Pomeron and Gauge/String Duality†

Sokendai, Japan--- March 9, 2006

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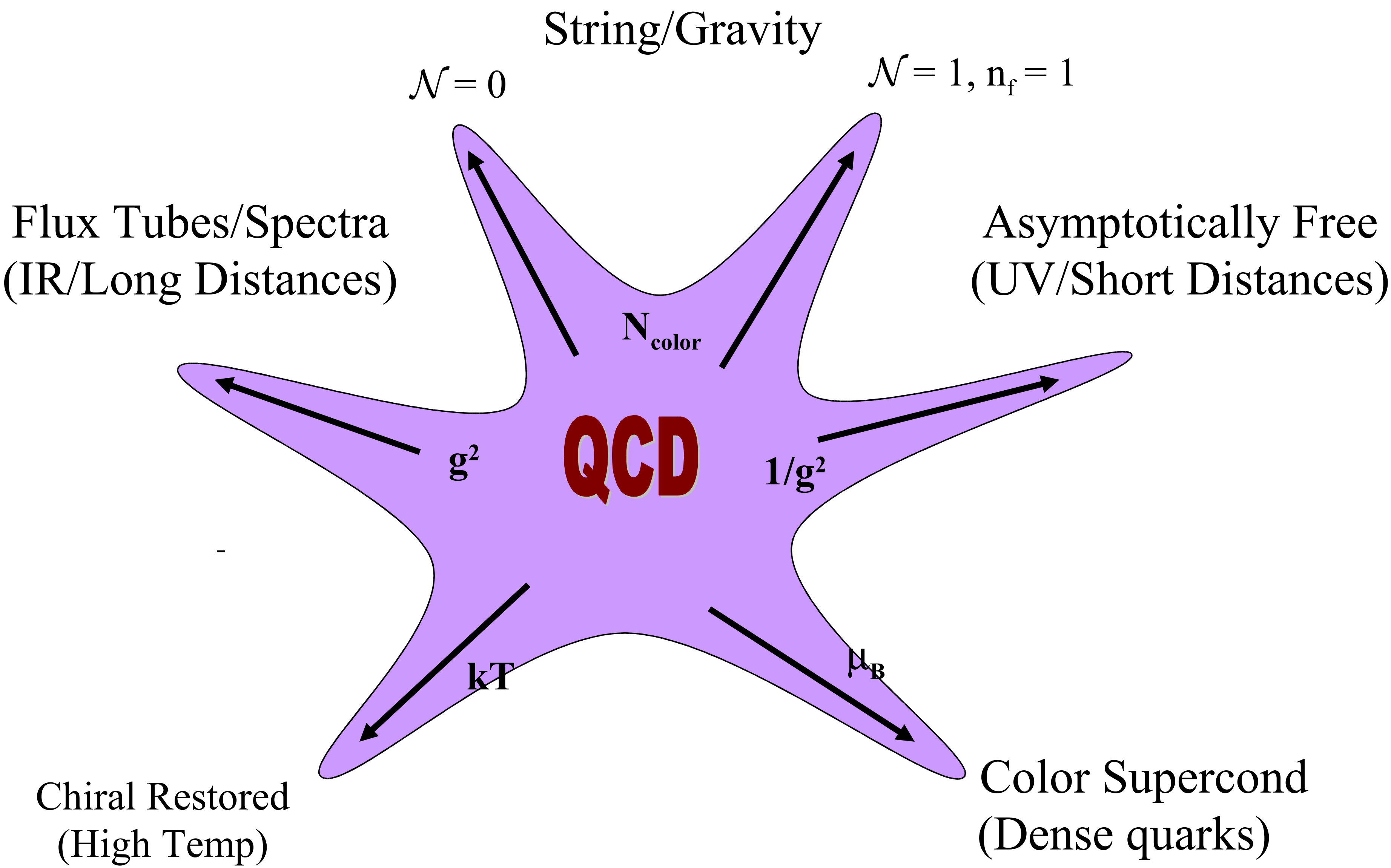
hep-th 0603xxx

QCD Theory Space!



Outline

- I. Tutorial: Regge, String theory & AdS/CFT
- II. Synthesis of Hard(BFKL) & Soft (Regge) Pomeron
- III. Lattice QCD ≡ String theory experimental data
- VI. Possible impact on New Algorithms

See also **KITP Conference: QCD and String Theory (Nov 15-19, 2004)**
http://online.itp.ucsb.edu/online/qcd_04

I Tutorial

Regge trajectory:

$\alpha(t) \simeq \alpha' t + \alpha_0$

where $\alpha(t = M^2) = J$

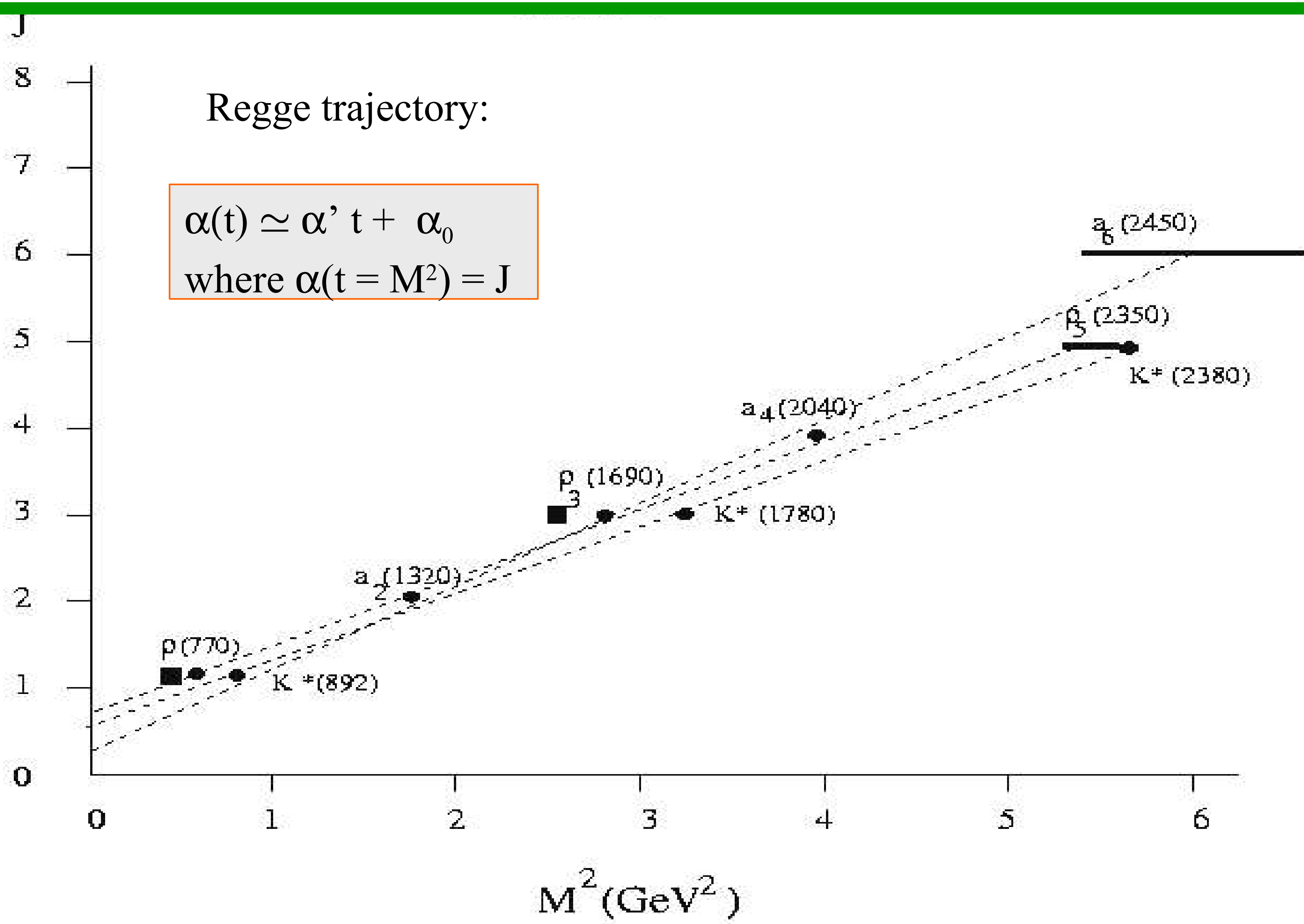
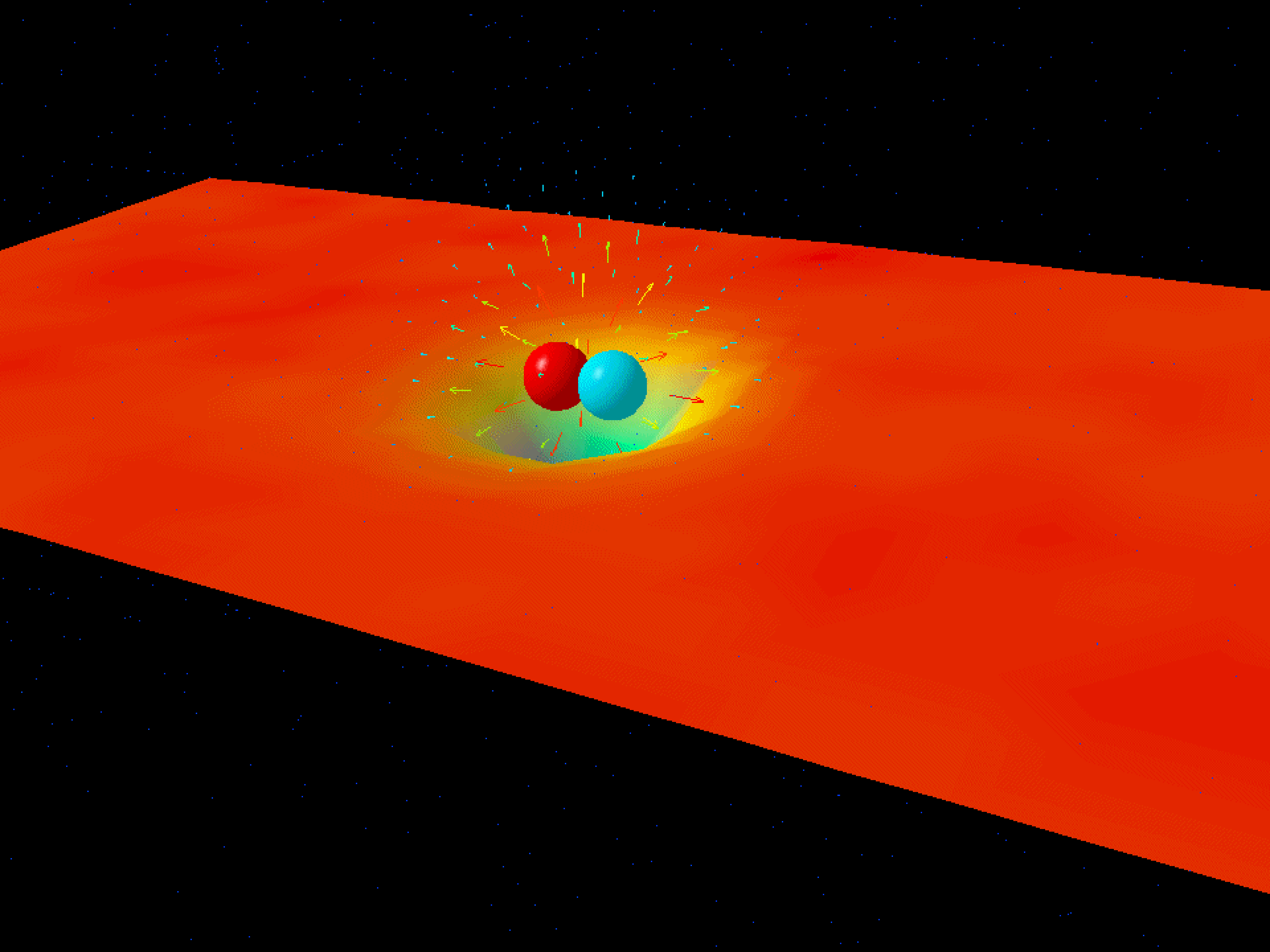


FIG. 1. Meson (ρ, K^* and a) Regge trajectories constructed from recent tabulated data (dark circles and error bars, PDG 2000). Boxes are model TDA predictions for the ρ trajectory.



STRING THEORIST'S REGGE THEORY:

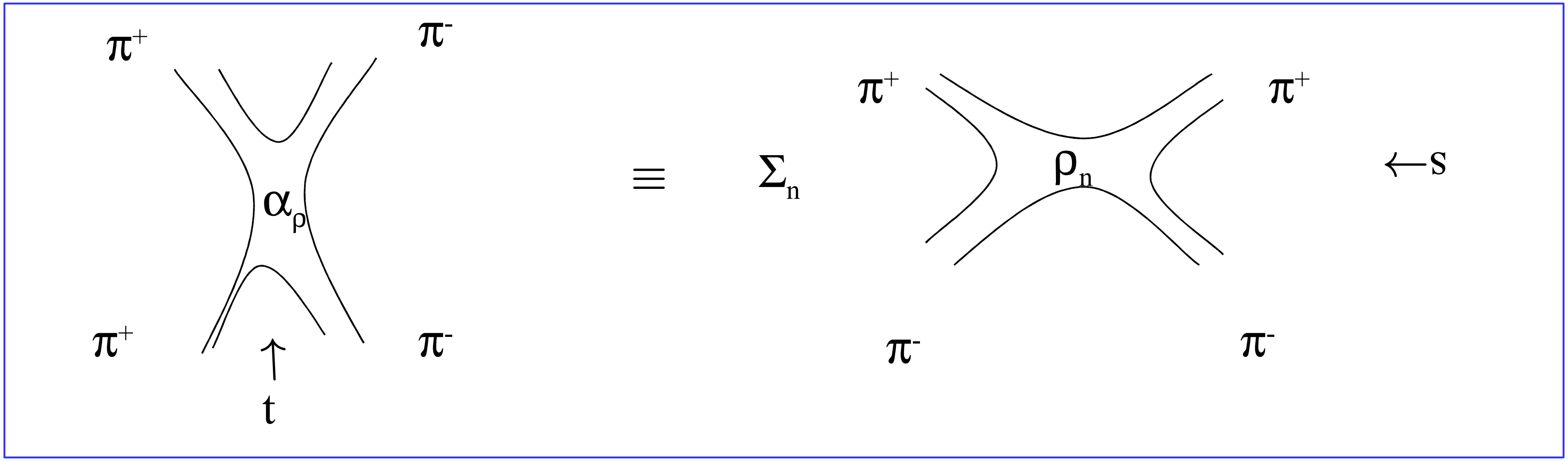
$$J = \alpha_\rho(t) \equiv \alpha' t + \alpha(0)$$

$$A_{\pi^+\pi^- \to \pi^+\pi^-}(s,t) \simeq \Gamma[1-\alpha_\rho(t)](-\alpha' s)^{\alpha_\rho(t)}$$

Dolan-Horn-Schmid ***duality*** *(Phys.Rev. 166, 1768 (1968:* t-channel Regge amplitude

$A \simeq (-s)^{\alpha(t)}$ smoothly interpolates s-channel resonances (analyticity / unitarity)

$$\beta(t)(-\alpha' s)^{\alpha_\rho(t)} \simeq \sum_n \frac{g_n^2}{s-(M_n - i\Gamma_n)^2}$$



Dual Pion Amplitude (aka NS string[†])

$$A_{\pi^+\pi^-\to\pi^+\pi^-}(s,t) = \frac{\Gamma[1-\alpha_\rho(t)]\Gamma[1-\alpha_\rho(s)]}{\Gamma[1-\alpha_\rho(s)-\alpha_\rho(t)]}$$

$$= (1-\alpha_\rho(s)-\alpha_\rho(t))\frac{\Gamma[1-\alpha_\rho(t)]\Gamma[1-\alpha_\rho(s)]}{\Gamma[2-\alpha_\rho(s)-\alpha_\rho(t)]} \sim \alpha'(s+t)$$

If $\alpha_{rho}(0) = ½$ then

χ Lagrangian implies low energy (Adler) zero

$A(p_1 \to 0) = 0$ or $s = t \to m^2_\pi = 0$

† Neveu-Schwarz "Quark model of dual pions", 1971

Failures of (flat space) String for QCD

(i) ZERO MASS STATE (gauge/graviton)

(ii) EXTRA SUPER SYMMETRY

(iii) EXTRA DIMENSION 4+6 = 10

(iv) NO HARD PROCESSES! (totally wrong dynamics)

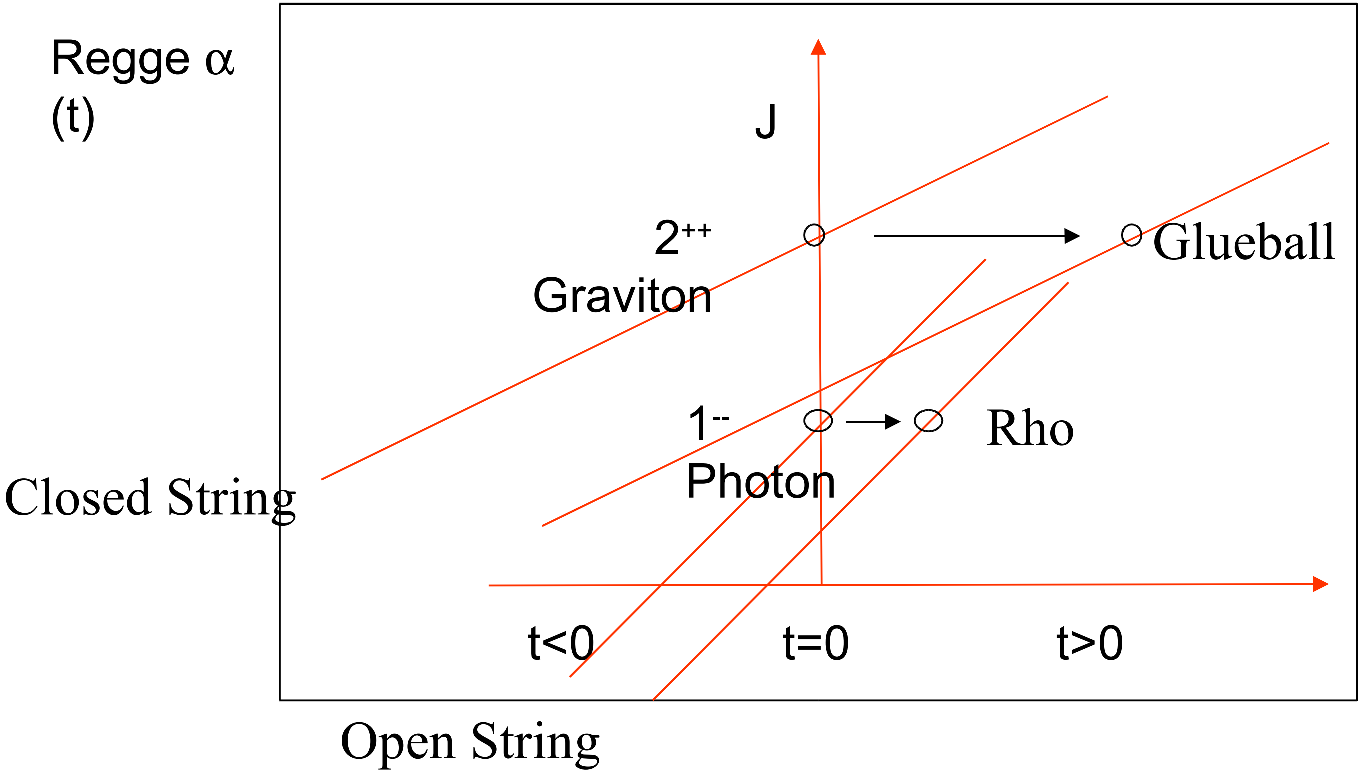
Wide angle is ridiculous: $A(s,t) \to \exp\,[\,-\,\alpha'(s\ln s + t\ln t)]$

Strings are too soft: $\langle X_\perp^2\rangle \simeq \alpha' \log[\,N_{\text{modes}}\,]$

Form Factors do not exist! $F[q^2] \simeq \exp[-q_\perp^2\, log(\infty)]$

No longitudinal modes on the Flux tube, etc.

Need to give mass to Graviton to turn into a the 2^{++} Glueball



Maldacena: "Solution put 10-d (super) strings in curved space"

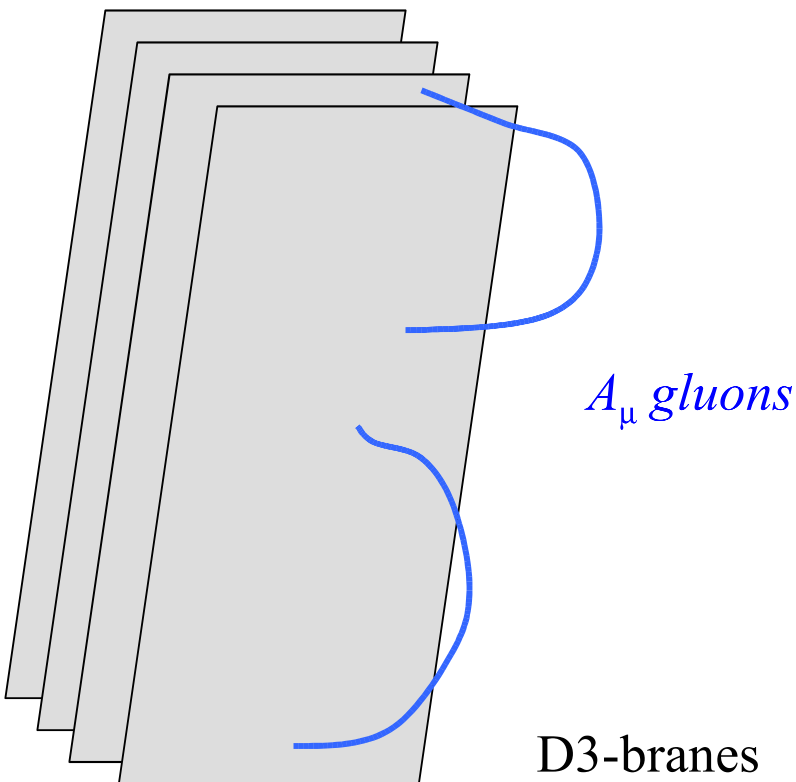
first example: $AdS^5 \times S^5$ string $\equiv$ $\mathcal{N} = 4$ Super Conformal YM in 4-d

D brane Picture: Two Descriptions

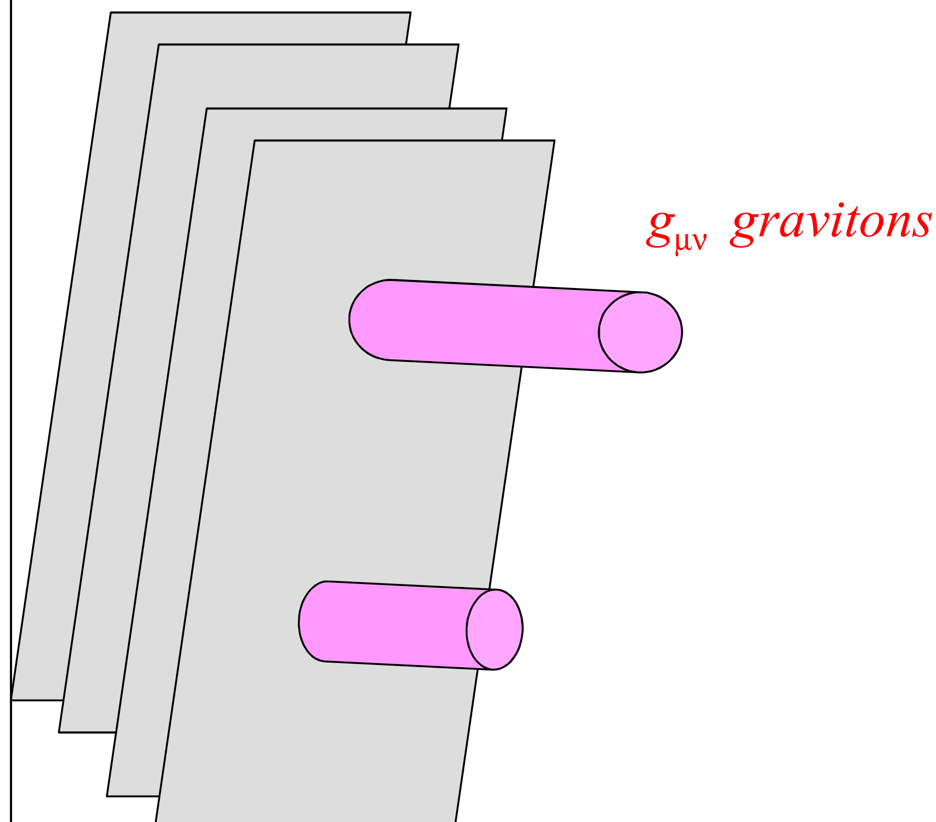
Open stings are Gluons ***dual*** *to closed string Gravity.*

- 3-branes (1+3 world volume) -- Source for open strings and closed strings:

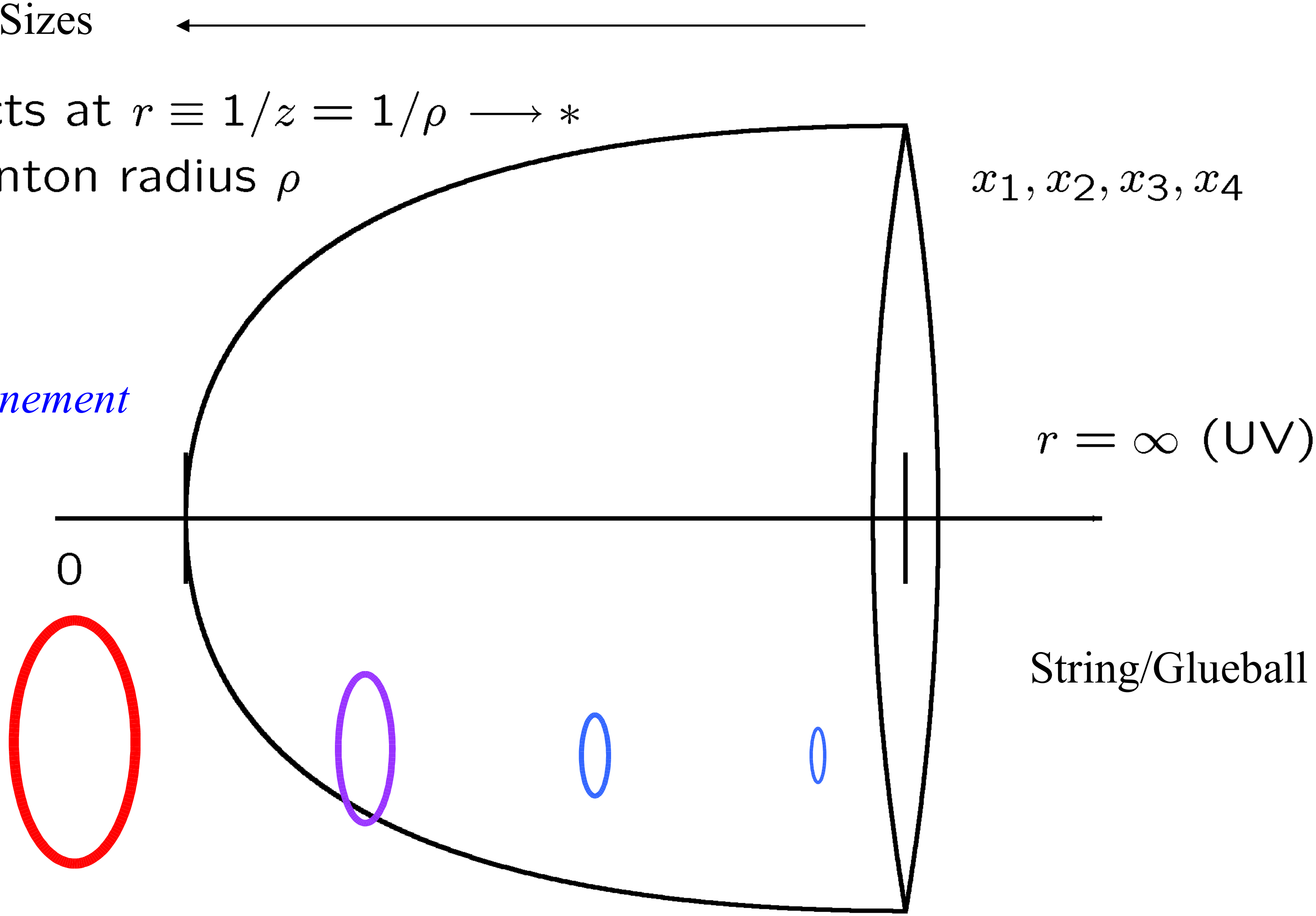
Dynamics of N D3 branes at low energies is (Super) SU(N) YM.



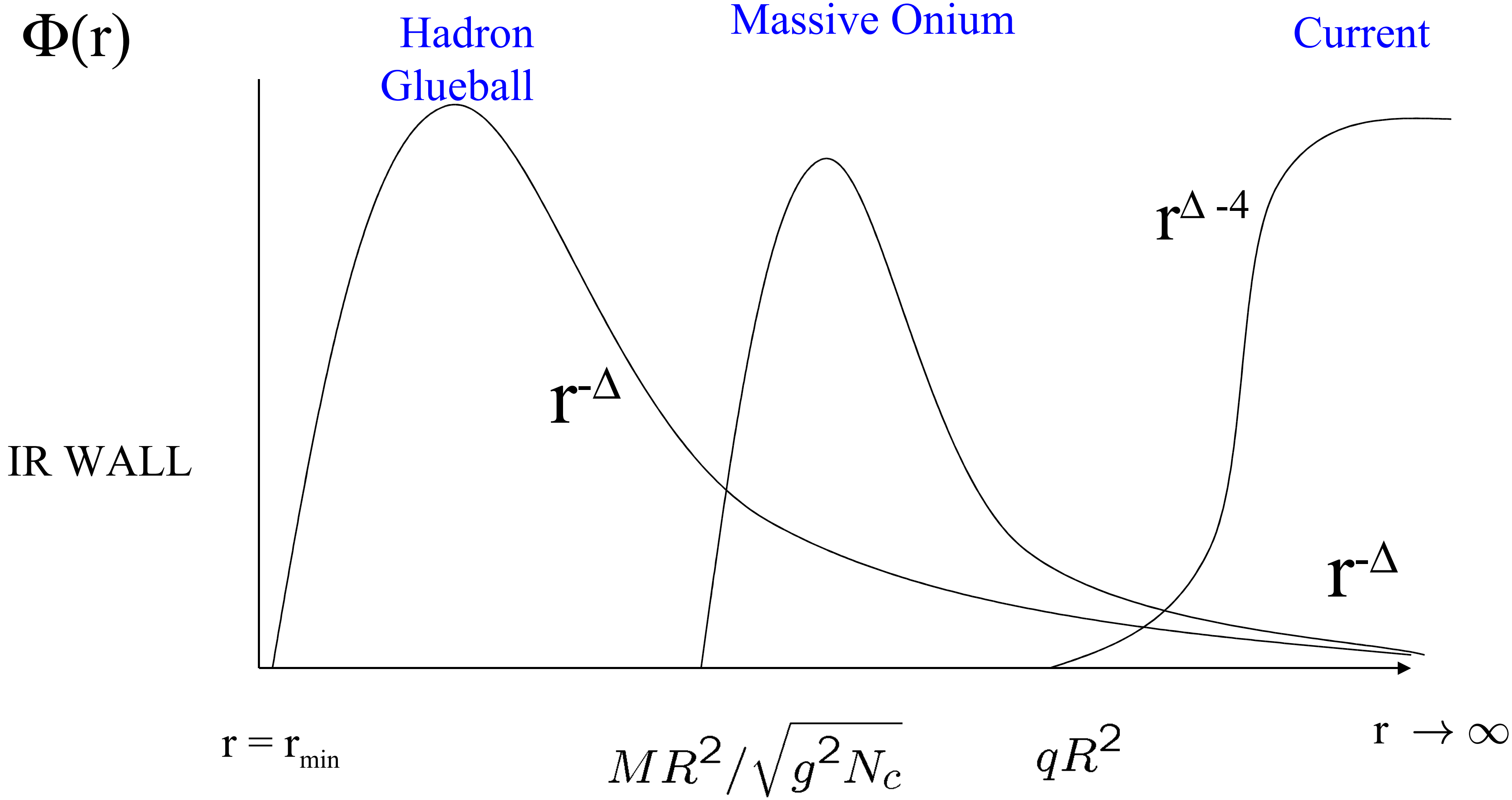
Their mass curves the space (near horizon) into AdS^5 and emits closed string (graviton)



Scale Invariance and the 5th dimension



Scale Invariance and the 5th dimension



$$ds^2 = \frac{r^2}{R^2}[dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2] + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2$$

II Pomeron and String/Gauge Duality

BFKL (Balinsky-Lipatov-Fadin-Kuraev)

- Weak perturbation theory: 1^{st} order in α_s and all orders $(\alpha_s \log s)^n$
- Implies "planar" diagrams (e.g. $N_c = \infty$) and conformal scaling
- BFKL is essentially a large N_c CFT results!

$$A(s,t=0) \simeq \int \frac{dk_\perp}{k_\perp} \int \frac{dk'_\perp}{k'_\perp} \Phi_1(k_\perp) K(s; k_\perp, k'_\perp) \Phi_2(k'_\perp)$$

$$K(s,k_\perp,k'_\perp) \approx \frac{s^{\alpha(0)-1}}{\sqrt{\pi \ln s}} e^{-\left[(\ln k'_\perp - \ln k_\perp)^2/4\mathcal{D} \ln s\right]}$$

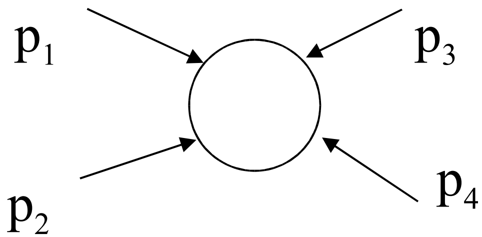
Diffusion in "virtuality" $k_\perp$

Weak Coupling:

$$\alpha(0) = 1 + \ln(2) g^2 N/\pi^2$$

$$\mathcal{D} = \frac{14\zeta(3)}{\pi} g^2 N/4\pi^2.$$

Diffusion in $\log(k_\perp)$ is familiar in Regge but ...!



$s = (p_1+p_2)^2 \simeq m_1\, m_2 \exp[y]$

$t = (p_1+p_3)^2 \equiv -q^2_\perp$

$$A_{closed\ string}(s,t) \simeq (e^{-i\pi/2} s)^{\alpha_G(t)}$$

Take Fourier transform:

$$\exp[-\alpha' q_\perp^2 \log(s)/2] \to \exp[-\alpha' x_\perp^2/2\alpha' \log(s)]$$

Regge "Form Factor" shrinks due to diffusion in impact parameter space as you increase "time" ($y = \log[s]$ 🡐 the rapidity)

How do we combine diffusion in $x_\perp$ and $\log(k_\perp)$?

Intuitive Approach: Soft vs Hard in M QCD

(RCB & C-I Tan hep-th/Tan 0207144)

- Red Shift:

  Proper Length: $\Delta s = (r/R)\ \Delta x$

  Local Momentum: $p^{local}_\mu = (R/r)\ p_\mu$ (large p in IR!)

- Wide angles has power (Polcinki & Strassler)

  $A_{string}(\alpha' R^2 s/r^2, \alpha' R^2 t/r^2) \sim \exp[-R^2 s \log(s)/r^2]$

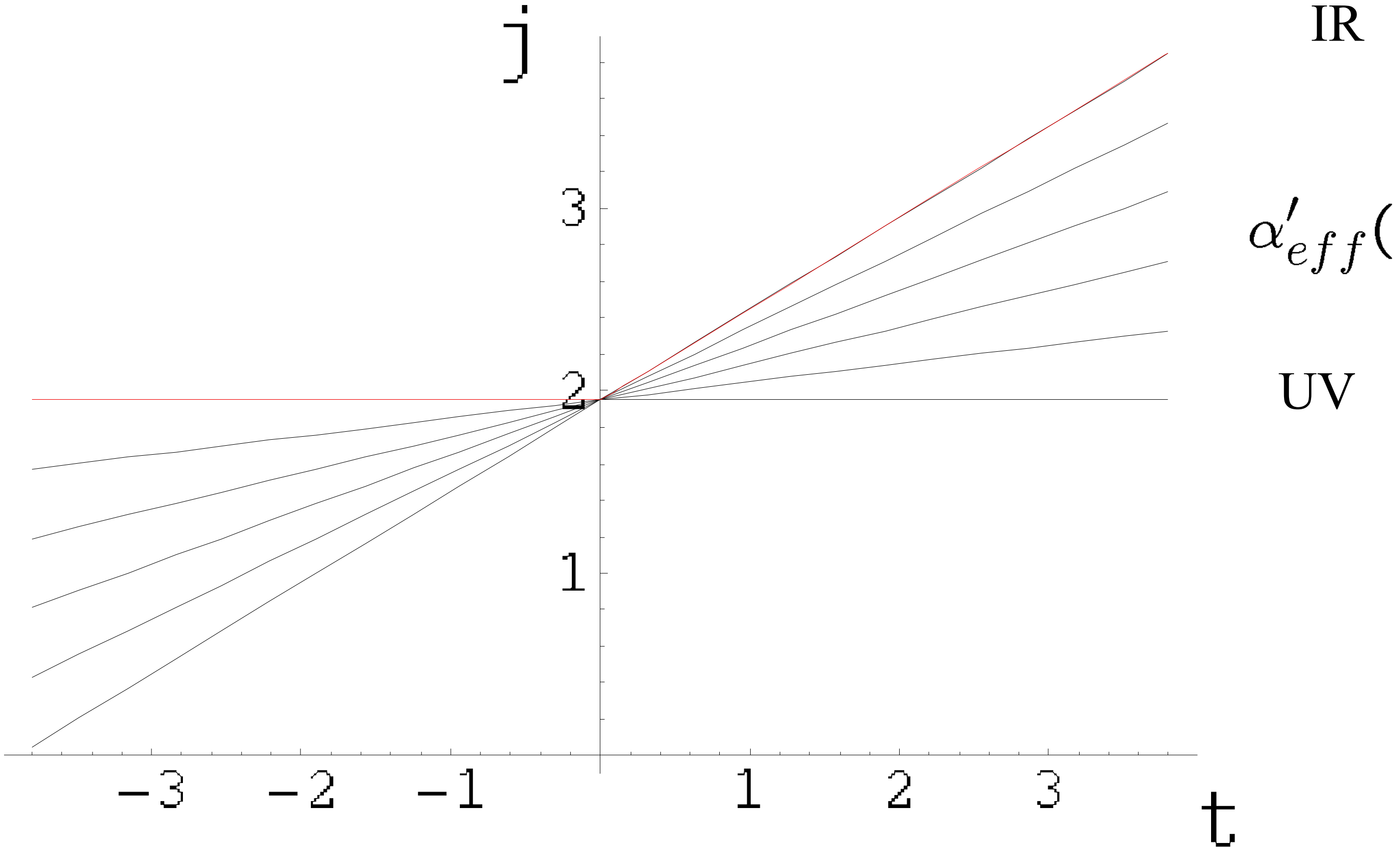
  Domant piece is conformal scaling for $r \to \infty$

- Regge region is an average for r:

$$T(s,t) = \int_{r_{min}}^{\infty} dr\ \Phi(r) (\alpha' s)^{\alpha(0) + \alpha'_{eff}(r) t}.$$

with $\alpha'_{eff}(r) = \alpha' R^2/r^2$

Ultra local Model in AdS^5



$$\alpha'_{eff}(r) = \alpha' R^2/r^2$$

- Soft: IR region: $r \simeq r_{min,}$ gives Regge pole with slope $\alpha'_{qcd} \sim \alpha' R^3/r^3_{min}$

$$T(s,t) \sim \exp[+\alpha' t \log(s)](\alpha'_{qcd} s)^{\alpha_s(0)}$$

- The ``shrinkage'' is caused the soft stringy ``form factor'' in impact parameter:

$$< X_\perp^2 > \simeq \alpha'_{qcd} \log(s) \sim \alpha'_s \log(\text{No. of d.o.f})$$

- Hard IR region: BFKL-like Pomeron with almost flat cut in the j-plane

$$T(s,t) \sim (\alpha' s)^{\alpha_s(0)} / (\log s)^{\gamma+1}$$

Strong Coupling YM is computed in String Theory

- Semi classical 2-d conformal String theory in AdS^5 background

Strong Coupling:

at t =0 $$\mathcal{K}(r, r', s) = \frac{s^{j_0}}{\sqrt{4\pi\mathcal{D}\ln s}} e^{-(\ln r - \ln r')^2/4\mathcal{D}\ln s}$$

Diffusion in "warped co-ordinate"

$$j_0 = 2 - \frac{2}{\sqrt{g^2 N}} + O(1/g^2 N) \qquad \mathcal{D} = \frac{1}{2\sqrt{g^2 N}} + O(1/g^2 N) \ .$$

Compare with weak Coupling:

$$K(s, k_\perp, k'_\perp) \approx \frac{s^{\alpha(0)-1}}{\sqrt{\pi \ln s}} e^{-[(\ln k'_\perp - \ln k_\perp)^2/4\mathcal{D}\ln s]}$$

$$j_0 = 1 + \ln(2) g^2 N/\pi^2 \qquad \mathcal{D} = \frac{14\zeta(3)}{\pi} g^2 N/4\pi^2.$$

Main Lesson from AdS/CFT dual description of Diffraction

Here $\lambda \equiv R^4/\alpha'^2 = g^2_{YM} N = 4\pi\alpha N$ in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory — the numerical coefficient can differ in other theories but the proportionality always holds — so large λ is large 't Hooft coupling.

The identification of r and $k_\perp$ has its source in the UV/IR correspondence and has been suggested in numerous contexts, but here appears as a nontrivial and precise match. The effective diffusion time, $\ln s$, holds for both the BFKL and the Regge diffusions, at both large and small λ.

General form depends on Conformal Symmetry.

Hard versus Soft Diffraction
(Lightcone Derivation)

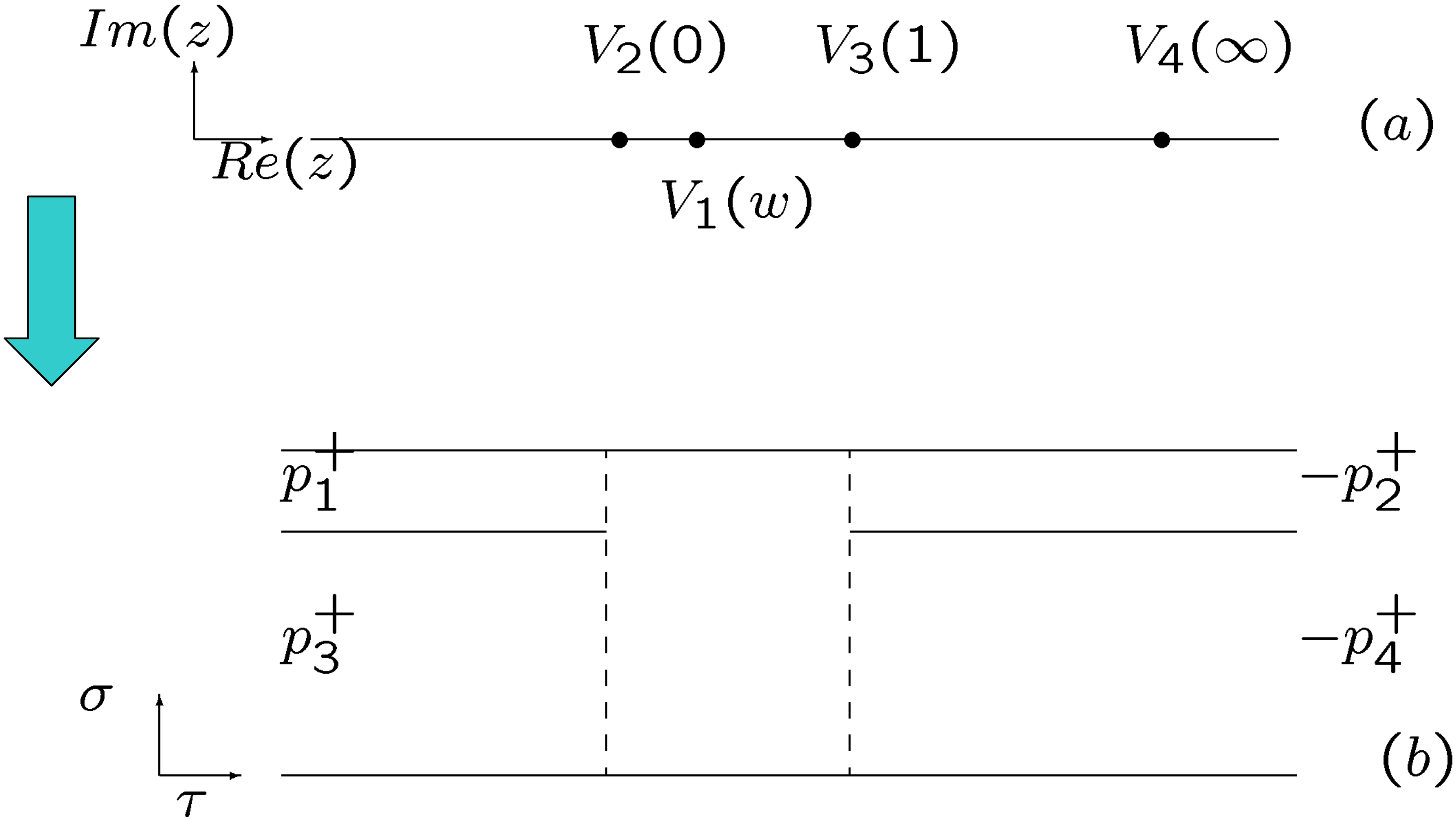
$$\mathcal{A}(s,t) = \int_0^1 dw\,(1-w)^{-2\alpha' p_1 p_3}\, w^{-2\alpha' p_1 p_2} = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}\,,$$

With $X^+ = \tau$

$$A(s,t)\delta^2(p_1^\perp + p_2^\perp + p_3^\perp + p_4^\perp) \quad \sim$$

$$\int d\tilde{\tau}\mathcal{D}X_\perp(\sigma,\tau)\; V_1V_2V_3V_4\;\; e^{-\frac{1}{2}\int d\tau \int_0^{p^+} d\sigma(\dot{X}_\perp^2 + \frac{1}{(2\pi\alpha')^2}X_\perp'^{\,2})}$$

The Schwarz-Christoffel trans maps
the upper half plane (a) into the light-cone strip σ + iτ (b):



$$\rho = \tau + i\sigma = \frac{1}{\pi}[p_1^+ \log(z-w) - |p_2^+| \log(z) + p_3^+ \log(z-1)] + \text{const}$$

Reduction to 1-d Path Integral

$$A \sim \int d\tilde{\tau}\, \mathcal{D}X_\perp^{(in)}(\sigma)\mathcal{D}X_\perp^{(out)}(\sigma)$$

$$\Phi(X_\perp^{(2)})\ \Phi(X_\perp^{(4)})\ \ G_{int}(X_\perp^{(out)}, X_\perp^{(in)}, \tau)\ \ \Phi(X_\perp^{(1)})\ \Phi(X_\perp^{(3)})$$

where

$$\Phi(X_\perp^{(r)}(\sigma)) = e^{-\frac{1}{2}\sum_{n=1}^{\infty}\omega_n X_n^{(r)} X_n^{(r)}}\ e^{ip_\perp^{(r)} x_\perp^{(r)}}\ ,$$

$$G_{int}(X_\perp^{(out)}, X_\perp^{(in)}, \tau) \sim \delta(R_\perp)\,\exp[-\frac{\tau}{2}\int_0^{p^+} d\sigma \dot{X}^-]\ ,$$

$$\omega_n^{(r)} = \frac{n}{2\alpha'|p_r^+|}$$

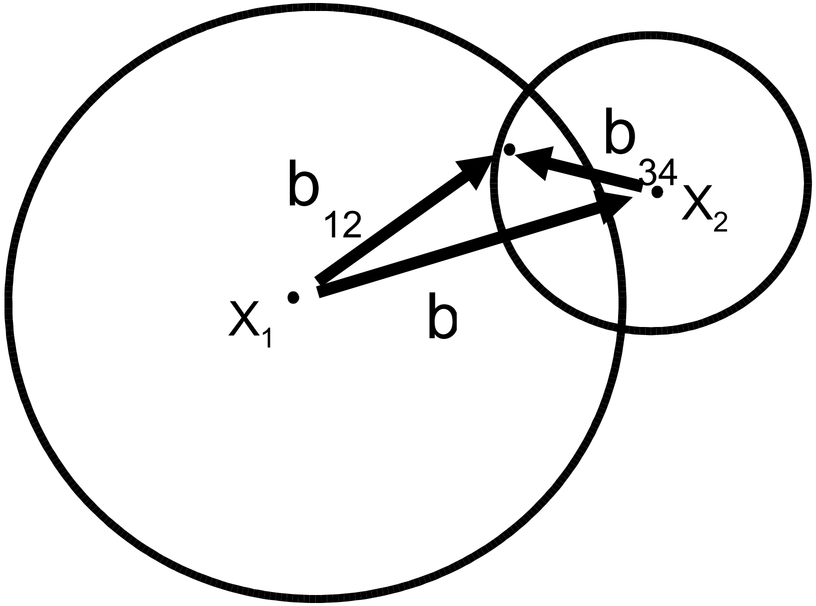
Regge Behavior is diffusion for time log(s) in impact parameter space (and AdS radial space)

$$A(s,q_\perp) = s\int db_{34}db_{12}e^{-iq_\perp(b_{34}-b_{12})}\mathcal{K}(s,b_{34},b_{12})$$

$$[\partial_{\log(s)} - 1 - \alpha'\partial_x^2]K(y;x,x') = \delta(x-x')\delta(y)$$

Rapidity $y = \log(s/s_0)$ and $t = -q^2_\perp$

$$[\partial_y - 1 - \alpha' t]K(y;t) = \delta(y)$$



Boosts increases size of "hadronic string"

$\exp[-\alpha' q^2_\perp \log(s)] \rightarrow \exp[-b^2/(\alpha' \log(s))]$

AdS5 Modifications

$$L = \frac{1}{2}\int_0^{p^+} d\sigma[\dot{X}_\perp^2 + \dot{Z}^2 + \frac{1}{(2\pi\alpha'_{eff}(Z))^2}({X'_\perp}^2 + {Z'}^2)]$$

$$\alpha'_{eff} = \alpha' Z^2/R^2 = \alpha' \exp[-2u]$$

where $Z = 1/r$

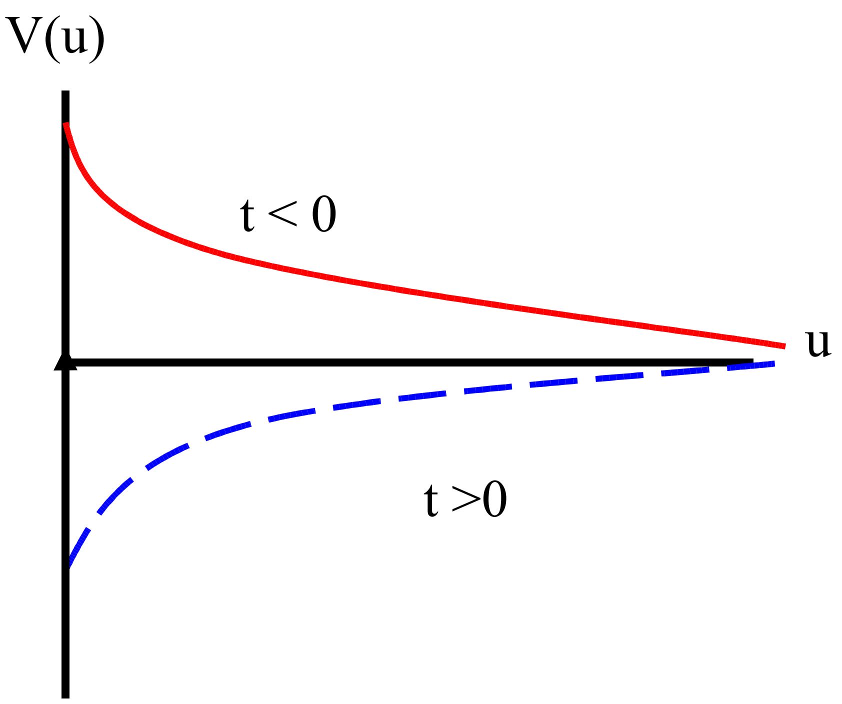
$$[\partial_y - 1 + \alpha'_{eff}(u) q^2 - (\alpha'/R^2)(\partial_u^2 - 1)]\mathcal{K}(y; q, u, u')$$

$$= \delta(u - u')\delta(y) \ .$$

Strong Coupling Pomeron

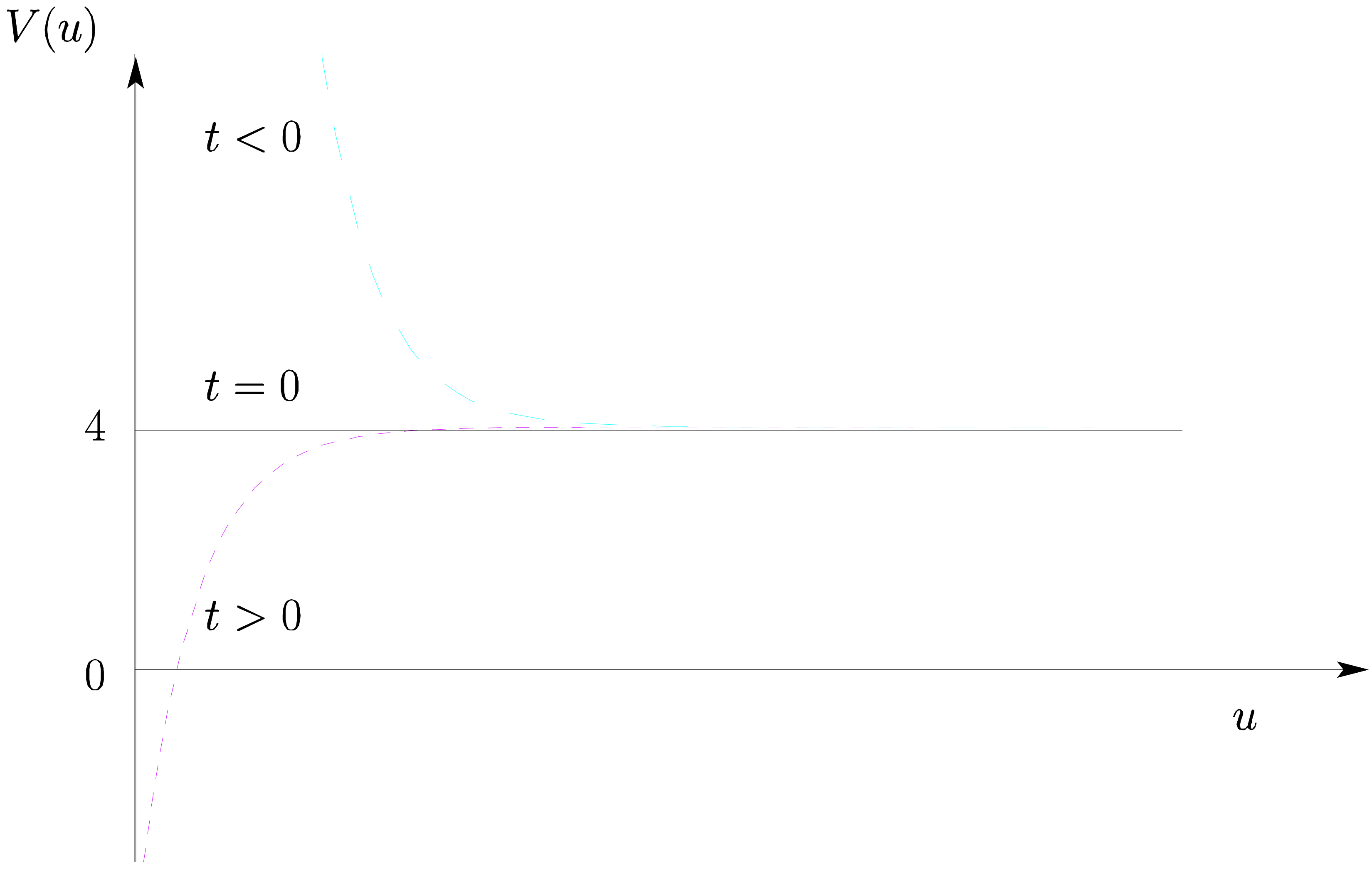
$$\frac{1}{2\sqrt{g^2N}}[-\frac{d}{du^2}-te^{-2u}]\Psi(u,J) = (2-J-\frac{2}{\sqrt{g^2N}})\Psi(u,J)$$

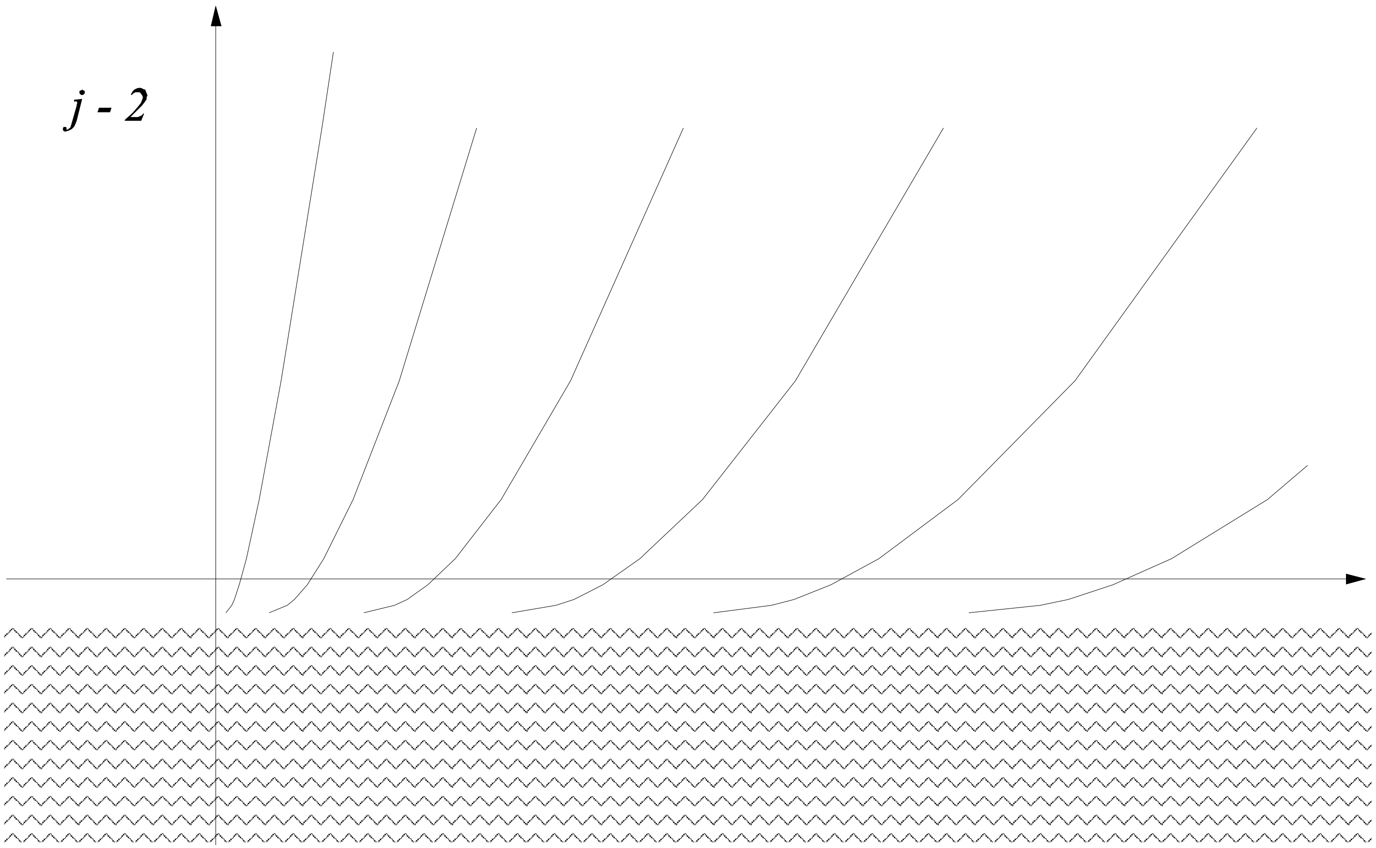
- V(u) = -t e^{-u} $0 < u < \infty$
- Attractive for t >0, Regge Pole +
- BKLF cut
- t < 0 only scattering state for BKLF



Hard Wall at r = r_{min}

Conformal Breaking by Hardwall Model

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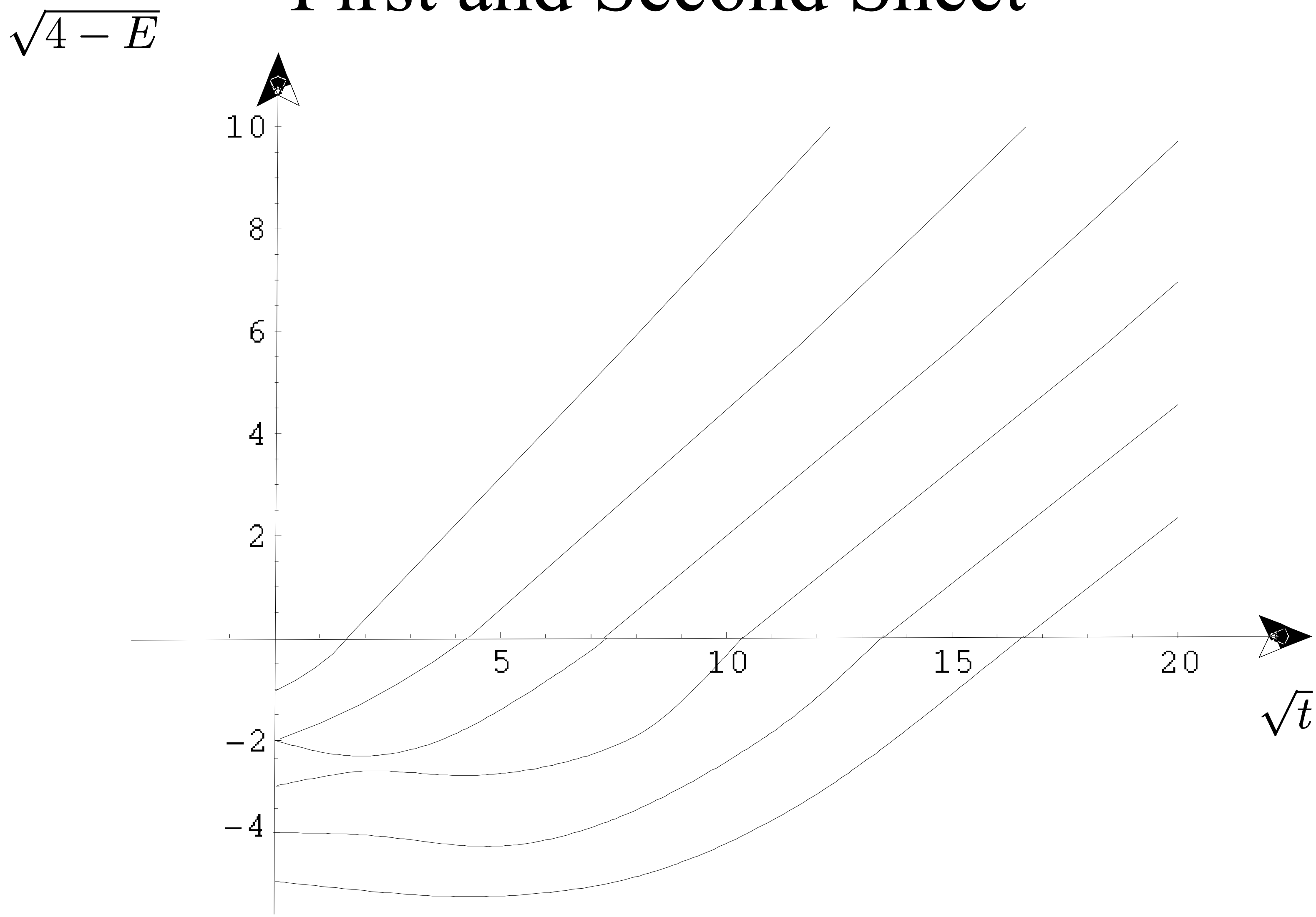
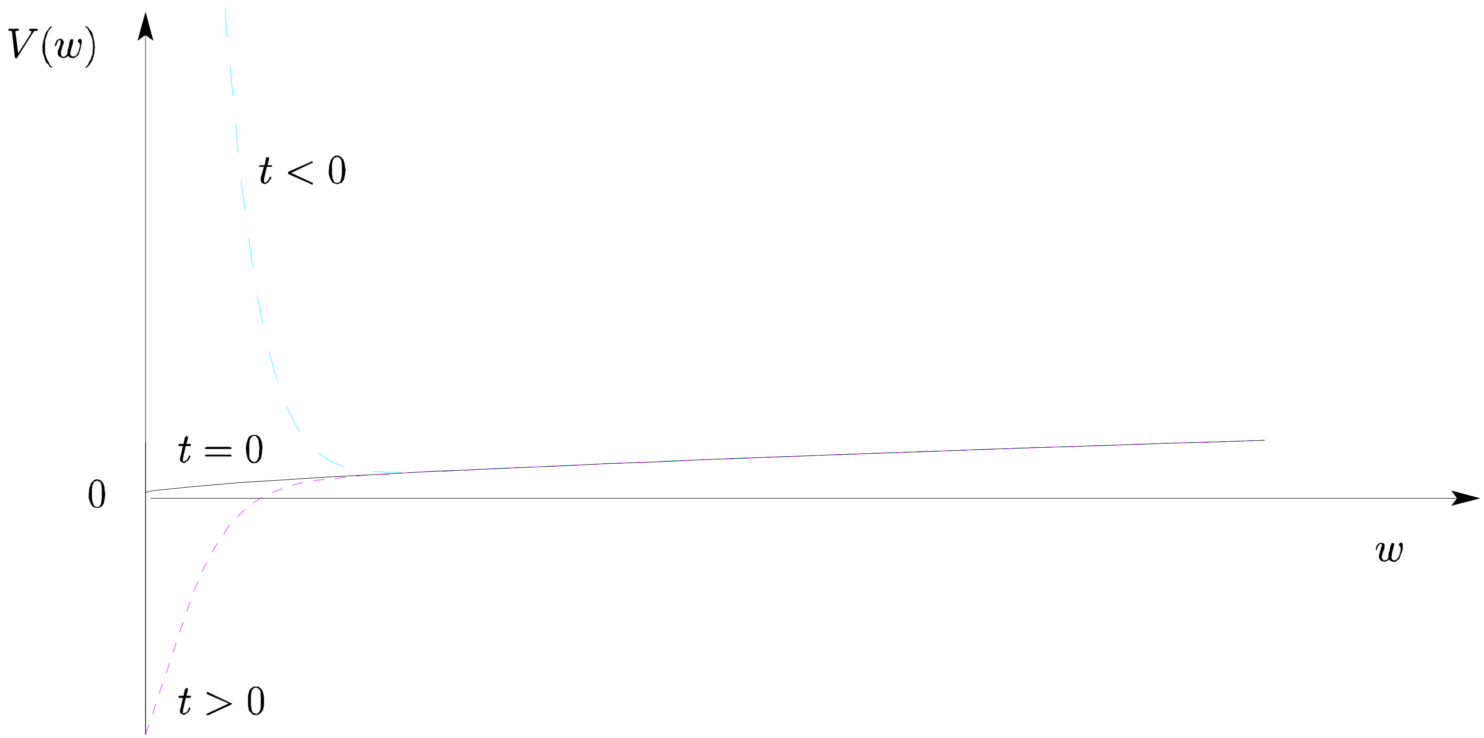
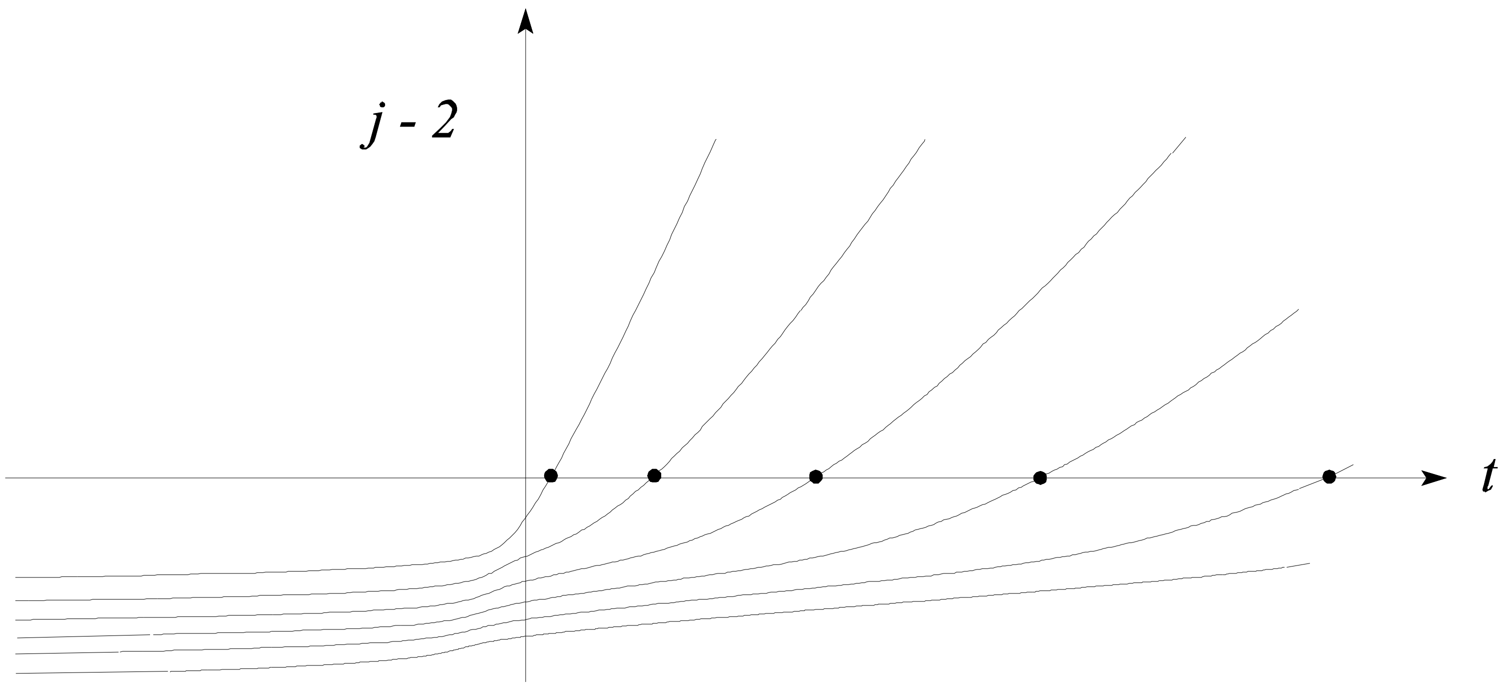
First and Second Sheet



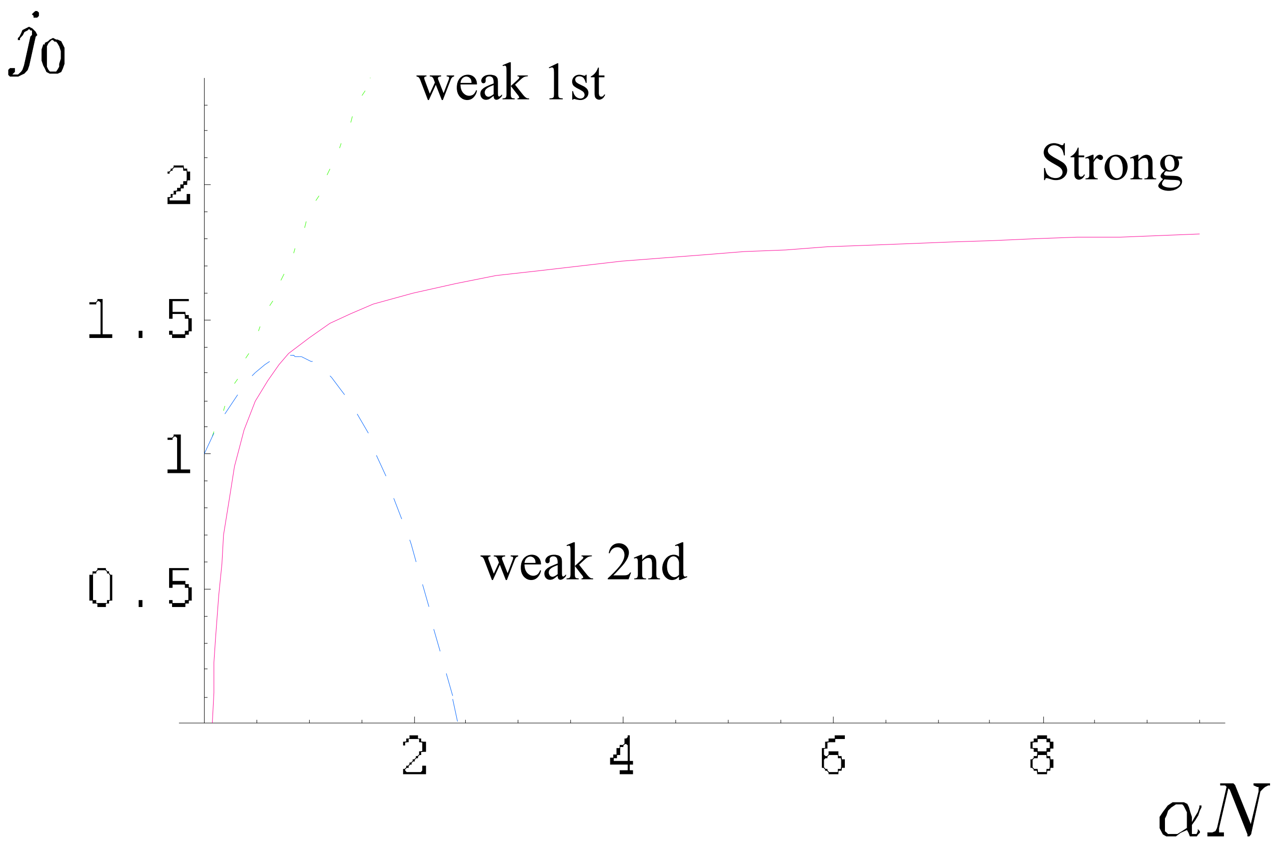
V running



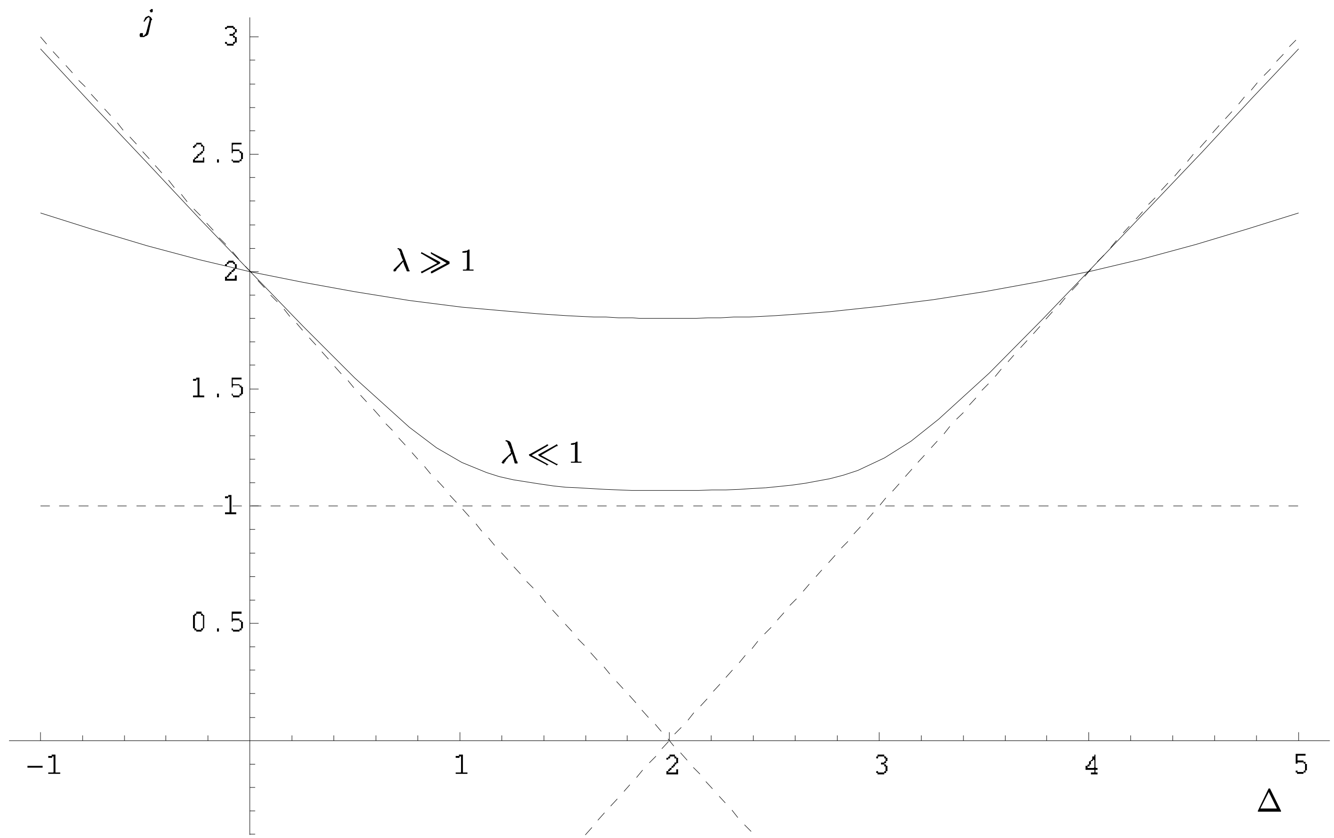
(Strong) Running Coupling



$\mathcal{N} = 4$ Strong vs Weak BFKL

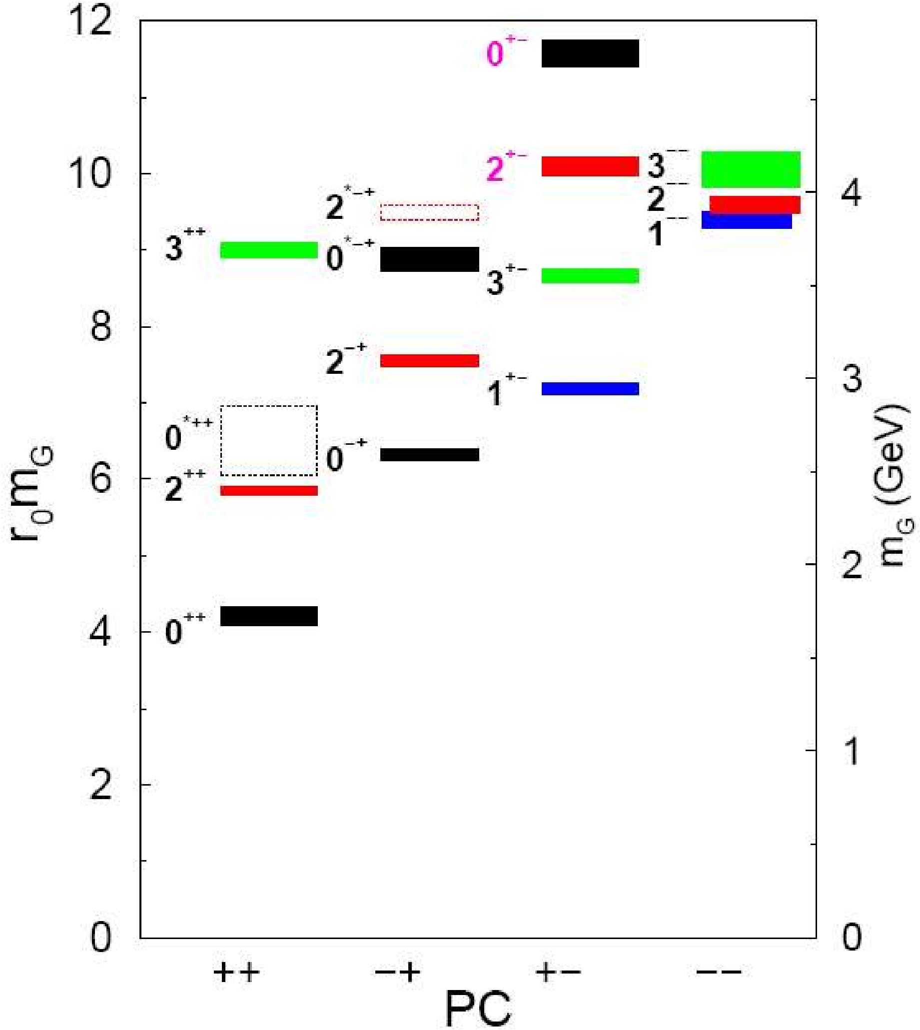


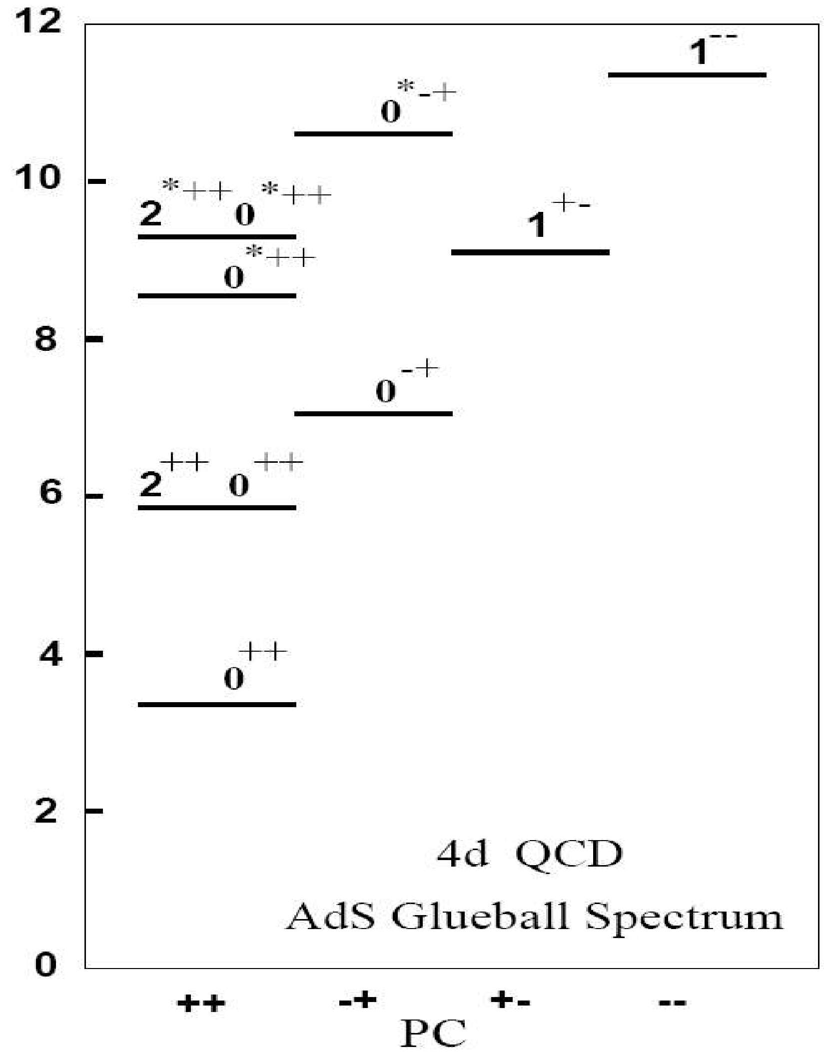
All coupling form: Δ(j) DGLAP vs BFKL



III. Lattice Data for String Theory

Lattice Data vs AdS Confining Gauge Theory at α' = 0





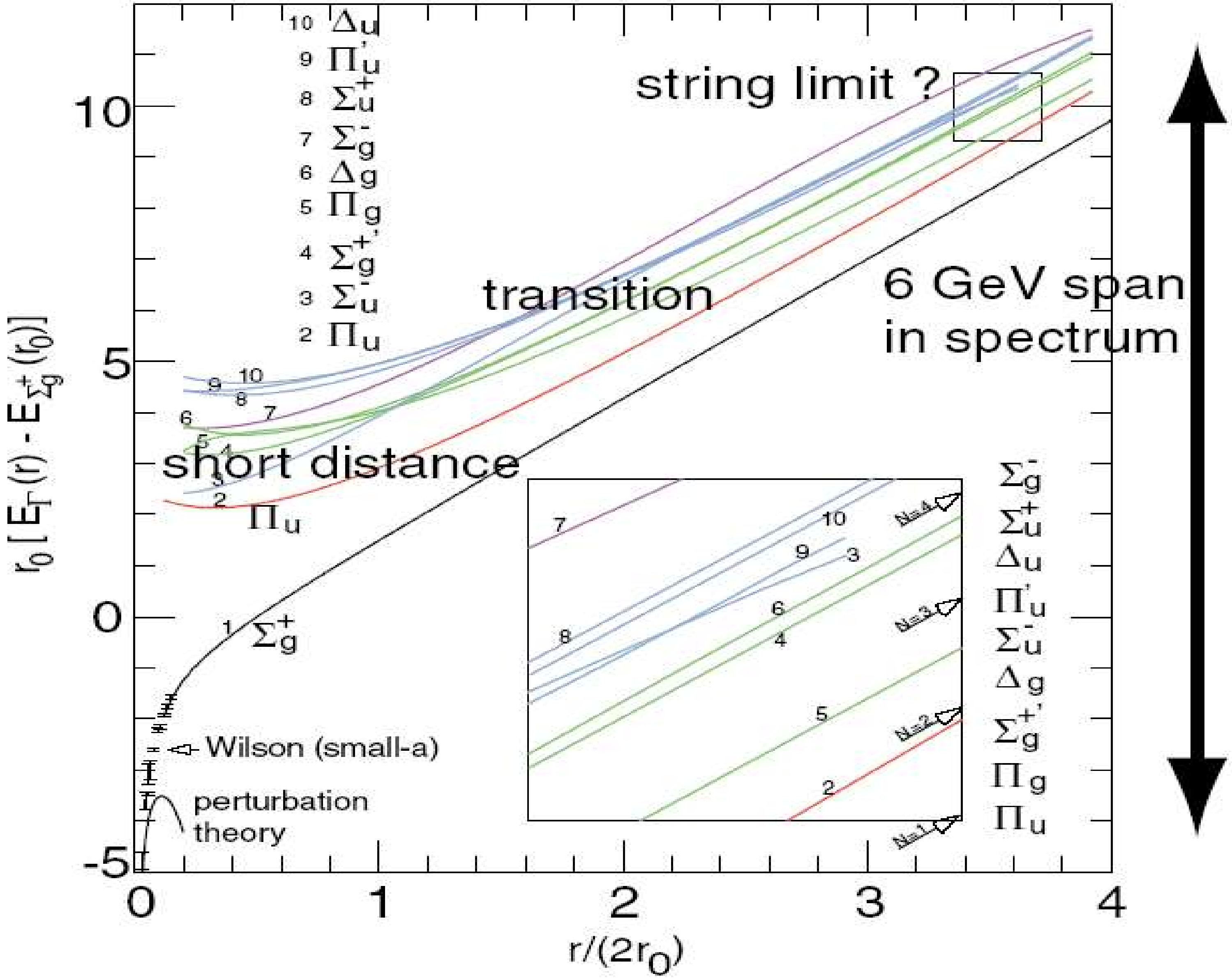
IIA Classification of QCD_4

States from 11-d G_{MN}				States from 11-d A_{MNL}		
$G_{\mu\nu}$	$G_{\mu,11}$	$G_{11,11}$	m_0 (Eq.)	$A_{\mu\nu,11}$	$A_{\mu\nu\rho}$	m_0(Eq.)
G_{ij} 2^{++}	C_i $1^{++}_{(-)}$	ϕ 0^{++}	4.7007 (T_4)	B_{ij} 1^{+-}	C_{123} $0^{+-}_{(-)}$	7.3059(N_4)
$G_{i\tau}$ $1^{-+}_{(-)}$	C_τ 0^{-+}		5.6555 (V_4)	$B_{i\tau}$ $1^{--}_{(-)}$	$C_{ij\tau}$ 1^{--}	9.1129(M_4)
$G_{\tau\tau}$ 0^{++}			2.7034(S_4)		$G^{\alpha}_{\ \alpha}$ 0^{++}	10.7239(L_4)

Subscripts to J^{PC} refer to $P_\tau = -1$ states

Lattice QCD_4 Glueball Spectrum

Moringstar and Peardon



Transverse String excitations

N	m	$\|n_{m+},n_{m-}>$	Λ	States
1	1	$\|1_{1+}>$, $\|1_{1-}>$	1	Π_u
2	2	$\|1_{2+}>$, $\|1_{2-}>$	1	Π_g
	1	$\|2_{1+}>$, $\|2_{1-}>$	2	Δ_g
	1	$\|1_{1+},1_{1-}>$	0	$\Sigma^{+'}_g$
3	1,2	$\|1_{1+},1_{2+}>$, $\|1_{1-},1_{2-}>$	2	Δ_u
	1,2	$\|1_{1+},1_{2-}> + \|1_{1-},1_{2+}>$	0	Σ^{+}_u
	1,2	$\|1_{1+},1_{2-}> - \|1_{1-},1_{2+}>$	0	Σ^{-}_u
	3	$\|1_{3+}>$, $\|1_{3-}>$	1	Π'_u
	1	$\|1_{1+},2_{1-}>$, $\|2_{1+},1_{1-}>$	1	Π'_u
	1	$\|3_{1+}>$, $\|3_{1-}>$	3	Φ_u
4	1,3	$\|1_{1+},1_{3-}> - \|1_{1-},1_{3+}>$	0	Σ^{-}_g

Excited states (Semi-classical limit)

2 Transverse (Goldstone) Modes

$$-\partial_t^2 X_\perp + v^2(z) X_\perp'' = 0$$

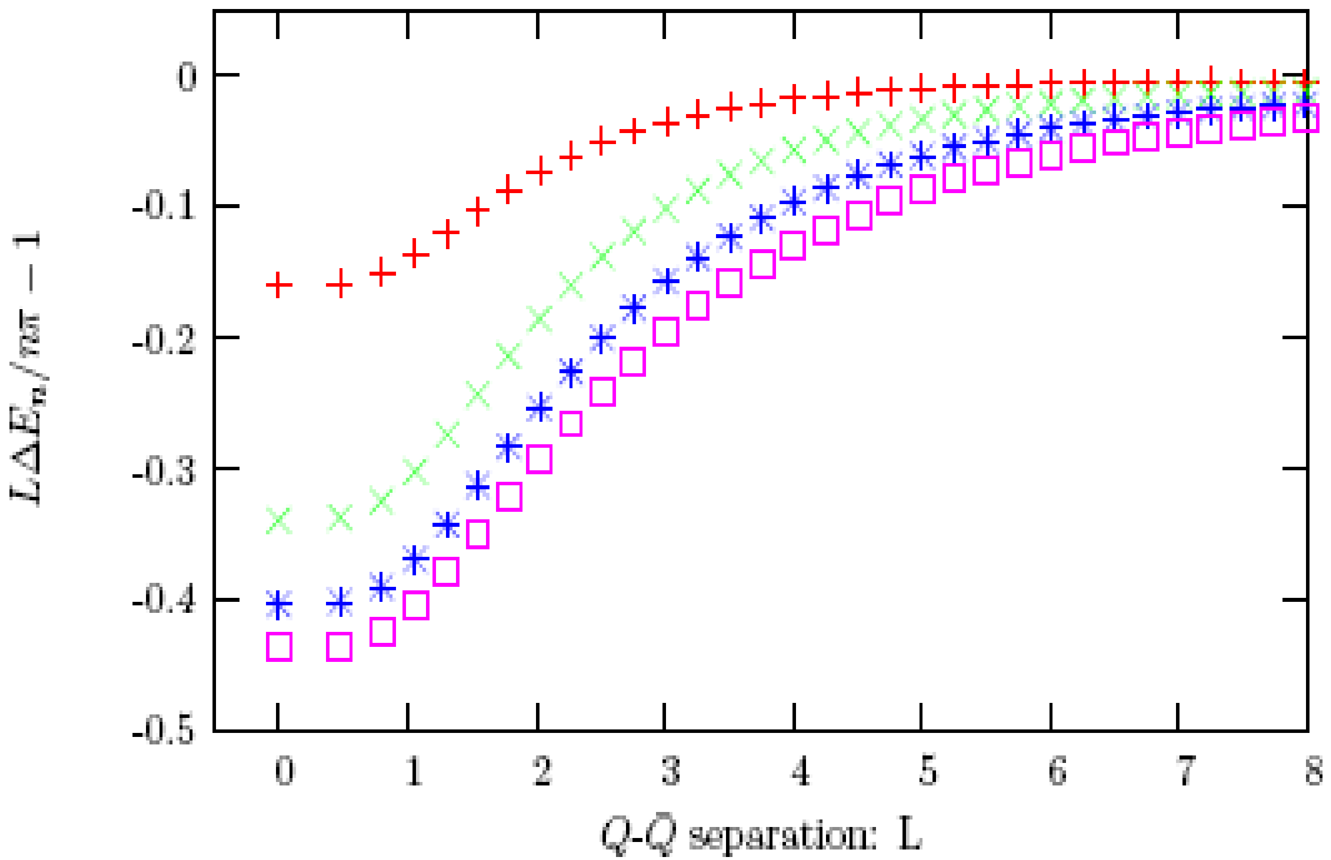
$$\Delta E_n = \frac{\omega_n(L)\pi}{L} \simeq \frac{n\pi}{L}$$

Radial (longitudinal) Mode

$$-\partial_t^2 \xi + v^2(z)\xi'' = M^2(z)\xi$$

$$\Delta E_n = \sqrt{(\omega_n/L)^2 + M_{BG}^2} \simeq M_{GB} + \frac{\omega_n^2}{2L^2 M_{GB}}$$

String Level struture



Fit to Ground State of Lattice Data

- Fit is essential perfect

$$V(r) - V(r_0) = T_0 r - \frac{g_{eff}^2}{4\pi}\frac{1}{r}$$

where $T_0 = 5.04$ /fermi^2 and $g^2_{eff}/4\pi = .26^\dagger$

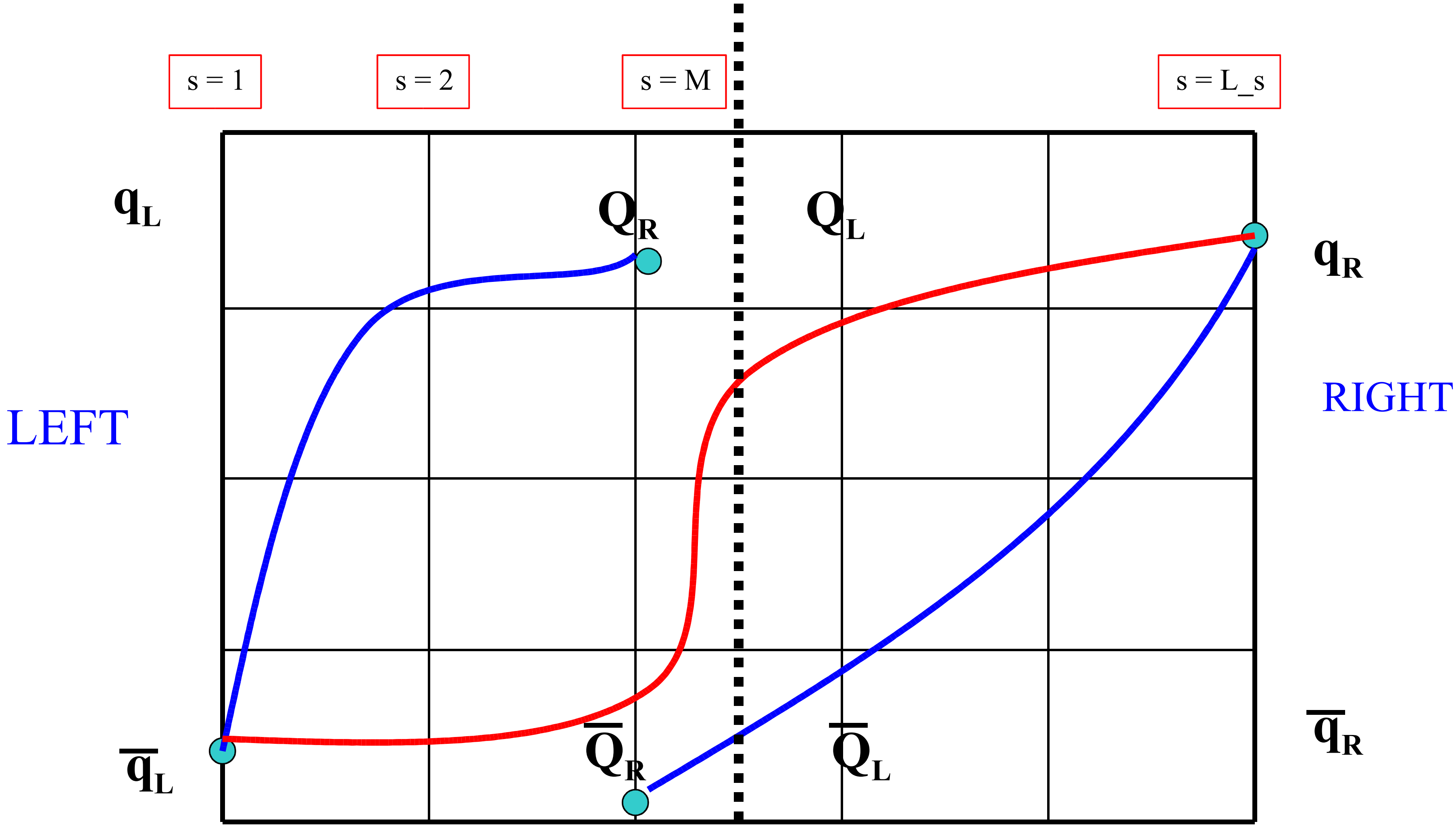
Lattice Summer scale: $r_0 \simeq 0.5$ fermi.
$r^2_0 \; dV(r_0)/dr = 1.65$

Adobe Acrobat Document

† Comment: In strong coupling AdS^5 both term are actually $\sim (g^2_{YM}N)^{1/2}$

IV Possible Impact on Algorithms?

4. Taking the 5th Dimension Seriously



$\langle Q_x \bar{q}_y \rangle$

$$\langle q_x \bar{q}_y \rangle = [D_{ov}^{-1}(m)]_{xy}$$

$\langle q_x \bar{Q}_y \rangle$

What is best use of 5th Dimension?

- ***Let glue be a true 5-d (warped) Gauge theory?***

  Improved isolation of Left and Right domain walls by "localization"?

- ***Should the 5-d theory be SUSY YM broken by domain walls boundaries ?***

- ***Quantum Links uses replaces U_μ by fermionic bilinears.***

  (R.Brower, S.Chandrasekharan, S.Riederer, U.-J.Wiese
  D-Theory: Field Quantization by Dimensional Reduction of Discrete Variables
  hep-lat/0309182)

- ***What is hadronic content of 5-d DW QCD?***

  Hadronic AdS^5/CFT works pretty well. Why?

5-d Vector Current ➔ 4-d Vector/Axial Current

$$\Delta_\mu \mathcal{J}^a_\mu(x,s) + \Delta_5 \mathcal{J}^a_5(x,s) = 0 \Rightarrow$$

Vector:

$$\Delta_\mu V^{a,DW}_\mu(x) = \sum_s \mathcal{J}^a_\mu(x,s) = 0$$

Axial:

$$\begin{aligned}\Delta_\mu A^{a,DW}_\mu(x) &= \sum_s^{L_s/2} [\mathcal{J}^a_\mu(s,x) - \mathcal{J}^a_\mu(L_s - s, x)] \\ &= -2m\,\bar{q}_x \lambda^a \gamma_5 q_x + 2\bar{Q}_x \gamma_5 \lambda^a Q_x\end{aligned}$$

Define Overlap Axial by the decent relation:

$$\langle A^{ov}_\mu(x)\psi_y\bar{\psi}_z\rangle_c \equiv \langle A^{DW}_\mu(x) q_y \bar{q}_z\rangle_c$$

Mesons: A generalized weak coupling (chiral theory) 5-d theory

$$S = \int d^4x \int_{-L_s/2}^{L_s/2} ds[\frac{1}{4\sqrt{f(s)}} F_{\mu\nu}F_{\mu\nu} + \frac{r^4\sqrt{f(s)}}{2R^4} F_{\mu 5}F_{\mu 5} + m_q(\ldots)]$$

where $\Sigma(x) = P\exp[i\int_{-\infty}^{\infty} \lambda^a A_5^a(x,s)ds]$ obey Chiral L

Observable	Measured (MeV)	Model A (MeV)	Model B (MeV)
m_π	139.6±0.0004	139.6*	140
m_ρ	775.8±0.5	775.8*	793
m_{a_1}	1230±40	1363	1256
f_π	92.4±0.35	92.4*	86.5
$F_\rho^{1/2}$	345±8	329	337
$F_{a_1}^{1/2}$	433±13	452	449
$g_{\rho\pi\pi}$	6.03±0.07	5.43	6.05

"QCD and a Holographic Model of Hadrons" Erlich, Katz, Son, Stephanov, hep-ph/05011

FINI