Present and Future of Lattice QCD

Akira Ukawa
Center for Computational Physics
University of Tsukuba

- overview
- hadron spectrum and fundamental constants of QCD
- topics in hadron physics
- issues and progress with chiral symmetry
- weak interactions of hadrons
- QCD thermodynamics
- conclusions and prospects
Center for computational Physics
University of Tsukuba

- Founded in 1992
- Emphasis on
  - Development of HPC systems suitable for computational physics
  - Close collaboration of physicists and computer scientists

- Computing facility
  - CP-PACS parallel system
    - MPP with 2048PU/0.6Tflops peak
    - Developed at the Center with Hitachi Ltd.
    - #1 of Top500-November 1996
  - GRAPE-6 system
    - Dedicated to gravity calculations
    - Developed at U. Tokyo
    - 8Tflops equivalent
Quantum Chromodynamics

- Quantum Chromodynamics (QCD)
  - Fundamental theory of quarks and gluons and their strong interactions

\[ S_{\text{QCD}} = \frac{1}{8\pi\alpha_s} \text{Tr}(F_{\mu\nu}F_{\mu\nu}) + \sum_f \bar{\psi}_f (\gamma_\mu (\partial_\mu - iA_\mu) + m_f)\psi_f \]

- Knowing

\[ \langle O(A,\bar{\psi},\psi) \rangle = \frac{1}{Z} \int dA d\bar{\psi} d\psi O(A,\bar{\psi},\psi) e^{-S} \]

1 coupling constant
and
6 quark masses

\[ m_u, m_d, m_s, m_c, m_b, m_t \]

will allow full understanding of strong interactions
“Yukawa’s dream (1935) in modern form”
QCD on a Lattice

- **Lattice QCD**
  - Powerful mean to calculate the QCD Feyman path integral
    \[ S_{QCD} = \frac{1}{\alpha_s} \sum_p \text{tr}(UUUU) + \sum_f \bar{\psi}_f \left( \gamma \cdot U + m_f \right) \psi_f \]
    \[ \langle O(U, \bar{\psi}, \psi) \rangle = \frac{1}{Z} \int dU d\bar{\psi} d\psi O(U, \bar{\psi}, \psi) e^{-s} \]

- **From computational point of view**
  - Relatively simple calculation
    - Uniform mesh
    - Single scale
  - Requires much computing power due to
    - 4-dim. Problem
    - Fermions essential
    - Physics is at lattice spacing a=0
  - Precision required(<a few % error in many cases)
Subjects of lattice QCD

Hadron spectrum and Fundamental constants of QCD
- Strong coupling constant
- Quark masses
  \[ \alpha_s, m_u, m_d, m_s, m_c, m_b, m_t \]

Finite-temperature/density behavior
- Order of transition
- Critical temperature/density
- Equation of state

Physics of quark-gluon plasma

Hadron physics
- \( \eta' \) meson mass and U(1) problem
- Exotic states: glueball, dibaryon, hybrids, ...
- Hadronic matrix elements: proton spin, sigma term, ...
- Structure functions/form factors

\[ \int_0^1 dx x^{n-1} F(x, q^2) = (\ln q^2)^{-\gamma_E} \langle N | \mathcal{O}_n | N \rangle \]

Weak interaction matrix elements
- \( B_K \) amplitudes
- \( K \to \pi \pi \) decays
- \( B \) meson amplitudes
  - \( f_B, B_B \), form factors

CKM matrix and CP violation
Physics subjects of this talk

- Hadron spectrum and fundamental constants of QCD
  - Fundamental verification of QCD at low energies
  - Determination of quark mass and strong coupling constant
- Topics in hadron physics
  - Nucleon structure functions
- Issues and progress with chiral symmetry
  - Chiral extrapolation of observables
  - Chirally symmetric fermion formulations
- Weak amplitudes of hadrons
  - K meson decays and CP violation
  - K and B meson amplitudes and constraints on the CKM matrix
- Finite temperature/density QCD
  - Status at zero density
  - Progress toward non-zero density
Light hadron mass spectrum

- Benchmark calculation to verify QCD
- Indispensable for determination of QCD scale and quark masses
- Essential to control various systematic errors down to a few % level
  - Finite lattice size \( L > 3 \text{fm} \)
  - Finite quark mass \( m_q \not= 0 \)
  - Finite lattice spacing \( a \not= 0 \)

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![Experimental spectrum]
CP-PACS result for the quenched spectrum’98

- Sea quark effects ignored

- General pattern reproduced, but clear systematic deviation beyond 10% precision

- Completes the calculation started in ’81 (Hamber-Parisi/Weingarten)

\[ \text{use } m_\pi, m_\rho \text{ for fixing } a \text{ and } m_{ud} \]
\[ \text{use } m_K \text{ for fixing } m_s \]

Calculated quenched spectrum
Input dependence in quenched QCD

- Details of disagreement depends on input, but overall agreement not possible
  
  *use $m_K$ for fixing $m_s$*
  *use $m_\phi$ for fixing $m_s$*

- Predictions in quenched QCD suffer from uncertainties depending on input

Calculated quenched spectrum
Strange quark mass in quenched QCD

- 25% discrepancy in the predicted value of strange quark mass

- Clearly illustrates
  - limitation of quenched QCD
  - necessity of full QCD with dynamical quarks

\[ m_s(MS, 2GeV) = \begin{cases} 142_{-6}^{+28} & \text{MeV, } \phi \text{ meson mass input} \\ 114_{-6}^{+8} & \text{MeV, } K \text{ meson mass input} \end{cases} \]
QCD simulation with dynamical quarks

- Spectrum of quarks
  - 3 light quarks (u,d,s) $m < 1\text{GeV}$
  - Need dynamical simulation
  - 3 heavy quarks (c,b,t) $m > 1\text{GeV}$
  - Quenching sufficient

- Dynamical quark simulation (full QCD)
  - Costs 100-1000 times more computing power
  - Algorithm for odd number of quarks now available

- Two-flavor full QCD (since around 1996) $N_f = 2$
  - u and d quark dynamical simulation
  - s quark quenched approximation
  - Number of studies: SESAM/UKQCD/MILC/CP-PACS/JLQCD

- Three-flavor full QCD (since around 2000) $N_f = 2 + 1$
  - s quark also treated dynamically
  - Extensive studies have begun: MILC/CP-PACS-JLQCD
Sea quark effects in the spectrum

- K*-K and f0-K mass difference (Meson hyperfine splitting)
  - too small in quenched QCD
  - Much closer agreement for two-flavor full QCD
Sea quark effects in quark masses

- Significant decrease by inclusion of sea quark effects
- Input dependence of strange quark mass also reduced

up&down quark  Quenched value  strange quark

Two-flavor full QCD  K-input
**Light quark masses (u, d, s) $m_q^{\overline{MS}}(2\text{GeV})$**

- **Significant sea quark effects**
  - Large uncertainty (~20%) depending on input in quenched theory
  - Sizable decrease (~25%) from quenched to two-flavor full QCD

- Lighter than naïve quark model values

- Nf=3 simulations being pursued to obtain physical values of light quark masses, e.g., Hein et al hep-lat/0209077

**CP-PACS Collab. Hep-lat/0004010**

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Real world; three flavors?
Heavy quark masses (c, b)

- **Charm quark mass**
  - J. Rolf and S. Sint  Lattice01
  - Fully non-perturbative determination in the continuum in quenched QCD
  
  \[ m_c^{\overline{MS}}(m_c) = 1.314(45)\text{GeV} \]

- **Bottom quark mass**
  - Not straightforward since  \( m_b > 1/a \)
  - Use
    
    \[ M_B = m_b^{\text{pole}} + \left( \frac{E_B}{\delta m} \right) \]
    
    \( M_B, E_B \) Monte Carlo, \( \delta m \) pert. theory
  
  - G. Gimenez et al (’00)  \( m_b^{\overline{MS}}(m_b) = 4.27(9)\text{GeV} \)
Strong coupling constant $\alpha_s(\mu)$

- Another fundamental parameter of QCD
- Large number of high energy determinations from experiments
- Lattice determinations:
  - Method I (Lepage et al ’91):
    - Calculate short-distance observables as a function of $\alpha_s(1/a)$ at cutoff scale 1/a
    - the scale 1/a is fixed from hadron mass
  - Method II (Luescher et al ’93)
    - Non-perturbative determination of the RG evolution by Schrodinger functional finite-size technique
Scale dependence of $\alpha_s(\mu)$

- Non-perturbative determination by Alpha Collaboration for two-flavor full QCD (Lattice02)
- Indicates perturbative evolution for $p > a$ few GeV
- Similar results for quenched QCD ('94)
- Physical scale yet to be determined

M. Della Morte et al hep-lat/0209023
Determination of $\alpha_s^{\overline{MS}}(M_Z)^{N_f=5}$

**Comments**

- Davies et al '97(hep-lat/9703010):
  Involved extrapolation of $N_f=0$(quenched) and $N_f=2$ data to $N_f=3$

- QCDSF-UKQCD '01(hep-lat/0103023)
  Continuum estimate with systematic $N_f=2$ simulations

- Davies et al '02(hep-lat/0209121):
  Preliminary result based on MILC $N_f=3$ configurations at $a=0.13\text{fm}$

**Systematic $N_f=3$ full QCD determination expected in a few years**
Topics in hadron physics

- Progress in hadron spectroscopy
  - Eta’ meson mass and U(1) problem
  - Glueballs
  - Multiquark states
  - Excited string states etc

- Hadron structure
  - Moments of nucleon structure functions
  - Form factors

- Hadron scattering amplitudes
  - Scattering length
  - Phase shift $I=2 \, \pi\pi$ channel Ishizuka et al. 2002
Moments of nucleon structure functions (I)

\[
\int_0^1 dx x^{n-1} F(x, q^2) = (\ln q^2)^{-\gamma_n} \langle N | O_n | N \rangle \quad \langle x \rangle_{u-d}
\]

- A number of calculations
  - QCDSF (quenched)'96
  - Alpha(quenched)'98
  - QCDSF+UKQCD (full)'01-'02
  - LHPC+SESAM (full)'02

- No large sea quark effects observed for non-singlet moments

- No large scaling violation

- Lattice predictions do not agree with experiment

Compiled in M. Goeckeler et al hep-lat/0209160
Moments of nucleon structure functions (II)

- **Linear chiral extrapolation misses experiment**

- **Possible reasons:**
  - Quenching? No
  - O(a) error? No
  - Chiral extrapolation itself?

- **Chiral perturbation theory:**

\[
\langle x^n \rangle = a_n \left( 1 - \frac{3g_A^2 + 1}{(4\pi f)^2} m^2 \ln \frac{m^2}{\Lambda^2} + cm^2 \right)
\]

- Pion not light enough to see curvature?
- An effective model can reproduce experiment

\[
\begin{pmatrix}
\langle x \rangle_{u-d}
\end{pmatrix}
\]

From D. Dolgov et al hep-lat/0201021

\[
\begin{pmatrix}
W. Detmold et al hep-lat/0103006
\end{pmatrix}
\]
Moments of nucleon structure functions (II)

- **Linear chiral extrapolation misses experiment**
- Possible reasons:
  - Quenching? No
  - O(a) error? No
  - Chiral extrapolation itself?
- **Chiral perturbation theory:**
  \[ \langle x^n \rangle = a_n \left(1 - \frac{3g_A^2 + 1}{(4\pi f)^2} m_\pi^2 \ln \frac{m_\pi^2}{\Lambda_\chi^2} + cm_\pi^2 \ldots \right) \]
  - Pion not light enough to see curvature?
  - An effective model can reproduce experiment

From D. Dolgov et al hep-lat/0201021

\[ \langle x \rangle_{u-d} \]

\[ m_\pi^2 \]

W. Detmold et al hep-lat/0103006
A general issue with chiral extrapolation

- Current lattice data often fails to see logarithmic singularity expected from chiral perturbation theory
- Often causes sizable (10-20%) uncertainties in the extrapolated result
- Pion mass in current simulations (~500MeV) too heavy; needs to be reduced
- Lattice fermion action with exact chiral symmetry much desired (conventional Wilson and KS action breaks chiral symmetry)
Lattice fermion with exact chiral symmetry

- Theoretically based on the Ginsparg-Wilson relation:
  \[ D\gamma_5 + \gamma_5 D = 2aDR\gamma_5 D \]
  - Domain-wall fermion Kaplan('92)/Furman-Shamir('94)
  - Overlap formalism Neuberger-Narayanan('92,'97)
  - Fixed point action Hasenfratz-Neidermyer('94)

- Avoids the Nieslen-Ninomiya Theorem by using “infinitely” many fields (hence needs more computer power)

- Quenched calculations show very promising results: good chiral property, small scaling violation, …
A test in quenched QCD

- Chiral logarithm behavior of pion mass in quenched QCD
  Sharpe/Bernard-Golterman ’91

\[ m_\pi^2 = A m_0 \left(1 - \delta \ln m_0 + b m_0 + \cdots \right) \]

- Nice confirmation with the new fermion formalism
  - T. Draper et al : overlap fermion
  - C. Gattringer et al : fixed point fermion

- Reached very light pion mass
  - \( m_\pi \sim 170 \text{MeV} \) (T. Draper et al)
  - Similar results from other chiral formalisms

\[ \delta = 0.26(3) \]

T. Draper et al hep-lat/0208045
Subjects of lattice QCD

Hadron spectrum and Fundamental constanst of QCD

- Strong coupling constant
- Quark masses
  \( \alpha_s \)

\( m_u, m_d, m_s, m_c, m_b, m_t \)

Finite-temperature/ density behavior

- order of transition
- critical temperature/density
- equation of state

Physics of quark-gluon plasma

Weak interaction matrix elements

- K meson amplitudes
  \( B_K \)
  \( K \to \pi \pi \) decays
- B meson amplitudes
  \( f_B, B_B \), form factors

CKM matrix and CP violation

\[ \int_0^1 \! dx x^{n-1} F(x, q^2) = \left( \ln q^2 \right)^{\gamma_n} \langle N | \mathcal{O}_n | N \rangle \]
Weak amplitudes of hadrons

- First principles calculation of strong interaction corrections to weak amplitudes of hadrons
- Understand old and new issues in hadronic weak interactions
  - $I=1/2$ rule and direct CP violation in $K \to \pi\pi\pi$ decay
- Constraints on the CKM mixing matrix
  - Neutral K and B meson mixings
  - B meson decay form factors

$$\text{Measured weak amplitude} = \text{Known factor including CKM matrix} \times \langle h'|H_{weak}|h \rangle$$

QCD matrix element
I=1/2 rule and CP violation in K decays

- Weak interaction decays of K mesons
  \[ \frac{\text{Re} A_0(K \rightarrow \pi\pi (I = 0))}{\text{Re} A_2(K \rightarrow \pi\pi (I = 2))} \approx 22 \]
  - I=1/2 rule

- CP violation
  \[ \frac{\varepsilon'}{\varepsilon} = \begin{cases} (20.7 \pm 2.8) \times 10^{-3} & \text{KTeV experiment (FNAL)} \\ (15.3 \pm 2.6) \times 10^{-3} & \text{NA48 experiment (CERN)} \end{cases} \]

- Crucial numbers to verify the Standard Model understanding of CP violation

- Chiral symmetry crucial because of the chiral structure of weak interactions

- Two large-scale calculations using domain-wall QCD
  - RIKEN-BNL-Columbia: T. Blum et al. hep-lat/0110124
  - CP-PACS: J. Noaki et al. hep-lat/0108013
I=1/2 rule

- Reasonable agreement with experiment for I=2
- About half of experiment for I=0
- RIKEN-BNL-Columbia obtains a somewhat different result (smaller I=2 and larger I=0)
CP violation parameter $\epsilon'/\epsilon$

- Small and negative in disagreement with experiment
- Similar result from RIKEN-BNL-Columbia

Possible reasons
- connected with insufficient enhancement of $I=1/2$ rule
- Method of calculation (K $\rightarrow \pi$ reduction) may have serious problems

Still a big problem requiring further work
Constraints on the CKM matrix

\[ \text{Measured weak amplitude} = \text{Known factor including CKM matrix} \times \left\langle h' | H_{\text{weak}} | h \right\rangle \]

- **\( K^0 - \bar{K}^0 \) mixing**
  \[ \epsilon_K \propto \eta \left( (1 - \rho)A + B \right)^2 \hat{B}_K \]
  \[ \left\langle K^0 | \bar{s} \gamma_\mu (1 - \gamma_5) d \bar{s} \gamma_\mu (1 - \gamma_5) d | K^0 \right\rangle = \frac{8}{3} f_K^2 B_K M_K^2 \]
  \[ \rightarrow B_K \]

- **\( B_{d,s}^0 - \bar{B}_{d,s}^0 \) mixing**
  \[ \Delta M_{B_q} \propto \left( \rho^2 + \eta^2 \right) f_{B_q}^2 B_{B_q} \]
  \[ \left\langle B_q^0 | \bar{b} \gamma_\mu (1 - \gamma_5) q \bar{b} \gamma_\mu (1 - \gamma_5) q | B_q^0 \right\rangle = \frac{8}{3} f_{B_q}^2 B_{B_q} M_{B_q}^2 \]
  \[ \xi = \frac{f_{B_s}}{f_{B_d}} \sqrt{\frac{B_{B_s}}{B_{B_d}}} \]
  \[ \rightarrow B_B \quad f_B \]
Lattice results for $B_K$

- Previous best result obtained with conventional KS fermion action
- Recent Domain-wall results indicates a smaller value
- Full QCD calculation yet to be made with domain-wall fermion
- Current best estimate:

$$B_K = 0.628(42) - 0.532(11)$$

$\hat{B}_K = 0.87^{+0.06}_{-0.13}$

RG-invariant B parameter

CP-PACS hep-lat/0105020
RBC hep-lat/0110075
Full QCD results for $f_B$

- Two-flavor full QCD result begins to accumulate
- $f_{B_d}$: possibly large uncertainty due to chiral extrapolation
- Best estimate from two-flavor full QCD:
  
  $f_{B_d} = 198(30)^{+0}_{-34}\text{ MeV}$

  $f_{B_d} / f_{B_s} = 1.16(5)^{+24}_{-0}$

  N. Yamada at Lattice2002
Results for $B_B$

- Still mostly quenched (only one calculation in full QCD)
- Sea quark effects small
- Uncertainty with chiral extrapolation is small
- Current best estimate:

$$B_{B_d} = 1.33(12)$$

$$B_{B_d} / B_{B_s} = 1.00(3)$$

N. Yamada at Lattice2002
Summary of lattice results for CKM matrix

\[
\hat{B}_K = 0.87^{+0.06}_{-0.13}
\]
\[
f_{B_d} \sqrt{B_{B_d}} = 0.227(37)^{+0}_{-34} \text{GeV}
\]
\[
\xi = 1.16(5)^{+24}_{-0}
\]

Cf. numbers used in the figure left

\[
\hat{B}_K = 0.87^{+0.06}_{-0.13}
\]
\[
f_{B_d} \sqrt{B_{B_d}} = 0.227(28) \text{GeV}
\]
\[
\xi = 1.16^{+3}_{-5}
\]

Better control of chiral extrapolation needed

status 2002  http://www.ckmfitter.in2p3.fr/
Subjects of lattice QCD

Hadron spectrum and Fundamental constants of QCD

- Strong coupling constant
- Quark masses

\[ \alpha_s, m_u, m_d, m_s, m_c, m_b, m_t \]

Finite-temperature/density behavior

- order of transition
- critical temperature/density
- equation of state

Physics of quark-gluon plasma

Hadron physics

- eta’ meson mass and U(1) problem
- exotic states
  - glueball, dibaryon, hybrids, ...
- hadronic matrix elements
  - proton spin, sigma term, ...
- structure functions/form factors

\[ \int_0^1 dx x^{n-1} F(x, q^2) = \left( \ln q^2 \right)^{-n} \langle N | O_n | N \rangle \]

Weak interaction matrix elements

- K meson amplitudes
  - \( B_K \)
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CKM matrix and CP violation
Finite temperature/density QCD

- Status for $T>0$ and $\mu=0$
  - Expected phase diagram
  - Recent results

- Progress toward $\mu \neq 0$
  - Reweighting
  - Taylor expansion
  - Analytic continuation

Remarks
- Still mostly (improved) KS fermion action
- Still mostly Temporal size $N_t=4-8$
  i.e., coarse lattice $a^{-1} \approx 0.6-1.2 GeV$
Phase diagram expected at $\mu=0$

$N_f = 2 + 1 \, QCD$

Tricritical point

$m_{ud} \propto (m^*_s - m_s)^{5/2}$

D=3 Z(3) Potts universality

Second-order

D=3 Ising universality

Where is the physical point?
Nature of the 2\textsuperscript{nd} order endline

- Existence of the endline well established
  - JLQCD/Bielefeld/Columbia
- Binder cumulant test to distinguish universality class
  \[ B_4 = \frac{\langle (\delta \bar{\psi} \psi)^4 \rangle}{\langle (\delta \bar{\psi} \psi)^2 \rangle^2} \]
- Clear evidence of Ising universality as predicted by S. Gavin et al
  - S. Gavin et al hep-ph/9311350
  - Karsch et al hep-lat/0107020
Location of the physical point

- Previous work by JLQCD/Columbia
- Recent work by
  - Schmidt (Bielefeld - Swansea)
  - Christ (Columbia)

Cross-over at the physical point indicated with the KS fermion simulations

NB first-order with Wilson fermion; old controversy still remains

Ch. Schmidt et al hep-lat/0209009
Progress with finite chemical potential

- Difficulties with finite density:
  - Monte Carlo methods fail for complex quark determinant for \( \mu \neq 0 \)

- New developments:
  - Reweighting method to move from \( \mu=0 \) to \( \mu \neq 0 \)
    - Budapest (Fodor et al)
  - Taylor expansion around \( \mu=0 \)
    - Bielefeld-Swansea
  - Analytic continuation from \( \text{Im} \mu \neq 0 \) to \( \text{Re} \mu \neq 0 \)
    - Forcrand et al/Lombardo et al

Schematic phase diagram
(assuming cross-over at \( T=0 \))
Reweighting in chemical potential $\mu$

- Fodor-Katz strategy
  - Z. Fodor et al. hep-lat/0104001
  - Reweight in $\beta$ and $\mu$ such that width of $\omega$ is minimal
  - Turned out to work for small volume:
    $$\mu a \leq \left( N_t \cdot N_s^3 \right)^{-1/4}$$
  - Use Lee-Yang zero analysis to locate the end-point $E$

From Fodor et al. hep-lat/0208078
Results

- End point:
  \[ T_E = 160+/-3.5 \text{ MeV} \]
  \[ \mu_E = 725+/-35 \text{ MeV} \]
  \[ NF=2+1 \]
  \[ [m_{ud}=0.025, m_s=0.2] \]
  \[ (4, 6, 8)^3\times4 \]
  Z. Fodor et al hep-lat/0106002

- Equation of state
  Pressure \( p \)
  Energy density \( e \)
  \[ NF=2+1 \]
  \[ [m_{ud}=0.025, m_s=0.2] \]
  \[ (8, 10, 12)^3\times4 \]
  Z. Fodor et al hep-lat/0208078
Taylor expansion in chemical potential $\mu$

- Taylor expansion should converge up to the endpoint
  $$Tr \ln D(\mu) = Tr \ln D(\mu = 0) + Tr D(\mu = 0)^{-1} \frac{dD}{d\mu}(\mu = 0) \cdot \mu + \cdots$$

- Calculate
  $$\left. \frac{d^2 \beta_c(\mu)}{d\mu^2} \right|_{\mu=0} \Rightarrow \left. \frac{d^2 T_c(\mu)}{d\mu^2} \right|_{\mu=0}$$

- Simulation
  - P4-improved KS
  - $N_f=2$ [$m_{ud}=0.01, 0.02$]
  - $16^3 \times 4$
Analytic continuation from Imaginary to Real $\mu$

- Determinant real for Imaginary $\mu$, hence amenable to Monte Carlo
- Fit observables in polynomials of $\mu$
- Analytically continue in $\mu$

From Ph. Forcrand et al hep-lat/0205016
Future direction of lattice QCD

- From 2-flavor QCD to 3-flavor QCD
  - Dynamical treatment of all light quarks (u, d, s)
  - “Light” light quarks
  - Non-perturbative improvement coefficients and renormalization factors

- From non-chiral to chiral action for quark
  - Domain-wall/overlap/perfect actions

**Truly realistic and exact simulation of QCD**

- Polynomial HMC algorithm

\[
\frac{m_\pi}{m_\rho} \approx 0.6 \ (m_\pi \approx 500\text{MeV})
\]

\[
\Rightarrow \frac{m_\pi}{m_\rho} \approx 0.4 \ (m_\pi \approx 300\text{MeV})
\]
Scale of QCD simulations

- **Typical lattice size**
  - Quenched QCD  64^3x112
  - 2-flavor Full QCD  24^3x48

- **Total CPU time with CP-PACS**
  - 0.6Tflops peak
  - 53% of peak for quenched QCD (0.32Tflops effective)
  - 34% of peak for 2-flavor full QCD (0.20Tflops effective)
  - Quenched QCD  199 days of full machine
  - 2-flavor full QCD  415 days of full machine
  - K decay  180 days of full machine

- **Scaling law for 2-flavor QCD**

\[
#\text{FLOPS} = C \cdot \left[ \frac{\text{#conf}}{1000} \right] \cdot \left( \frac{m_\pi}{m_\rho} \right)^{-6} \cdot \left( \frac{L}{3 \text{fm}} \right)^5 \cdot \left( \frac{1/a}{2 \text{GeV}} \right)^7 \text{ Tflops} \cdot \text{year}
\]

\[C \approx 2.8\]
## Prospects with Computers

The table below summarizes the peak performance of various supercomputers and their corresponding years:

<table>
<thead>
<tr>
<th>Year</th>
<th>Machine</th>
<th>Peak Performances</th>
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<tbody>
<tr>
<td>88-90</td>
<td>Columbia</td>
<td>16 GFLOPS</td>
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<td>89-90</td>
<td>QCDPAX</td>
<td>14 GFLOPS</td>
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<td>91</td>
<td>GF11</td>
<td>11 GFLOPS</td>
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<td>88-94</td>
<td>APE / APE-100</td>
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<td>ACPMAPS</td>
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<td>CP-PACS</td>
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<td>410, 600</td>
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<td>APEmille</td>
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<td>10 TFLOPS?</td>
</tr>
<tr>
<td>03?</td>
<td>apeNEXT</td>
<td>10 TFLOPS?</td>
</tr>
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</table>

### Diagram

- **CP-PACS (1996)**: 614 GFLOPS
- **QCDPAX (1990)**: 14 GFLOPS
- **Columbia (1990)**: 16 GFLOPS
- **QCDSP (1998)**: 600 GFLOPS
- **APEmille (2000)**: 520 FLOPS
- **APE-100 (1994)**: 25 FLOPS
- **GF11 (1991)**: 11 GFLOPS

### Additional Notes
- Prospects with computers suggest an upward trend in computational power, indicating advancements in technology and parallel computing capabilities.
Prospects toward 3-flavor simulations

- Polynomial HMC algorithm

- Assumption for Scaling law for 3-flavor QCD
  - FLOPS = 1.5*(2-flavor case)

- $O(5-10) \text{Tflops}$ computer needed for $L=2.4 \text{fm}$ simulations

QCDOC/apeNEXT (fall 2003) machines well suited for the job
Worldwide prospects

- Regional developments and competition
  - Asia-Pacific Mini-Workshop on lattice QCD (23-24 Jan 03)
    China/Taiwan/Korea/Australia/Japan/

- International collaboration
  - Sharing of resources
    - International Workshop on Lattice Data Grid (19-20 Dec 02)
  - Exchange of people
Conclusions

- Visible shift from quenched to full QCD simulations with dynamical quarks
  - Important effects observed in physical observables
  - Crucial for consistent predictions from lattice QCD

- Development of lattice fermion with exact chiral symmetry
  - Both conceptual and practical advantages
  - Need O(10) times more computer power; awaits next generation of computers for full QCD

- Notable progress in
  - Study of finite chemical potential

- Require further effort to understand
  - $K$ meson decays
Prospects

- Full QCD simulations with dynamical up, down and strange quark
  - Already underway with staggered quark action
  - Simulations with Wilson quark action will follow

- Definitive prospect toward exact QCD predictions with realistic quark spectrum over the next few years
  - Firm numbers to our phenomenology/experiment colleagues
  - Quantitative understanding of the full range of strong interactions

1935 meson theory (Yukawa)
1951 strangeness (Gell-Mann-Nishijima)
1961 chiral symmetry and pion (Nambu-Jona-Lasinio)
1973 QCD and asymptotic freedom (Gross-Wilczek-Politzer)
1974 Lattice QCD (Wilson)
1981 Monte Carlo simulation of QCD (Creutz-Jacobs-Rebbi)