Neutron Electric Dipole Moment(NEDM)

Toward the calculation on the lattice

Sinya Aoki, Yoshio Kikukawa, Yoshinobu Kuramashi,
Eigo Shintani
for CP-PACS collaboration

theta term in QCD

CP odd

$$i\theta \frac{1}{32\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}(x)F_{\alpha\beta}(x) \equiv i\theta q(x)$$

NEDM: experimental bound

$$|\vec{d_n}| \le 6.3\times 10^{-26} e\cdot cm$$

Model estimate

$$|\vec{d_n}|/\theta \simeq 10^{-15} \sim 10^{-17} e\cdot cm$$

$$\theta = \theta_{\rm QCD} + \theta_{\rm EW} \le O(10^{-8})$$

Strong CP problem

1st principle calculation of NEDM from lattice QCD

Some History

Aoki-Gocksch quenched QCD with Wilson fermion $me^{i\theta\gamma_5}$

Incorrect ⇐ Aoki, Gocksch, Manohar, Sharpe

⇒ twisted mass QCD

⇒ some of model estimates are also incorrect
New estimate by ChPT(Aoki-Hatsuda)

Kuramashi((unpublished) Wilson fermion $\bar\psi(x)\gamma_5\psi(x) \Leftrightarrow q(x)$
No signal !

Anomalous flavor singlet chiral WT identities

theta term ⇔ complex mass term $me^{i\theta\gamma_5}$

$|\vec{d}_n| = 0$ at $m = 0$

Chiral symmetry is important $\Longrightarrow$ DW/GW fermions !

Method for small theta

$$\vec{d_n} \propto \quad \theta \int d^3x \langle N | \vec{x} J_0^{EM}(\vec{x}) Q | N \rangle$$

↑

$$\langle N(p) | J_\mu^{EM}(q = p - p') Q | N(p') \rangle$$

Form factor

How can I extract this form factor from lattice data ?

Naive reduction formula can not be used. (Ishizuka)

Formulation at finite θ

Parity-even Nucleon interpolating operators

$$\chi_+(\vec{0},0) = Z_+^\theta N_+^\theta(\vec{0},0) + Z_-^\theta \gamma_5 N_-^\theta(\vec{0},0)$$
$$= Z_+ N_+(\vec{0},0) + Z_- \gamma_5 N_-(\vec{0},0)$$

$$N_\pm^\theta(\vec{x},t) = \sum_{\vec{p},s} \left[a_\pm^\theta(\vec{p},s) u_\pm^\theta(\vec{p},s) e^{ipx} + \cdots\right]$$

$$N_\pm(\vec{0},0) = \sum_{\vec{p},s} \left[a_\pm(\vec{p},s) u_\pm(\vec{p},s) + \cdots\right]$$

Assumption

$$e^{-i\theta Q_5} a_\pm(P) e^{i\theta Q_5} = a_\pm(P) - i\theta C_\mp(P,K) a_\mp(K) + O(\theta^2)$$

Ex. $C_\pm(P,K) = c\delta_{P.K}$

Q_5 : Anomalous chiral charge

(Note $a_\pm^\theta(P) = e^{i\theta Q_5} a_\pm(P) e^{-i\theta Q_5}$)

Consequence of the assumption on 2-pt function

$$|\theta\rangle = e^{i\theta Q_5}|0\rangle$$

$$G^{\theta}_{NN}(\vec{p},t) \equiv \sum_{\vec{x}} \langle\theta|\chi_+(\vec{0},0)\bar{\chi}_+(\vec{x},t)|\theta\rangle e^{i\vec{p}\vec{x}}$$

$$= \sum_{s} \langle\theta|\chi_+(\vec{0},0)|N^{\theta}_+(P)\rangle \langle N^{\theta}_+|\bar{\chi}_+(\vec{0},0)|\theta\rangle e^{-E^{\theta}_+ t} + \cdots$$

$$\langle\theta|\chi_+(\vec{0},0)|N^{\theta}_+(P)\rangle = Z^{\theta}_+ u^{\theta}_+(P)$$

$$= Z_+\langle\theta|N_+|N^{\theta}_+(P)\rangle + Z_-\gamma_5\langle\theta|N_-|N^{\theta}_+(P)\rangle$$

$$\langle\theta|N_+|N^{\theta}_+(P)\rangle = \langle 0|e^{-i\theta Q_5}N_+ e^{i\theta Q_5}|N_+(P)\rangle = u_+(P) + O(\theta^2)$$

$$\langle\theta|N_-|N^{\theta}_+(P)\rangle = \langle 0|e^{-i\theta Q_5}N_- e^{i\theta Q_5}|N_+(P)\rangle$$

$$= -i\theta C_-(P,K)u_-(K) + O(\theta^3)$$

CP symmetry at $\theta = 0$ ⇒ $Z_+^\theta = Z_+ + O(\theta^2)$

Therefore

$$u_+^\theta(P) = u_+(P) - i\theta\gamma_5 D(P,K) u_-(K) + O(\theta^2)$$

$$D(P,K) \equiv \frac{Z_-}{Z_+} C_-(P,K)$$

After a little algebra

$$G_{NN}^\theta(\vec{p},t) = G_{NN}(\vec{p},t) + i\theta G_{NN}^Q(\vec{p},t) + O(\theta^2)$$

$$G_{NN}(\vec{p},t) = |Z_+|^2 e^{-E_+ t} \frac{E_+\gamma_0 + m_+ - i\vec{\gamma}\cdot\vec{p}}{2E_+}$$

$$G_{NN}^Q(\vec{p},t) = -|Z_+|^2 e^{-E_+ t}\, \gamma_5 \left[D_P + i\gamma_0 D_0 - \vec{\gamma}\cdot\vec{p} D_k\right]$$

$$G_{NN}^Q(\vec{p},t) = \langle 0|\chi(0)\bar{\chi}(t) Q|0\rangle$$

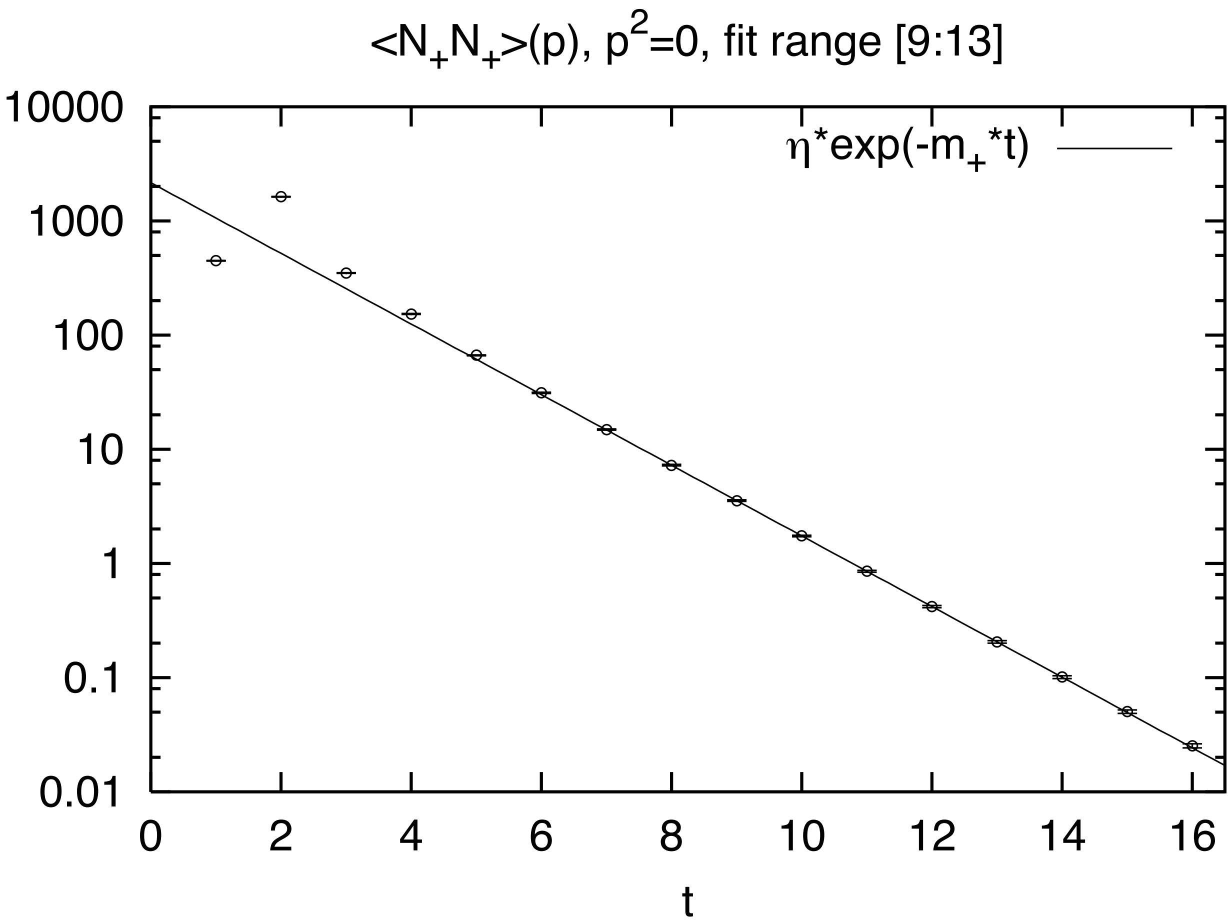
$D_{P,0,k}$: real constants

Numerical Results

Iwasaki-RG(Quenched) at $\beta = 2.6 (a \simeq 0.1\mathrm{fm})$ + DWF

$16^3 \times 32 \times 16,\ m_f a = 0.03,$ 716 conf.

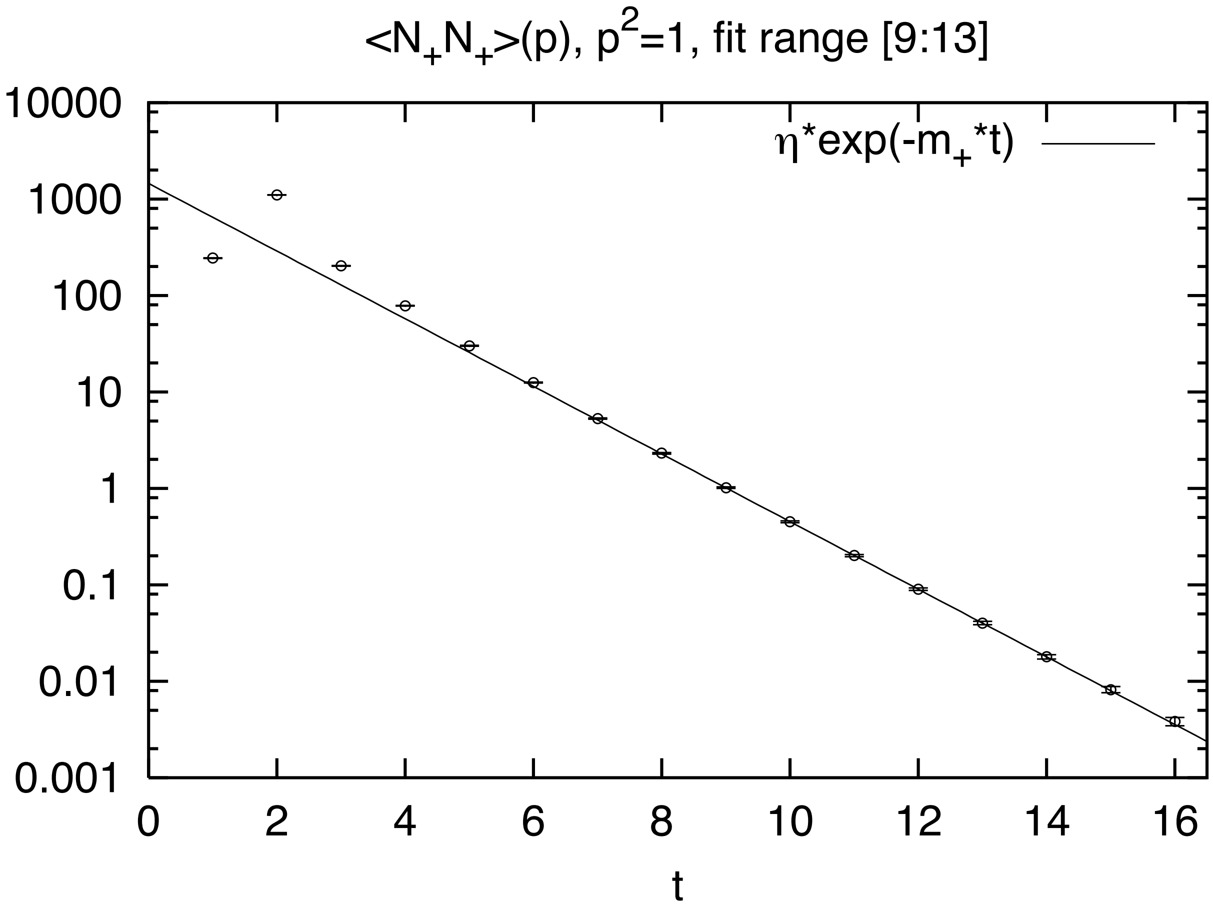
$$G_{NN}(\vec{0}, t)$$



$$m_+ = 0.712(4)$$

$$Z_+^2 = 2162(87)$$

$$G_{NN}(\vec{p}, t) \qquad |\vec{p}| = \frac{2\pi}{L}$$



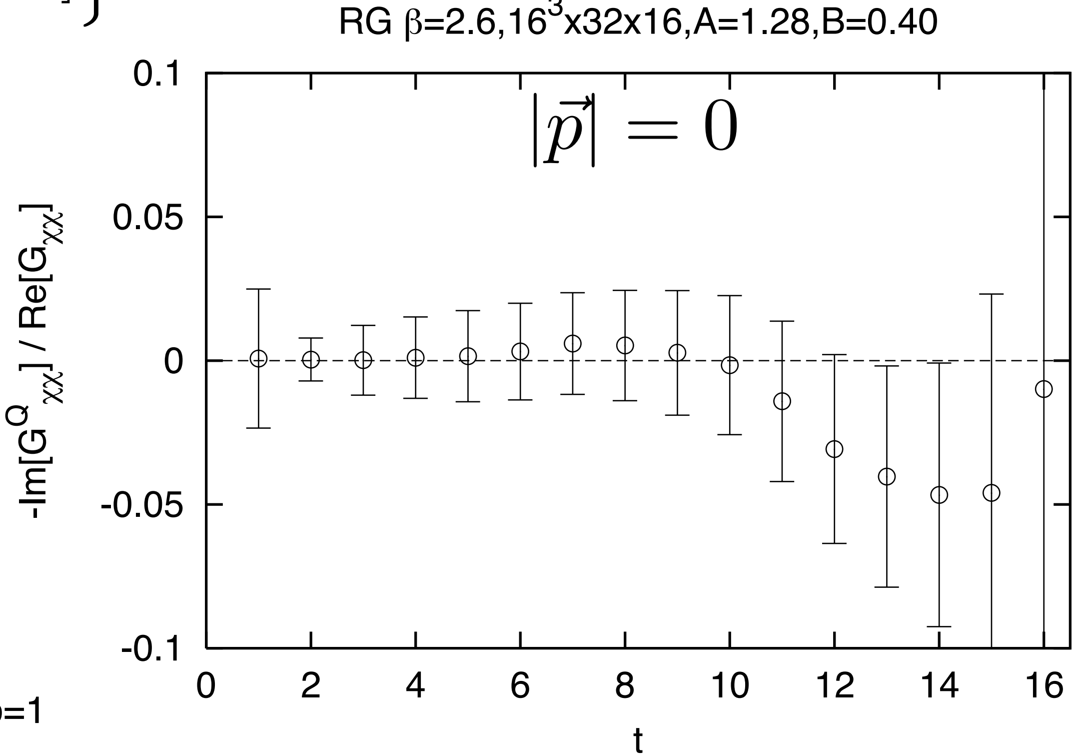
$$E_+ = 0.807(8)$$

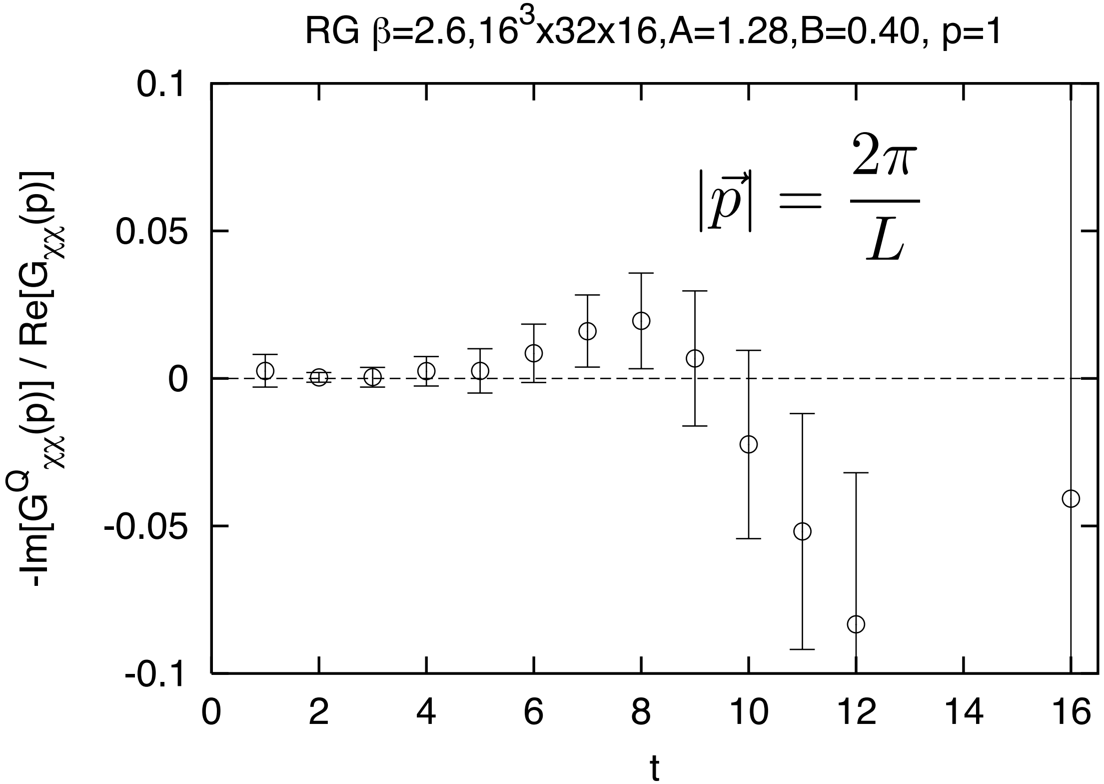
$$(\sqrt{m_+^2 + \vec{p}^2} = 0.813)$$

$$Z_+^2 = 1453(104)$$

$$\mathrm{Im}\left\{\mathrm{tr}[G^Q_{NN}(\vec{p},t)\gamma_5\Gamma_0]/\mathrm{tr}[G_{NN}(\vec{p},t)\Gamma_0]\right\}$$

$$D_0 \simeq 0$$





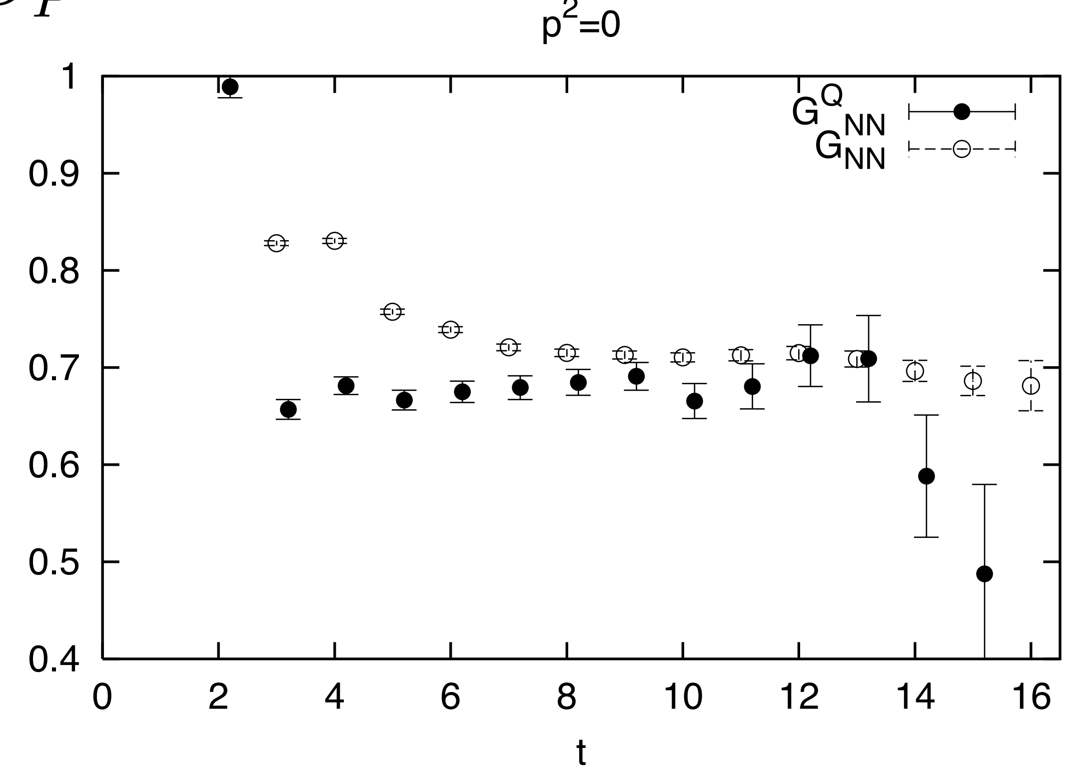
Similarly

$$D_k \simeq 0$$

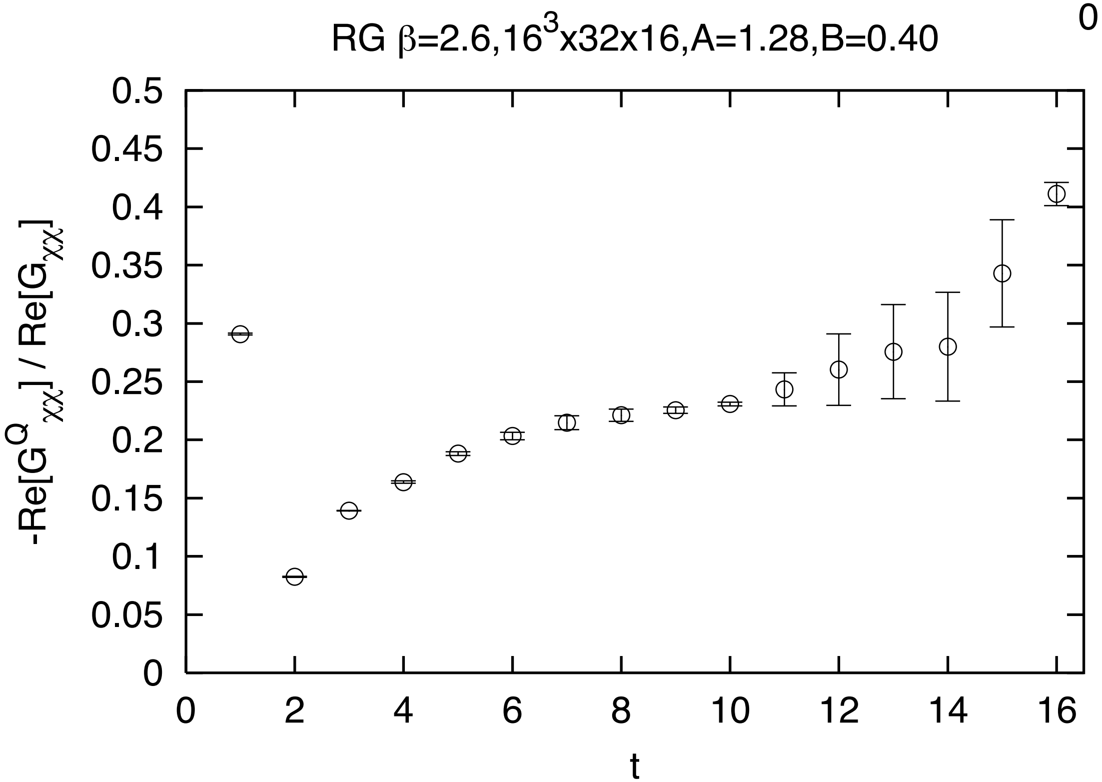
$$G^{Q}_{NN}(\vec{p},t) = -\,|Z_+|^2 e^{-E_+ t}\,\gamma_5 D_P$$

$$|\vec{p}| = 0$$

Effective mass

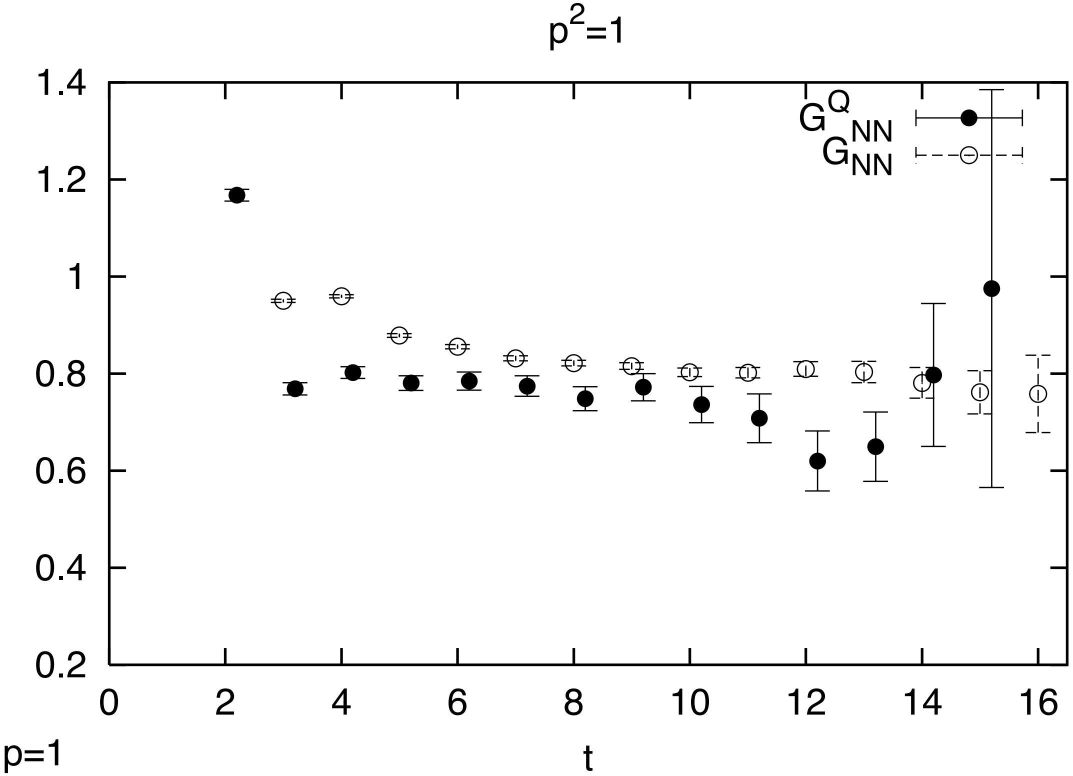


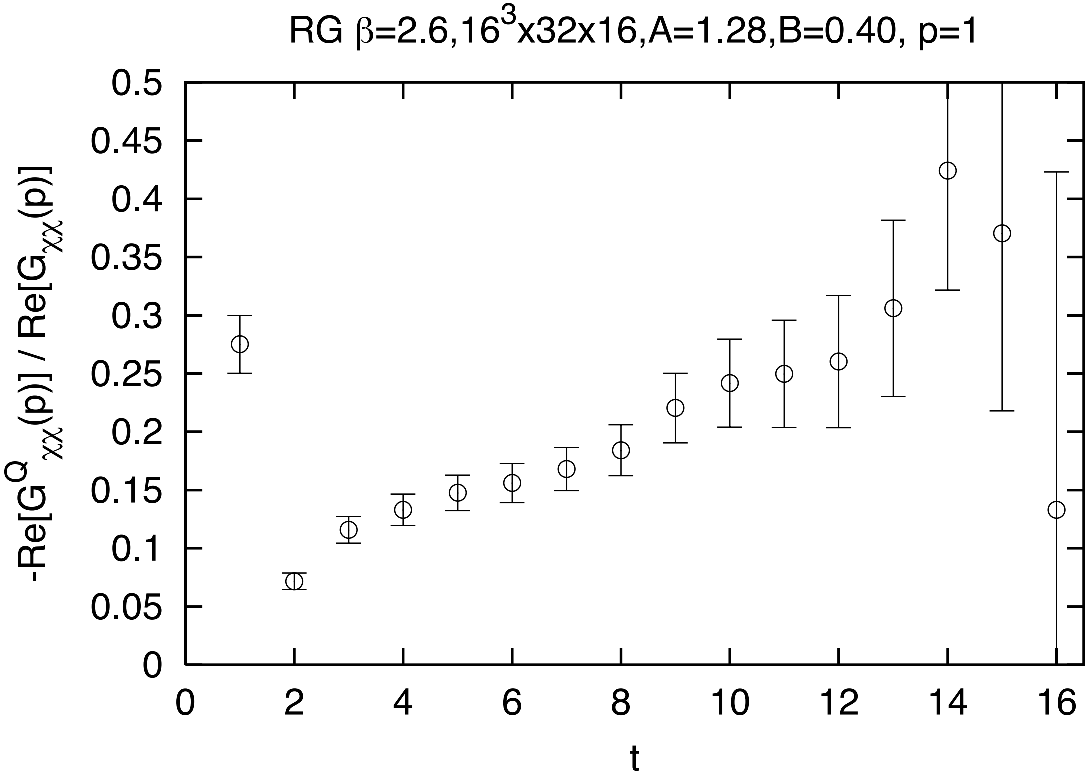
Ratio



Signal for $D_P \neq 0$!

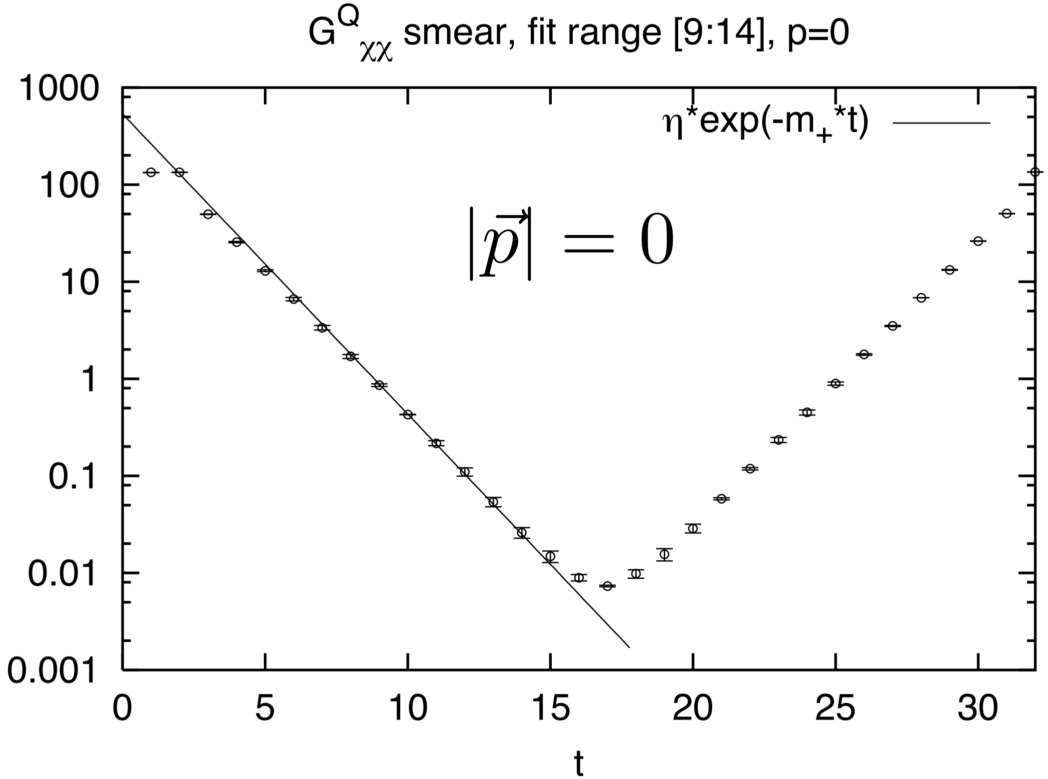
$$|\vec{p}| = \frac{2\pi}{L}$$



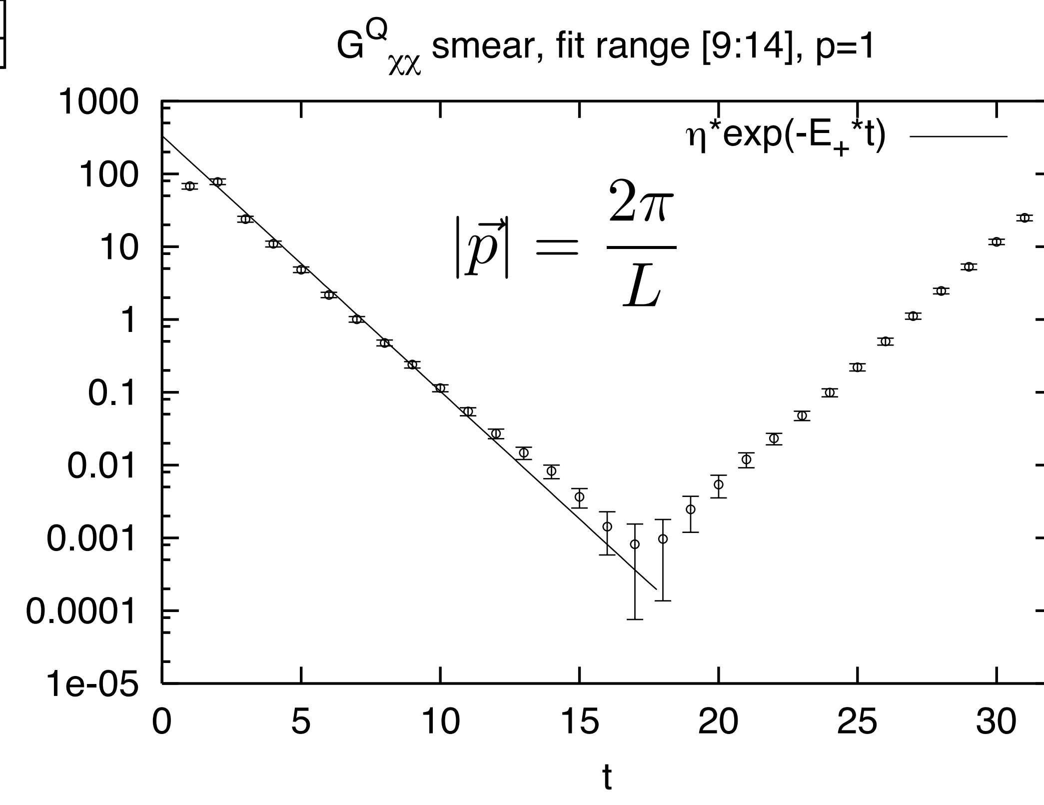


$$G^Q_{NN}(\vec{p},t) = -|Z_+|^2 e^{-E_+ t}\,\gamma_5 D_P$$

Fixed E_+



$$D_P = 0.248(17)$$



$$D_P = 0.216(18)$$

Correlation between Q and quark is very important to get signals

$$\mathrm{tr}[G_{NN}^{Q}(\vec{p},t)\gamma_5] = -D_P\,|Z_+|^2 e^{-E_+ t} \quad \Leftrightarrow \quad \mathrm{tr}[G_{NN}(\vec{p},t)\gamma_5] = 0$$

Same E_+

$$\mathrm{tr}[G_{NN}(\vec{p},t)] = |Z_+|^2 e^{-E_+ t}$$

use of DWF

enough sampling of Q

3-pt function and form factor

$$\langle N_+^\theta(P')|J_\mu(q)|N_+^\theta(P)\rangle = \bar{u}_+^\theta(P')W_\mu^{\theta++}(q)u_+^\theta(P)$$

Target

$$W_\mu^{\theta++}(q) = W_\mu^{0++}(q) + i\theta W_\mu^{1++}(q) + O(\theta^2)$$

$$W_\mu^{0++}(q) = F_1(q^2)\gamma_\mu + F_2(q^2)\frac{q_\nu\sigma_{\mu\nu}}{2m_+}$$

$$W_\mu^{1++}(q) = F_3(q^2)\gamma_5\frac{q_\nu\sigma_{\mu\nu}}{2m_+} + G_2(q^2)(\gamma_\mu q^2 - q_\mu\gamma\cdot q)$$

Lattice calculation

$$\langle\theta|\chi_+(\vec{p'},0)J_\mu(q,\tau)\bar{\chi}_+(\vec{p},t)|\theta\rangle \;\; = |Z_+^\theta|^2 e^{-E_+^{\theta\prime}\tau}e^{-E_+^\theta(t-\tau)}$$

$$\times\sum_{s,s'} u_+^\theta(P')\bar{u}_+^\theta(P')\; W_\mu^{\theta++}(q)\;\; u_+^\theta(P)\bar{u}_+^\theta(P) \quad + \cdots$$

By expanding in terms of θ in both sides

$$\langle 0|\chi(0)J_\mu(\tau)\bar{\chi}(t)|0\rangle = |Z_+|^2 e^{-E'_+\tau}e^{-E_+(t-\tau)} \times \frac{-i\gamma\cdot p' + m_+}{2E'_+} W_\mu^{0++}(q) \frac{-i\gamma\cdot p + m_+}{2E_+}$$

$$\langle 0|\chi(0)J_\mu(\tau)\bar{\chi}(t)Q|0\rangle = |Z_+|^2 e^{-E'_+\tau}e^{-E_+(t-\tau)}$$

$$\times \Big[\frac{-i\gamma\cdot p' + m_+}{2E'_+} W_\mu^{1++}(q) \frac{-i\gamma\cdot p + m_+}{2E_+}$$

$$- \frac{-i\gamma\cdot p' + m_+}{2E'_+} W_\mu^{0++}(q)\ D_P(\gamma_5 X_{-+} + X_{+-}\gamma_5)$$

$$- D_P(\gamma_5 X_{-+} + X_{+-}\gamma_5)'\, W_\mu^{0++}(q) \frac{-i\gamma\cdot p + m_+}{2E_+} \Big]$$

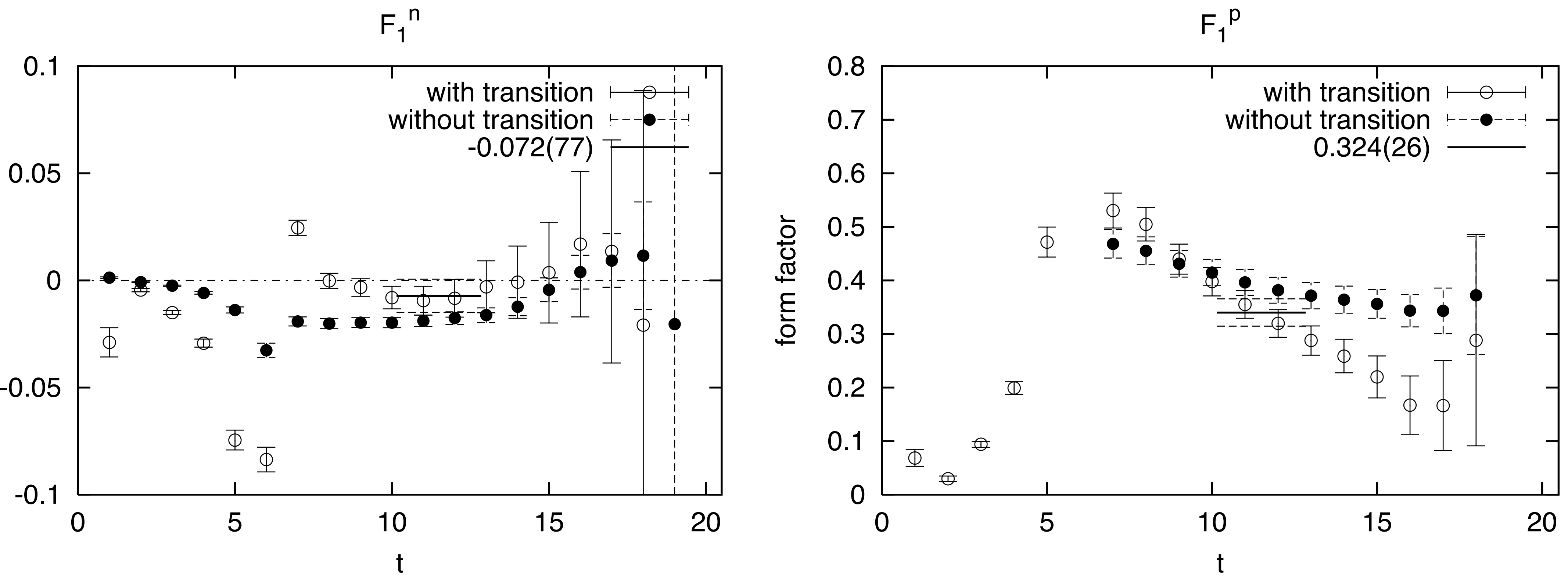
$$X_{\pm\mp} = \sum_s u_\pm(P)\bar{u}_\mp(P)$$

We have to subtract the effect of mixing due to $W_\mu^{0++}(q)$ from

$$\langle 0|\chi(0)J_\mu(\tau)\bar{\chi}(t)Q|0\rangle$$ to obtain $W_\mu^{1++}(q)$

Work in progress (Shintani)

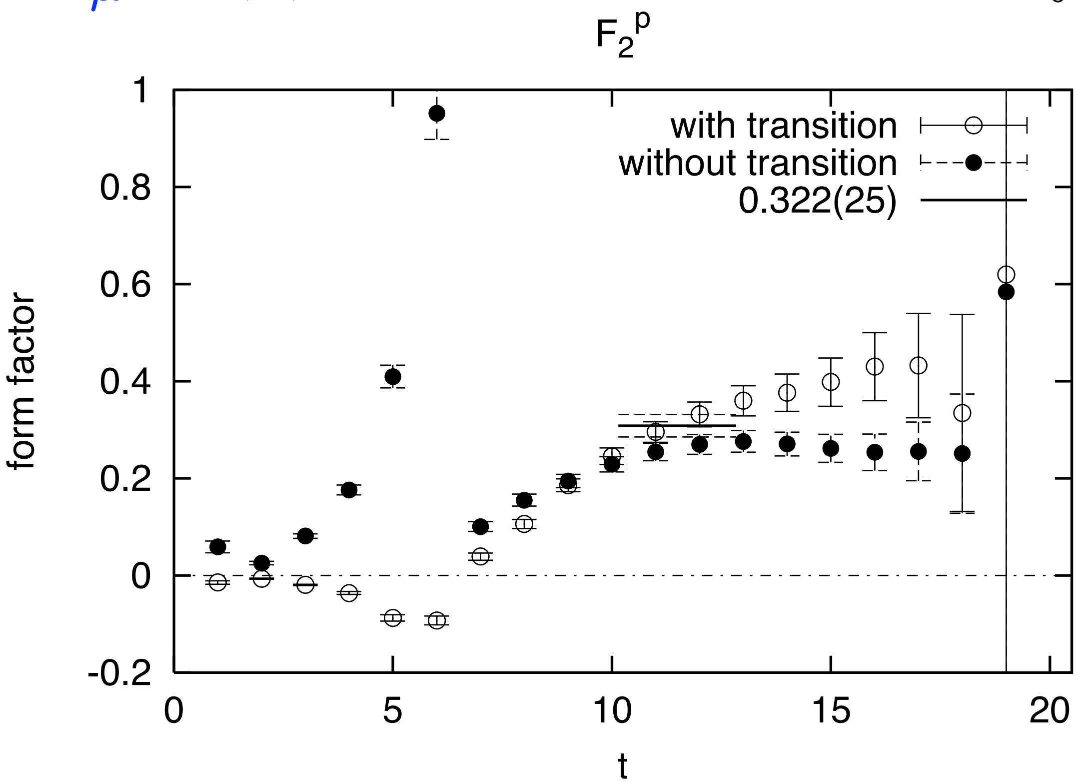
$f_1(q^2)$



Neutron Proton

$W_\mu^{0++}(q)$

$f_2(q^2)$



Proton

Neutron