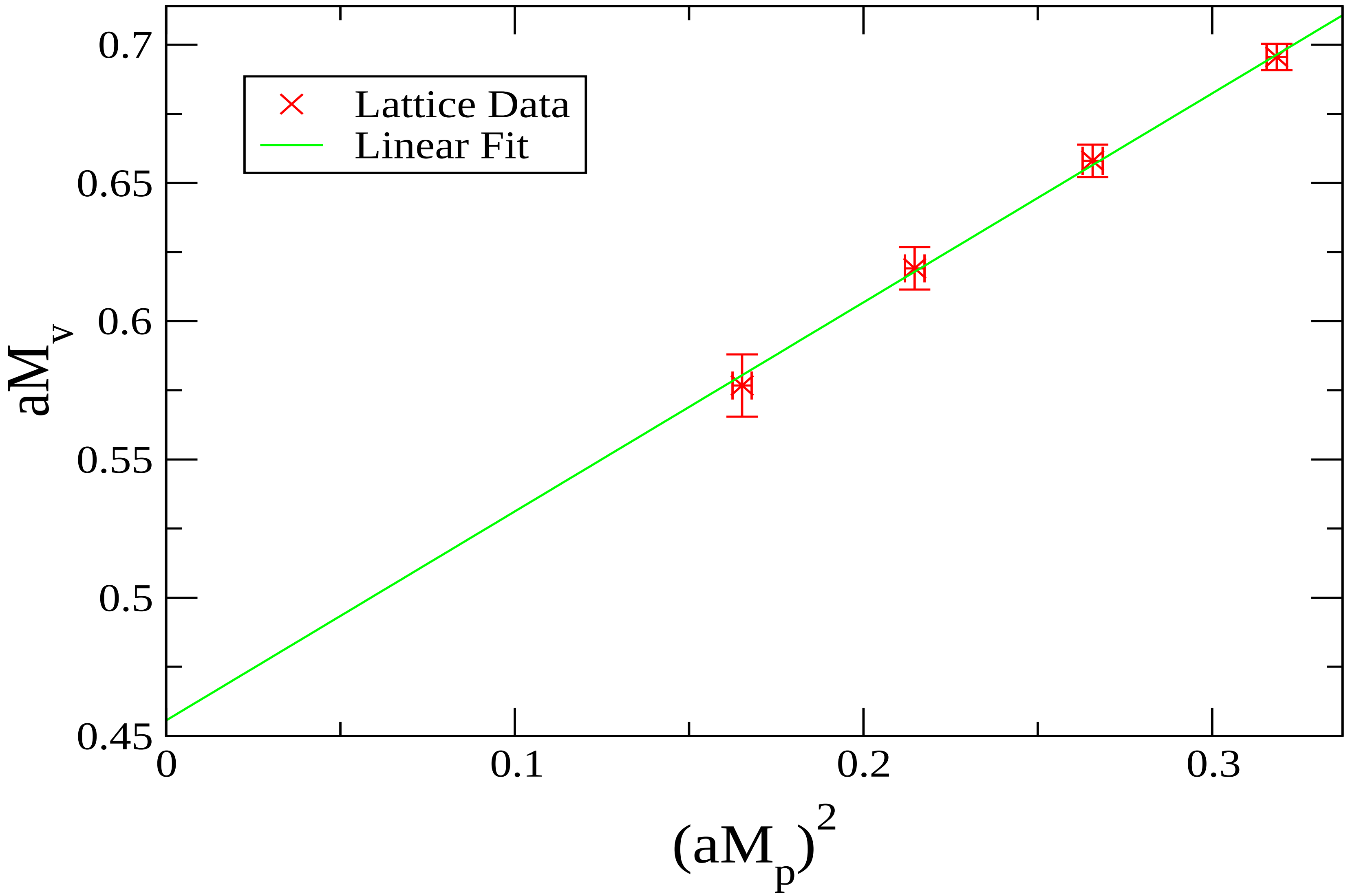
Chiral Extrapolation of the Rho and Nucleon Masses

W. Armour,[a] **C.A.,**[a] **D. Leinweber,**[b]
A.W. Thomas,[b,c] **R. Young**[b]

a Department of Physics, University of Wales Swansea
b CSSM, University of Adelaide
c JLab

hep-lat/0309053 (Wes Armour's Lat03 talk)

Typical Lattice Data



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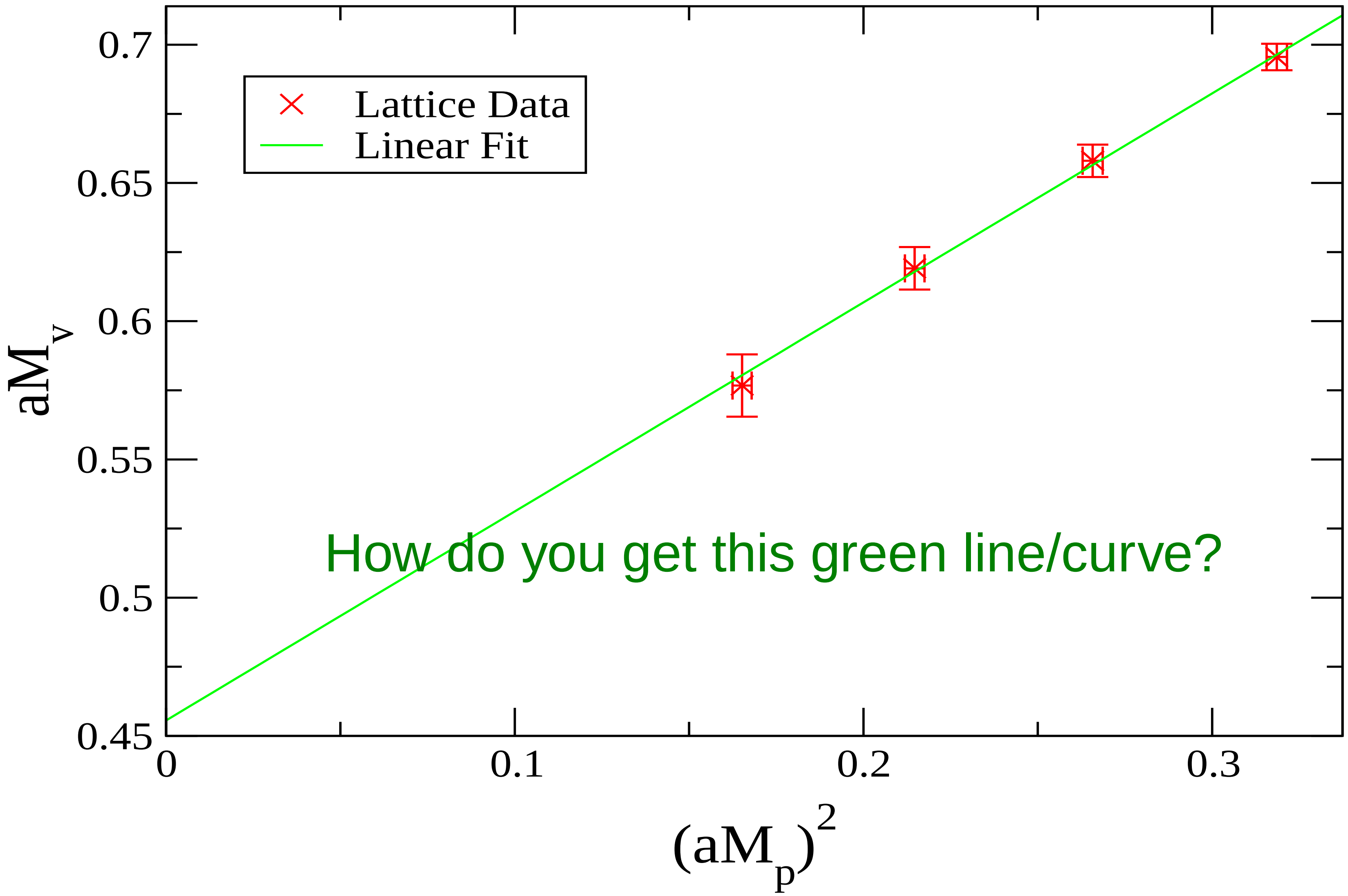

Extrapolations required

Lattice simulations don't solve real QCD:
Recall $< \mathcal{O} >= f(g_0, m, \mu, L, N_f, N_{\{U\}})$

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Lattice parameters

$a \approx 0.1$ fm

$m_q \approx$ 50 MeV

$L \approx 2$ fm

$N_f =$ 0, 2 or 2+1

$N_{\{U\}} = \mathcal{O}(100)$

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Lattice parameters		*Nature's parameters*
$a \approx 0.1$ fm	$\longrightarrow$	0
$m_q \approx$ 50 MeV	$\longrightarrow$	few MeV
$L \approx 2$ fm	$\longrightarrow$	∞
$N_f =$ 0, 2 or 2+1	$\longrightarrow$	"2+1+1+1+1"
$N_{\{U\}} = \mathcal{O}(100)$	$\longrightarrow$	∞

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Outline

- Traditional chiral extrapolations
- Introduction to the Adelaide approach to chiral extrapolations
- Results for the ρ meson
- Results for the nucleon
- Adelaide method bonza or not?

χPT Approach

use χPT to get, e.g.

$$M_n = c_0 + c_2 M_{PS}^2 + c_3 M_{PS}^3 + \ldots$$

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$$M_n = c_0 + c_2 M_{PS}^2 + c_3 M_{PS}^3 + \ldots$$

where c_{odd} are determined from χPT arguments

- i.e. they are *fixed*

here $\qquad c_3 = -5.5 GeV^{-3}$

Truncated χPT Fit

$$M_n = c_0 + c_2 M_{PS}^2 - 5.5\text{GeV}^{-2} M_{PS}^3$$

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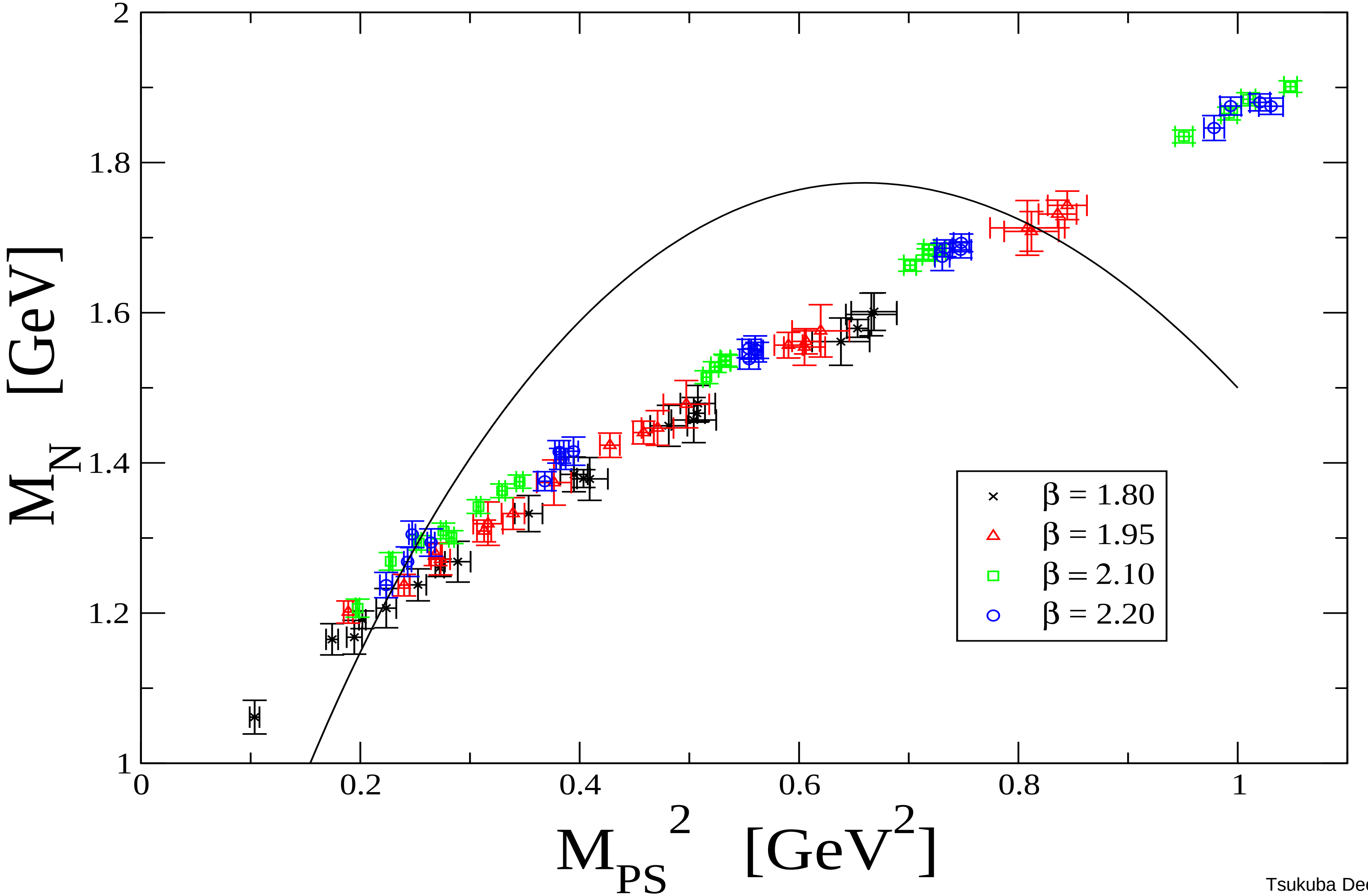

Adelaide Formula: M_ρ

$$
\begin{aligned}
& M_V^2(\beta, s; v, v) \\
= \quad & a_0 + a_2 M_{PS}^2(\beta, s; v, v) + a_4 M_{PS}^4(\beta, s; v, v) + \ldots \\
+ \quad & \Sigma^{\rho}_{\pi\pi}(M_{PS}^2(\beta, s; s, v)) + \Sigma^{\rho}_{\pi\omega}(M_{PS}^2(\beta, s; s, v)) \\
+ \quad & \Sigma_{DHP}(M_{PS}^2(\beta, s; s, v), M_{PS}^2(\beta, s; v, v), M_{PS}^2(\beta, s; s, s))
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e.g. $$\Sigma^{\rho}_{\pi\omega} = -\frac{g^2_{\omega\rho\pi}\mu_\rho}{12\pi^2}\int_0^\infty \frac{k^4 u^2_{\pi\omega}(k)\, dk}{\omega^2_\pi(k)}$$

with $$\omega^2_\pi(k) = k^2 + M_{PS}^2(\beta, s; s, v)$$

and $$u(k) = \frac{\Lambda^4}{(\Lambda^2 + k^2)^2}$$

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Note it introduces a Λ parameter

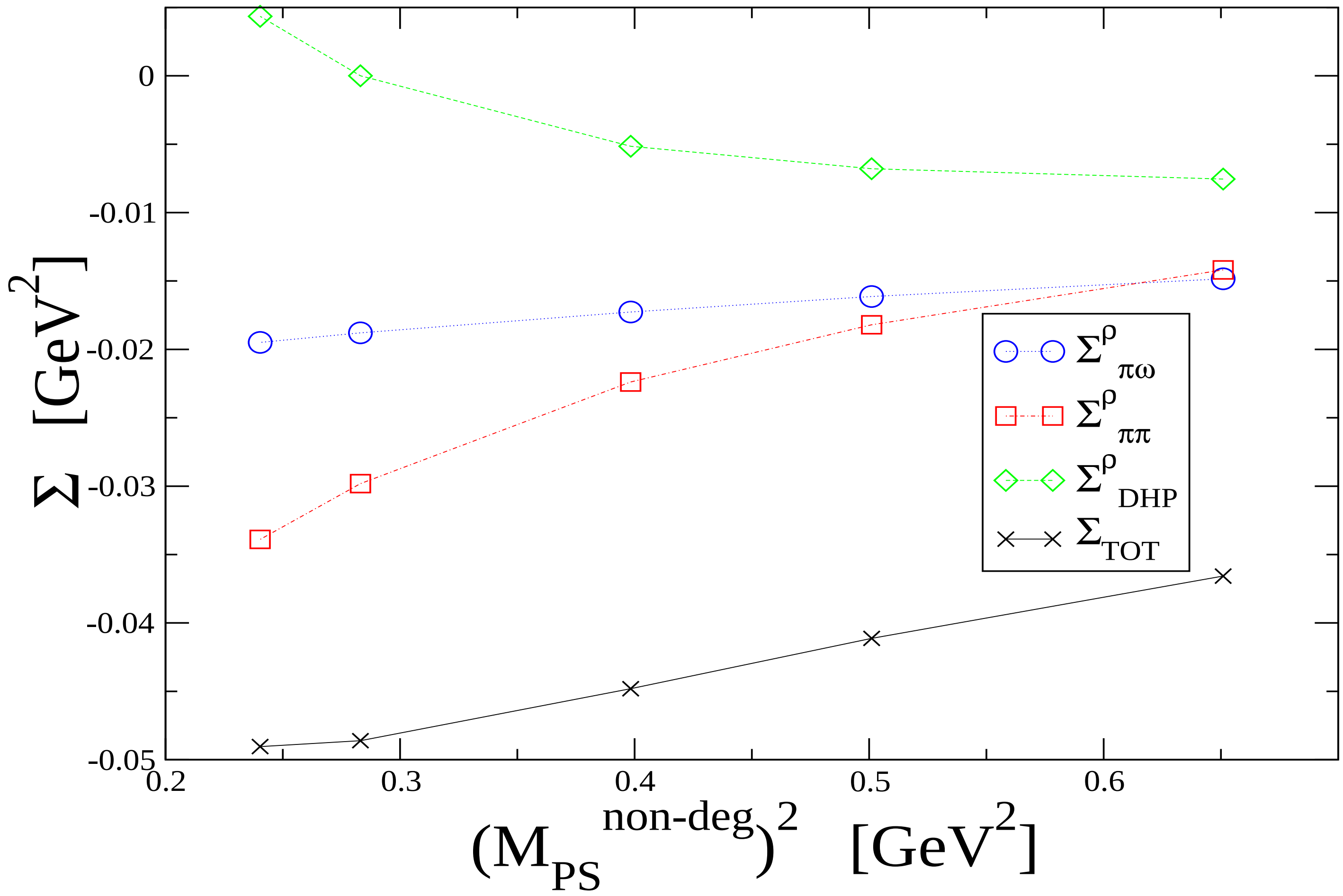
Momentum discretisation

The self energy equations are discretised using:

$$4\pi \int_0^\infty k^2 dk = \int d^3k \approx \frac{1}{V}\left(\frac{2\pi}{a}\right)^3 \sum_{k_x,k_y,k_z}$$

With k_μ being constrained by the finite periodic volume of the lattice.

Self-energies



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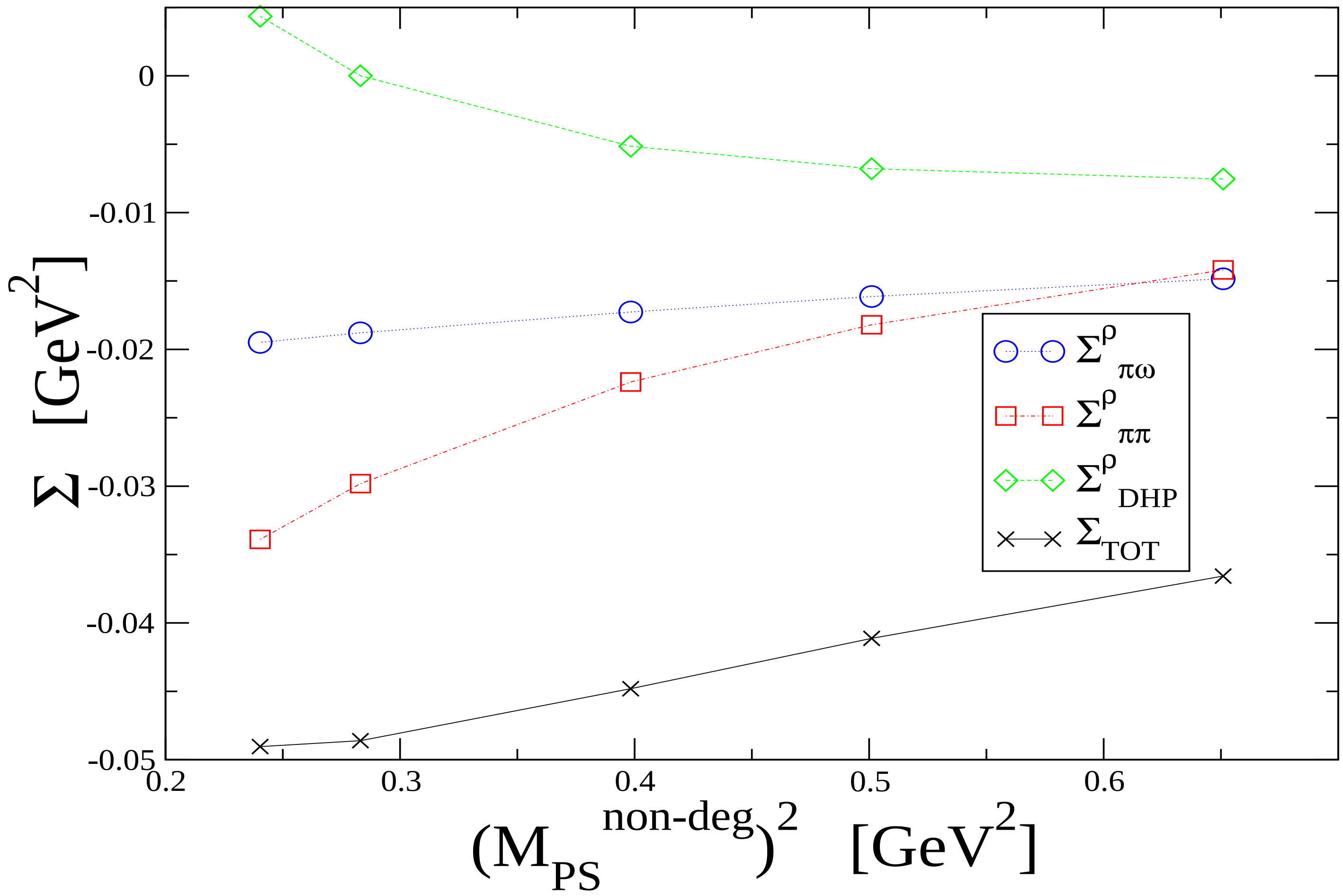


- representative ensemble: $(\beta, \kappa_{\text{sea}}) = (2.10, 0.1382)$.

Overview of CP-PACS Data

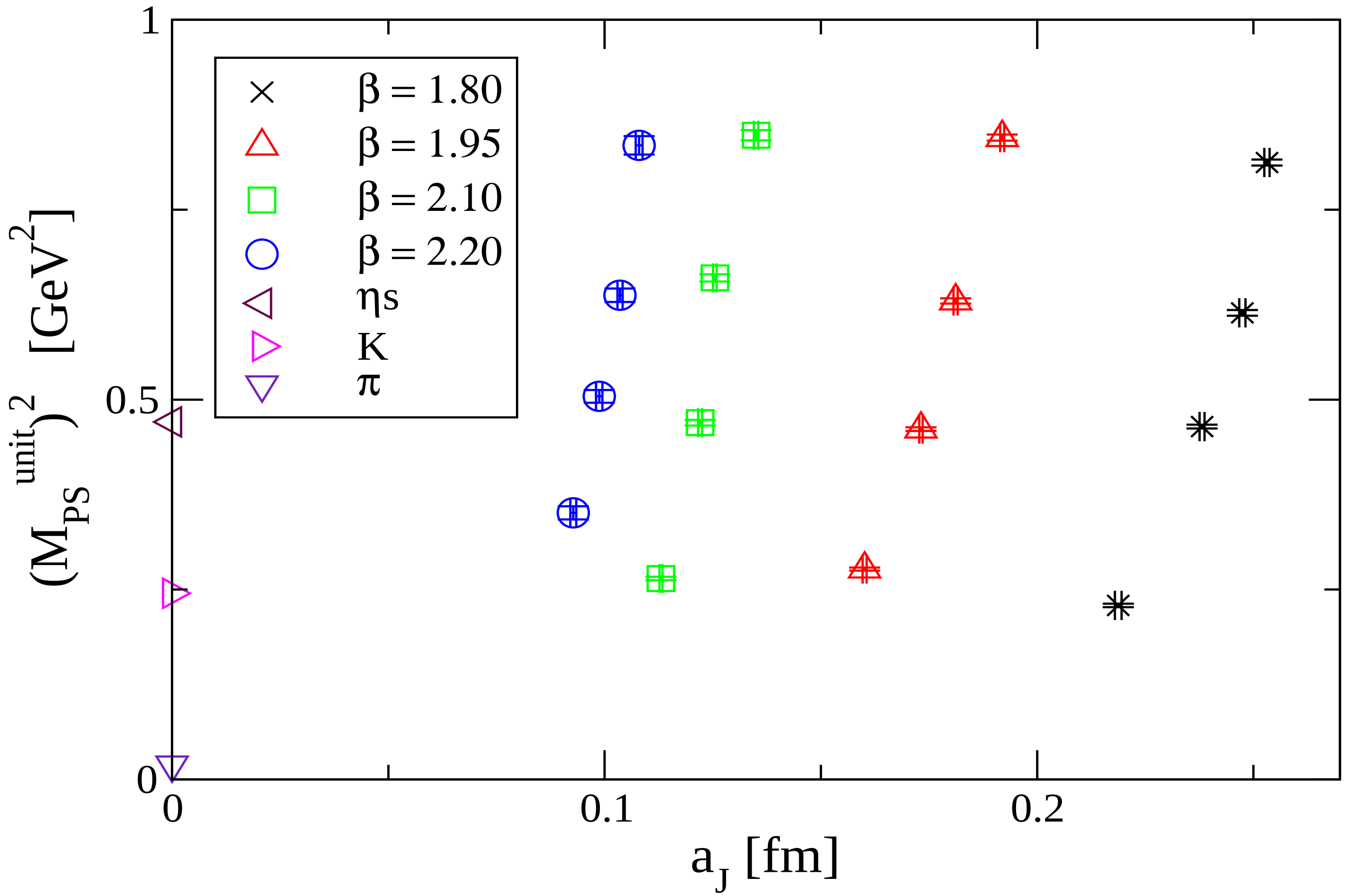
Data used from CP-PACS Collaboration:
A.A. Khan et al., Phys.Rev. D65 (2002) 054505

- mean-field improved Wilson fermions with improved gluons
- 4×4 different $(\beta, \kappa_{\mathrm{sea}})$ combinations
- we generated 1000 bootstrap ensembles for hadron masses with FWHM = quoted error
- fully uncorrelated ensembles
  $\longrightarrow$ no cancellation of systematic errors in our analysis

CP-PACS lattice parameters

β	κ_{sea}	Volume	a_J [fm]	M_{PS}^{unit}/M_V^{unit}
1.8000	0.1409	$12^3 \times 24$	0.2531$\pm$6	0.8067$\pm$9
1.8000	0.1430	$12^3 \times 24$	0.2474$\pm$7	0.7526$\pm$15
1.8000	0.1445	$12^3 \times 24$	0.2381$\pm$6	0.694$\pm$2
1.8000	0.1464	$12^3 \times 24$	0.2188$\pm$8	0.547$\pm$4
1.9500	0.1375	$16^3 \times 32$	0.1919$\pm$5	0.8045$\pm$11
1.9500	0.1390	$16^3 \times 32$	0.1811$\pm$5	0.752$\pm$2
1.9500	0.1400	$16^3 \times 32$	0.1731$\pm$4	0.690$\pm$2
1.9500	0.1410	$16^3 \times 32$	0.1601$\pm$5	0.582$\pm$3
2.1000	0.1357	$24^3 \times 48$	0.1350$\pm$5	0.806$\pm$2
2.1000	0.1367	$24^3 \times 48$	0.1255$\pm$4	0.755$\pm$2
2.1000	0.1374	$24^3 \times 48$	0.1221$\pm$5	0.691$\pm$3
2.1000	0.1382	$24^3 \times 48$	0.1130$\pm$3	0.576$\pm$4
2.2000	0.1351	$24^3 \times 48$	0.1080$\pm$8	0.799$\pm$3
2.2000	0.1358	$24^3 \times 48$	0.1036$\pm$6	0.753$\pm$4
2.2000	0.1363	$24^3 \times 48$	0.0988$\pm$8	0.705$\pm$6
2.2000	0.1368	$24^3 \times 48$	0.0928$\pm$7	0.632$\pm$8

CP-PACS data range



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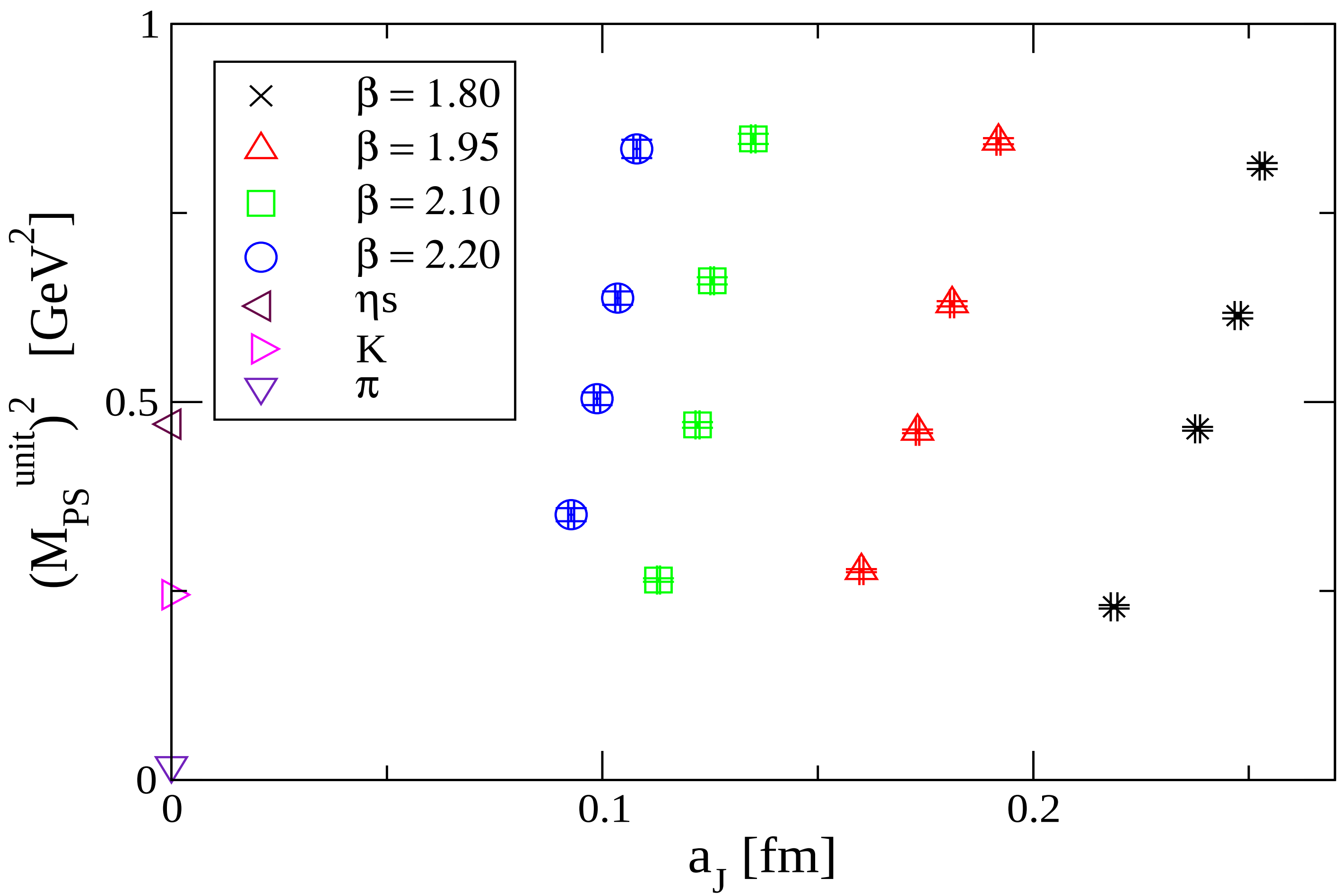

- M_{PS}^{unit} is the "unitary" pseudo-scalar mass
- a determined at the (M_K, M_{K^*}) point (c.f.J)

Fitting Philosophy

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$$M^{dimful} = M^{\#} \times a_{\Omega}^{-1} \equiv M^{\#}/M_{\Omega}^{\#} \times M_{\Omega}^{expt}$$

Fitting Procedure

In our chiral extrapolations we use the Adelaide ansatz, as well as a naive polynomial fit:

$$\begin{aligned} M_V(\beta, \kappa_{\rm sea}; \kappa_{\rm val}, \kappa_{\rm val}) = a_0 &+ a_2 M_{PS}^2(\beta, s; v, v) \\ &+ a_4 M_{PS}^4(\beta, s; v, v) \\ &+ a_6 M_{PS}^6(\beta, s; v, v) \end{aligned}$$

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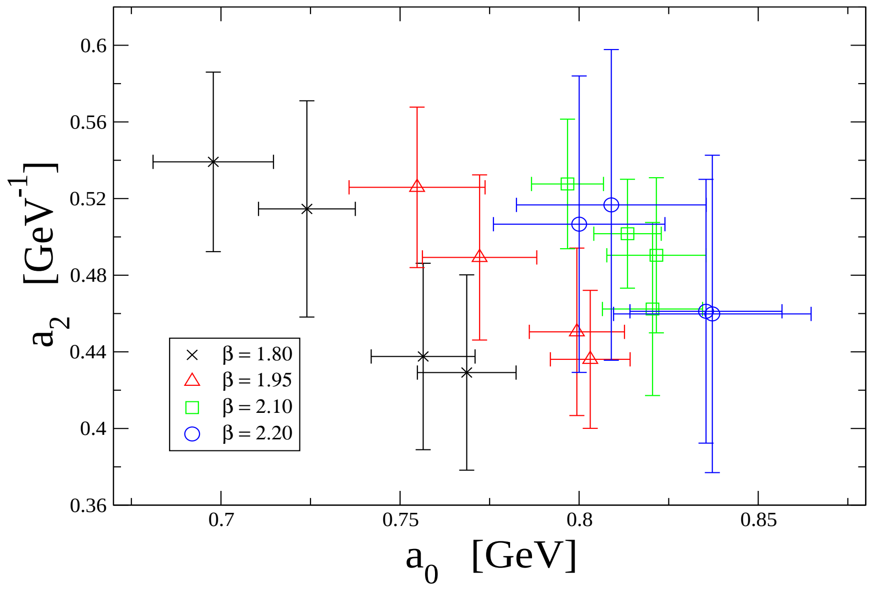
- fit to each individual ensemble's data as a warmup
- then fit to all ensembles globally
  $\rightarrow \quad 4 \times 4$ ensembles $\times\, 5\ \kappa_{\rm val}$ each $= 80$ points

Individual Fits: a_0, a_2



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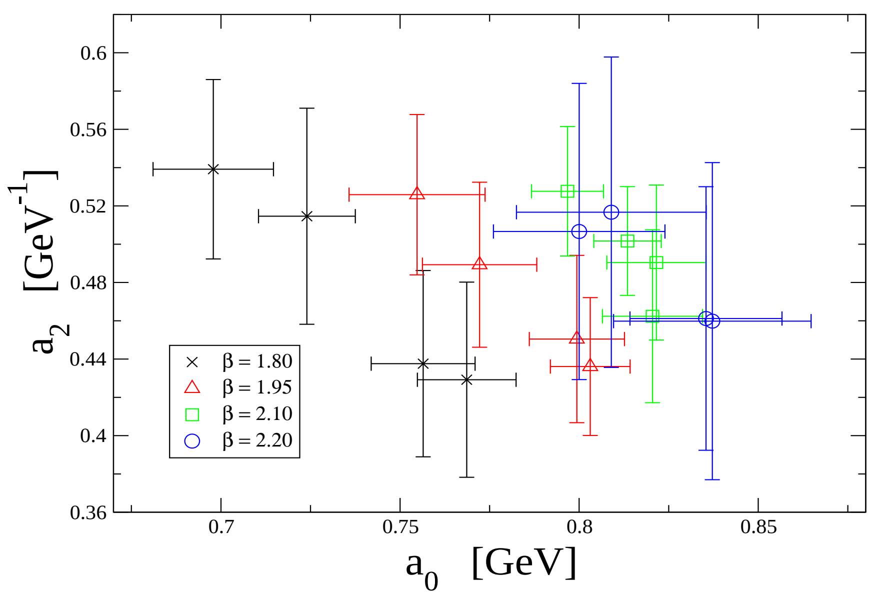

Scatter plot of a_2 against a_0 (Scale set from r_0.)

- Adelaide fit

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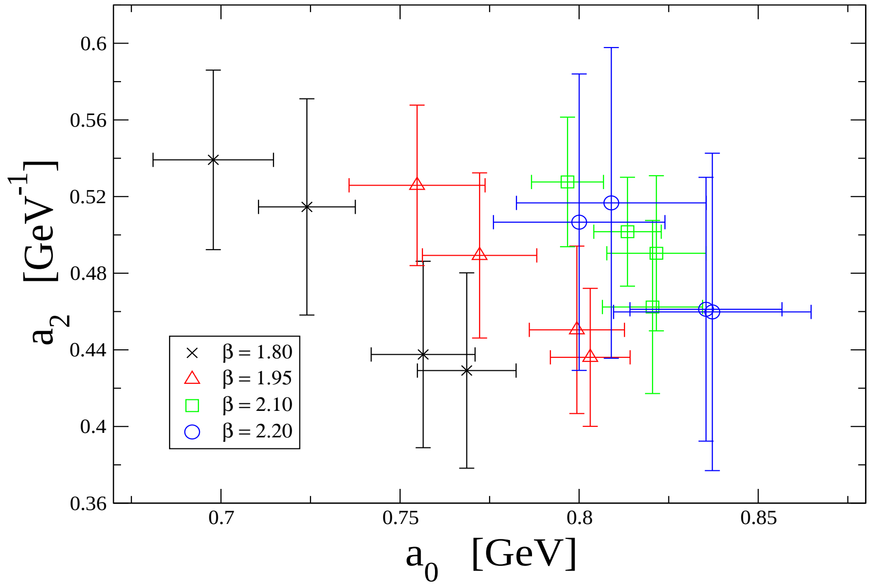

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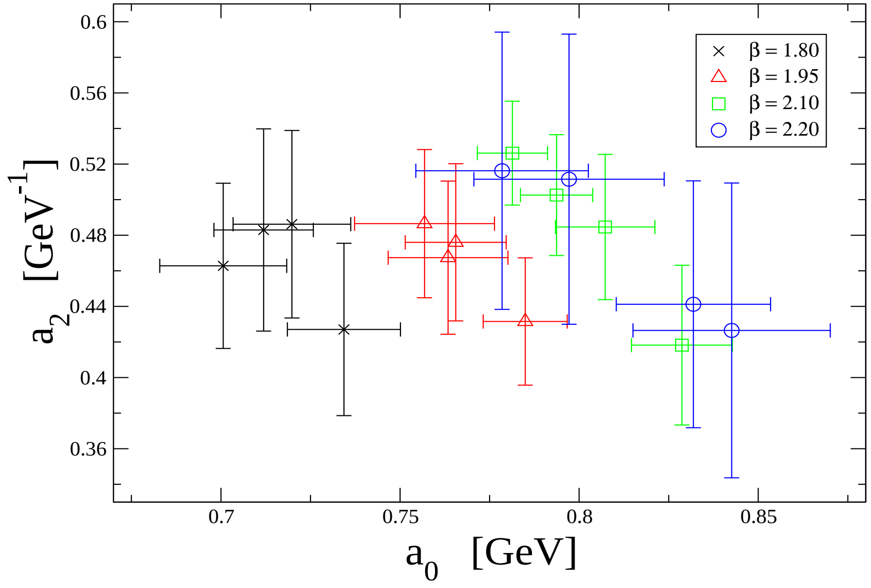

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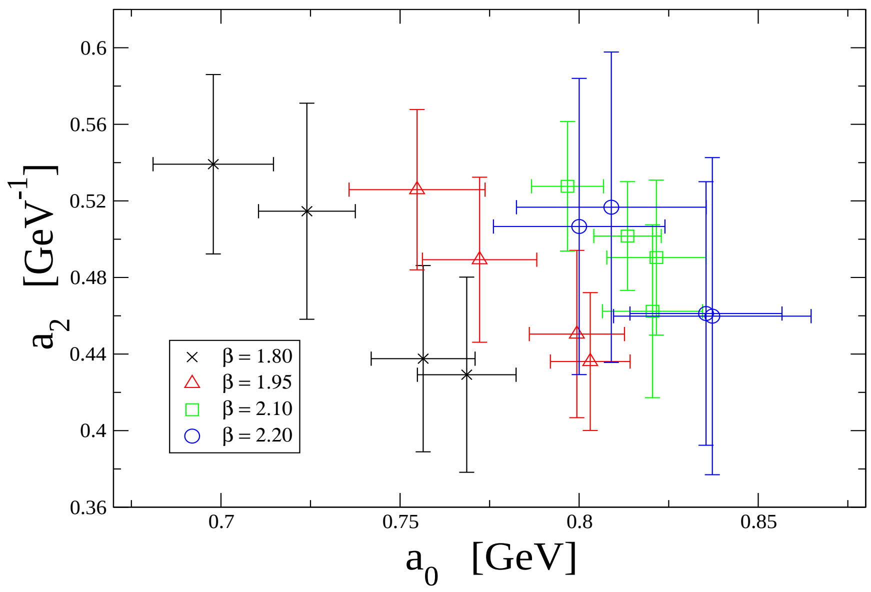

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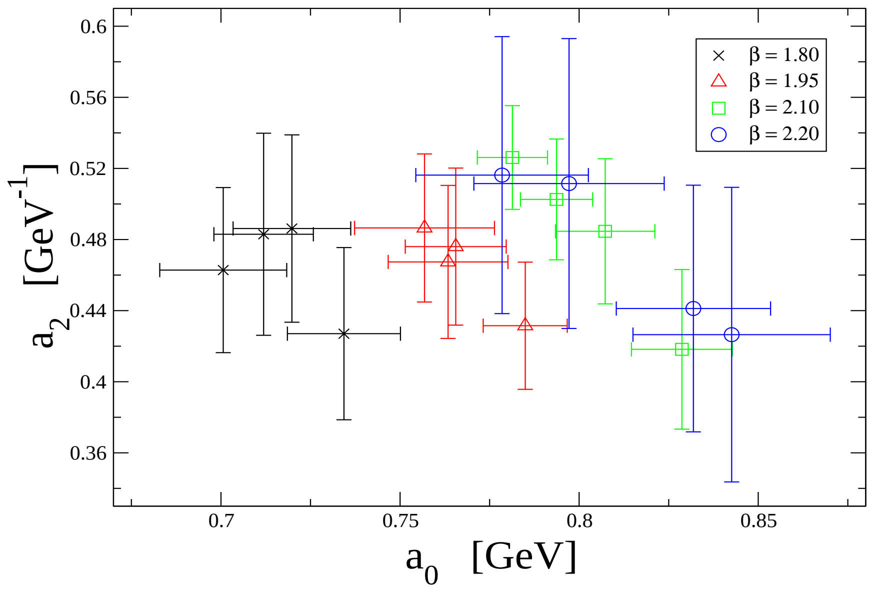
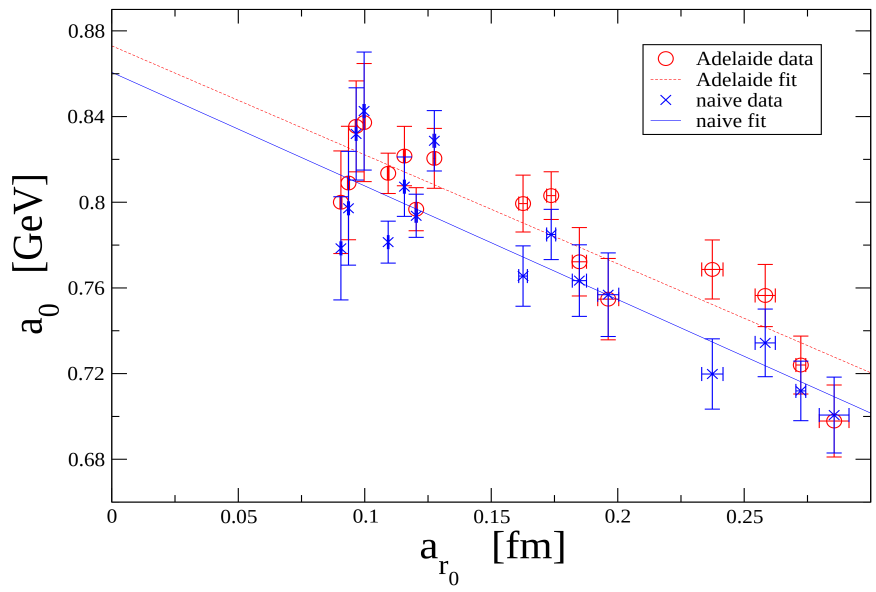


- Naive Polynomial Fit

$\rightarrow$ suggestive of $\mathcal{O}(a^n)$ systematics

Continuum Extrapolation of a_0 and a_2



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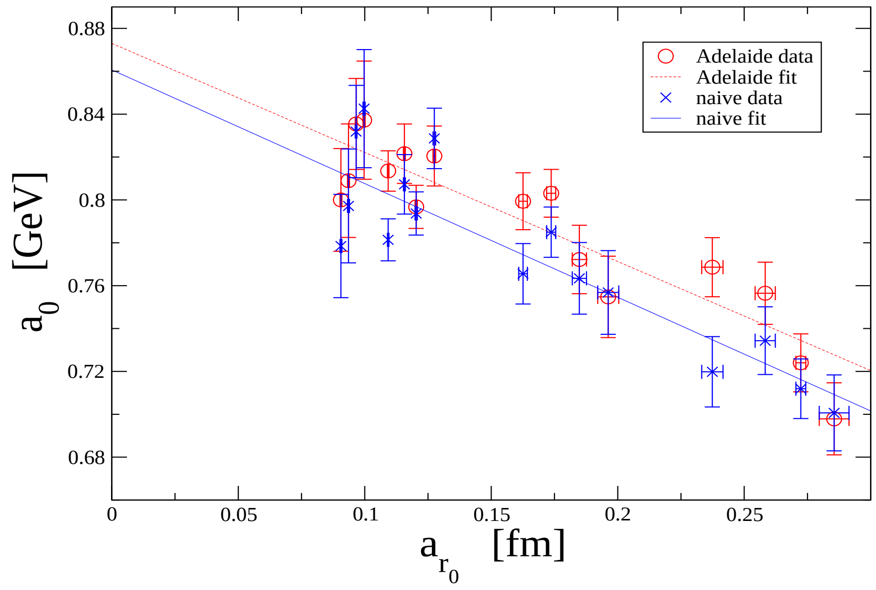

(linear) Continuum extrapolation
for Adelaide and Naive fits

- a_0 continuum extrapolation (significant)

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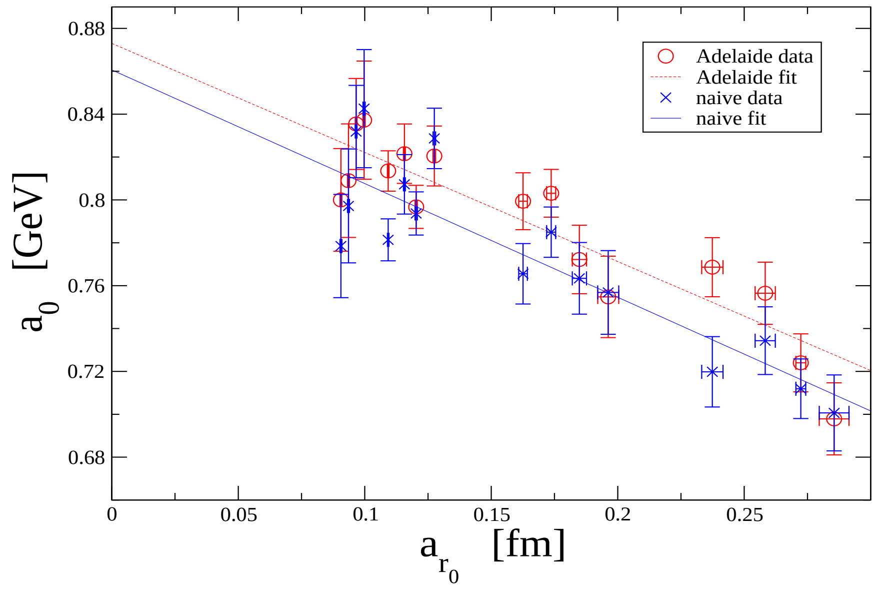

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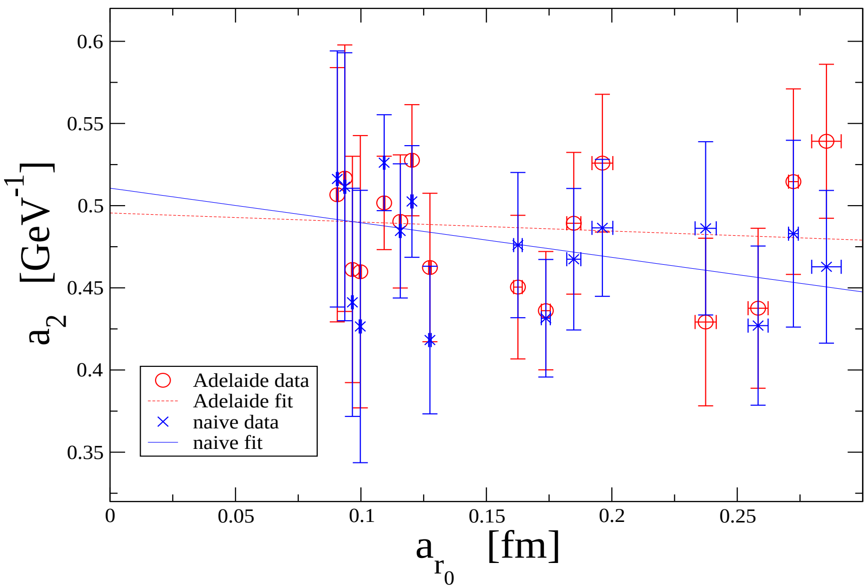

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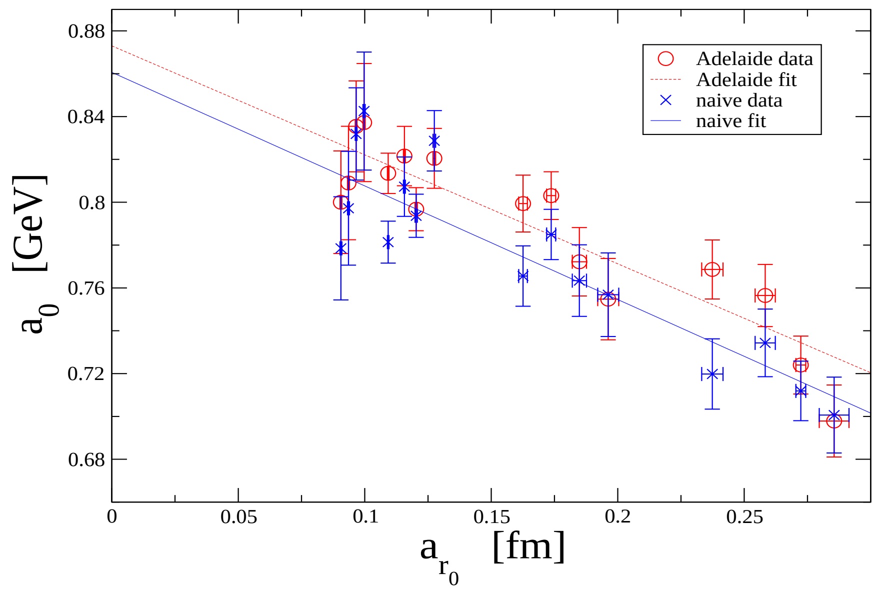

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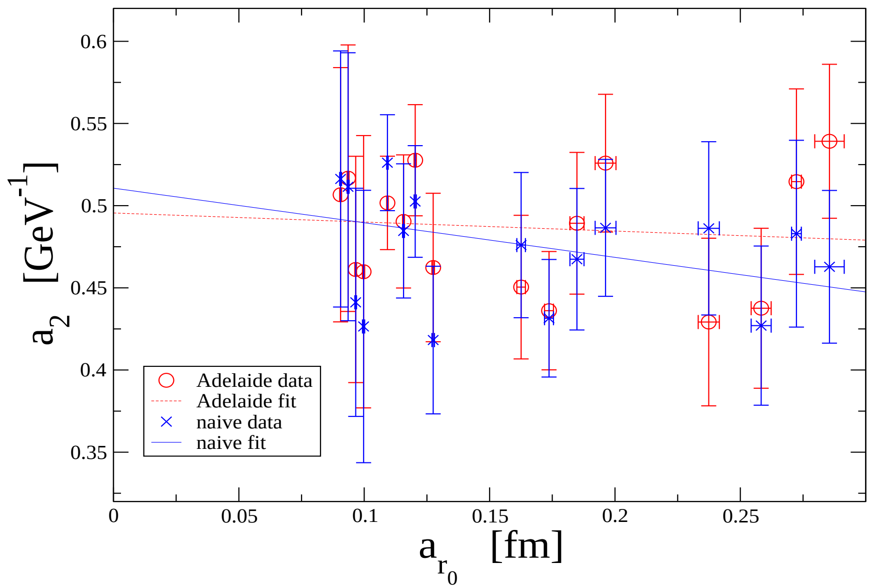
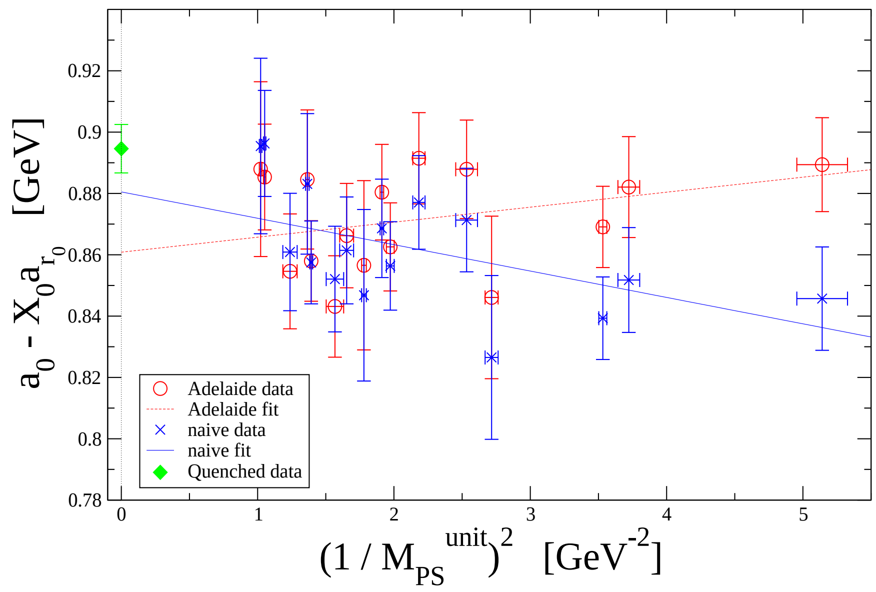


- a_2 continuum extrapolation (not significant)

Chiral extrapolation in m^q_{sea}



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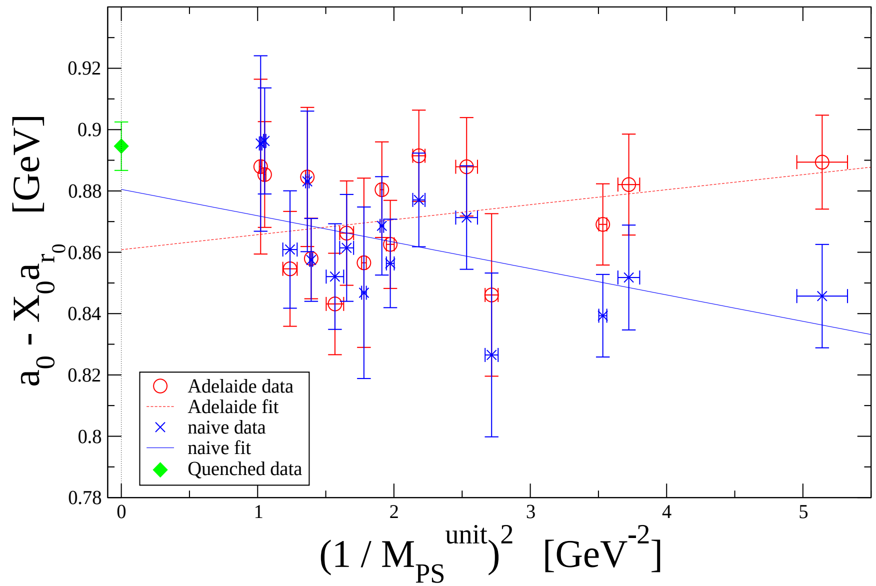

Chiral extrapolation of

$a_0 - X_0 a$ versus $1/m_{sea}^q$

- No sea quark effect observed ...

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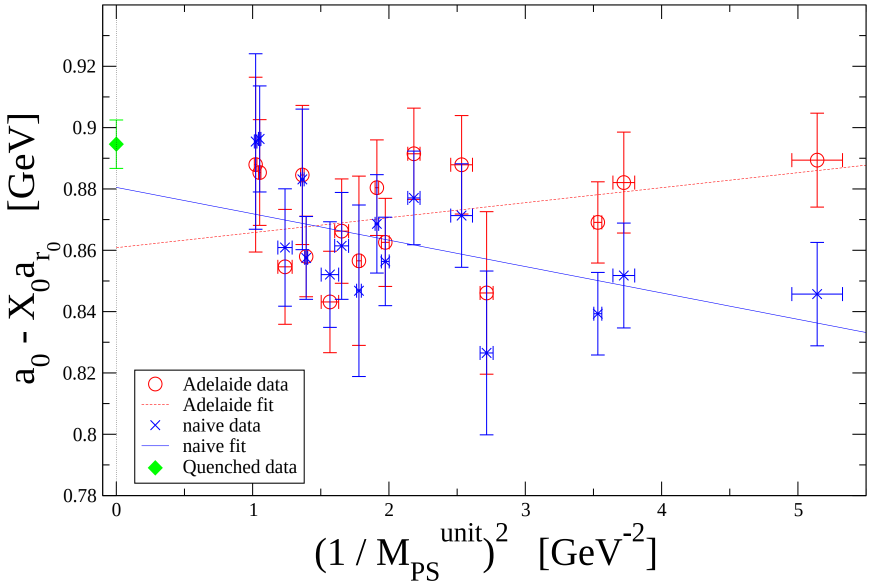

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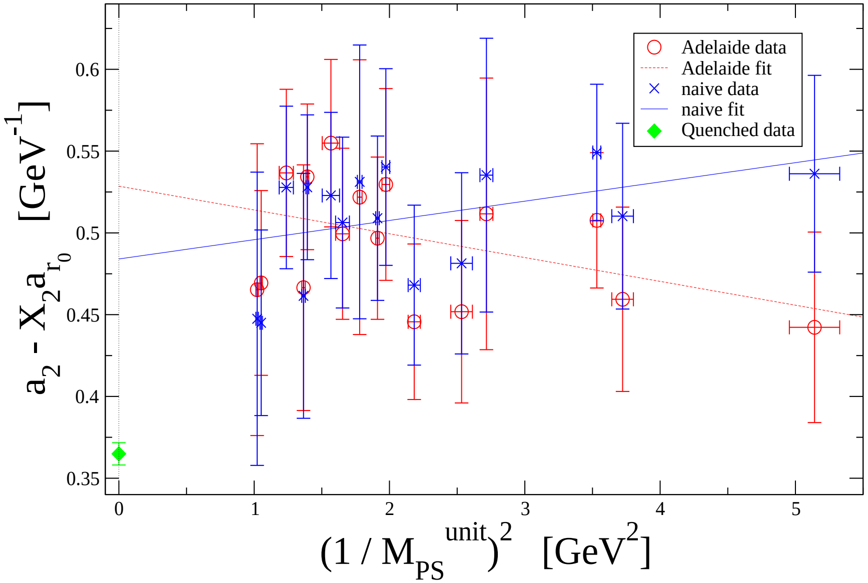

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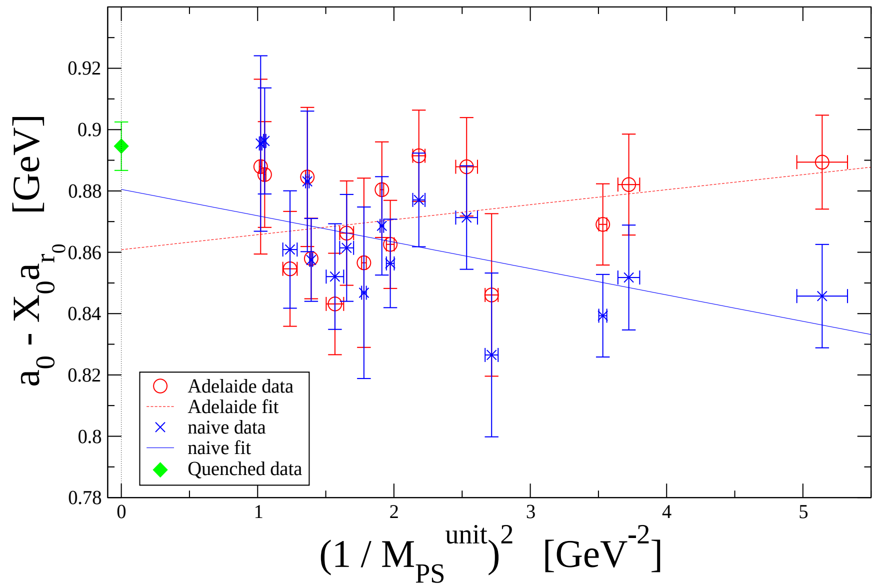

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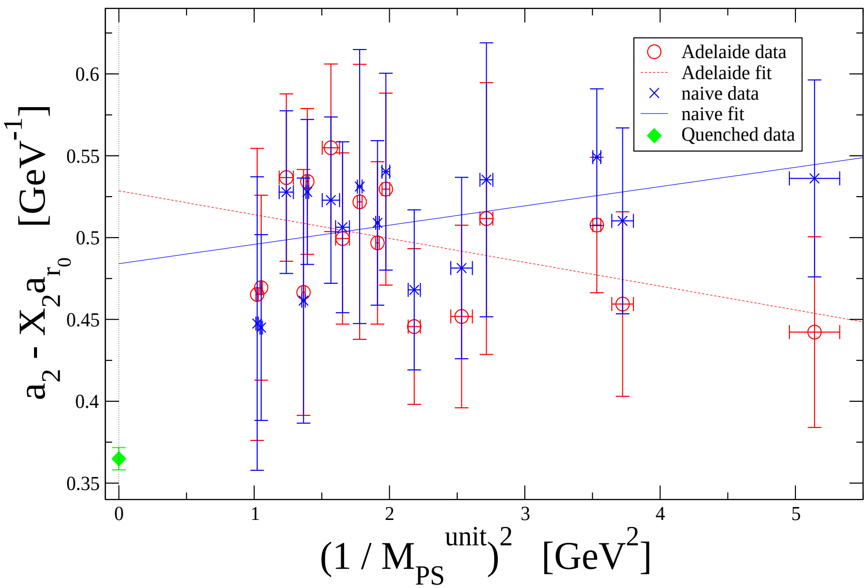

- ditto for $a_2 - X_2 a$

Global analysis

- Individual ensemble fits suggestive of $\mathcal{O}(a^n)$ systematics in a_0 only
  - motivates the fitting form (to $80 = 5 \times 4 \times 4$ data points)

Adelaide:

$$\begin{aligned}\sqrt{M_V(v,v)^2 - \Sigma_{TOT}} &= (a_0^{cont} + X_1 a + X_2 a^2) \\ &+ a_2 M_{PS}(v,v)^2 + a_4 M_{PS}(v,v)^4 + a_6 M_{PS}(v,v)^6\end{aligned}$$

Naive Polynomial:

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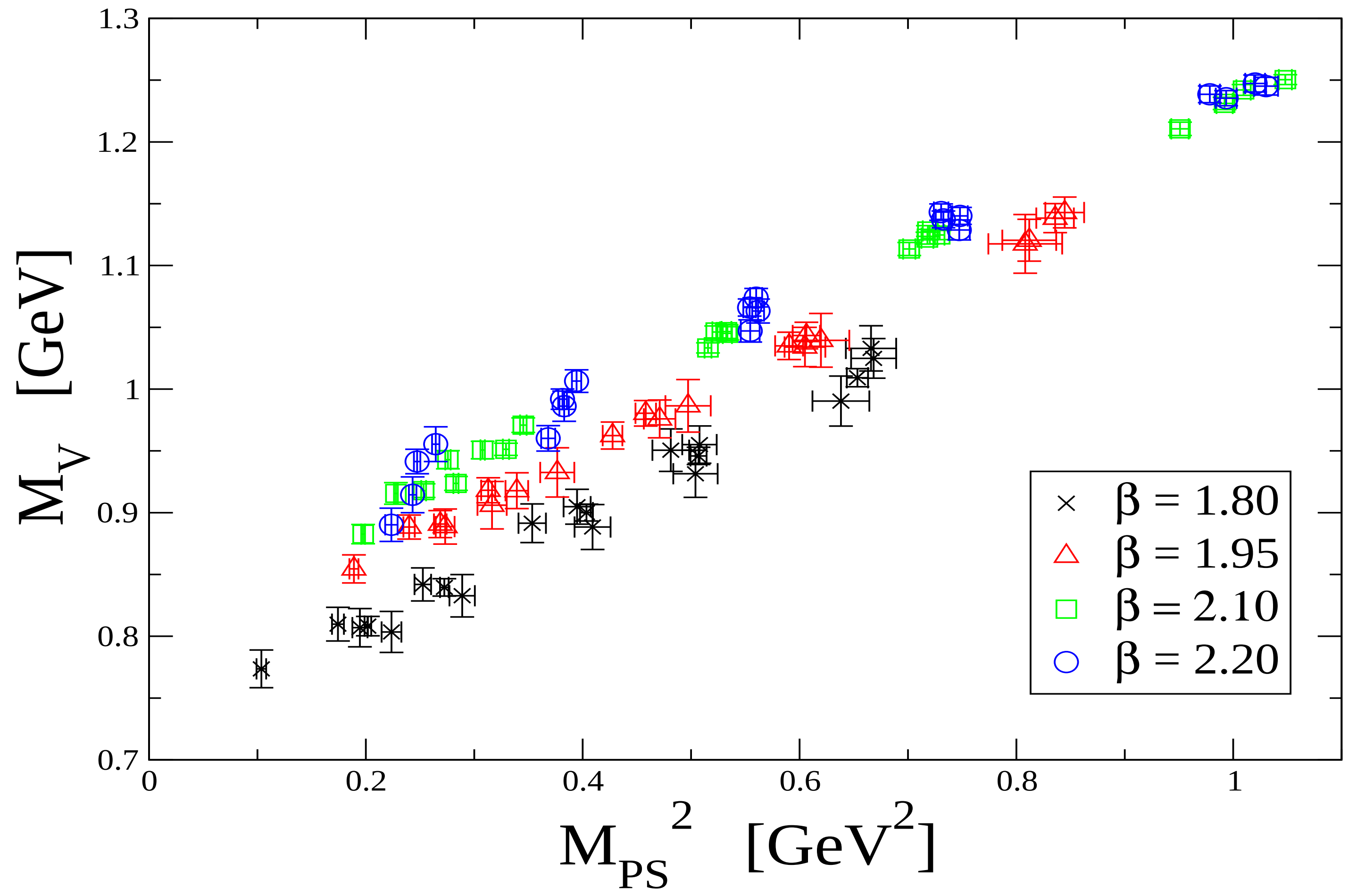
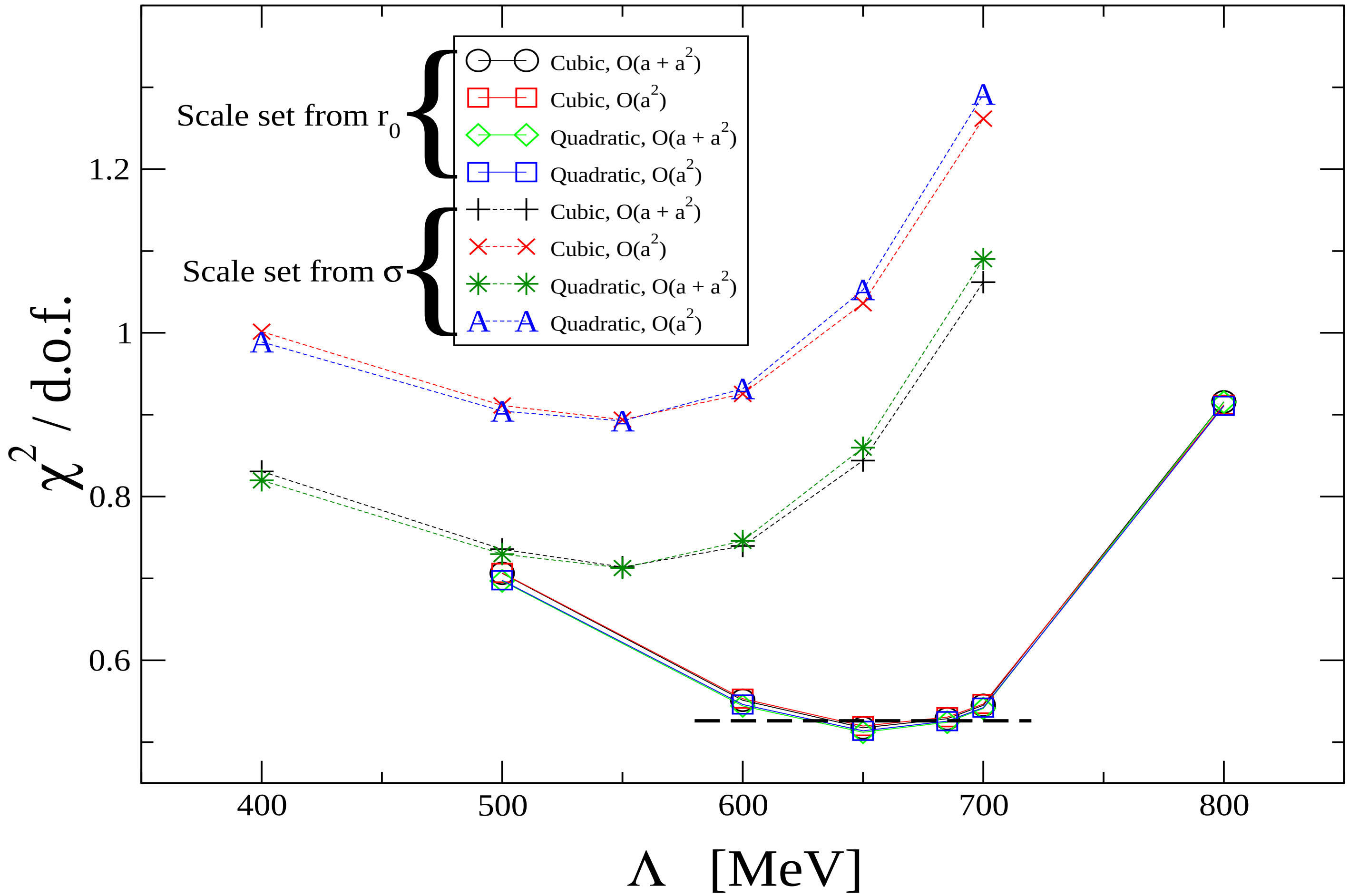


Global fit types

Approach	Chiral Extrapolation	Treatment of Lattice Spacing Artefacts in a_0	Lattice Spacing set from
Adelaide	Cubic i.e. $\mathcal{O}(M_{PS}^6)$ included	a_0 term has $\mathcal{O}(a + a^2)$ corrections	r_0
Naive	Quadratic i.e. no $\mathcal{O}(M_{PS}^6)$ term	a_0 term has only $\mathcal{O}(a^2)$ corrections	σ

Determining Λ

Recall Adelaide approach introduces Λ parameter



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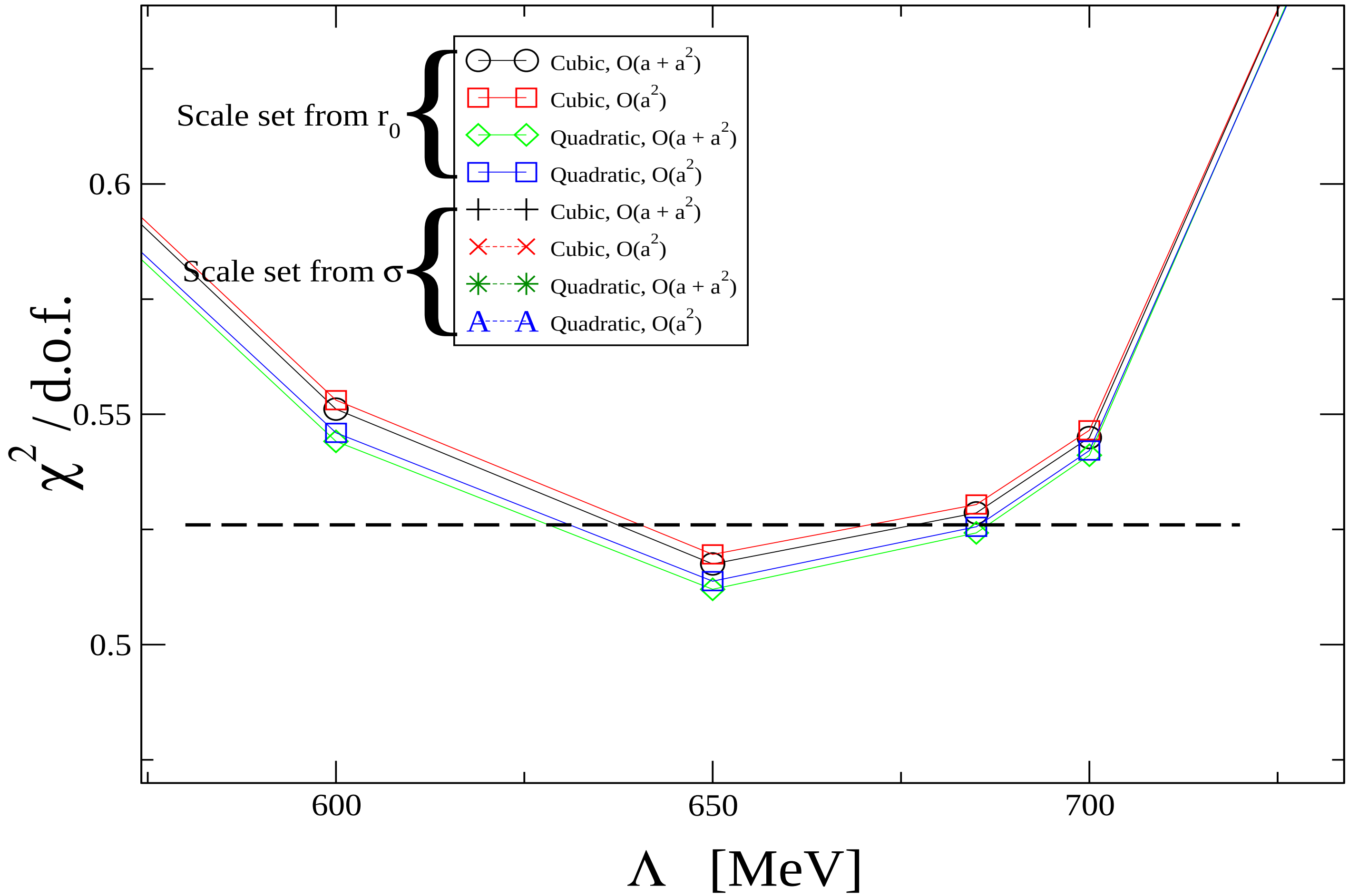

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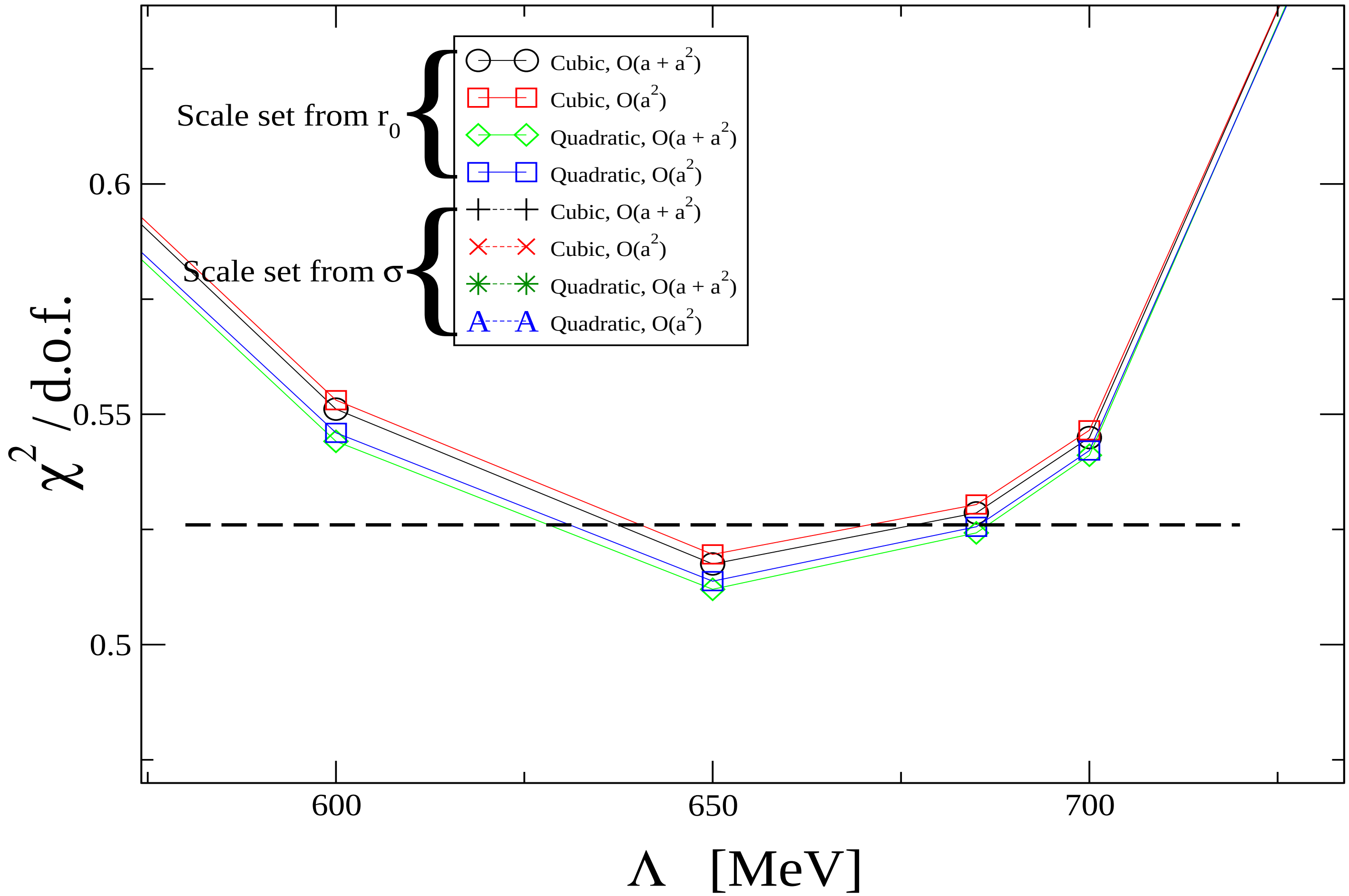

$\rightarrow \quad \Lambda = 650$ MeV

Global fit results

Fit Approach	Scale from	a_0^{cont} [GeV]	X_1 [GeV/fm]	X_2 [GeV/fm^2]	a_2 [1/GeV]	a_4 [1/GeV^3]	a_6 [1/GeV^5]	$\chi^2/d.o.f.$
		Cubic chiral extrapolation			a_0 contains $\mathcal{O}(a+a^2)$			
Adelaide	r_0	0.844±16	-0.11±13	-1.1±4	0.47±4	-0.02±10	-0.02±4	38 / 74
Adelaide	σ	0.836±11	-0.37±9	-0.2±3	0.44±4	0.04±9	-0.06±4	53 / 74
Naive	r_0	0.819±17	-0.15±13	-1.1±4	0.56±5	-0.16±10	0.05±4	77 / 74
Naive	σ	0.805±12	-0.38±9	-0.3±3	0.57±4	-0.18±10	0.06±5	73 / 74
		Cubic chiral extrapolation			a_0 contains $\mathcal{O}(a^2)$ only			
Adelaide	r_0	0.835±9	-	-1.40±4	0.48±4	-0.03±10	-0.02±4	39 / 75
Adelaide	σ	0.807±8	-	-1.24±3	0.43±4	0.06±9	-0.06±4	67 / 75
Naive	r_0	0.806±10	-	-1.49±4	0.56±5	-0.17±10	0.06±4	78 / 75
Naive	σ	0.775±8	-	-1.31±4	0.56±4	-0.16±10	0.05±5	87 / 75
		Quadratic chiral extrapolation			a_0 contains $\mathcal{O}(a+a^2)$			
Adelaide	r_0	0.840±12	-0.11±13	-1.1±4	0.493±11	-0.061±9	-	38 / 75
Adelaide	σ	0.829±9	-0.37±9	-0.2±3	0.490±11	-0.052±11	-	54 / 75
Naive	r_0	0.828±13	-0.16±13	-1.1±4	0.505±11	-0.068±10	-	78 / 75
Naive	σ	0.812±9	-0.37±9	-0.3±3	0.523±12	-0.075±11	-	74 / 75
		Quadratic chiral extrapolation			a_0 contains $\mathcal{O}(a^2)$ only			
Adelaide	r_0	0.832±4	-	-1.40±4	0.494±11	-0.061±9	-	39 / 76
Adelaide	σ	0.799±4	-	-1.23±3	0.486±11	-0.046±11	-	68 / 76
Naive	r_0	0.815±4	-	-1.49±4	0.506±11	-0.068±10	-	79 / 76
Naive	σ	0.781±4	-	-1.31±4	0.520±12	-0.069±11	-	88 / 76

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Method of choice: quadratic chiral extrapolation, scale set from r_0, $\mathcal{O}(a^2)$ corrections in the a_0 coefficient

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M_ρ, M_{K^*}, M_ϕ predictions obtained by setting

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Vector Meson	$M_{PS}(s;v,v)$	$M_{PS}(s;s,v)$	$M_{PS}(s;s,s)$
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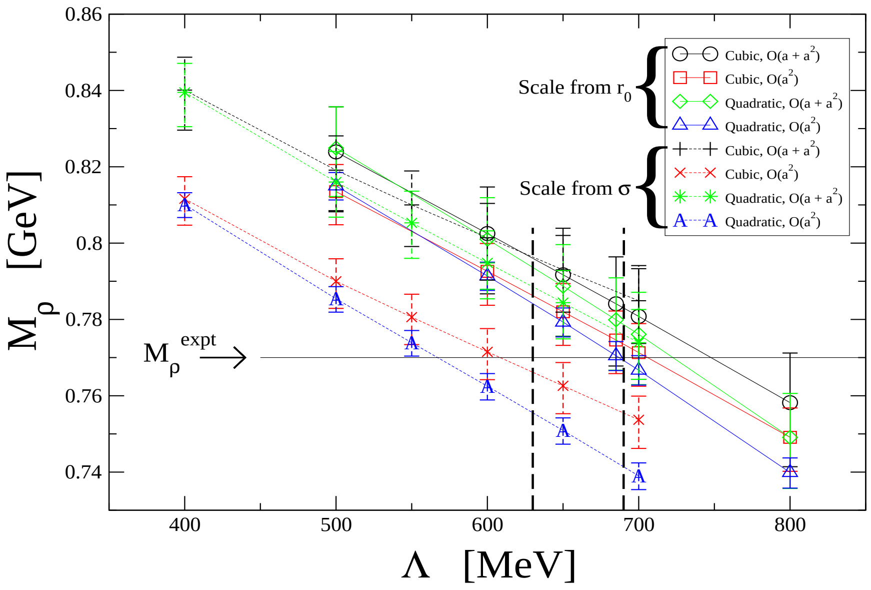
- Note that Σ^ρ_{DHP} vanishes, as required, for the calculation of M_ρ.
- Self energy integrals calculated directly (rather than using the lattice interpretation of the integral)
- Continuum predictions obtained for all 2^4 fitting types

Physical predictions

Source	Procedure	Scale	M_ρ [GeV]	M_{K*} [GeV]	M_ϕ	J^{discrete}
Experiment			0.770	0.892	1.0194	0.487
Quenched	Naive	r_0	0.902± 7	0.984± 7	1.066± 7	0.359± 8
"	Naive	σ	0.861± 8	0.947± 8	1.033± 8	0.361± 9
			Cubic chiral extrapolation		a_0 contains $\mathcal{O}(a + a^2)$	
Dynamical	Adelaide	r_0	0.792± 16	0.889± 13	1.029± 12	0.38± 3
"	Adelaide	σ	0.810± 11	0.886± 9	1.026± 9	0.29± 2
"	Naive	r_0	0.829± 16	0.947± 12	1.051± 12	0.49± 3
"	Naive	σ	0.815± 12	0.936± 9	1.042± 9	0.50± 2
			Cubic chiral extrapolation		a_0 contains $\mathcal{O}(a^2)$ only	
Dynamical	Adelaide	r_0	0.782± 9	0.879± 2	1.0198± 15	0.38± 2
"	Adelaide	σ	0.781± 7	0.853± 2	0.9946± 14	0.27± 2
"	Naive	r_0	0.817± 9	0.935± 2	1.039± 2	0.49± 3
"	Naive	σ	0.786± 7	0.905± 2	1.0109± 15	0.48± 2
			Quadratic chiral extrapolation		a_0 contains $\mathcal{O}(a + a^2)$	
Dynamical	Adelaide	r_0	0.789± 13	0.889± 13	1.029± 12	0.392± 9
"	Adelaide	σ	0.805± 9	0.886± 9	1.026± 9	0.316± 9
"	Naive	r_0	0.837± 13	0.948± 13	1.051± 12	0.462± 10
"	Naive	σ	0.822± 9	0.935± 9	1.041± 9	0.471± 10
			Quadratic chiral extrapolation		a_0 contains $\mathcal{O}(a^2)$ only	
Dynamical	Adelaide	r_0	0.779± 4	0.879± 2	1.0200± 14	0.389± 8
"	Adelaide	σ	0.774± 3	0.853± 2	0.9950± 14	0.299± 7
"	Naive	r_0	0.825± 4	0.935± 2	1.0381± 14	0.456± 8
"	Naive	σ	0.791± 3	0.905± 2	1.0106± 14	0.453± 8

Variation of Mass Prediction with Λ



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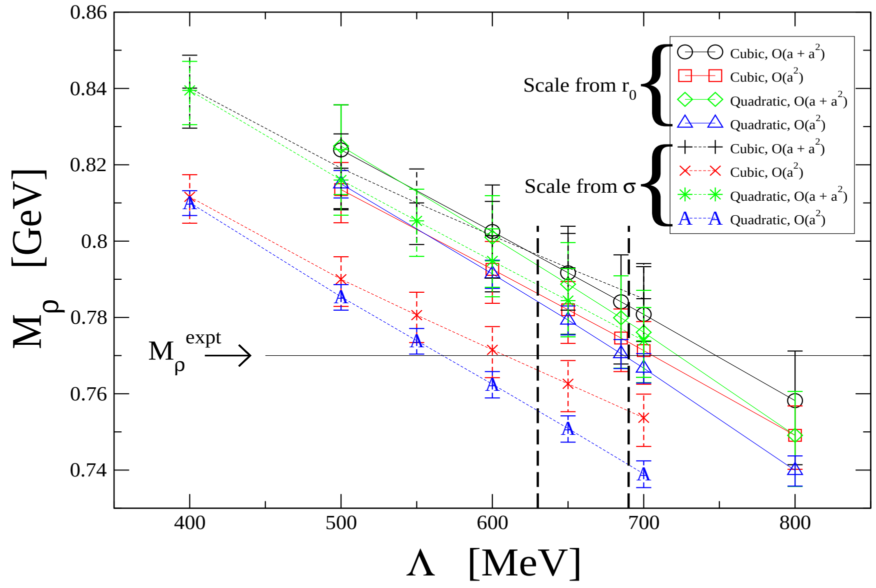

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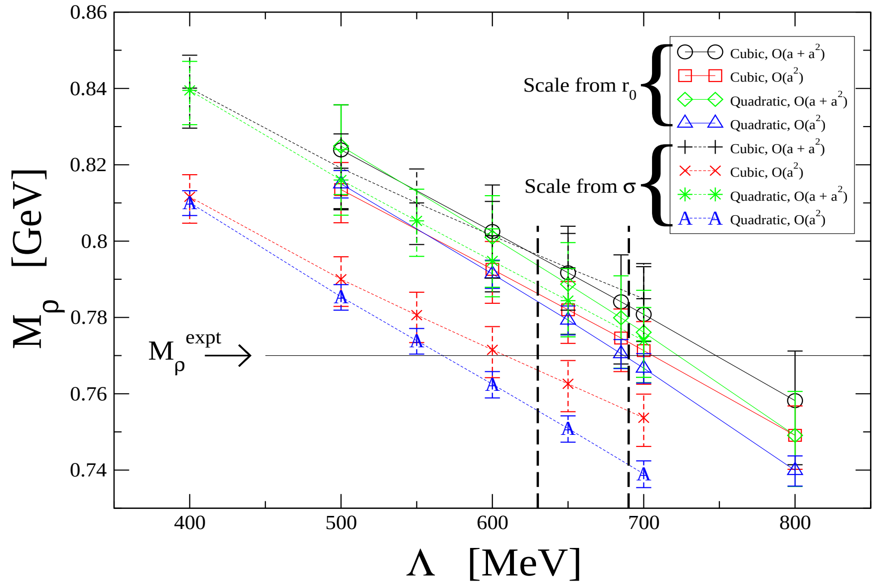

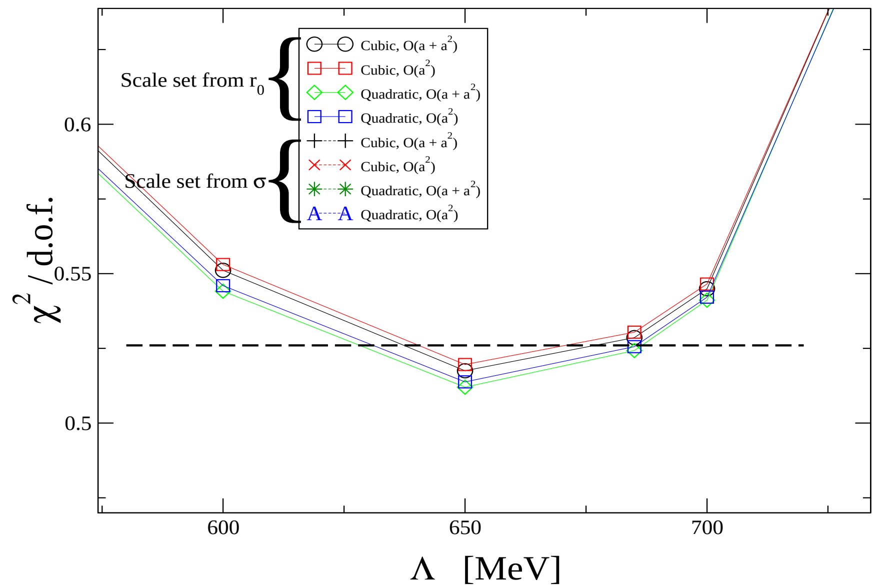

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- The naive polynomial fits have both a larger spread of values and are further from the experimental value than the Adelaide procedure.
- The quenched results significantly overestimate M_ρ.

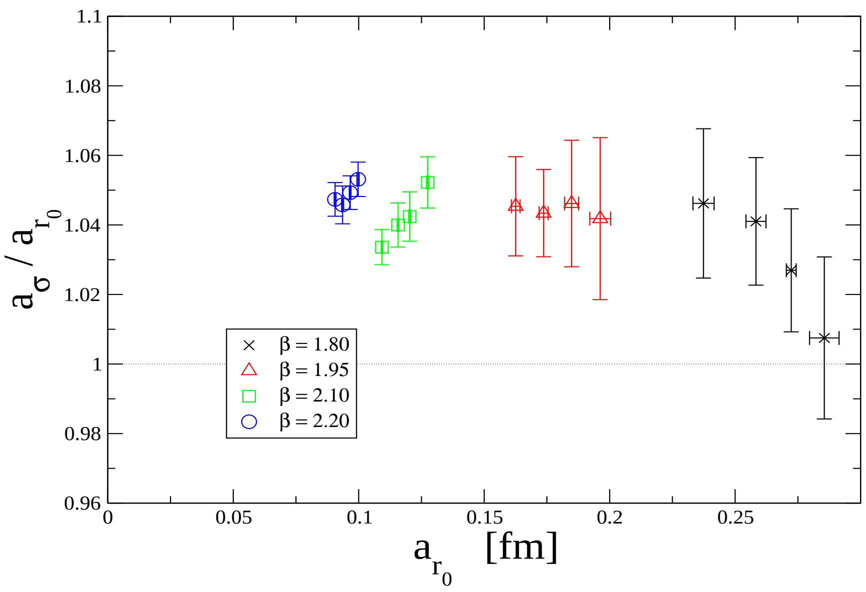
Final estimate of M_ρ

- Final estimates of M_ρ by taking the the quadratic chiral extrapolation, the scale set from r_0, and $\mathcal{O}(a^2)$ corrections in the a_0 coefficient in both the Adelaide and naive fitting procedure.

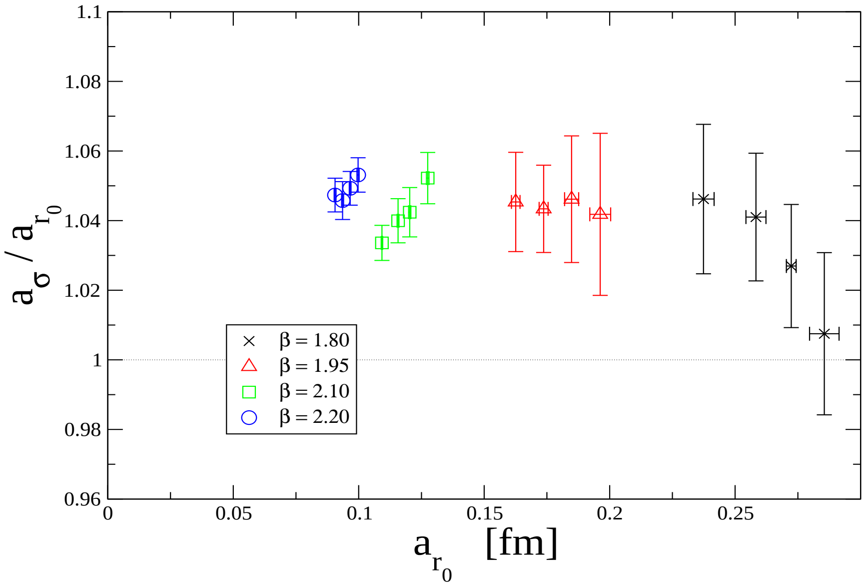
$$
\begin{aligned}
M_\rho^{Adelaide} &= 779(4)^{+13}_{-0}{}^{+5}_{-10}\text{MeV},\\
M_\rho^{Naive} &= 825(4)^{+12}_{-8}\text{MeV},
\end{aligned}
$$

Errors: statistical, fit procedure, Λ (Adelaide case)
No error estimate from r_0 itself, nor finite volume, nor $N_f \neq 2+1$

Setting the Scale

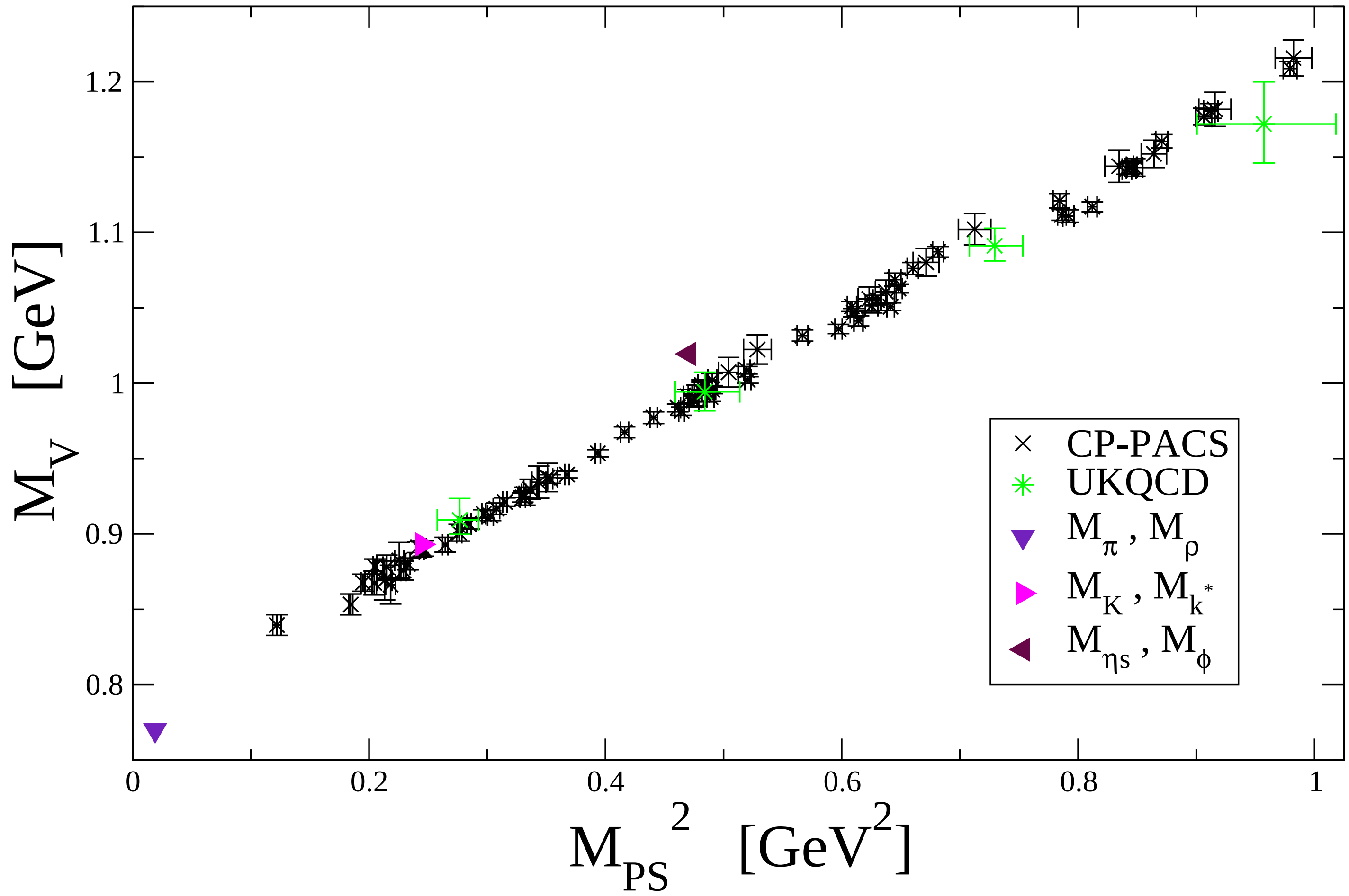


Setting the Scale



- ratio $a_{r0}/a_\sigma \approx$ constant
- little evidence of $\mathcal{O}(a)$ or m_q systematics
- a possible explanation is $\sqrt{\sigma} r_0 = 440$ MeV $\times\ 0.49$ fm is $\approx$ 5 % below true value.

Setting the Scale from a_J



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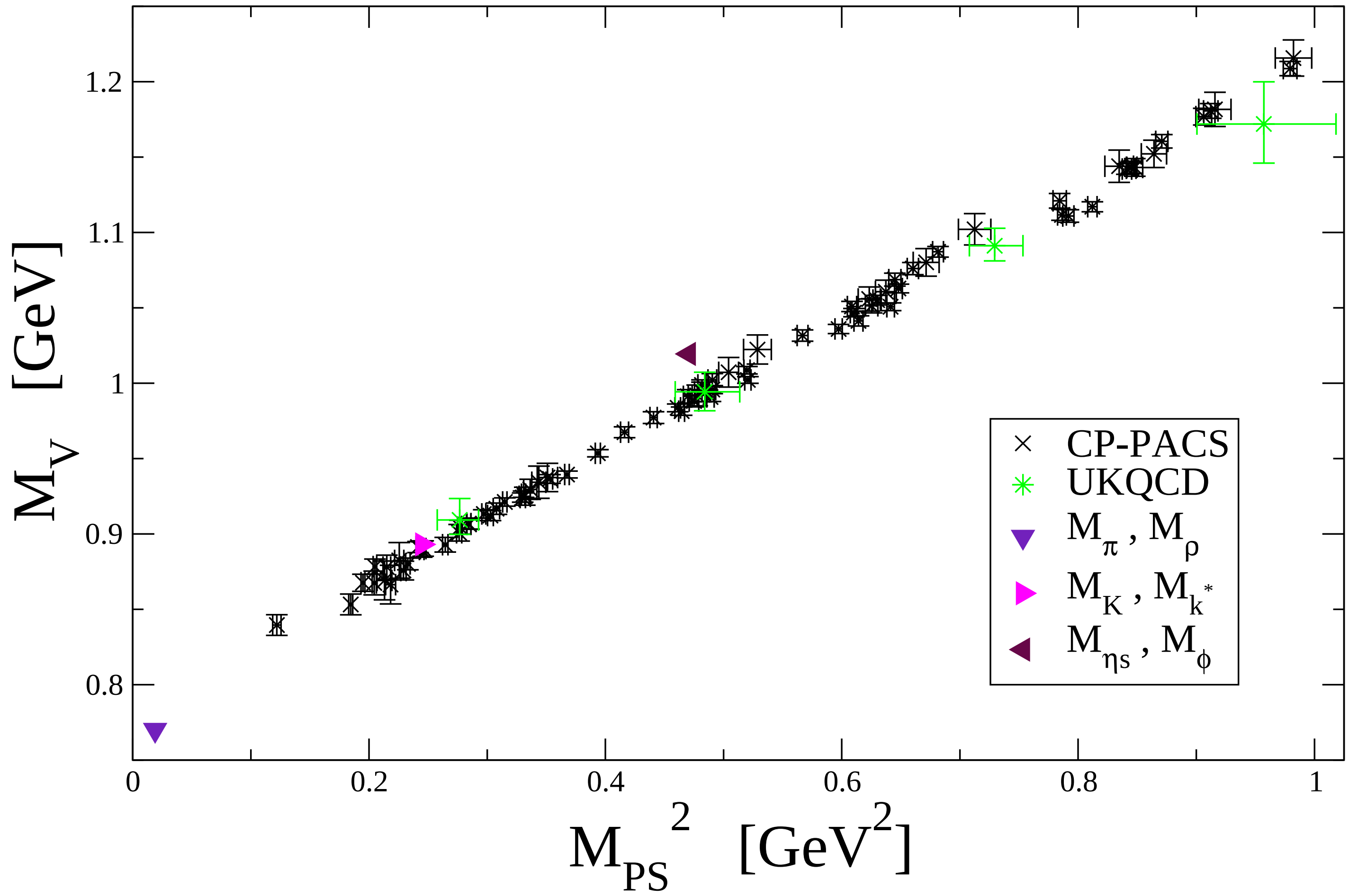
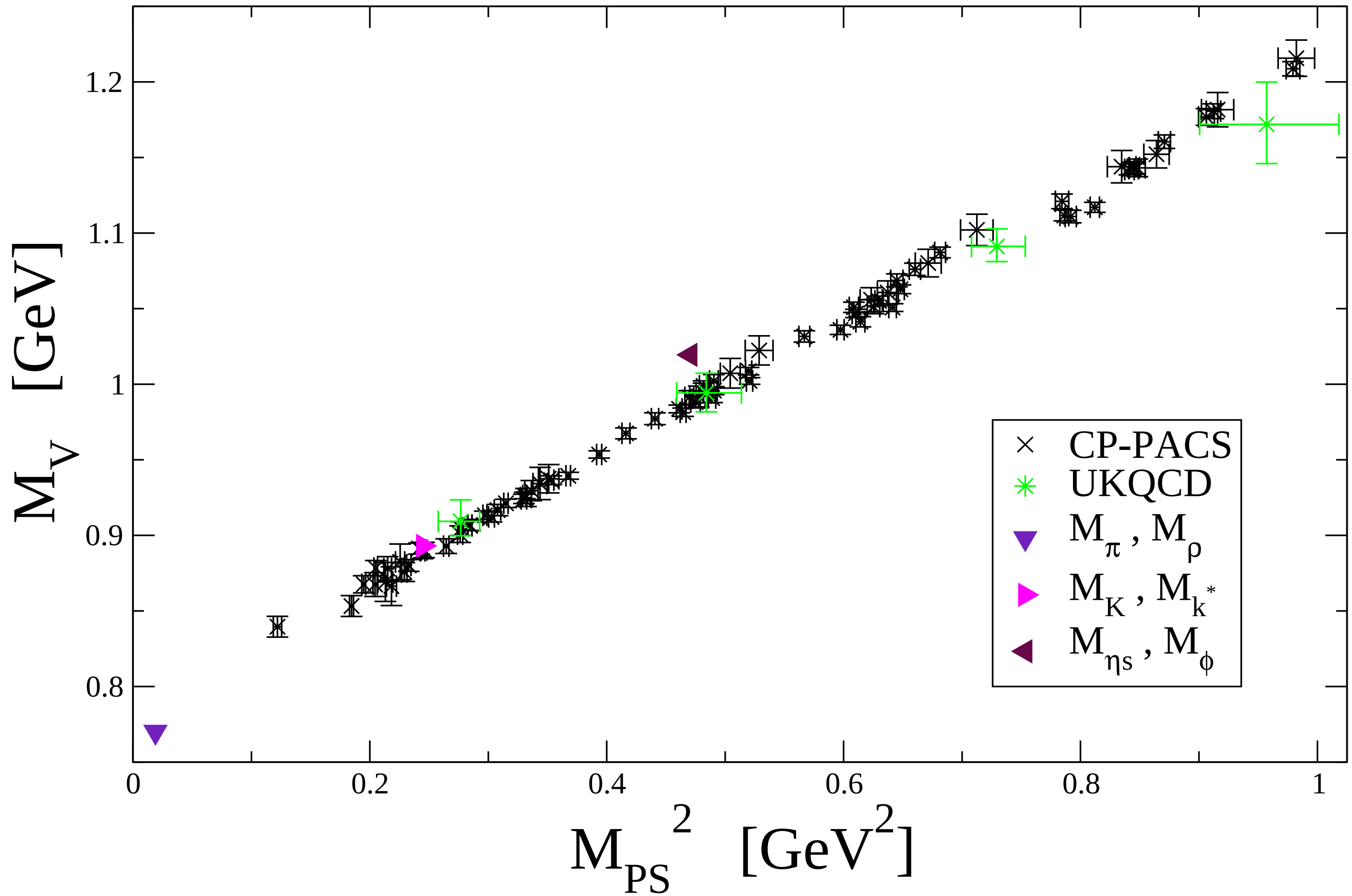


- a_J uses the (K, K^*) mass point (cf J parameter)
- 80 CP-PACS data points plotted and the unitary UKQCD points

Setting the Scale from a_J



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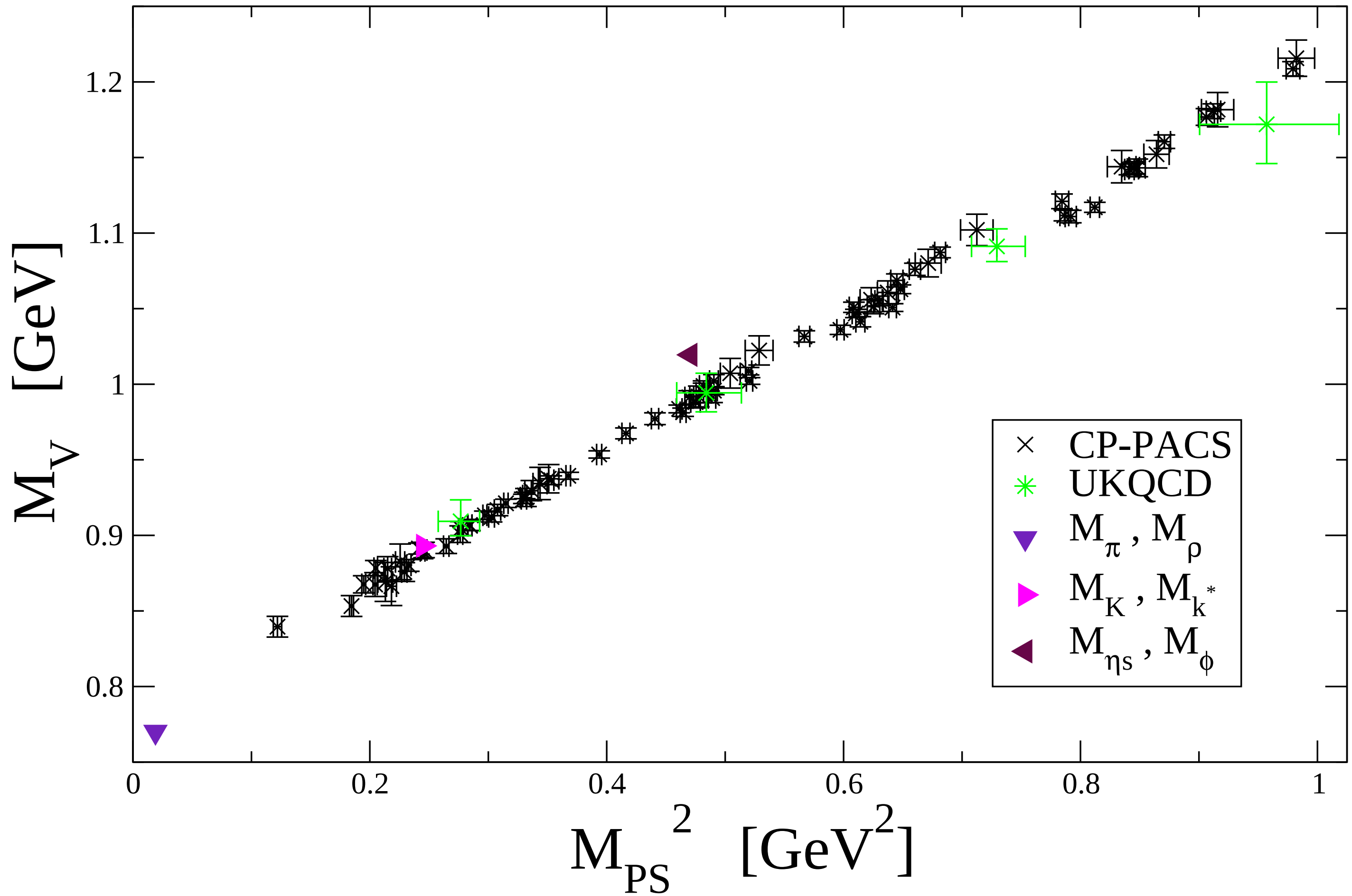
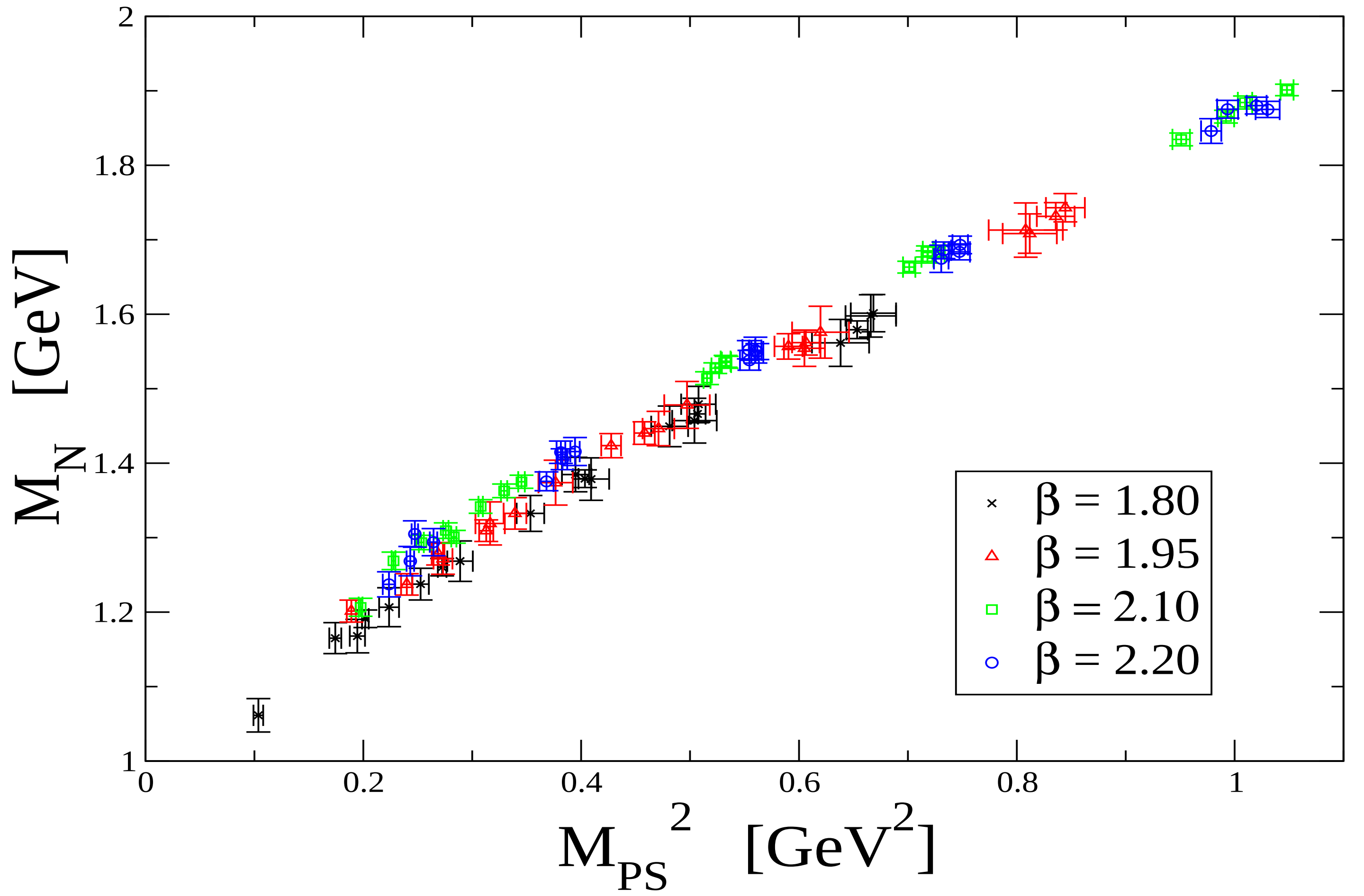


- constrains fit to go through (M_K, M_{K^*}) point
  - $\longrightarrow$ gradient is (only) free parameter

- normalises away a part of Σ_{TOT}
  - $\longrightarrow$ so can't be used in Adelaide method

Nucleon Data



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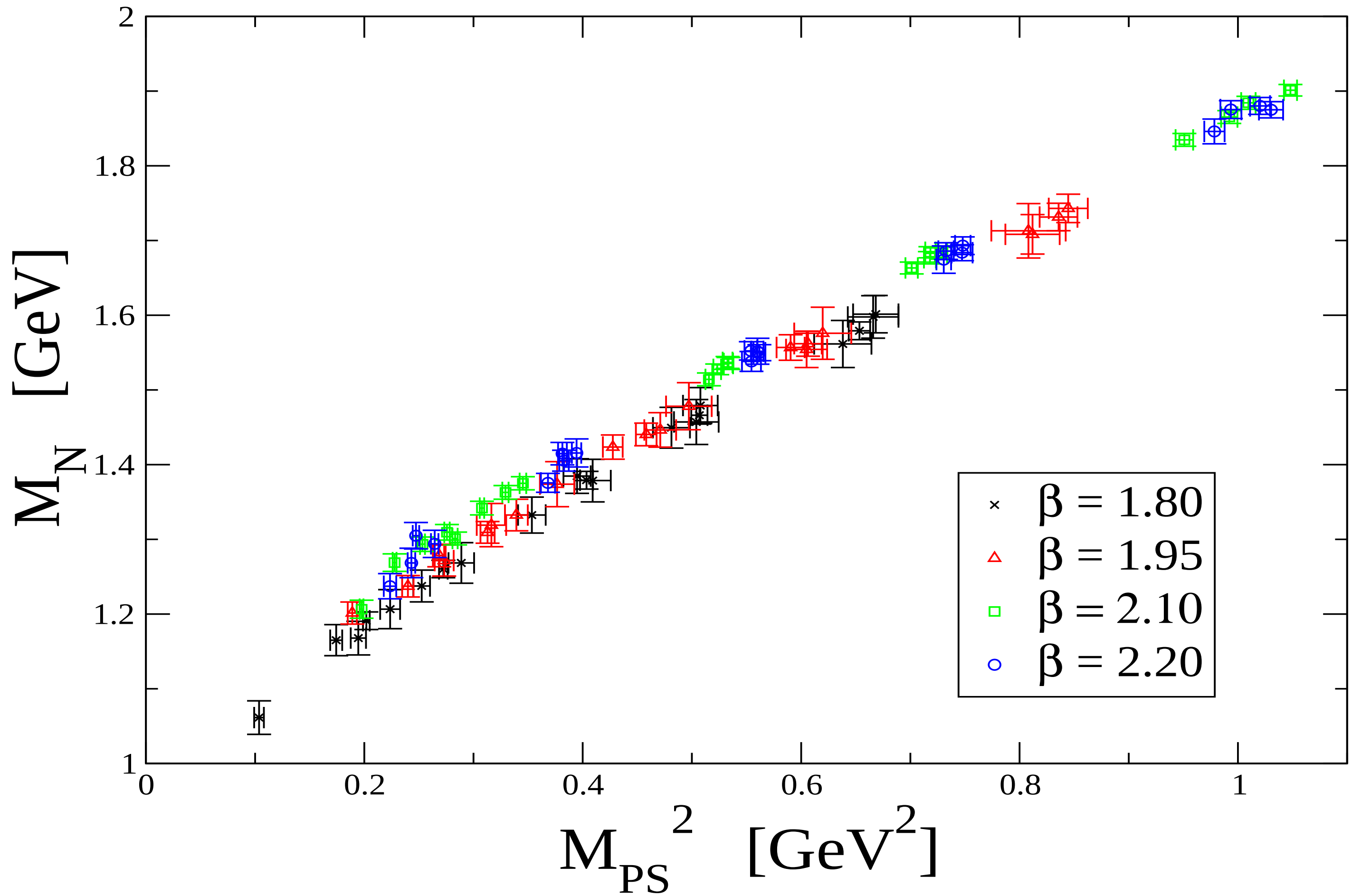
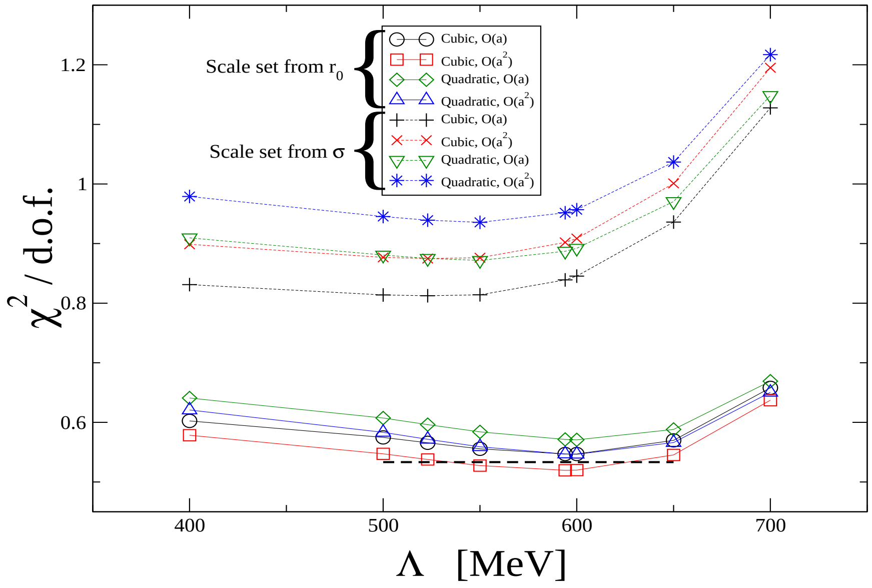


- Same CP-PACS ensembles
- Considered *both* dipole and Gaussian Form Factors

χ^2 variation with Λ and Form Factor



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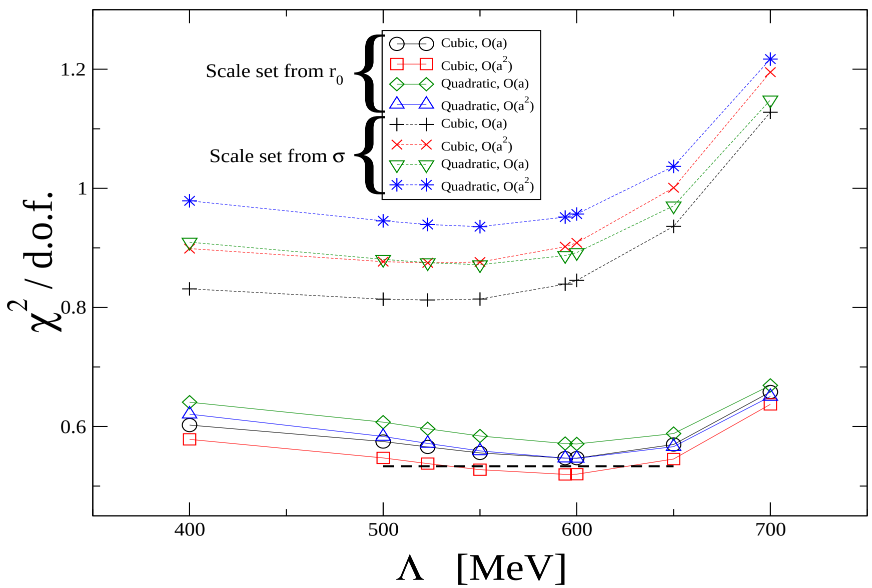

- Dipole Form Factor

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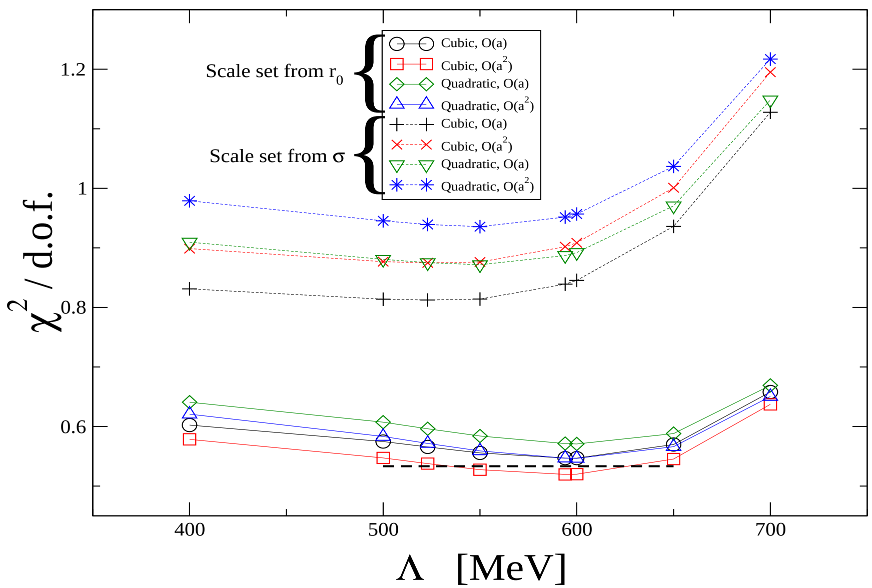

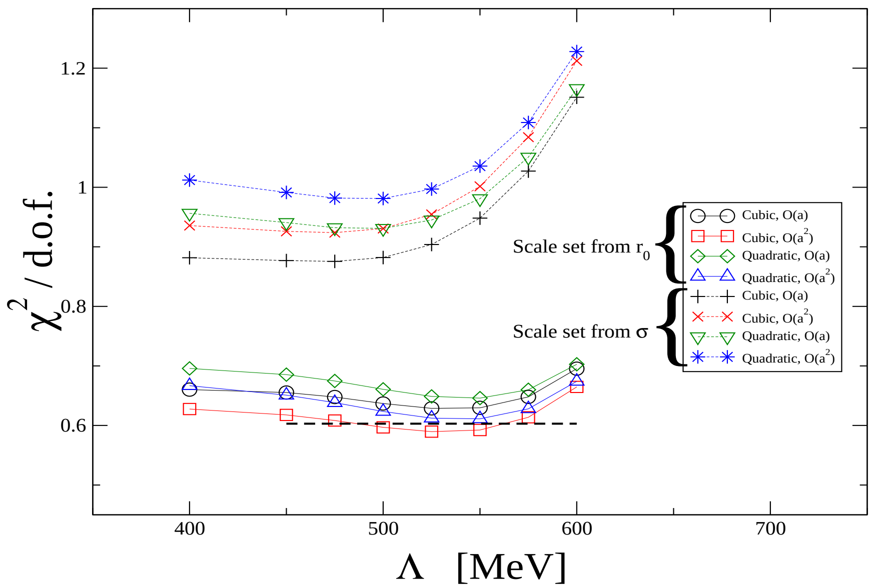

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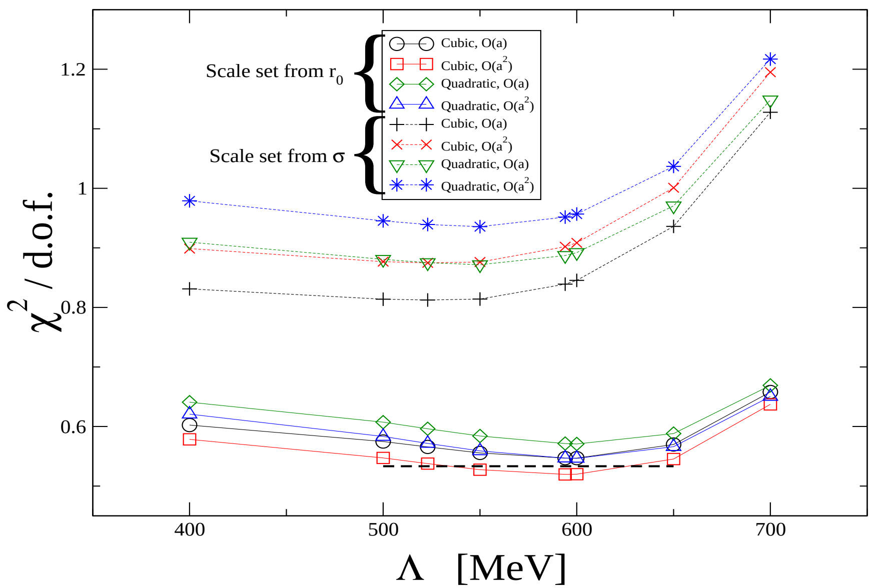

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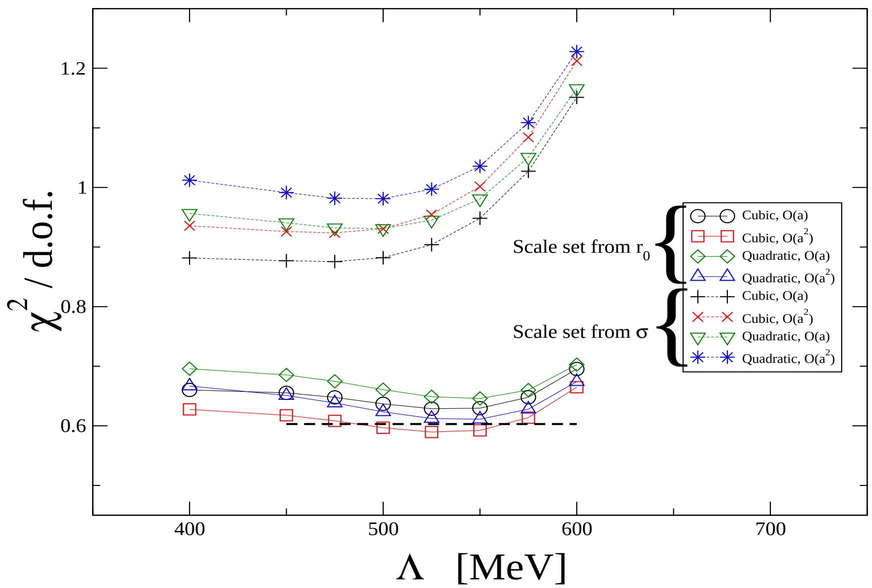
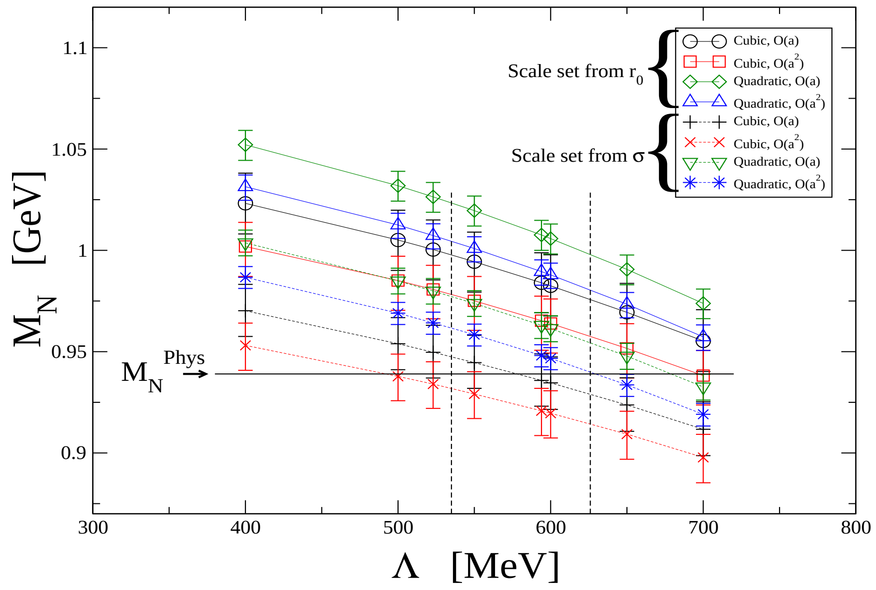


- Gaussian Form Factor

Nucleon Mass variation with Λ and F.F.



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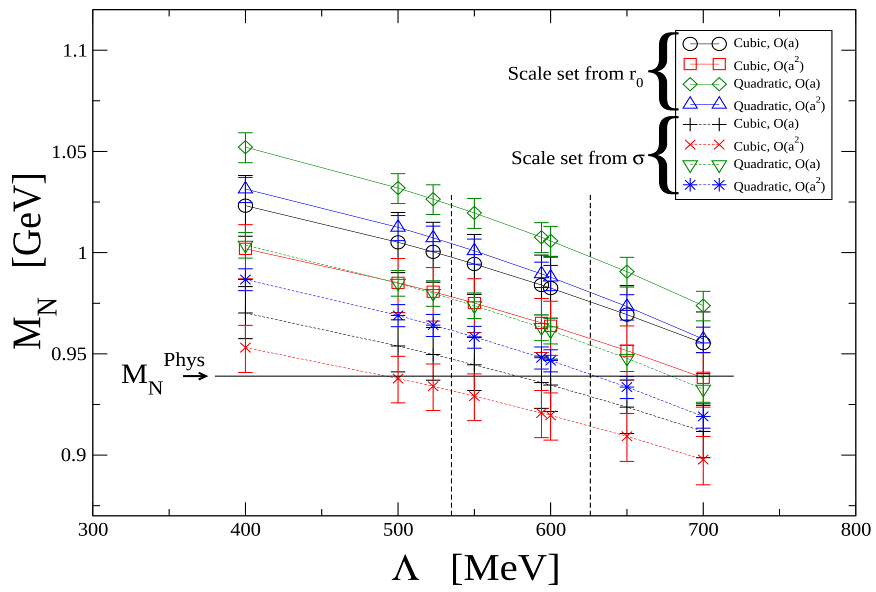

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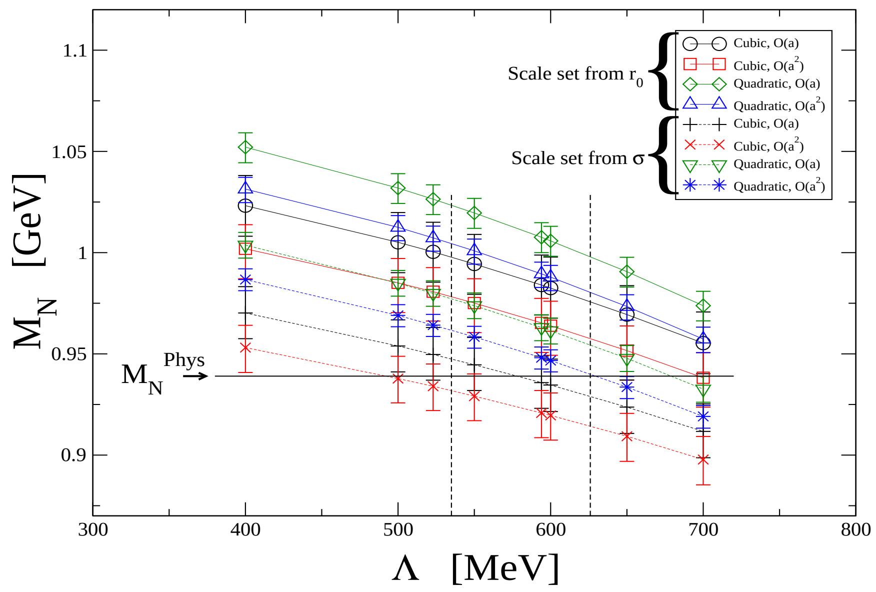

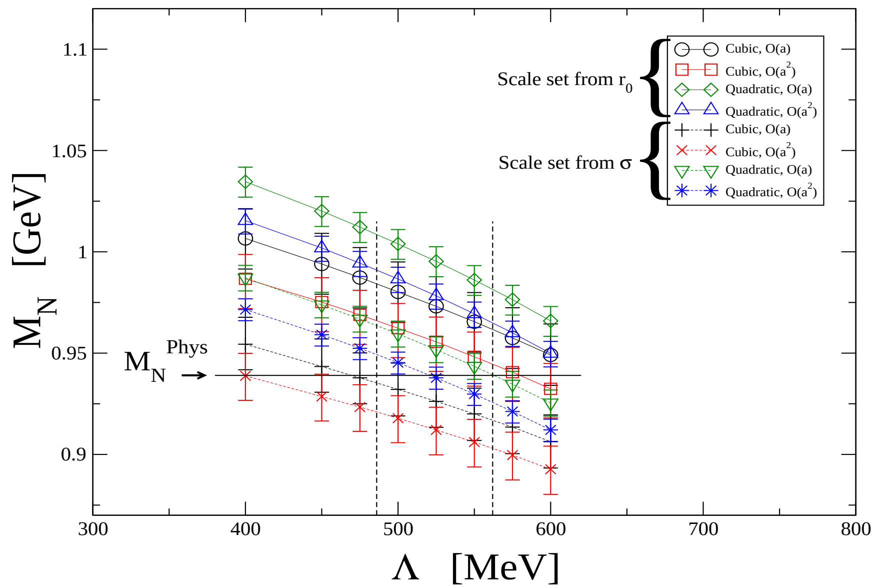

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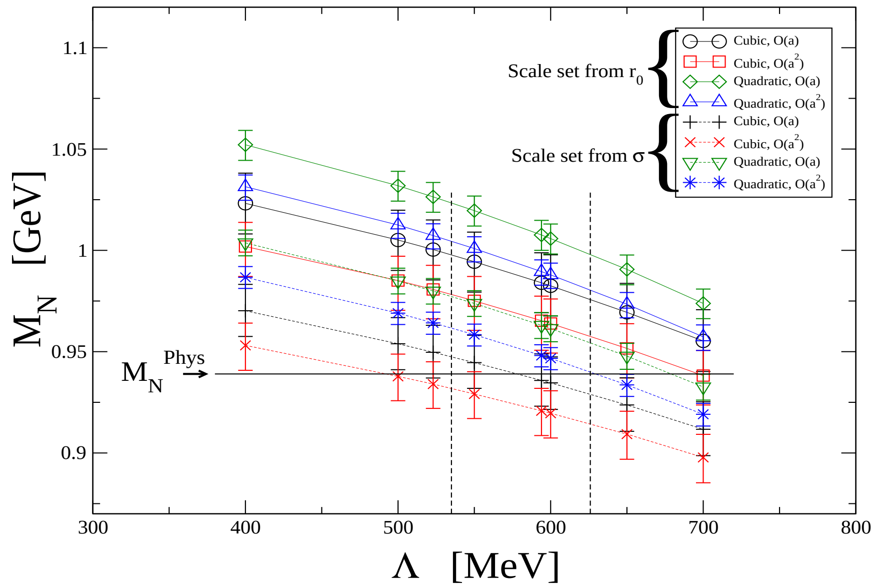

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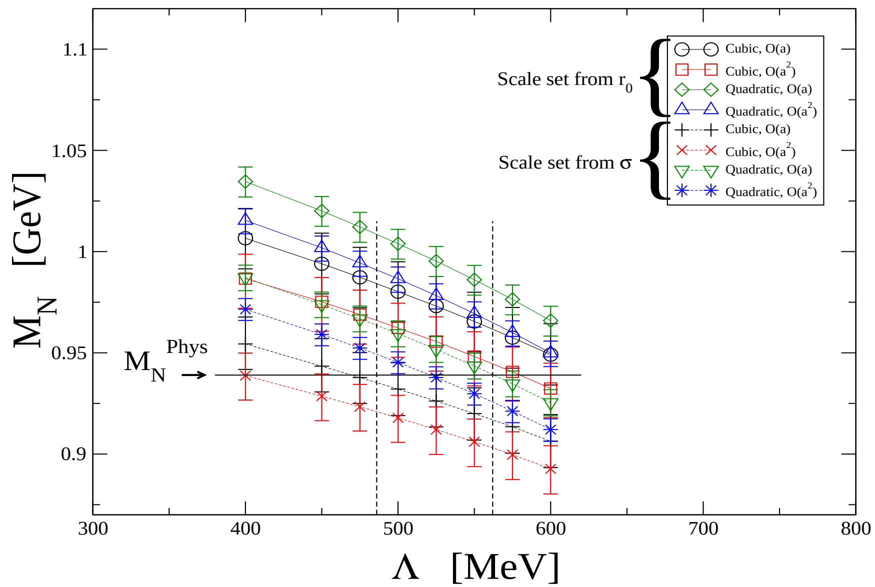

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\begin{aligned}
M_N^{Adelaide} &= 965(15)^{+41}_{-0}{}^{+13}_{-8}\text{[MeV]} \\
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Errors: statistical, fit procedure, Λ (Adelaide case)

Conclusions

- generalised Adelaide chiral ansatz to "pseudo-quenched" case
- obtained continuum estimate of ρ and nucleon masses
- estimated systematic errors in masses from fitting procedure (both chiral and continuum fitting procedure).

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- found Adelaide approach valid ("model independent")
- We haven't modelled finite-size effects
- $N_f = 2 \neq 2 + 1$

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