Quarkonia at finite temperature

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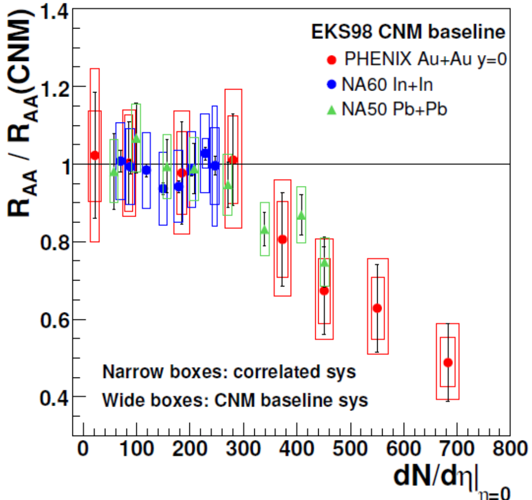
Plan of this talk

- Introduction
- Part I（Studies in WHOT-QCD）
  - A variational analysis on charmonia at finite temperature
- Part II（An ongoing study）
  - Charmonia and bottomonia at finite temperature on large quenched lattices
- Summary and outlook

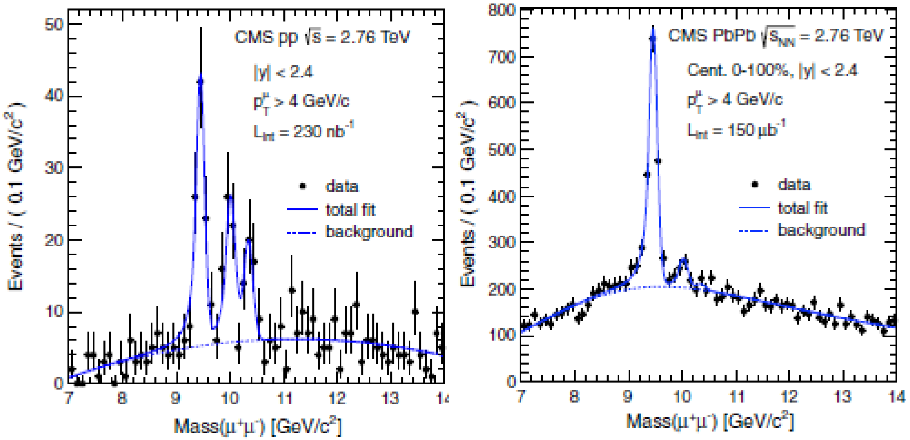
Quarkonia at finite temperature

- Bound states of heavy $q\bar{q}$
- At a certain temperature T_D, the dissociation should occur due to the color Debye screening
- An important probe of the quark-gluon plasma created in relativistic heavy ion collisions at RHIC, LHC

→ Theoretical investigation of in-medium properties of quarkonia plays an important role to understand experimental results.



N. Brambilla *et al.*, EPJ C71 (2011) 1534



S. Chatrchyan *et al.*, PRL 109 (2012) 222301

Meson correlator and spectral function

Temporal Euclidian meson correlator

$$G_H(\tau,\vec{p}) \equiv \int d^3x e^{-i\vec{p}\cdot\vec{x}} \langle J_H(\tau,\vec{x}) J_H(0,\vec{0}) \rangle$$

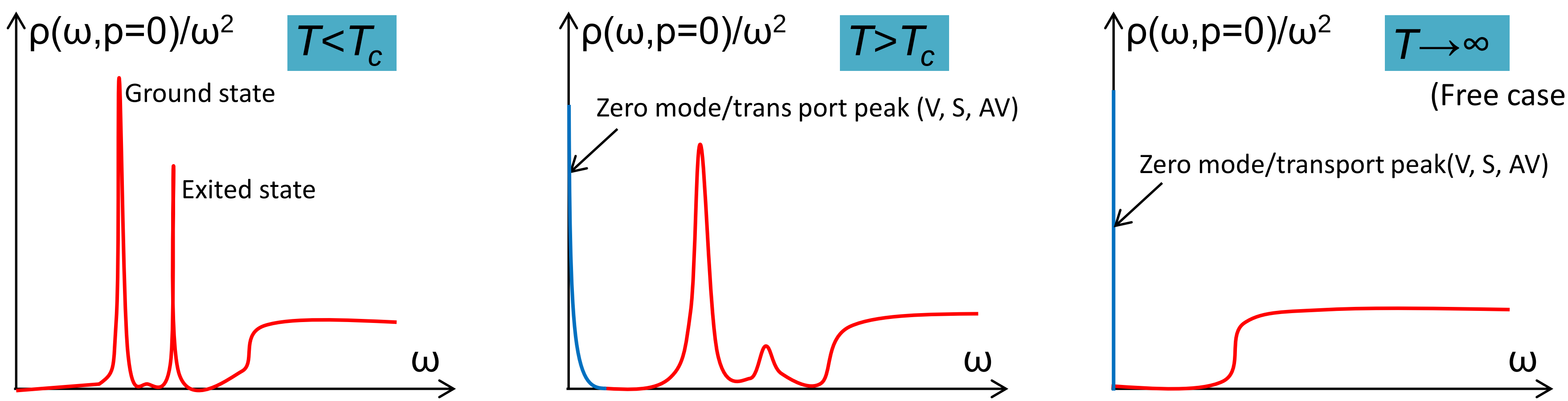
$$= \int_0^\infty \frac{d\omega}{2\pi} \boxed{\rho_H(\omega,\vec{p})} K(\omega,\tau)$$

Spectral function (SPF) has all information about in-medium meson properties

$$J_H(\tau,\vec{x}) \equiv \bar{\psi}(\tau,\vec{x}) \Gamma_H \psi(\tau,\vec{x})$$

$$K(\omega,\tau) \equiv \frac{\cosh[\omega(\tau-1/2T)]}{\sinh(\omega/2T)}$$

Channel	Γ_H	$^{2S+1}L_J$	J^{PC}	Quarkonia
Pseudoscalar (PS)	γ_5	$^1S_0$	0^{-+}	$\eta_c,\ \eta_b$
Vector (V)	γ_i	$^3S_1$	1^{--}	$J/\psi,\ \Upsilon$
Scalar (S)	$\mathbf{1}$	$^1P_0$	0^{++}	$\chi_{c0},\ \chi_{b0}$
Axialvector (AV)	$\gamma_i\gamma_5$	$^3P_1$	1^{++}	$\chi_{c1},\ \chi_{b1}$



PART I

A variational analysis on charmonia at finite temperature

HO, T. Umeda and K. Kanaya (WHOT-QCD Collaboration), J.Phys. G36 (2009) 064027
HO *et al*. (WHOT-QCD Collaboration), Phys.Rev. D84 (2011) 094504

Spectral function in finite volume

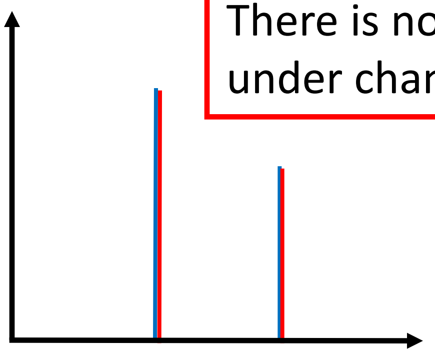
A spectral function consists of **discrete spectra** due to the finite spatial lattice extent.

$T<T_C$ Bound states

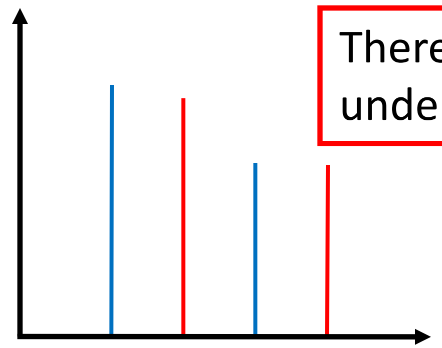
$T>T_C$ Scattering states

PBC
APBC

Spectral function

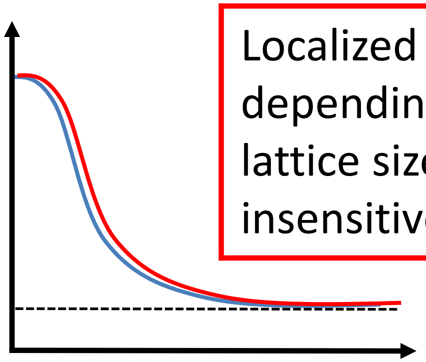


There is no mass shift under changing BC

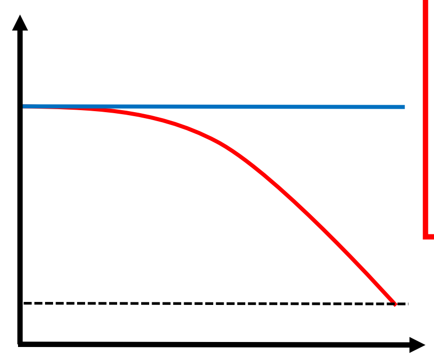


There is some mass shift under changing BC

Wave function



Localized shape not depending on spatial lattice size and insensitive to BC



Extended shape depending on spatial lattice size and sensitive to BC

Variational analysis

- A suitable method to study discrete spectra.
- Excited states also can be investigated.
  - Dissociation of charmonium excited states are important for the sequential J/Ψ suppression. L. Antoniazzi et al. [E705 Collaboration] (1993)
- Construct a matrix of correlators from a certain operator set with a same quantum number

$$\mathbf{C}_\Gamma(t) \equiv \left[C_\Gamma(t)_{ij} = \sum_{\vec{x}} \langle \mathcal{O}_\Gamma(\vec{x},t)_i \, \mathcal{O}_\Gamma^\dagger(\vec{0},0)_j \rangle \right] \quad i,j = 1,2,\cdots,n$$

E.g. Gaussian smeared operators

$$\mathcal{O}_\Gamma(\vec{x},t)_i \equiv \sum_{\vec{y},\vec{z}} \omega_i(\vec{y})\omega_i(\vec{z})\bar{q}(\vec{x}+\vec{y},t)\Gamma q(\vec{x}+\vec{z},t) \quad \omega_i(\vec{x}) \equiv e^{-A_i||\vec{x}||^2}$$

- Solve a generalized eigenvalue problem

$$\mathbf{C}_\Gamma(t)\,\mathbf{v}^{(k)} = \lambda_k(t;t_0)\,\mathbf{C}_\Gamma(t_0)\,\mathbf{v}^{(k)} \quad k = 1,2,\cdots,n$$

Variational analysis (cont'd)

- Mass spectra

$$\lambda_k(t;t0) = \frac{\cosh[m_k(t;t_0)(t-N_t/2)]}{\cosh[m_k(t;t_0)(t_0-N_t/2)]}$$

- Bethe-Salpeter wave function

$$\Psi_k(\vec{r};\vec{r}_0,t) = \frac{\sum_i A_i(\vec{r},t)V_{ik}}{\sum_i A_i(\vec{r}_0,t)V_{ik}}$$

$$A_i(\vec{r},t) = \sum_{\vec{x}} \langle \bar{q}(\vec{x},t)\Gamma q(\vec{x}+\vec{r},t)\mathcal{O}^\dagger_\Gamma(\vec{0},0)_i\rangle$$

$$\mathbf{V} = [\mathbf{v}^{(1)}\cdots\mathbf{v}^{(n)}]$$

- Spectral Weight

$$\rho_\Gamma(m_k(t;t_0)) = (\mathbf{C}_\Gamma(t_0)\mathbf{V})_{1k}(\mathbf{V}^{-1})_{k1}\frac{\sinh[m_k(t;t_0)\,N_t/2]}{\cosh[m_k(t;t_0)\,(t_0-N_t/2)]}$$

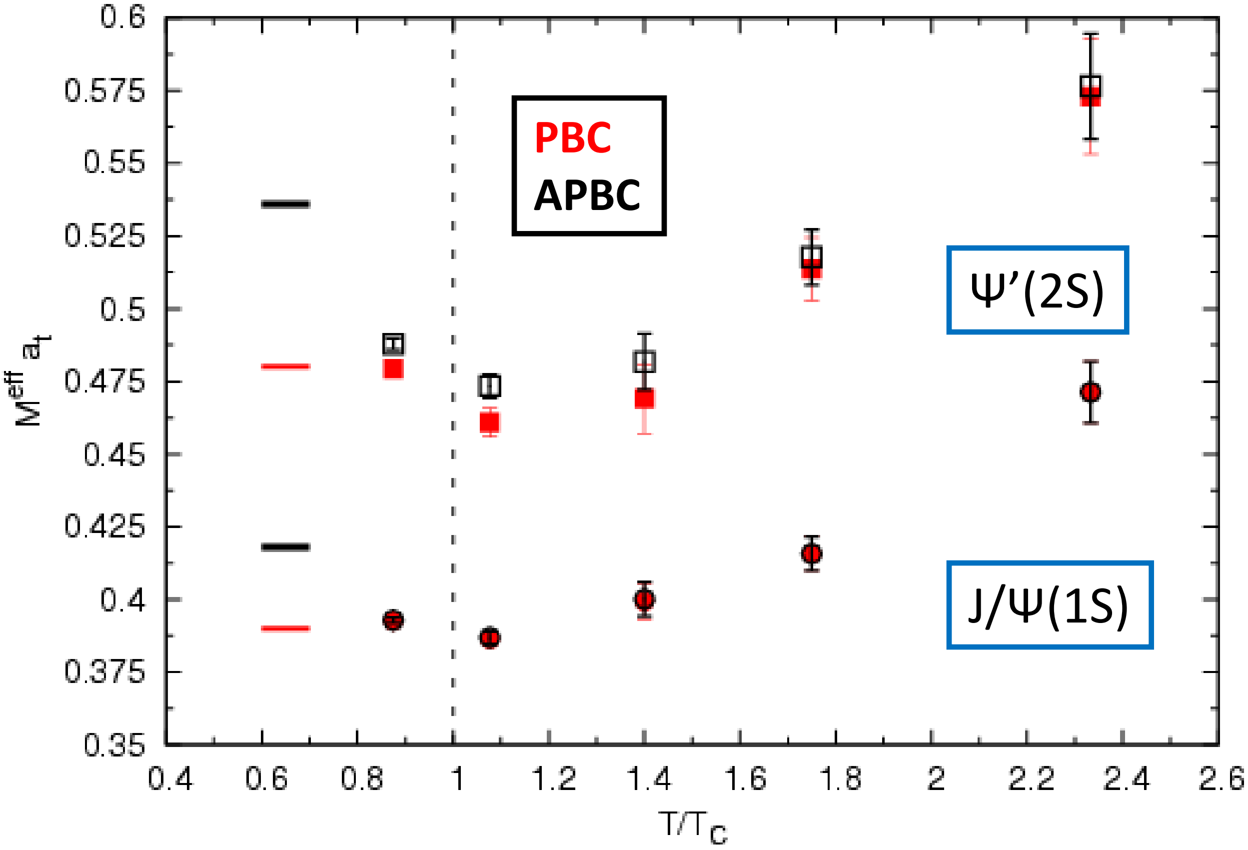
Assuming that the (1,1)-component of the correlator matrix corresponds to the point source-point sink operator

Lattice setup

- Standard plauette gauge & O(a)-improved Wilson quark actions
- In quenched QCD
- On anisotropic lattices (a_σ/a_τ = 4)
- β = 6.10 (α_σ = 0.0970(5) fm, α^{-1}_σ = 2.030(13) MeV)
- N_σ = 20 (, 16, 32)
- N_τ = 12, 16, 20 , 26, 32 (, 160) (T = 0.88—2.3T_c)
- Quark mass has been tuned so that J/Ψ mass becomes almost equal to its experimental value

Mass spectra

- Temperature and spatial BC dependence (Ve channel)



***n* = 4**
$20^3 \times N_t$ lattice

: mass shift in the free quark case

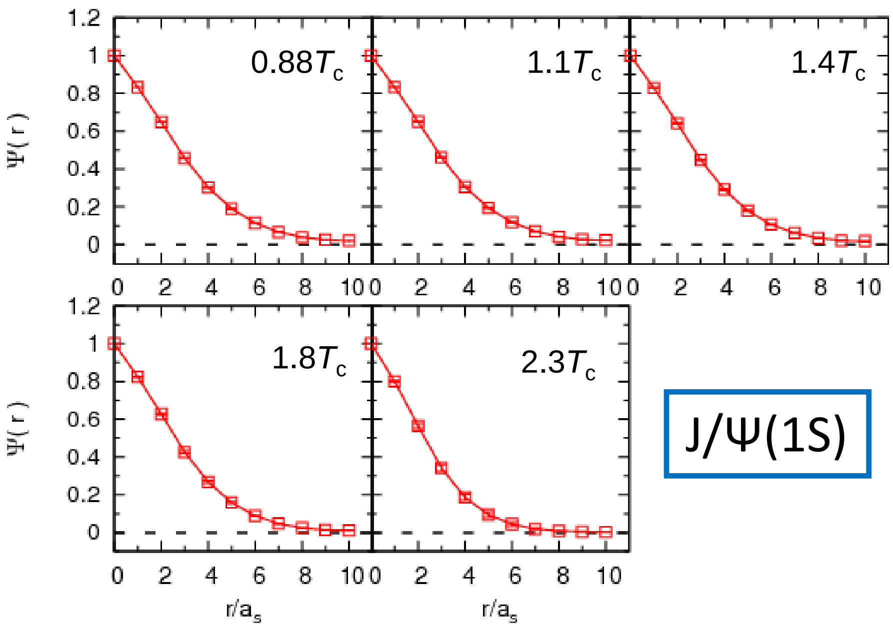
There is no clear BC dependence up to 2.3 T_c.

There seems to be no singal of scattering states up to 2.3T_c.

Wave function

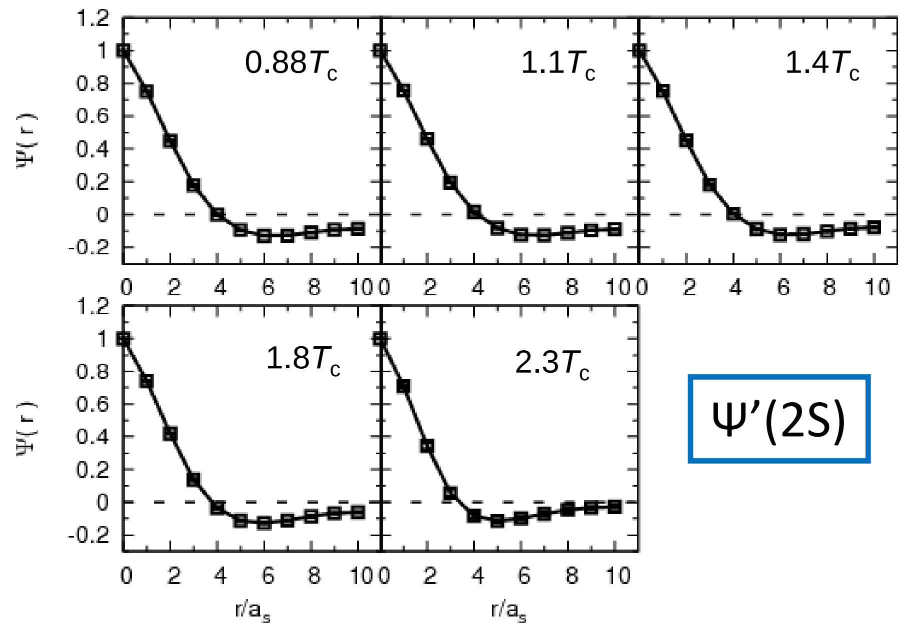
- Temperature dependence (Ve channel)

The ground state



J/Ψ(1S)

The first excited state



Ψ’(2S)

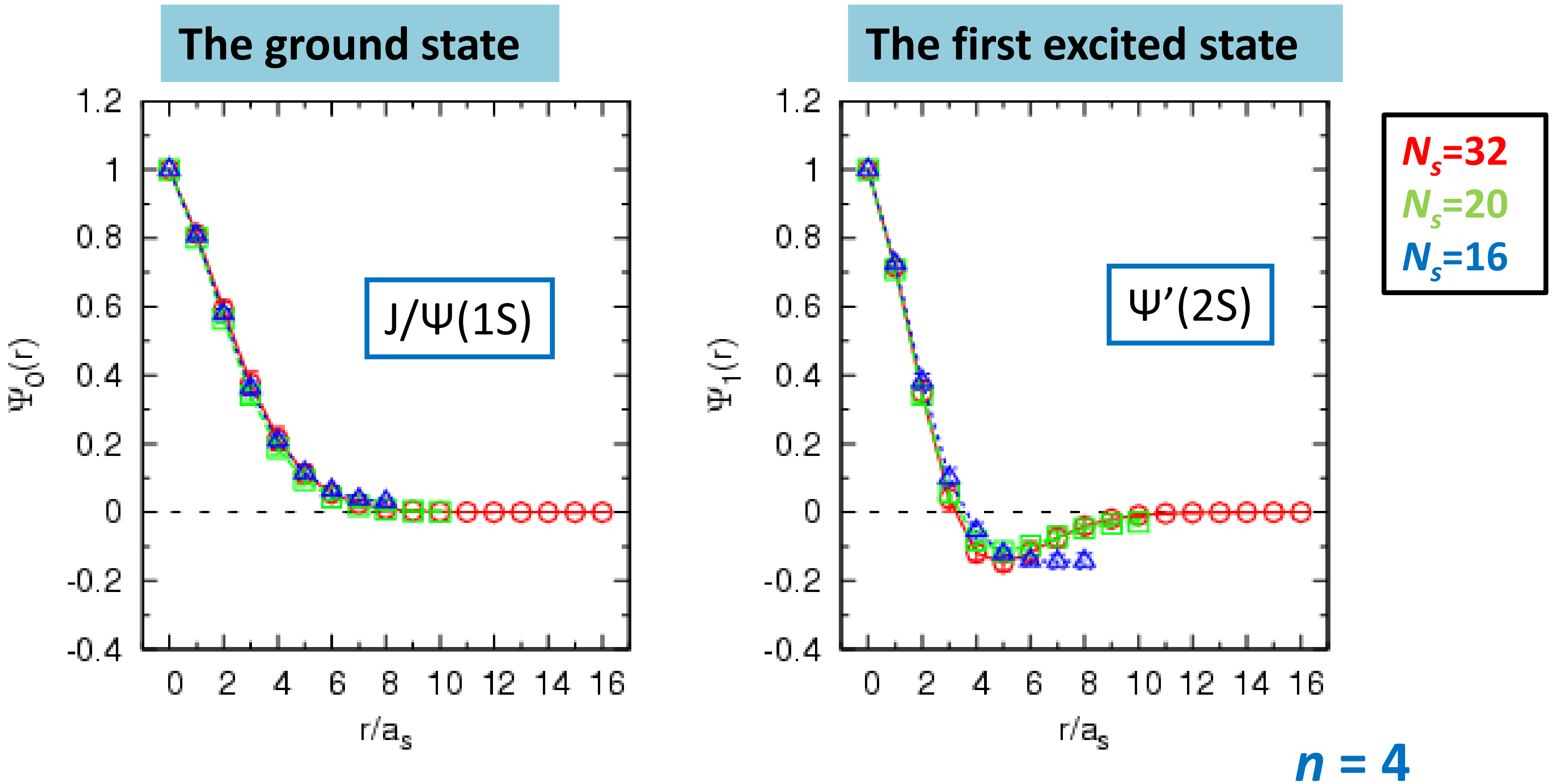
***n* = 4**

$20^3 \times N_t$ lattice

The wave functions of the ground and the first excited state keep their shapes up to 2.3 T_c.

Wave function (cont'd)

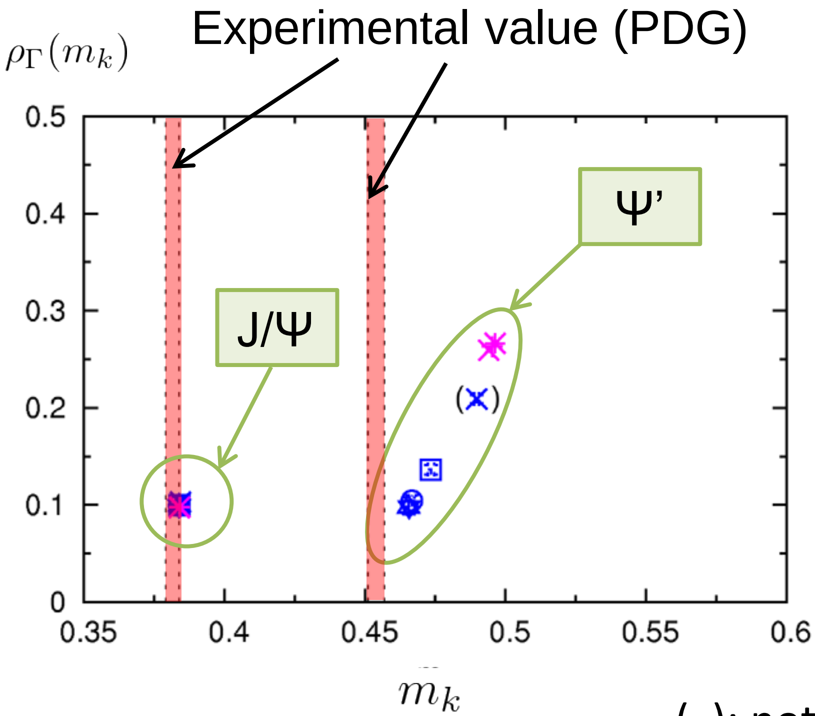
- Volume dependence at 2.3 T_c(Ve channel)



- **Not sensitive to the volume**
- **Spatially localized even at T=2.3T_c for both ground state and 1st excited state**

Spectral function

Comparison with the Maximum Entropy Method (Ve channel)



MEM

+ × ✳

variational method

× $n = 3$

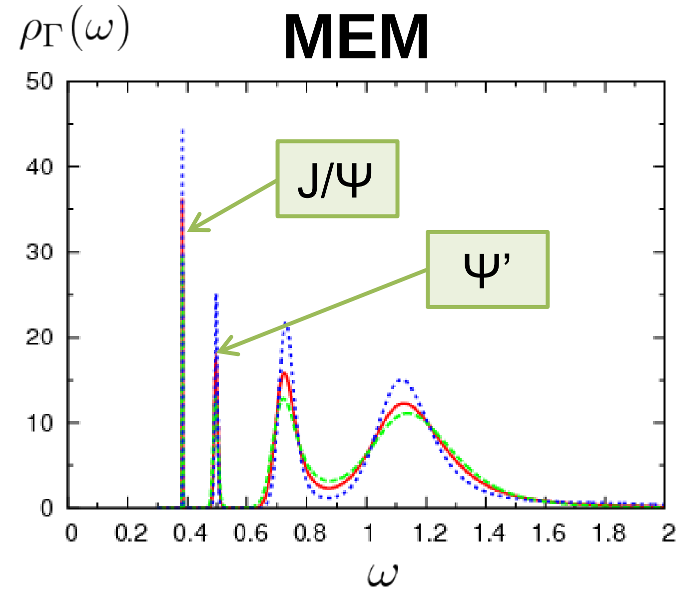
□ $n = 4$

○ $n = 5$

△ $n = 6$

▽ $n = 7$

( ): not asymptotic signals



m_k ← location of each peak

$\rho_\Gamma(m_k)$ ← area of each peak

Ground state → all data almost consistent with experimental value

1st excited state → there is difference between variational method results and MEM results

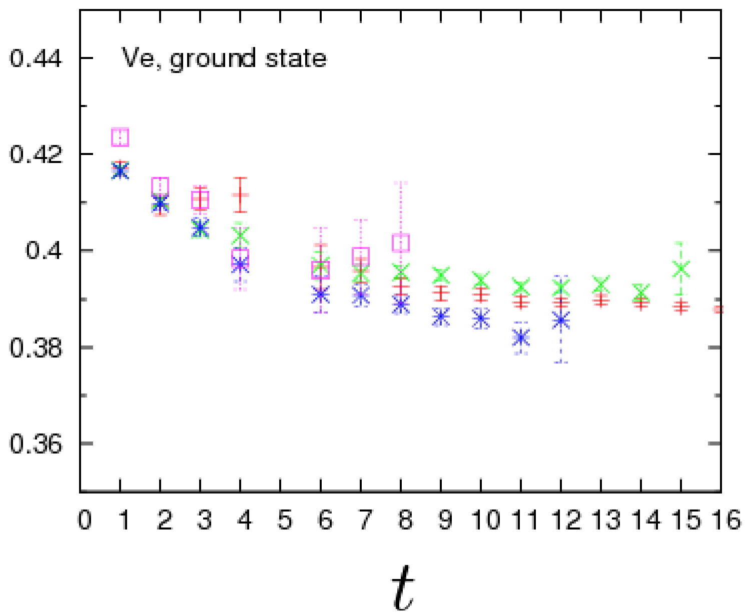
→ variational method data get closer to experimental value as *n* increases

It seems that variational method can improve data accuracy for excited states.

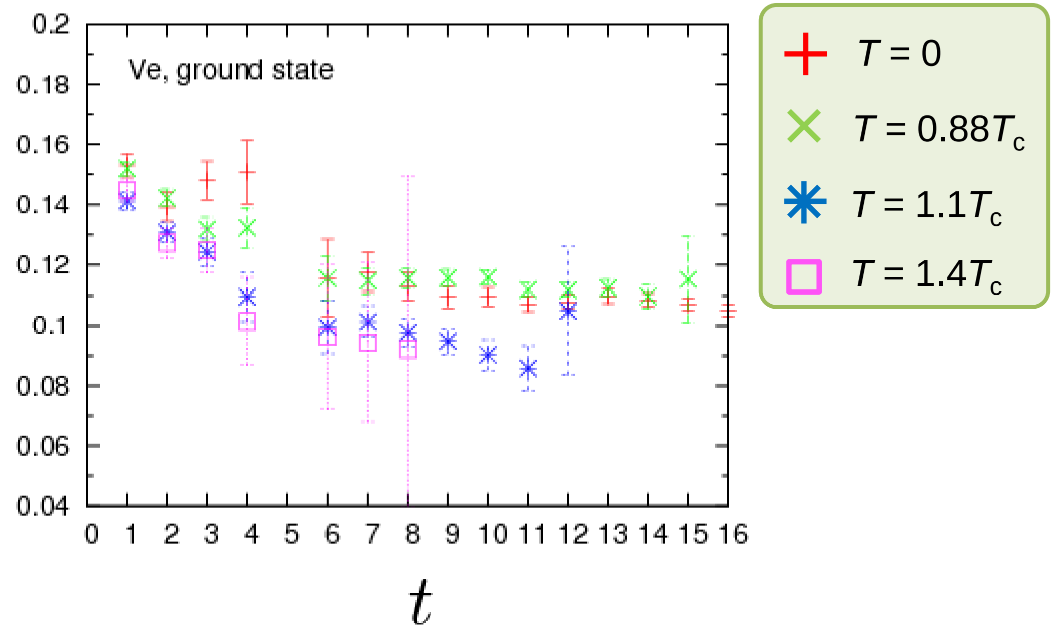
Spectral function (cont'd)

Temperature dependence (Ve channel, ground state) $n = 7$

m_1 Effective mass



$\rho_\Gamma(m_1)$ Spectral weight



No clear temperature dependence for the effective masses.

The spectral weight may change but the modification is quite small.

There is no clear evidence of dissociation for J/Ψ up to 1.4T_c

Summary on Part I

- Charmonia at finite temperature have been studied with a variational analysis in quenched lattice QCD.
  - **Spatial boundary condition dependence of effective masses was investigated.**
  - **Temperature and volume dependences of wave function were also investigated.**
  - **Discrete spectral functions were constructed**
  - **There is no clear evidence of dissociation of charmonia up to 2.3T_c so far.**

PART II
Charmonia and bottomonia at finite temperature on large quenched lattices

HO, PoS LATTICE2013 (2014) 172
HO, H.-T. Ding and O. Kaczmarec, PoS LATTICE2014 (2014) 219

Simulation Setup

- Standard plauette gauge & O(a)-improved Wilson quark actions
- In quenched QCD
- On fine and large isotropic lattices
- $T = 0.7 — 1.5T_c$
- Both charm & bottom

β	N_σ	N_τ	T/T_c	# confs.
7.192	96	48	0.7	259
		32	1.1	476
		28	1.2	336
		24	1.5	336
7.793	192	96	0.7	69
		56	1.2	190
		48	1.5	210

β	a [fm]	a^{-1} [GeV]	κ_{charm}	κ_{bottom}	$m_{J/\Psi}$ [GeV]	m_Υ [GeV]
7.192	0.0188	10.5	0.13194	0.12257	3.140(3)	9.574(3)
7.793	0.00942	20.9	0.13221	0.12798	3.175(5)	9.687(5)

The scale has been set by r_0=0.49fm and with a formula for r_0/a in

A. Francis, O. Kaczmarec, M. Laine, T. Neuhaus, HO, PRD 91 (2015) 9, 096002

Experimental values: $m_{J/\Psi}$ = 3.096.916(11) GeV, m_Υ = 9.46030(26) GeV

J. Beringer *et al*. [PDG], PRD 86 (2012) 010001

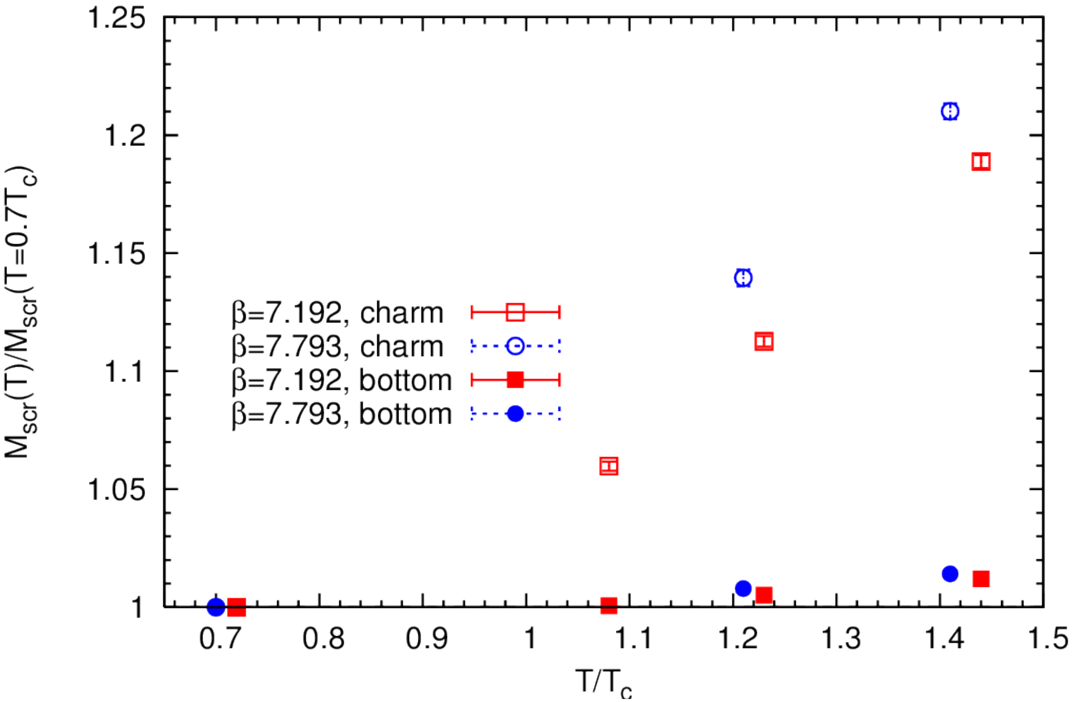
Screening mass

Spatial meson correlation function

Screening mass

$$G(z) \equiv \int dxdy \int_0^{1/T} d\tau \langle J_H(\tau, \vec{x}) J_H(0, \vec{0}) \rangle \underset{z >> 1/T}{\longrightarrow} e^{-M_{\rm scr} z}$$

Ve channel



If there is a lowest lying bound state

$$M_{\rm scr} = M$$

High *T* limit (free case)

$$M_{\rm scr} = 2\sqrt{(\pi T)^2 + m_q^2}$$

Quark mass

***M_{scr}* increases monotonically as increasing temperature.**

Small temperature dependence for bottom.

Reconstructed correlator

$$G_{\rm rec}(\tau, T; T') \equiv \int_0^\infty d\omega \rho(\omega, T') K(\omega, \tau, T)$$

S. Datta *et al.*, PRD 69 (2004) 094507

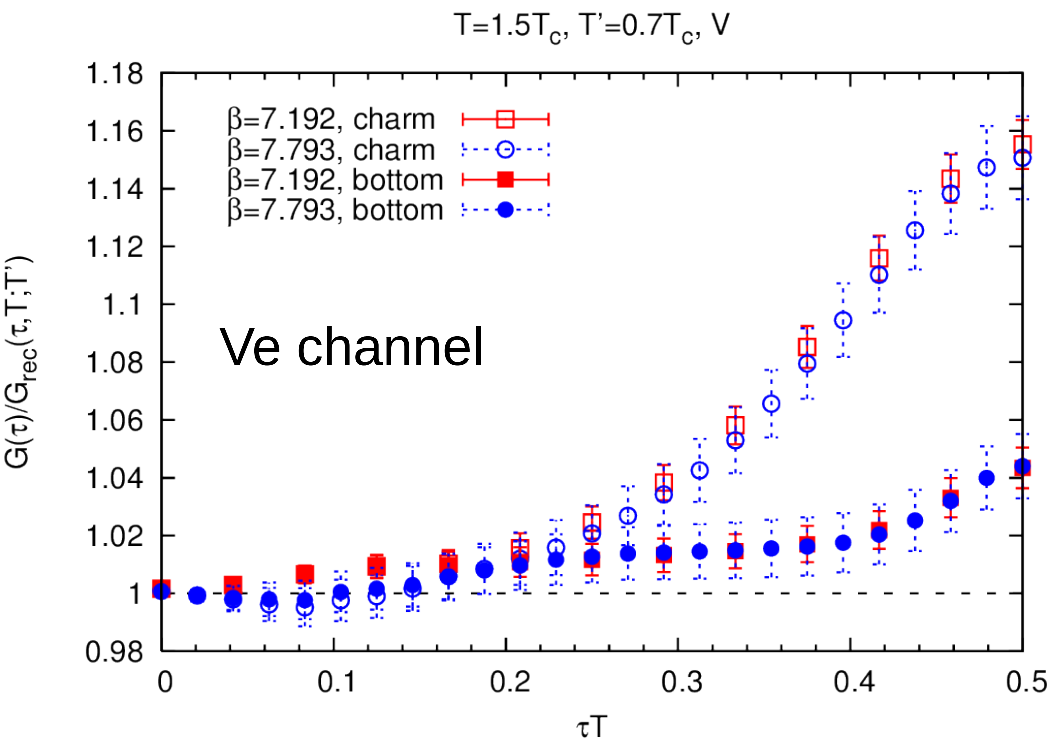
If the spectral function doesn't vary with temperature

$\dfrac{G(\tau,T)}{G_{\rm rec}(\tau,T;T')}$ equals to unity at all τ

$$\frac{\cosh[\omega(\tau - N_\tau/2)]}{\sinh[\omega N_\tau/2]} = \sum_{\tau'=\tau;\Delta\tau'=N_\tau}^{N'_\tau - N_\tau + \tau} \frac{\cosh[\omega(\tau' - N'_\tau/2)]}{\sinh[\omega N'_\tau/2]}$$

$T = 1/(N_\tau a)$ $\quad N'_\tau = mN_\tau$ $\quad m = 1, 2, 3, \cdots$

$$G_{\rm rec}(\tau, T; T') = \sum_{\tau'=\tilde{\tau};\Delta\tau'=N_\tau}^{N'_\tau - N_\tau + \tau} G(\tau', T')$$

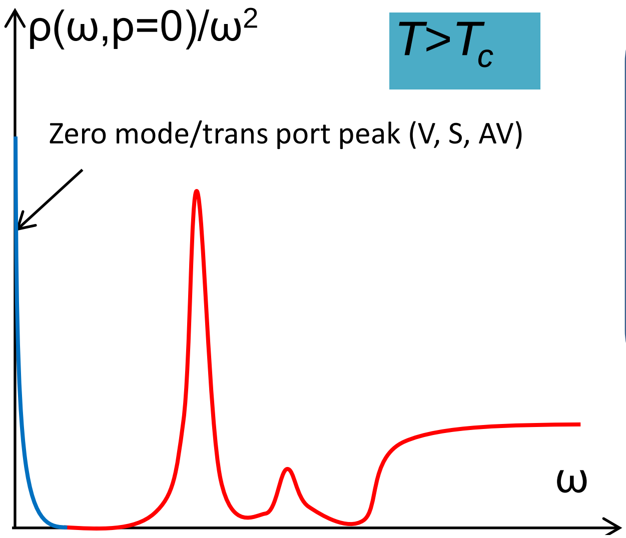


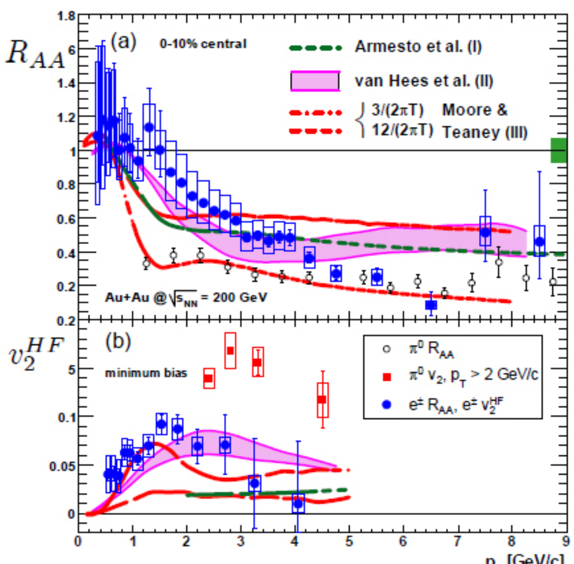
There is strong modification at large τ/a, especially for charm.

Large τ ↔ Small ω

→ This strong modification might be related to the transport peak.

Transport coefficients





Adare *et al*. [PHENIX Collaboration], PRL 98 (2007) 172301

Heavy quark diffusion constant

$$D = \frac{1}{6\chi_{00}} \lim_{\omega\to 0} \sum_{i=1}^{3} \frac{\rho^V_{ii}(\omega,\mathbf{0})}{\omega}$$

$\rho^V_{ii}(\omega)$: spatial component of vector spectral function

χ_{00} : Quark number susceptibility

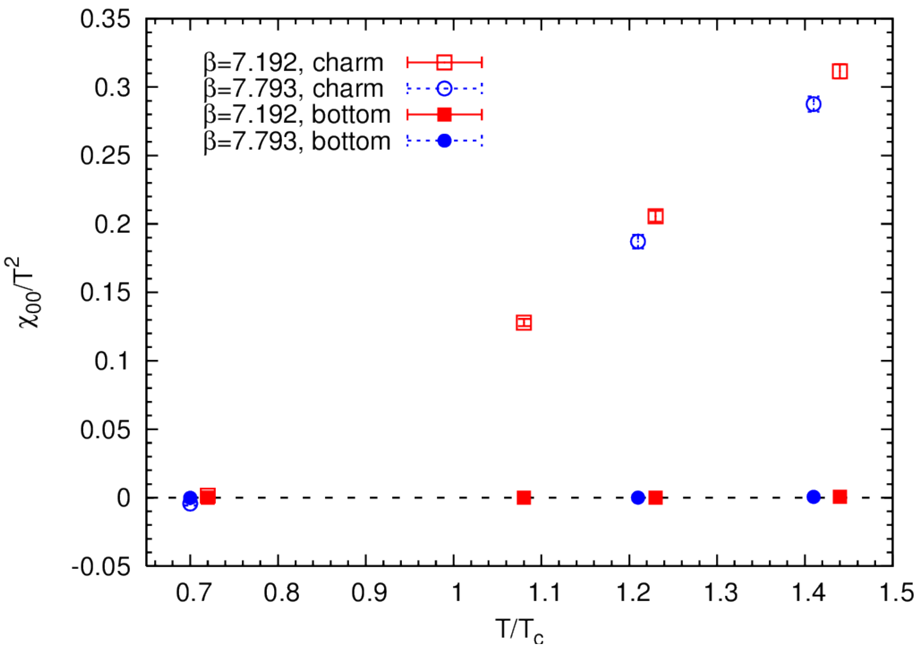
$\rho^V_{00}(\omega) = 2\pi\chi_{00}\omega\delta(\omega)$ ➡ $G^V_{00}(\tau) = T\chi_{00}$

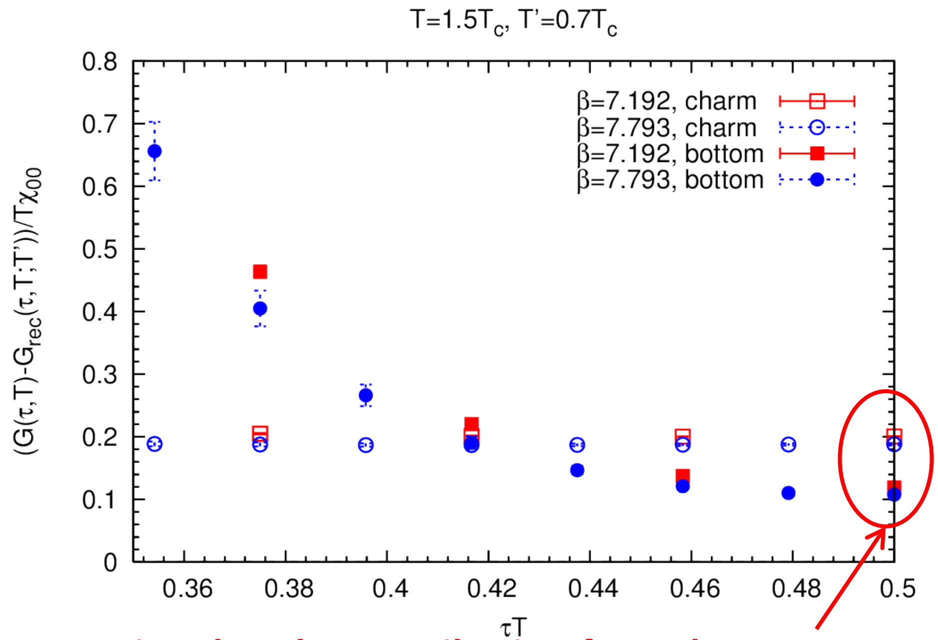
The evolution of the system in hydro models

→ **Transport coefficients are important.**

Determination by first principle calculations in QCD is needed.

Transport coefficient (cont'd)





Ve channel

Assuming that the contribution from the transport peak would be dominant in G – G_{rec} at τT = ½.

Ansatz: $$\rho_{ii}^{V}(\omega << T) = 2\chi_{00}\frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2} \qquad \eta \equiv \frac{T}{MD} \qquad M \equiv m_q a$$

P. Petreczky and D. Teaney, PRD 73 (2006) 01458

Charm: 2πTD ≈ 0.6—4 (β = 7.192), 2πTD ≈ 0.5—2 (β = 7.793) for m_q = 1—2 GeV

Bottom: there is no intersection for m_q = 4—5 GeV → D is infinitely large

Summary on Part II

- We calculate meson correlation functions
  - on fine and large isotropic lattices
  - With 2 different cutoffs & quark masses for charm and bottom
- Screening masses
  - **Increase monotonically as increasing temperature for V channel**
  - **Small temperature dependence for bottomonia**
- Meson spectral functions are investigated in terms of reconstructed correlators
  - **There is strong modification at large τ for V channel, which would be related to the transport peak.**
  - **From the difference between the ordinary and reconstructed correlation functions, the heavy quark diffusion constant is roughly estimated in the charm case.**

Outlook

- Reconstruction of spectral functions
- Searching dissociation temperatures of quarkonia
- Estimating transport coefficients more accurately
- Taking continuum limit

金谷さん、還暦おめでとうございます。