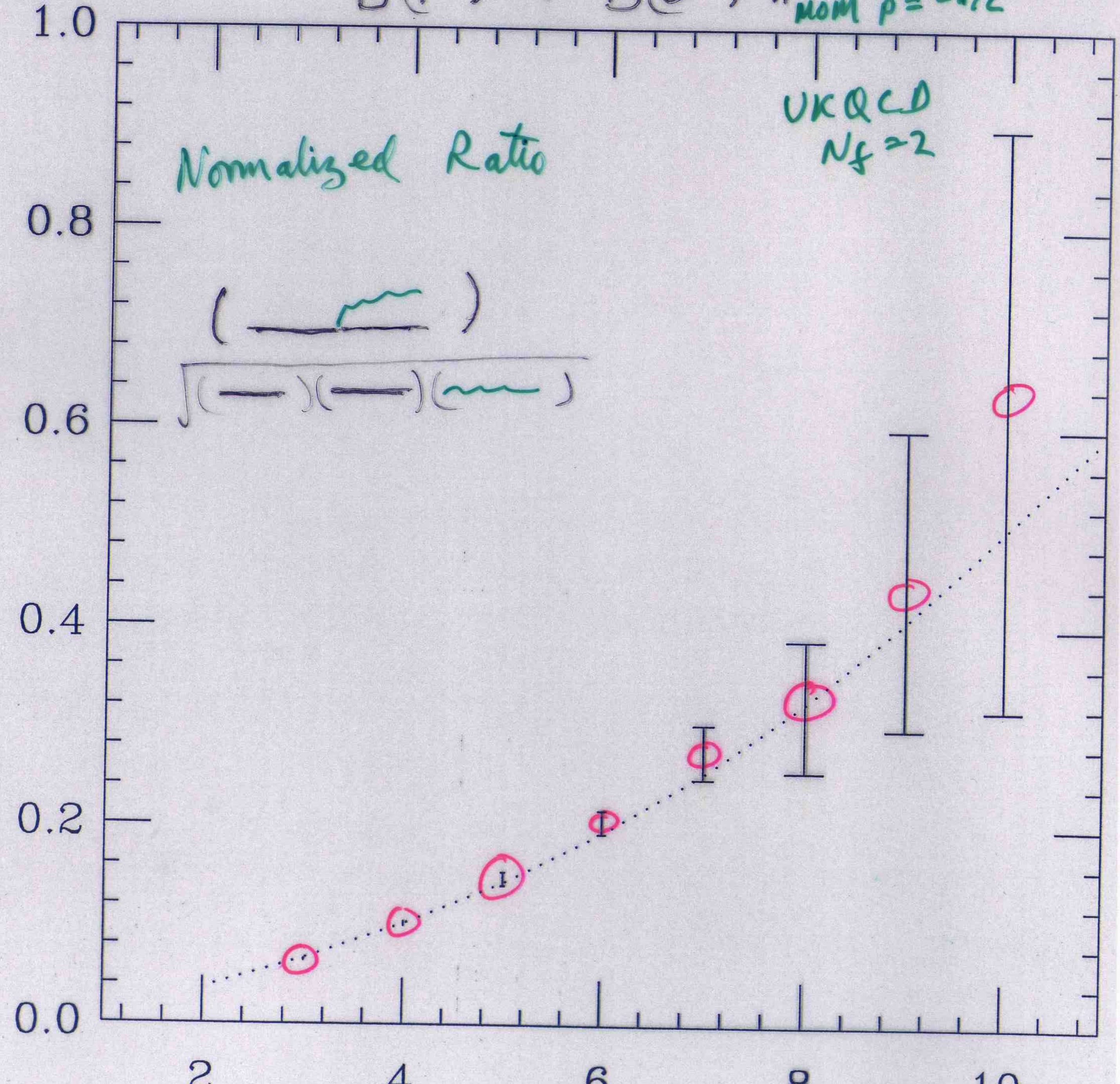
we are reaching the region where χPT effects should be observable on a lattice.



5·1
f_B
$g^2 = .35$
+
Donoghue et al
$\Lambda = .4$, 1.0 GeV
UKQCD Nf = 2
Naive Static
Fuzzed Static
Λ=0.4 GeV
Λ=1.0 GeV
π
K
ss
$r_0^{3/2}\, Z_L$
$(r_0 m_\pi)^2$
1
2
3
0
1
2
3
4



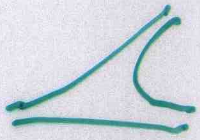
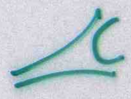
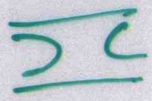
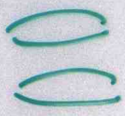
$B^*(1^-) \to B(0^-)\,\pi_{\text{MOM } p = 2\pi/L}$



f_B static

- signal still limited.
  (even using 5×5 ; MVR stochastic)
- "less static" static helps
  self energy less so signal better
- still sizable uncertainty as $m_q \to 0$

Summary

- evaluation of lattice transition  does give useful information NOW. "on shell" if $E_{initial} \approx E_{final}$
- stochastic methods are very efficient for evaluating   

Prospects for future

Underway $\quad a_0 \to \eta\pi \quad K\bar{K}$

$\quad f_0 \to \pi\pi \quad K\bar{K}$ $\quad$ – fate of the scalar glueball

Feasible $\quad H(1^{-+}) \to \eta\pi$, $\rho\pi$ $\quad$ hybrid meson

Care : disconnected (flavour singlet) contributions more noisy on lattice (f_0, η)

Electromagnetic transitions are feasible (easier than hadronic transitions)