Lattice calculation of Nucleon Dipole Moments

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Preface

- Work in progress (Shoji's directive)

- In light of Sinya's talk on Tuesday, calculation of neutron electric dipole moment (d_N) is incomplete

- Calculation has (at least) two parts:

  1. Naive calculation (this talk)

  2. Subtraction of mixing term (Sinya's talk)

Introduction

T- and P-odd term allowed in QCD Lagrangian:

$$
\begin{aligned}
S_{QCD,\theta} &= i\theta \int d^4x \, \frac{g^2}{32\pi^2} \mathrm{tr}\left[G(x)\tilde{G}(x)\right] = i\theta Q, \\
(G(x)\tilde{G}(x) &\sim E \cdot B)
\end{aligned}
$$

where Q is the topological charge of the QCD vacuum.

θ-term is CP-*odd* $\rightarrow$ neutron electric dipole moment, d_N.

Weak interactions: Also violate CP: CKM mechanism: $d_N \leq 10^{-30}\, e$-cm (vanishes at one-loop), many orders of magnitude below the experimental bound [1], $|\vec{d}_N| < 6.3 \times 10^{-26}\, e$-cm.

Experimental bound + model calculations imply $\theta \leq 10^{-10}$, which is *unnaturally* small. However, no known symmetry to say it vanishes. This is often called the *Strong CP problem*.

To translate the above experimental bound to a constraint on the fundamental θ parameter requires evaluation of nucleon matrix elements.

Lattice method is first-principles technique for calculation.

Previous calculations of d_N

1. V. Baluni [2] computed d_N in the framework of the MIT bag model obtaining $d_N \simeq 8.2 \cdot 10^{-16}\, e\,\theta$-cm

2. Crewther *et al.* [3], using an effective chiral lagrangian found $d_N \propto \theta\ M_\pi^2\ \ln(M_\pi^2) \simeq 5.2 \cdot 10^{-16}\, e\,\theta$-cm

3. Pospelov and Ritz [4], using QCD sum rules techniques, found $d_N = 1.2 \times 10^{-16}\, e\,\theta$-cm (40-50% error estimate)

4. Aoki and Gocksch [5] were the first to pioneer lattice QCD calculations of d_N (quenched approximation).

5. Faccioli, Guadagnoli, and Simula [12] recently found $d_N = (6 \div 14) \times 10^{-16}\, e\,\theta$-cm in the instanton liquid model

(What is the meaning of "$\div$"?)

Using the axial anomaly, one can replace the CP violating gauge action above with the fermionic action,

$$
\begin{aligned}
S'_\theta &= -i\theta\,\overline{m}\int d^4x P(x) \\
P(x) &= \bar{u}(x)\gamma_5 u(x) + \bar{d}(x)\gamma_5 d(x) + \bar{s}(x)\gamma_5 s(x) \\
\overline{m} &= \left(m_u^{-1} + m_d^{-1} + m_s^{-1}\right)^{-1} \\
&= \frac{m_u\ m_d\ m_s}{m_u + m_d + m_s}
\end{aligned}
$$

Note, that the θ term *does* vanish if one of the quark masses vanishes, provided $P(x)$ does not go like $\sim 1/\overline{m}$.

Remarks on the quenched case

The QCD partition function in the presence of explicit CP violation is

$$Z = \int \mathcal{D}A_\mu \ \mathrm{det}[D(m) + i\theta\overline{m}\gamma_5] \ e^{-S_G}.$$

Setting $\mathrm{det}[D(m) + i\theta\overline{m}\gamma_5] = 1$, we lose CP violating physics. However, if θ is small,

$$\mathrm{det}\,[D(m) + i\theta\overline{m}\gamma_5] = \mathrm{det}[D(m)]\,[1 + i\theta\overline{m}\ \mathrm{tr}(\gamma_5 D(m)^{-1})\,] + \mathcal{O}(\theta^2),$$

and we quench as usual by setting $\mathrm{det}\,[D(m)] = 1$.

The chiral limit

The spectral decomposition of $D^{-1}(m)$ leads to

$$
\begin{aligned}
D(m)|\lambda_i\rangle &= (\lambda_i + m)|\lambda_i\rangle \\
\sum_{f=1}^{N_f} \mathrm{Tr}\left[\gamma_5 D^{-1}(m_f)\right] &= \frac{n_+ - n_-}{\overline{m}} = \frac{Q}{\overline{m}}
\end{aligned}
$$

for N_f flavors and n_+ and n_- the number of right- and left-handed zero modes of $D(m)$.

If we trade Q for $-\overline{m}P$ (using the anomaly), $\bar{m}$ dependence cancels. Correct mass dependence of d_N requires $(\det D(m))^{N_f}$. d_N *does not vanish in the quenched chiral limit* (*c.f.* topological susceptibility). Recall that $\det D(m) \sim m$ for $Q \neq 0$, and contributions to d_N vanish for $Q = 0$.

more on the $m_f \to 0$ limit

Faccioli, *et al.*:

- In the *quenched* instanton liquid model $d_N \sim 1/m_f^{N_f}$
- $d_N \sim m_f$ for unquenched
- Find d_N quenched is 2-4 times larger than d_N unquenched

Computational Methodology

Compute the matrix elements of the electromagnetic current between nucleon states,

$$
\begin{aligned}
\langle p', s | J^{\mu} | p, s \rangle_{\theta} &= \bar{u}(p', s) \Gamma_{\mu}(q^2) u(p, s) \\
\Gamma_{\mu}(q^2) &= \gamma_{\mu} F_1(q^2) \\
& + i \, \sigma_{\mu\nu} q^{\nu} \frac{F_2(q^2)}{2m} \\
& + \left(\gamma_{\mu} \gamma_5 \, q^2 - 2m \gamma_5 \, q_{\mu} \right) F_A(q^2) \\
& + \sigma_{\mu\nu} q^{\nu} \gamma_5 \frac{F_3(q^2)}{2m},
\end{aligned}
$$

$q^2 = -2E(\vec{p}) m_N + 2m_N^2 < 0$

Four terms consistent with Lorentz, gauge, CPT symmetry

The above terms have the following transformation properties under C,P,T:

	$\bar{u}\gamma_\mu u$	$i\bar{u}\sigma_{\mu\nu}q^\nu u$	$\bar{u}(q_\mu - \gamma_\mu)\gamma_5 u$	$\bar{u}\sigma_{\mu\nu}q^\nu\gamma_5 u$
P	$(-1)^\mu$	$(-1)^\mu$	$-(-1)^\mu$	$-(-1)^\mu$
T	$(-1)^\mu$	$(-1)^\mu$	$(-1)^\mu$	$-(-1)^\mu$
C	-1	-1	$+1$	-1
CPT	-1	-1	-1	-1

Peskin's notation: $(-1)^\mu = -1$ for $\mu = 1, 2, 3$, $+1$ for $\mu = 0$.

(multiply the amplitude by i to get CPT even)

Now we move to the lattice and Euclidean space, $\gamma^i \to i\gamma^i \equiv \gamma_E^i$, $\gamma^0 \to \gamma^4 \equiv \gamma_E^4$. (same as the Pauli metric used in FORM)

(ignore Aoki-Kuramashi-Shintani mixing problem for now)
compute the correlation function

$$\begin{aligned}
G(t,t') &= \langle \chi_N(t',\vec{p}')\, J^\mu(t,q)\, \chi_N^\dagger(0,\vec{p})\rangle \\
&= \sum_{s,s'} \langle 0|\chi_N|p',s'\rangle \langle p',s'|J^\mu|p,s\rangle \langle p,s|\chi_N^\dagger|0\rangle \\
&\quad \times \frac{1}{2E\,2E'} e^{-(t'-t)E'} e^{-tE} + \ldots \\
&= G^\mu(q) * f(t,t',E,E') + \ldots
\end{aligned}$$

where "..." represent excited state contributions.

Using the spinor relation

$$\sum_s u^s(p)\bar{u}^s(p) \;=\; -i\not{p} + m$$

and setting the initial state momentum $\vec{p} = 0$, we find

$$\begin{aligned} G^\mu(q^2) \;=\; & \Big(E'(p)\gamma_4 - i\vec{\gamma}\cdot\vec{p}\,' + m\Big) \times \\ & \Bigg(\gamma_\mu\, F_1(-q^2) + \sigma_{\mu\nu}q^\nu\,\frac{F_2(-q^2)}{2m} \\ & + \Big(i\gamma_\mu\,\gamma_5\, q^2\; + 2m\gamma_5\, q_\mu\Big)\, F_A(-q^2) - i\sigma_{\mu\nu}q^\nu\gamma_5\,\frac{F_3(-q^2)}{2m}\Bigg) \\ & m\;(1+\gamma_4) \end{aligned}$$

where $q^2 = -2m^2 + 2E(p')\,m > 0$ (Pauli, Euclidean metric).

Use projectors to obtain linear combinations of F_1 and F_2, and F_3. From FORM:

γ_μ	projector P	$\mathrm{Tr} P\, G^\mu$
γ_x	$\frac{i}{4}\frac{1}{2}(1+\gamma_4)\gamma_x\gamma_y$	$-p_y\, m(F_1(-q^2)+F_2(-q^2))$ $(-\frac{1}{2}q_z p_x\, F_3(-q^2)+2p_x p_z m^2 F_A(-q^2))$
γ_4	$\frac{i}{4}\frac{1}{2}(1+\gamma_4)\gamma_x\gamma_y$	$-\frac{i}{2}p_z(E+m)F_3(-q^2)$
γ_4	$\frac{1}{4}\frac{1}{2}(1+\gamma_4)$	$m\,(E+m)(F_1(-q^2)-\frac{q^2}{(2m)^2}F_2(-q^2))$ $=m\,(E+m)G_E(-q^2)$
γ_x	$\frac{1}{4}\frac{1}{2}(1+\gamma_4)$	$-i\,p_x(mF_1(-q^2)+\frac{E-m}{2}F_2(-q^2))$

(projectors multiplied by $\frac{1}{2}(1+\gamma_4)$ to project onto the positive parity state)

Conventional electric and magnetic form factors:

$$
\begin{aligned}
G_E(q^2) &= F_1(q^2) + \frac{q^2}{(2m)^2} F_2(q^2) \\
G_M(q^2) &= F_1(q^2) + F_2(q^2)
\end{aligned}
$$

$F_1(0) = 1$, (0) is the charge of the proton (neutron) in units of e

$F_2(0)$ and $F_3(0)$ are related to the magnetic and electric dipole moments

Form a ratio of the neutron and proton correlation functions:

$$
\begin{aligned}
\lim_{t' \gg t \gg 0} \frac{i}{p_z} \frac{\mathrm{Tr} P^{xy} G_N^4(t, t', E, \vec{p})|_{\theta \neq 0}}{\mathrm{Tr} P^4 G_P^4(t, t', m, \vec{p})} &= \frac{i}{p_z} \frac{\mathrm{Tr} P^{xy} G_N^4(q^2)|_{\theta \neq 0}}{\mathrm{Tr} P^4 G_P^4(q^2)} \\
&= \frac{1}{2m} \frac{F_3(-q^2)}{G_E^{(P)}(-q^2)} \\
\lim_{q \to 0} &= \frac{d_N}{a\, e\, \theta}
\end{aligned}
$$

The ratio is the electric dipole moment form factor to the electric form factor. In the limit $q \to 0$ this is just the electic dipole moment of the neutron in units of $a\,\theta\,e$.

Note that (implicit) factors of Z_V cancel in the ratio.

Similarly, the magnetic dipole moments can be obtained from the $\lim_{q\to 0}$ of the ratio's

$$\lim_{t'\gg t\gg 0} \frac{1}{p_y}\frac{\mathrm{Tr}P^{xy}G^x_{P,N}(t,t',E,\vec{p})|_{\theta=0}}{\mathrm{Tr}P^4G^4_P(t,t',E,\vec{p})} = \frac{1}{p_y}\frac{\mathrm{Tr}P^{xy}G^x_{P,N}(q^2)|_{\theta=0}}{\mathrm{Tr}P^4G^4_P(q^2)}$$

$$= \frac{1}{E+m}\frac{F_1(q^2)+F_{2(P,N)}(-q^2)}{G_E^{(P)}(-q^2)}$$

$$\lim_{q\to 0} = \begin{cases} \frac{1}{2m}\left(1+a_{\mu,P}\right) \\ \frac{1}{2m}a_{\mu,N} \end{cases}$$

where a_μ denotes the anomalous part

(again, I have used the fact that $F_1(0)=1$ (0) is the charge of the proton (neutron) in units of e.)

interpolating operators

Standard "positive" parity nucleon:

$$\begin{aligned} B_1^+ &= \epsilon_{abc}\left[u_a^T C\gamma_5 d_b\right] u_c \\ B_2^+ &= \epsilon_{abc}\left[u_a^T C d_b\right] \gamma_5 u_c \end{aligned}$$

B_2 couples weakly to the nucleon, vanishes as $m_f \to \infty$ (non-relativistic limit).

Under Parity transformation

$$\mathcal{P} B_{1,2}^+ \mathcal{P} = +\gamma_4 B_{1,2}^+$$

Negative parity:

$$\begin{aligned} B_{1,2}^- &= \gamma_5 B_{1,2}^+ \\ \mathcal{P} B_{1,2}^- \mathcal{P} &= -\gamma_4 B_{1,2}^+ \end{aligned}$$

In fact, both sets couple to both parities in correlation functions. We use B_1^+ only.

For $\theta \neq 0$, neutron has admixture of the negative parity state:

$$\begin{aligned} |n\rangle &= e^{i\alpha\gamma_5}|+\rangle = \cos\alpha|+\rangle + i\gamma_5 \sin\alpha|+\rangle \\ &= \cos\alpha|+\rangle + i\sin\alpha|-\rangle \end{aligned}$$

Should compute the correlation function

$$\langle[\cos\alpha B_1^+ + i\sin\alpha B_1^-](t',\vec{p}')\, J^\mu(t,q)\, [\cos\alpha B_1^+ + i\sin\alpha B_1^-](0,\vec{p})\rangle$$

Interested in the case $\theta \ll 1$. Contributions from the negative parity state to d_N will go like θ^2, so ignore.

This is *not* the mixing discussed by Aoki-Kuramashi-Shintani.

Computing with $\theta \neq 0$

Complex action. But since $\theta \ll 1$ in Nature, expand $\langle p', s'|J^{\mu}|p, s\rangle_{\theta}$ to lowest order in θ.

$$\begin{aligned} \langle \mathcal{O} \rangle &= \frac{1}{Z(\theta)} \int_{fields} \mathcal{O}\, e^{-S + i\theta \int d^4x \; \frac{g^2}{32\pi^2} \mathrm{tr}[G(x)\tilde{G}(x)]} \\ &= \frac{i\theta}{Z(0)} \int_{fields} Q\, \mathcal{O}\, e^{-S} + \mathcal{O}(\theta^2) \end{aligned}$$

Compute $F_3(q^2)$ term in each topological sector ν, and then average over all sectors with weight Q_{ν}

$$\langle p', s'|J^{\mu}|p, s\rangle_{\theta} = \sum_{\nu} iQ_{\nu} \langle p', s'|J^{\mu}|p, s\rangle_{Q_{\nu}}$$

(the right hand side is computed in CP *even* vacuum)

Disadvantage: unlike the background-electric-field-method [8] used in [5]: our method does not allow a direct calculation at $q^2 = 0$ (because of the explicit factors of q^ν for F_2 and F_3 terms)

On a finite lattice only the form factor F_1 can be computed at $q^2 = 0$ [9].

Our method requires extrapolation of the form factors to $q^2 = 0$ from non-vanishing values of q^2.

Numerical Results

Computed on 233 $N_f = 2$, $m_{\text{sea}} = m_{\text{val}} = 0.04$, domain wall fermion configurations (separated by 20 trajectories).
See `hep-lat/0411006` (RBC Collaboration) for basic details of the $N_f = 2$ DWF simulations.

Lattice: $16^3 \times 32$, $L_s = 12$, and the inverse lattice spacing in the $m_{sea} = 0$ limit is $a^{-1} \approx 1.7$ GeV.

Computed for one source/sink combination $t = 0$ and 10. Currently running another with $t = 15$ and 25. Using Gaussian smeared source and sink. Operator (J^μ) is inserted between source and sink.

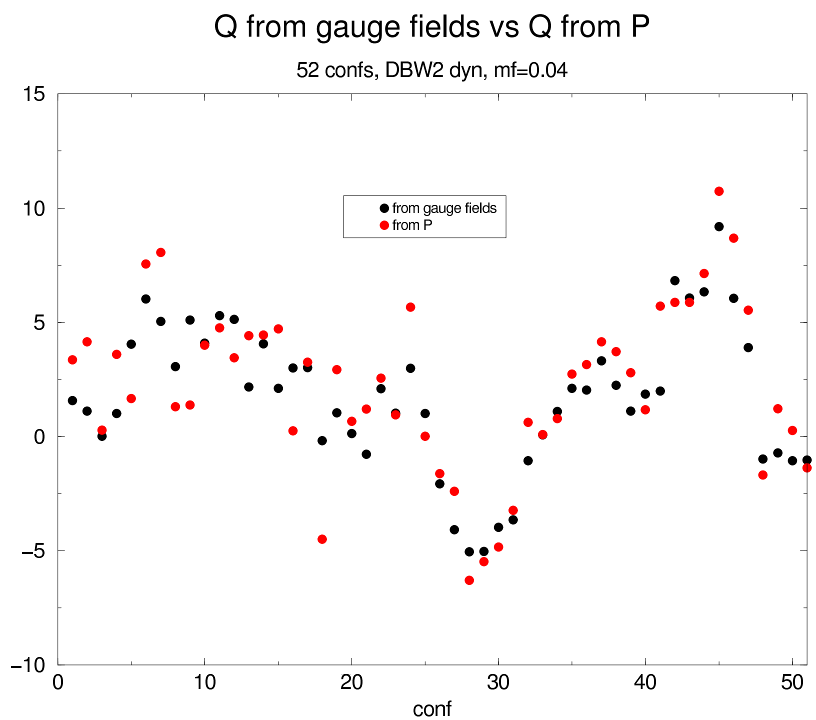
We have averaged over time slices 4-7 and (equivalent) permutations of the momenta $\vec{p} = (1,0,0)$, (1,1,0), and (1,1,1).

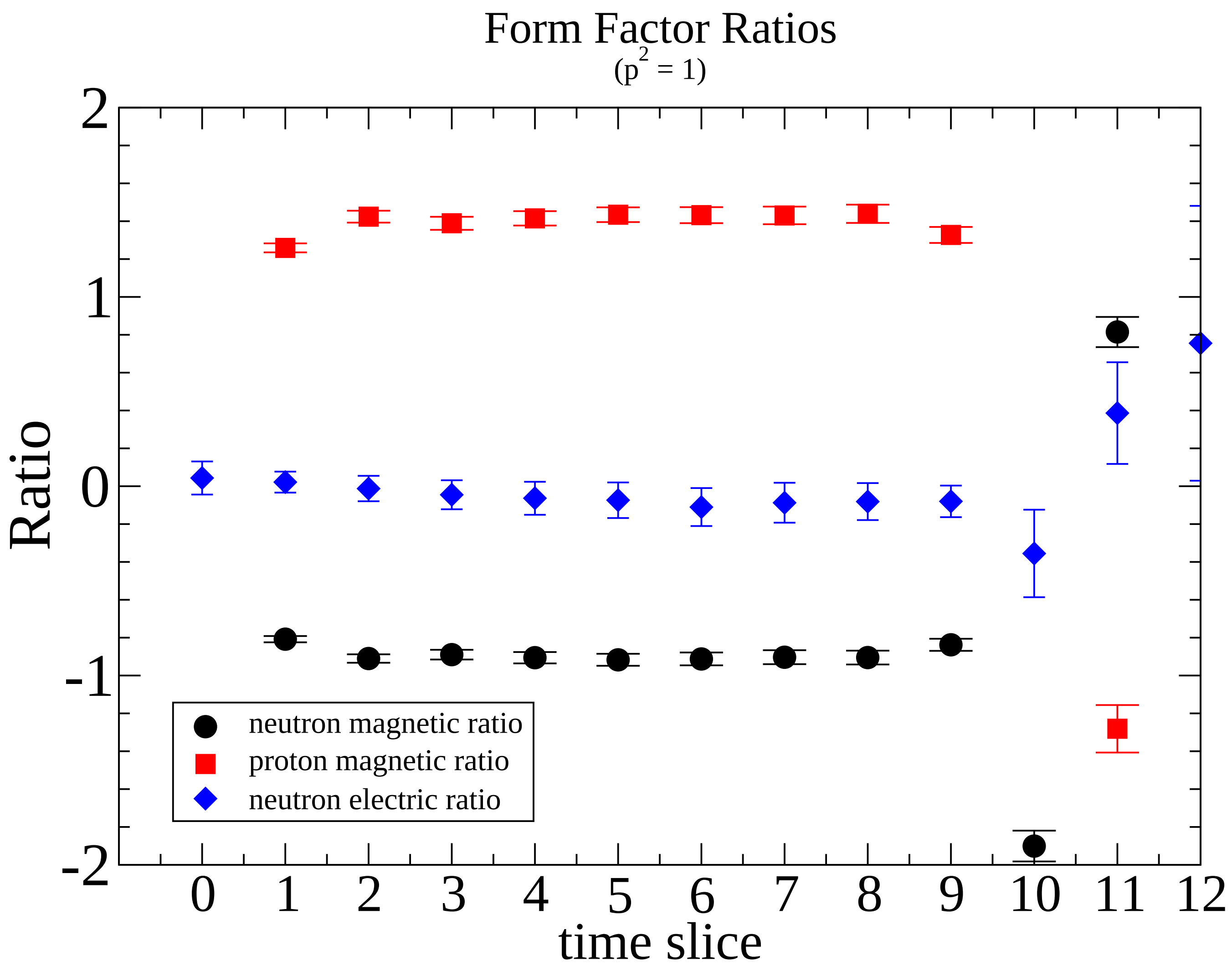
Topological charge:

Q was computed by integrating the topological charge density after APE smearing the gauge fields (20 sweeps with ape weight 0.45) [11].

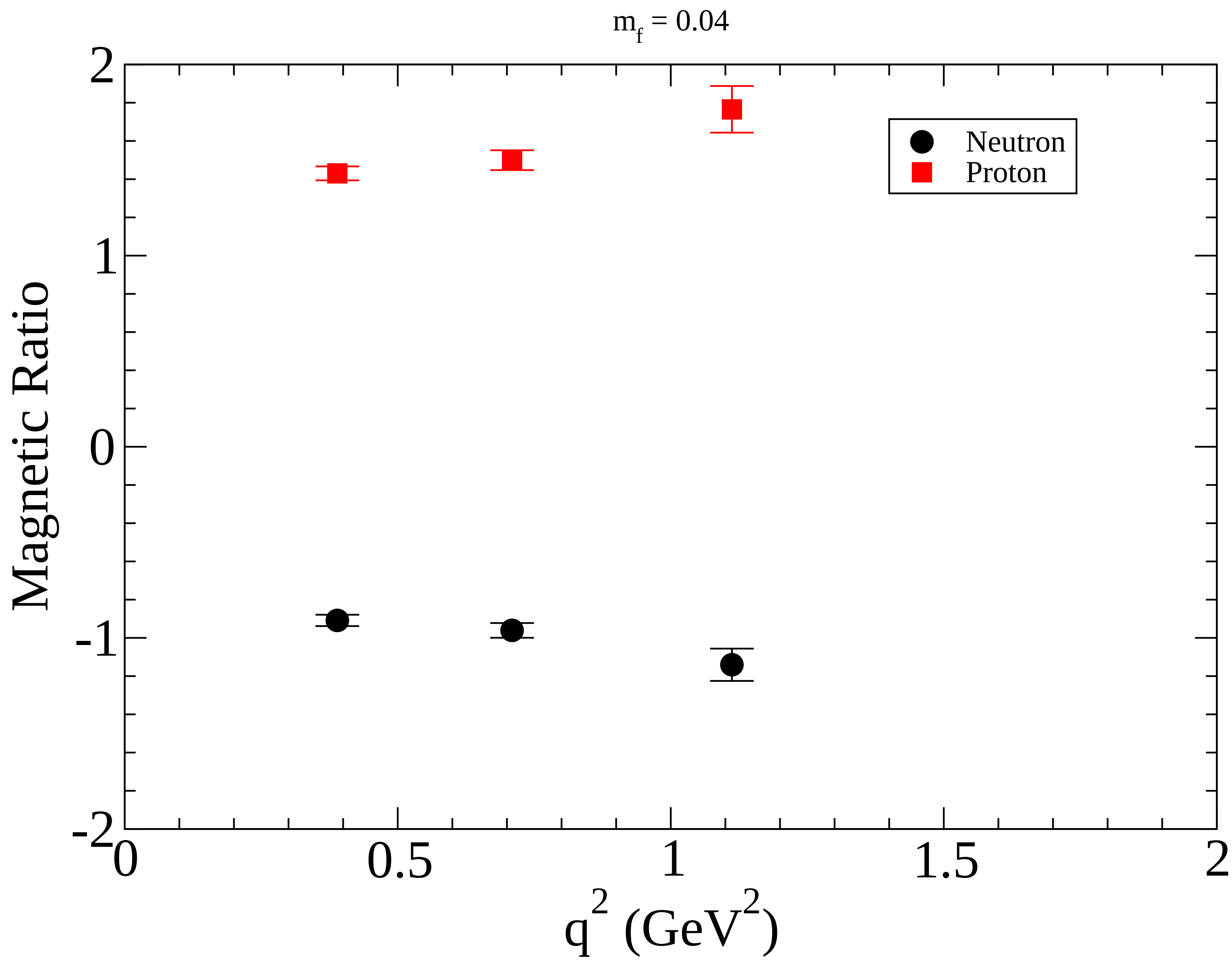
We are also investigating computing the topological charge from the index defined from the domain wall fermion Dirac operator (strictly valid in the limit $L_s \to \infty$).

The correspondence between the two has been high in previous (quenched) calculations and seems to be the case here as well.


Q from gauge fields vs Q from P
52 confs, DBW2 dyn, mf=0.04
from gauge fields
from P
15
10
5
0
−5
−10
0
10
20
30
40
50
conf

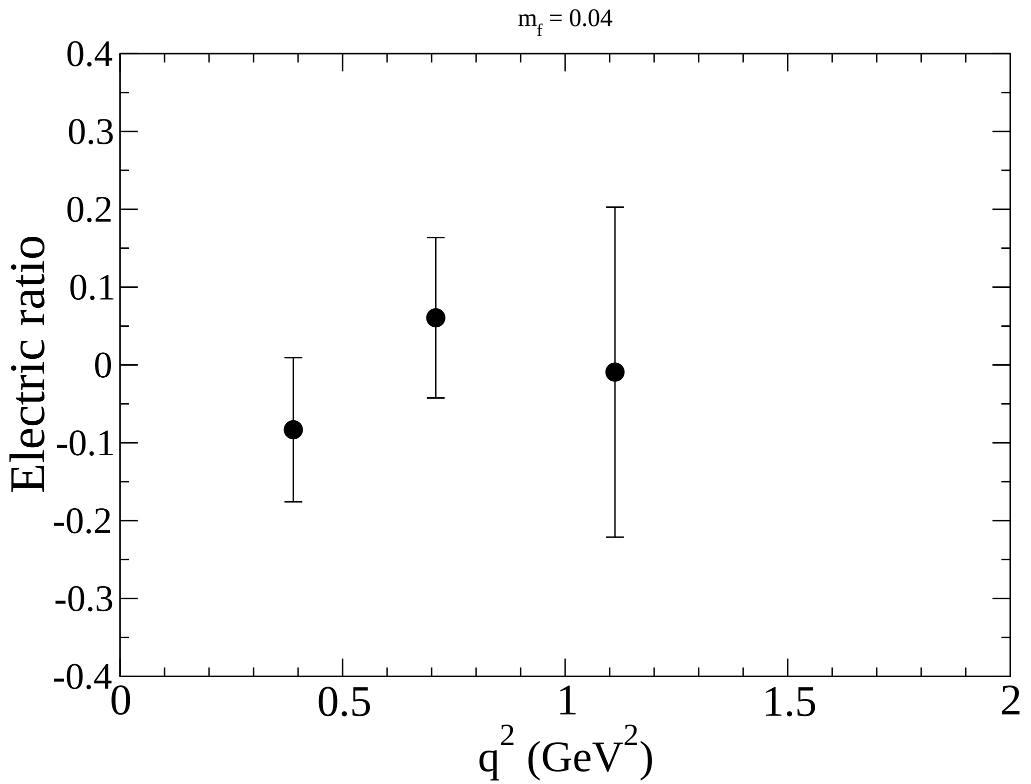


Form Factor Ratios
(p^2 = 1)
Ratio
time slice
2
1
0
-1
-2
0 1 2 3 4 5 6 7 8 9 10 11 12
neutron magnetic ratio
proton magnetic ratio
neutron electric ratio



m_f = 0.04
Neutron
Proton
Magnetic Ratio
q^2 (GeV^2)
2
1
0
-1
-2
0
0.5
1
1.5
2

$$\lim_{q^2 \to 0} \text{electric ratio} = d_N/(a\,e\,\theta)$$



Approximating the $q^2 \to 0$ limit with smallest value of $\vec{p}^2$ $(= (2\pi/16)^2)$, taking $m_f = 0.04$ as physical yields

$$\begin{aligned} a_\mu^P &\approx 1.75(7) \\ a_\mu^N &\approx -1.75(6) \\ d_N/a(e\,\theta) &\approx -0.083(93). \end{aligned}$$

roughly consistent with the experimental values $a_\mu^P = 1.79$ and $a_\mu^N = -1.91$ (and of course $d_N \sim 0$). d_N is the naive value, *i.e.* (may) need to subtract unwanted mixing term.

In physical units, $d_N = -10.2(11.2) \times 10^{-16}\,\theta\, e - \mathrm{cm}$, consistent with model calculations.

Error estimates are statistical uncertainties only.

Summary/Outlook

We are striving to reduce the statistical error on our determination of d_N.

Do this by measuring d_N on all available $N_f = 2$ lattices ($\sim 1000/m_{sea}$), measuring more than once on each lattice, and using smeared sources.

Computing the sea quark mass dependence as well.

Topological charge distribution will limit the accuracy of the current run (need longer HMC evolutions)

Future: 2+1 flavor DWF calculation, and of course, include subtraction of mixing term a'la Aoki-Kuramshi-Shintani (and try Chris Michael's short-cut).

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