Present and Future of Lattice QCD

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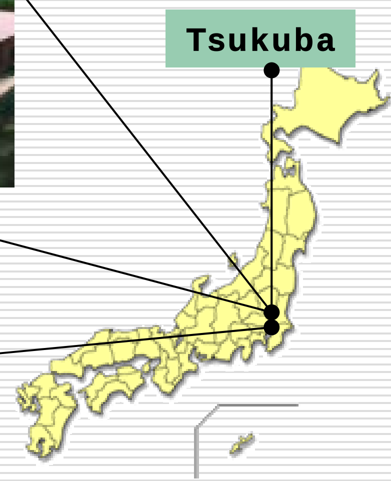
- overview
- hadron spectrum and fundamental constants of QCD
- topics in hadron physics
- issues and progress with chiral symmetry
- weak interactions of hadrons
- QCD thermodynamics
- conclusions and prospects

Center for computational Physics
University of Tsukuba

- Founded in 1992
- Emphasis on
  - Development of HPC systems suitable for computational physics
  - Close collaboration of physicists and computer scientists



- Computing facility
  - CP-PACS parallel system
    - MPP with 2048PU/0.6Tflops peak
    - Developed at the Center with Hitachi Ltd.
    - #1 of Top500-November 1996
  - GRAPE-6 system
    - Dedicated to gravity calculations
    - Developed at U. Tokyo
    - 8Tflops equivalent

Quantum Chromodynamics

- Quantum Chromodynamics (QCD)
  - Fundamental theory of quarks and gluons and their strong interactions

$$S_{QCD}=\frac{1}{8\pi\alpha_s}Tr\left(F_{\mu\nu}F_{\mu\nu}\right)+\sum_f \overline{\psi}_f\left(\gamma_\mu\cdot\left(\partial_\mu-iA_\mu\right)+m_f\right)\psi_f$$

  - Knowing

$$\left\langle O\left(A,\overline{\psi},\psi\right)\right\rangle=\frac{1}{Z}\int dAd\overline{\psi}d\psi O\left(A,\overline{\psi},\psi\right)e^{-S}$$

1 coupling constant α_s

and

6 quark masses $m_u, m_d, m_s, m_c, m_b, m_t$

will allow full understanding of strong interactions

"Yukawa's dream(1935) in modern form"

QCD on a Lattice

- Lattice QCD
  - Powerful mean to calculate the QCD Feyman path integral

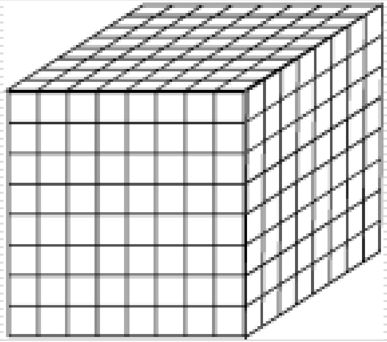
$$S_{QCD} = \frac{1}{\alpha_s}\sum_P tr(UUUU) + \sum_f \overline{\psi}_f(\gamma \cdot U + m_f)\psi_f$$

$$\langle O(U,\overline{\psi},\psi)\rangle = \frac{1}{Z}\int dU d\overline{\psi} d\psi O(U,\overline{\psi},\psi) e^{-S}$$

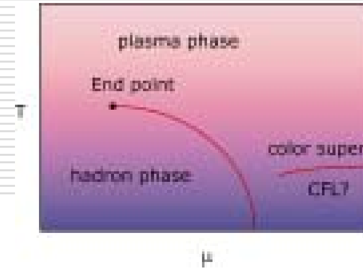
- From computational point of view
  - Relatively simple calculation
    - Uniform mesh
    - Single scale
  - Requires much computing power due to
    - 4-dim. Problem
    - Fermions essential
    - Physics is at lattice spacing a=0
  - Precision required(<a few % error in many cases)

lattice spacing a

QCD scale parameter Λ_{QCD}



Subjects of lattice QCD

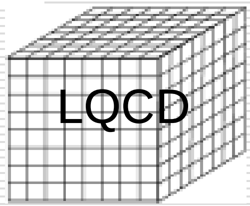


Hadron spectrum and Fundamental constanst of QCD

- Strong coupling constant
- Quark masses

α_s

$m_u, m_d, m_s, m_c, m_b, m_t$



Finite-temperature/ density behavior

- order of transition
- critical temperature/density
- equation of state

Physics of quark-gluon plasma

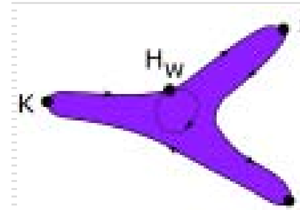
Hadron physics

- eta' meson mass and U(1) problem
- exotic states
  glueball, dibaryon, hybrids,…
- hadronic matrix elements
  proton spin, sigma term, ….
- structure functions/form factors

$$\int_0^1 dx x^{n-1} F(x,q^2) = (\ln q^2)^{-\gamma_n} \langle N | O_n | N \rangle$$

Weak interaction matrix elements

- K meson amplitudes
  B_K
  $K \rightarrow \pi\pi$ decays
- B meson amplitudes
  f_B, B_B, form factors



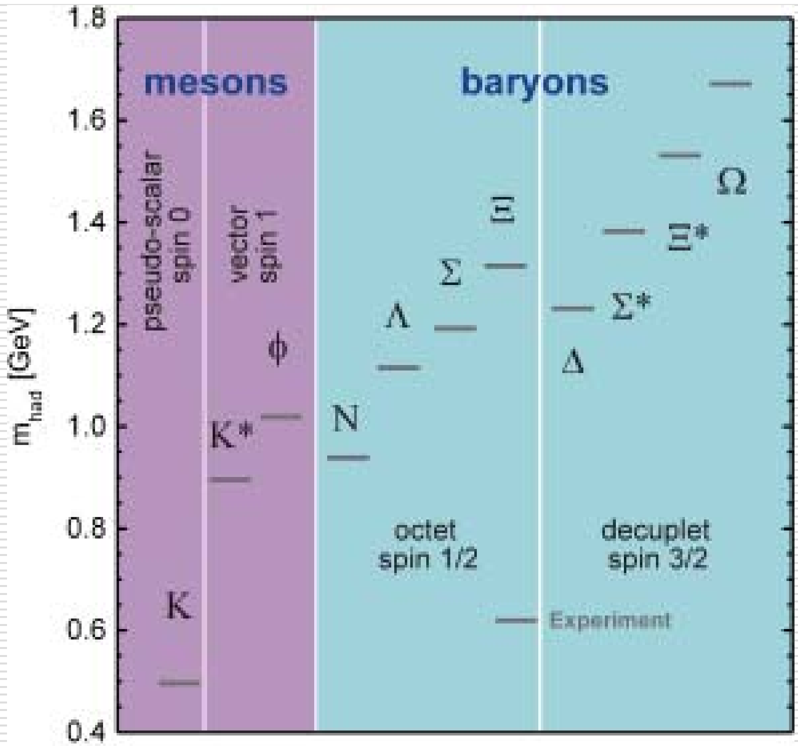
CKM matrix and CP violation

Physics subjects of this talk

- Hadron spectrum and fundamental constants of QCD
  - Fundamental verification of QCD at low energies
  - Determination of quark mass and strong coupling constant
- Topics in hadron physics
  - Nucleon structure functions
- Issues and progress with chiral symmetry
  - Chiral extrapolation of observables
  - Chirally symmetric fermion formulations
- Weak amplitudes of hadrons
  - $K \rightarrow \pi\pi$ decays and CP violation
  - K and B meson amplitudes and constraints on the CKM matrix
- Finite temperature/density QCD
  - Status at zero density
  - Progress toward non-zero density

Light hadron mass spectrum

- Benchmark calculation to verify QCD
- Indispensable for determination of QCD scale and quark masses
- Essential to control various systematic errors down to a few % level
  - Finite lattice size L>3fm
  - Finite quark mass mq→0
  - Finite lattice spacing a→0



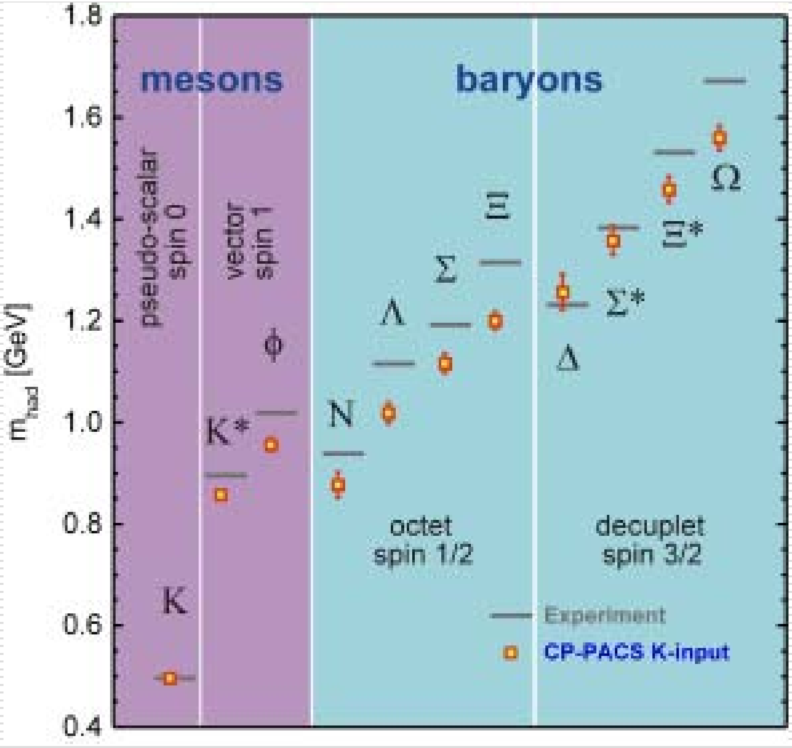
Experimental spectrum

CP-PACS result for the quenched spectrum'98

- Sea quark effects ignored
- General pattern reproduced, but clear systematic deviation beyond 10% precision
- Completes the calculation started in '81 (Hamber-Parisi/Weingarten)

$use\ m_\pi, m_\rho\ \ for\ \ fixing\ a\ and\ m_{ud}$

$use\ m_K\ \ for\ \ fixing\ m_s$



Calculated quenched spectrum

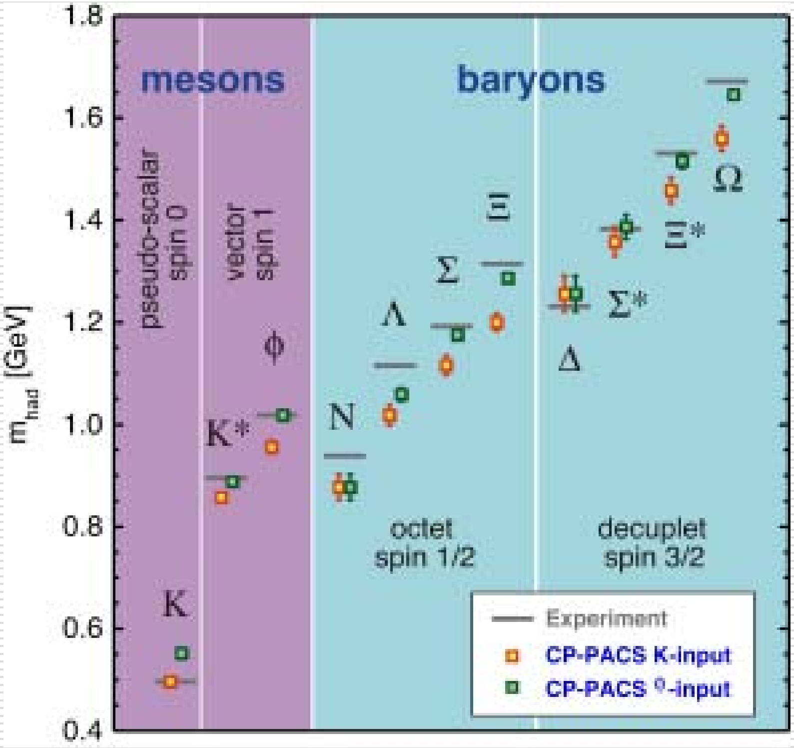
Input dependence in quenched QCD

- Details of disagreement depends on input, but overall agreement not possible

$use\ m_K\ \ for\ \ fixing\ m_s$

$use\ m_\phi\ \ for\ \ fixing\ m_s$

- *predictions in quenched QCD suffer from uncertainties depending on input*

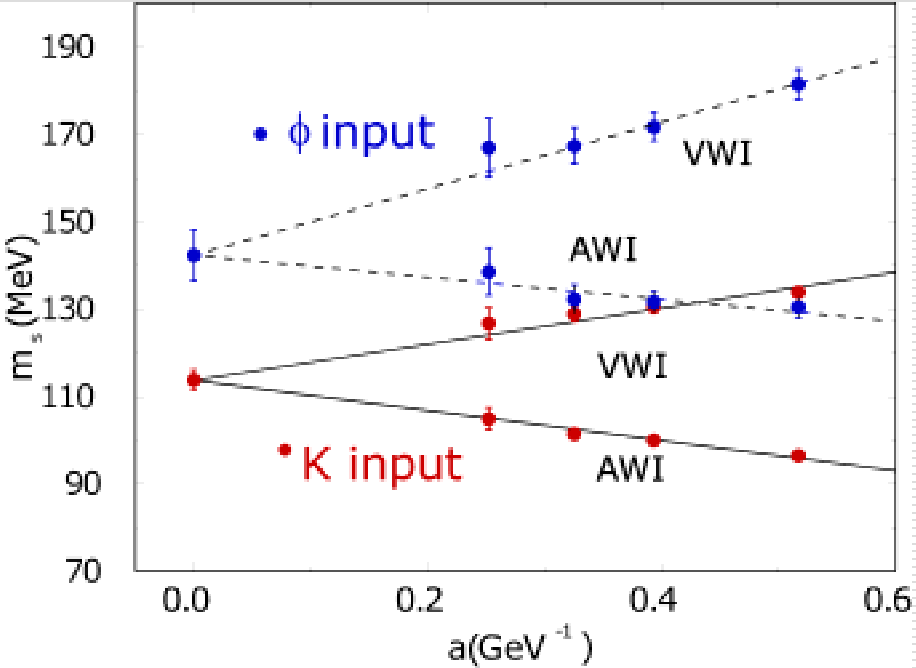


Calculated quenched spectrum

Strange quark mass in quenched QCD

- 25% descrepancy in the predicted value of strange quark mass

- Clearly illustrates
  - *limitation of quenched QCD*
  - *necessity of full QCD with dynamical quarks*



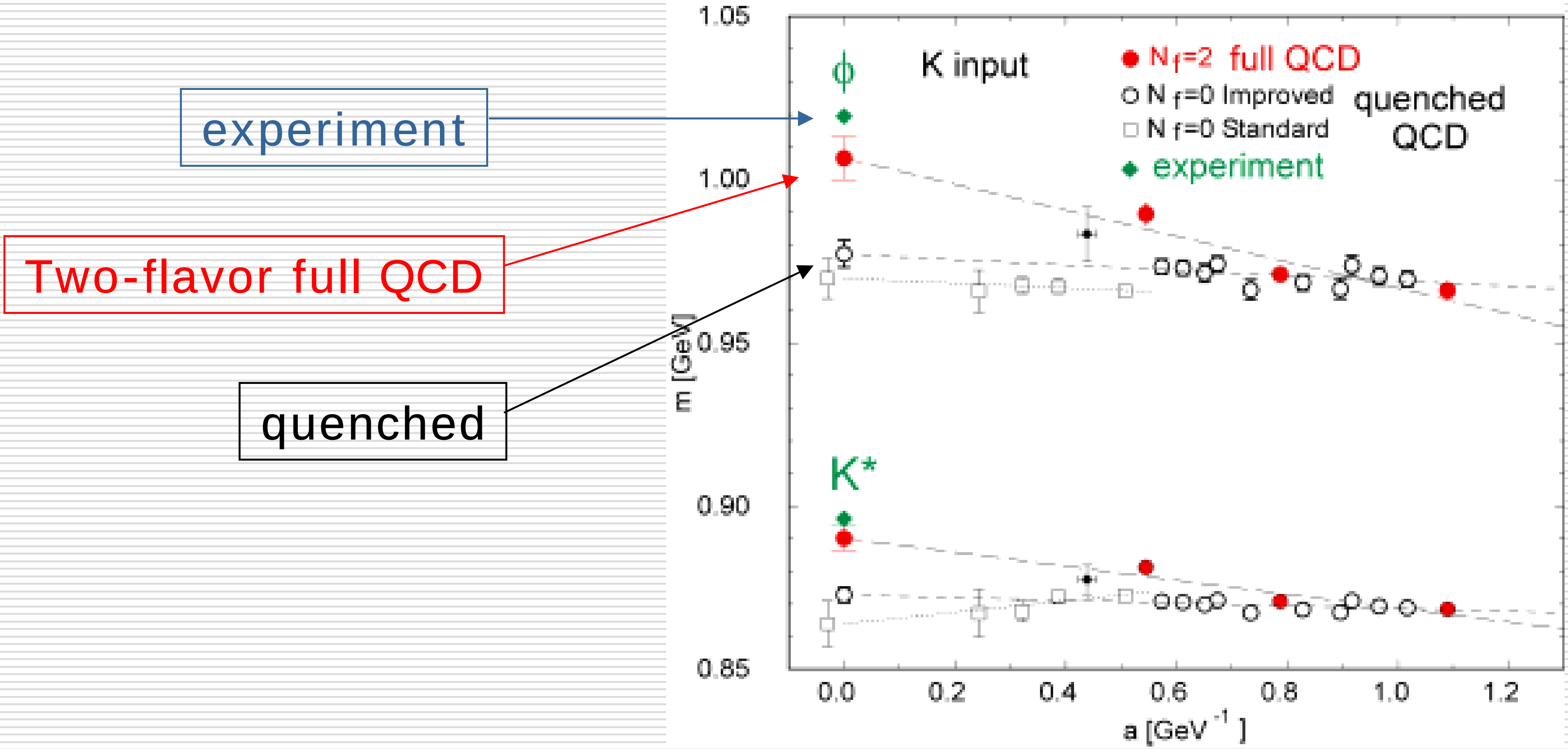
$$m_s\left(\overline{MS}, 2GeV\right) = \begin{cases} 142^{+28}_{-6}\ MeV & \phi\ meson\ mass\ input \\ 114^{+8}_{-6}\ MeV & K\ meson\ mass\ input \end{cases}$$

QCD simulation with dynamical quarks

- Spectrum of quarks
  - *3 light quarks (u,d,s)* *m < 1GeV*
    - *Need dynamical simulation*
  - 3 heavy quarks (c,b,t) m >1GeV
    - Quenching sufficient
- Dynamical quark simulation (full QCD)
  - costs 100-1000 times more computing power
  - Algorithm for *odd number of quarks* now available
- *Two-flavor full QCD (since around 1996)* $N_f = 2$
    - u and d quark dynamical simulation
    - s quark quenched approximation
  - Number of studies: SESAM/UKQCD/MILC/CP-PACS/JLQCD
- *Three-flavor full QCD (since around 2000)* $N_f = 2+1$
    - s quark also treated dynamically
  - Extensive studies have begun : MILC/CP-PACS-JLQCD

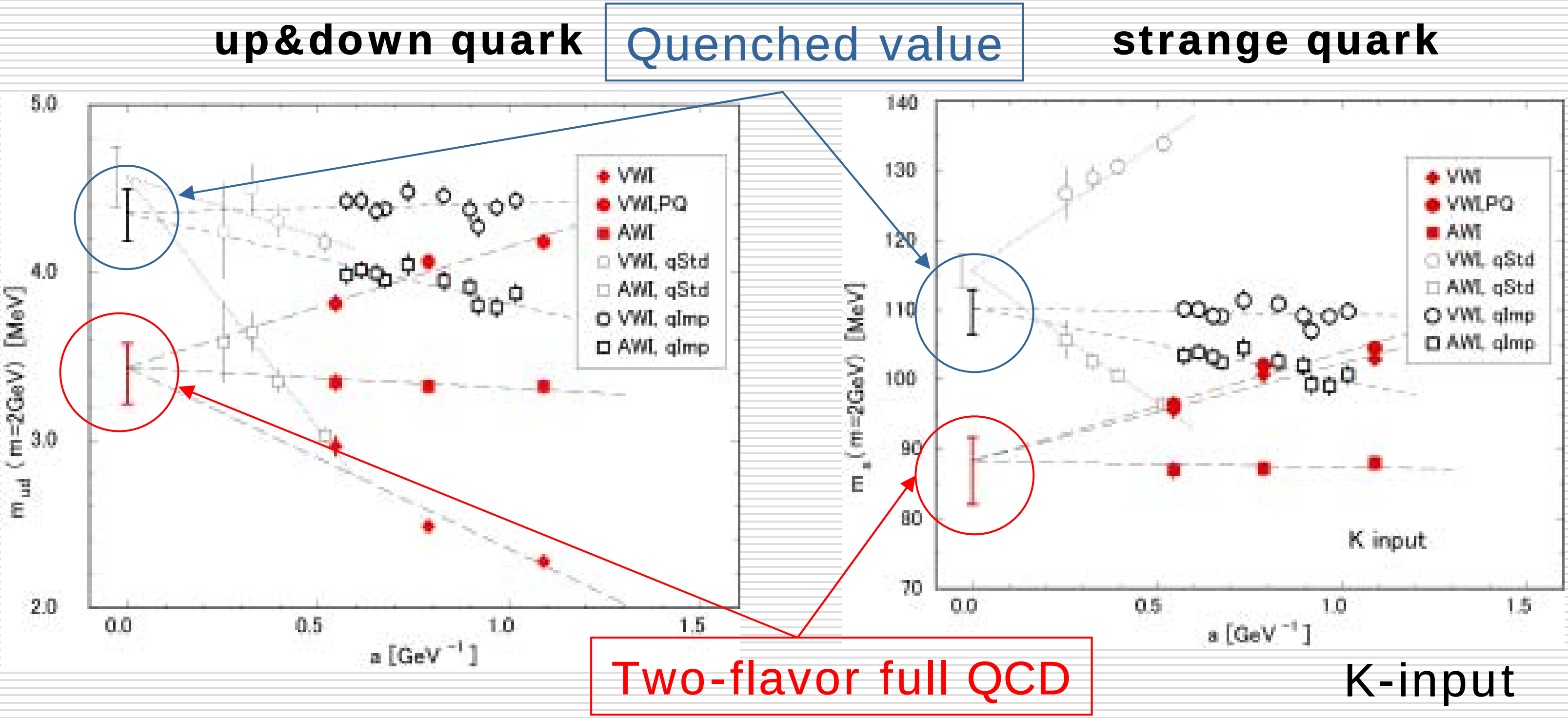
Sea quark effects in the spectrum

- K*-K and ϕ-K mass difference (Meson hyperfine splitting)
  - too small in quenched QCD
  - Much closer agreement for two-flavor full QCD



Sea quark effects in quark masses

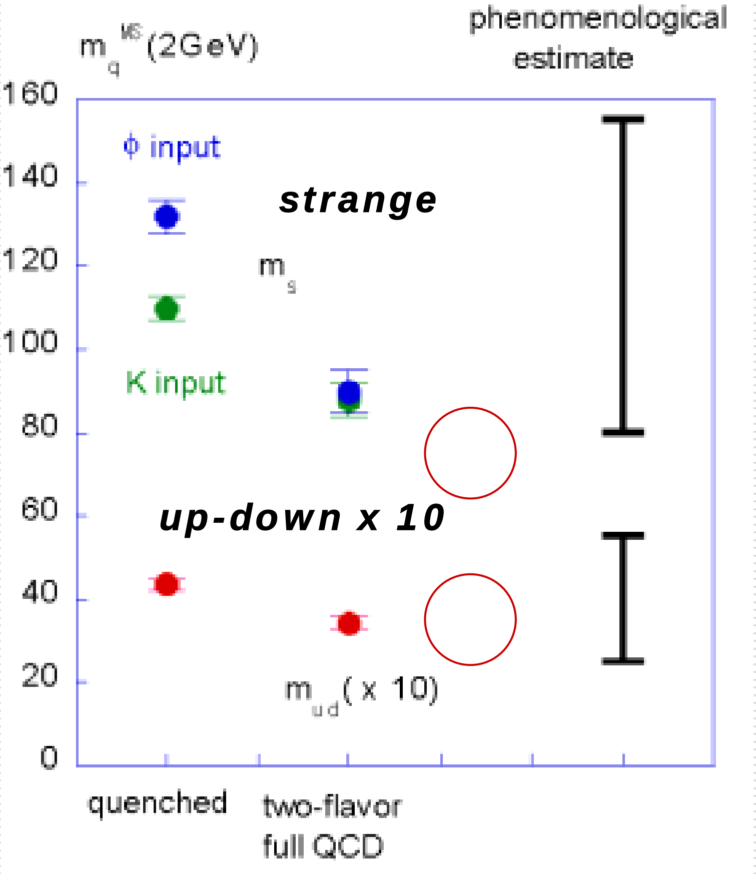
- Significant decrease by inclusion of sea quark effects
- Input dependence of strange quark mass also reduced



Light quark masses (u, d, s) $m_q^{\overline{MS}}(2GeV)$

- Significant sea quark effects
  - Large uncertainty (∼20%) depending on input in quenched theory
  - Sizable decrease (∼25%) from quenched to two-flavor full QCD
- Lighter than naïve quark model values
- Nf=3 simulations being pursued to obtain physical values of light quark masses, e.g., Hein et al hep-lat/0209077

CP-PACS Collab. Hep-lat/0004010



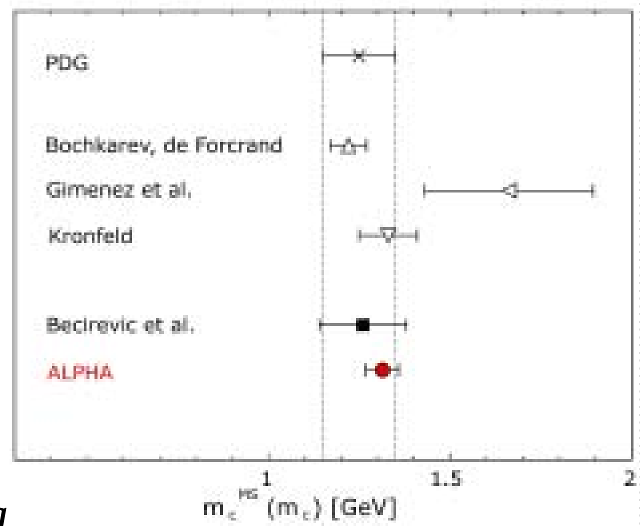
Real world;three flavors?

Heavy quark masses (c, b)

- **Charm quark mass**
  - J. Rolf and S. Sint Lattice01
    - Fully non-perturbative determination in the continuum in quenched QCD

$$m_c^{\overline{MS}}(m_c) = 1.314(45)GeV$$

compiled by T. Kaneko (Lattice '01)
Hep-lat/0111005



- **Bottom quark mass**
  - Not straightforward since $m_b > 1/a$
  - Use

$$M_B = m_b^{pole} + \underbrace{E_b}_{binding\ energy} - \underbrace{\delta m}_{radiative\ corrections}$$

$$\begin{cases} M_B, E_B & Monte\ Carlo \\ \delta m & pert.\ theory \end{cases}$$

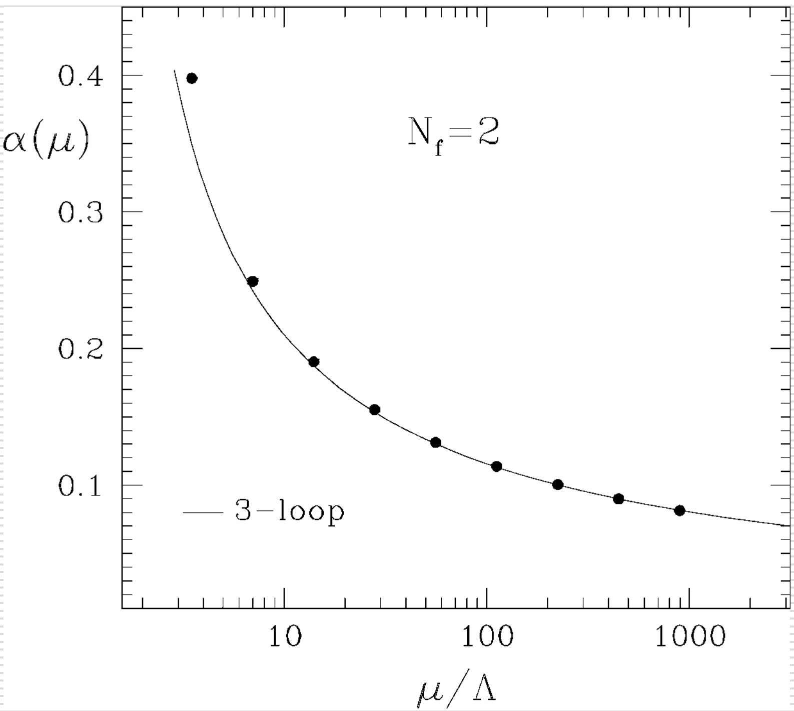
  - G. Gimenez et al ('00) $m_b^{\overline{MS}}(m_b) = 4.27(9)GeV$

Strong coupling constant $\alpha_s(\mu)$

- Another fundamental parameter of QCD
- Large number of high energy determinations from experiments
- Lattice determinations:
  - Method I (Lepage et al '91):
    - Calculate short-distance observables as a function of $\alpha_s(1/a)$ at cutoff scale 1/a
    - the scale 1/a is fixed from hadron mass
  - Method II (Luescher et al '93)
    - Non-perturbative determination of the RG evolution by Shcrodinger functional finite-size technique

Scale dependence of $\alpha_s(\mu)$

SF scheme

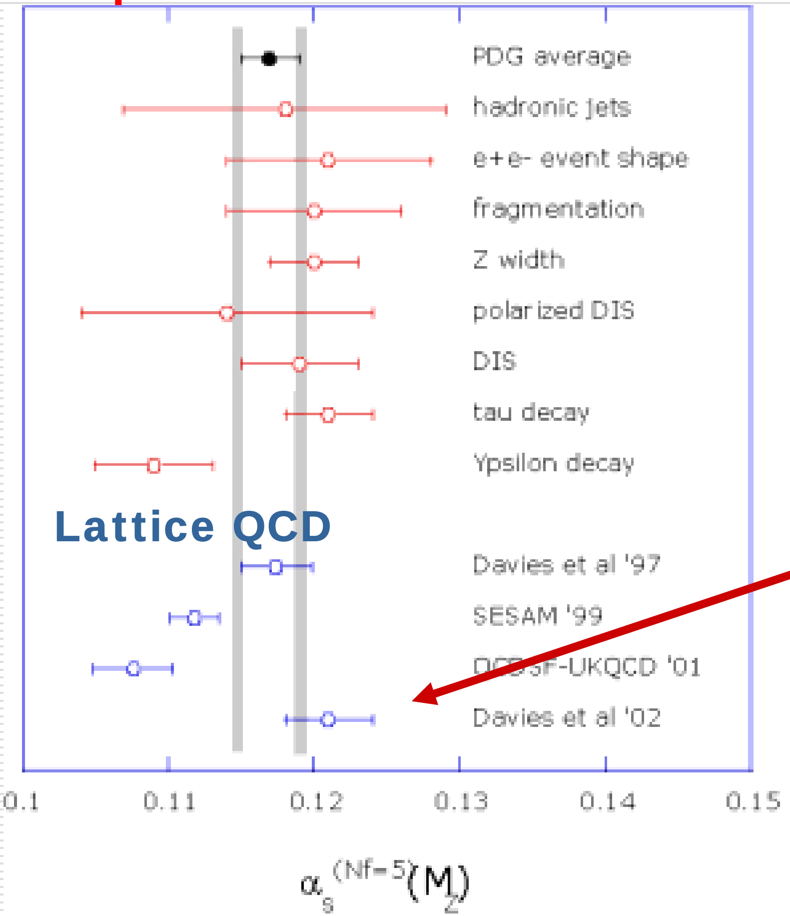


M. Della Morte et al hep-lat/0209023

- Non-perturbative determination by Alpha Collaboration for two-flavor full QCD (Lattice02)
- Indicates perturbative evolution for p>a few GeV
- Similar results for quenched QCD ('94)
- Physical scale yet to be determined

Determination of $\alpha_s^{\overline{MS}}(M_Z)^{N_f=5}$

experiment



- Comments
  - Davies et al ’97(hep-lat/9703010):
    Involved extrapolation of Nf=0(quenched) and Nf=2 data to Nf=3
  - QCDSF-UKQCD ’01(hep-lat/0103023)
    Continuum estimate with systematic Nf=2 simulations
  - Davies et al ’02(hep-lat/0209121):
    Preliminary result based on MILC **Nf=3** configurations at a=0.13fm
- Systematic Nf=3 full QCD determination expected in a few years

Topics in hadron physics

- Progress in hadron spectroscopy
  - Eta' meson mass and U(1) problem
  - Glueballs
  - Multiquark states
  - Excited string states etc

- Hadron structure
  - *Moments of nucleon structure functions*
  - Form factors

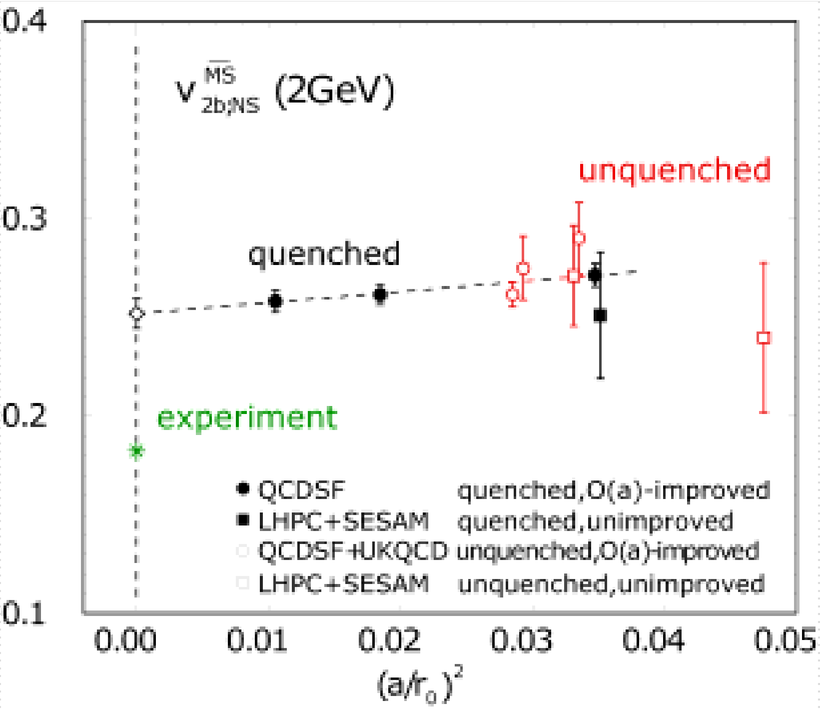
- Hadron scattering amplitudes
  - Scattering length
  - *Phase shift* *I=2 $\pi\pi$ channel Ishizuka et al* ***2002***

Moments of nucleon structure functions (I)

$$\int_0^1 dx x^{n-1} F\left(x, q^2\right) = \left(\ln q^2\right)^{-\gamma_n} \langle N | O_n | N \rangle$$

- A number of calculations
  - QCDSF (quenched)’96
  - Alpha(quenched)’98
  - QCDSF+UKQCD (full)’’01-’02
  - LHPC+SESAM (full)’02
- No large sea quark effects observed for non-singlet moments
- No large scaling violation
- Lattice predictions do not agree with experiment

$\langle x \rangle_{u-d}$



Compiled in M. Goeckeler et al hep-lat/0209160

Moments of nucleon structure functions (II)

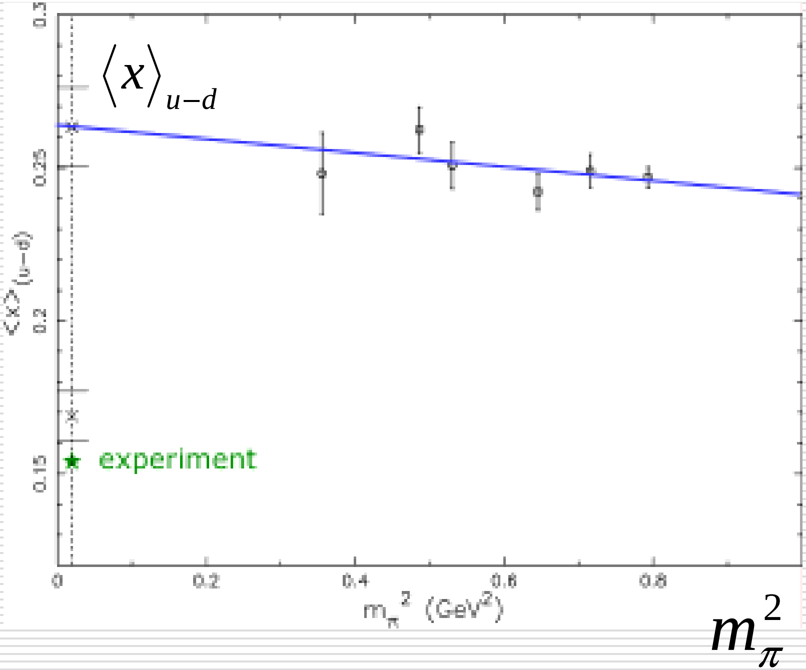
- *Linear chiral extrapolation misses experiment*
- Possible reasons:
  - Quenching? No
  - O(a) error? No
  - Chiral extrapolation itself?
- Chiral perturbation theory:

$$\langle x^n \rangle = a_n \left(1 - \frac{3g_A^2+1}{(4\pi f)^2} m_\pi^2 \ln \frac{m_\pi^2}{\Lambda_\chi^2} + c m_\pi^2 \cdots \right)$$

  - Pion not light enough to see curvature?
  - An effective model can reproduce experiment

$$\langle x^n \rangle = a_n \left(1 - \frac{3g_A^2+1}{(4\pi f)^2} m_\pi^2 \ln \frac{m_\pi^2}{\Lambda_\chi^2 + m_\pi^2} + d m_\pi^2 \right)$$

From D. Dolgov et al hep-lat/0201021



W. Detmold et al hep-lat/0103006

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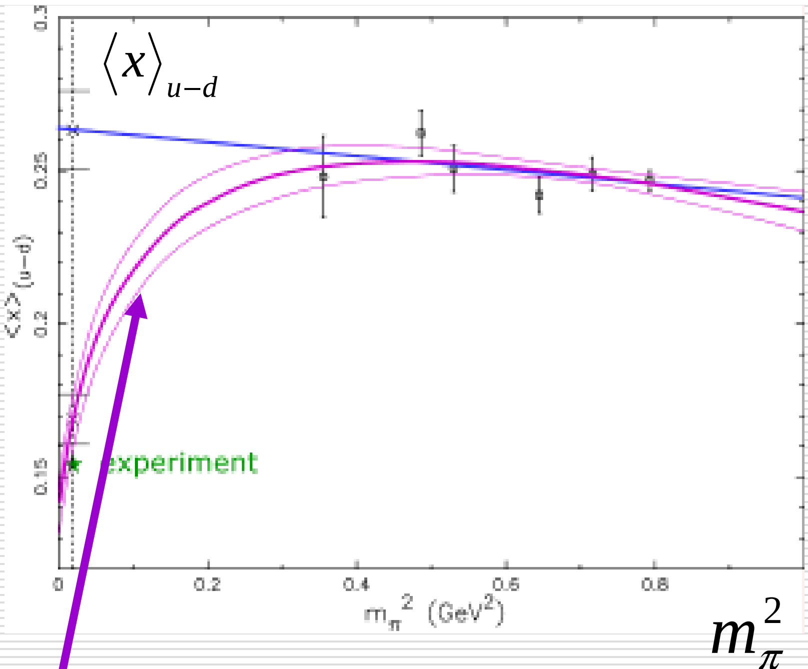


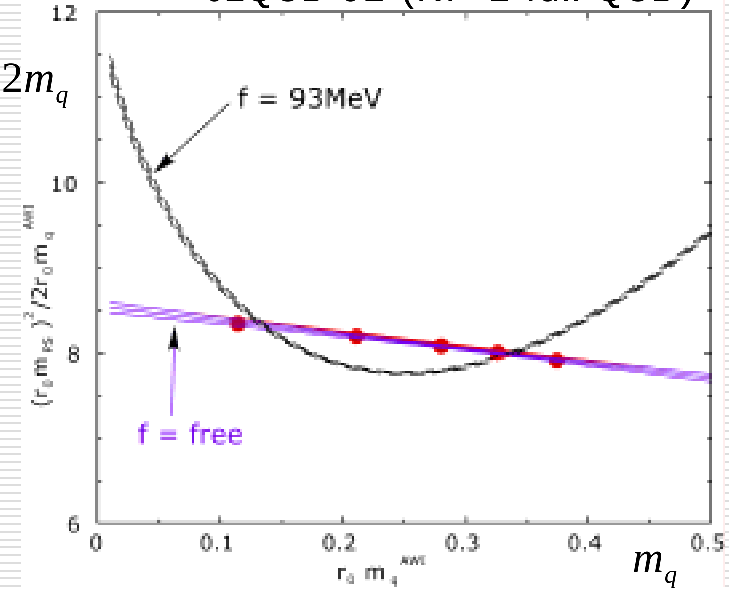
W. Detmold et al hep-lat/0103006

A general issue with chiral extrapolation

- Current lattice data often fails to see logarithmic singularity expected from chiral perturbation theory
- Often causes sizable (10-20%) uncertainties in the extrapolated result
- Pion mass in current simulations (∼500MeV) too heavy;needs to be reduced
- Lattice fermion action with exact chiral symmetry much desired (conventional Wilson and KS action breaks chiral symmetry)

JLQCD'02 (Nf=2 full QCD)

$m_\pi^2/2m_q$



m_q

$$m_\pi^2 = A m_q \left(1 + \frac{1}{N_f (4\pi f)^2} m_q \ln m_a + b m_q + \cdots \right)$$

Lattice fermion with exact chiral symmetry

- Theoretically based on the Ginsparg-Wilson relation:

$$D\gamma_5 + \gamma_5 D = 2aDR\gamma_5 D$$

  - Domain-wall fermion Kaplan('92)/Furman-Shamir('94)
  - Overlap formalism Neuberger-Narayanan('92,'97)
  - Fixed point action Hasenfratz-Neidermyer('94)

- Avoids the Nielesen-Ninomiya Theorem by using "infinitely" many fields (hence needs more computer power)

- quenched calculations show very promising results: good chiral property, small scaling violation, ...

A test in quenched QCD

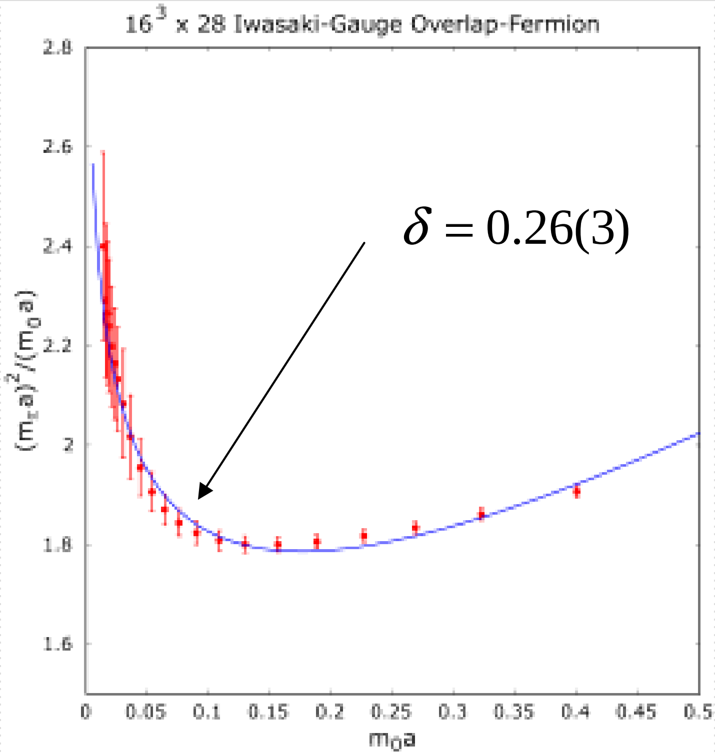
- chiral logarithm behavior of pion mass in quenched QCD

  Sharpe/Bernard-Golterman '91

$$m_\pi^2 = A m_0 \left(1 - \delta \ln m_0 + b m_0 + \cdots\right)$$

- Nice confirmation with the new fermion formalism
  - T. Draper et al : overlap fermion
  - C. Gattringer et al : fixed point fermion
- Reached very light pion mass
  - m_π~170MeV (T. Draper et al)
  - Similar results from other chiral formalisms

$m_\pi^2 / 2m_q$



m_q

T.Draper et al hep-lat/0208045

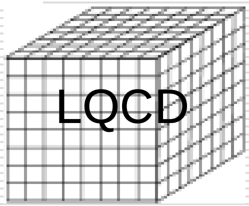
Subjects of lattice QCD

Hadron spectrum and Fundamental constanst of QCD

- Strong coupling constant
- Quark masses

α_s

$m_u, m_d, m_s, m_c, m_b, m_t$

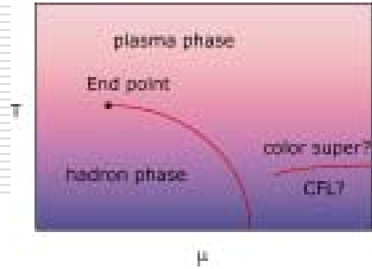


Hadron physics

- eta' meson mass and U(1) problem
- exotic states
  glueball, dibaryon, hybrids,...
- hadronic matrix elements
  proton spin, sigma term, ...
- structure functions/form factors

$$\int_0^1 dx x^{n-1} F(x, q^2) = (\ln q^2)^{-\gamma_n} \langle N | O_n | N \rangle$$

Finite-temperature/ density behavior



- order of transition
- critical temperature/density
- equation of state

Physics of quark-gluon plasma

Weak interaction matrix elements

- K meson amplitudes
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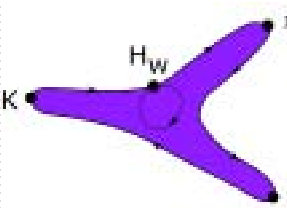


CKM matrix and CP violation

Weak amplitudes of hadrons

- First principles calculation of strong interaction corrections to weak amplitudes of hadrons
- Understand old and new issues in hadronic weak interactions
  - $\Delta I=1/2$ rule and direct CP violation in $K\to\pi\pi$ decay
- Constraints on the CKM mixing matrix
  - Neutral K and B meson mixings
  - B meson decay form factors

Measured weak amplitude	=	Known factor including CKM matrix	X	$\langle h' \vert H_{weak} \vert h \rangle$

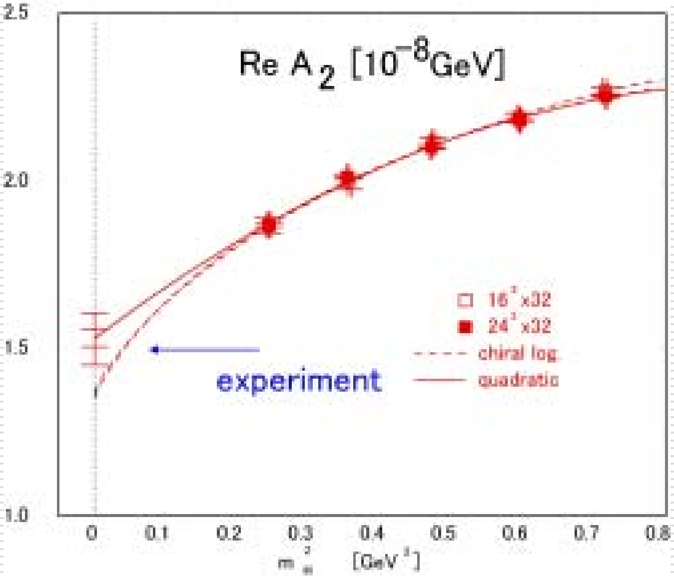
QCD matrix element

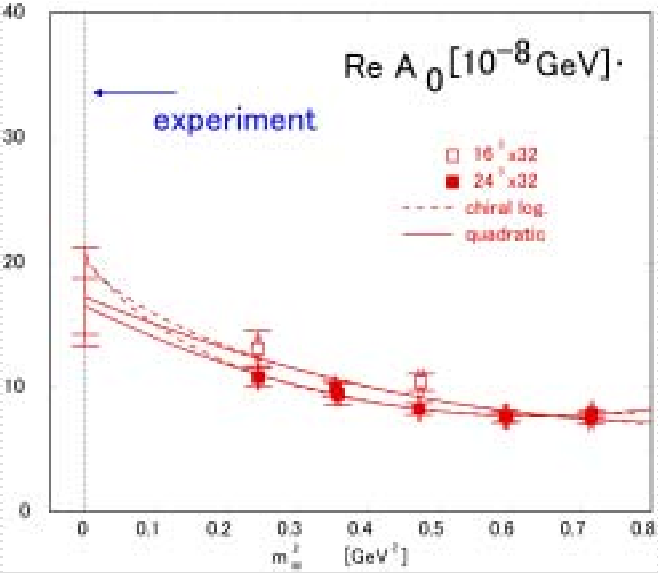
$\Delta I = 1/2$ rule and CP violation in K decays

- Weak interaction decays of K mesons
  - $\Delta I = 1/2$ rule $$\frac{\mathrm{Re}\,A_0\left(K \to \pi\pi(I=0)\right)}{\mathrm{Re}\,A_2\left(K \to \pi\pi(I=2)\right)} \approx 22$$
  - CP violation $$\frac{\varepsilon'}{\varepsilon} = \begin{cases} (20.7 \pm 2.8) \times 10^{-3} & KTeV \text{ experiment } (FNAL) \\ (15.3 \pm 2.6) \times 10^{-3} & NA48 \text{ experiment } (CERN) \end{cases}$$
- Crucial numbers to verify the Standard Model understanding of CP violation
- Chiral symmetry crucial because of the chiral structure of weak interactions
- Two large-scale calculations using domain-wall QCD
  - RIKEN-BNL-Columbia T. Blum et al hep-lat/0110124
  - CP-PACS J. Noaki et al hep-lat/0108013

ΔI=1/2 rule

- Reasonable agreement with experiment for I=2
- About half of experiment for I=0
- RIKEN-BNL-Columbia obtains a somewhat different result (smaller I=2 and larger I=0)



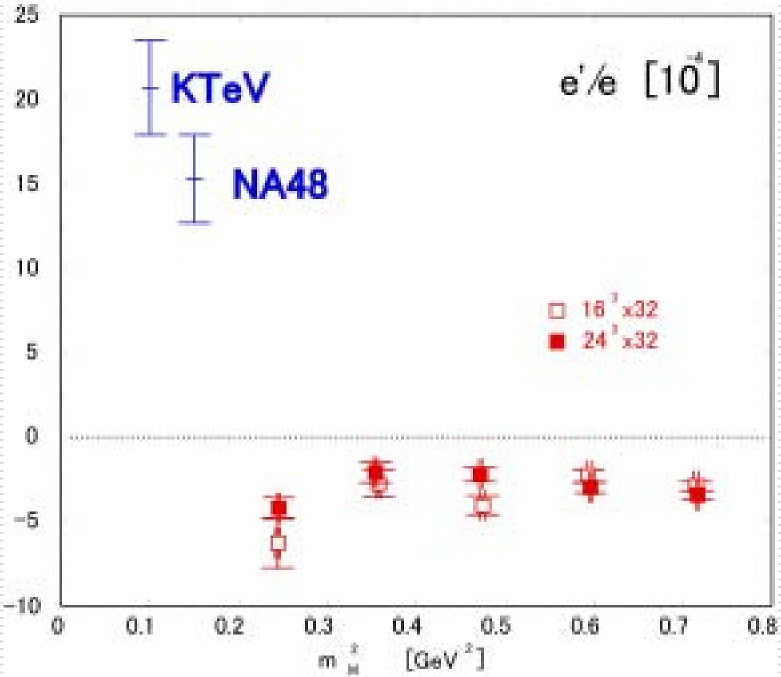


CP violation parameter ε'/ ε

- Small and negative in disagreement with experiment
- Similar result from RIKEN-BNL-Columbia

$$\frac{\varepsilon'}{\varepsilon} = \frac{\omega}{\sqrt{2}|\varepsilon|}\left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]$$

- Possible reasons
  - connected with insufficient enhancement of $\Delta I=1/2$ rule
  - Method of calculation ($K\to\pi$ reduction) may have serious problems



- Still a big problem requiring further work

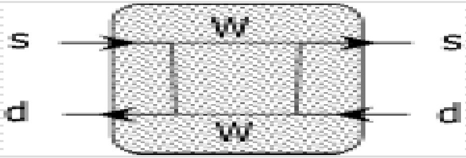
Constraints on the CKM matrix

Measured weak amplitude	=	Known factor including CKM matrix	X	$\langle h' \vert H_{weak} \vert h \rangle$

QCD matrix element

- $K^o - \overline{K}^0$ mixing $\quad \varepsilon_K \propto \overline{\eta}\left[\left(1-\overline{\rho}\right)A + B\right]^2 \hat{B}_K$

$$\left\langle \overline{K}^0 \left| \overline{s}\gamma_\mu (1-\gamma_5) d\, \overline{s}\gamma_\mu (1-\gamma_5) d \right| K^0 \right\rangle = \frac{8}{3} f_K^2 B_K M_K^2$$



⟹ B_K

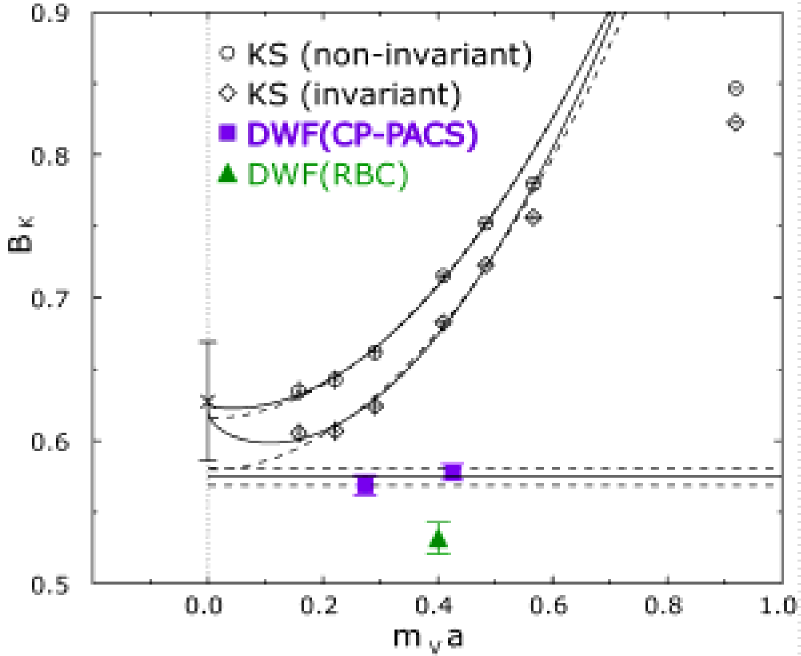
- $B^0_{d,s} - \overline{B}^0_{d,s}$ mixing $\quad \Delta M_{B_q} \propto \left(\overline{\rho}^2 + \overline{\eta}^2\right) f_{B_q}^2 B_{B_q}$

$$\left\langle \overline{B}_q^o \left| \overline{b}\gamma_\mu (1-\gamma_5) q\, \overline{b}\gamma_\mu (1-\gamma_5) q \right| B_q^0 \right\rangle = \frac{8}{3} f_{B_q}^2 B_{B_q} M_{B_q}^2 \qquad \xi = \frac{f_{B_s}}{f_{B_d}} \sqrt{\frac{B_{B_s}}{B_{B_d}}}$$

⟹ $B_B \quad f_B$

Lattice results for B_K

- Previous best result obtained with conventional KS fermion action
- Recent Domain-wall results indicates a smaller value
- Full QCD calculation yet to be made with domain-wall fermion
- Current best estimate:



$$B_K = 0.628(42) - 0.532(11)$$

CP-PACS hep-lat/0105020
RBC hep-lat/0110075

$$\hat{B}_K = 0.87^{+0.06}_{-0.13}$$

RG-invariant B parameter

Full QCD results for f_B

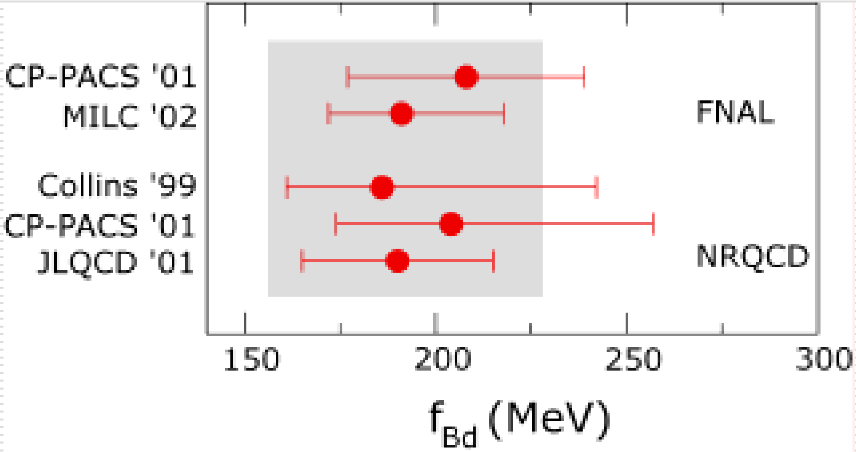
- Two-flavor full QCD result begins to accumulate
- f_{B_d} : possibly large uncertainty due to chiral extrapolation
- Best estimate from two-flavor full QCD:

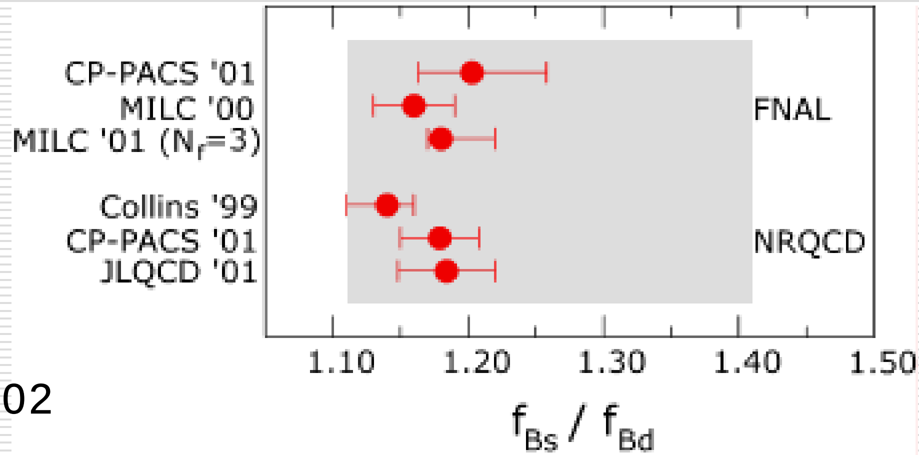
$$f_{B_d} = 198(30)^{+0}_{-34}\,MeV$$

$$f_{B_d} / f_{B_s} = 1.16(5)^{+24}_{-0}$$

N. Yamada at Lattice2002

Compilation by N. Yamada at Lattice02





Results for B_B

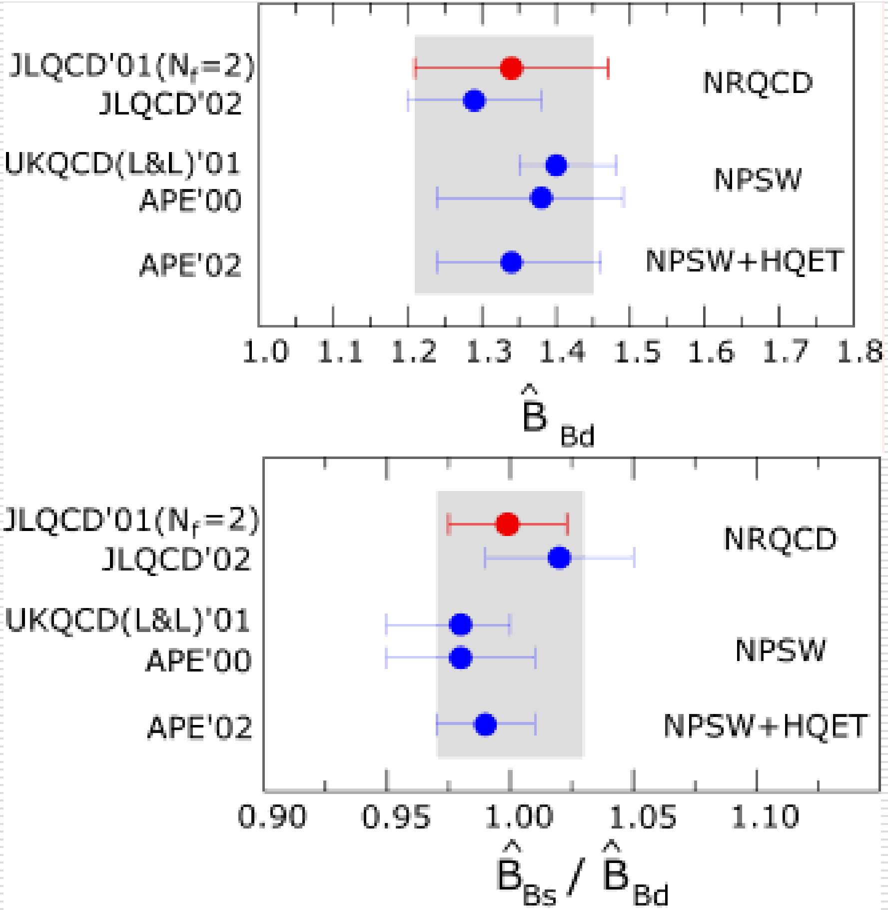
- Still mostly quenched (only one calculation in full QCD)
- Sea quark effects small
- Uncertainty with chiral extrapolation is small
- Current best estimate:

$$B_{B_d} = 1.33(12)$$

$$B_{B_d} / B_{B_s} = 1.00(3)$$

N. Yamada at Lattice2002

Compilation by N. Yamada at Lattice02



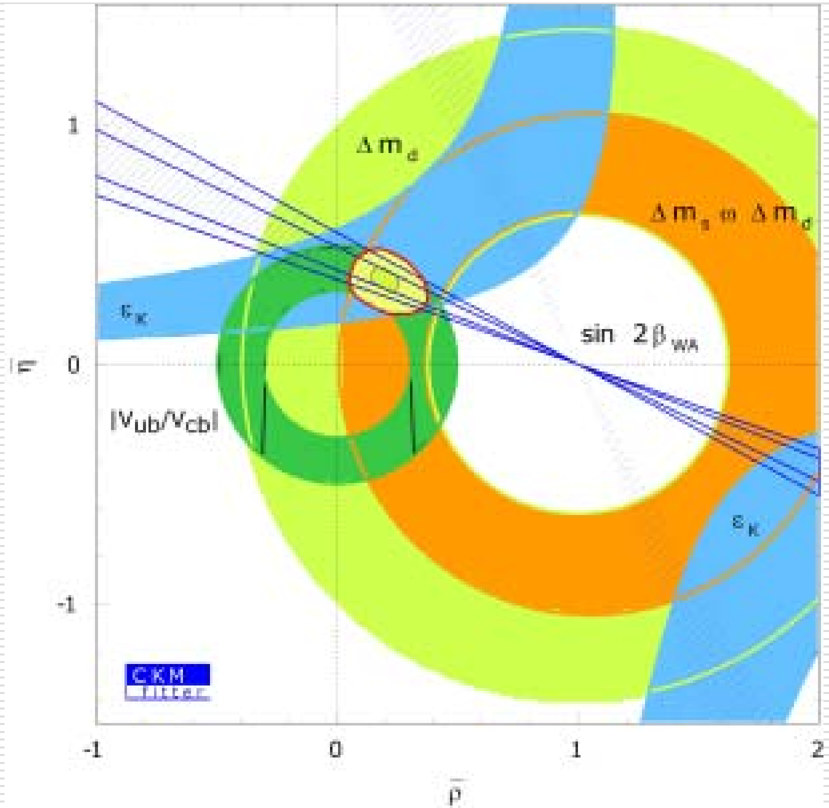
Summary of lattice results for CKM matrix

$$\hat{B}_K = 0.87^{+0.06}_{-0.13}$$

$$f_{B_d}\sqrt{B_{B_d}} = 0.227(37)^{+0}_{-34}\,GeV$$

$$\xi = 1.16(5)^{+24}_{-0}$$

status 2002 http://www.ckmfitter.in2p3.fr/



Cf. numbers used in the figure left

$$\hat{B}_K = 0.87^{+0.06}_{-0.13}$$

$$f_{B_d}\sqrt{B_{B_d}} = 0.227(28)\,GeV$$

$$\xi = 1.16^{+3}_{-5}$$

→ Better control of chiral extrapolation needed

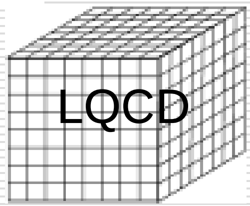
Subjects of lattice QCD

Hadron spectrum and Fundamental constanst of QCD

- Strong coupling constant
- Quark masses

α_s

$m_u, m_d, m_s, m_c, m_b, m_t$

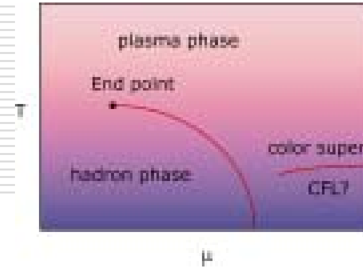


Hadron physics

- eta' meson mass and U(1) problem
- exotic states
  glueball, dibaryon, hybrids,...
- hadronic matrix elements
  proton spin, sigma term, ...
- structure functions/form factors

$$\int_0^1 dx\, x^{n-1} F(x,q^2) = (\ln q^2)^{-\gamma_n} \langle N | O_n | N \rangle$$

Finite-temperature/ density behavior

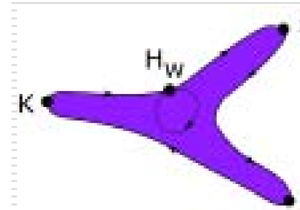


- order of transition
- critical temperature/density
- equation of state

Physics of quark-gluon plasma

Weak interaction matrix elements

- K meson amplitudes
  B_K
  $K \rightarrow \pi\pi$ decays



- B meson amplitudes
  f_B, B_B, form factors

CKM matrix and CP violation

Finite temperature/density QCD

- Status for $T>0$ and $\mu=0$
  - Expected phase diagram
  - Recent results
- Progress toward $\mu \neq 0$
  - Reweighting
  - Taylor expansion
  - Analytic continuation

Remarks

- Still mostly (improved) KS fermion action
- Still mostly Temporal size $N_t=4\text{-}8$

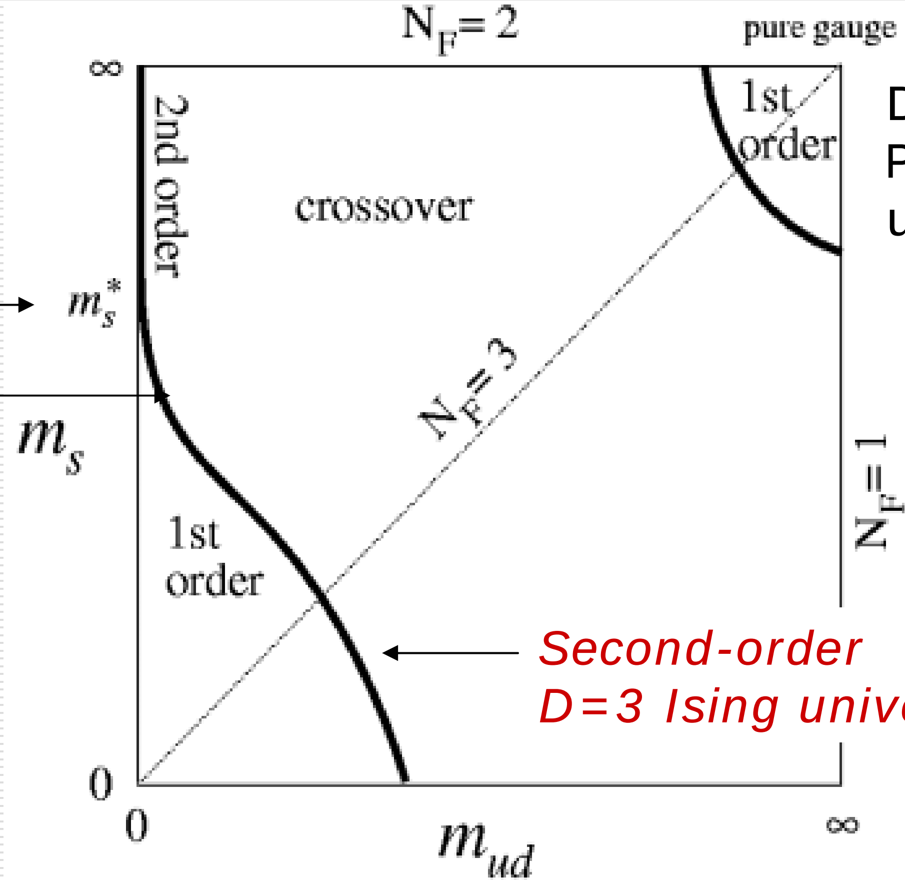
  i.e., coarse lattice $a^{-1} \approx 0.6 - 1.2 GeV$

Phase diagram expected at μ=0

$N_f = 2+1\,QCD$

Tricritical point →

$m_{ud} \propto \left(m_s^* - m_s\right)^{5/2}$



D=3 Z(3) Potts universality

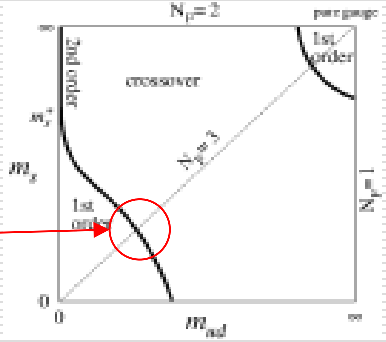
Second-order
D=3 Ising universality

Where is the physical point?

Nature of the 2nd order endline

- Existence of the endline well established

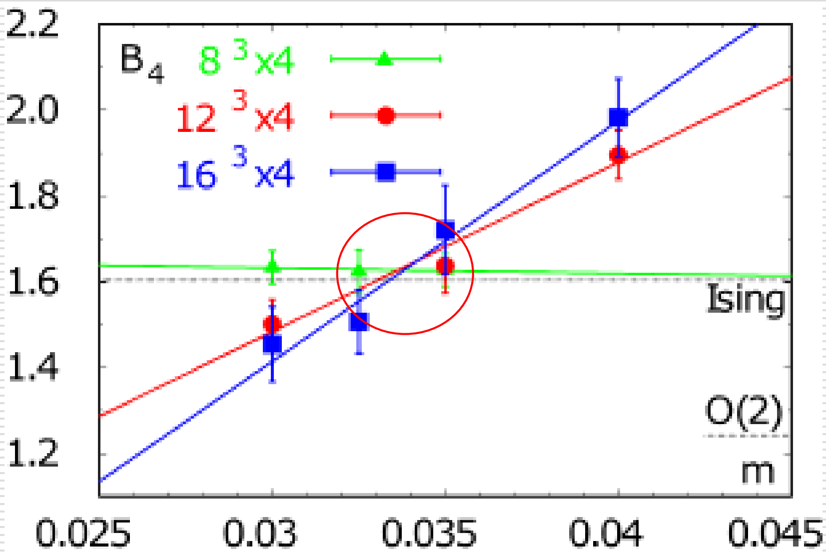
  JLQCD/Bielefeld/Columbia
- Binder cumulant test to distinguish universality class



$$B_4 = \frac{\left\langle \left(\delta\overline{\psi}\psi\right)^4 \right\rangle}{\left\langle \left(\delta\overline{\psi}\psi\right)^2 \right\rangle^2}$$

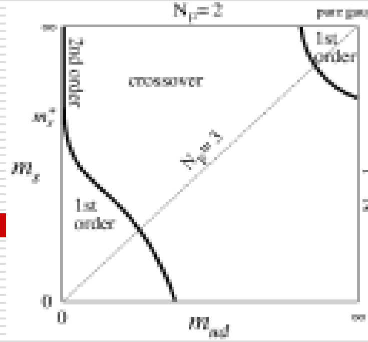
- Clear evidence of Ising universality as predicted by S. Gavin et al

S. Gavin et al hep-ph/9311350

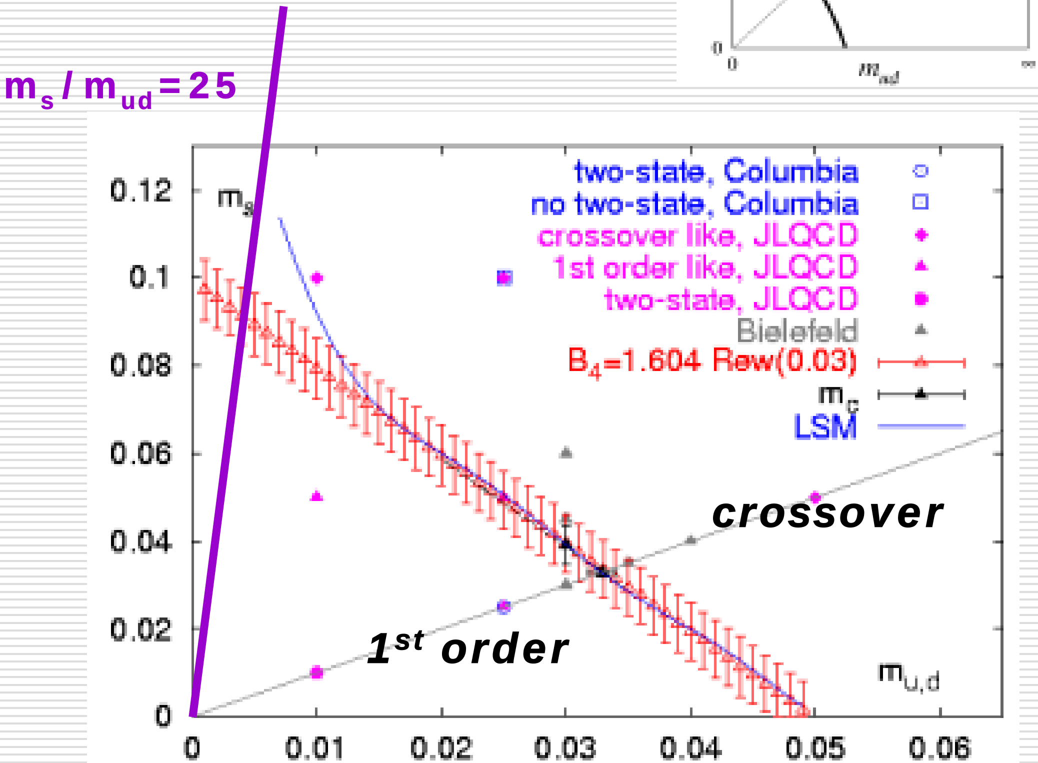


Karsch et al hep-lat/0107020

Location of the physical point



- Previous work by JLQCD/Columbia
- Recent work by
  - Schmidt(Bielefeld -Swansea)
  - Christ(Columbia)



Ch. Schmidt et al hep-lat/020900

→ Cross-over at the physical point indicated with the KS fermion simulations

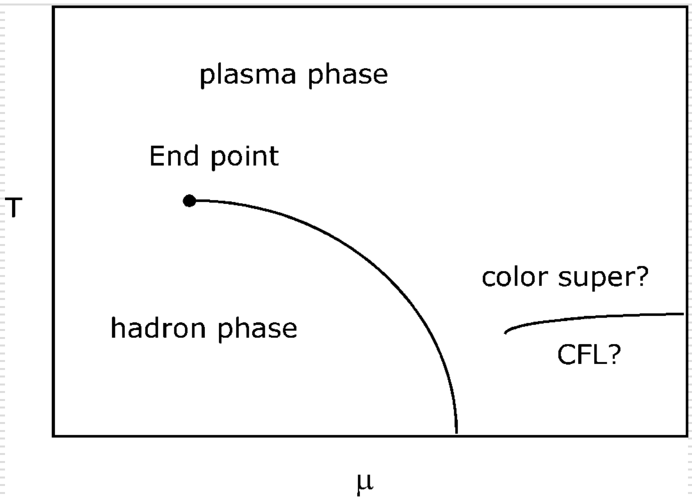
NB first-order with Wilson fermion;old controversy still remains

Progress with finite chemical potential

- Difficulties with finite density:
  - Monte Carlo methods fail for comlex quark determinant for $\mu \neq 0$
- New developments:
  - Reweighting method to move from $\mu=0$ to $\mu \neq 0$
    Butapest (Fodor et al)
  - Taylor expansion around $\mu=0$
    Bielefeld-Swansea
  - analytic continuation from Im$\mu \neq 0$ to Re$\mu \neq 0$
    Forcrand et al/Lombardo et al

$$Z_{QCD} = \int dU \det D(\mu) e^{-\beta S_{gluon}}$$

Schematic phase diagram
(assuming cross-over at T=0)



Reweighting in chemical potential μ

- Fodor-Katz strategy

  Z.Fodor et al hep-lat/0104001

  - Reweight in β and μ such that width of ω is minimal
  - Turned out to work for small volume;

    $$\mu a \leq \left(N_t \cdot N_s^3\right)^{-1/4}$$

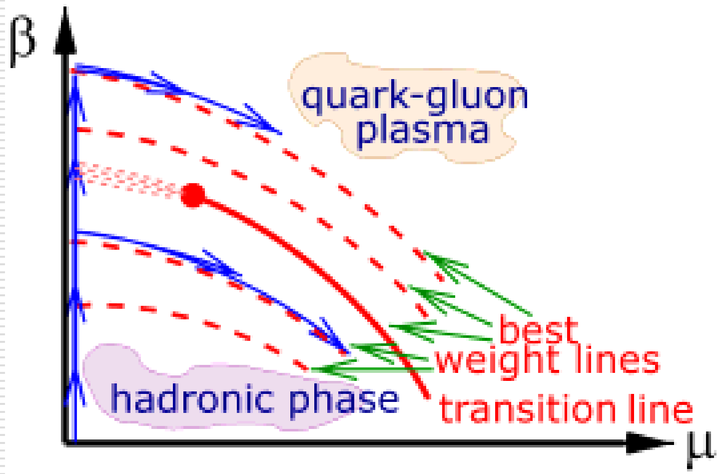
  - Use Lee-Yang zero analysis to locate the end-point E

complex

$$\det D(\mu) e^{-\beta S_{gluon}}$$

$$= \det D(\mu=0) e^{-\beta_0 S_{gluon}} \cdot \underbrace{\frac{\det D(\mu)}{\det D(\mu=0)} e^{-(\beta-\beta_0) S_{gauge}}}_{\omega}$$

real

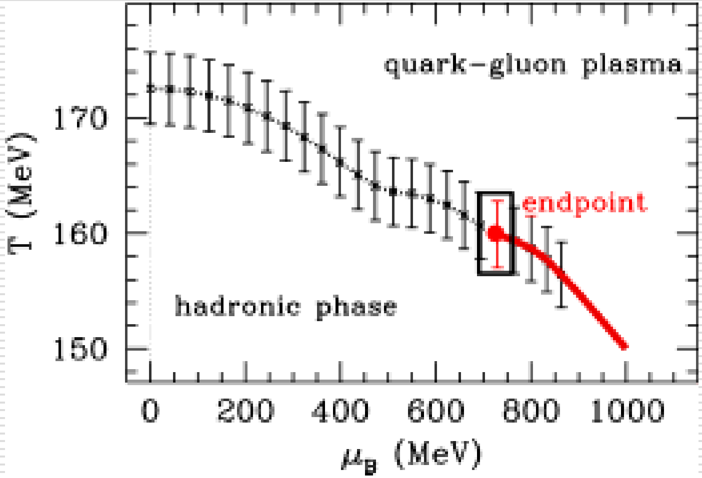


From Fodor et al hep-lat/0208078

Results

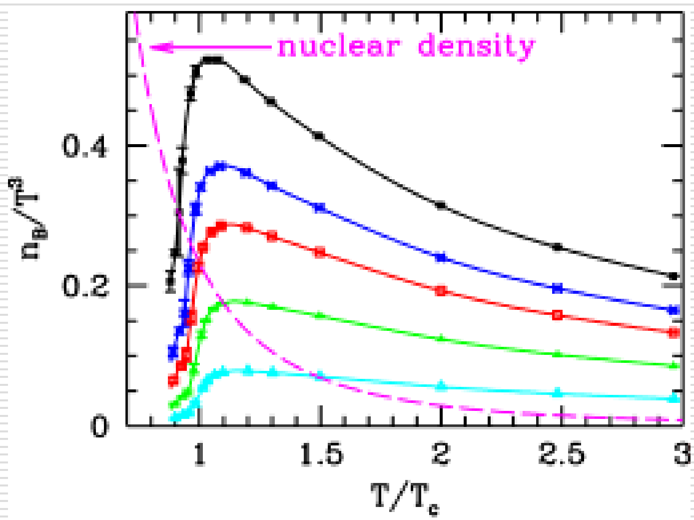
- End point:
  - T_E = 160+/-3.5 MeV
  - μ_E = 725+/-35 MeV
  - *NF*=2+1
  - [m_{ud}=0.025, m_s=0.2]
  - $(4, 6, 8)^3$x4

Z. Fodor et al hep-lat/0106002



- Equation of state
  - Pressure p
  - Energy density e
  - *NF*=2+1
  - [m_{ud}=0.025, m_s=0.2]
  - $(8, 10, 12)^3$x4

Z. Fodor et al hep-lat/0208078



Taylor expansion in chemical potential μ

S. Ejiri et al heo-lat/0209012
C. R. Allton et al hep-lat/0204010

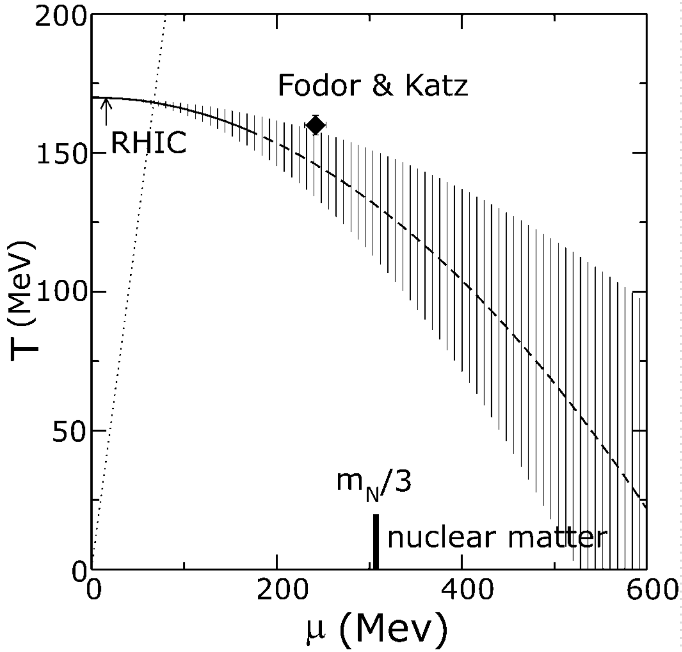
- Taylor expansion should converge up to the endpoint

$$Tr \ln D(\mu) = Tr \ln D(\mu = 0) + Tr D(\mu = 0)^{-1} \frac{\partial D}{\partial \mu}(\mu = 0) \cdot \mu + \cdots$$

- Calculate

$$\left.\frac{d^2 \beta_c(\mu)}{d\mu^2}\right|_{\mu=0} \Rightarrow \left.\frac{d^2 T_c(\mu)}{d\mu^2}\right|_{\mu=0}$$

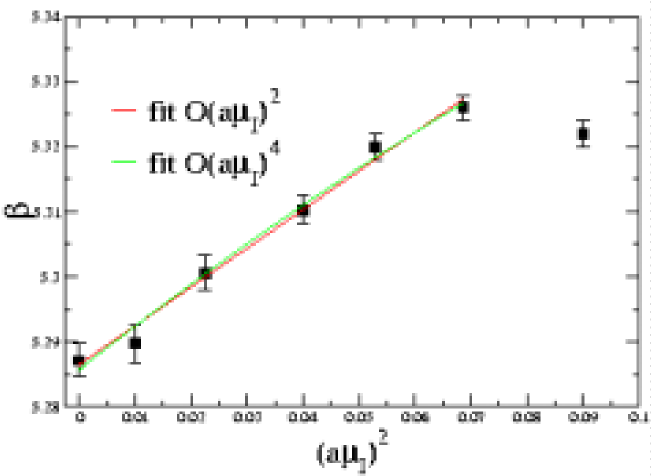
- Simulation
  - P4-improved KS
  - $N_f=2$ [m_{ud}=0.01, 0.02]
  - 16^3x4



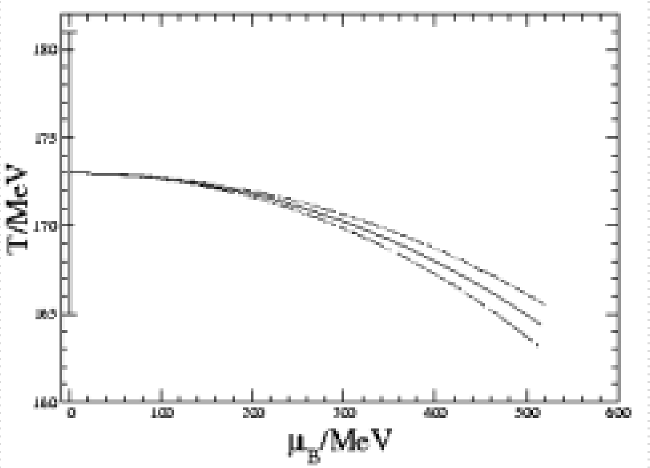
Analytic continuation from Imaginary to Real μ

- Determinant real for Imaginary μ, hence amenable to Monte Carlo
- Fit obervables in polynomials of μ
- Analytically continue in μ

M. Alford et al hep-lat/9807039
Ph. Forcrand et al hep-lat/020501
M. D'Elia et al hep-lat/0209146



$$i\mu_I \Rightarrow \mu$$



$$\beta_c(\mu) = \beta_c(0) + c(a\mu_I)^2$$

$$\beta_c(\mu) = \beta_c(0) - c(a\mu)^2$$

From Ph. Forcrand et al hep-lat/0205016

Future direction of lattice QCD

- From 2-flavor QCD to 3-flavor QCD
  - Dynamical treatment of all light quarks (u, d, s)
  - "Light" light quarks
  - Non-perturbative improvement coefficients and renormalization factors

Polynomial HMC algorithm

$$\frac{m_\pi}{m_\rho} \approx 0.6 \left(m_\pi \approx 500MeV\right)$$

$$\Rightarrow \frac{m_\pi}{m_\rho} \approx 0.4 \left(m_\pi \approx 300MeV\right)$$

$O(a^2)$ continuum extrapolation

- From non-chiral to chiral action for quark
  - Domain-wall/overlap/perfect actions

Truly realistic and exact simulation of QCD

Scale of QCD simulations

- Typical lattice size
  - Quenched QCD 64^3x112
  - 2-flavor Full QCD 24^3x48
- Total CPU time with CP-PACS
  - 0.6Tflops peak
  - 53% of peak for quenched QCD (0.32Tflops effective)
  - 34% of peak for 2-flavor full QCD (0.20Tflops effective)
  - Quenched QCD 199 days of full machine
  - 2-flavor full QCD 415 days of full machine
  - K decay 180 days of full machine
- Scaling law for 2-flavor QCD

$$\#FLOP's = C \cdot \left[\frac{\#conf}{1000}\right] \cdot \left[\frac{m_\pi / m_\rho}{0.6}\right]^{-6} \cdot \left[\frac{L}{3\,fm}\right]^{5} \cdot \left[\frac{1/a}{2GeV}\right]^{7} Tflops \cdot year$$

$$C \approx 2.8$$

Prospects with computers

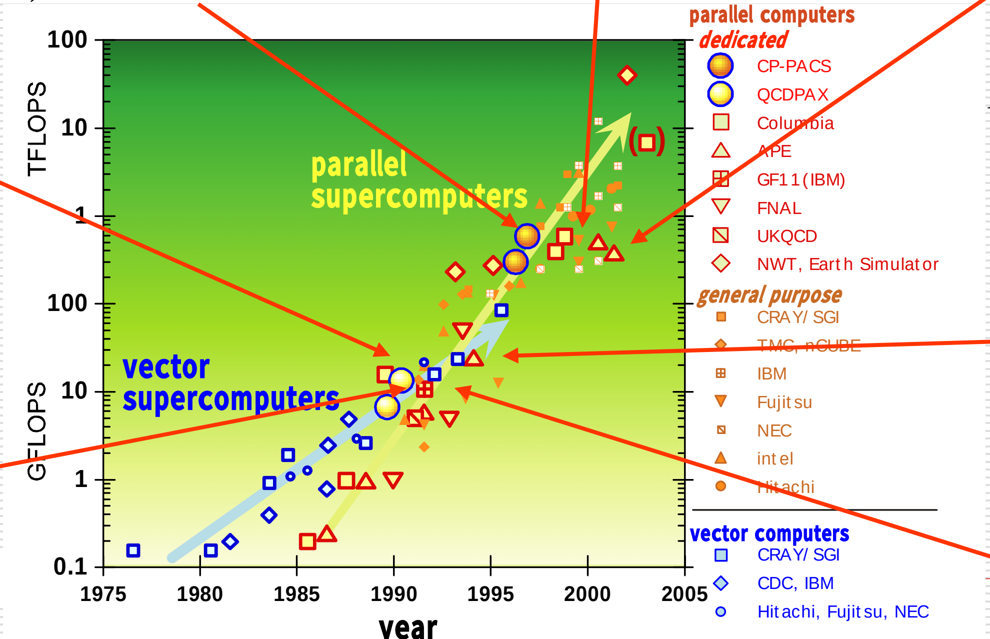
year	machine	peak
88-90	Columbia	16 GFLOPS
89-90	QCDPAX	14 GFLOPS
91	GF11	11 GFLOPS
88-94	APE / APE-100	25 GFLOPS
89-93	ACPMAPS	50 GFLOPS
96	CP-PACS	614GFLOPS
98-99	QCDSP	410, 600
00-01	APEmille	520 GFLOPS
03?	QCDOC	10TFLOPS?
03?	apeNEXT	10TFOPLS?

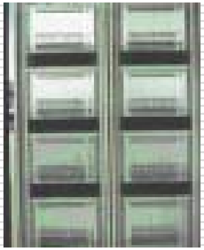
CP-PACS (1996) 614 GFLOPS

QCDSP (1998) 600 GFLOPS

APEmille (2000) 520 FLOPS

lumbia (1990) 16 GFLOPS





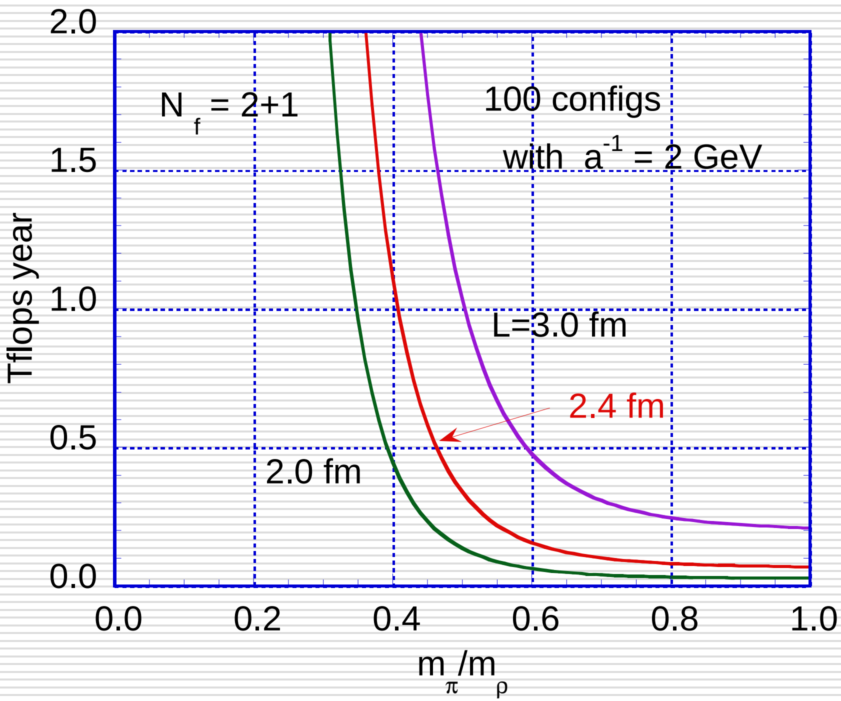
APE-100 (1994) 25FLOPS

CDPAX (1990) 14 GFLOPS

GF11 (1991) 11 GFLOPS

Prospects toward 3-flavor simulations

- Polynomial HMC algorithm
- Assumption for Scaling law for 3-flavor QCD
  - FLOPS=1.5*(2flavor case)
- O(5-10)Tflops computer needed for L=2.4fm simulations



QCDOC/apeNEXT (fall 2003) machines well suited for the job

Worldwide prospects

- Regional developments and competition
  - Asia-Pacific Mini-Workshop on lattice QCD (23-24 Jan 03)
    China/Taiwan/Korea/Australia/Japan/
- International collaboration
  - Sharing of resources
    - International Workshop on Lattice Data Grid (19-20 Dec 02)
  - Exchange of people

Conclusions

- *Visible shift from quenched to full QCD simulations with dynamical quarks*
  - *Important effects observed in physical observables*
  - *Crucial for consistent predictions from lattice QCD*
- *Development of lattice fermion with exact chiral symmetry*
  - *Both conceptual and practical advantages*
  - *Need O(10) times more computer power; awaits next generation of computers for full QCD*
- *Notable progress in*
  - *Study of finite chemical potential*
- *Require further effort to understand*
  - *K meson decays*

Prospects

- Full QCD simulations with dynamical up, down and strange quark
  - Already underway with staggered quark action
  - Simulations with Wilson quark action will follow

- *Definitive prospect toward exact QCD predictions with realistic quark spectrum over the next few years*
  - *Firm numbers to our phenomenology/experiment colleagues*
  - *Quantitative understanding of the full range of strong interactions*

1935 meson theory (Yukawa)
1951 strangeness (Gell-Mann-Nishijima)
1961 chiral symmetry and pion(Nambu-Jona-Lasinio)
1973 QCD and asymptotic freedom(Gross-Wilczek-Politzer)
1974 Lattice QCD(Wilson)
1981 Monte Carlo simulation of QCD(Creutz-Jacobs-Rebbi)