The Hadron Properties on the Lattice

4^{th} ILFTN Workshop

"Lattice QCD via International Reseach Network"

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http://www.ccs.tsukuba.ac.jp/workshop/ILFT05/

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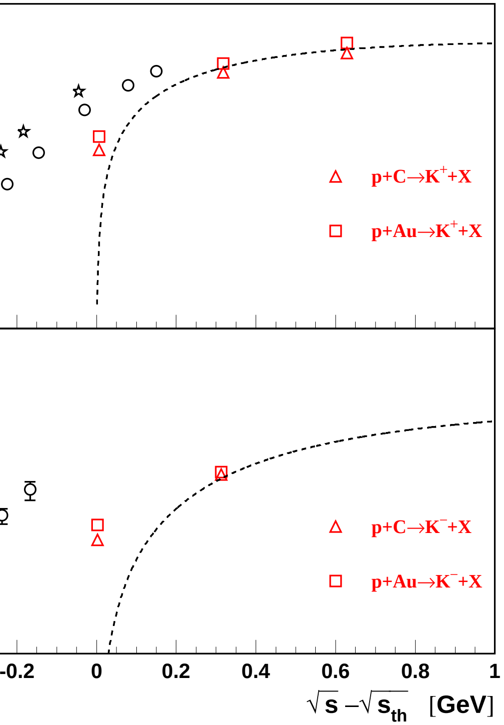
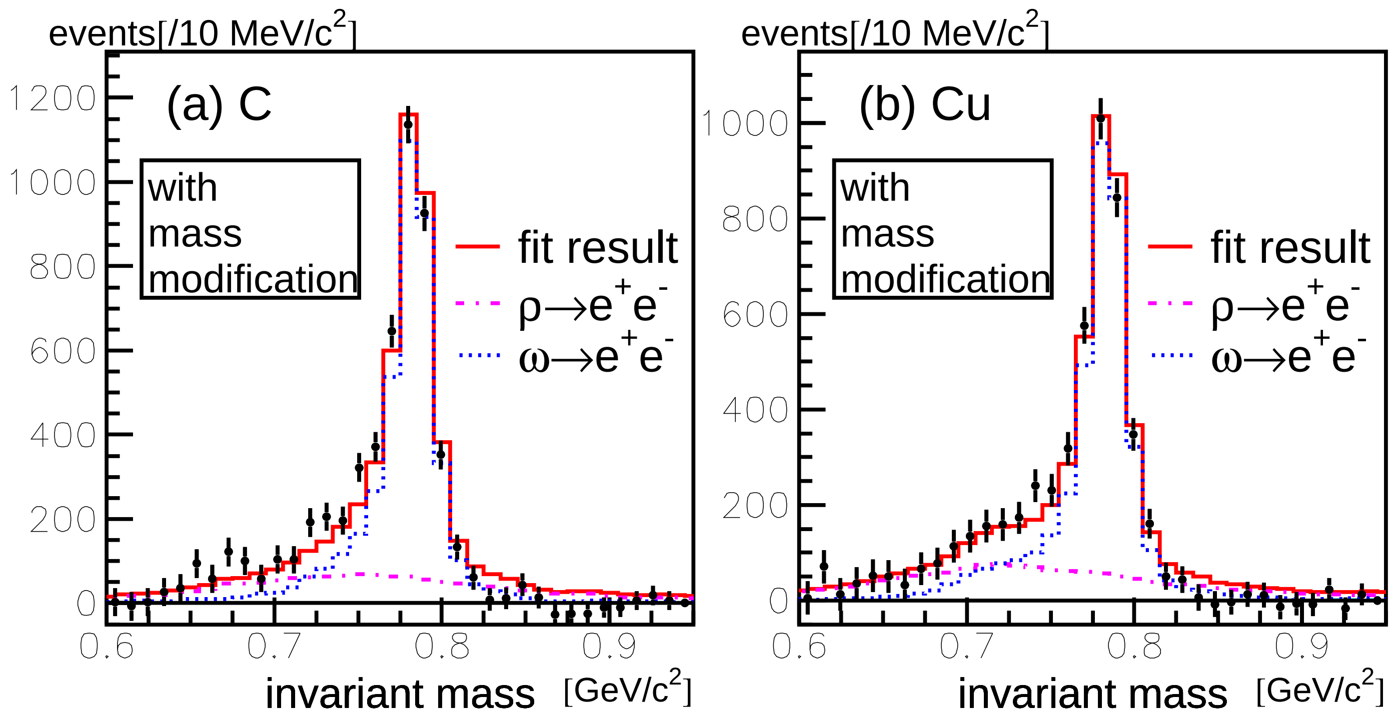
Outline

- The importance of in-medium modifications
- The QCD phase diagram
- Lattice QCD at finite density
- Two-point correlation functions
- Decay constants
- Hadron masses
- Debye screening masses
- Wave functions
- Summery

Heavy-ion collisions





Invariant mass spectrum in two different targets. The access can only be explained by assuming a mass reduction in the medium.

- Starting from effective (Skyrme) Lagrangian which is based on spontaneously broken χ-symmetry. It describes well the low-energy and low-density hadron interactions.
- Putting hadrons in the medium $\equiv$ changing the vacuum, modifying $< qq >$ and $< GG >$ condensates and fi nally changing the relevant scale.
- In the free-space, the size and mass of baryons are given as $< r_N^2 > \sim g_A/f_\pi^2$, $m_N \sim g_A^{1/2} f_\pi$
- The in-medium effective condensate reads $< \bar{q}q >^* \; / < \bar{q}q >_0 = (\chi^*/\chi_0)^3$. The effective pion decay constant reads $f_\pi^*/f_\pi = \chi^*/\chi_0$. Here χ is the density.
- **Result:** $m_N^*/m_N \approx m_V^*/m_V \approx f_\pi^*/f_\pi$
- **Result:** Both parametric and pole m_ρ smoothly goes to zero, i.e., m_ρ can be used as order parameter for χ-symmetry restoration

A.T., D.Toublan, Phys.Lett.B623, 48 (2005)

In vacuum, $| < \bar{q}q > | \sim (240MeV)^3 \approx 1.8$ nucleons/fm^3, $n_0 = 0.16$ nucleons/fm^3

$< G^2 > \sim (850 \text{ MeV})^4 \approx 70\ m_N/\text{fm}^3$ vacuum is strongly correlated

$$< \bar{q}q > = \frac{\partial p_v}{\partial m_q}, \qquad < G^2 > = \frac{32\pi^2}{b}\left(4p_v + m_q \frac{\partial p_v}{\partial m_q}\right), \qquad b = (11N_c - 2N_f)/3$$

At fi nite T and μ

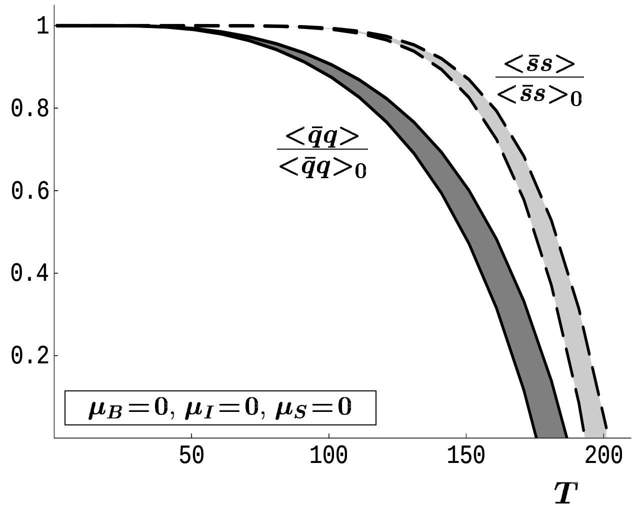
$$< \bar{q}q(T,\mu) > = < \bar{q}q > + \frac{\partial p}{\partial m_q}, \qquad < G^2(T,\mu) > = < G^2 > - \frac{32\pi^2}{b}\left(\epsilon - 3p + m_q \frac{\partial p}{\partial m_q}\right)$$

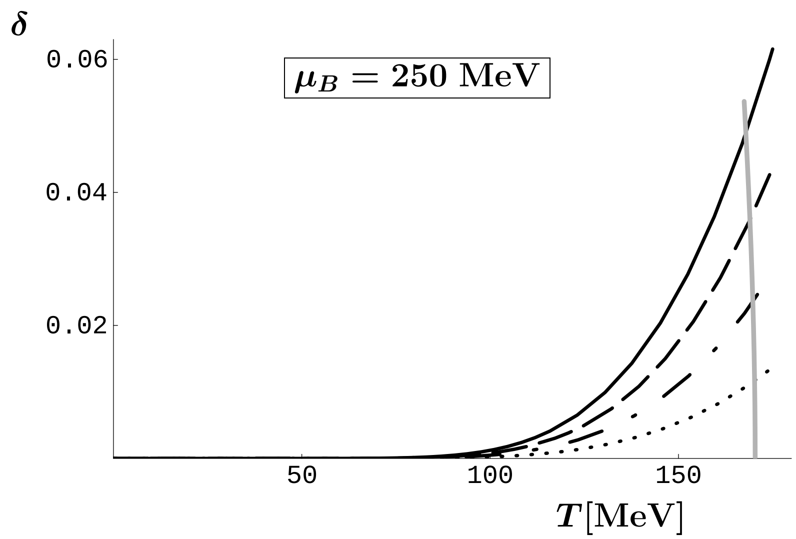
$$\Delta < \bar{q}q(T,\mu) > = - < \bar{q}q > \frac{g}{2\pi^2} \frac{ATm_h}{F_\pi^2} \sum_{n=1}^{\infty} \frac{(-\eta)^{n+1}}{n} e^{n\frac{\mu}{T}} K_1\left(n\frac{m_h}{T}\right),$$

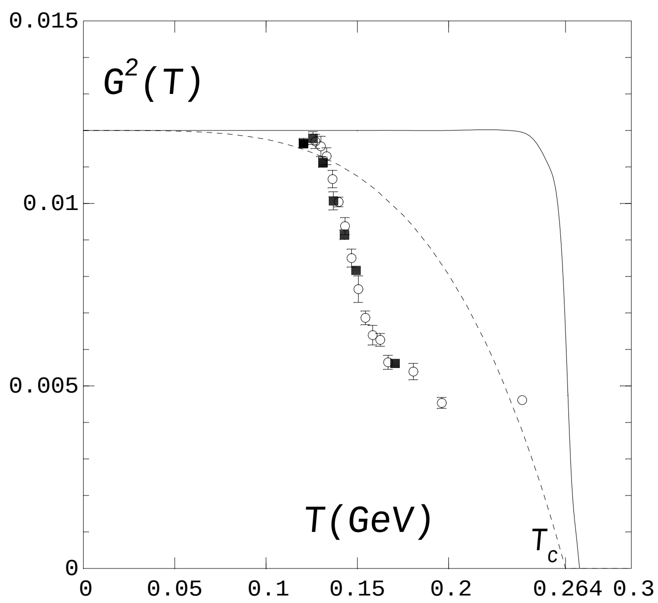
$$< G^2(T,\mu) > = < G^2 > + m_q < \bar{q}q > - m_q < \bar{q}q(T,\mu) > - < \Theta^\mu_\mu(T,\mu)$$

The second term acts as the "impurity". It increasingly "dopes" the vacuum until the ordered state ultimately destroyed.

Quark Condensates







where

$\delta = < \bar{u}u - \bar{d}d > / < \bar{q}q >$.

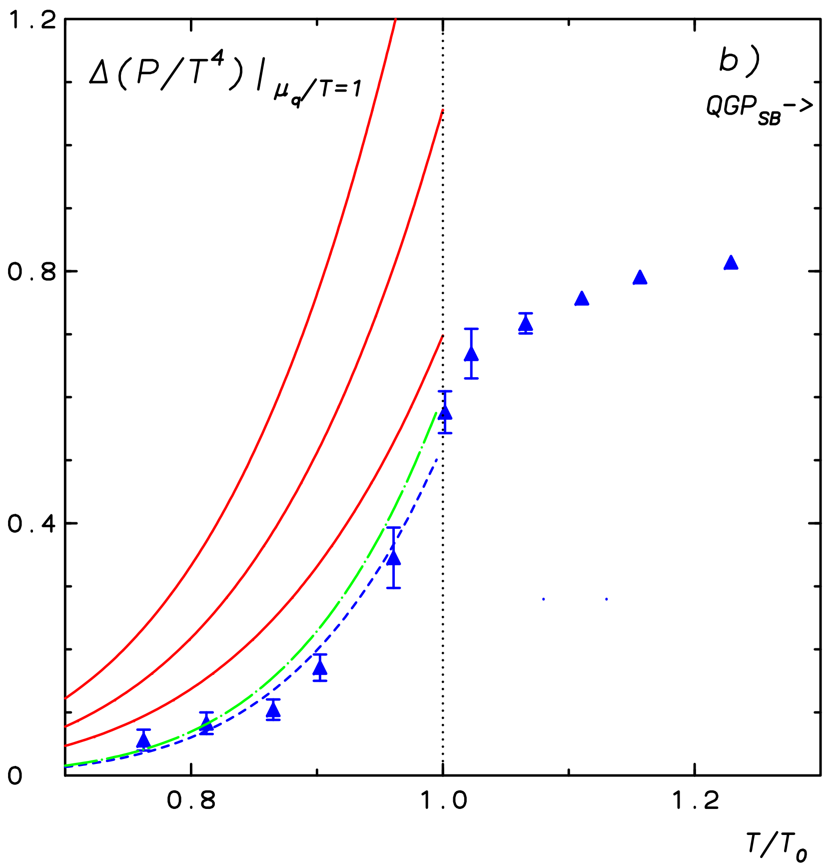
The up and down quark condensates can significantly differ at finite T and μ.

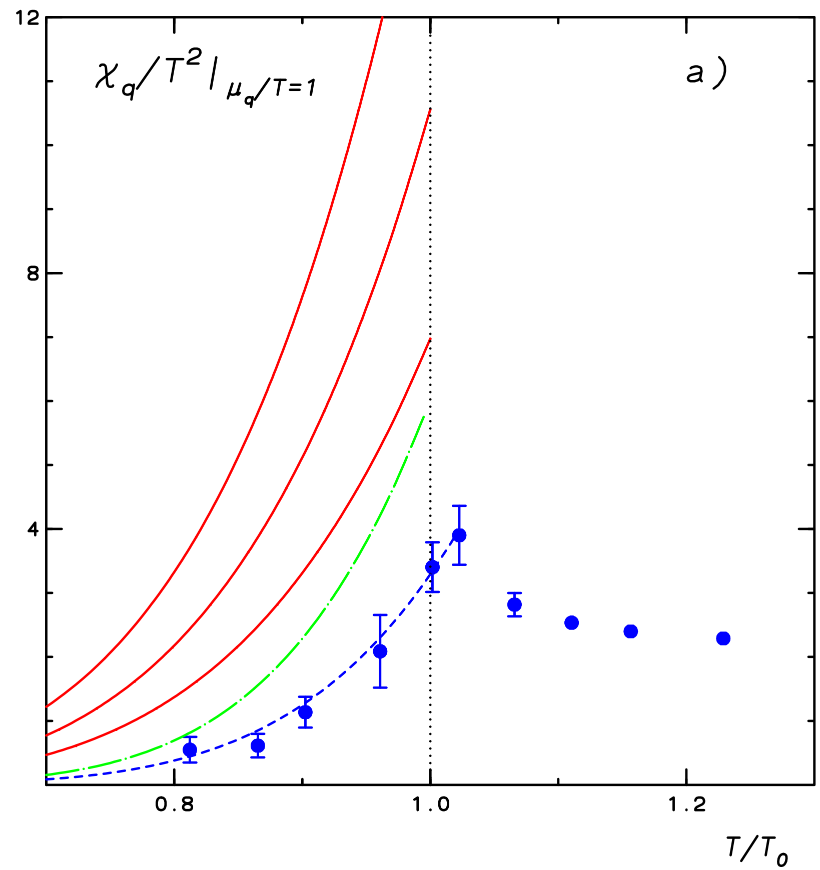
↑D. Tounlan, hep-ph/0511138

$\Longleftarrow$ D.E.Miller, hep-ph/0008031

"physical" world on lattice

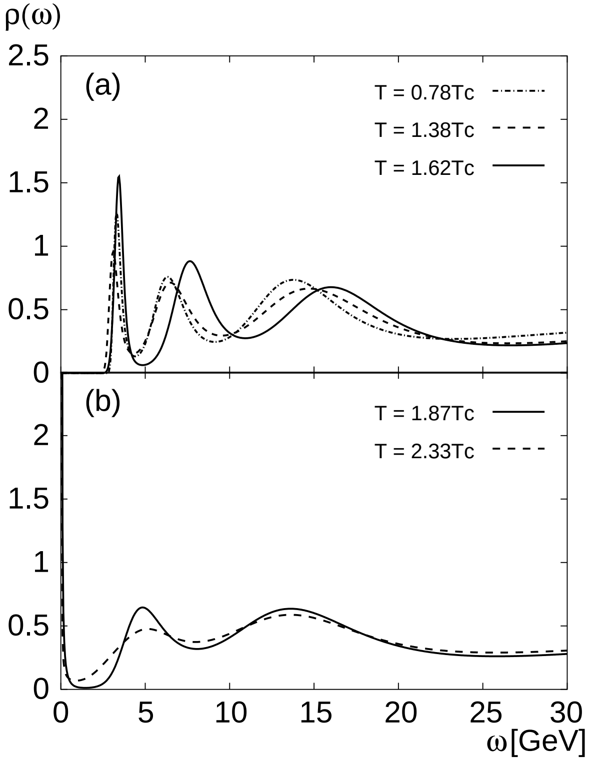
F.Karsch, K.Redlich, A.T., PLB571,74,2003





Charmonium:

J/Ψ-suppresion due to color screening. Recent lattice simulations: J/Ψ survives above T_c



(Asakawa, Hatsuda, PRL92)

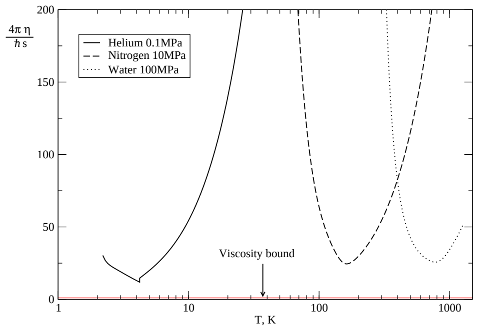
What is the new QGP:

"most" ideal perfect fluid (NOT free gas)

regular fluid with viscosity (NOT superfluid)

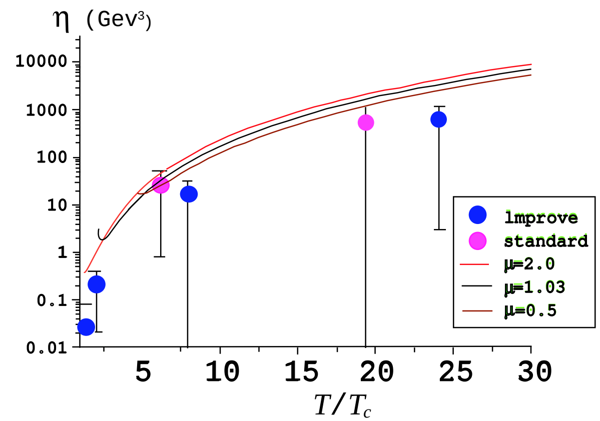
has an electric resistance (NOT superconductor)

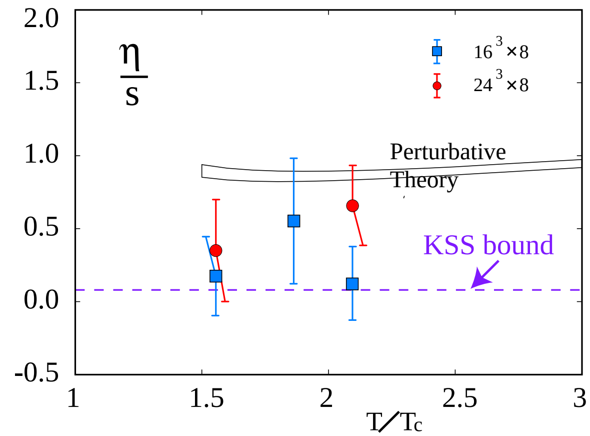
$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi}$$



(Kovtun, Son, Starinets, PRL94)

Lattice Confirmation:





(Nakamura, Sakai, LAT2005)

- Direct simulation at $\mu \neq 0$ is *very hard*

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- Response of $\mathcal{O}$ with respect to $\mu = 0$ [Gottlieb,*et al.* PRD55:6852]
  Derivatives of screening mass and $< \bar{q}q >$ [Choe,*et al.* PRD65:054501]
  Taylor expansion at $\mu = 0$: Dependence of p on μ [Gavai, Gupta, PRD68:034506]

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- **Glasgow:** Zero in complex μ-plane, Lee-Yang zero [Barbour, NP60A:229]

$$Z = \frac{\int \mathcal{D}U \det \mathcal{M}[\mu] \exp(-\beta S_G)}{\int \mathcal{D}U \det \mathcal{M}[0] \exp(-\beta S_G)} = \left\langle \frac{\det \mathcal{M}[\mu]}{\det \mathcal{M}[0]} \right\rangle_{\mu=0} = \sum_{n=-3N_\sigma^3}^{+3N_\sigma^3} < b_n > e^{n\mu}/T$$

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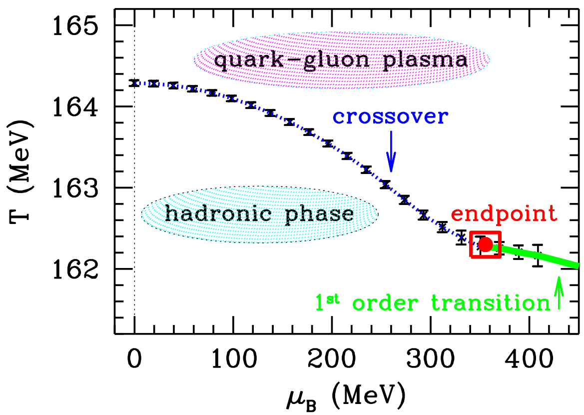
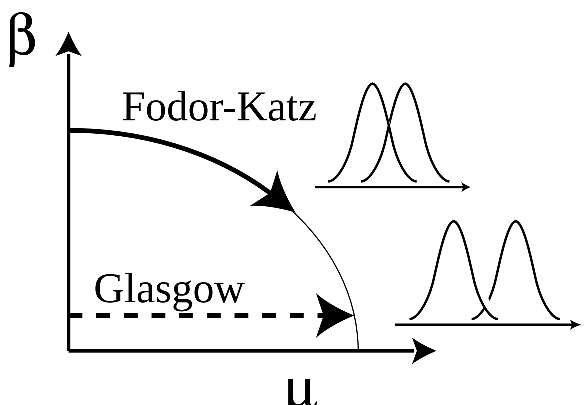
- **Multi-parameter reweighting method:** Transition line at $\mu \neq 0$ [Fodor and Katz JHEP0203:014]

$$< \mathcal{O} > = \frac{\int \mathcal{D}U \mathcal{O} \frac{\det \mathcal{M}[\mu]}{\det \mathcal{M}[0]} e^{-(\beta-\beta_0)S_G} \det \mathcal{M}[0] e^{-\beta_0 S_G}}{\mathcal{Z}(\mu)} = \frac{\left\langle \mathcal{O} \frac{\det \mathcal{M}[\mu]}{\det \mathcal{M}[0]} e^{-\Delta\beta S_G} \right\rangle_0}{\left\langle \frac{\det \mathcal{M}[\mu]}{\det \mathcal{M}[0]} e^{-\Delta\beta S_G} \right\rangle_0}$$





Bielefeld-Swansea: Derivatives with respect to $\mu = 0$ [Alton, *et al*, PRD66:074507]

Reweighting for the gauge and fermion parts of Wilson action read, respectively

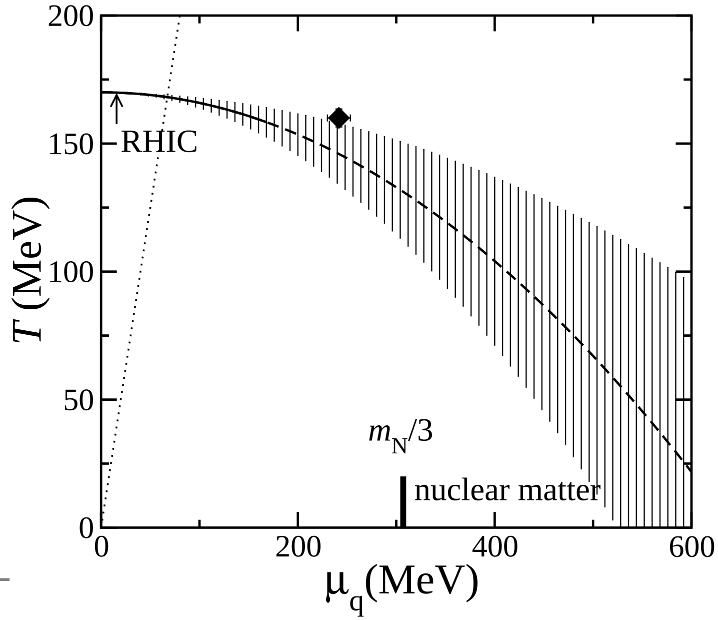
$$S_G(\beta) - S_G(\beta_0) = (\beta - \beta_0) \sum_{x,\mu>\nu} P_{\mu\nu}(x), \qquad \text{plaquette}$$

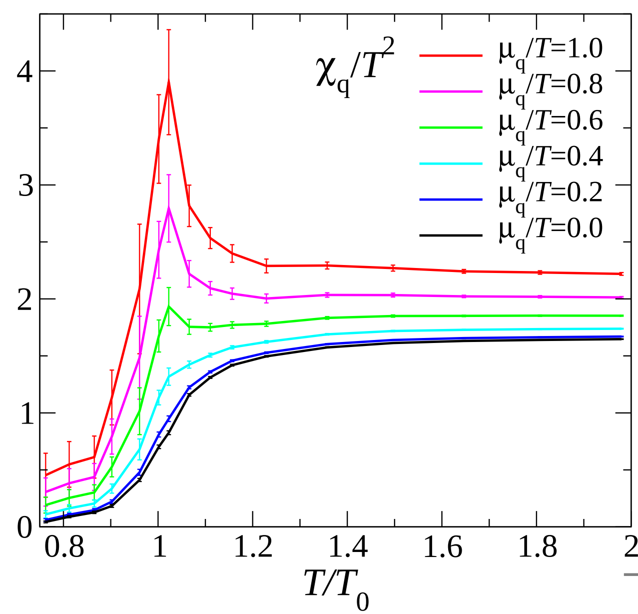
$$\ln\left(\frac{\det \mathcal{M}[\mu]}{\det \mathcal{M}[0]}\right) = \sum_{n=1}^{\infty} \frac{\mu^n}{n!} \frac{\partial^n \ln\det \mathcal{M}}{\partial \mu^n} \equiv \sum_{n=1}^{\infty} \mathcal{R}_c \mu^n$$

It is easier to calculate the phase μ Im Tr$\mathcal{M}^{-1}\frac{\partial \mathcal{M}}{\partial \mu}$ than the determinant itself.

Expand the fermionic observables such as chiral condensate $< \bar{\psi}\psi >= \partial \ln \mathcal{Z}/\partial m_q = c <$ Tr $\mathcal{M}^{-1} >$

with the identity $\frac{\partial \mathcal{M}^{-1}}{\partial x} = -\mathcal{M}\frac{\partial \mathcal{M}}{\partial x}\mathcal{M}^{-1}$ one can get expressions for $\frac{\partial^n \ln\det \mathcal{M}}{\partial \mu^n}$ and $\frac{\partial^n \text{Tr}\, \mathcal{M}^{-1}}{\partial \mu^n}$.





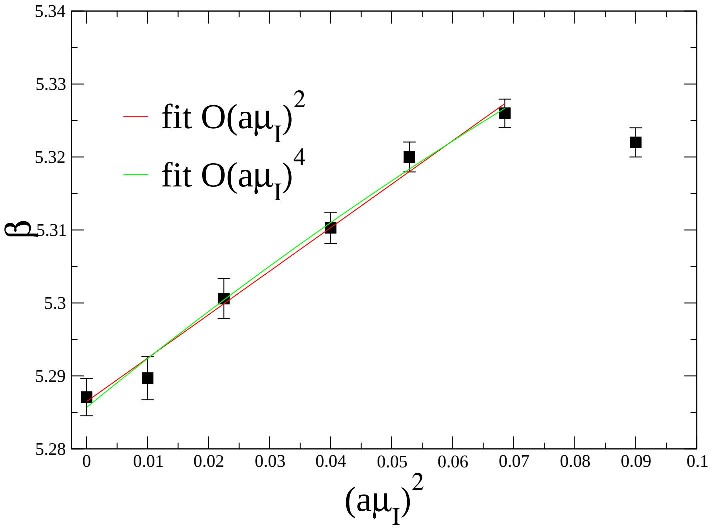
Imaginary μ: Pure imaginary $\mu \to i\mu_I$ [de Forcrand, Philipsen, NPB642:290]

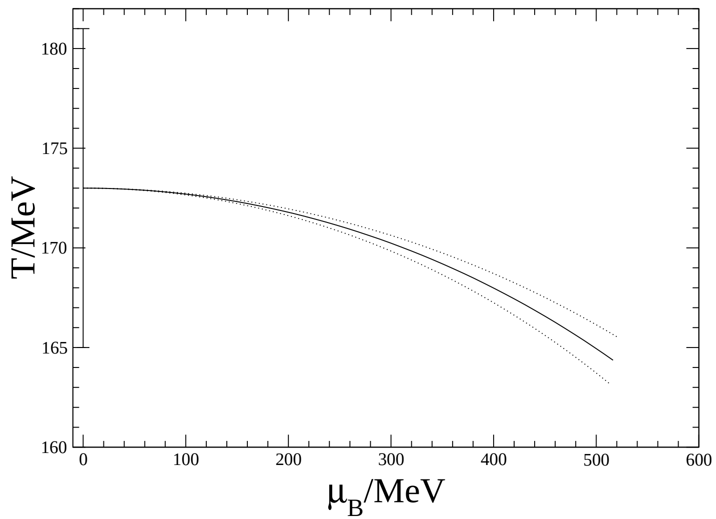
$$\mathcal{M} = \mathcal{D}_\nu \gamma_\nu + m + i\mu_I \gamma_0, \quad \Longrightarrow \quad \mathcal{M}^\dagger = \gamma_5 \mathcal{M} \gamma_5$$

The connection to real chemical potential is provided by

$$\mathcal{Z}(T, m_q) = \int_{-\pi T}^{+\pi T} \frac{d\mu_I}{2\pi T} \mathcal{Z}(T, i\mu_I) e^{-i\mu_I m_q / T}$$

Using the analyticity of the partition function to continue expectation values computed with





$$T = (N_\tau a_\tau(\beta))^{-1}, \qquad V = (N_\sigma a_\sigma(\beta))^3,$$

$$\frac{\epsilon - 3p}{T^4} = -\frac{1}{VT^3}\left(a\frac{\partial\beta}{\partial a}\frac{\partial \ln \mathcal{Z}}{\partial \beta} + a\frac{\partial m}{\partial a}\frac{\partial \ln \mathcal{Z}}{\partial m} \right), \qquad -pV = \frac{T}{V}\ln \mathcal{Z} = E - T\mathcal{S} - \mu_q n_q$$

From the Euclidean action $S(\beta, m, \mu)$

$$a\frac{dS}{da} = 3V\frac{\partial S}{\partial V} - T\frac{\partial S}{\partial T}$$

$$V\frac{\partial \Omega}{\partial V} = VT\left\langle \frac{\partial S}{\partial V} \right\rangle = -pV$$

$$T\frac{\partial \Omega}{\partial T} = \Omega + T^2\left\langle \frac{\partial S}{\partial T} \right\rangle = -T\mathcal{S} = \Omega - E + \mu_q N_q$$

$$\frac{T}{V}\left\langle a\frac{dS}{da} \right\rangle = \epsilon - 3p - \mu_q n_q = -\frac{T}{V}\left(a\frac{\partial\beta}{\partial a}\frac{\partial \ln \mathcal{Z}}{\partial \beta} + a\frac{\partial m}{\partial a}\frac{\partial \ln \mathcal{Z}}{\partial m} + a\frac{\partial \mu}{\partial a}\frac{\partial \ln \mathcal{Z}}{\partial \mu} \right)$$

Taylor expansion about $\mu = \mu_q a = 0$ leads to

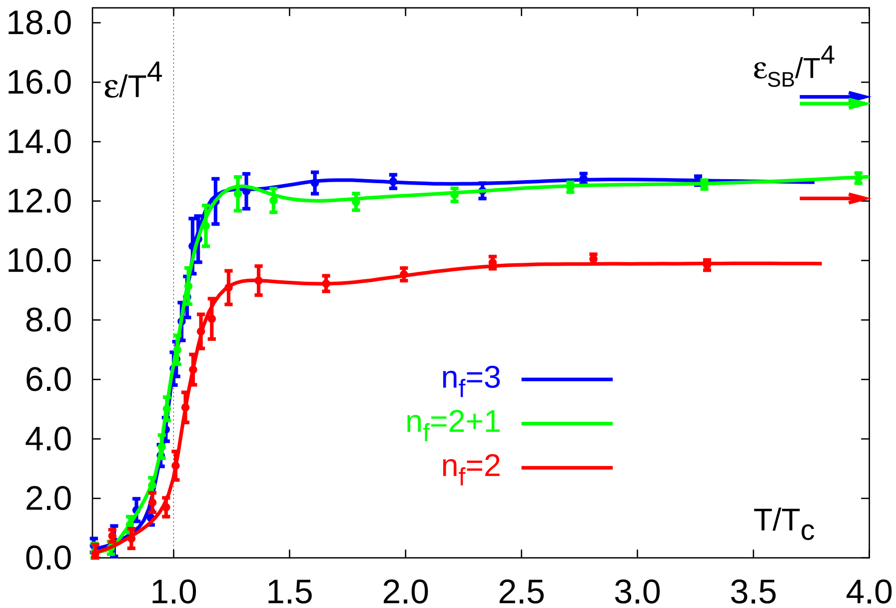
$$\Delta\left(\frac{p}{T^4}(\mu)\right) = \left.\frac{p}{T^4}\right|_{T,\mu_q} - \left.\frac{p}{T^4}\right|_{T,0} = \frac{1}{2!}\frac{N_\tau^3}{N_\sigma^3}\mu^2\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu^2} + \frac{1}{4!}\frac{N_\tau^3}{N_\sigma^3}\mu^4\frac{\partial^4 \ln \mathcal{Z}}{\partial \mu^4} + \cdots$$

$$= \sum_{p=1}^{\infty} c_p(T)\left(\frac{\mu}{T}\right)^p$$

Lattice QCD predicts a phase transition to QGP at hight temperatures where the number of degrees of freedom is signifi cantly increases. $\epsilon = g\frac{\pi^2}{30}T^4$

Phase transition:

$$T = 170 MeV,$$
$$\epsilon = 0.7 \text{ GeV/fm}^3$$



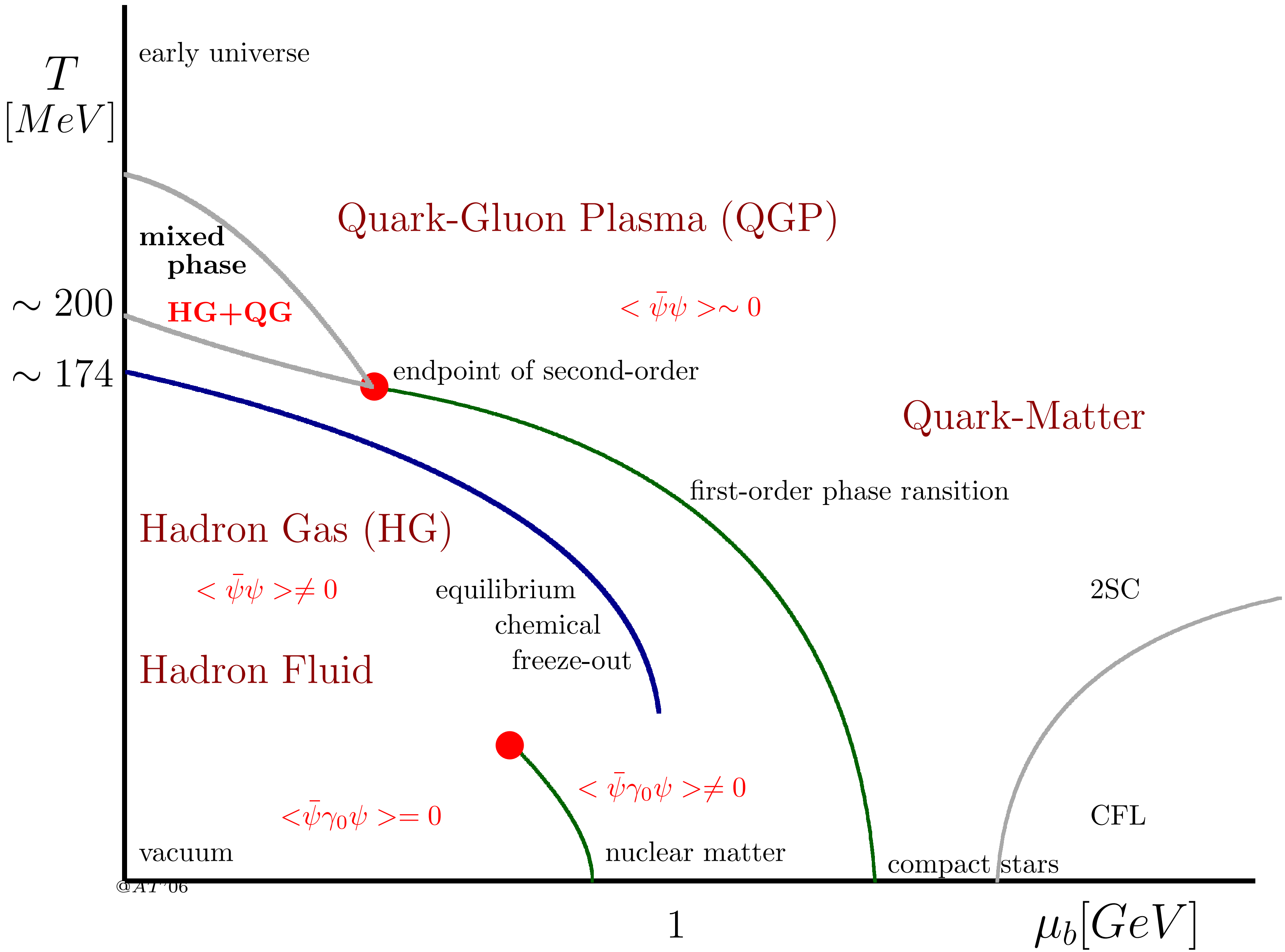
At SPS, this value has been reached J.Pisut, N.Pisutov, B.Tomasik, Eur.Phys.J.C29,79(2003)
At RHIC, $\epsilon \sim 10$ GeV/fm^3, T.Ludlam, L.McLerran, Physics Today,Vol. 75,49

A "weakly" interacting system has been suggested according to screening the Lagrangian confi ning potential.

Recent RHIC results indicate that QGP seems to be a strongly interacting system.

Lattice results shows that the thermodynamics is still far away from Boltzmann limit.

"new" phase diagram



Why is it important?

- Characters of the modifi ed states are related to the ground state of the medium.
Landau in 1950's
- The hadron resonances in thermal and dense medium can't remain the same as in "vacuum" remember resonance-gas-, bootstrap- and dual-model!
- The hadrons can be seen as elementary excitations and dynamical fluctuations of the medium. In QCD they are given as

$$J^{(M)} = \bar{q}(x)\Gamma_{\alpha\beta}q(x); \qquad \Gamma_i = 1, \gamma^\mu, \gamma^5, \gamma^\mu\gamma^5 \cdots$$
$$J^{(B)} = \epsilon_{abc}\bar{q}(x)^a C\gamma_\mu q(x)^b \gamma^5 \gamma^\mu q(x)^c$$

- Phenomenologically: The modifi cations of mass/width – even below T_c – should have consequences in the heavy-ion collisions; pre-deconfi ned dilepton enhancement due to broadening of ρ-mesons or shift of their masses
The χ-symmetry restoration should reflect itself in the degeneracies of the hadron spectrum

Pion decay constant at $T \neq 0$ and $\mu \neq 0$

- The pion decay constant sets the scale for the χ-symmetry breaking.
- At $T = 0$, the pion decay constant has been measured and calculated on the lattice.
- Therefore, the thermal behavior of the pion decay constant is related to the χ-symmetry restoration and to the modifi cation of the bound states.
- The lattice simulations for the thermal hadronic bound states are encouraging, BUT we still need to understand fundamental quantities, like wave function, decay constant, etc.

$$G^{(A)} \approx < A_\mu(x) P(y) > \; = \; C_A e^{-m_A(x_0 - y_0)} + \cdots ; \qquad A_\mu = \bar{\psi}\gamma_\mu\gamma_5\psi \ \text{Axial-Vector}$$

$$G^{(\pi)} = < \pi(x)\pi(y) > \; = \; C_P e^{-m_\pi(x_0 - y_0)} + \cdots$$

$$f_\pi^{latt} = \mathcal{O}(C_A, A_P); \qquad f_\pi = z f_\pi^{latt}$$

$$\frac{f_\pi^{latt}(T)}{f_\pi^{latt}(0)} = \frac{f_\pi^{phys}(T)}{f_\pi^{phys}(0)}$$

$$C(\mu) = C_0 + C_1\mu + C_2\mu^2 + \cdots$$

Decay Constant at $T \neq 0$ and $\mu \neq 0$

Pion decay constant in the continium theory

$$\sqrt{2} f_\pi m_\pi = \langle 0 | \bar{u} \gamma_4 \gamma_5 d | \pi^+ (\vec{p} = 0) \rangle$$

From the temporal correlation functioms

$$< \mathcal{O}(\vec{p}, \tau) > = \frac{\mathcal{C}(\vec{p})}{2E_h(\vec{p})} \frac{\cosh(E_h(\vec{p})(\tau - \tau_0))}{\sinh(E_h(\vec{p})\tau_0)} \quad \text{(-16)}$$

where $\mathcal{C}(\vec{p})$ is the residue of the current when $\vec{p} \to 0$. Applying the GMOR relation,

$$< \mathcal{O}(\tau) >_\pi = f_\pi^2 \frac{m_\pi^2}{8m_q^2} \frac{\cosh(m_\pi(\tau - \tau_0))}{\sinh(m_\pi \tau_0)}$$

The Dependence of decay constant on the medium

$$(f_\pi^*)^2 (m_\pi^*)^2 = -2m_q < \bar{q}q >^*$$

Decay Constant at $T \neq 0$ and $\mu \neq 0$

Temporal meson correlation functions in coordinate space with Euclidian time $\tau \in [1, 1/T)$

$$G_h(\tau,\vec{x}) \quad \equiv \quad < \mathcal{J}_h(\tau,\vec{x})\mathcal{J}_h^\dagger(0,\vec{0}) >= T \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d^3p}{(2\pi)^3} e^{-i\omega_n\tau + i\vec{p}\vec{x}} \tilde{G}_h(i\omega_n\vec{p})$$

Here, we introduced the spectral function

$$\tilde{G}_h(i\omega_n\vec{p}) \quad \equiv \quad \int_{-\infty}^{\infty} d\omega \frac{\sigma_h(\omega,\vec{p},T)}{\omega - i\omega_n}$$

Then, we can project G to a fi xed momentum $\vec{p}$

$$G_h(\tau,\vec{p}) \quad = \quad \int_0^{\infty} d\omega\sigma_h(\omega,\vec{p},T) \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T}.$$

For large T and large x, G is dominated by the pole contribution

$$G_h(\tau,\vec{p}) \quad = \quad \frac{| < O|\mathcal{J}_h|h(\vec{p}) > |^2}{2E_h(\vec{p})} \frac{\cosh(E_h(\vec{p})(\tau - 1/2T)}{\sinh(E_h(\vec{p})/2T)}$$

$P\equiv\bar\Psi\gamma_5\Psi, J^{PC}=0^{-+}$, in χ-limit, $<0|\mathcal{J}_P|P(\vec p)>=f_\pi\frac{E_P^2(\vec p)}{2m_q}=f_\pi\frac{m_P^2}{2m_q^2}$, Pion

$$\sum_{\vec n}e^{-i\vec p\vec n}<P(\vec n,\tau)P^\dagger(0,0)>\Longrightarrow\frac{|<0|P|P(\vec p)>|^2}{E_p(\vec p)}e^{E_P(\vec p)N_\tau/2}\cosh\left(E_P(\vec p)\left(\frac{N_\tau}{2}-\tau\right)\right)$$

The mass is recovered for $\vec p=\vec 0$.

$V\equiv\sum_i+1^3\bar\Psi\gamma_i\Psi, J^{PC}=1^{--}$. For $i=1$ we get m_ρ Vector Scalar

$$\sum_{\vec n}e^{-i\vec p\vec n}<V(\vec n,\tau)V^\dagger(0,0)>\Longrightarrow\frac{|<0|V|V(\vec p)>|^2}{E_p(\vec p)}e^{E_V(\vec p)N_\tau/2}\cosh\left(E_V(\vec p)\left(\frac{N_\tau}{2}-\tau\right)\right)$$

$A_\mu\equiv\bar\Psi\gamma_\mu\gamma_5\Psi, J^{PC}=0^{-+},(a_1(J^{PC}=1^{++}))$, $<0|A^\mu(0)|P(p)>=p^\mu\frac{f_P}{Z_A}$ Axial Vector

$$\sum_{\vec n}<A_4(\vec n,\tau)A_4^\dagger(\vec n,\tau)>\quad\Longrightarrow\quad\frac{f_\pi^2M_P}{Z_A^2}e^{M_P(\vec p)N_\tau/2}\cosh\left(M_P\left(\frac{N_\tau}{2}-\tau\right)\right)$$

$$\sum_{\vec n}<A_4(\vec n,\tau)P_4^\dagger(\vec n,\tau)>\quad\Longrightarrow\quad\frac{f_\pi^2}{Z_A^2}|<0|P|P(\vec p=\vec 0)>|e^{M_PN_\tau/2}\sinh\left(M_P\left(\frac{N_\tau}{2}-\tau\right)\right)$$

Temporal correlation functions

The existance of excited states or cut-off effects will modify the correlation functions. Adding an auxiliary bound state minimizes these effects.

$$G_h(\tau,\vec{p}) = \frac{|<O|\mathcal{J}_h|h(\vec{p})>|^2}{2E_h(\vec{p})} \cdot \frac{\cosh(E_h(\vec{p})(\tau - 1/2T)}{\sinh(E_h(\vec{p})/2T)} + a\frac{\cosh(\mathcal{E}[\tau - 1/2T])}{\sinh(\mathcal{E}/2T)}$$

To extract the properties of this state, a large time direction τ is required. Increasing T shrinkes τ. An anisotrpic lattice has to be used, $\zeta = a_\sigma/a_\tau$
Then the hadron effective mass is

$$m_h^{eff}(\tau) = \log\left(\frac{G_h(\tau)}{G_h(\tau+1)}\right), \text{ when } \tau \to \infty, m_h^{eff} = m_h$$

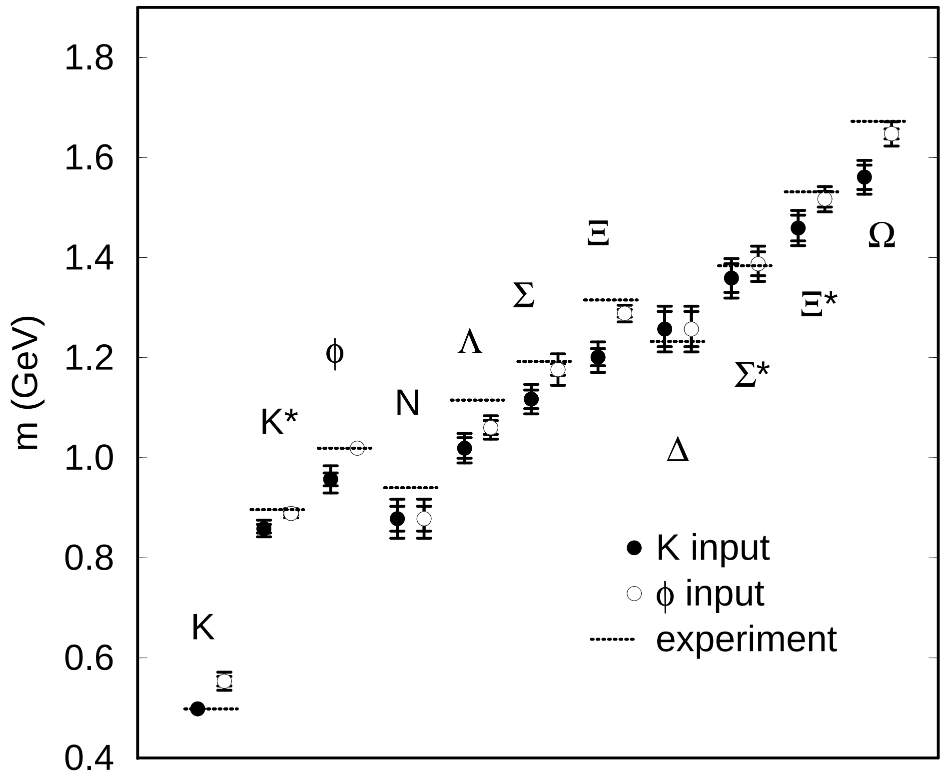
The screening mass can be obtained by a fi t including one or two poles (absorb the contaminations from excited states)

We aim to investigate the *"origin of the mass"* from first principle. The current for the mesonic state is

$$\mathcal{O}_m(x) \quad = \quad \bar{\psi}^a_\alpha(x)\Gamma_{\alpha\beta}\psi^b_\beta(x)$$

where $\Gamma_{\alpha\beta}$ stands for the Dirac matrices $(1, \gamma_\mu, \gamma_5, \gamma_\mu\gamma_5, \sigma_{\mu\nu})$. For the baryonic state is

$$\mathcal{O}_b(x) \quad = \quad \epsilon_{ijk}\left(\psi^{i,a}_\alpha(x)\Gamma_{\alpha\beta}\psi^{j,b}_\beta(x)\right)\psi^{k,c}_\gamma$$

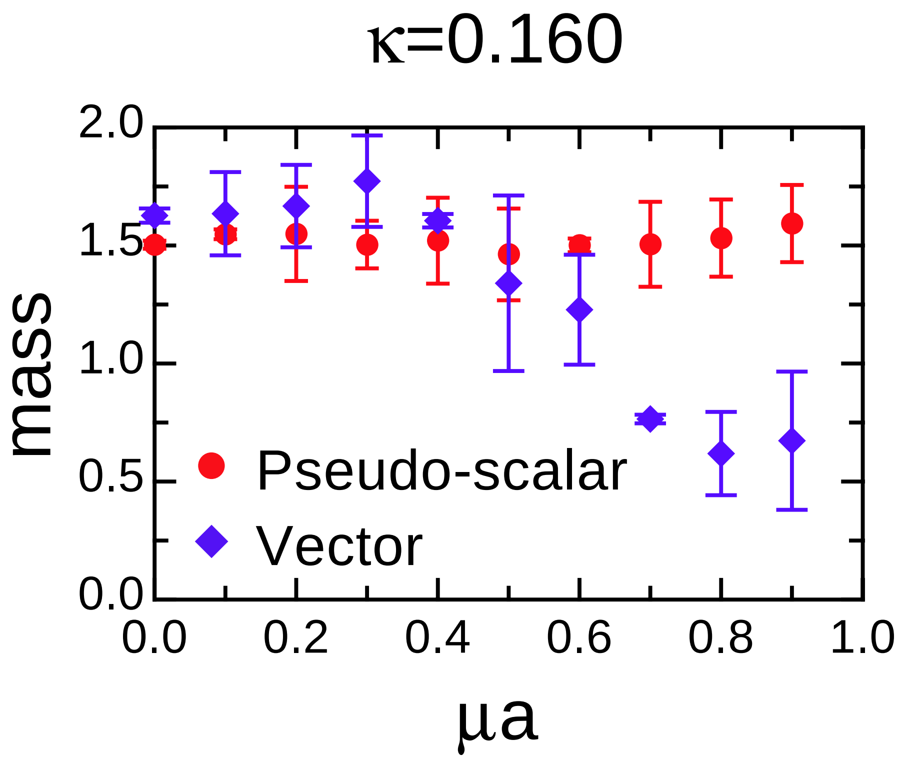


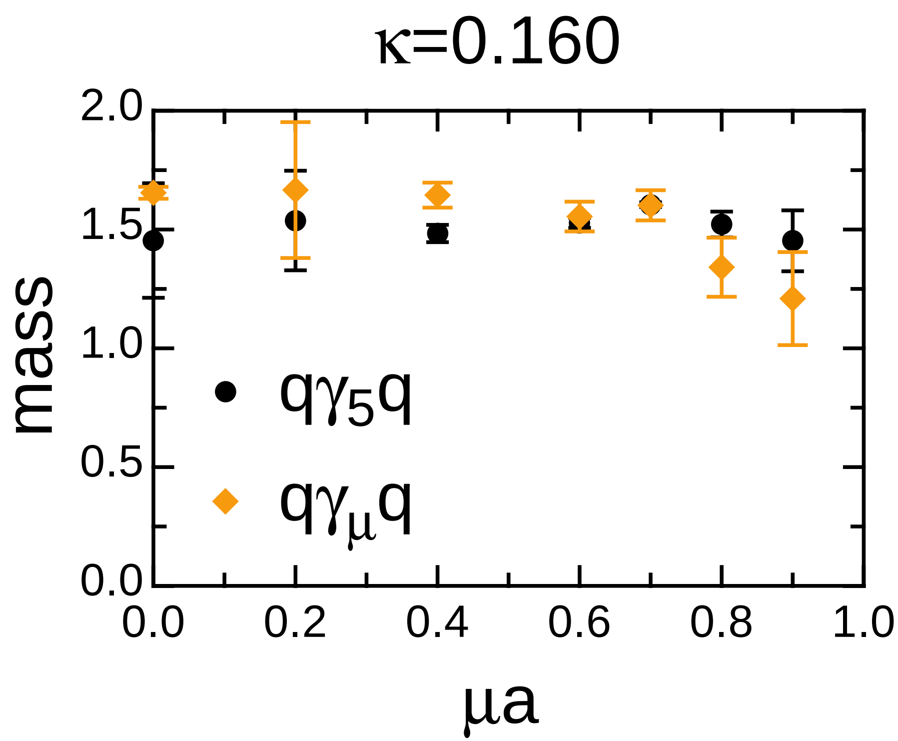
Cp-PACS, PRD67, 034503 (2003)

Hadron masses at $T \neq 0$ and $\mu \neq 0$

The Tayler series of $m(T,\mu=0)$

$$\frac{m(T,\mu)}{T} = \frac{m(T,0)}{T}|_{\mu=0} + \left(\frac{\mu}{T}\right)\frac{\partial m(T,0)}{\partial\mu}|_{\mu=0} + \left(\frac{\mu}{T}\right)^2\frac{T}{2}\frac{\partial^2 m(T,0)}{\partial\mu^2}|_{\mu=0} + \cdots$$





S. Muroya, A. Nakamura, C. Nonaka, PTP149, 247 (2003)

Screening mass

In perturbation theory the screening mass (next-to-leading order) reads

$$
\begin{aligned}
M_D(T) &= M_D^{(LO)} + \frac{Ng(T)T}{4\pi}\ln\left(\frac{M_D^{(LO)}}{g^2(T)T}\right) + c_N g^2(T)T + \mathcal{O}(g^3(T)T) \\
M_D^{(LO)} &= \left(\frac{N_c}{3}+\frac{N_f}{6}\right)^{1/2} g(T)T \\
c_N &= \frac{m}{g^2(T)T} + \frac{N}{8\pi}\left(\ln\frac{g^4(T)T^2}{8} - 1\right)
\end{aligned}
$$

where c_N is a numerically computed constant. Including the chemical potential is a non-trivial task.

$$
M_D(T,\mu_b) = \left(\frac{N_c}{3}+\frac{N_f}{6}+\left(\frac{\mu_b}{T}\right)^2\frac{N_f}{2\pi^2}\right)^{1/2} g(T)T
$$

QCD-TARO, PLB609, 265, 2005

Taylor expansion at a fixed temperature, coupling and bare quarks

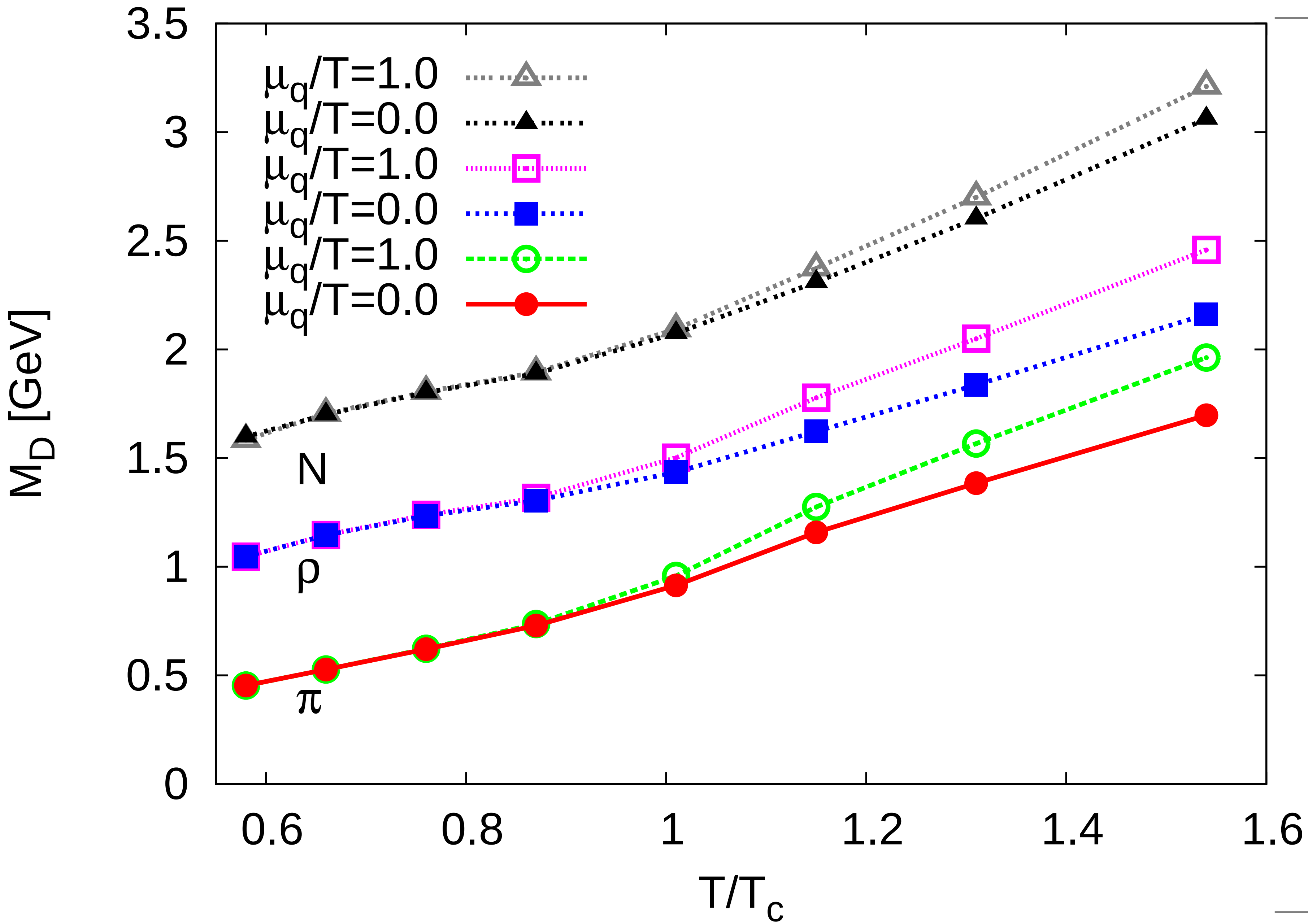
$$\frac{M(\mu)}{T} = \left.\frac{M}{T}\right|_{\mu=0} + \left(\frac{\mu}{T}\right)\left.\frac{\partial M}{\partial\mu}\right|_{\mu=0} + \frac{1}{2}\left(\frac{\mu}{T}\right)^2 T\left.\frac{\partial^2 M}{\partial\mu^2}\right|_{\mu=0} + \cdots \quad \text{(-35)}$$

The coefficients are determined by measuring the hadron propagators and their derivatives at $\mu = 0$ by using the standard MC method.

$$\begin{aligned}
C_\pi(z) &= C_1\left(e^{-\hat{m}_1\,\hat{z}} + e^{-\hat{m}_1\,(N_z-\hat{z})}\right) + C_2\left(e^{-\hat{m}_2\,\hat{z}} + e^{-\hat{m}_2\,(N_z-\hat{z})}\right) \\
C_\rho(z) &= C_1'\left(e^{-\hat{m}_1'\,\hat{z}} + e^{-\hat{m}_1'\,(N_z-\hat{z})}\right) + C_2'\,(-1)^z\left(e^{-\hat{m}_2'\,\hat{z}} + e^{-\hat{m}_2'\,(N_z-\hat{z})}\right) \\
C_N(z) &= C_1''\left(e^{-\hat{m}_1''\,\hat{z}} + (-1)^z\,e^{-\hat{m}_1''\,(N_z-\hat{z})}\right) + C_2''\left((-1)^z\,e^{-\hat{m}_2''\hat{z}} + e^{-\hat{m}_2''\,(N_z-\hat{z})}\right)
\end{aligned}$$

N_z is the length of the lattice in the z-direction.

Screening mass at finite μ



Wave function

Hadronic states fulfill the lattice symmetry. But they still need a spatial dimension.

Numerically, this requirement is convenient, **smeared gauge fields**

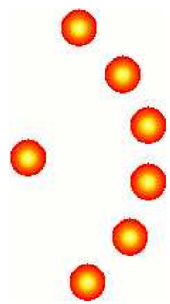
$$\psi^r(x) \;=\; \frac{1}{6}\sum_{\mu=1}^{3} U^\dagger_\mu(x-\hat{\mu})\cdots U^\dagger_\mu(x-r\hat{\mu})\psi(x-r\hat{\mu}) + U(x)\cdots U(x+(r-1)\hat{\mu})\psi(x-r\hat{\mu})$$

r has to be chosen so that the exited states get suppressed and *therefore*, the hadron properties are improved.

$$G_h(\tau,\vec{x}) \;=\; \int\int dy_1\, dy_2 F(y_1)F(y_2) < \mathcal{J}_h(\tau,\vec{x};\vec{y_1},0)\mathcal{J}_h(\vec{y_2},\vec{0};\vec{x},\tau) > \qquad (-38)$$

For example, the pion states can be expressed as

$$|\pi > \;=\; \alpha_1|q\bar{q} > +\alpha_2|q\bar{q}g > +\alpha_3|q\bar{q}gg > + \cdots$$



The gauge invariant BS amplitude with $\vec{p}$ is $A_\pi(\vec{p},\vec{x}) =< 0|\bar{d}(\vec{0})\gamma_5\mathcal{G}(\vec{0},\vec{x})u(\vec{x})|\pi(\vec{p}) >$. $\mathcal{G}(\vec{0},\vec{p})$ is the path-ordered product of the gauge links that connect $\vec{0}$ with $\vec{x}$.

- Fundamental questions, mass, CKM, phase transitions, etc.
- Phenomenology of the particle production in heavy-ion collisions
- Matter under extreme conditions
- Dynamics controlling the phase transition
- Properties of the matter below and above T_c
- Anisotropic lattice,
  exact algorhithms,
  ”new” simulation concepts at $\mu \neq 0$.