PaNic02 Osaka
30 September 2002

Lattice QCD and Hadrons

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- overview
- hadron spectrum and fundamental constants of QCD
- topics in hadron physics
- issues and progress with chiral symmetry
- weak interactions of hadrons
- QCD thermodynamics
- conclusions

Center for computational Physics
University of Tsukuba

- Founded in 1992
- Emphasis on
  - Development of HPC systems suitable for computational physics
  - Close collaboration of physicists and computer scientists


City of Tsukuba

- Computing facility
  - CP-PACS parallel system
    - MPP with 2048PU/0.6Tflops peak
    - Developed at the Center with Hitachi Ltd.
    - #1 of Top500-November 1996
  - GRAPE-6 system
    - Dedicated to gravity calculations
    - Developed at U. Tokyo
    - 8Tflops equivalent

Lattice QCD

- Quantum Chromodynamics (QCD)
  - Fundamental theory of quarks and gluons and their strong interactions
  - Knowing 1 coupling constant
    and 6 quark masses
    will allow full understanding of strong interactions (Yukawa's dream)

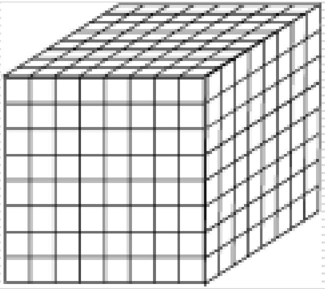
α_s

$m_u, m_d, m_s, m_c, m_b, m_t$

$$S_{QCD} = \frac{1}{\alpha_s}\sum_P tr(UUUU) + \sum \overline{\psi}(\gamma \cdot U + m_q)\psi$$

$$\langle O(U,\overline{\psi},\psi)\rangle = \frac{1}{Z}\int dU d\overline{\psi} d\psi O(U,\overline{\psi},\psi) e^{-S}$$

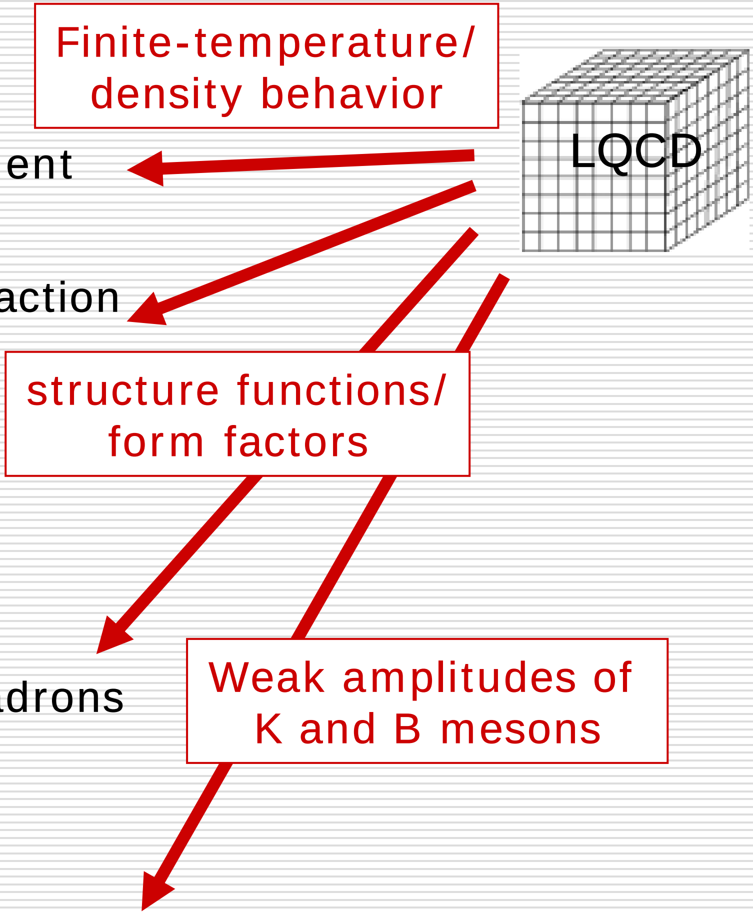
- Lattice QCD and numerical simulation
  - Provides the mean for first-principles realization of the dream
  - With computers instead of pencils in your hand



Lattice QCD and PaNic2002

PaNic02 Plenary program

- Sep.30
  - Quark confinement and deconfinement
- Oct.1
  - Nucleon structure and hadron interaction
  - Nuclear matter at high density
- Oct.2
  - Flavored matter and astrophysics
  - Neutrino physics
- Oct.3
  - Hadron physics with leptons and hadrons
  - Structure functions of nucleon
- Oct.4
  - CP violation and rare K-decays
  - Precision frontier and beyond standard model



Finite-temperature/ density behavior

structure functions/ form factors

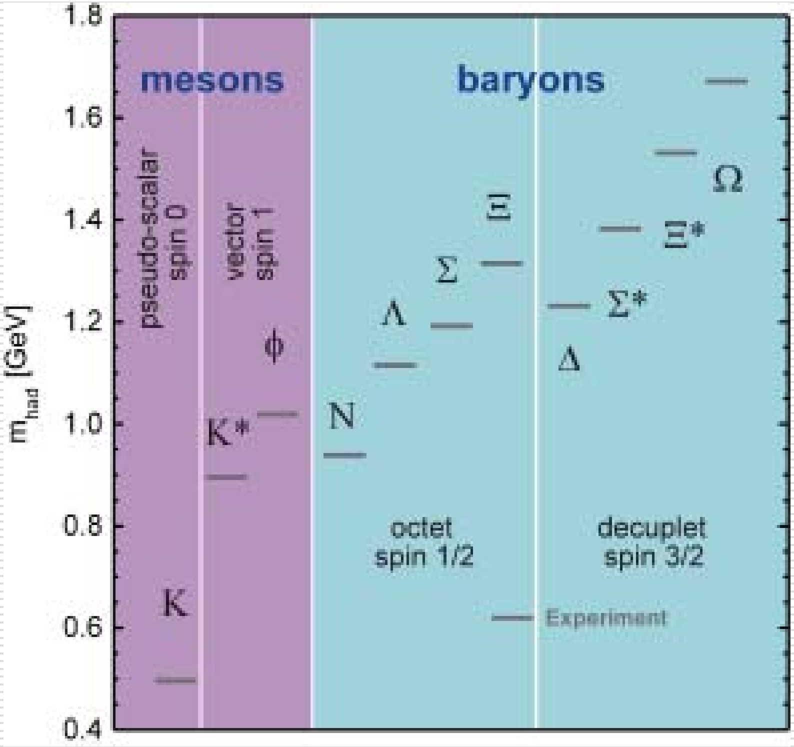
Weak amplitudes of K and B mesons

Organization of this talk

- Hadron spectrum and fundamental constants of QCD
  - Fundamental verification of QCD at low energies
  - Determination of quark mass and strong coupling constant
- Topics in hadron physics
  - Nucleon structure functions
- Issues and progress with chiral symmetry
  - Chiral extrapolation of observables
  - Chirally symmetric fermion formulations
- Weak amplitudes of hadrons
  - $K \rightarrow \pi\pi$ decays and CP violation
  - K and B meson amplitudes and constraints on the CKM matrix
- Finite temperature/density QCD
  - Status at zero density
  - Progress toward non-zero density

Light hadron mass spectrum

- ☐ Benchmark calculation to verify QCD
- ☐ Indispensable for determination of QCD scale and quark masses
- ☐ Essential to control various systematic errors down to a few % level
  - Finite lattice size L>3fm
  - Finite quark mass mq→0
  - Finite lattice spacing a→0



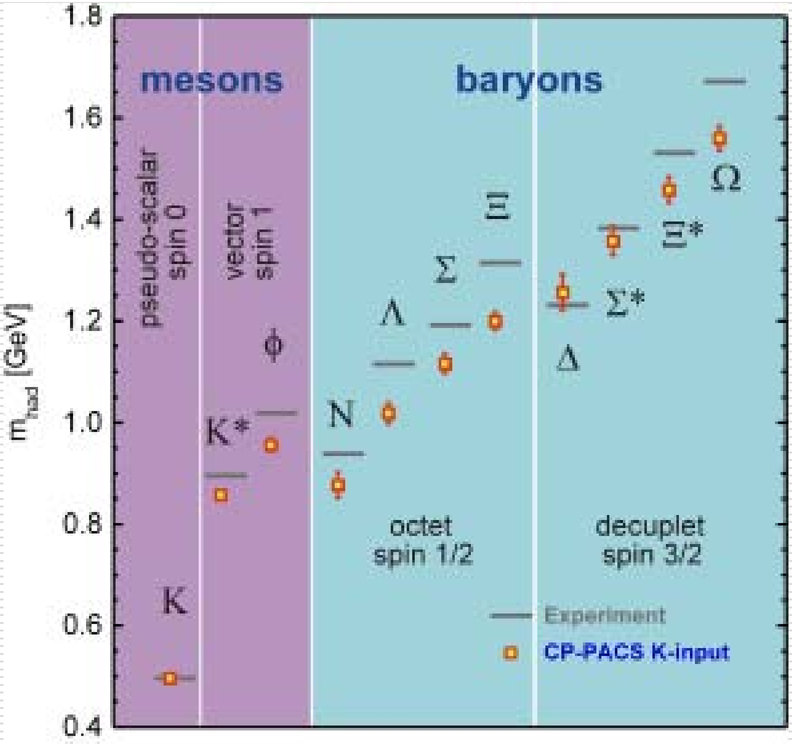
Experimental spectrum

CP-PACS result for the quenched spectrum’98

- Sea quark effects ignored
- General pattern reproduced, but clear systematic deviation beyond 10% precision
- Completes the calculation started in ’81 (Hamber-Parisi/Weingarten)

$use\ m_\pi, m_\rho\ \ for\ \ fixing\ a\ and\ m_{ud}$

$use\ m_K\ \ for\ \ fixing\ m_s$



Calculated quenched spectrum

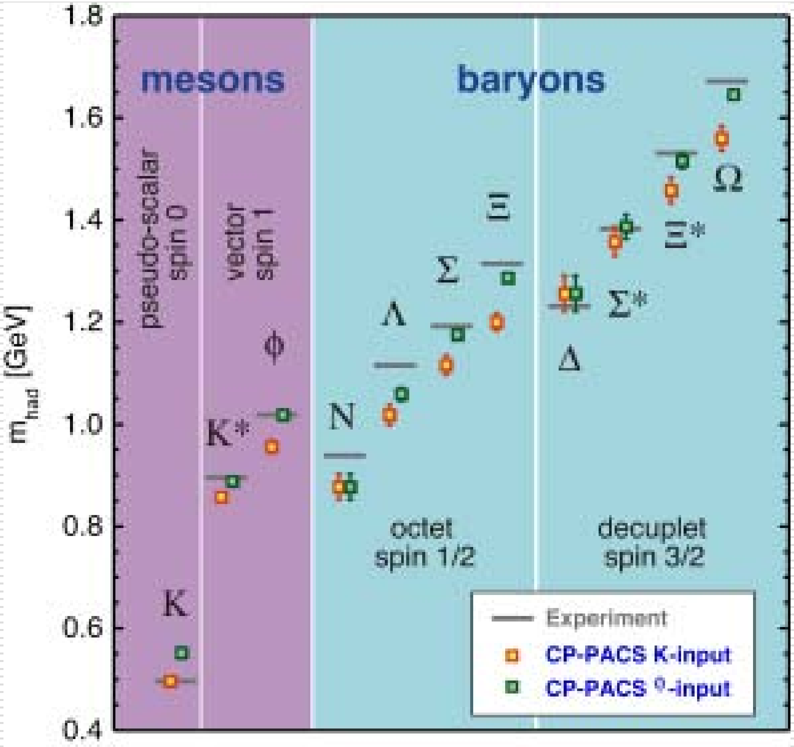
Input dependence in quenched QCD

- ☐ Details of disagreement depends on input, but overall agreement not possible

  $use\ m_K\ \ for\ \ fixing\ m_s$

  $use\ m_\phi\ \ for\ \ fixing\ m_s$

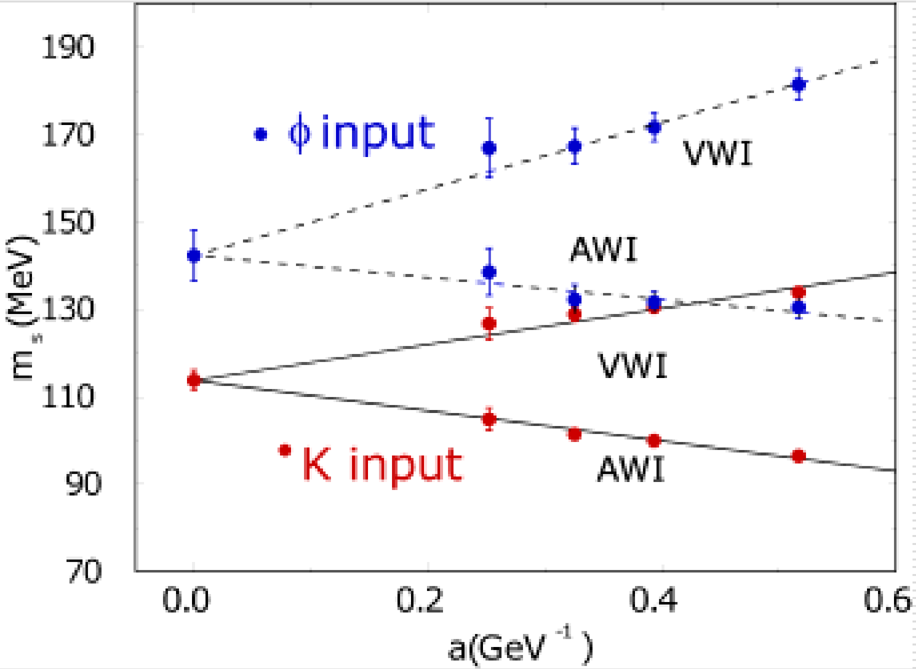
- ☐ *predictions in quenched QCD suffer from uncertainties depending on input*



Calculated quenched spectrum

Strange quark mass in quenched QCD

- 25% descrepancy in the predicted value of strange quark mass
- Clearly illustrates
  - *limitation of quenched QCD*
  - *necessity of full QCD with dynamical quarks*



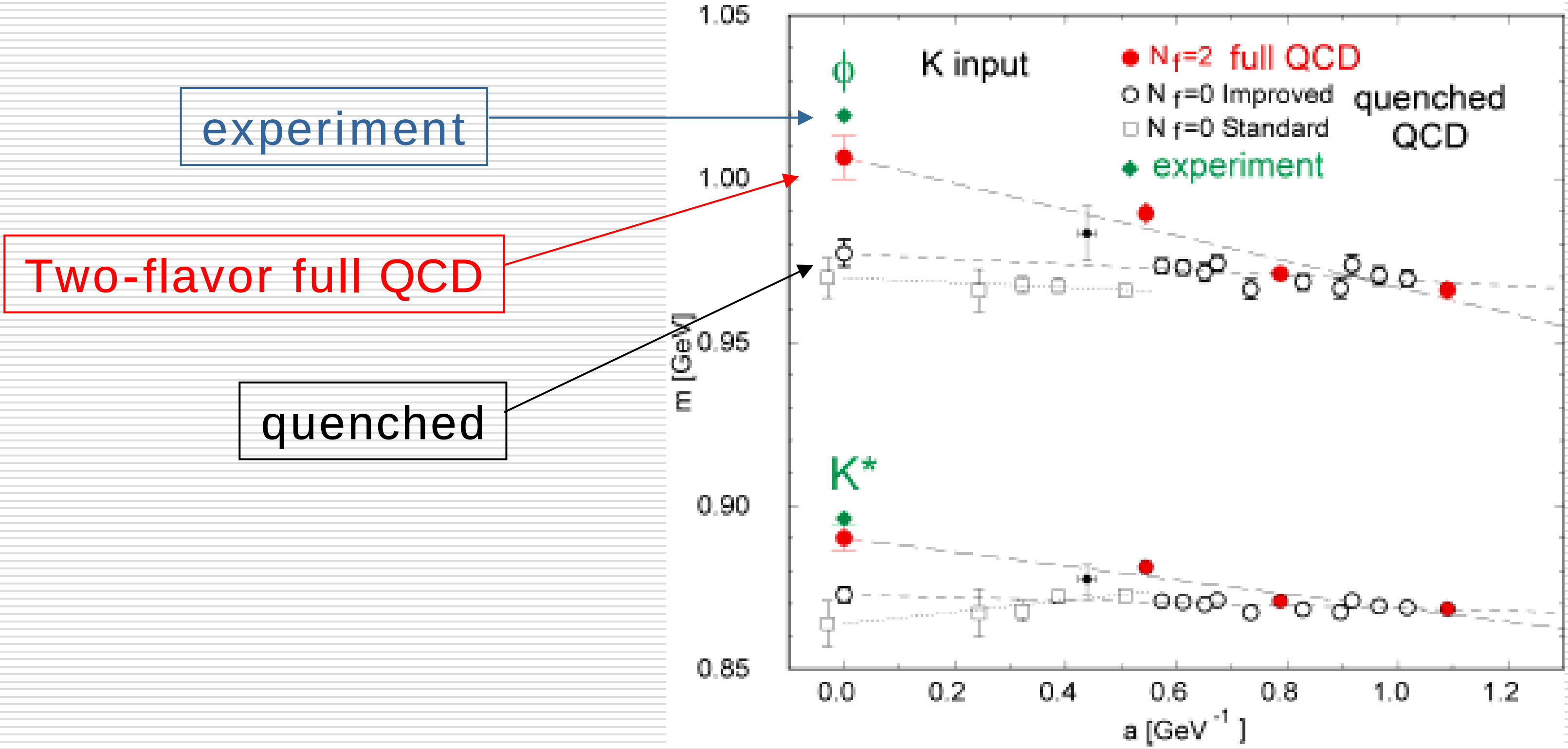
$$m_s\left(\overline{MS},2GeV\right)=\begin{cases}142^{+28}_{-6}\ MeV & \phi\ meson\ mass\ input\\ 114^{+8}_{-6}\ MeV & K\ meson\ mass\ input\end{cases}$$

QCD simulation with dynamical quarks

- Spectrum of quarks
  - *3 light quarks (u,d,s)* *$m < 1\text{GeV}$*
    - *Need dynamical simulation*
  - 3 heavy quarks (c,b,t) $m > 1\text{GeV}$
    - Quenching sufficient
- Dynamical quark simulation (full QCD)
  - costs 100-1000 times more computing power
  - Algorithm for *odd number of quarks* now available
- *Two-flavor full QCD* $N_f = 2$
    - u and d quark dynamical simulation
    - s quark quenched approximation
  - Number of studies: SESAM/UKQCD/MILC/CP-PACS/JLQCD
- *Two+one-flavor full QCD* $N_f = 2+1$
    - s quark also treated dynamically
  - Extensive studies have begun : MILC/CP-PACS-JLQCD

Sea quark effects in the spectrum

- K*-K and **ϕ**-K mass difference (Meson hyperfine splitting)
  - too small in quenched QCD
  - Much closer agreement for two-flavor full QCD

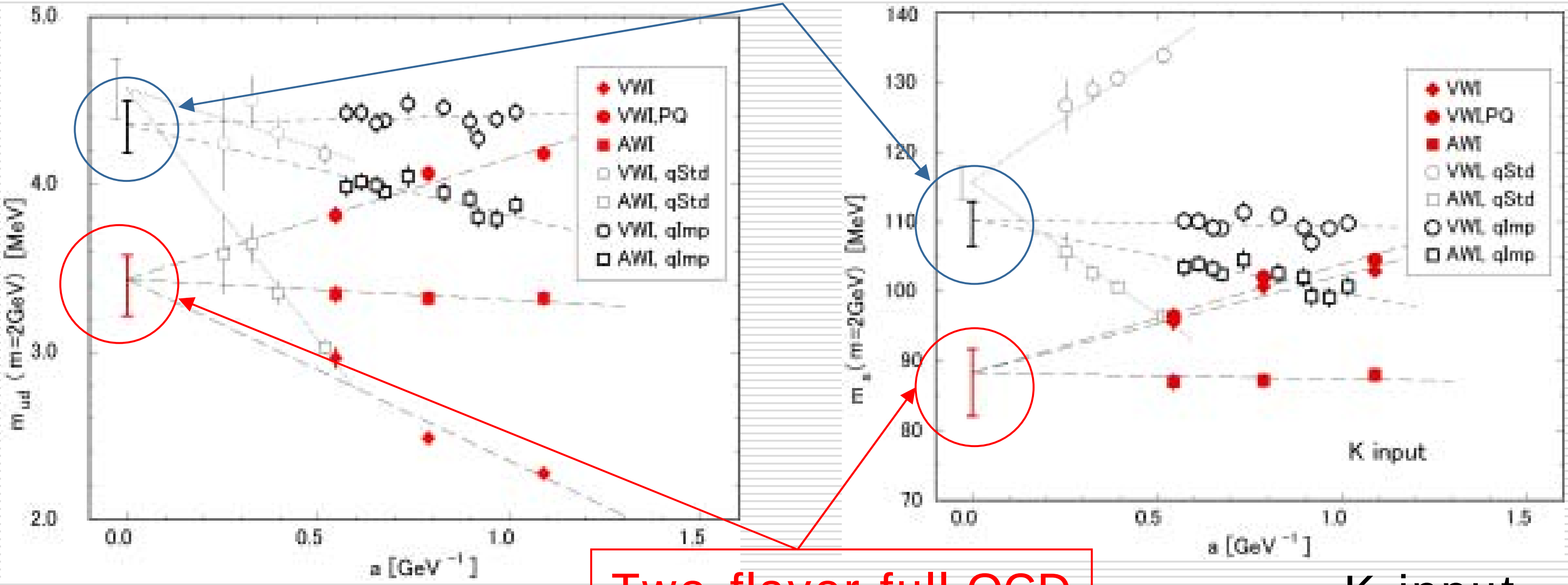

experiment

Two-flavor full QCD

quenched

Sea quark effects in quark masses

- ☐ Significant decrease by inclusion of sea quark effects
- ☐ Input dependence of strange quark mass also reduced



up&down quark

Quenched value

strange quark

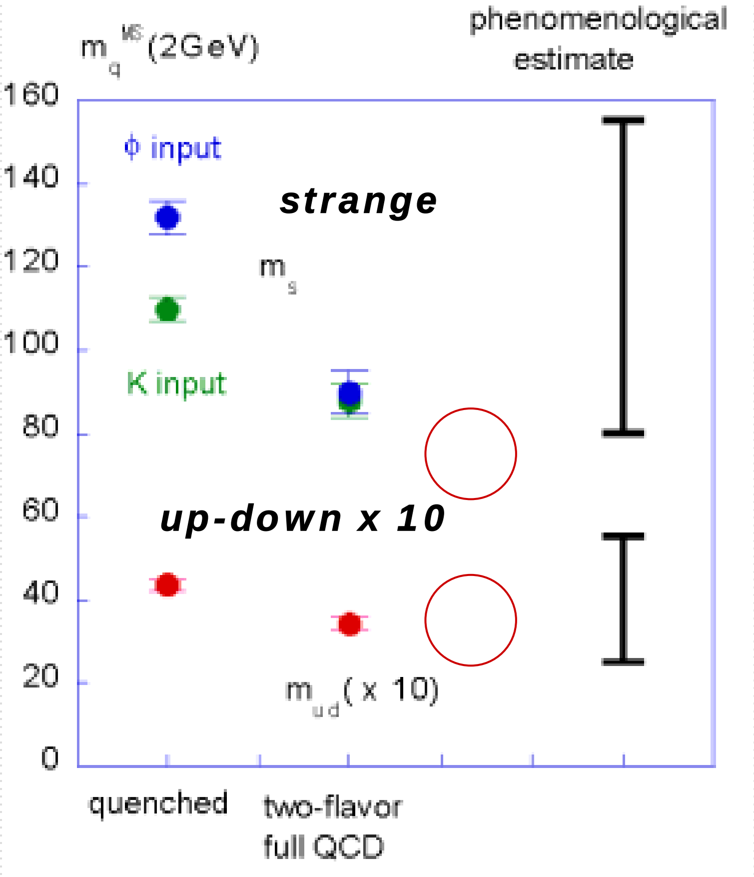
Two-flavor full QCD

K-input

Light quark masses (u, d, s) $m_q^{\overline{MS}}(2GeV)$

- Significant sea quark effects
  - Large uncertainty (∼20%) depending on input in quenched theory
  - Sizable decrease (∼25%) from quenched to two-flavor full QCD
- Lighter than naïve quark model values
- Nf=3 simulations being pursued to obtain physical values of light quark masses, e.g., Hein et al hep-lat/0209077

CP-PACS Collab. Hep-lat/0004010



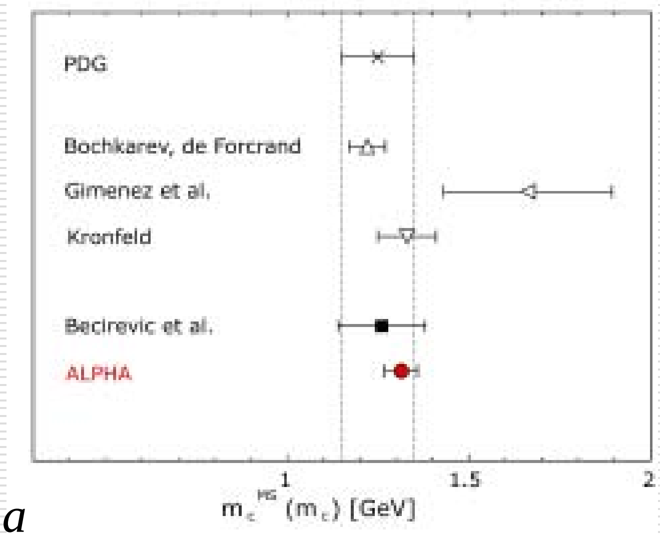
Real world;three flavors?

Heavy quark masses (c, b)

- ☐ Charm quark mass
  - J. Rolf and S. Sint Lattice01
    - ☐ Fully non-perturbative determination in the continuum in quenched QCD

$$m_c^{\overline{MS}}(m_c) = 1.314(45)GeV$$

compiled by T. Kaneko (Lattice '01)
Hep-lat/0111005



- ☐ Bottom quark mass
  - Not straightforward since $m_b > 1/a$
  - Use

$$M_B = m_b^{pole} + \underbrace{E_b}_{binding\ energy} - \underbrace{\delta m}_{radiative\ corrections}$$

$$\begin{cases} M_B, E_B & Monte\ Carlo \\ \delta m & pert.\ theory \end{cases}$$

  - G. Gimenez et al ('00)

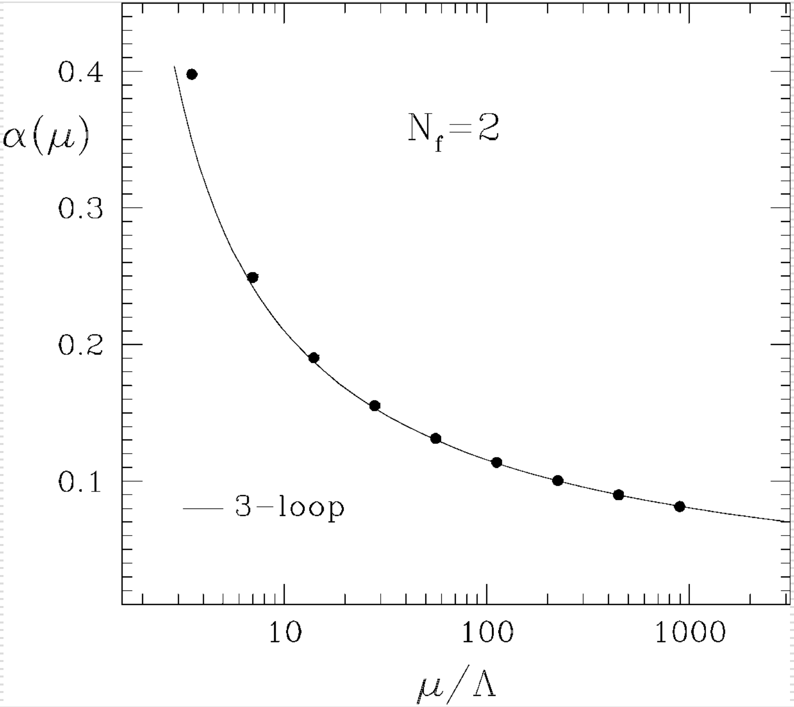
$$m_b^{\overline{MS}}(m_b) = 4.27(9)GeV$$

Strong coupling constant $\alpha_s(\mu)$

- Another fundamental parameter of QCD
- Large number of high energy determinations from experiments
- Lattice determinations:
  - Method I (Lepage et al ’91):
    - Calculate short-distance observables as a function of $\alpha_s(1/a)$ at cutoff scale 1/a
    - the scale 1/a is fixed from hadron mass
  - Method II (Luescher et al ’93)
    - Non-perturbative determination of the RG evolution by Shcrodinger functional finite-size technique

Scale dependence of $\alpha_s(\mu)$

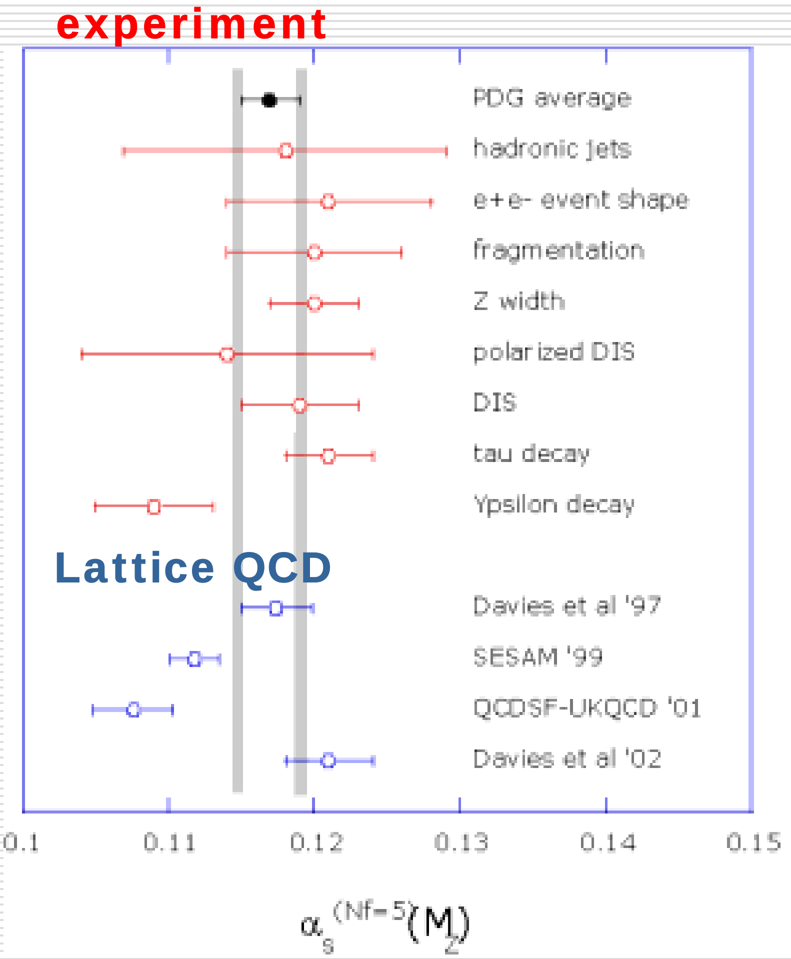
SF scheme



M. Della Morte et al hep-lat/0209023

- ☐ Non-perturbative determination by Alpha Collaboration for two-flavor full QCD (Lattice02)
- ☐ Indicates perturbative evolution for p>a few GeV
- ☐ Similar results for quenched QCD ('94)
- ☐ Physical scale yet to be determined

Determination of $\alpha_s^{\overline{MS}}(M_Z)^{N_f=5}$



- Comments
  - Davies et al ’97(hep-lat/9703010):
    Involved extrapolation of Nf=0(quenched) and Nf=2 data to Nf=3
  - QCDSF-UKQCD ’01(hep-lat/0103023)
    Continuum estimate with systematic Nf=2 simulations
  - Davies et al ’02(hep-lat/0209121):
    Preliminary result based on MILC **Nf=3** configurations at a=0.13fm
- Systematic Nf=3 full QCD determination expected in a few years

Topics in hadron physics

- Progress in hadron spectroscopy
  - Eta’ meson mass and U(1) problem
  - Glueballs
  - Multiquark states
  - Excited string states etc

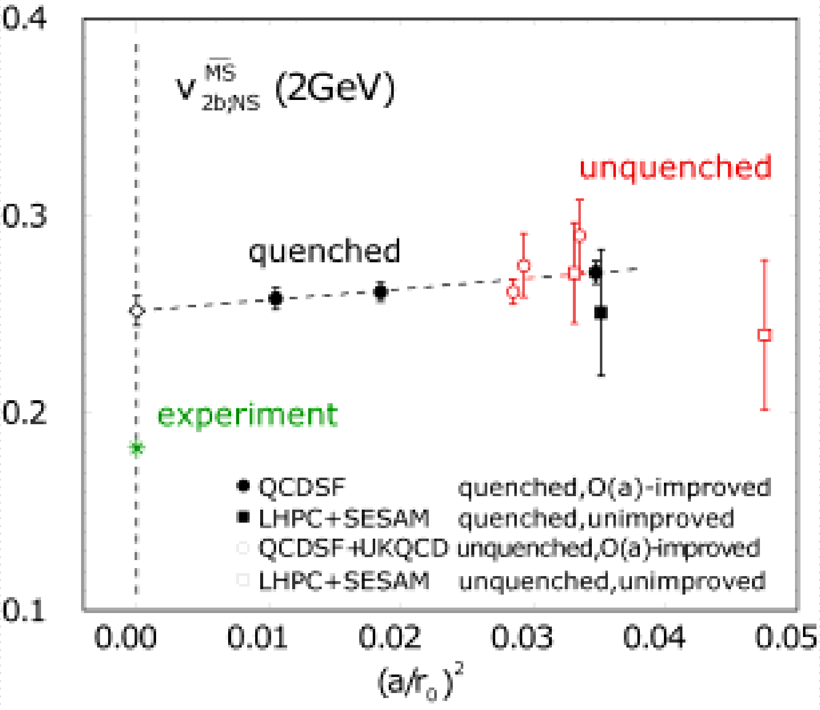
- Hadron structure
  - *Moments of nucleon structure functions*
  - Form factors

- Hadron scattering amplitudes
  - Scattering length
  - Phase shift

Moments of nucleon structure functions (I)

- A number of calculations
    - QCDSF (quenched)'96
    - Alpha(quenched)'98
    - QCDSF+UKQCD (full)''01-'02
    - LHPC+SESAM (full)'02
- No large sea quark effects observed for non-singlet moments
- No large scaling violation
- Lattice predictions do not agree with experiment

$\langle x \rangle_{u-d}$



Compiled in M. Goeckeler et al hep-lat/0209160

Moments of nucleon structure functions (II)

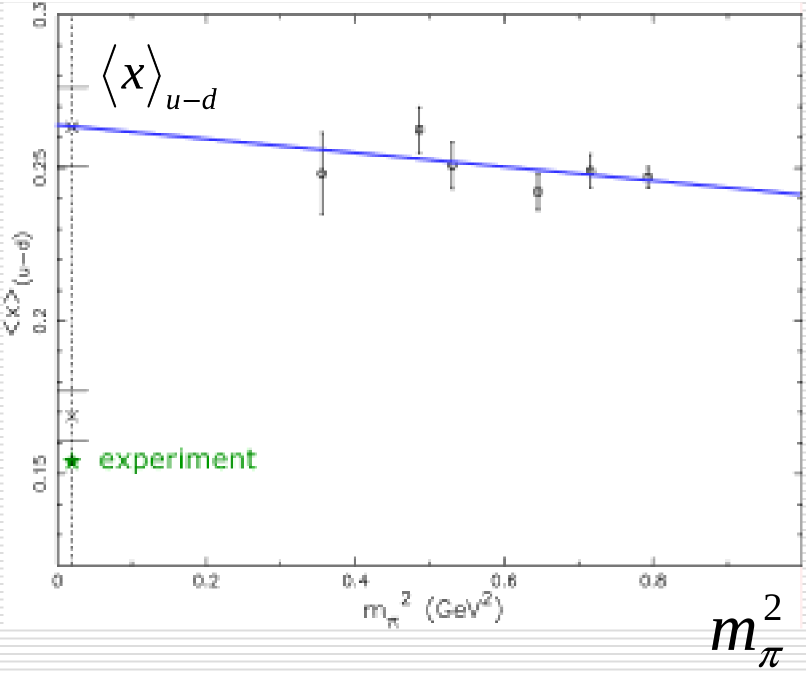
- *Linear chiral extrapolation misses experiment*
- Possible reasons:
  - Quenching? No
  - O(a) error? No
  - Chiral extrapolation itself?
- Chiral perturbation theory:

$$\langle x^n \rangle = a_n \left(1 - \frac{3g_A^2+1}{(4\pi f)^2} m_\pi^2 \ln \frac{m_\pi^2}{\Lambda_\chi^2} + c m_\pi^2 \cdots \right)$$

  - Pion not light enough to see curvature?
  - An effective model can reproduce experiment

$$\boxed{\langle x^n \rangle = a_n \left(1 - \frac{3g_A^2+1}{(4\pi f)^2} m_\pi^2 \ln \frac{m_\pi^2}{\Lambda_\chi^2 + m_\pi^2} + d m_\pi^2 \right)}$$

From D. Dolgov et al hep-lat/0201021



W. Detmold et al hep-lat/0103006

Moments of nucleon structure functions (II)

- *Linear chiral extrapolation misses experiment*
- Possible reasons:
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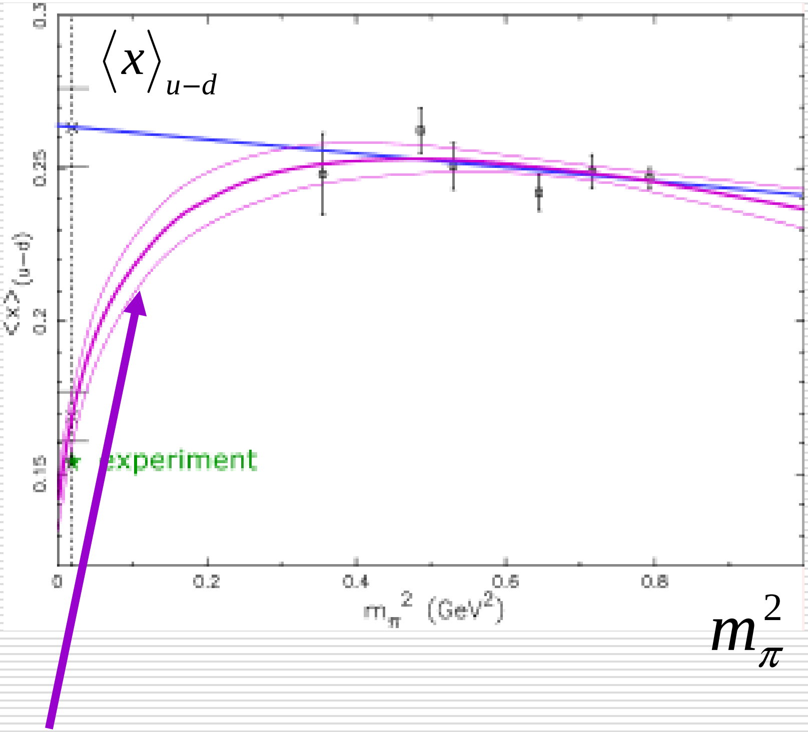
  - Chiral extrapolation itself?
- Chiral perturbation theory:

$$\langle x^n\rangle = a_n\left(1-\frac{3g_A^2+1}{(4\pi f)^2}m_\pi^2\ln\frac{m_\pi^2}{\Lambda_\chi^2}+cm_\pi^2\cdots\right)$$

  - Pion not light enough to see curvature?
  - An effective model can reproduce experiment

From D. Dolgov et al hep-lat/0201021



$$\boxed{\langle x^n\rangle = a_n\left(1-\frac{3g_A^2+1}{(4\pi f)^2}m_\pi^2\ln\frac{m_\pi^2}{\Lambda_\chi^2+m_\pi^2}+dm_\pi^2\right)}$$

W. Detmold et al hep-lat/0103006

A general issue with chiral extrapolation

- ☐ Current lattice data often fails to see logarithmic singularity expected from chiral perturbation theory

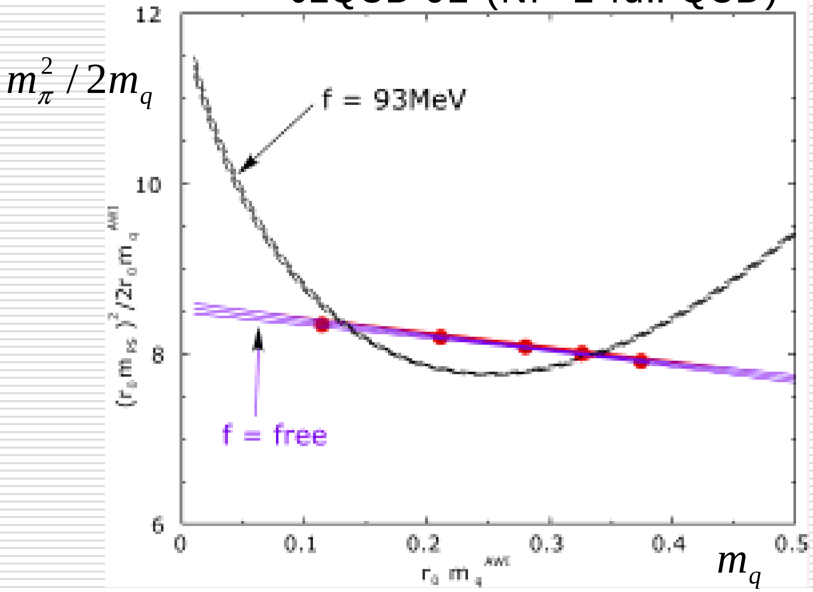
- ☐ Often causes sizable (10-20%) uncertainties in the extrapolated result

- ☐ Pion mass in current simulations (∼500MeV)

  too heavy;needs to be reduced

- ☐ Lattice fermion action with exact chiral symmetry much desired (conventional Wilson and KS action breaks chiral symmetry)

JLQCD'02 (Nf=2 full QCD)



$$m_\pi^2 = Am_q\left(1 + \frac{1}{N_f(4\pi f)^2} m_q \ln m_a + bm_q + \cdots\right)$$

Lattice fermion with exact chiral symmetry

- Theoretically based on the Ginsparg-Wilson relation:

$$D\gamma_5 + \gamma_5 D = 2aDR\gamma_5 D$$

Domain-wall fermion	Kaplan('92)/Furman-Shamir('94)
Overlap formalism	Neuberger-Narayanan('92,'97)
Fixed point action	Hasenfratz-Neidermyer('94)

- Avoids the Nielesen-Ninomiya Theorem by using "infinitely" many fields (hence needs more computer power)

- quenched calculations show very promising results: good chiral property, small scaling violation, …

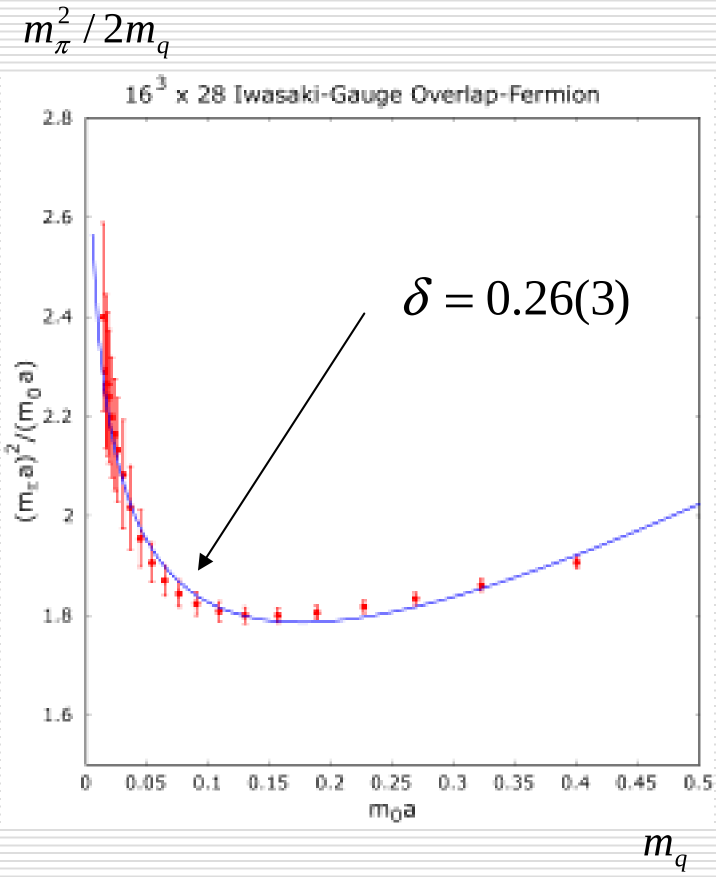
A test in quenched QCD

- chiral logarithm behavior of pion mass in quenched QCD

  Sharpe/Bernard-Golterman ’91

$$m_\pi^2 = Am_0\left(1-\delta\ln m_0 + bm_0 + \cdots\right)$$

- Nice confirmation with the new fermion formalism
  - T. Draper et al : overlap fermion
  - C. Gattringer et al : fixed point fermion
- Reached very light pion mass
  - m_π∼170MeV (T. Draper et al)
  - Similar results from other chiral formalisms



$\delta = 0.26(3)$

T.Draper et al hep-lat/0208045

Weak amplitudes of hadrons

- First principles calculation of strong interaction corrections to weak amplitudes of hadrons
- Understand old and new issues in hadronic weak interactions
  - $\Delta I=1/2$ rule and direct CP violation in $K\rightarrow\pi\pi$ decay
- Constraints on the CKM mixing matrix
  - Neutral K and B meson mixings
  - B meson decay form factors

Measured weak amplitude = Known factor including CKM matrix X $\langle h'|H_{weak}|h\rangle$

QCD matrix element

ΔI=1/2 rule and CP violation in K decays

- Weak interaction decays of K mesons
  - ΔI=1/2 rule

$$\frac{\mathrm{Re}\,A_0\left(K\to\pi\pi(I=0)\right)}{\mathrm{Re}\,A_2\left(K\to\pi\pi(I=2)\right)}\approx 22$$

  - CP violation

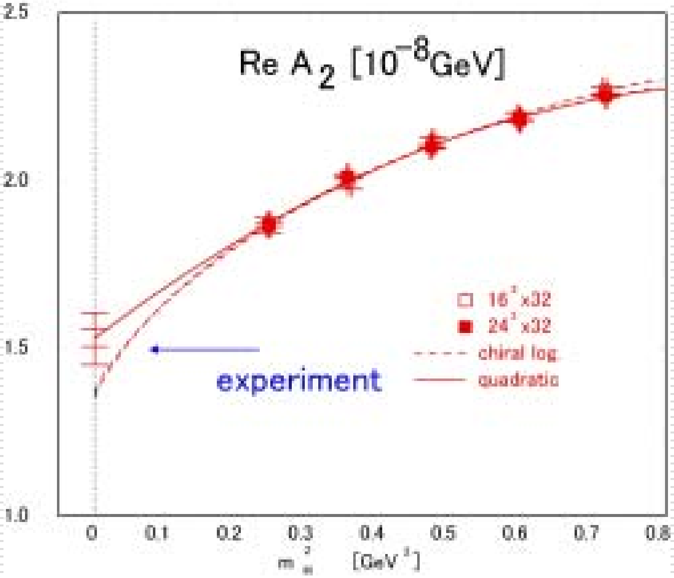
$$\frac{\varepsilon'}{\varepsilon}=\begin{cases}(20.7\pm 2.8)\times 10^{-3} & KTeV\ \mathrm{exp}eriment\ (FNAL)\\ (15.3\pm 2.6)\times 10^{-3} & NA48\,\mathrm{exp}eriment\,(CERN)\end{cases}$$

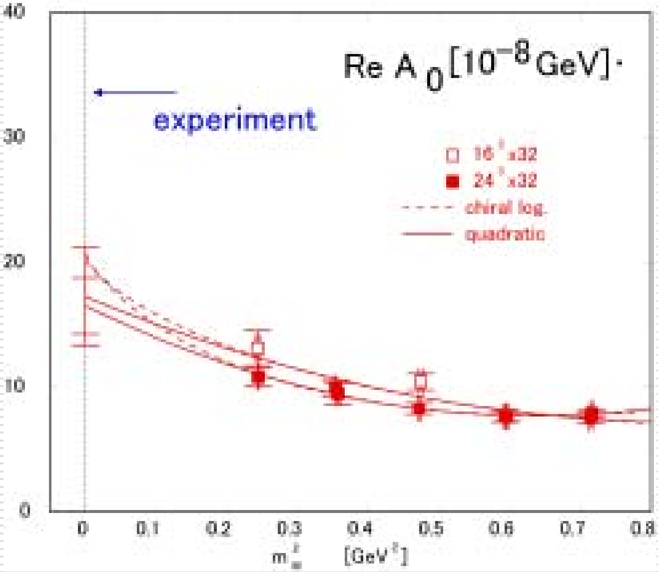
- Crucial numbers to verify the Standard Model understanding of CP violation

- Chiral symmetry crucial because of the chiral structure of weak interactions
- Two large-scale calculations using domain-wall QCD
  - RIKEN-BNL-Columbia — T. Blum et al hep-lat/0110124
  - CP-PACS — J. Noaki et al hep-lat/0108013

ΔI=1/2 rule

- Reasonable agreement with experiment for I=2
- About half of experiment for I=0
- RIKEN-BNL-Columbia obtains a somewhat different result (smaller I=2 and larger I=0)



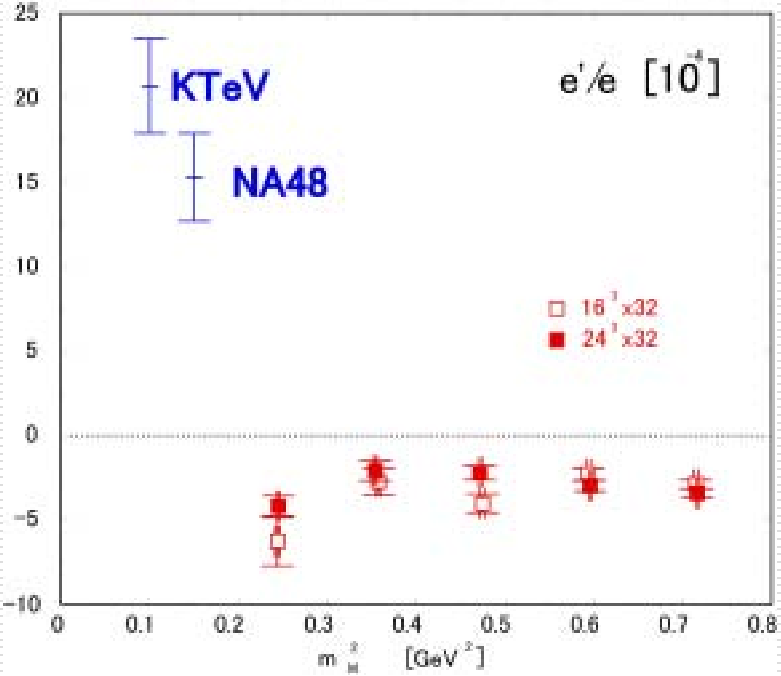


CP violation parameter ε'/ε

- Small and negative in disagreement with experiment
- Similar result from RIKEN-BNL-Columbia

$$\frac{\varepsilon'}{\varepsilon}=\frac{\omega}{\sqrt{2}|\varepsilon|}\left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2}-\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]$$

- Possible reasons
  - connected with insufficient enhancement of ΔI=1/2 rule
  - Method of calculation (K→π reduction) may have serious problems
- Still a big problem requiring further work



Constraints on the CKM matrix

Measured weak amplitude = Known factor including CKM matrix X $\langle h' | H_{weak} | h \rangle$

QCD matrix element

- $K^o - \overline{K}^0$ *mixing* $\varepsilon_K \propto \overline{\eta}\left[\left(1-\overline{\rho}\right)A + B\right]^2 \hat{B}_K$

$$\langle \overline{K}^0 | \bar{s}\gamma_\mu(1-\gamma_5)d\,\bar{s}\gamma_\mu(1-\gamma_5)d | K^0 \rangle = \frac{8}{3} f_K^2 B_K M_K^2$$

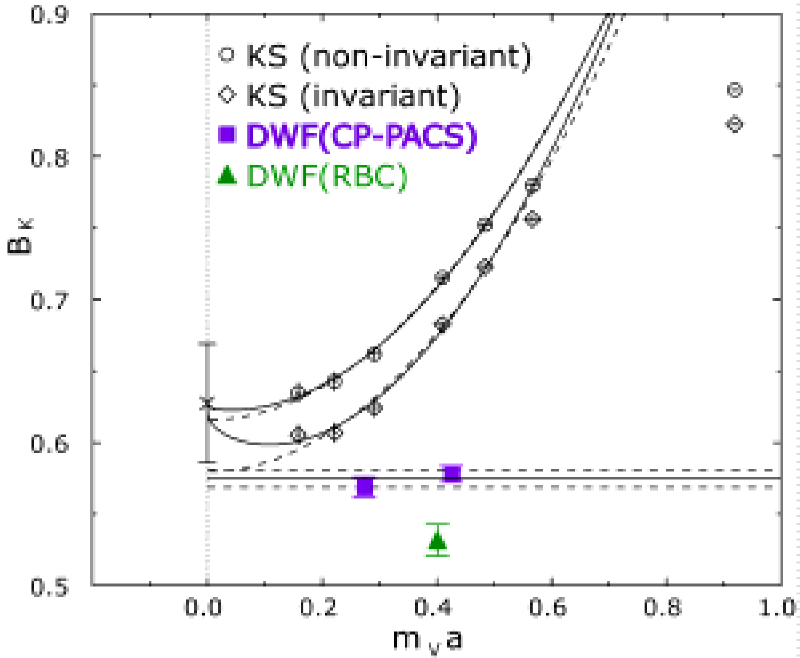
- $B^0_{d,s} - \overline{B}^0_{d,s}$ *mixing* $\Delta M_{B_q} \propto \left(\overline{\rho}^2 + \overline{\eta}^2\right) f_{B_q}^2 B_{B_q}$

$$\langle \overline{B}_q^o | \bar{b}\gamma_\mu(1-\gamma_5)q\,\bar{b}\gamma_\mu(1-\gamma_5)q | B_q^0 \rangle = \frac{8}{3} f_{B_q}^2 B_{B_q} M_{B_q}^2$$

$$\xi = \frac{f_{B_s}}{f_{B_d}}\sqrt{\frac{B_{B_s}}{B_{B_d}}}$$

Lattice results for B_K

- ☐ Previous best result obtained with conventional KS fermion action
- ☐ Recent Domain-wall results indicates a smaller value
- ☐ Full QCD calculation yet to be made with domain-wall fermion
- ☐ Current best estimate:



$$B_K = 0.628(42) - 0.532(11)$$

CP-PACS hep-lat/0105020
RBC hep-lat/0110075

$$\Rightarrow \hat{B}_K = 0.87^{+0.06}_{-0.13}$$

RG-invariant B parameter

Full QCD results for f_B

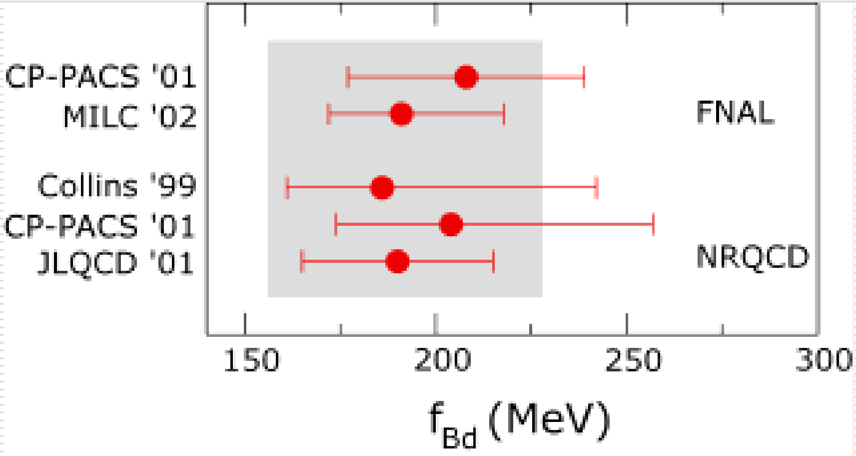
- ☐ Two-flavor full QCD result begins to accumulate
- ☐ f_{B_d} : possibly large uncertainty due to chiral extrapolation
- ☐ Best estimate from two-flavor full QCD:

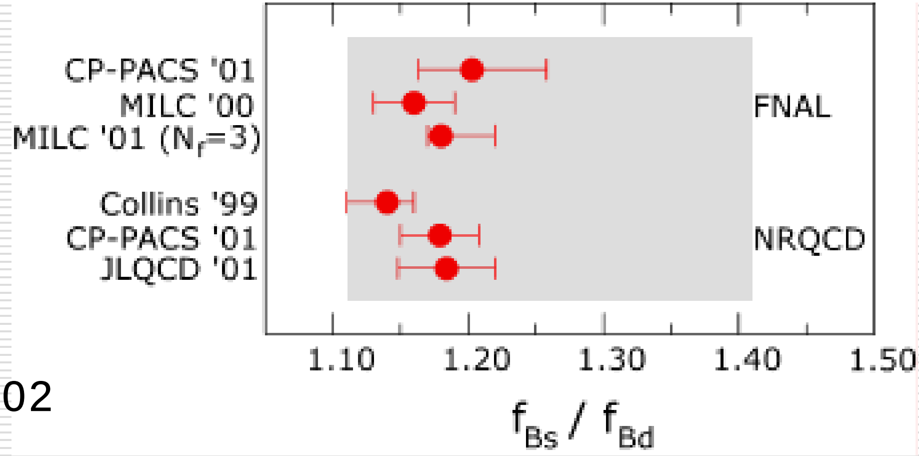
$$f_{B_d} = 198(30)^{+0}_{-34}\, MeV$$

$$f_{B_d} / f_{B_s} = 1.16(5)^{+24}_{-0}$$

N. Yamada at Lattice2002

Compilation by N. Yamada at Lattice02





Results for B_B

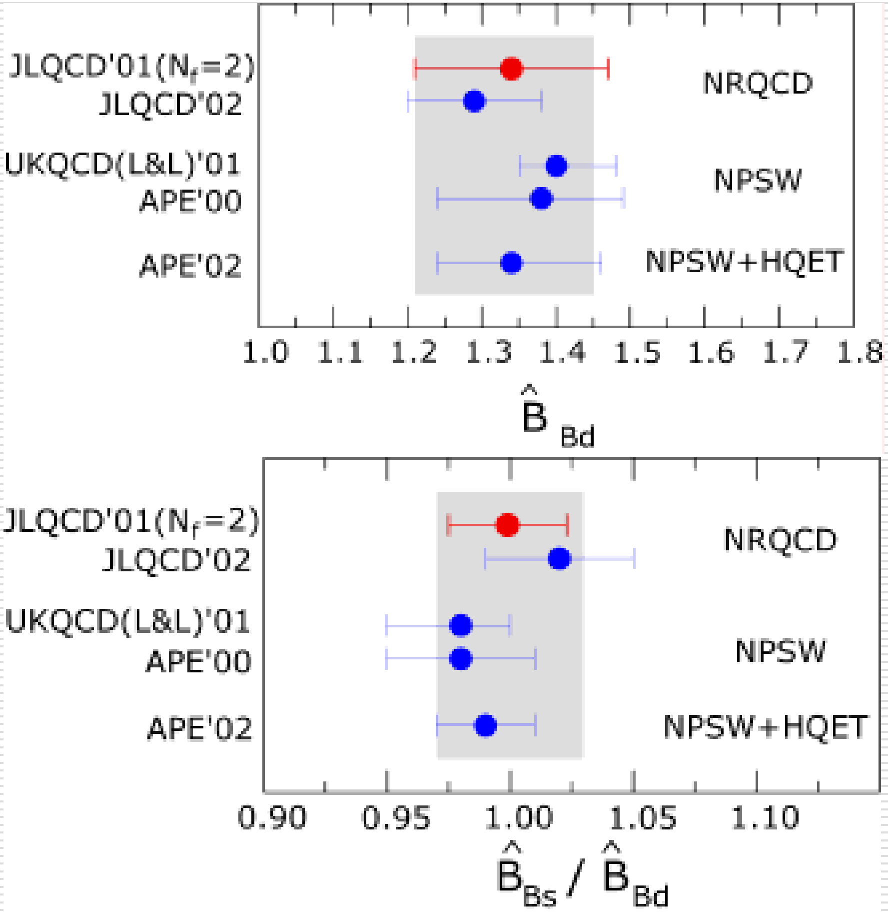
- Still mostly quenched (only one calculation in full QCD)
- Sea quark effects small
- Uncertainty with chiral extrapolation is small
- Current best estimate:

$$B_{B_d} = 1.33(12)$$

$$B_{B_d} / B_{B_s} = 1.00(3)$$

N. Yamada at Lattice2002

Compilation by N. Yamada at Lattice02



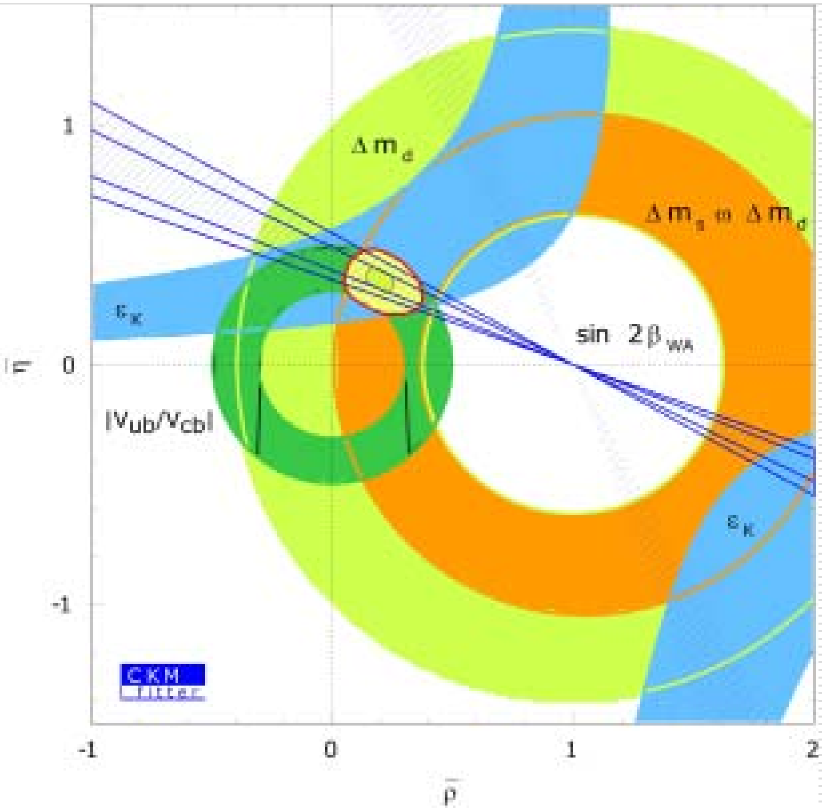
Summary of lattice results for CKM matrix

$$\hat{B}_K = 0.87^{+0.06}_{-0.13}$$

$$f_{B_d}\sqrt{B_{B_d}} = 0.227(37)^{+0}_{-34}\,GeV$$

$$\xi = 1.16(5)^{+24}_{-0}$$

status 2002 http://www.ckmfitter.in2p3.fr/



Cf. numbers used in the figure left

$$\hat{B}_K = 0.87^{+0.06}_{-0.13}$$

$$f_{B_d}\sqrt{B_{B_d}} = 0.227(28)\,GeV$$

$$\xi = 1.16^{+3}_{-5}$$

→ Better control of chiral extrapolation needed

Finite temperature/density QCD

- Status for T>0 and $\mu=0$
  - Expected phase diagram
  - Recent results
- Progress toward $\mu \neq 0$
  - Reweighting
  - Taylor expansion
  - Analytic continuation

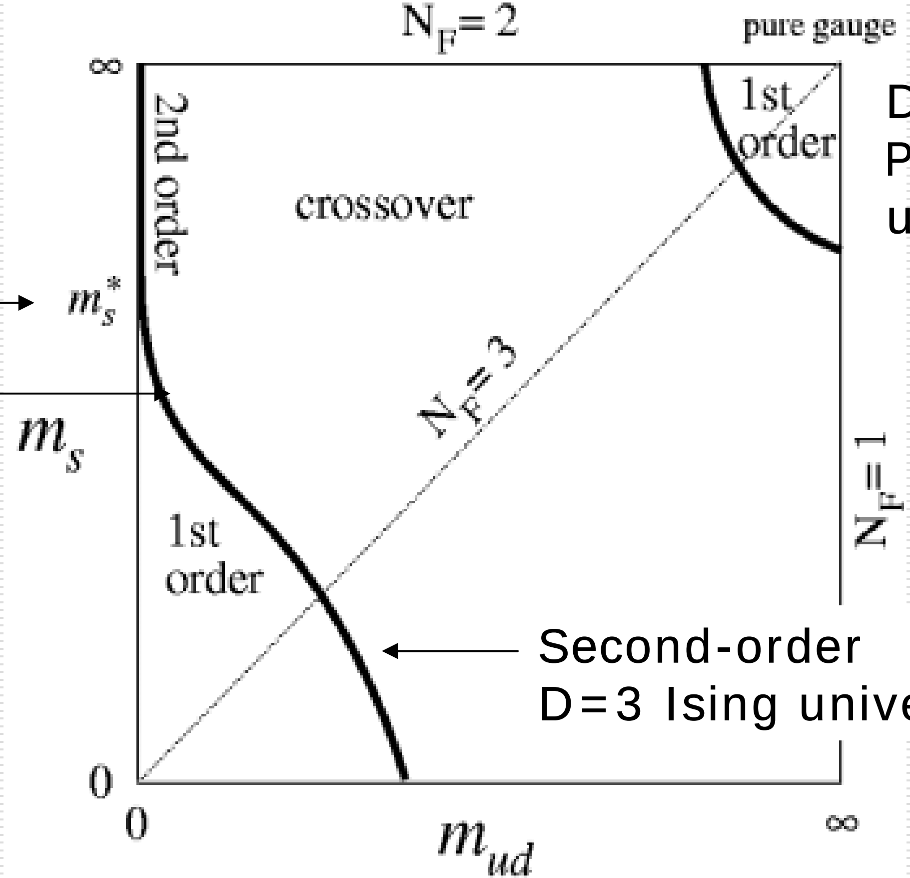
Remarks

- Still mostly (improved) KS fermion action
- Still mostly Temporal size N_t=4-8

  i.e., coarse lattice $a^{-1} \approx 0.6 - 1.2 GeV$

Phase diagram expected at $\mu=0$

$$N_f = 2+1\,QCD$$



D=3 Z(3) Potts universality

Tricritical point →

$$m_{ud} \propto \left(m_s^* - m_s\right)^{5/2}$$

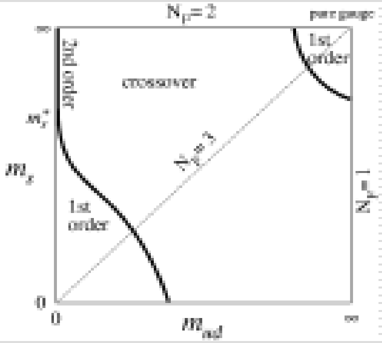
← Second-order D=3 Ising universality

Where is the physical point?

Nature of the 2nd order endline

- ☐ Existence of the endline well established

  JLQCD/Bielefeld/Columbia

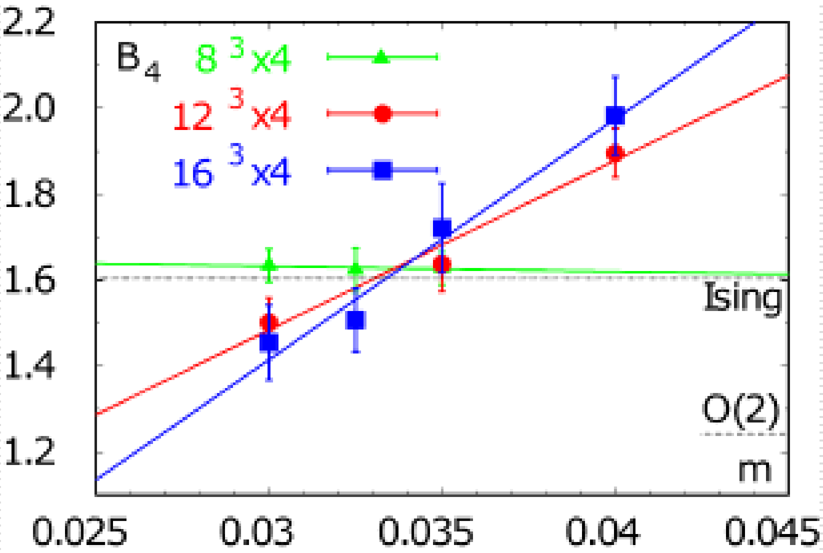


- ☐ Binder cumulant test to distinguish universality class

$$B_4 = \frac{\left\langle (\delta\bar{\psi}\psi)^4 \right\rangle}{\left\langle (\delta\bar{\psi}\psi)^2 \right\rangle^2}$$

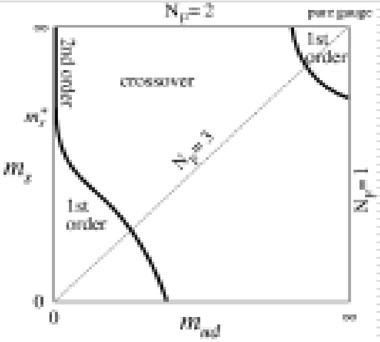
  - Clear evidence of Ising universality as predicted by S. Gavin et al

  S. Gavin et al hep-ph/9311350

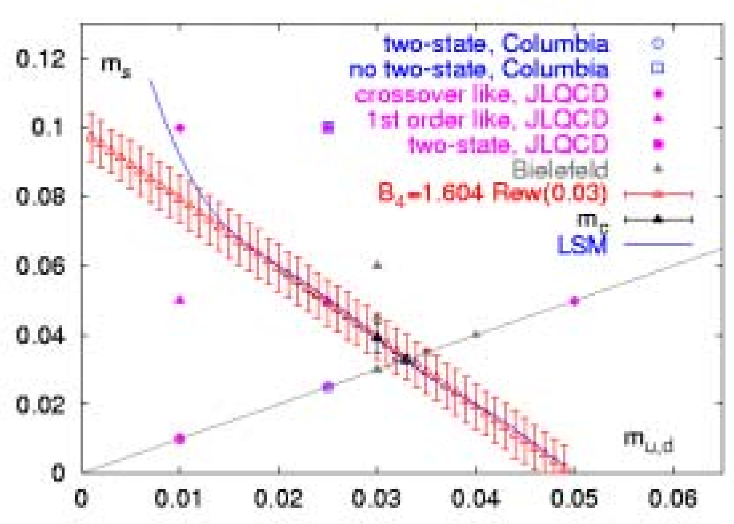


Karsch et al hep-lat/0107020

Location of the physical point



- Previous work by JLQCD/Columbia
- Schmidt(Bielefeld-Swansea) @Lattice2002
  - $m_{ud}=0.03, m_s=0.045, 0.06$
  - 12^3x4
  - Binder cumulant
- Christ(Columbia)@Lattice2002
  - $m_{ud}=m_s=0.015-0.045$
  - 8^3x4, 16^3x4 , 32^3x4



Ch. Schmidt et al hep-lat/0209009

Cross-over at the physical point indicated with the KS fermion simulations

NB first-order with Wilson fermion;old controversy still remains

Progress with finite chemical potential

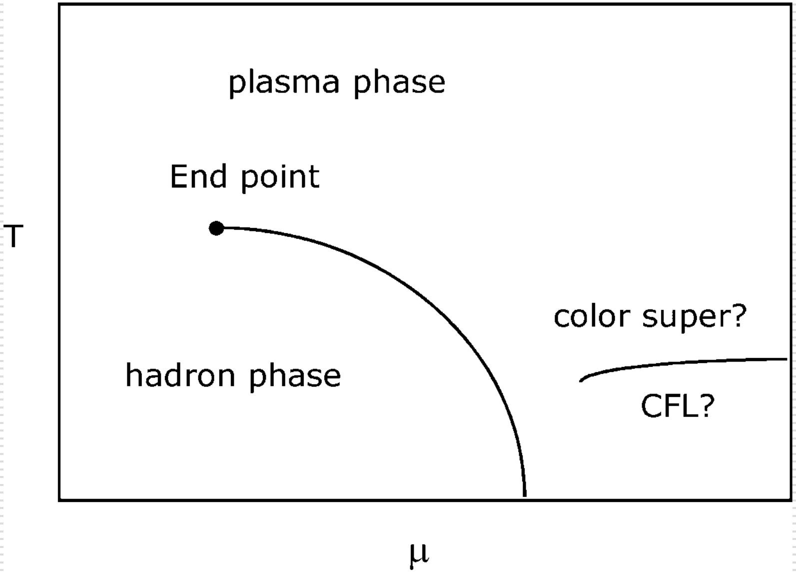
- Reweighting method to move from $\mu=0$ to $\mu \neq 0$

  Butapest (Fodor et al)
- Taylor expansion around $\mu=0$

  Bielefeld-Swansea
- analytic continuation from Im$\mu \neq 0$ to Re$\mu \neq 0$

  Forcrand et al/Lombardo et al
- Glasgow attempts ('92-'98) failed at T=0, but resurrected at $T \neq 0$ by Fodor-Katz

Schematic phase diagram
(assuming cross-over at T=0)



Reweighting in chemical potential μ

- Fodor-Katz strategy

  Z.Fodor et al hep-lat/0104001

  - Reweight in β and μ such that width of ω is minimal
  - Turned out to work for small volume;

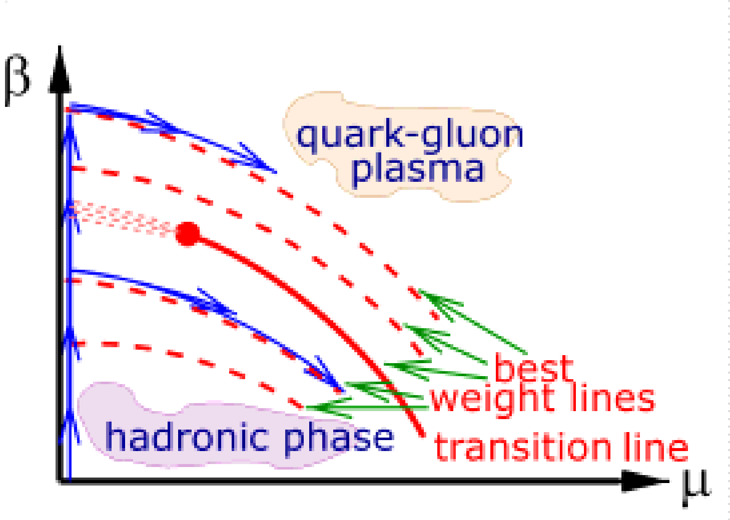
    $$\mu a \leq \left(N_t \cdot N_s^3\right)^{-1/4}$$

  - Use Lee-Yang zero analysis to locate the end-point E

complex

$$\det D(\mu) e^{-\beta S_{gluon}} = \det D(\mu=0) e^{-\beta_0 S_{gluon}} \cdot \underbrace{\frac{\det D(\mu)}{\det D(\mu=0)} e^{-(\beta-\beta_0) S_{gauge}}}_{\omega}$$

real

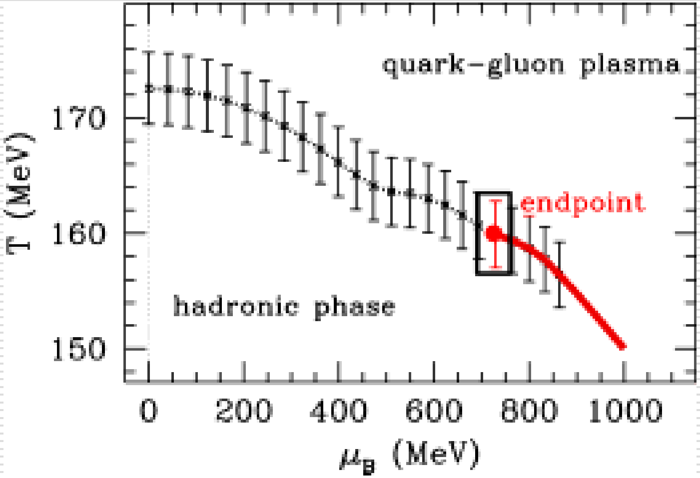


From Fodor et al hep-lat/0208078

Results

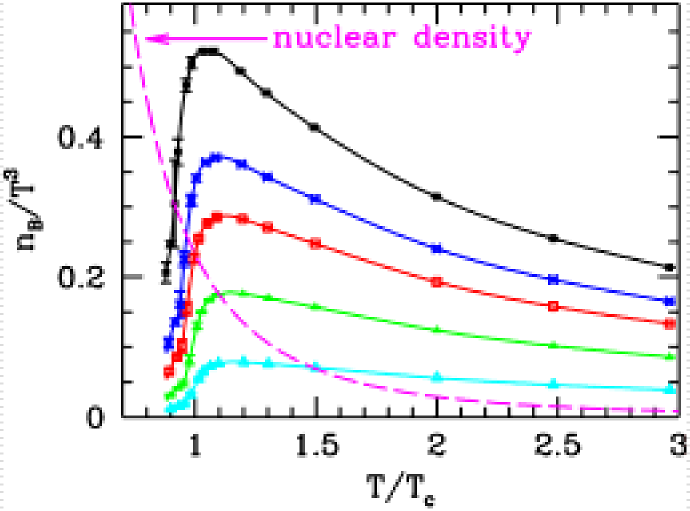
- End point:
  - T_E = 160+/-3.5 MeV
  - μ_E = 725+/-35 MeV
  - *NF*=2+1
    [m_{ud}=0.025, m_s=0.2]
  - $(4, 6, 8)^3$x4

Z. Fodor et al hep-lat/0106002



- Equation of state
  - Pressure p
  - Energy density e
  - *NF*=2+1
    [m_{ud}=0.025, m_s=0.2]
  - $(8, 10, 12)^3$x4

Z. Fodor et al hep-lat/0208078



Taylor expansion in chemical potential μ

S. Ejiri et al heo-lat/0209012
C. R. Allton et al hep-lat/0204010

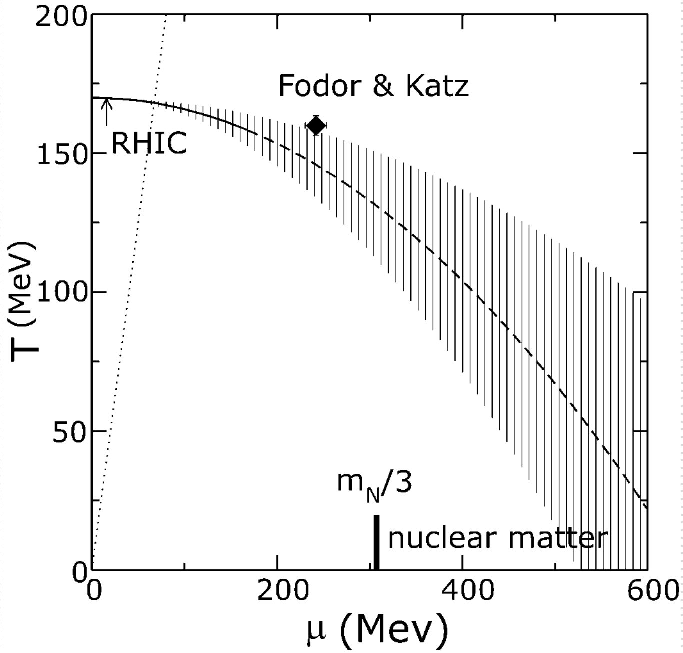
- Taylor expansion should converge up to the endpoint

$$Tr\ln D(\mu) = Tr\ln D(\mu=0) + TrD(\mu=0)^{-1}\frac{\partial D}{\partial \mu}(\mu=0)\cdot\mu + \cdots$$

- Calculate

$$\left.\frac{d^2\beta_c(\mu)}{d\mu^2}\right|_{\mu=0} \Rightarrow \left.\frac{d^2T_c(\mu)}{d\mu^2}\right|_{\mu=0}$$

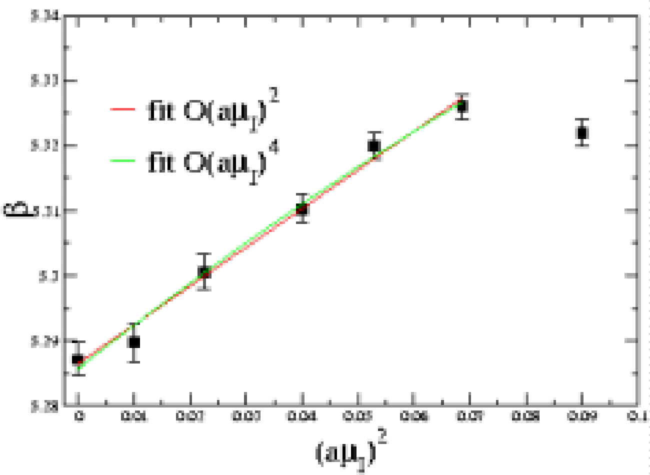
- Simulation
  - P4-improved KS
  - $N_f=2$ [$m_{ud}=0.01, 0.02$]
  - 16^3x4



Analytic continuation from Imaginary to Real μ

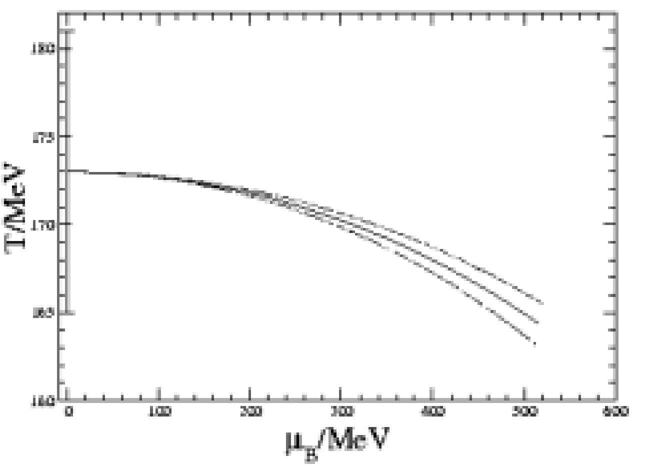
- ☐ Determinant real for Imaginary μ, hence amenable to Monte Carlo
- ☐ Fit obervables in polynomials of μ
- ☐ Analytically continue in μ

M. Alford et al hep-lat/9807039
Ph. Forcrand et al hep-lat/020501
M. D'Elia et al hep-lat/0209146



$$\beta_c(\mu) = \beta_c(0) + c(a\mu_I)^2$$

$$i\mu_I \Rightarrow \mu$$



$$\beta_c(\mu) = \beta_c(0) - c(a\mu)^2$$

From Ph. Forcrand et al hep-lat/0205016

Conclusions and prospects

- Visible shift from quenched to full QCD simulations
  - Important effects observed in physical observables
  - Crucial for consistent predictions from lattice QCD
- Development of lattice fermion with exact chiral symmetry
  - Both conceptual and practical advantages
  - Need O(10) times more computer power; awaits next generation of computers for full QCD
- Notable progress in
  - Study of finite chemical potential
- Require further effort to understand
  - K meson decays
- Expect substantial progress by the next PaNic onference with next negeration of computers
  - QCDOC/APENEXT
  - Development of clusters