4th ILFTN Workshop at Hayama, Mar. 8-11, 2006

Baryon semi-leptonic decay from lattice QCD with domain wall fermions

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Baryon semi-leptonic decay

The octet baryons (p,n,Λ,Σ,Ξ) admit various β-type decays.

$$B' \to B + l^{\pm} + \nu_l(\bar{\nu}_l)$$

- neutron beta decay

$$n \to p + e^- + \bar{\nu}_e$$

CVC (conserved vector current) hypothesis:

$$\langle p | V_\mu^+ - A_\mu^+ | n \rangle = 2 \langle p | V_\mu^3 - A_\mu^3 | p \rangle$$

Weak matrix element   Iso-vector nucleon matrix element

way to access the nucleon structure

Baryon semi-leptonic decay

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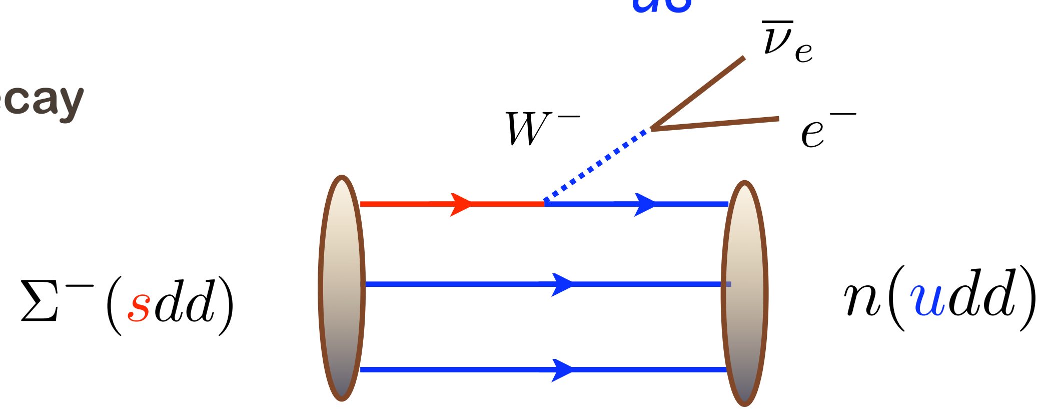
$$B' \to B + l^{\pm} + \nu_l(\bar{\nu}_l)$$

- hyperon beta decay
  - ✓ *Alternative way to determine $|V_{us}|$ other than K_{l3} decays*

Δs=1 decay

T. Kaneko's talk



the weak mixing element (CKM) V_{us}

Baryon semi-leptonic decay

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$$B' \to B + l^{\pm} + \nu_l(\bar{\nu}_l)$$

- hyperon beta decay
  - ✓ *Alternative way to determine $|V_{us}|$ other than K_{l3} decays*
  - ✓ *Vital input to analysis of strange quark spin fraction*

$$\Delta\Sigma (= \Delta u + \Delta d + \Delta s)_{\mathrm{Expt.}} = 0.213 \pm 0.138$$

$$\left.\begin{aligned}(g_A/g_V)_{np} &= \Delta u - \Delta d \\ (g_A/g_V)_{\Lambda p} &= (2\Delta u - \Delta d - \Delta s)/3 \\ (g_A/g_V)_{\Xi\Sigma} &= (\Delta u + \Delta d - 2\Delta s)/3 \\ (g_A/g_V)_{\Sigma n} &= \Delta d - \Delta s\end{aligned}\right\} \text{Assumption : SU(3) symmetry}$$

Baryon semi-leptonic decay

The octet baryons (p,n,Λ,Σ,Ξ) admit various β-type decays.

$$B' \to B + l^{\pm} + \nu_l(\bar{\nu}_l)$$

- hyperon beta decay
  - ✓ *Alternative way to determine $|V_{us}|$ other than K_{l3} decays*
  - ✓ *Vital input to analysis of strange quark spin fraction*

$$\Delta s = -0.124 \pm 0.046$$

The hidden uncertainty of Δs coming from unknown SU(3) breaking in hyperon beta decays.

Baryon semi-leptonic decay

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- hyperon beta decay
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SU(3) breaking in hyperon beta decays

Baryon semi-leptonic decay

$$B' \to B + l^{\pm} + \nu_l(\bar{\nu}_l)$$

These decays are described by 6 form factors

$$\langle B|V_\alpha - A_\alpha|B'\rangle = \bar{u}_B(p)[f_1(q^2)\gamma_\alpha + \frac{f_2(q^2)}{2M_{B'}}\sigma_{\alpha\beta}q_\beta + \frac{f_3(q^2)}{2M_{B'}}q_\alpha$$

$$+g_1(q^2)\gamma_\alpha\gamma_5 + \frac{g_2(q^2)}{2M_{B'}}\sigma_{\alpha\beta}\gamma_5 q_\beta + \frac{g_3(q^2)}{2M_{B'}}q_\alpha\gamma_5]u_{B'}(p')$$

1st class: $f_1(q^2),\ \ f_2(1^2),\ \ g_1(q^2),\ \ g_3(q^2)$

2nd class: $f_3(q^2),\ \ g_2(q^2)$

Baryon semi-leptonic decay

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1st class: $f_1(q^2), \; f_2(1^2), \; g_1(q^2), \; g_3(q^2)$

Sach's form factors
$$G_E(q^2) = f_1(q^2) + \frac{q^2}{2M_{B'}}f_2(q^2)$$
$$G_M(q^2) = f_1(q^2) + f_2(q^2)$$

Baryon semi-leptonic decay

$$B' \to B + l^{\pm} + \nu_l(\bar{\nu}_l)$$

These decays are described by 6 form factors

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1st class: $f_1(q^2),\ \ f_2(1^2),\ \ g_1(q^2),\ \ g_3(q^2)$

$$g_V = \lim_{q^2\to 0} f_1(q^2) \qquad g_A = \lim_{q^2\to 0} g_1(q^2)$$ forward limit

Baryon semi-leptonic decay

$$B' \to B + l^{\pm} + \nu_l(\bar{\nu}_l)$$

These decays are described by **6 form factors**

$$\langle B|V_\alpha - A_\alpha|B'\rangle = \bar{u}_B(p)[f_1(q^2)\gamma_\alpha + \frac{f_2(q^2)}{2M_{B'}}\sigma_{\alpha\beta}q_\beta + \frac{f_3(q^2)}{2M_{B'}}q_\alpha$$

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2nd class: $f_3(q^2), \quad g_2(q^2)$

SU(3) limit (neutron beta decay) $f_3(q^2) = 0, \quad g_2(q^2) = 0$

• Contents

- ✳ Nucleon axial charge — Finite size effect, $O(a^2)$ correction

  Sasaki, Orginos, Ohta, Blum, Phys. Rev. D68 (03) 054509

- ✳ Nucleon form factors — Finite size effect

  ✓ Nf=2 & Nf=0 DWF results (preliminary)

- ✳ Hyperon beta decay — SU(3) breaking effect

  ✓ An exploratory study in quench (Nf=0) DWF calculation

Nucleon axial charge

Nucleon axial charge g_A

- Well measured quantity in experiment

  Iso-symmetry gives rise to the relation

$$\langle p|V_\mu^+ - A_\mu^+|n\rangle = 2\langle p|V_\mu^3 - A_\mu^3|p\rangle$$

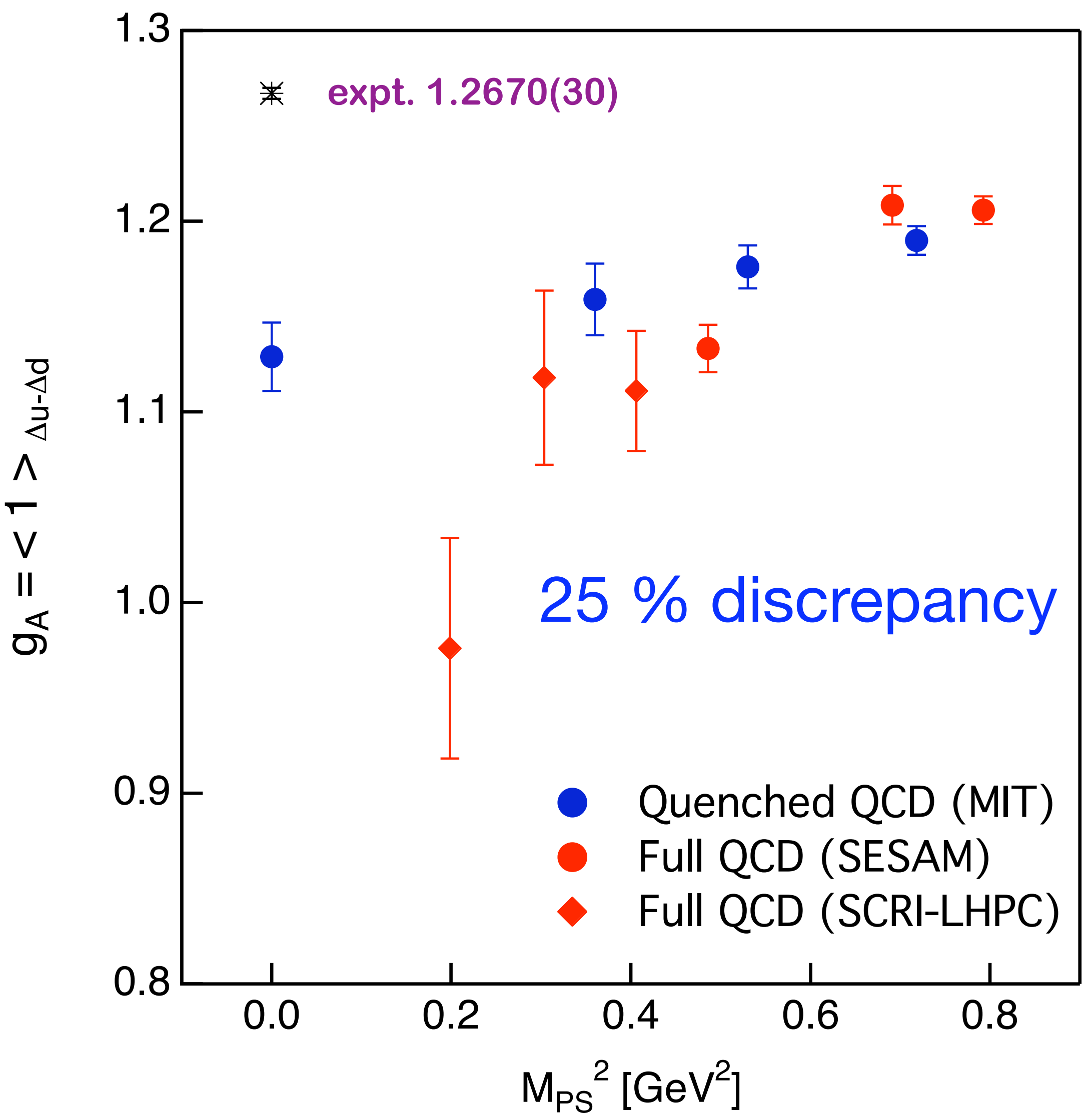
  from neutron beta decay; g_A/g_V= 1.2670(35).

- The simplest nucleon matrix elements

  - lowest moment (no covariant derivative)
  - zero momentum transfer
  - no disconnected diagram

a benchmark calculation = a “gold plated” test

But . . .



$$n \to p + e^- + \bar{\nu}_e$$

$$g_A = \lim_{q^2 \to 0} g_1(q^2)$$

$$\left(\frac{g_A}{g_V}\right)_{\text{expt.}} = 1.2670(30)$$

long-standing problem

Lattice calculation of g_A (before 2002)

type	group	fermion	lattice	β	volume	configs	$m_\pi L$	g_A
quench	KEK [1)]	Wilson	16^3x20	5.7	$(2.2\ \mathrm{fm})^3$	260	≥ 5.9	0.985(25)
	Kentuchy [2)]	Wilson	16^3x24	6.0	$(1.5\ \mathrm{fm})^3$	24	≥ 5.8	1.20(10)
	DESY [3)]	Wilson	16^3x32	6.0	$(1.5\ \mathrm{fm})^3$	1000	≥ 4.8	1.074(90)
	LHPC-SESAM [7)]	Wilson	16^3x32	6.0	$(1.5\ \mathrm{fm})^3$	200	≥ 4.8	1.129(98)
			16^3x32	6.0	$(1.5\ \mathrm{fm})^3$	O(500)		
	QCDSF [4)]	Wilson	24^3x48	6.2	$(1.6\ \mathrm{fm})^3$	O(300)		1.14(3)
			32^3x48	6.4	$(1.6\ \mathrm{fm})^3$	O(100)		
			16^3x32	6.0	$(1.5\ \mathrm{fm})^3$	O(500)		
	QCDSF-UKQCD [5)]	Clover	24^3x48	6.2	$(1.6\ \mathrm{fm})^3$	O(300)		1.135(34)
			32^3x48	6.4	$(1.6\ \mathrm{fm})^3$	O(100)		
full	LHPC-SESAM [7)]	Wilson	16^3x32	5.5	$(1.7\ \mathrm{fm})^3$	100	≥ 4.2	0.914(106)
	SESAM [6)]	Wilson	16^3x32	5.6	$(1.5\ \mathrm{fm})^3$	200	≥ 4.5	0.907(20)

1. M. Fukugita et al., Phys. Rev. Lett. 75 (1995) 2092.
2. K.F. Liu et al., Phys. Rev. D49 (1994) 4755.
3. M. Göckeler et al., Phys. Rev. D53 (1996) 2317.
4. S. Capitani et al., Nucl. Phys. B (Proc. Suppl.) 79 (1999) 548.
5. R. Horsley et al., Nucl. Phys. B (Proc. Suppl.) 94 (2001) 307.
6. S. Güsken et al., Phys. Rev. D59 (1999) 114502.
7. D. Dolgov et al., Phys. ReV.D66 (2002)034506.

Low value of g_A in lattice QCD

- Possible systematic errors
  - ✓ Quenching

    $g_A^{Full} \lesssim g_A^{Quench}$ at a~0.1 fm ~ 5-10 % ↘

  - ✓ Finite lattice spacing

    $(g_A)_{at\ a \to 0} > (g_A)_{at\ a \sim 0.1\ fm}$ ~ 5 % ↗

  - ✓ Determination of Z_A

    $Z_A^{Non\text{-}pert} < Z_A^{Pert}$ (Clover) ~ 10 % ↘

  - ✓ Finite volume

    No estimation ?

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DWF calculation of g_A

- Big advantage in dealing with the axial symmetry
  - g_A are supposed to respect the axial WT identity
  - Empirically known as Goldberger-Treiman relation: $M_N g_A = f_\pi g_{\pi N}$

✓ Excellent chiral properties of DWF

✳ lighter pion mass

✳ Especially, a significant relation $Z_V = Z_A$ for local lattice currents

T. Blum et al, PRD66 (02) 014504

- up to $O(a^2)$

A ratio g_A^{latt} / g_V^{latt} directly yields the renormalized value of g_A

➡ Just require to calculate the ratio of three-point functions!

CVC hypothesis

- The vector weak current is the iso-spin rotation of the electromagnetic current, $j_\mu^{\rm em} = \frac{2}{3}V_\mu^u - \frac{1}{3}V_\mu^d + \cdots$

$$[I_+, j_\mu^{\rm em}] = -\bar{d}\gamma_\mu u$$

$$\langle p|\bar{d}\gamma_\mu u|n\rangle = -\langle p|[I_+, j_\mu^{\rm em}]|n\rangle$$
$$= \langle p|j_\mu^{\rm em}|p\rangle - \langle n|j_\mu^{\rm em}|n\rangle$$

$$g_V = \lim_{q^2\to 0}\langle p|\bar{d}\gamma_\mu u|n\rangle = \lim_{q^2\to 0}\langle p|j_\mu^{\rm em}|p\rangle = 1$$

way to calculate the renormalization factor $Z_V = g_V^{\rm ren}/g_V^{\rm lat} = 1/g_V^{\rm lat}$

CVC hypothesis (cont'd)

- Further consideration of iso-spin symmetry provides

$$\lim_{q^2 \to 0} \langle p | j_\mu^{\rm em} | p \rangle = \lim_{q^2 \to 0} \langle p | V_\mu^d | p \rangle = \lim_{q^2 \to 0} \langle p | V_\mu^u - V_\mu^d | p \rangle = 1$$

in the continuum

- For the local lattice currents in the chiral limit,

$$\mathcal{V}_\mu^f = Z_V V_\mu^f + \mathcal{O}(a^2)$$

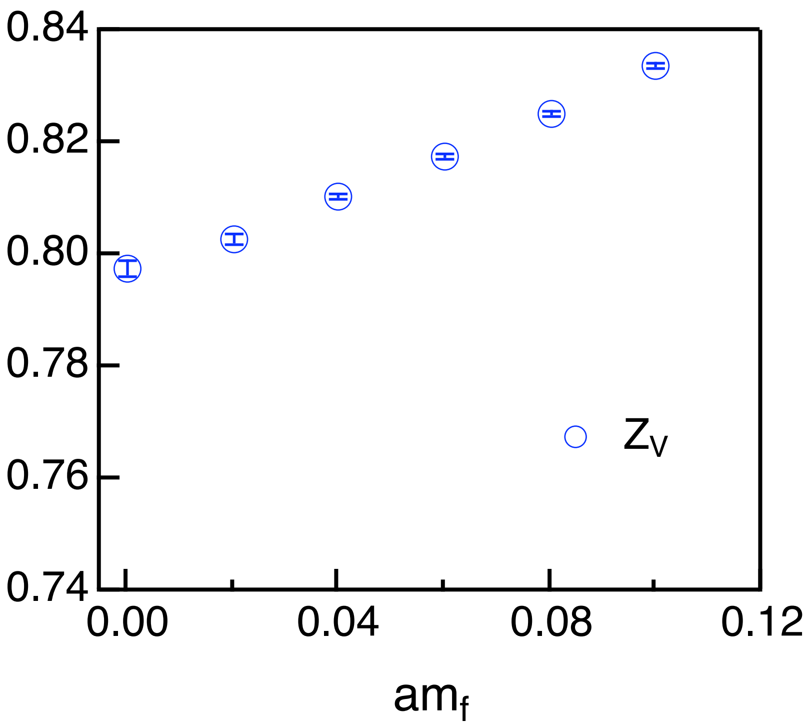
➡ Three different determination of Z_V can expose an $O(a^2)$ lattice artifact.

Check of the relation $Z_A=Z_V$

- Nf0: DWF-DBW2 at $\beta=0.87$ ($a^{-1}=1.3\text{GeV}$)
  - $16^3 \times 32 \times 16$ with $M_5=1.8$

coarse lattice

Sasaki-Orginos-Ohta-Blum, PRD68 (03) 054509



$$Z_V = \frac{1}{g_V^{\rm latt}} \propto \frac{\langle N(t')\bar{N}(0)\rangle}{\langle N(t')V_4(t)\bar{N}(0)\rangle}$$

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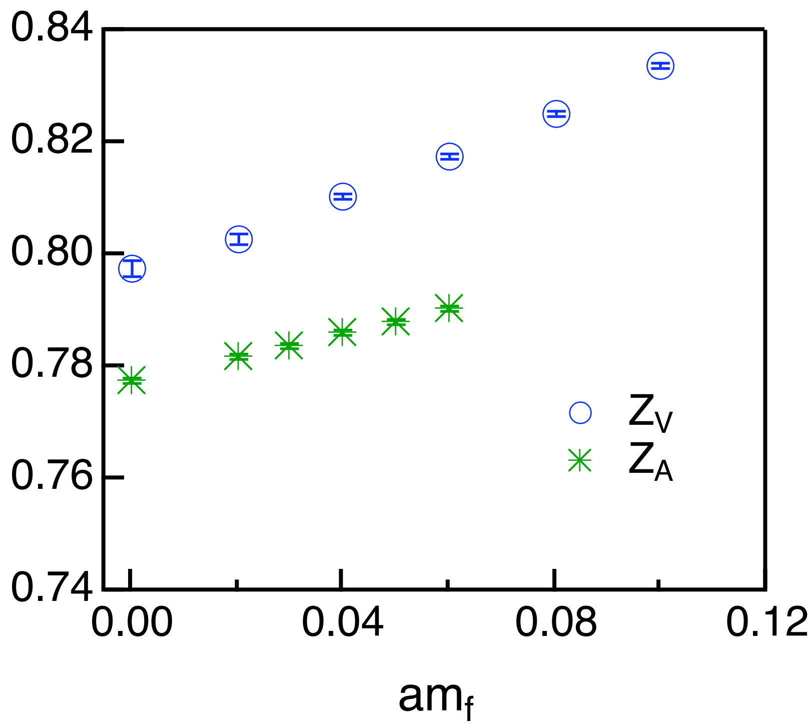
Sasaki-Orginos-Ohta-Blum, PRD68 (03) 054509



$$Z_V = \frac{1}{g_V^{\rm latt}} \propto \frac{\langle N(t')\bar{N}(0)\rangle}{\langle N(t')V_4(t)\bar{N}(0)\rangle}$$

$$Z_A \propto \frac{\langle \mathcal{A}_0^{\rm con}(t)\pi(0)\rangle}{\langle A_0(t)\pi(0)\rangle}$$

Y.Aoki et al., PRD69 (04) 074504

Check of the relation $Z_A=Z_V$

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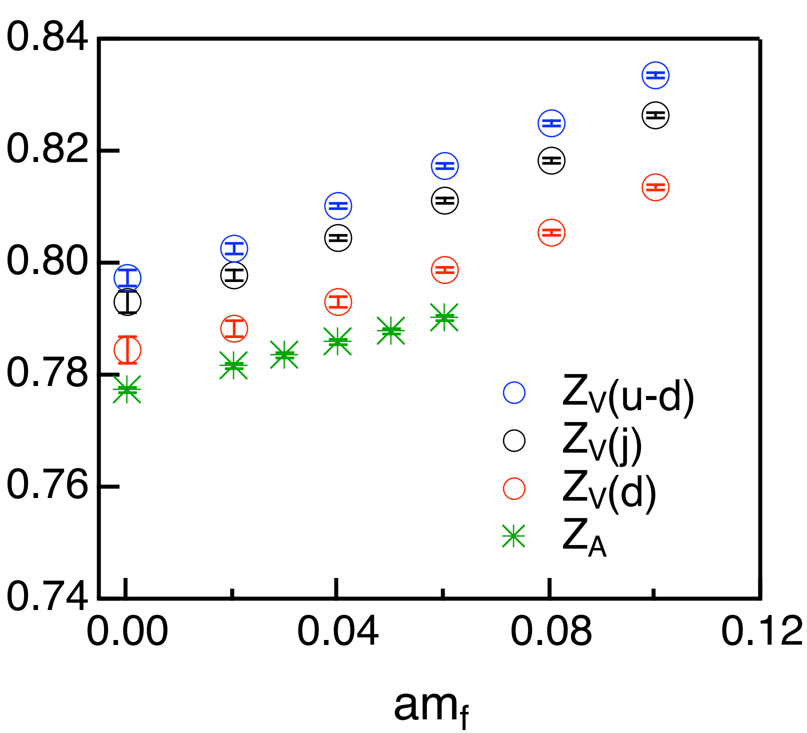


$$Z_V = \frac{1}{g_V^{\rm latt}} \propto \frac{\langle N(t')\bar{N}(0)\rangle}{\langle N(t')V_4(t)\bar{N}(0)\rangle}$$

Violation of

$$\lim_{q^2\to 0}\langle p|j_\mu^{\rm em}|p\rangle = \lim_{q^2\to 0}\langle p|V_\mu^d|p\rangle = \lim_{q^2\to 0}\langle p|V_\mu^u - V_\mu^d|p\rangle$$

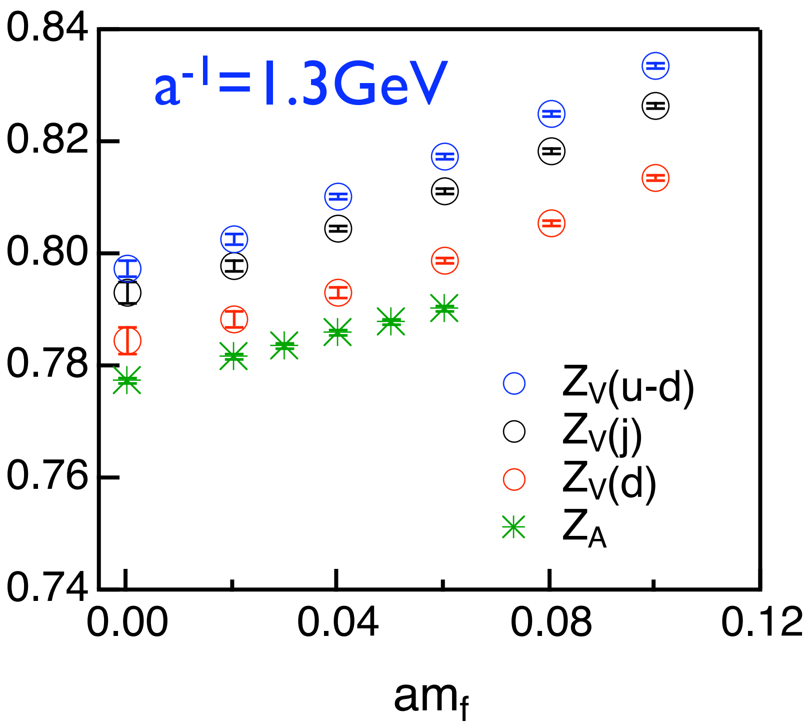
implies an $O(a^2)$ lattice artifact

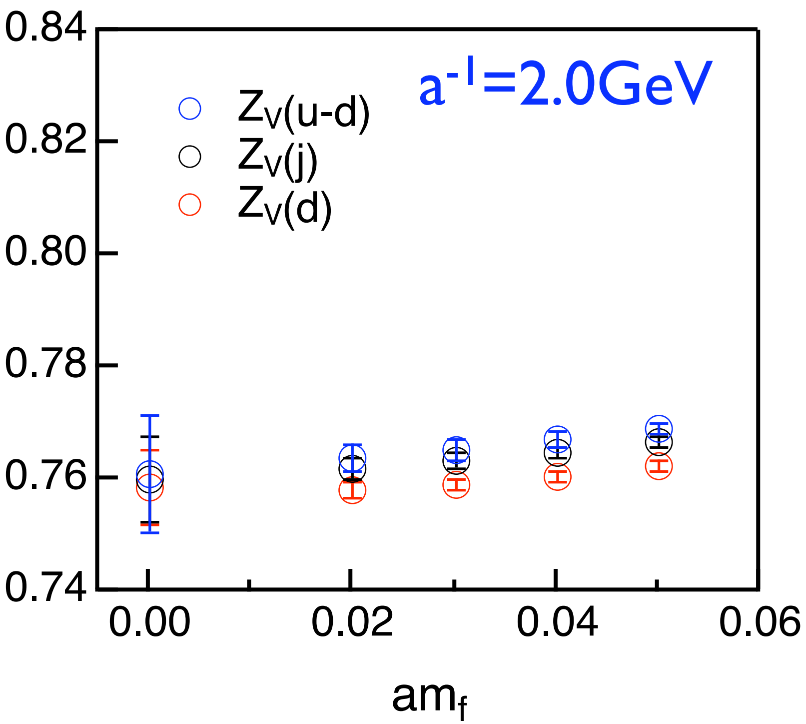
Check of the relation $Z_A=Z_V$

- Nf0: DWF-DBW2 at β=0.87 (a^{-1}=1.3GeV)
  - 16^3 x 32 x 16 with M_5=1.8

coarse lattice

Sasaki-Orginos-Ohta-Blum, PRD68 (03) 054509





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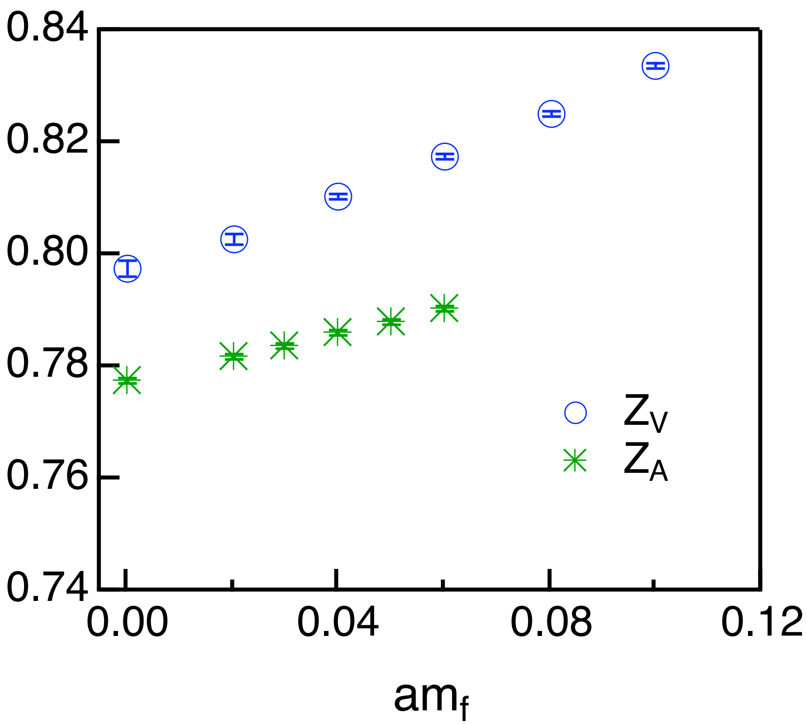


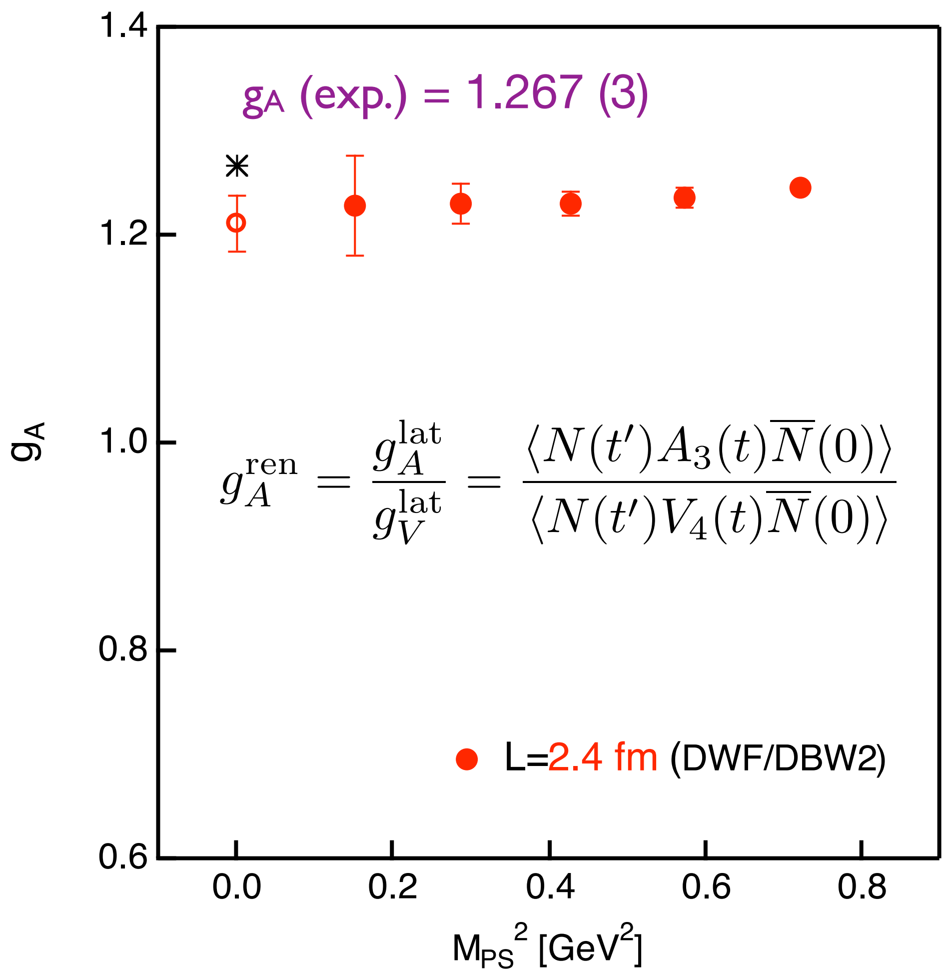
In the chiral limit

$$Z_V = 0.796(3)$$

$$Z_A = 0.7776(5)$$

2-3 % systematic error stemming from determination of Z-factor mainly due to $O(a^2)$ corrections

Quench DWF calculation of g_A



$$g_A^{\rm ren} = \frac{g_A^{\rm lat}}{g_V^{\rm lat}} = \frac{\langle N(t')A_3(t)\overline{N}(0)\rangle}{\langle N(t')V_4(t)\overline{N}(0)\rangle}$$

Sasaki-Orginos-Ohta-Blum, PRD68 (03) 054509

✓ the lightest pion mass, $M_\pi \sim 0.39$ GeV

✓ relatively large volume, $V \sim (2.4\ \mathrm{fm})^3$

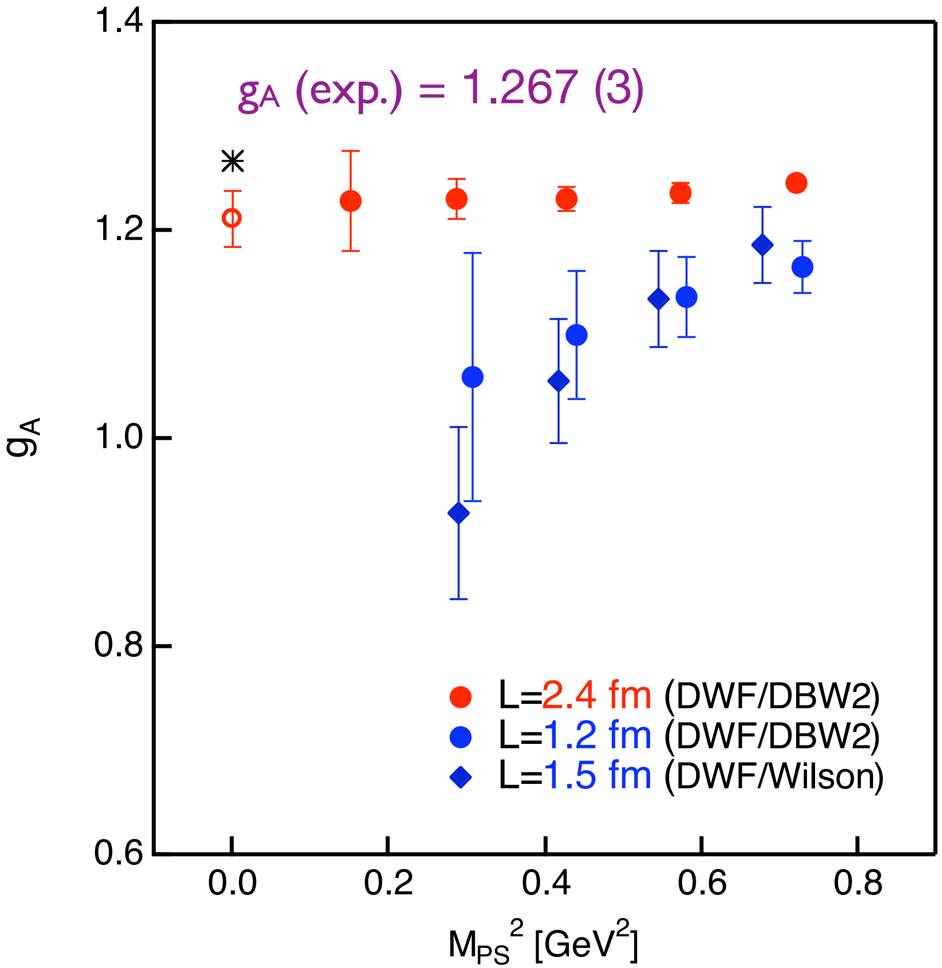
✓ large statistics, 416 configs

✳ mild quark mass dependence

➡ Linear extrapolation yields

g_A = 1.212 (27)

Quench DWF calculation of g_A



Sasaki-Orginos-Ohta-Blum, PRD68 (03) 054509

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✓ relatively large volume, $V \sim (2.4\ \text{fm})^3$

✓ large statistics, 416 configs

✳ mild quark mass dependence

✳ clear finite volume dependence

- a 20 % increase from 1.2 fm to 2.4 fm

resolve the long-standing problem!

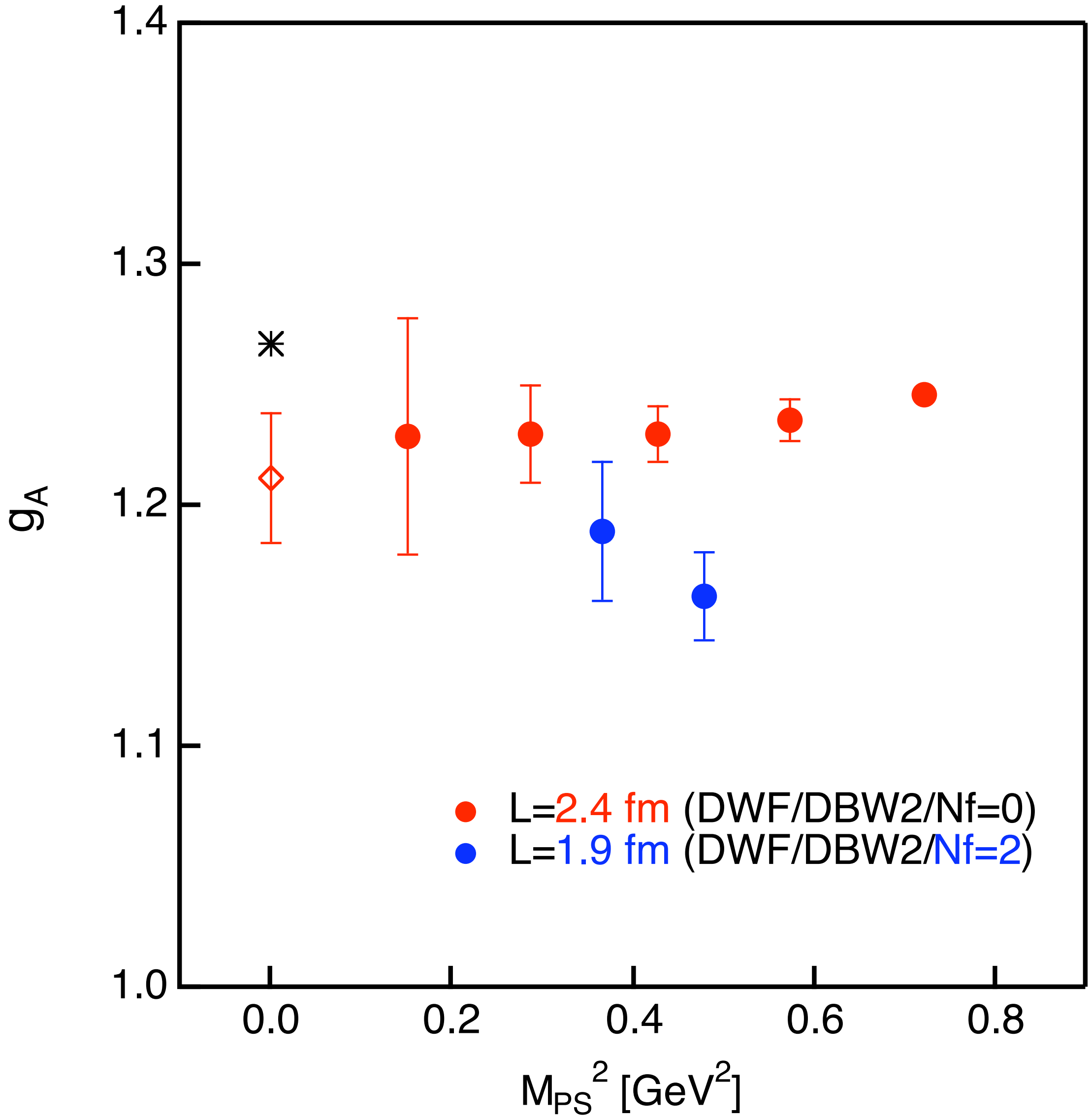
- Nf2: DWF-DBW2 at beta=0.80 (a^{-1}=1.7GeV)
  - 16^3x32x12 (L=1.9 fm): O(5000) MC trajectories
  - m_{sea}=0.02, 0.03, 0.04 (M_π=0.49, 0.61, 0.70 GeV)
  - m_{res}=0.00137(5)
  - f_π= 134.0(42), f_K=157.4(38) MeV
  - $B_K^{\overline{MS}}$(2 GeV)=0.495(18)

RBC collaboration, Phys. Rev. D72, 114505 (05)

- Nf2: DWF-DBW2 at beta=0.80 (a^{-1}=1.7GeV)
  - 16^3x32x12 (L=1.9 fm): 220 statistics
  - m_{sea}=0.03, 0.04
  - m_{sea}=0.02 (underway)
  - Nucleon structure function

Blum, Lin, Ohta, Orginos, Sasaki

Nf=2 DWF calculation of g_A



Nucleon form factors

Neutron beta decay

$$n \longrightarrow p + e^- + \bar{\nu}_e$$

$$\langle B|V_\alpha - A_\alpha|B'\rangle = \bar{u}_B(p)[f_1(q^2)\gamma_\alpha + \frac{f_2(q^2)}{2M_B}\sigma_{\alpha\beta}q_\beta + g_1(q^2)\gamma_\alpha\gamma_5 + \frac{g_3(q^2)}{2M_B}q_\alpha\gamma_5]u_{B'}(p')$$

Under CVC hypothesis (rigid isospin symmetry)

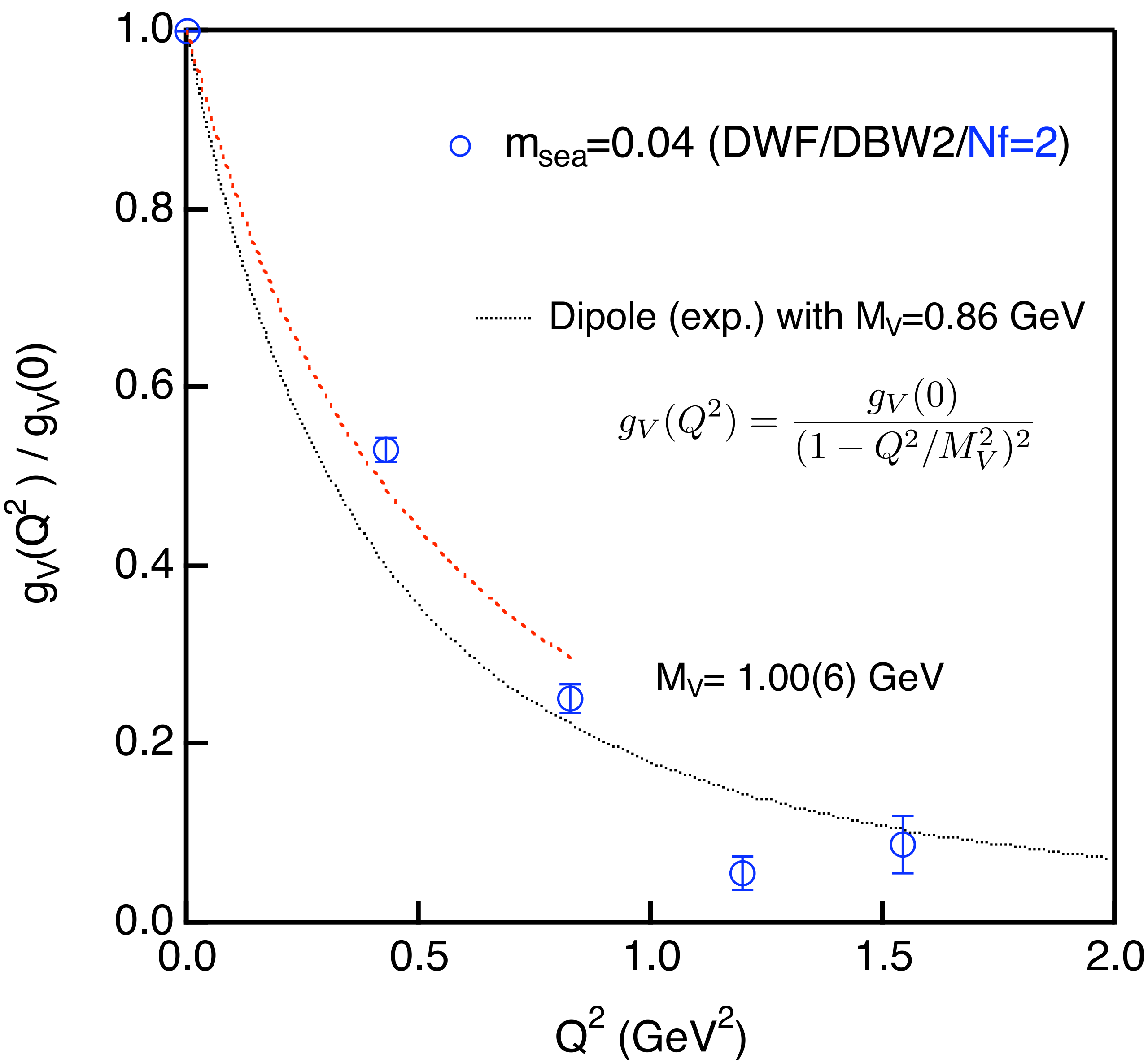
✓ $f_3(q^2) = 0$ $\quad$ $g_2(q^2) = 0$

✓ $\langle p|V_\mu^+ - A_\mu^+|n\rangle = 2\langle p|V_\mu^3 - A_\mu^3|p\rangle$

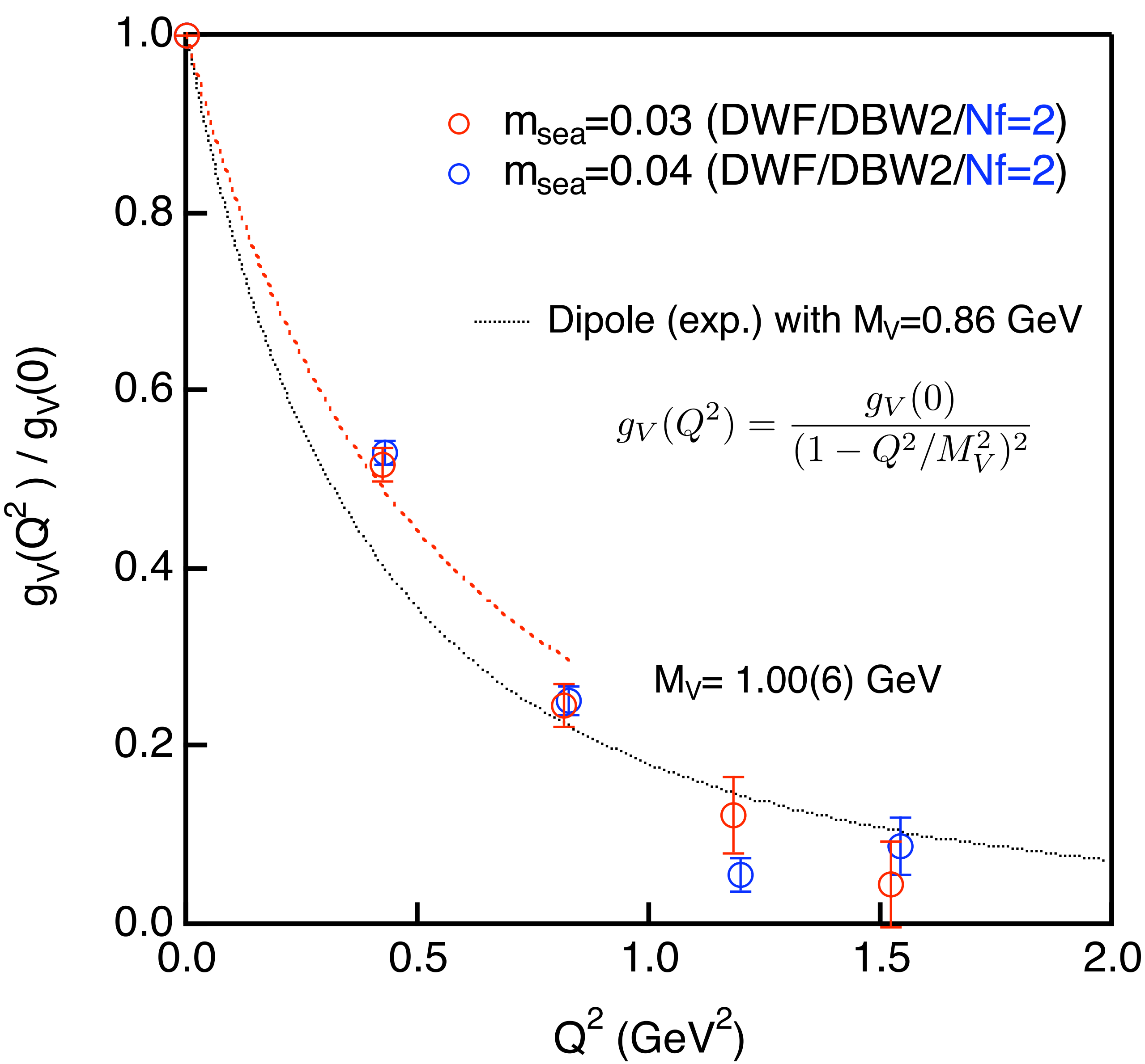
weak matrix element $\quad$ isovector nucleon matrix element

- Nf2: DWF-DBW2 at beta=0.80 (a^{-1}=1.7GeV)
  - 16^3x32x12 (L=1.9 fm): 220 statistics
  - m_{sea}=0.03, 0.04 (M_{π}=0.61, 0.70 GeV)

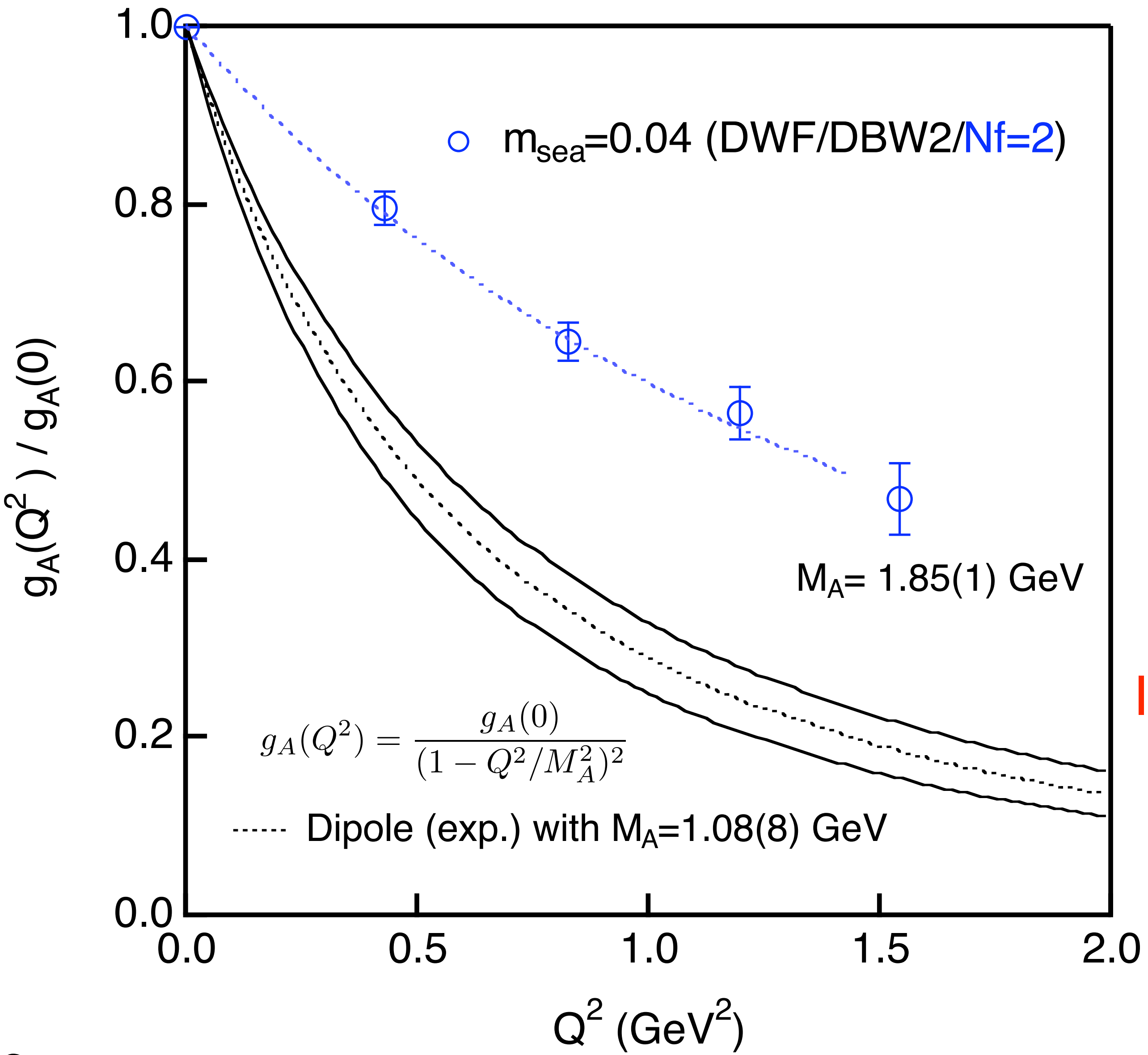
Vector (Dirac form factor; f_1=g_V)



Vector (Dirac form factor; $f_1=g_V$)



Axial-vector ($g_1 = g_A$)



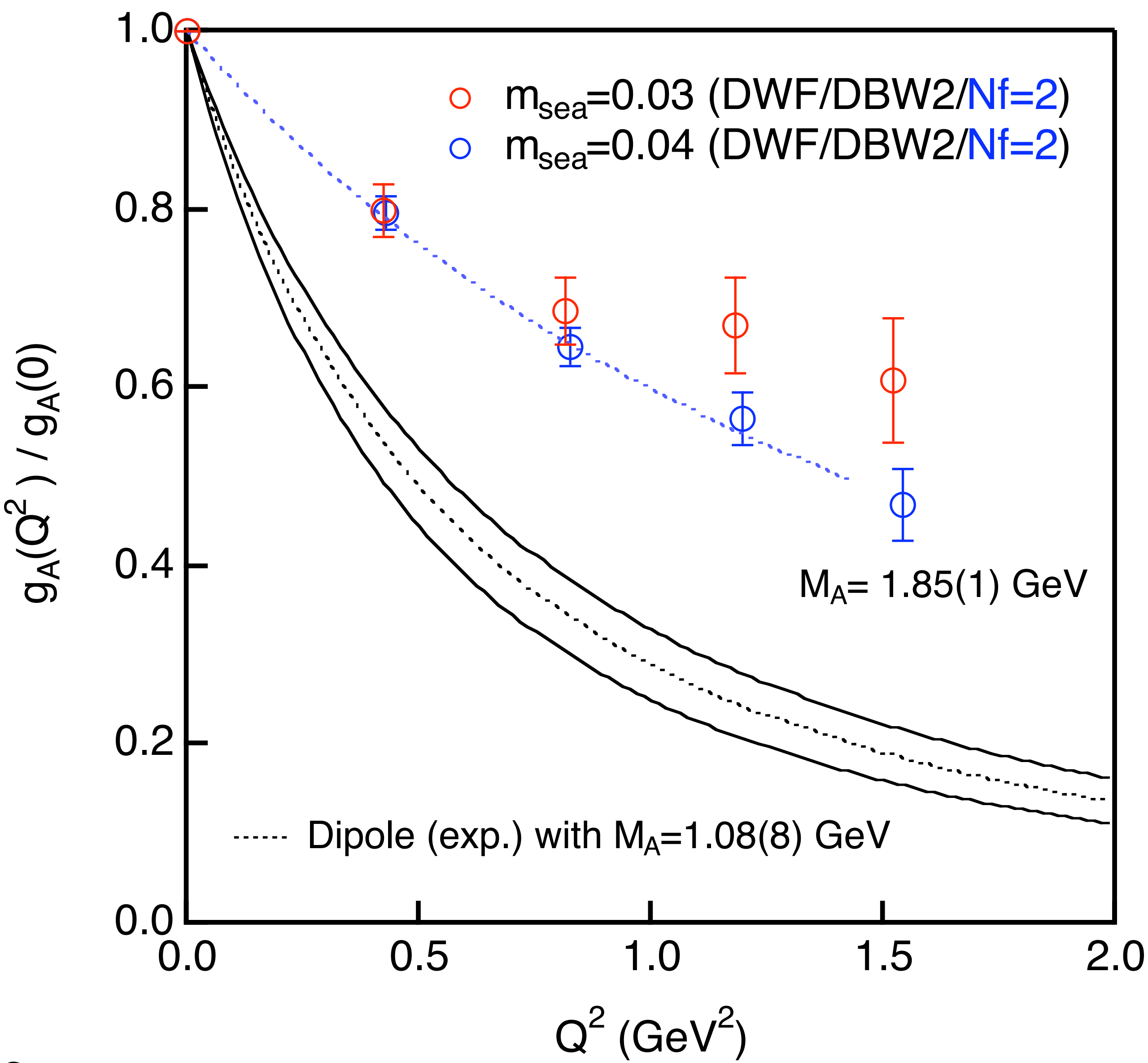
r_A~0.38fm

large discrepancy

r_A~0.63fm

$$\langle r_A^2 \rangle = \frac{12}{M_A^2}$$

Axial-vector ($g_1 = g_A$)

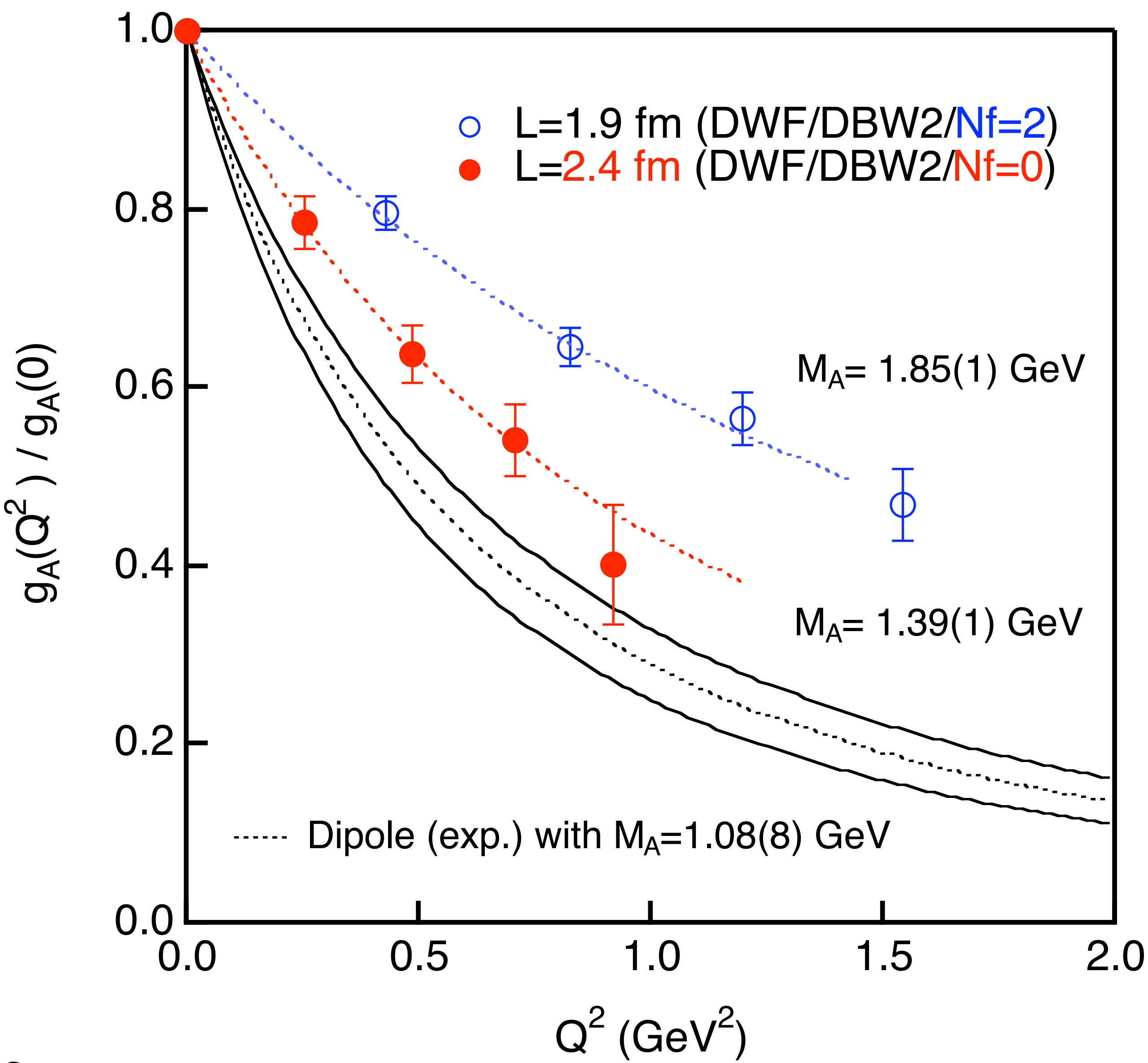


$$\langle r_A^2 \rangle = \frac{12}{M_A^2}$$

No sea-quark mass dependence!

- Nf0: DWF-DBW2 at beta=0.87 (a^{-1}=1.3GeV)
    - 16^3x32x16 (L=2.4 fm): 119 statistics
    - m_f=0.04, 0.05, 0.06, 0.08 (M_π=0.53 - 0.78 GeV)

Axial-vector ($g_1 = g_A$)



r_A~0.38fm

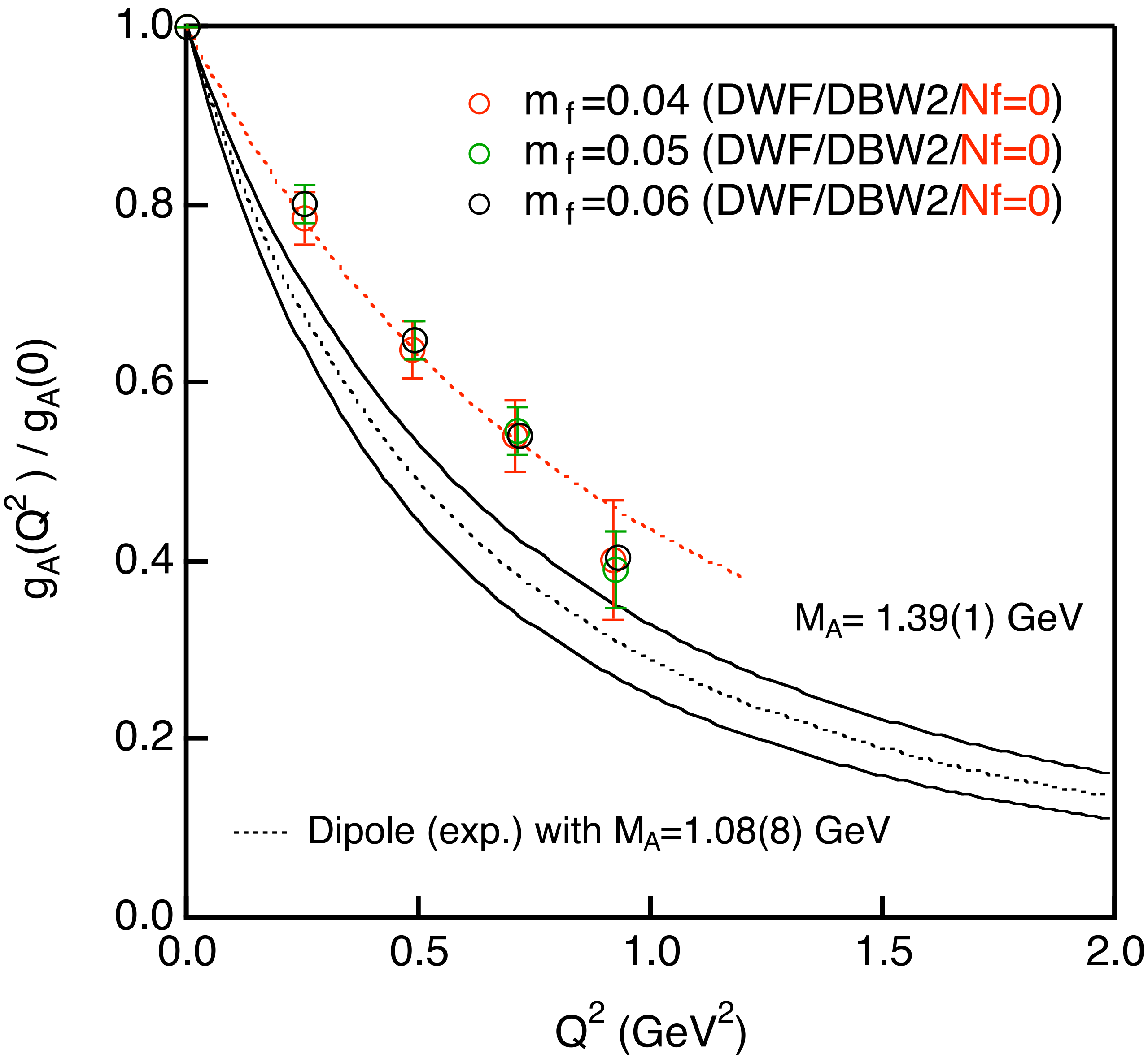
r_A~0.50fm

r_A~0.63fm

$$\langle r_A^2 \rangle = \frac{12}{M_A^2}$$

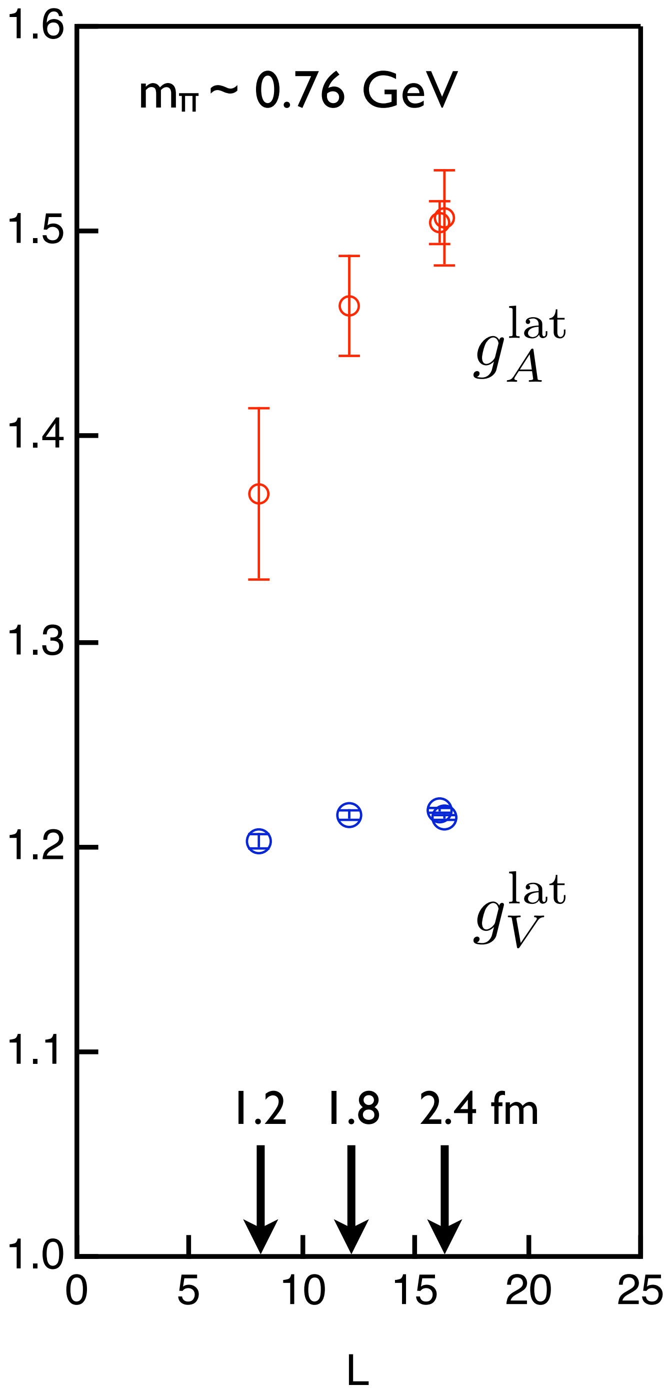
Finite volume effect ?

No quark mass dependence!



- Nf0: DWF-DBW2 at beta=0.87 (a^{-1}=1.3GeV)
  - 12^3x32x16 (L=1.8 fm): 400 statistics

Finite volume effect on g_A an g_V



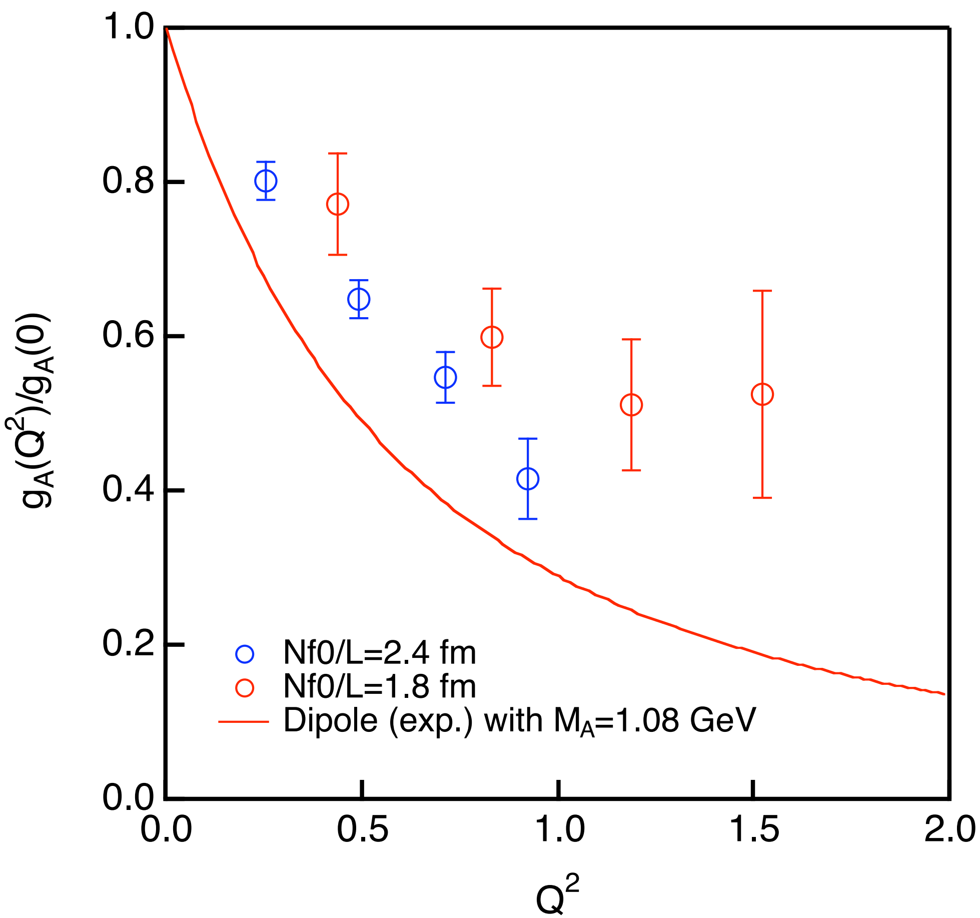
Nf0: DWF-DBW2 at beta=0.87 (a^{-1}=1.3GeV)

- Large finite volume effect on nucleon axial charge
- less volume dependence for nucleon vector charge

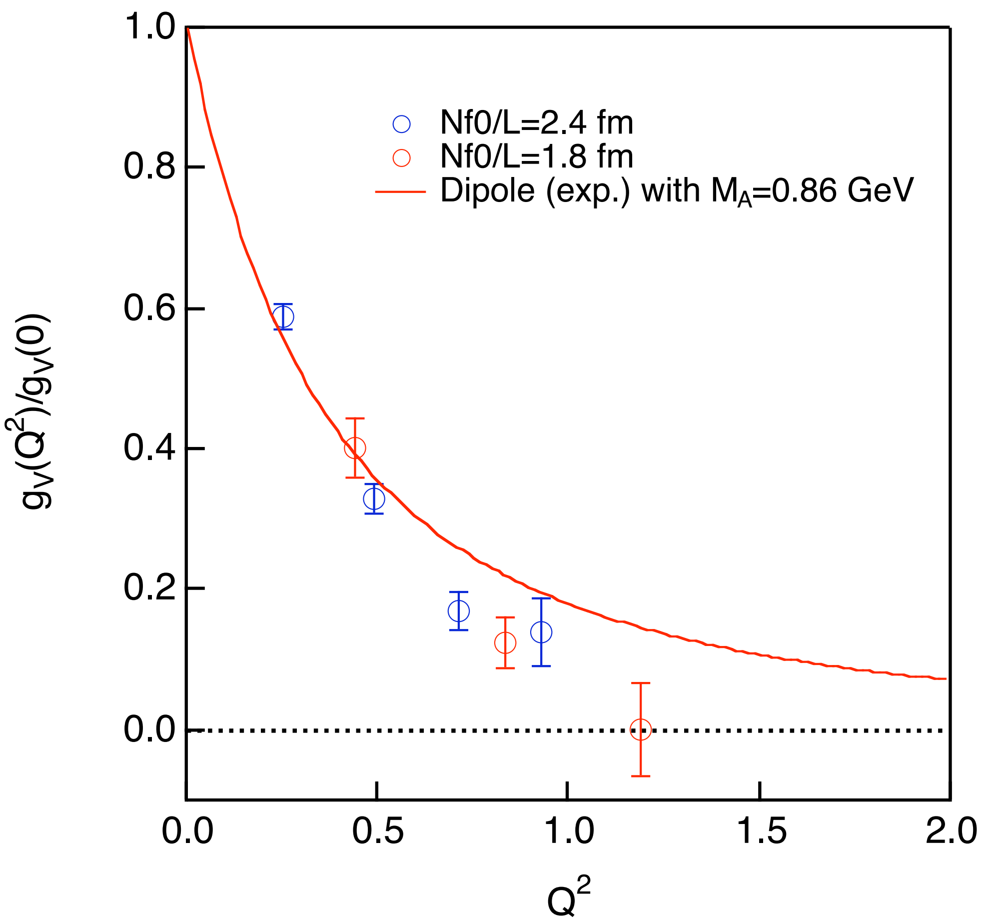
$$g_A^{lat} = \frac{\langle N(t') A_3(t) \overline{N}(0) \rangle}{\langle N(t') \overline{N}(0) \rangle}$$

$$g_V^{lat} = \frac{\langle N(t') V_4(t) \overline{N}(0) \rangle}{\langle N(t') \overline{N}(0) \rangle}$$

Axial-vector

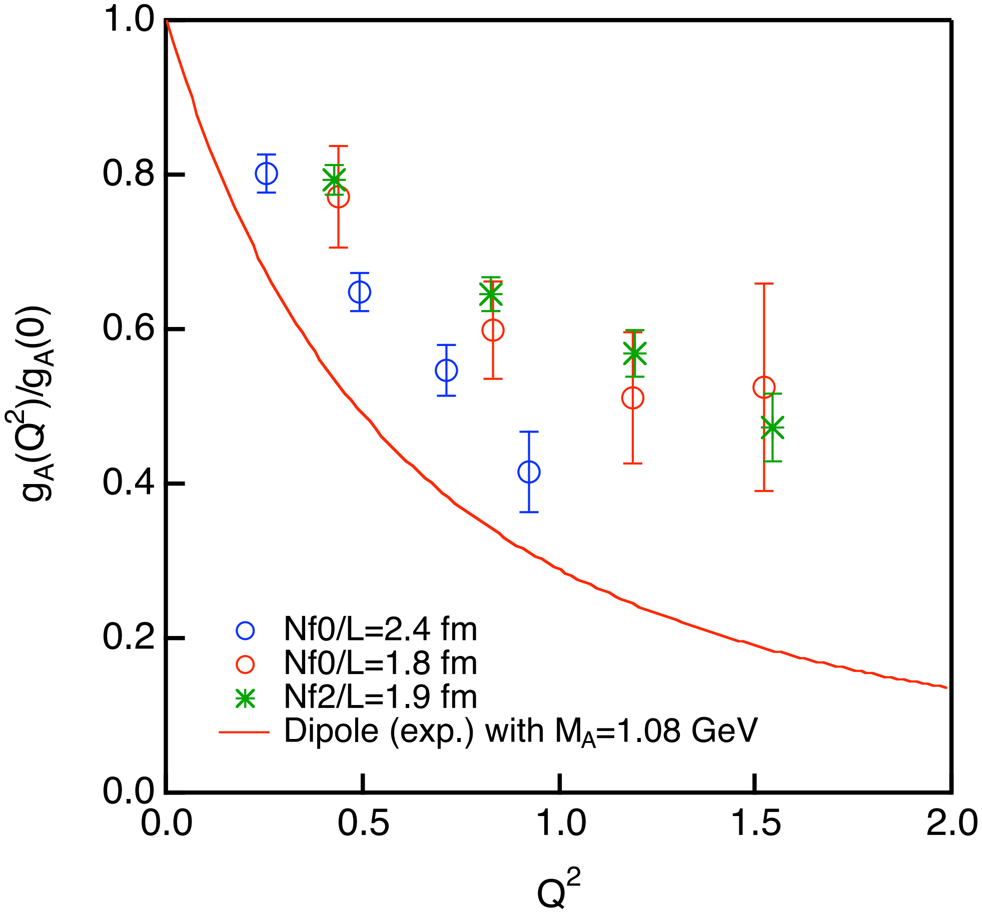


Vector

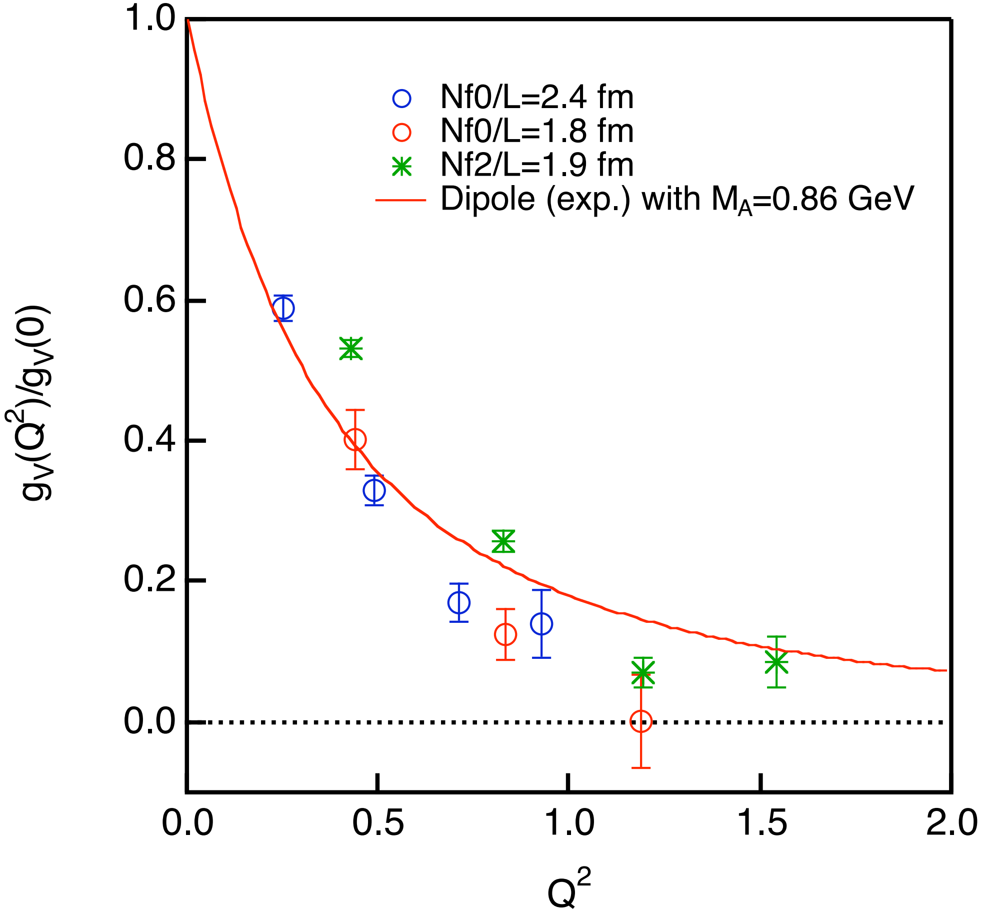


Finite volume effect is observed!

Axial-vector



Vector



Hyperon beta decay

Baryon semi-leptonic decay

- According to DWF study of neutron beta decay
  - ✓ Better control of determination of Z factor, thanks to excellent chiral properties of DWF
  - ✓ g_A/g_V in neutron beta decay is well reproduced within about 5 % accuracy even in quenched calculation
  - ✓ The lightest baryon (nucleon) seems to be fitted in 2.4 fm lattice size (quench)

In the next stage,
we explore the hyperon beta decay in lattice QCD

Hyperon Beta Decay (Expt.)

$B' \to Bl\nu$	$f_1^{SU(3)}$	g_1/f_1 (Exp.)	$(g_1/f_1)^{SU(3)}$
$n \to p$	1	1.2670± 0.0030	$F + D$
$\Lambda \to p$	$-\frac{\sqrt{6}}{2}$	0.718± 0.015	$F + \frac{1}{3}D$
$\Xi^- \to \Lambda$	$\frac{\sqrt{6}}{2}$	0.25 ± 0.05	$F - \frac{1}{3}D$
$\Sigma^- \to n$	-1	-0.340 ± 0.017	$F - D$
$\Xi^0 \to \Sigma^+$	1	1.32± 0.21	$F + D$
$\Xi^- \to \Xi^0$	-1	N/A	$F - D$

$g_V = \lim_{q^2 \to 0} f_1(q^2)$ $\quad g_A = \lim_{q^2 \to 0} g_1(q^2)$

Hyperon Beta Decay (Expt.)

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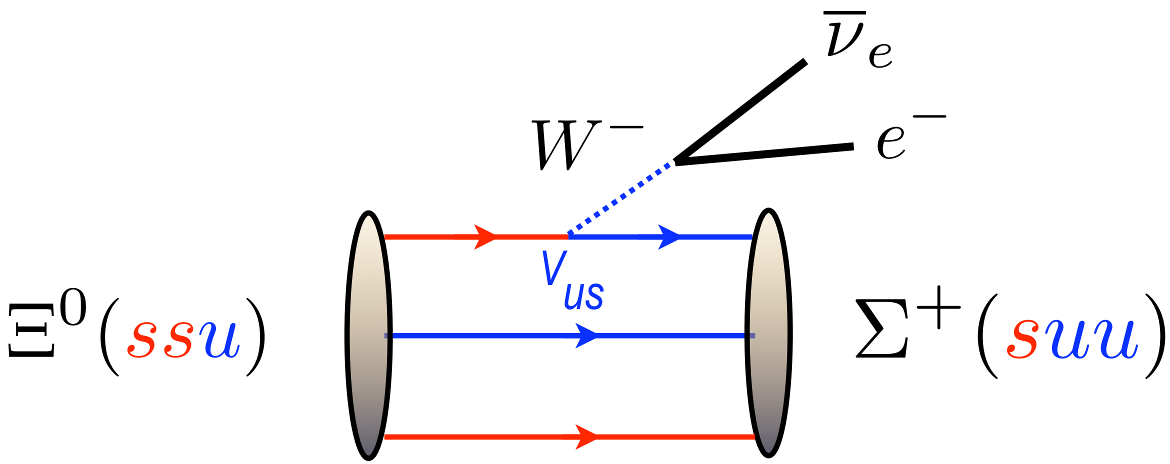
$g_V = \lim_{q^2 \to 0} f_1(q^2)$ $\quad g_A = \lim_{q^2 \to 0} g_1(q^2)$

$\Xi^0 \to \Sigma^+$ is the direct analogue of $n \to p$ under d ↔ s

$\Xi^- \to \Xi^0$ is the direct analogue of $\Sigma^- \to n$ under d ↔ s

Hyperon Beta Decay ($\Xi^0 \rightarrow \Sigma^+$)

$\Xi^0 \rightarrow \Sigma^+$ is the direct analogue of $n \rightarrow p$ under $d \leftrightarrow s$



highly sensitive to SU(3) breaking

Center of mass correction approach (Ratcliffe)

$(g_A/g_V)_{np} > (g_A/g_V)_{\Xi\Sigma}$ 8-10%

$1/N_c$ expansion approach (Flores-Mendieta-Jenkins-Manohar)

$(g_A/g_V)_{np} > (g_A/g_V)_{\Xi\Sigma}$ 20-30%

Summary on $\Xi^0 \rightarrow \Sigma^+$ (exp.)

- First and Single experiment at KTeV@FNAL
  - $g_1 / f_1 = 1.17 \pm 0.28(\text{stat}) \pm 0.05(\text{syst})$
  - $g_2 / f_1 = -1.7 \pm 2.0(\text{stat}) \pm 0.5\ (\text{syst})$

    ➡ no evidence for a nonzero value of the g_2 form factor (2nd-class)

✓ Assumming $g_2 / f_1 = 0$

$n \rightarrow p$: $g_1 / f_1 = 1.2670(35)$

- $g_1 / f_1 = 1.32 \pm 0.21(\text{stat}) \pm 0.05(\text{syst})$

  ➡ no indication of flavor SU(3) breaking effects

2nd-class current

$$\langle B|V_\alpha - A_\alpha|B'\rangle = \bar{u}_B(p)[f_1(q^2)\gamma_\alpha + \frac{f_2(q^2)}{2M_{B'}}\sigma_{\alpha\beta}q_\beta + \frac{f_3(q^2)}{2M_{B'}}q_\alpha$$

$$+g_1(q^2)\gamma_\alpha\gamma_5 + \frac{g_2(q^2)}{2M_{B'}}\sigma_{\alpha\beta}\gamma_5 q_\beta + \frac{g_3(q^2)}{2M_{B'}}q_\alpha\gamma_5]u_{B'}(p')$$

- Time reversal invariance requires all 6 form factors to be real
- With respect to transformation under (extended) G-parity,

1st class $\quad Gf_{1,2}(Q^2)G^{-1} = +f_{1,2}(Q^2) \quad Gg_{1,3}(Q^2)G^{-1} = -g_{1,3}(Q^2)$

2nd class $\quad Gf_3(Q^2)G^{-1} = -f_3(Q^2) \quad Gg_2(Q^2)G^{-1} = +g_2(Q^2)$

- (extended) G-parity invariance requires $\quad G = Ce^{-i\pi T_{2,5,7}}$

$$f_3(Q^2) = 0 \qquad g_2(Q^2) = 0$$

2nd-class current

$$\langle B|V_\alpha - A_\alpha|B'\rangle = \bar{u}_B(p)[f_1(q^2)\gamma_\alpha + \frac{f_2(q^2)}{2M_{B'}}\sigma_{\alpha\beta}q_\beta + \frac{f_3(q^2)}{2M_{B'}}q_\alpha$$

$$+g_1(q^2)\gamma_\alpha\gamma_5 + \frac{g_2(q^2)}{2M_{B'}}\sigma_{\alpha\beta}\gamma_5 q_\beta + \frac{g_3(q^2)}{2M_{B'}}q_\alpha\gamma_5]u_{B'}(p')$$

- Time reversal invariance requires all 6 form factors to be real
- With respect to transformation under (extended) G-parity,

1st class $Gf_{1,2}(Q^2)G^{-1} = +f_{1,2}(Q^2)$ $Gg_{1,3}(Q^2)G^{-1} = -g_{1,3}(Q^2)$

2nd class $Gf_3(Q^2)G^{-1} = -f_3(Q^2)$ $Gg_2(Q^2)G^{-1} = +g_2(Q^2)$

- ~~(extended) G parity invariance~~ requires $G = Ce^{-i\pi T_{2,5,7}}$

SU(3) breaking $f_3(Q^2) \neq 0$ $g_2(Q^2) \neq 0$

An exploratory study

- ✳ Extract the 2nd-class form factors (g_2 and f_3)
  - ✓ SU(3) breaking = existence of non-zero g_2 and f_3
- ✳ Quantify the SU(3) breaking effect on g_1 / f_1
  - ✓ double ratio:

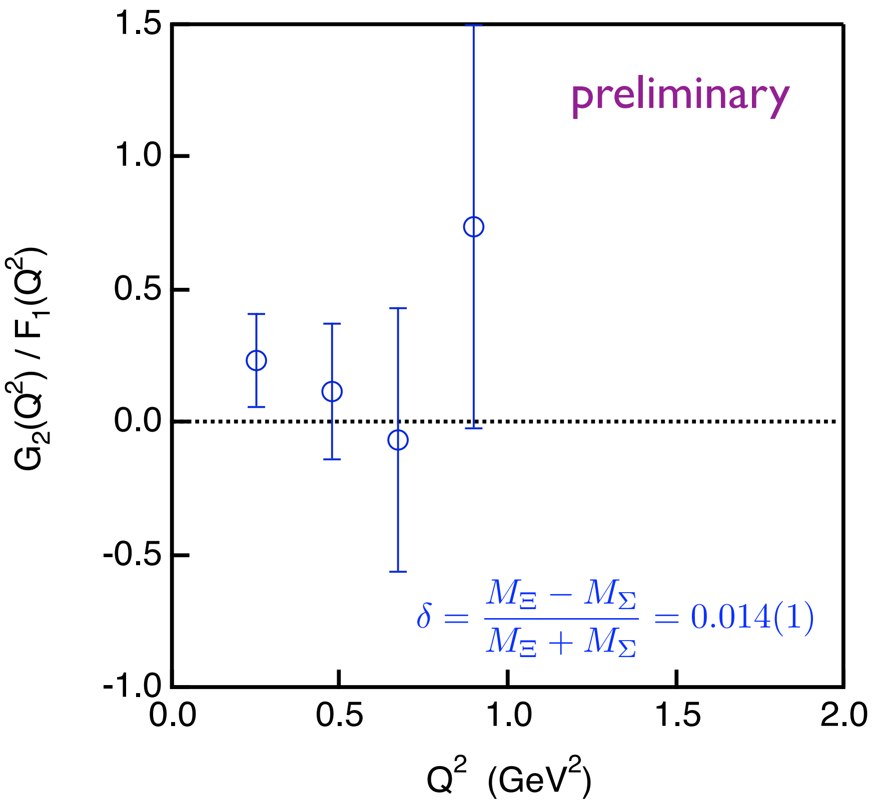
$$\frac{\langle \Sigma(t')A_3(t)\overline{\Xi}(0)\rangle}{\langle \Sigma(t')V_4(t)\overline{\Xi}(0)\rangle}\frac{\langle N(t')V_4(t)\overline{N}(0)\rangle}{\langle N(t')A_3(t)\overline{N}(0)\rangle} = \frac{g_1(q^2_{\rm max})}{f_1(q^2_{\rm max}) - \delta f_3(q^2_{\rm max})}\left(\frac{f_1(0)}{g_1(0)}\right)_{\rm SU(3)} \approx \left(\frac{g_1(0)}{f_1(0)}\right)\left(\frac{f_1(0)}{g_1(0)}\right)_{\rm SU(3)}$$

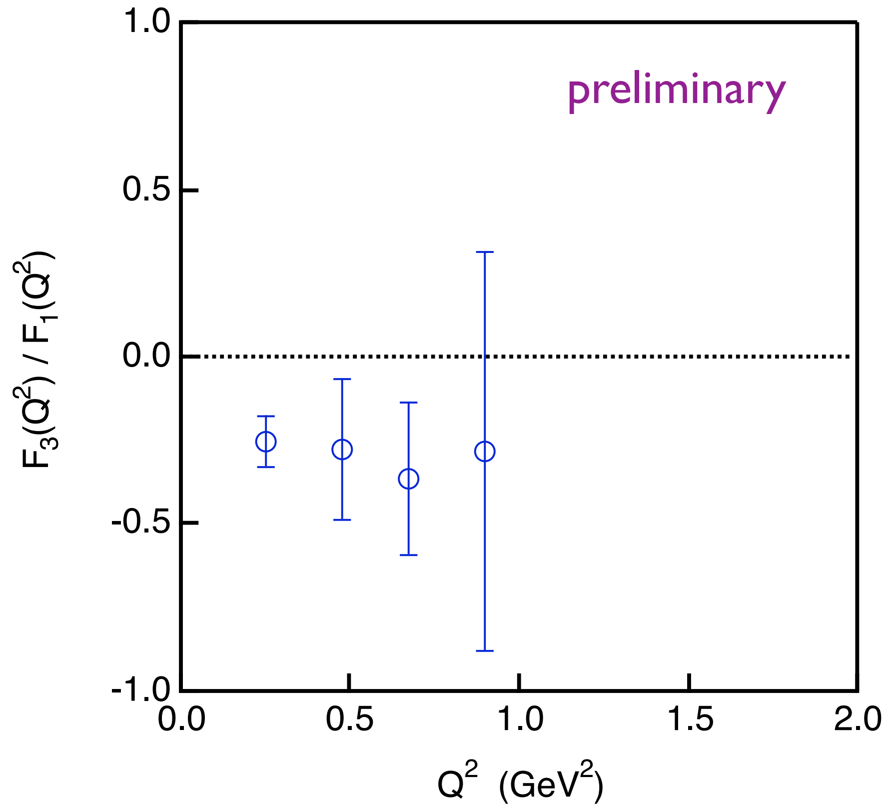
$$\delta = \frac{M_\Xi - M_\Sigma}{M_\Xi + M_\Sigma} \qquad q^2_{\rm max} = -(M_\Xi - M_\Sigma)^2$$

- Nf0: DWF-DBW2 at beta=0.87 (a^{-1}=1.3GeV)
  - 16^3x32x16 (L=2.4 fm): 119 statistics
  - m_l=0.04, 0.05, 0.06 (M_π=0.53, 0.60, 0.65 GeV)
  - fixed strange quark mass at m_s=0.08

2nd-class form factors

m_l=0.05, m_s=0.08



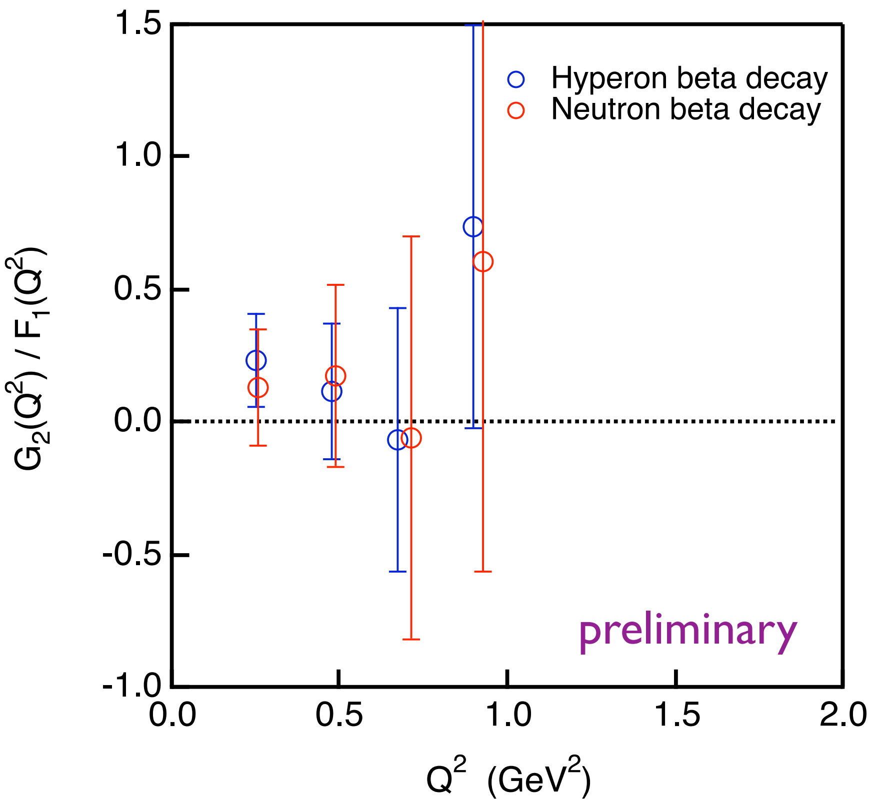


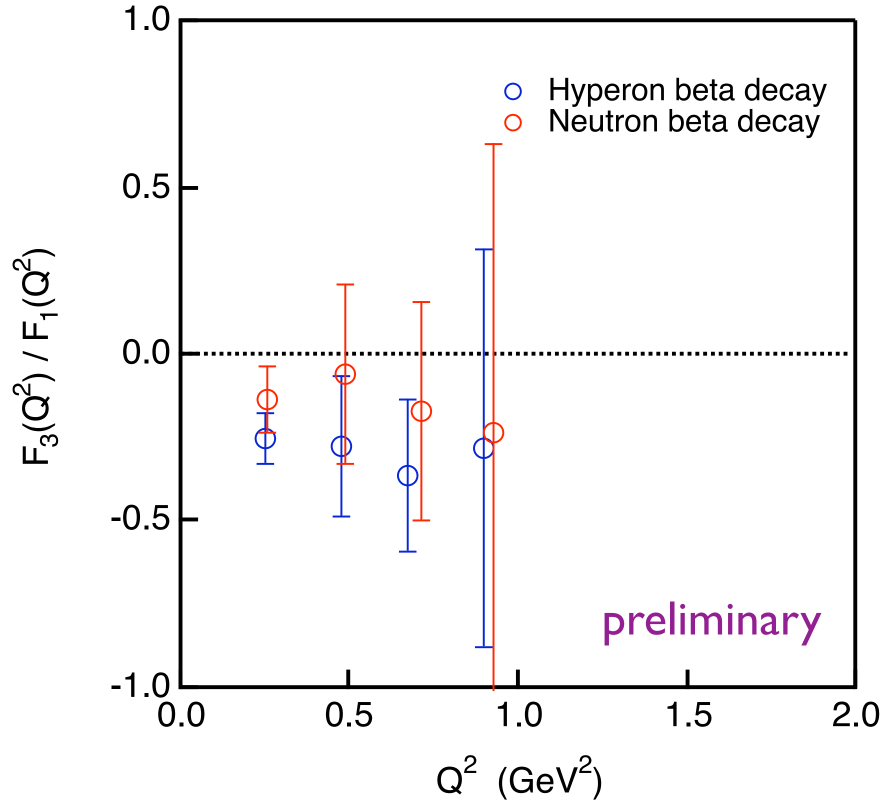
$$\left|\frac{g_2(0.25\ \mathrm{GeV}^2)}{f_1(0.25\ \mathrm{GeV}^2)}\right| = 0.24(18)$$

$$\left|\frac{f_3(0.25\ \mathrm{GeV}^2)}{f_1(0.25\ \mathrm{GeV}^2)}\right| = 0.25(8)$$

2nd-class form factors

m_l=0.05, m_s=0.08





$$\left|\frac{g_2(0.25\ \mathrm{GeV}^2)}{f_1(0.25\ \mathrm{GeV}^2)}\right| = 0.24(18)(13)$$

$$\left|\frac{f_3(0.25\ \mathrm{GeV}^2)}{f_1(0.25\ \mathrm{GeV}^2)}\right| = 0.25(8)(13)$$

2nd-class form factors

✓ Preliminary result (quenched lattice QCD)

- at δ=0.014(1) (m_l=0.05, m_s=0.08)

$$\left|\frac{g_2(0.25\ \mathrm{GeV}^2)}{f_1(0.25\ \mathrm{GeV}^2)}\right| = 0.24 \pm 0.18 \pm 0.13$$

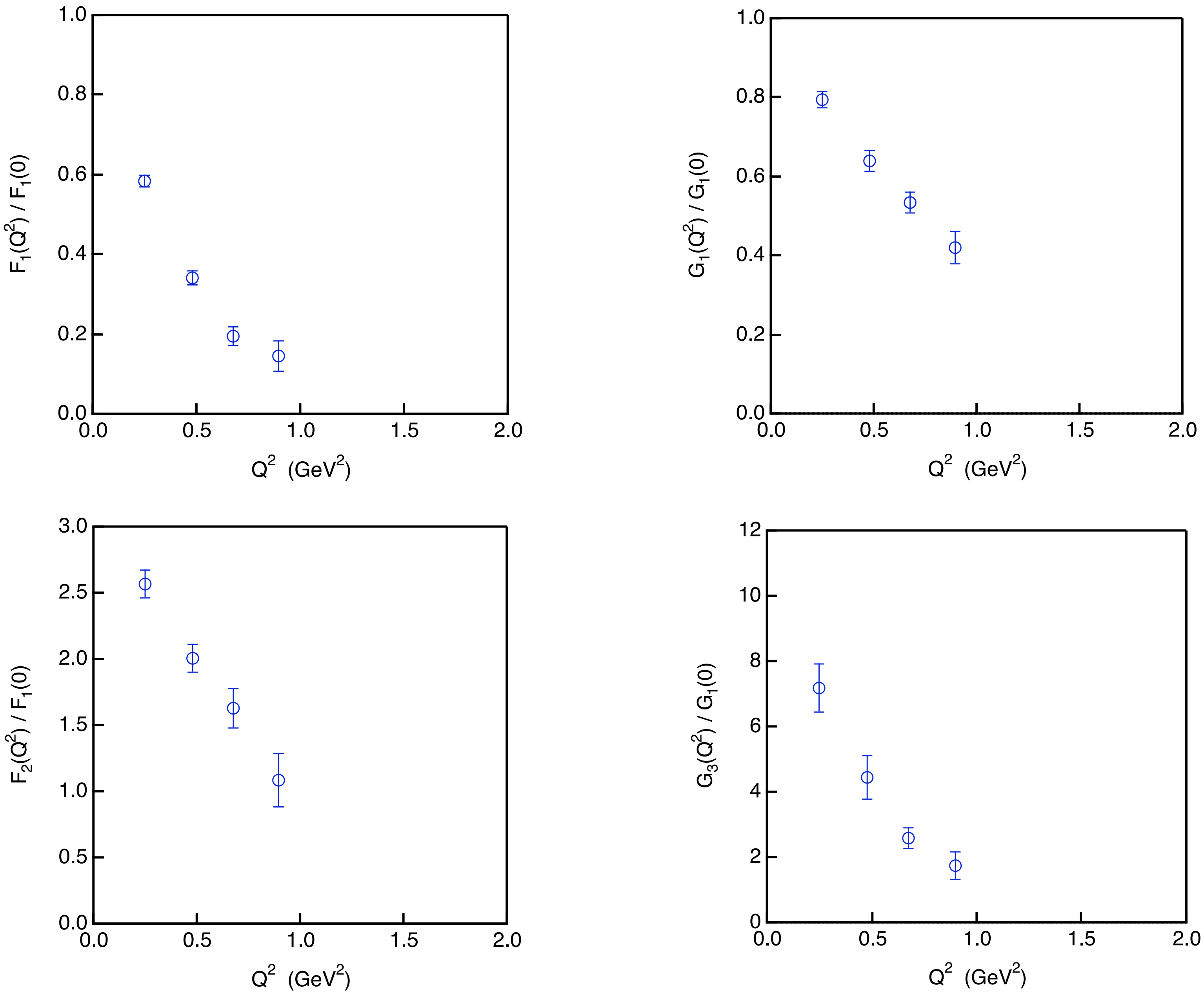
✴ KTeV@FNAL

- δ=0.05 (phys.)

$$\left|\frac{g_2(0)}{f_1(0)}\right| = 1.7 \pm 2.0 \pm 0.5$$

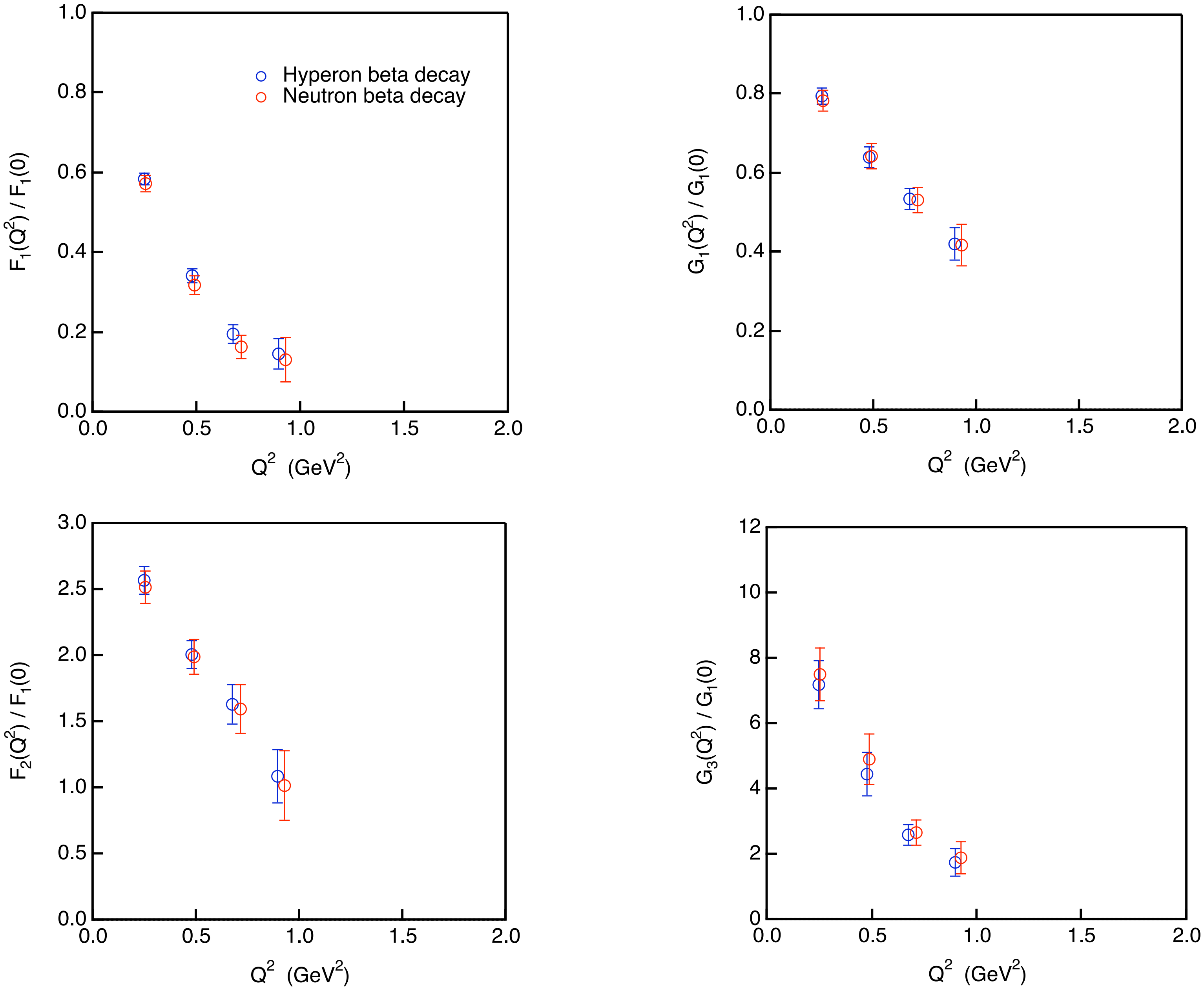
Other form factors

m_l=0.05, m_s=0.08



Other form factors

m_l=0.05, m_s=0.08



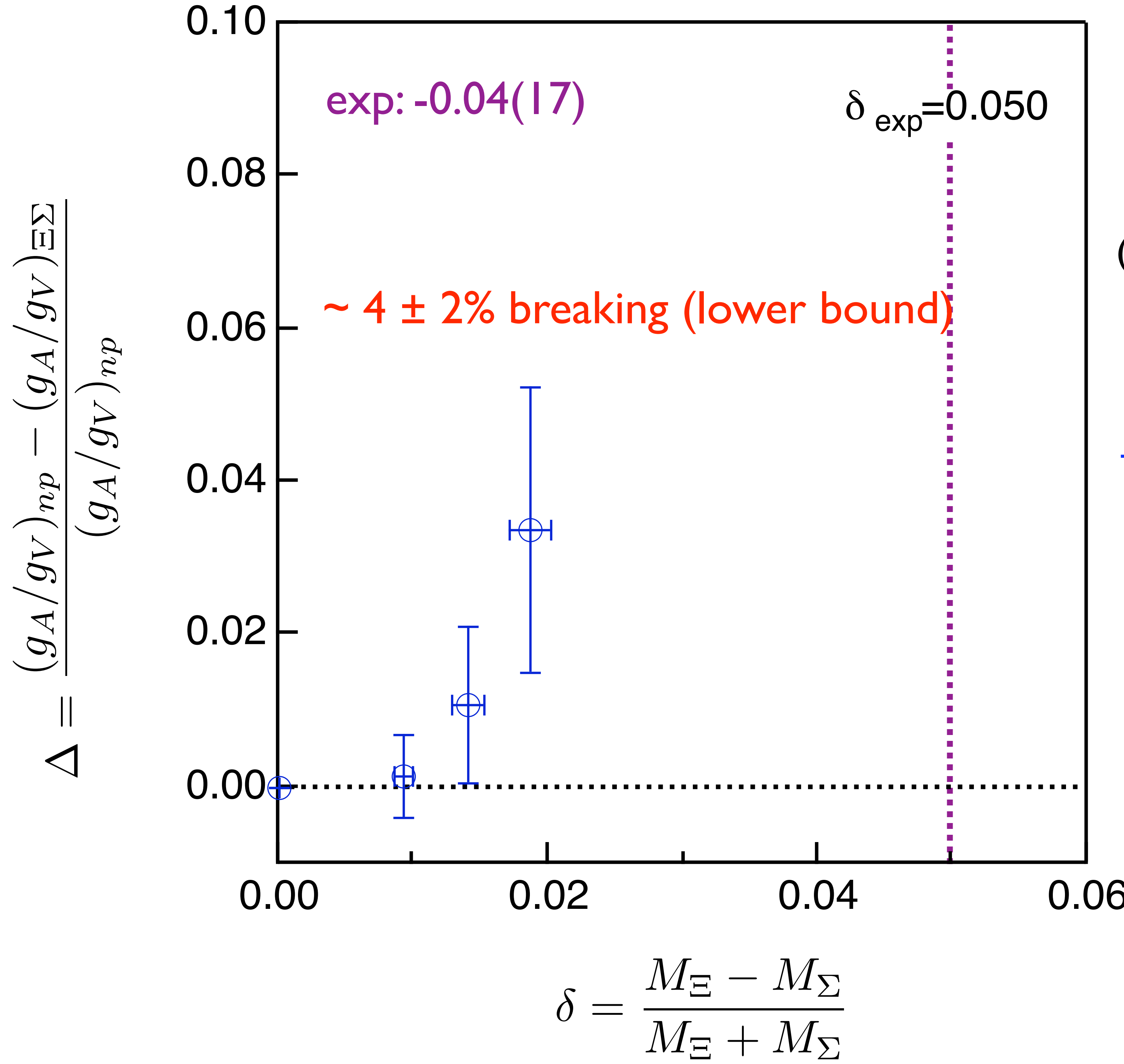
the SU(3) breaking effect on g_1 / f_1

- Consider the following double ratio at the rest frame (p,k=0)

$$\frac{\langle\Sigma(t')A_3(t)\overline{\Xi}(0)\rangle}{\langle\Sigma(t')V_4(t)\overline{\Xi}(0)\rangle}\frac{\langle N(t')V_4(t)\overline{N}(0)\rangle}{\langle N(t')A_3(t)\overline{N}(0)\rangle} = \frac{g_1(q^2_{\rm max})}{f_1(q^2_{\rm max})-\delta f_3(q^2_{\rm max})}\left(\frac{f_1(0)}{g_1(0)}\right)_{\rm SU(3)}$$

$$= \left(\frac{g_1(0)}{f_1(0)}\right)\left(\frac{f_1(0)}{g_1(0)}\right)_{\rm SU(3)} + \mathcal{O}(\delta^2)$$

where $\delta = \frac{M_\Xi - M_\Sigma}{M_\Xi + M_\Sigma}$ $\quad q^2_{\rm max} = -(M_\Xi - M_\Sigma)^2$



$(m_l, m_s) = (0.04, 0.08), (0.05, 0.08), (0.06, 0.08)$

Theoretical expectation

$$\Delta \propto \delta$$

Summary/Outlook

✳ The computation of weak matrix elements in lattice QCD is now progressing with steadily increasing accuracy by utilizing domain wall fermions (DWF).

- DWF has a big advantage in dealing with the axial symmetry
- It is easy to determine Z-factors for V(A) local currents

✓ Neutron beta decay corresponds to a "gold plated" test

- The axial-vector channel significantly suffers from the finite volume effect, while it is hardly observed in the vector channel.

Summary/Outlook

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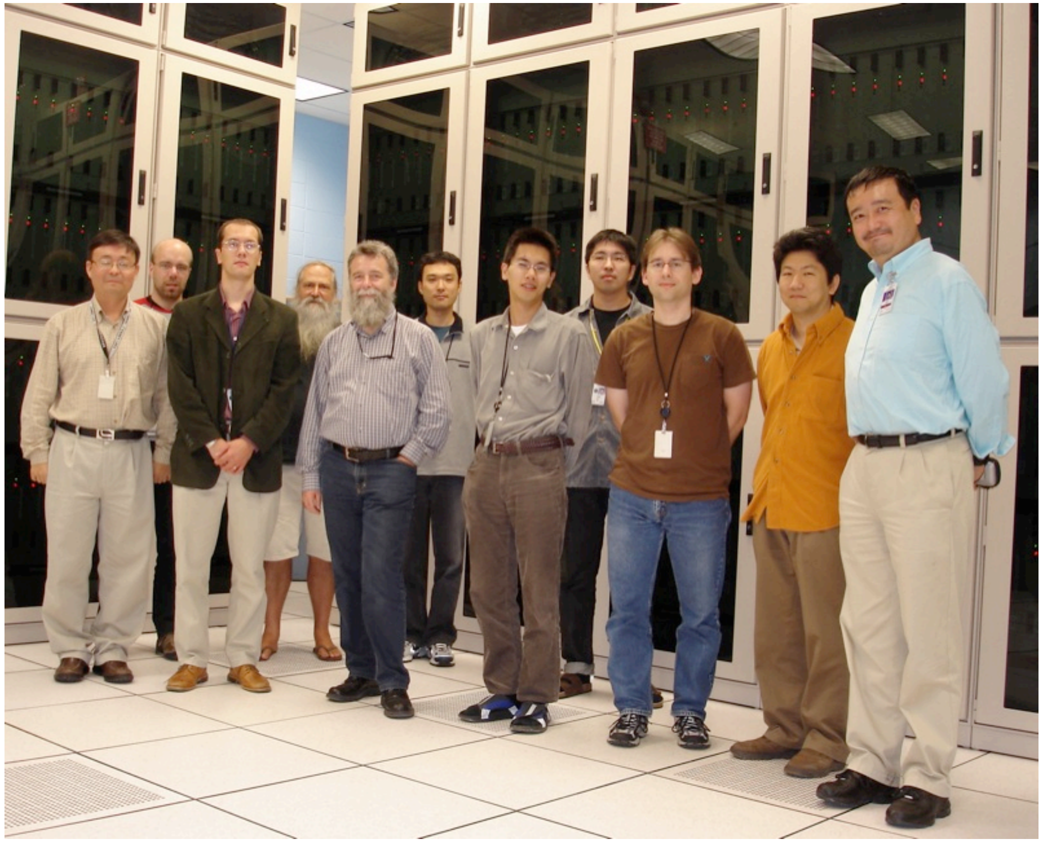
✓ Neutron beta decay corresponds to a "gold plated" test

✓ There is a challenging issue to explore the SU(3) breaking effect in hyperon beta decay through a first principles calculation.

➡ Our exploratory study in quenched DWF calculation:

- Succeeded in evaluating 2nd-class form factors from lattice QCD
- Observed the SU(3) breaking effect with higher accuracy than expt.

Generating 2+1 flavor DWF configurations



QCDOC with RBRC & BNL Lattice theorists

Large scale production run

- DWF + Iwasaki gauge action
- Lattice cutoff: $a^{-1} \sim 1.7$ GeV ($\beta = 2.13$, $c_1 = -0.331$)
- Box size: $V = 24^3 \times 64 \times 16$

  $L \sim 2.8$ fm
- $m_{light} = 3/4, 1/2, 1/4$ of $m_{strange}$

  $M_\pi \sim 350, 500, 750$ MeV

in collaboration with Columbia, UKQCD

Summary/Outlook

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  - DWF has a big advantage in dealing with the axial symmetry
  - It is easy to determine Z-factors for V(A) local currents

- ✓ Neutron beta decay corresponds to a "gold plated" test
- ✓ There is a challenging issue to explore the SU(3) breaking effect in hyperon beta decay through a first principles calculation.

Future:

2+1 flavors DWF calculation (RBC+UKQCD) is promising for theoretical research on the SU(3) breaking effect in baryon semi-leptonic decays.