Nf=3 QCD 相図

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「有限温度密度系の物理と格子ＱＣＤシミュレーション」研究会

理化学研究所
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RIKEN Advanced Institute for Computational Science

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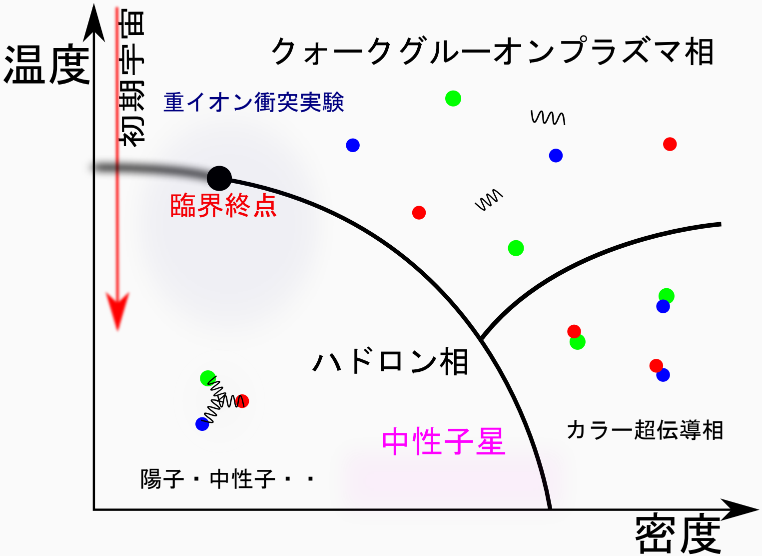
内容

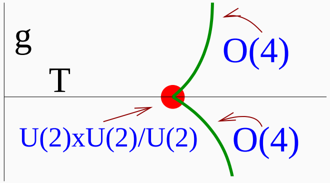
- QCD **の相図の概観**
- phase diagram for $N_f = 3$, $\mu = 0$
  - Critical endpoint of finite temperature phase transition for three flavor QCD, Phys. Rev. D 91, 014508 (2015)
- phase diagram for $N_f = 3$, $\mu \neq 0$
  - Curvature of the critical line on the plane of quark chemical potential and pseudo scalar meson mass for three-flavor QCD, arXiv:1504.00113[hep-lat]

QCD **の相図の概観**

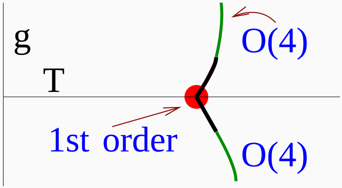
有限温度有限密度（物理点）



3D ϕ^4 theory



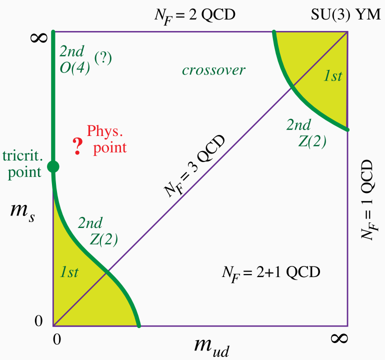
strong $\mathbf{U(1)_A}$ anomaly



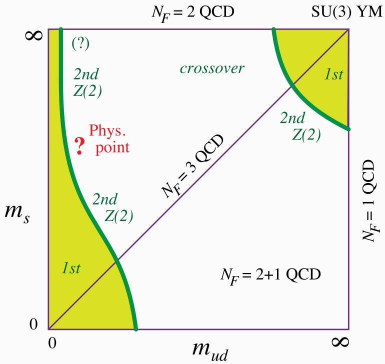
weak $\mathbf{U(1)_A}$ anomaly

Basile et al. (hep-lat/0509018)

Columbia plot（possible）



standard scenario



alternative scenario
(weak $\mathbf{U(1)_A}$ anomaly)

Kanaya (arXiv:1012.4247[hep-lat])

phase diagram for $N_f = 3$, $\mu = 0$

Motivation

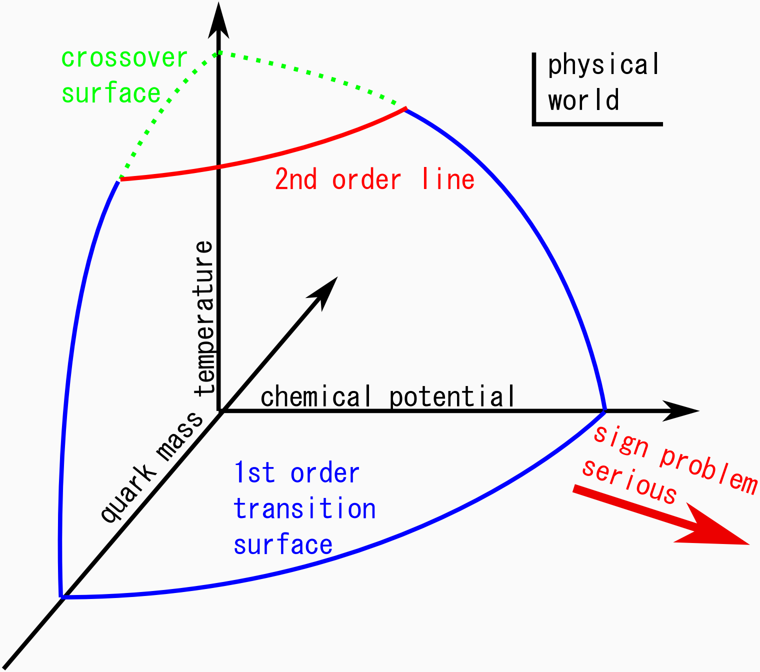
- Critical endpoint (CEP) obtained with staggered and Wilson type fermions is inconsistent. → Results in the continuum limit is necessary

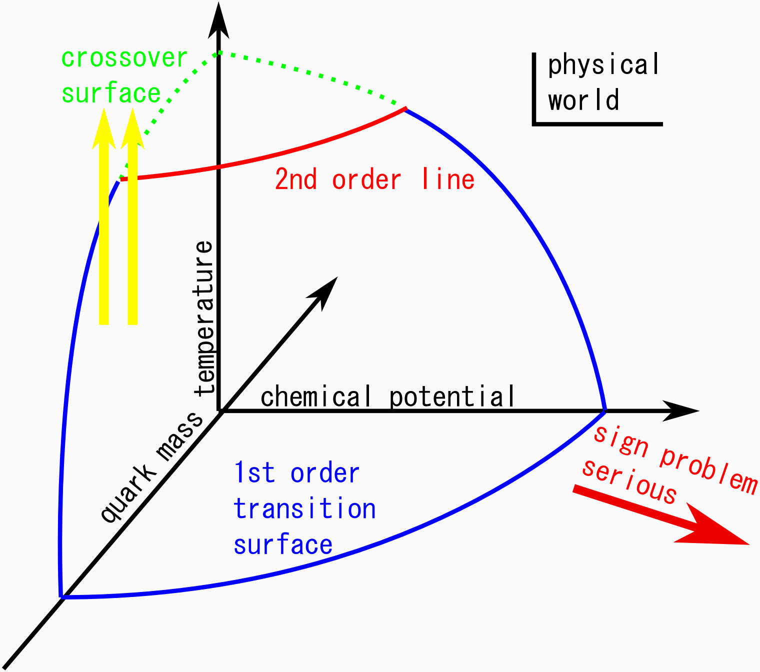
m_π at the endpoint at $\mu = 0$ (bottom-left corner of Columbia plot)

N_t	action	m_π^E [MeV]	
4	unimproved staggered	260	de Forcrand,
6	unimproved staggered	150	Philipsen '07
4	p4-improved staggered	70	Karsch et al. '03
6	stout-improved staggered	$\lesssim 50$	Endrődi et al. '07
6	HISQ	$\lesssim 50$	Ding et al. '11,...'15
4	unimproved Wilson	~ 1100	Iwasaki et al. '96

- $N_f = 3$ study is a stepping stone
  - curvature of critical surface
  - to the physical point

We determine CEP on $m_l = m_s$ line with clover fermions in the continuum limit


crossover
surface
physical
world
2nd order line
temperature
chemical potential
quark mass
1st order
transition
surface
sign problem
serious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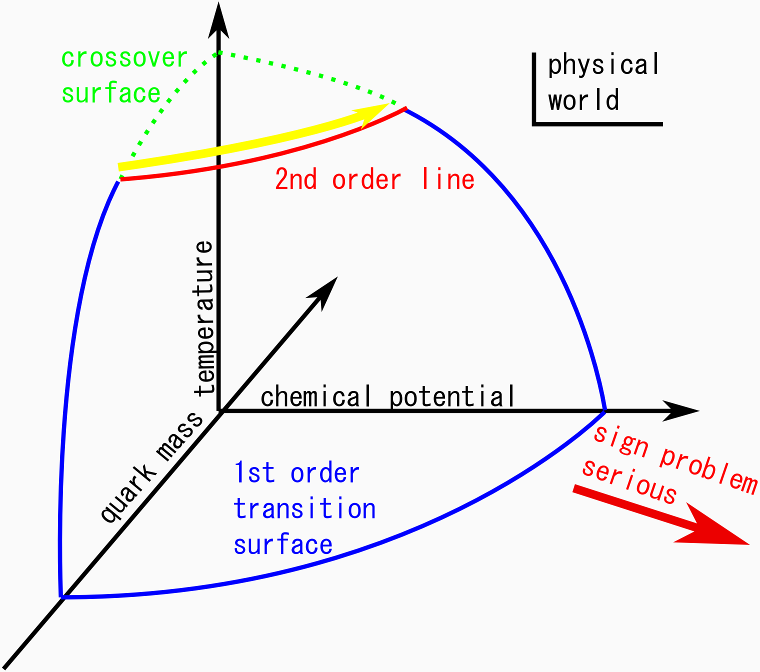

crossover
surface
physical
world
2nd order line
temperature
chemical potential
quark mass
1st order
transition
surface
sign problem
serious

Distinguishing between 1st, 2nd and crossover

criterion	first order	second order	crossover
distribution	double peak	single peak	singe peak
χ_{peak}	$\propto N_l^d$	$\propto N_l^{\gamma/\nu}$	-
$\beta(\chi_{\text{peak}}) - \beta_c$	$\propto N_l^{-d}$	$\propto N_l^{-1/\nu}$	-
kurtosis at $N_l \to \infty$	K= -2	-2 < K < 0	-

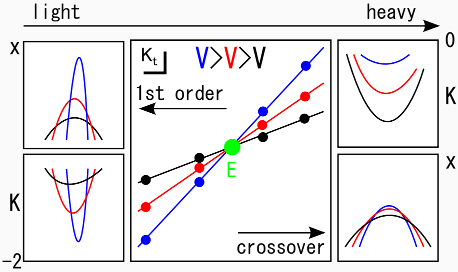
- scaling might work with wrong exponents near CEP
- peaks in histgram might emerge only at large N_l on weak 1st order
- K does not depend on volume at 2nd order phase transition point

$$M = N_l^{-\beta/\nu} f_M(tN_l^{1/\nu})$$

$$K + 3 = B_4(M) = \frac{N_l^{-4\beta/\nu} f_{M^4}(tN_l^{1/\nu})}{\left[N_l^{-2\beta/\nu} f_{M^2}(tN_l^{1/\nu})\right]^2} = f_B(tN_l^{1/\nu})$$

Method to determine CEP

- determine the transition point (peak position of susceptibility)
- determine kurtosis at transition point at each spatial lattice size
- find intersection point of kurtosis by fit, $K_{\mathrm{E}} + aN_l^{1/\nu}(\beta - \beta_{\mathrm{E}})$



- interpolate/extrapolate $\sqrt{t_0}m_{\mathrm{PS,t}}$ measured at transition point to β_{E}
- extrapolate $\sqrt{t_0}m_{\mathrm{PS,E}}$ to the continuum limit
- use scale determined from Wilson flow $1/\sqrt{t_0} = 1.347(30)$ GeV [Borsanyi et al. '12]

Higher moments

i-th derivative of $\ln Z$ with respect to control parameter c:

$$E = \frac{\partial \ln Z}{\partial c}$$

- Variance

$$V = \frac{\partial^2 \ln Z}{\partial c^2} = \sigma^2$$

- Skewness (e.g. right-skewed → $S > 0$, left-skewed → $S < 0$)

$$S = \frac{1}{\sigma^3} \frac{\partial^3 \ln Z}{\partial c^3}$$

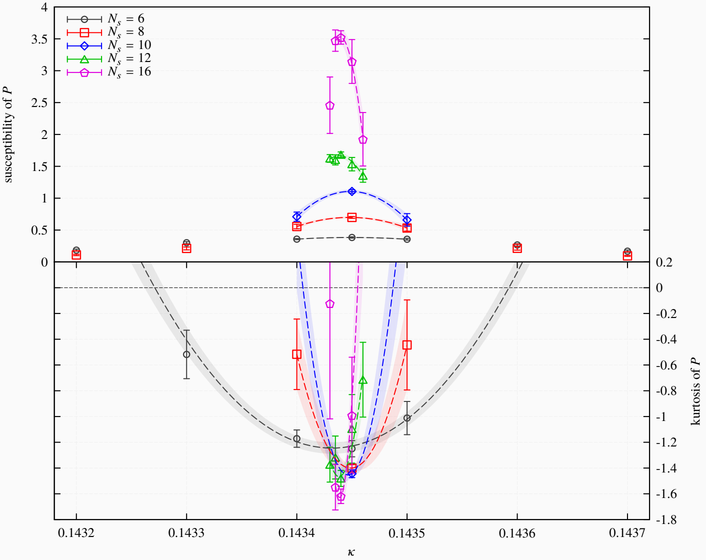
- Kurtosis(e.g. Gaussian → $K = 0$, 2δ func. → $K = -2$)

$$K = \frac{1}{\sigma^4} \frac{\partial^4 \ln Z}{\partial c^4} = B_4 - 3$$

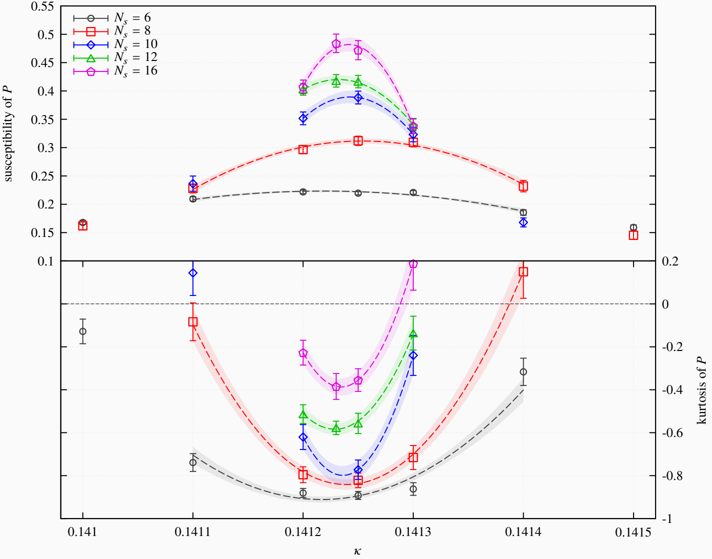
Simulations

- action: Iwasaki gluon + $N_f = 3$ clover (non perturbative c_{SW}, degenerate)
- observables
  - gauge action density, G
  - plaquette, P
  - Polyakov loop, L
  - chiral condensate, Σ
  - and their higher moments
- temporal lattice size $N_t = 4, 6, 8$
  - statistics: O(100K) traj
- preliminary $N_t = 10$
  - statistics: O(1K) traj

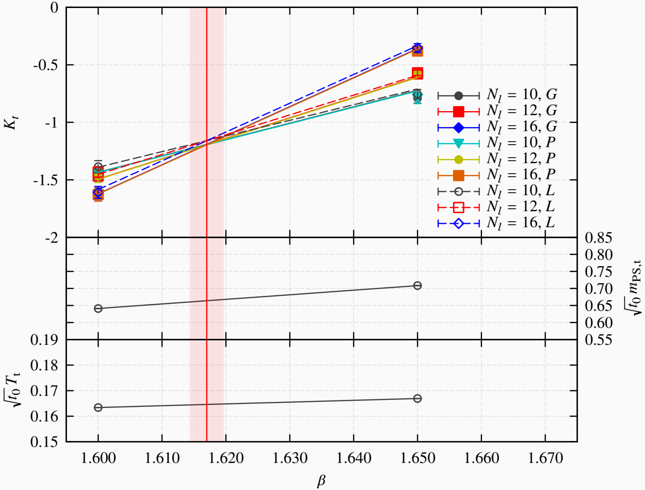
plaquette at $\beta = 1.60$, $N_t = 4$



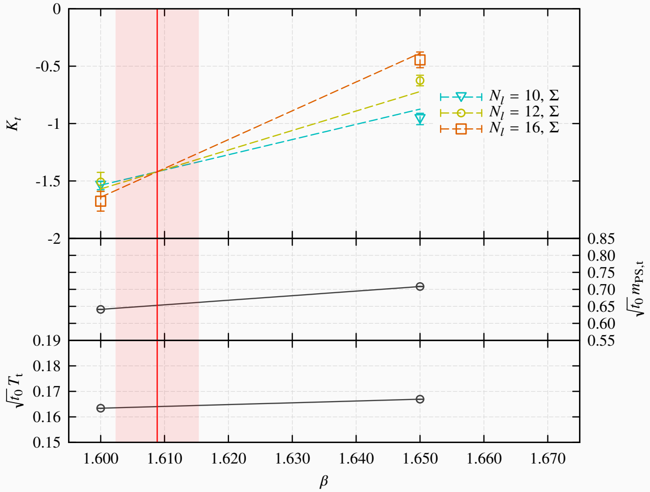
plaquette at $\beta = 1.65$, $N_t = 4$



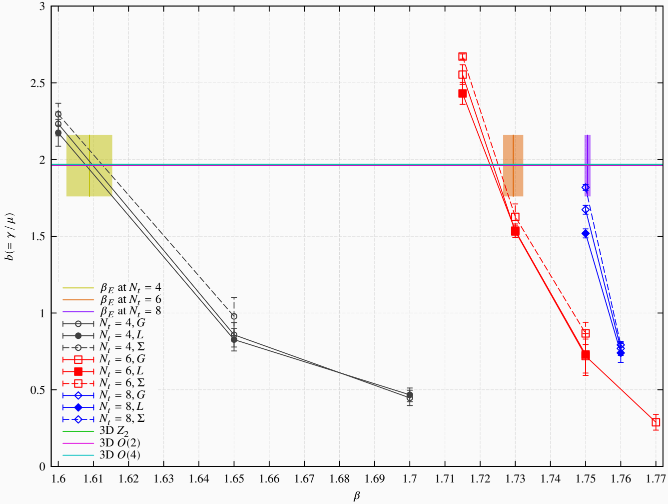
Kurtosis intersection at $N_t = 4$



Kurtosis intersection at $N_t = 4$

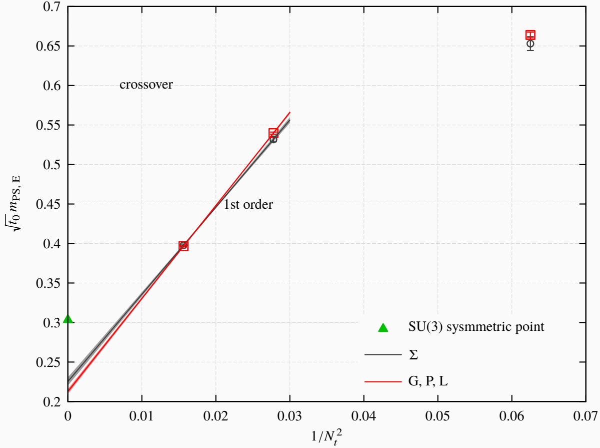


γ/ν v.s. β



$$\chi_{\max} = aN_l^{\gamma/\nu}$$

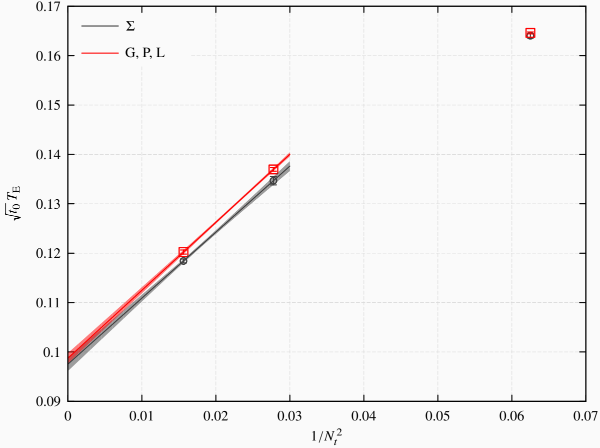
continuum extrapolation for $\sqrt{t_0} m_{\mathrm{PS,E}}$



▲ : $\sqrt{t_0} m_{PS}^{phy;sym} = \sqrt{t_0}\sqrt{(m_\pi^2 + 2m_K^2)/3} \sim 0.305$

$m_{\mathrm{PS,E}} = 304(7)(14)(7)$ MeV

continuum extrapolation for $\sqrt{t_0}T_\mathrm{E}$



$T_\mathrm{E} = 131(2)(1)(3)$ MeV

Summary at $N_t = 4, 6, 8$

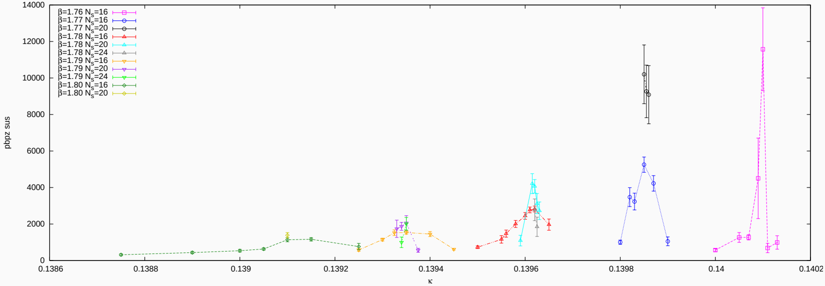
- kurtosis intersection analysis is consistent with χ_{max} analysis
- results at $N_t = 4$ is out of scaling region
- $\sqrt{t_0} m_{\mathrm{PS,E}}$ in the continuum limit is smaller than the SU(3) sysmmetric point,

$$m_{\mathrm{PS,E}} / m_{\mathrm{PS}}^{\mathrm{phys,sym}} = 0.739(17)(34)(17)$$

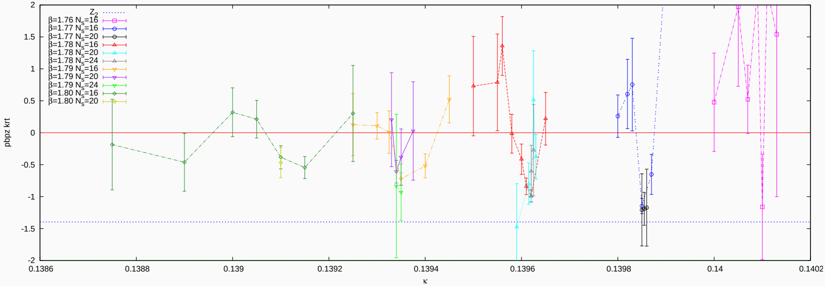
- further studies at larger temporal sizes to obtain conclusive results are needed

Phys. Rev. D 91, 014508 (2015)

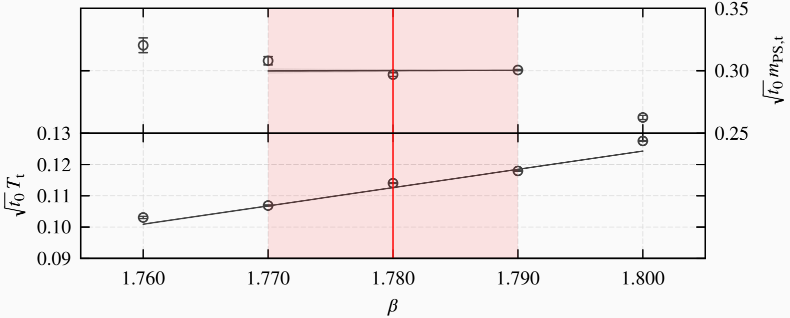
Σ at $N_t = 10$ (1/3, preliminary)



Σ at $N_t = 10$ (2/3, preliminary)

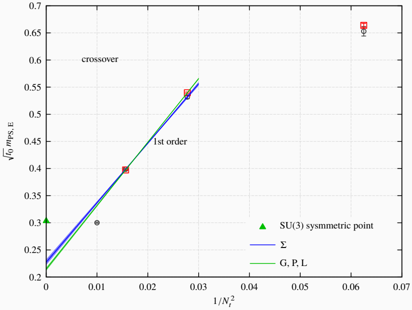


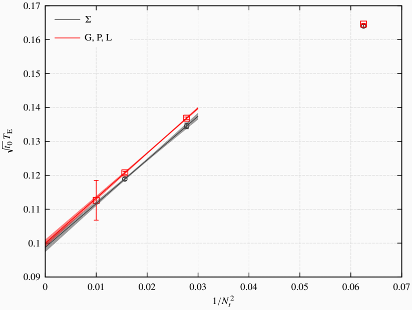
Σ at $N_t = 10$ (3/3, preliminary)



assuming $\beta_{\mathrm{E}} = \mathbf{1.78(1)}$

continuum extrapolation and results at $N_t = 10$ (preliminary)





- assuming $\beta_{\mathrm{E}} = 1.78(1)$ at $N_t = 10$
- excluding results at $N_t = 10$ from continuum extrapolation
- T_{E} would not change very much
- $m_{\mathrm{PS,E}}$ may become smaller than results at smaller N_t

Summary ($\mu = 0$)

We have investigated the critical endpoint of QCD with clover fermions and determined the critical endpoint by using the intersection points of the Binder cumulants and extrapolated to the continuum limit

- T_{E} in the continuum limit would not change very much

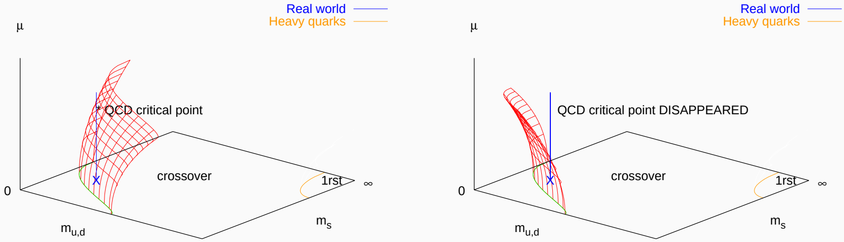
$$T_{\mathrm{E}} \approx 130 \text{ MeV}$$

- $m_{\mathrm{PS,E}}$ in the continuum limit may become smaller than results at smaller N_t

$$m_{\mathrm{PS,E}} < 304(7)(14)(7) \text{ MeV?}$$
$$m_{\mathrm{PS,E}}/m_{\mathrm{PS}}^{\mathrm{phys,sym}} < 0.739(17)(34)(17)?$$

- we are doing further studies with high statistics at larger temporal sizes to obtain conclusive results phase diagram for $\boldsymbol{N_f = 3}$, $\boldsymbol{\mu \neq 0}$

Motivation



de Forcrand and Philipsen
(JHEP 0701:077,2007)

We investigate curvature

Finite density simulation

$$
\begin{aligned}
Z &= \int DUe^{-S_g}(\det D(\mu))^{N_f} = \int DUe^{-S_g}|\det D(\mu)|^{N_f} e^{iN_f\theta} \\
&= \int DUe^{-S_g} \det D^\dagger(\mu)^{N_f/2} \det D(\mu)^{N_f/2} e^{iN_f\theta} \\
&= \int DUe^{-S_g}[\det D^\dagger(\mu) D(\mu)]^{N_f/2} e^{iN_f\theta}
\end{aligned}
$$

- generate gauge field configurations with weight:

$$
e^{-S_g + \ln \det[D^\dagger(\mu) D(\mu)]^{N_f/2}}
$$

- $a\mu = 0.1$
- at $N_t = 6$, $N_l = 8, 10, 12$, $\beta = 1.70 - 1.77$
- statistics: O(10,000) - O(100,000) traj.

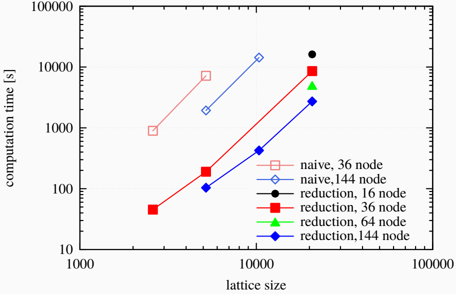
Finite density simulation

- reweight with phase θ

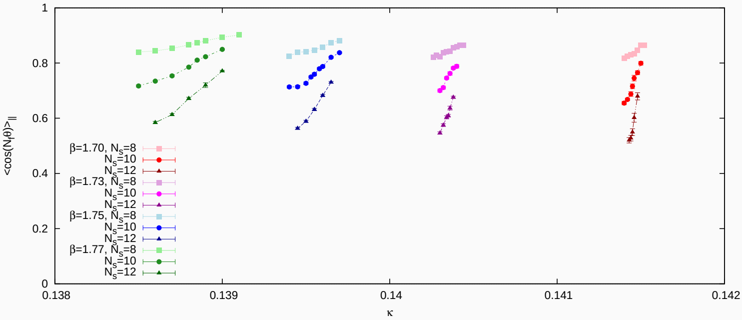
$$\langle \mathcal{O} \rangle = \frac{\langle \mathcal{O} e^{iN_f\theta} \rangle}{\langle \cos(N_f\theta) \rangle}$$

- cost reduction to calculate θ : (but cost is still expensive, $O(n^3)$)

$$\det \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \det[A]\det[D]\det[1 - D^{-1}CA^{-1}B]$$



$\cos(3\theta)$


1
0.8
0.6
0.4
0.2
0
0.138
0.139
0.14
0.141
0.142
κ
<cos(N_fθ)>_||
β=1.70, N_s=8
N_s=10
N_s=12
β=1.73, N_s=8
N_s=10
N_s=12
β=1.75, N_s=8
N_s=10
N_s=12
β=1.77, N_s=8
N_s=10
N_s=12

reweighting

We calculate

$$\frac{\det D_W(m_0', \mu')}{\det D_W(m_0, \mu)} = \exp\left[\ln \frac{\det D_W(m_0', \mu')}{\det D_W(m_0, \mu)} \right]$$

$$= \exp\Bigg[\sum_{j,k=0}^{\infty} \frac{\Delta_{m_0}^j \Delta_\mu^k}{j!k!} \left(\frac{\partial}{\partial m_0}\right)^j \left(\frac{\partial}{\partial \mu}\right)^k \ln \det D_W(m_0, \mu)$$

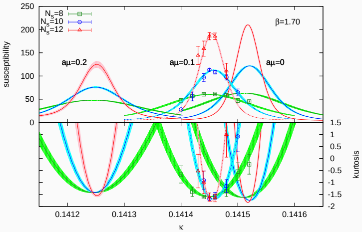
$$- \ln \det D_W(m_0, \mu) \Bigg]$$

with

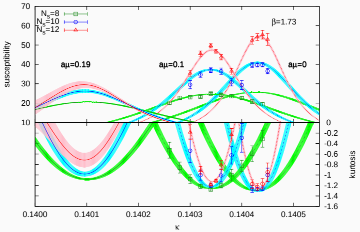
$$\Delta_{m_0} = m_0' - m_0$$
$$\Delta_\mu = \mu' - \mu$$

for the reweighting, where we use $m_0 = 1/(2\kappa) - 4$ to see easily mass derivative of D.

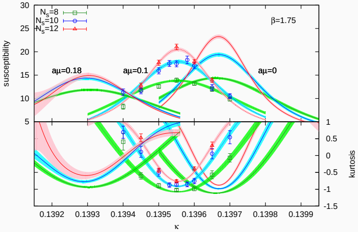
χ and K



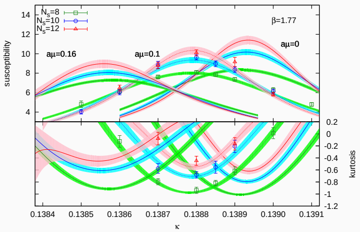
N_s=8
N_s=10
N_s=12
β=1.70
aμ=0.2
aμ=0.1
aμ=0
susceptibility
kurtosis
κ



N_s=8
N_s=10
N_s=12
β=1.73
aμ=0.19
aμ=0.1
aμ=0
susceptibility
kurtosis
κ

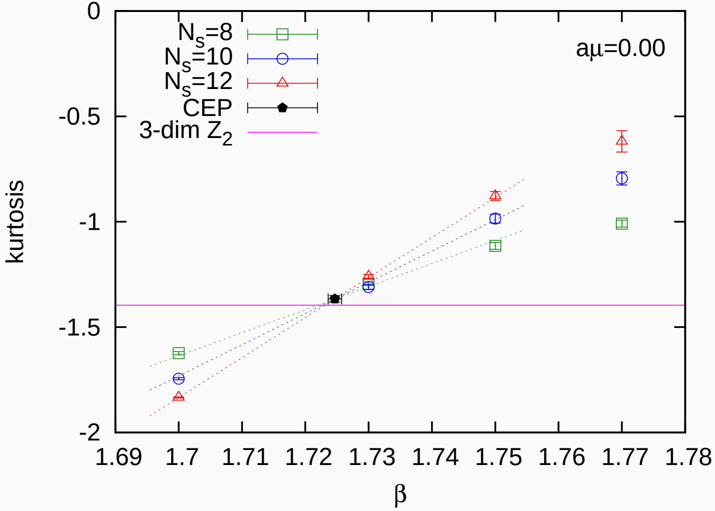


N_s=8
N_s=10
N_s=12
β=1.75
aμ=0.18
aμ=0.1
aμ=0
susceptibility
kurtosis
κ

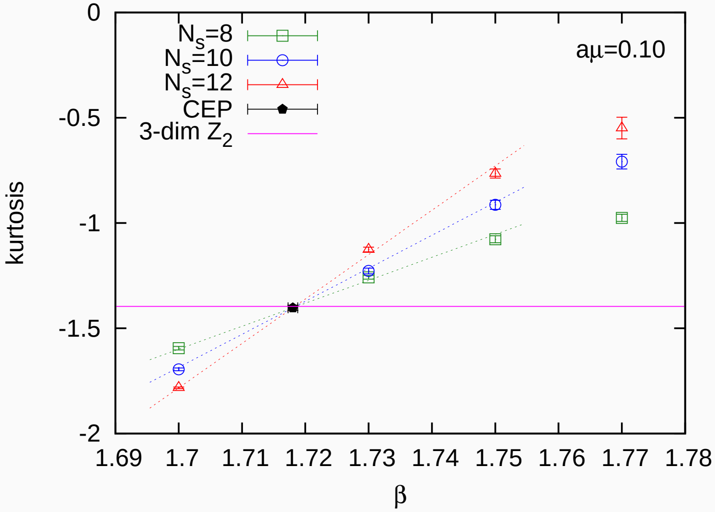


N_s=8
N_s=10
N_s=12
β=1.77
aμ=0.16
aμ=0.1
aμ=0
susceptibility
kurtosis
κ

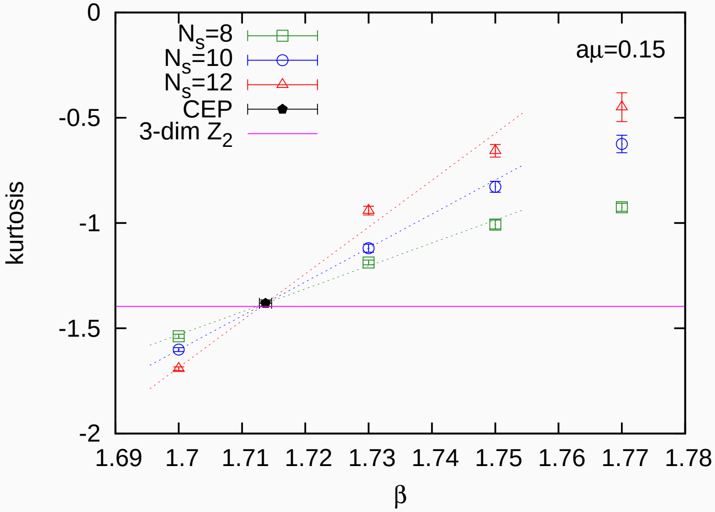
K intersection at $a\mu = 0.00$



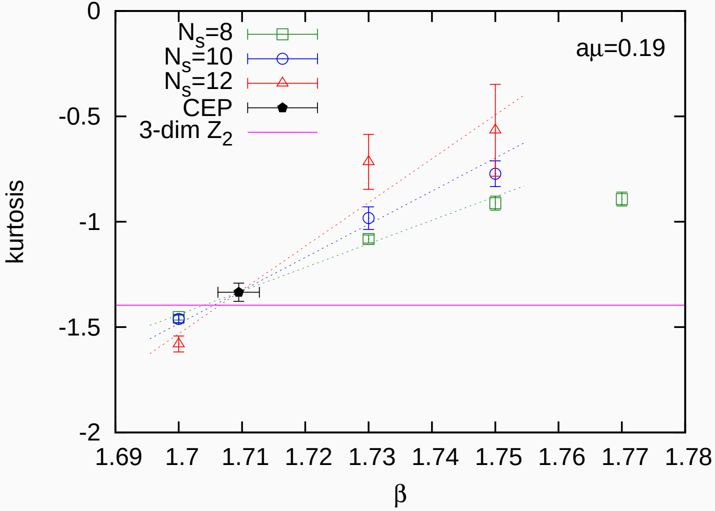
K intersection at $a\mu = 0.10$



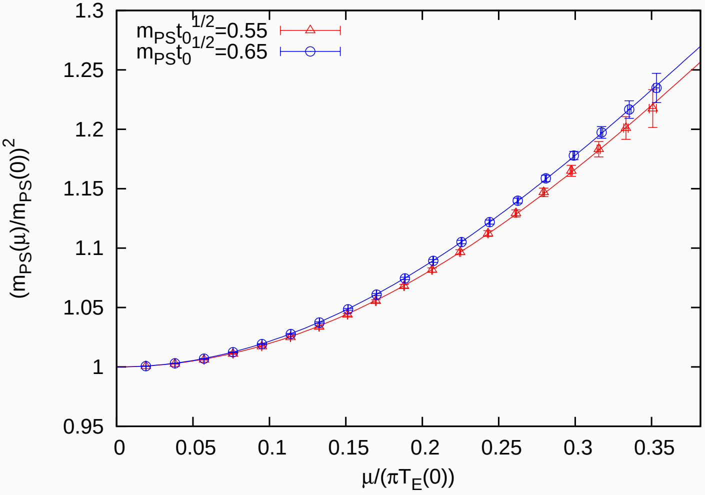
K intersection at $a\mu = 0.15$



K intersection at $a\mu = 0.19$



curvature



$m_{PS}t_0^{1/2}$=0.55
$m_{PS}t_0^{1/2}$=0.65
$(m_{PS}(\mu)/m_{PS}(0))^2$
$\mu/(\pi T_E(0))$
1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35

Summary

- We have investigated the critical endpoint of QCD with clover fermions
- We have determined the critical endpoint by using the intersection points of the Binder cumulants
- continuum extrapolation $a\mu = 0$

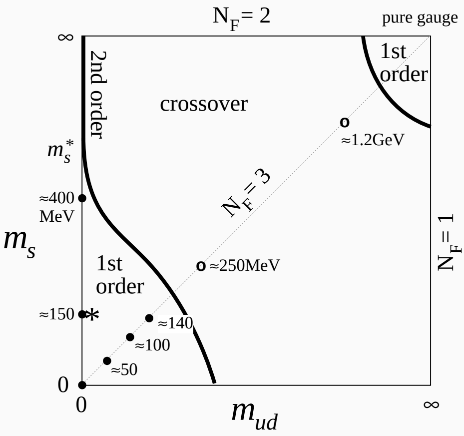
$$m_{\mathrm{PS,E}} < 304(7)(14)(7)\ \mathrm{MeV?}$$
$$m_{\mathrm{PS,E}}/m_{\mathrm{PS}}^{\mathrm{phys,sym}} < 0.739(17)(34)(17)?$$

- for $a\mu \neq 0$, we have found $\partial m_{\mathrm{E}}/\partial\mu > 0$ at heavier quark mass

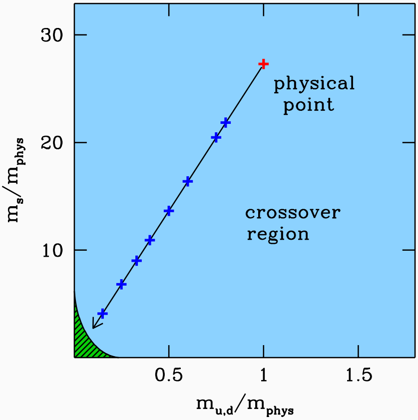
Backup slides

Columbia plot

- inconsistent results: Wilson and staggered type fermion

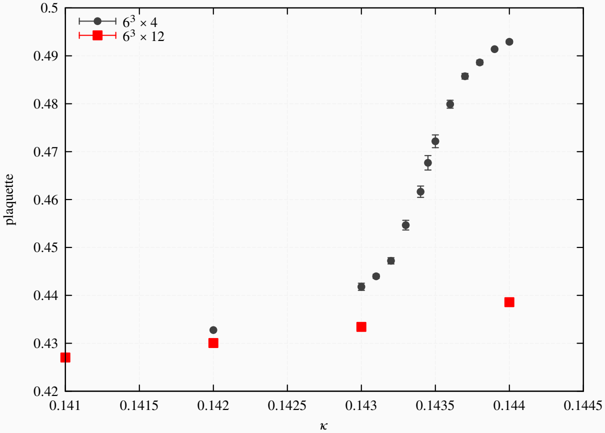


Wilson



staggered

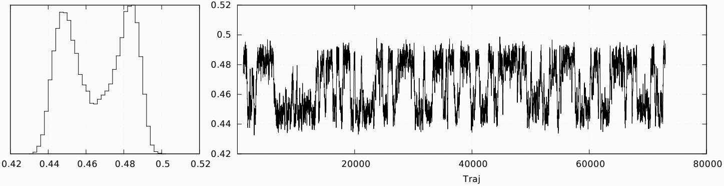
Finite temperature phase transition



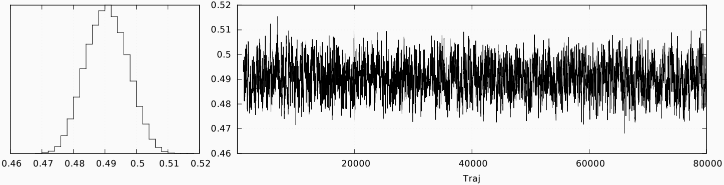
- Plaquette v.s. κ at lowest β **(= 1.60)**
- no bulk phase transition

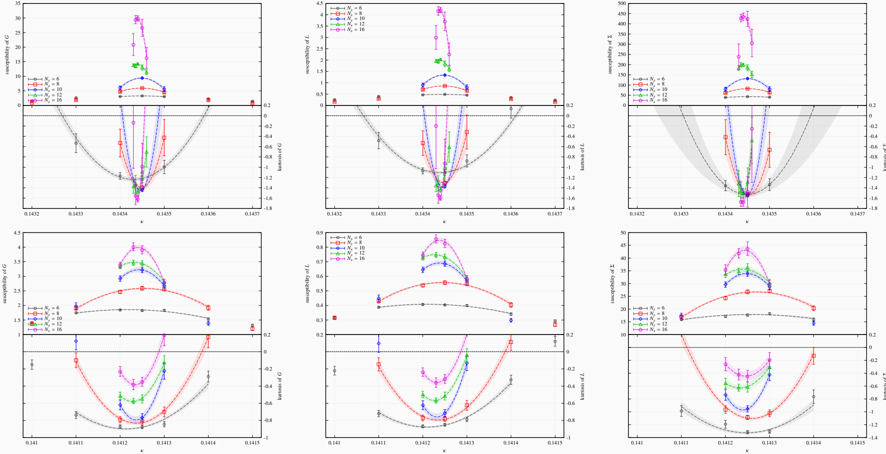
1st order phase transition and crossover (like)

$\beta = \mathbf{1.60}$ and $\kappa = \mathbf{0.14345}$ on $\mathbf{10^3 \times 4}$, clear two states,
K $\sim$ **–1.5**



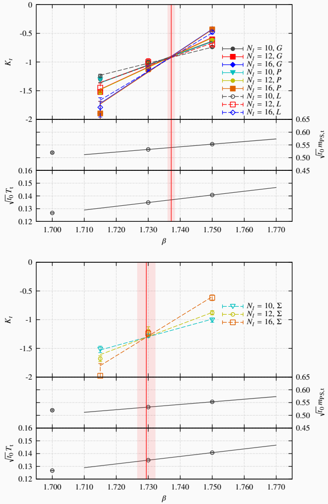
$\beta = \mathbf{1.70}$ and $\kappa = \mathbf{0.13860}$ on $\mathbf{10^3 \times 4}$, one state, ***K*** $\sim$ **–0.5**



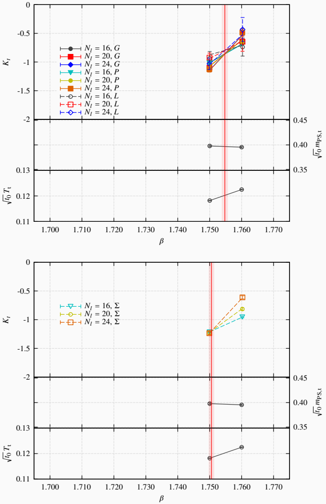


susceptibility of G
kurtosis of G
susceptibility of L
kurtosis of L
susceptibility of Σ
$N_s = 6$
$N_s = 8$
$N_s = 10$
$N_s = 12$
$N_s = 16$
κ

Critical endpoint at $N_t = 6, 8$



$N_t = 6$



$N_t = 8$