UTCCP-P-35
April 1998

Quantum Chromodynamics with Many Flavors*)

Y. Iwasaki, K. Kanaya, S. Kaya,* S. Sakai,† and T. Yoshié

Center for Computational Physics and Institute of Physics,
University of Tsukuba, Tsukuba, Ibaraki 305-8577, Japan

**Institute of Particle and Nuclear Studies,*
High Energy Accelerator Research Organization (KEK),
Tsukuba, Ibaraki 305-0801, Japan

†*Faculty of Education, Yamagata University,*
Yamagata 990-8560, Japan

Abstract

We investigate the phase structure of lattice QCD for general number of flavors N_F. Based on numerical data combined with the results of the perturbation theory we propose the following picture: When $N_F \geq 17$, there is only one IR fixed point at vanishing gauge coupling, i.e., the theory in the continuum limit is trivial. On the other hand, when $16 \geq N_F \geq 7$, there is a non-trivial fixed point. Therefore, the theory is non-trivial with anomalous dimensions, however, without quark confinement. Theories which satisfy both quark confinement and spontaneous chiral symmetry breaking in the continuum limit exist only for $N_F \leq 6$.

*) Talk presented by K. Kanaya at the 1997 Yukawa International Seminar (YKIS'97) on "Non-Perturbative QCD — Structure of the QCD Vacuum —", YITP, Kyoto, Japan, 2–12 Dec. 1997. To be published in the proceedings [Prog. Theor. Phys. Suppl.].

§1. Introduction

At low temperatures, QCD with a small number of light quarks is known to have three characteristic properties of asymptotic freedom, quark confinement and spontaneous breakdown of chiral symmetry. Among them, asymptotic freedom is reliably investigated by the perturbation theory. It is well-known that when the number of flavors N_F exceeds $16\frac{1}{2}$, the asymptotic freedom is lost. Then, a question which naturally arises is whether there is a constraint on N_F for quark confinement and/or the spontaneous breakdown of chiral symmetry. Because these properties are essentially non-perturbative in nature, we have to carry out a non-perturbative investigation.

In this paper, we study the N_F dependence of the QCD vacuum by numerical simulations on the lattice. Because the Wilson fermion formalism is the only formalism for lattice quarks that preserves manifest flavor symmetry, we use the Wilson fermion formalism in our study of N_F dependence in QCD. Based on the numerical data obtained for N_F from 2 up to 300, we elucidate the phase structure of QCD as a function of N_F. We conjecture the following picture: When $N_F \geq 17$, there is only one IR fixed point at vanishing gauge coupling. Therefore, the theory in the continuum limit is trivial. On the other hand, when $16 \geq N_F \geq 7$, there is a non-trivial IR fixed point. Therefore, for this case, the theory is non-trivial with anomalous dimensions, however, without quark confinement. Theories which satisfy both quark confinement and spontaneous chiral symmetry breaking in the continuum limit exist only for $N_F \leq 6$. Our results are in part reported in Refs. 1) and 2).

This paper is organized as follows. Before going into details of our results obtained from numerical simulations, in Sec. 2, we first summarize results and speculations from a perturbative study of the renormalization group beta function, and then present our conjecture for the N_F dependence of the beta function, based on our numerical data obtained on the lattice. In Sec. 3, our lattice model and simulation parameters are described. In order to investigate the N_F dependence of the QCD vacuum in the continuum limit, we study the phase structure of lattice QCD at zero temperature for general number of flavors, in particular, for $N_F \geq 7$. When the phase diagram becomes clear, we are able to see the nature of the QCD vacuum in the continuum limit, and eventually answer the question about the condition on N_F for confinement and spontaneous breakdown of the chiral symmetry. Results for the phase structure in the strong coupling limit are given in Sec. 4. Sec. 5 is devoted to the phase structure off the strong coupling limit. We then discuss the nature of the deconfining phase in Sec. 6. Our conclusions and conjectures concerning the phase structure of QCD for general number of flavors are summarized in Sec. 7.

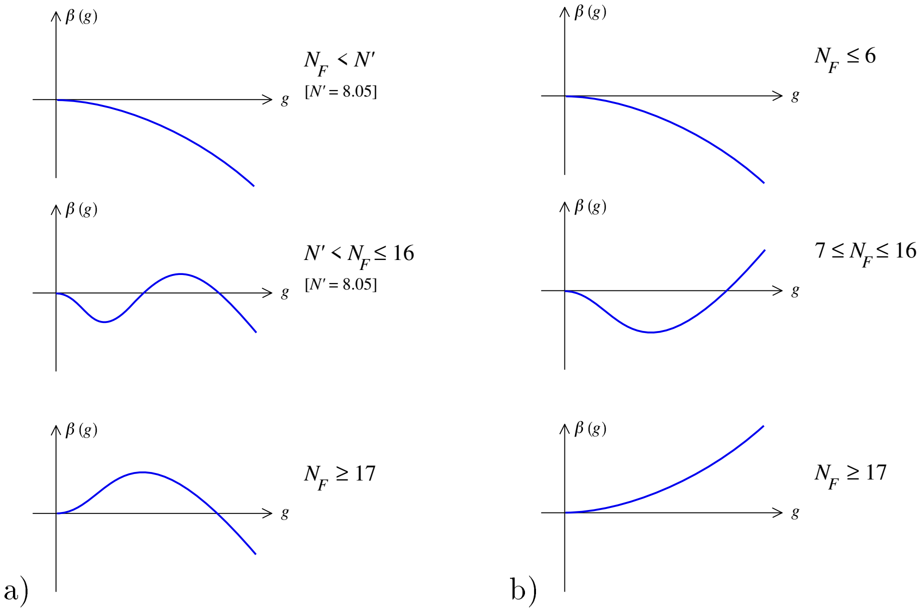

Fig. 1. Renormalization group beta function. (a) Conjecture by Banks and Zaks [3)] assuming confinement in the strong coupling limit for all N_F. (b) Our conjecture deduced from the results of lattice simulations.

§2. Beta function

The perturbative beta function of QCD is universal up to two-loop order: $\tilde{\beta}(g) = -b_0 g^3 - b_1 g^5 + \cdots$.where $b_0 = \frac{1}{16\pi^2}\left(11 - \frac{2}{3}N_F\right)$ and $b_1 = \frac{1}{(16\pi^2)^2}\left(102 - \frac{38}{3}N_F\right)$. The coefficient b_0 changes its sign at $N_F = 16\frac{1}{2}$. Therefore, for $N_F \geq 17$, asymptotic freedom is lost and the IR limit is free, i.e. we have no confinement. It is well-known that the second coefficient b_1 changes its sign already at $N_F \approx 8.05$. The values of the two coefficients imply that for $N_F \sim 9$, the IR fixed point appears at $g \sim 3$. However, this value of the gauge coupling constant is too large to trust the result from the perturbation theory. In contrast to this case, for $N_F \sim 16$, the IR fixed point appears in a perturbative region. Therefore, it is plausible that the beta function has a non-trivial IR fixed point for $N' \leq N_F \leq 16$ with some $N' \leq 16$. When such an IR fixed point exists, the coupling constant cannot become arbitrarily large in the IR region — this implies that quarks are not confined.

A pioneering study on the N_F dependence of the QCD vacuum was made by Banks and Zaks in 1982 [3)]. Based on an early result obtained on the lattice [8)], they assumed that the beta function is negative in the strong coupling limit for all N_F. Using the perturbative results for $N' \leq N_F \leq 16$ mentioned above, they conjectured Fig. 1(a) as the simplest N_F dependence of the beta function, and studied the phase structure of QCD based on this N_F dependence of the beta function.[*)] Because of an additional non-trivial UV fixed point for

[*)] For different approaches, see Refs. 4),5) and 6). When the quarks are confined, the sigma model might

$N_F \geq N'$, their conjecture for the phase structure is complicated.

The assumption of confinement in the strong coupling limit plays an essential role in the arguments of Banks and Zaks. Here, a quite important fact to be noted is that the lattice study cited by them is a study of pure gauge theory without quarks [8)].

When dynamical quarks are coupled, the vacuum structure become much complicated. In this case, we can formally rewrite the theory in terms of mesonic and baryonic fields [see, for example, Ref. 9)]. When mesons and baryons are physically relevant dynamical variables, i.e. when quarks are confined, the effective action for hadrons is useful to study the phase structure of the system. Oppositely, when the resulting effective action for hadrons is a well-defined action of weakly interacting particles, it is plausible that quarks are confined. In the strong coupling limit, computation of the effective theory for hadrons are carried out either in the large N_c limit [9)], using a meanfield approximation ($1/d$ expansion) [10), 11)], or using a heavy quark mass expansion [11), 12)]. In these cases, the effective action for mesons and baryons leads to a world with spontaneously broken chiral symmetry. Here, in contrast to these cases, we are interested in the case where $N_c = 3$ and quarks are light. In this case, the effective action is quite complicated for general N_F; diagrams with quark loops, which are suppressed at large N_c, large d or large m_q, become important. Therefore, in the realistic cases, it is difficult to analytically deduce the vacuum structure as a function of N_F even in the strong coupling limit.

Therefore, in our previous work [13)], we performed a series of numerical simulations of QCD in the strong coupling limit for various N_F, using the Wilson fermion formalism for lattice quarks. We found that, when $N_F \geq 7$, light quarks are deconfined and chiral symmetry is restored at zero temperature even in the strong coupling limit.

We extend this study to weaker couplings and to larger N_F. Based on the numerical results obtained on the lattice combined with the results of the perturbation theory (see the following sections for details), we conjecture Fig. 1(b) for the N_F dependence of the beta function: When N_F is smaller than a critical value, the beta function is negative for all values of g. Quarks are confined and the chiral symmetry is spontaneously broken at zero temperature. We find that the critical number of flavors is 6. (Corresponding critical N_F is 2 for the case $N_c = 2$ [1)].) On the other hand, when N_F is equal or larger than 17, we conjecture that the beta function is positive for all g, in contrast to the conjecture by Banks and Zaks shown in Fig. 1(a). The theory is trivial in this case. When N_F is between 7 and 16, the beta function changes sign from negative to positive with increasing g. Therefore, the theory has a non-trivial IR fixed point.

---

be a reasonable effective theory of QCD. The N_F-dependence of the vacuum in a sigma model is studied in Ref. 7).

§3. Model and simulation parameters

Lattice QCD is defined on a hypercubic lattice in 4-dimensional Euclidian space with lattice spacing a. Formulation of lattice QCD is described in Ref. 14).

In this study, we use the standard one plaquette gauge action combined with the standard Wilson quark action: $S = S_{gauge} + S_{quark}$ with

$$S_{gauge} = -\frac{\beta}{6} \sum_{x,\mu\neq\nu} \mathrm{Tr}\left(U_{x,\mu} U_{x+\hat{\mu},\nu} U^{\dagger}_{x+\hat{\mu}+\hat{\nu},\mu} U^{\dagger}_{x+\hat{\nu},\nu} \right), \tag{3·1}$$

$$S_{quark} = \sum_{f,x} \left[\bar{\psi}^f_x \psi^f_x - K \left\{ \bar{\psi}^f_x (1-\gamma_\mu) U_{x,\mu} \psi^f_{x+\hat{\mu}} + \bar{\psi}^f_{x+\hat{\mu}} (1+\gamma_\mu) U^{\dagger}_{x,\mu} \psi^f_x \right\} \right], \tag{3·2}$$

where $\beta = 6/g^2$ with g the bare gauge coupling constant.*) The index f $(= 1, \cdots, N_F)$ is for the flavors. We assume that all N_F quarks are degenerate with the bare mass m_0, which is related to the hopping parameter K by $m_0 = (1/2a)(1/K - 1/K_c)$, where a is the lattice spacing and $K_c = 1/8$ is the point where the bare mass vanishes. The point $K = 0$ corresponds to infinite quark mass.

In the formalism of Wilson quarks on the lattice, the flavor symmetry as well as C, P and T symmetries are exactly satisfied on the lattice with a finite a. However, chiral symmetry is explicitly broken even at $m_0 = 0$. [For details about the effects of broken chiral symmetry, see Ref. 16) and references cited there.] However, through a perturbative study of Ward identities using the Wilson fermion action [17)], it is shown that the effects of the broken chiral symmetry can be absorbed by appropriate renormalizations, including an additive renormalization of the quark mass, i.e. by a shift of K_c as a function of β.

We define the current quark mass in terms of an axial vector Ward identity:

$$2m_q \langle 0 \,|\, P \,|\, \pi(\vec{p}=0) \rangle = -m_\pi \langle 0 \,|\, A_4 \,|\, \pi(\vec{p}=0) \rangle \tag{3·3}$$

where P is the pseudoscalar density, A_4 the fourth component of the local axial vector current, and m_π the pion (screening) mass [18), 17)]. A multiplicative renormalization factor for the axial current, which is not important in this study, is absorbed into the definition of the quark mass. Numerical simulations show that the value of m_q does not depend on whether the system is in the high or the low temperature phase when β is large; $\beta \gtrsim 5.5$ for $N_F = 2$ [20)].

We define the chiral limit K_c by the condition $m_q = 0$. Numerical simulations show that, for small N_F, $K_c(\beta)$ forms a smooth curve connecting 1/8 at $\beta = \infty$ and 1/4 at $\beta = 0$. When we further define the pion decay constant f_π by $\langle 0 \,|\, A_4 \,|\, \pi(\vec{p}=0) \rangle = m_\pi f_\pi$, either $m_\pi = 0$

*) Many flavor QCD using the staggered fermion formalism is studied in Ref. 15).

or $f_\pi = 0$ is satisfied in the chiral limit. The chiral symmetry is restored when $f_\pi = 0$ in the chiral limit, while $m_\pi = 0$ if the chiral symmetry is spontaneously broken.

For QCD with a small N_F, the deconfinement transition is expected at high temperatures [14]. Therefore, we have to study the temperature dependence of the results to distinguish between the deconfinement transition due to high temperatures and a bulk transition due to the effects of many flavors. On a lattice with N_t sites in the Euclidian time direction, the temperature is given by $T = 1/N_t a$. When the beta function is negative, as in the case of small N_F, the lattice spacing a is a decreasing function of $\beta = 6/g^2$. In this case, T increases with increasing β for a fixed finite N_t. On the other hand, when the beta function is positive, T decreases with increasing β. Therefore, in order to distinguish between the deconfinement transition due to high temperatures and a bulk transition for general N_F, we have to vary N_t.

We perform simulations on lattices $8^2 \times 10 \times N_t$ ($N_t = 4$, 6, and 8), $16^2 \times 24 \times N_t$ ($N_t = 16$) and $18^2 \times 24 \times N_t$ ($N_t = 18$). We vary N_F from 2 to 300. For each N_F, we study the phase structure in the coupling parameter space (β, K). We adopt an anti-periodic boundary condition for quarks in the t direction and periodic boundary conditions otherwise. We use the hybrid R algorithm [19] for the generation of gauge configurations.[*)] Statistical errors are estimated by the jack-knife method.

It should be noted that, in QCD with dynamical quarks, there are no order parameters for quark confinement. We discuss about confinement by comparing the screening pion mass and the lowest Matsubara frequency, and, simultaneously, consulting the values of plaquette and the Polyakov loop. In the followings, we call the pion screening mass simply the pion mass, and similarly for the quark mass.

§4. Strong coupling limit

We first study the strong coupling limit, $\beta = 0$, where we expect the strongest confining force. In a previous work [13], we have shown that, when $N_F \geq 7$, light quarks are deconfined and chiral symmetry is restored at zero temperature even in the strong coupling limit: For $N_F \leq 6$, we have only one confining phase from the heavy quark limit $K = 0$ up to the chiral limit $K_c = 0.25$. On the other hand, for $N_F \geq 7$, we find a strong first order transition at $K = K_d < K_c$. When quarks are heavy ($K < K_d$), both plaquette and the Polyakov loop are small, and m_π satisfies the PCAC relation $m_\pi^2 \propto m_q$. Therefore, quarks are confined and

[*)] The R algorithm has discretization errors of $O(N_F \Delta\tau^2)$ for the step size $\Delta\tau$ of a molecular dynamic evolution. As N_F increases we have to decrease $\Delta\tau$, such as $\Delta\tau$ =0.0025 for $N_F = 240$, to reduce the errors. We check that the errors are sufficiently small with our choices of $\Delta\tau$ for typical cases.

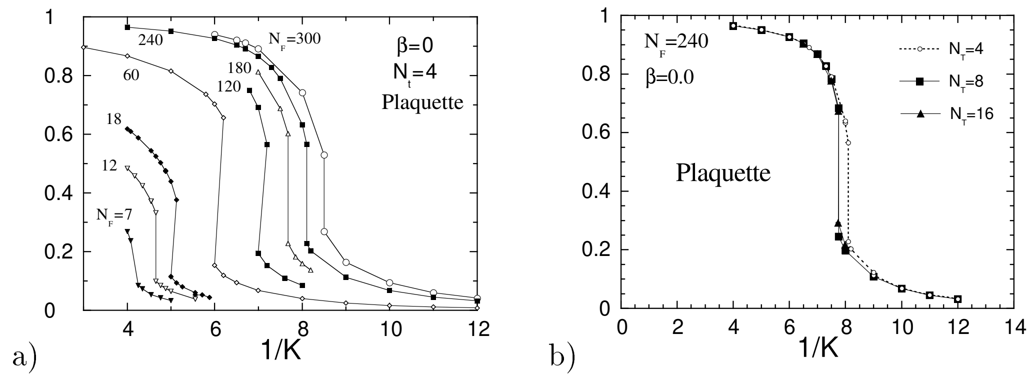

Fig. 2. Plaquette at $\beta = 0$ as a function of $1/K$. (a) For various N_F at $N_t = 4$. (b) For $N_F = 240$ at $N_t = 4$, 8 and 16. $1/K_d \simeq 8.1(1)$ for $N_t = 4$ and $1/K_d = 7.8(2)$ for $N_t = 8$ and 16.

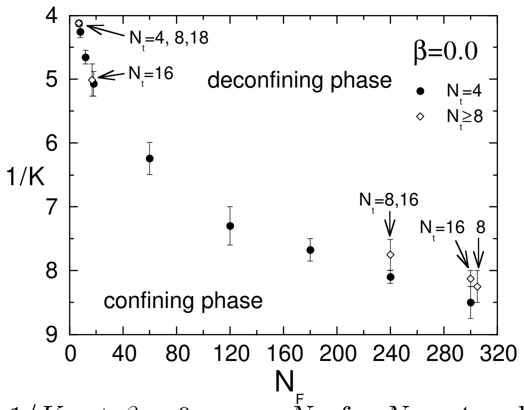

Fig. 3. The transition point $1/K_d$ at $\beta = 0$ versus N_F for $N_t = 4$ and $N_t \geq 8$. For clarity, data at $N_t = 8$ for $N_F = 300$ is slightly shifted to a larger N_F in the figure.

the chiral symmetry is spontaneously broken in this phase. We find that m_q in the confining phase is non-zero at the transition point K_d, i.e., the chiral limit does not belong to this phase. On the other hand, when quarks are light ($K > K_d$), plaquette and the Polyakov loop are large. In this phase, m_π remains large in the chiral limit and is almost equal to twice the lowest Matsubara frequency π/N_t. This implies that the pion state is a free two-quark state and, therefore, quarks are not confined in this phase. The pion mass is nearly equal to the scalar meson mass, and the rho meson mass to the axial vector meson mass. The chiral symmetry is also manifest within corrections due to finite lattice spacing.

We now extend the study to larger N_F. In Fig. 2(a), we show the results of plaquette at $N_t = 4$ for $N_F = 7$ – 300. Clear first order transition can be seen at K below 0.25. We then study the N_t dependence of the results, as shown in Fig. 2(b) for the case of $N_F = 240$. Results for $N_F = 7$ is given in Ref. 13). We find that, although the transition point shows a slight shift to smaller $1/K$ when we decrease $T = 1/N_t a$ from $N_t = 4$ to 8, it stays at the same point for $N_t \geq 8$. The chiral limit, $1/K = 4$, cannot be achieved in the confining phase even at $T = 0$. Therefore, we conclude that the transition is a bulk transition (transition

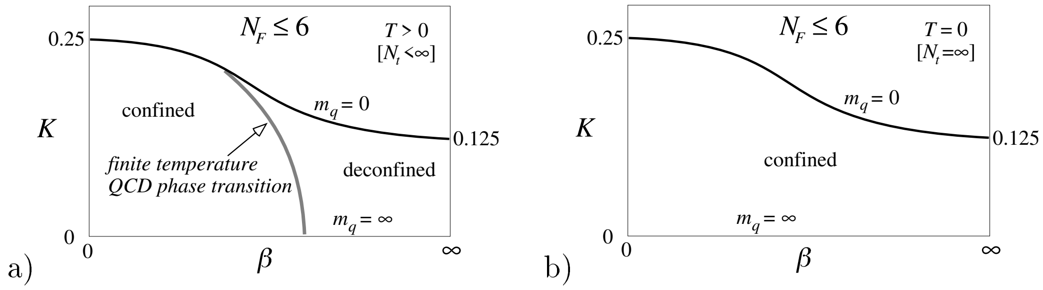

Fig. 4. The phase structure for $N_F \le 6$; (a) at finite temperatures, and (b) at zero temperature.

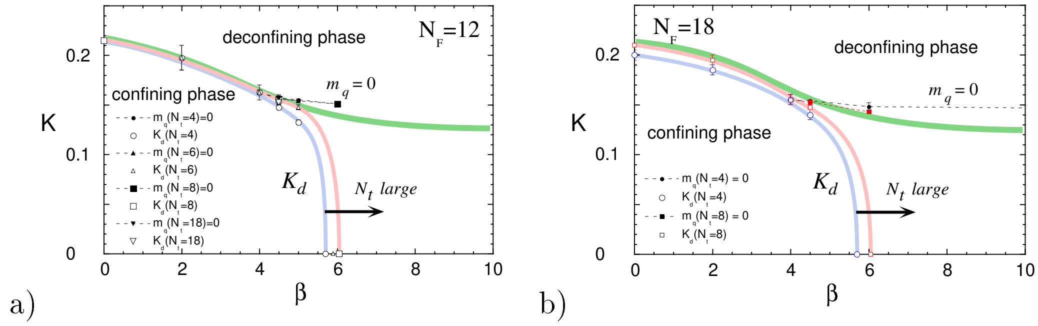

Fig. 5. Phase diagram for (a) $N_F = 12$ and (b) $N_F = 18$. Dark shaded lines represent our conjecture for the bulk transition line in the limit $N_t = \infty$.

that exists at zero-temperature). From these results, we obtain the phase diagram at $\beta = 0$ shown in Fig. 3.

§5. Phase structure at $\beta > 0$

Let us now study the case $\beta > 0$. On a lattice with a fixed finite N_t, we have the finite temperature deconfining transition at finite β when the quark mass is sufficiently heavy ($K \approx 0$). For $N_F \le 6$, our previous study shows that the finite temperature transition line crosses the K_c line at finite β[16]. A schematic diagram of the phase structure for this case is shown in Fig. 4.[*] The location of the finite temperature transition line moves towards a larger β as N_t is increased. In the limit $N_t = \infty$, the finite temperature transition line will shift to $\beta = \infty$ so that only the confining phase is realized for $N_F \le 6$ at $T = 0$. See Fig. 4(b). There, we have a monotonous RG flow towards the strong coupling limit on the K_c line.

When $N_F \ge 7$, we have seen that there exist a bulk deconfining transition in the strong coupling limit. We have to clarify the relation between the finite temperature transition at small K and the bulk transition at $\beta = 0$.

[*] See Ref. 16) and references cited there for discussions about complexities due to the lack of chiral symmetry with Wilson fermions.

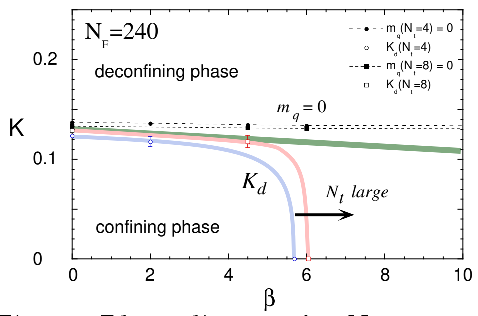

Fig. 6. Phase diagram for $N_F = 240$.

At $\beta = 0$, the location of the bulk deconfining transition K_d moves toward larger K with decreasing N_F. From Fig. 3, we find $1/K_d \simeq 8.5$, 8, 7.6, 7.2, 6.1, and 5.0 for $N_F = 300$, 240, 180, 120, 60, and 18, respectively, at $\beta = 0$. Recall that $1/K = 8$ is the point for massless free quarks.

We perform simulations at $\beta \approx 0.0$ – 6.0 varying N_F from 2 to 300. From the numerical data, we obtain the phase diagrams sown in Fig. 5 for $N_F = 12$ and 18, and in Fig. 6 for $N_F = 240$. When $N_t = 4$ or 8, the phase boundary line between confining and deconfining phase bends down at finite β due to the finite temperature phase transition of the confining phase. The dark shaded lines are our conjecture for the phase boundary between the confining phase and the deconfining phase at zero temperature ($N_t = \infty$).

In Figs. 5 and 6, the dashed lines in the deconfining phase represents the chiral limit, $m_q = 0$. This line also corresponds to the minimum point of m_π^2. We have checked that, for the case $N_F = 240$, the quark propagator in the Landau gauge actually shows the chiral symmetry, $\gamma_5 G(z) \gamma_5 = -G(z)$, on the $m_q = 0$ line. For $N_F \gtrsim 240$, we find that the $m_q = 0$ line in the deconfining phase, which starts at $1/K = 8$ in the weak coupling limit $\beta = \infty$, reaches the strong coupling limit, as shown in Fig. 6 for $N_F = 240$. For a smaller $N_F \lesssim 100$, because the bulk transition line K_d in the strong coupling region shifts toward larger K with decreasing N_F, the $m_q = 0$ line in the deconfining phase hits the K_d line at finite β. For example, in the case $N_F = 18$, it hits at $\beta = 4.0$ – 4.5, as shown in Fig. 5(b).

§6. Nature of the deconfining phase for large N_F

In order to study the nature of the deconfining phase, we first study the case of large N_F, because the range of the deconfining phase is wider at larger N_F (see Fig. 3) and therefore the parameter dependence of physical quantities becomes easier to study. In particular, for $N_F \gtrsim 240$, we are able to study the chiral limit in the deconfining phase at any values of β (see Fig. 6). Because of this, we first intensively investigate the cases $N_F = 240$ and 300, and then decrease N_F.

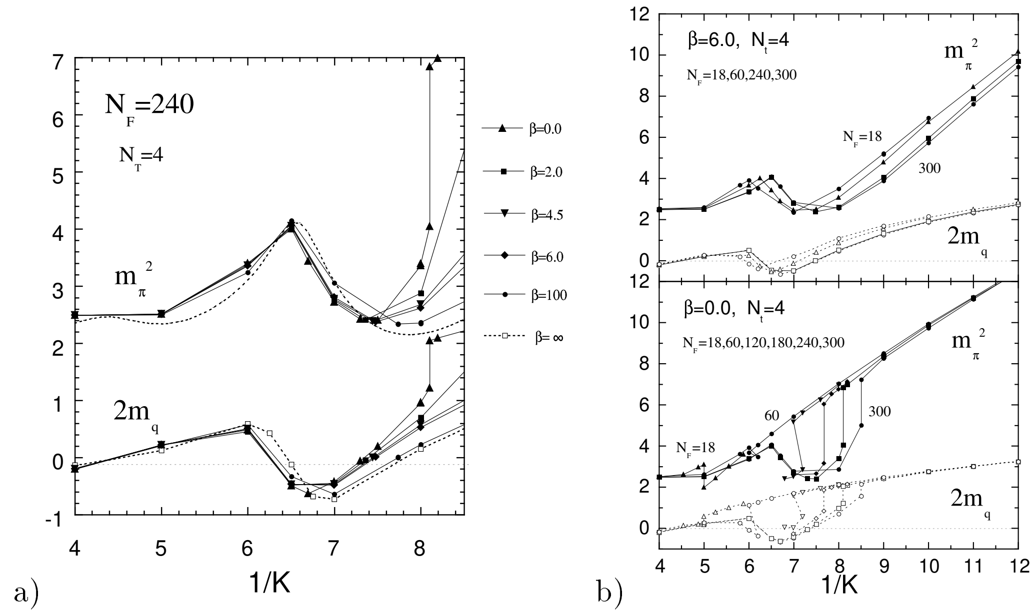

Fig. 7. Results of m_π^2 and $2m_q$ versus $1/K$ at $N_t = 4$: (a) $N_F = 240$ at various β. (b) $N_F = 18$ – 300 at $\beta = 6.0$ and 0.0.

6.1. $N_F = 240$ *and* 300

Fig. 7(a) shows the results of m_π^2 and $2m_q$ in the deconfining phase for $N_F = 240$ at $\beta = 0.0$, 2.0, 4.5, 6.0, and 100.0 on the $N_t = 4$ lattice. A very striking fact is that the shape of m_π^2 and $2m_q$ as a function of $1/K$ only slightly changes for $1/K < 8$ when the value of β decreases from ∞ down 0. [Note that $\beta = \infty$ corresponds to the case of free quarks. Due to the doublers, m_q defined by (3·3) shows a non-trivial $1/K$ dependence at $1/K < 8$ also for a free quark.] Only the position of the local minimum of m_π^2 at $1/K \simeq 8$, which corresponds to the vanishing point of m_q, slightly shifts toward smaller $1/K$. We obtain similar results also for $N_t = 8$. This suggests that quarks are almost free down to $\beta = 0$ in the deconfining phase.

The results for $N_F = 300$ are essentially the same as those for $N_F = 240$, except for very small shifts of the transition point and the minimum point of m_π^2.

6.2. *Renormalization group flow for* $N_F = 240$

From the perturbation theory, the $m_q = 0$ point at $\beta = \infty$ is a trivial IR fixed point for $N_F \geq 17$. The phase diagram shown in Fig. 6 suggests that there are no other fixed points on the $m_q = 0$ line at finite β. In order to confirm this, we investigate the direction of the renormalization group (RG) flow along the $m_q = 0$ line for $N_F = 240$, using a Monte Carlo renormalization group (MCRG) method.

We make a block transformation for a change of scale factor 2, and estimate the quantity

$\Delta\beta = \beta(2a) - \beta(a)$: We generate configurations on an 8^4 lattice on the $m_q = 0$ points at $\beta = 0$ and 6.0 and make twice blockings. We also generate configurations on a 4^4 lattice and make once a blocking. Then we calculate $\Delta\beta$ by matching the value of the plaquette at each step.*) From a matching using our data, we obtain $\Delta\beta \simeq 6.5$ at $\beta = 0$ and 10.5 at $\beta = 6.0$. The value obtained from the two-loop perturbation theory is $\Delta\beta \simeq 8.8$ at $\beta = 6.0$. The signs are the same and the magnitudes are comparable. This suggests that the direction of the RG flow on the $m_q = 0$ line at $\beta = 0$ and 6.0 is the same as that at $\beta = \infty$. This further suggests that there are no fixed points at finite β. All of the above imply that the theory is trivial for $N_F = 240$.

6.3. $240 \geq N_F \geq 17$

Now we decrease N_F from 240. As discussed in Sec. 5, the area of the deconfining phase decreases with decreasing N_F in the strong coupling region. However, except for this shift of the bulk transition point, the shape and the values of m_π^2 and m_q are quite similar when we vary N_F from 300 down to 17. Fig. 7(b) shows m_π^2 and $2m_q$ for $N_F = 18 - 300$ at $\beta = 6.0$ and 0. At $\beta = 6.0$, the shapes of m_π^2 are almost identical to each other, except for a small shift toward smaller $1/K$ as N_F is decreased.

These facts suggest that, for $N_F \geq 17$, the nature of physical quantities in the deconfining phase is almost identical to that observed for $N_F = 240$ and 300, i.e., quarks are almost free. Therefore, we conjecture that the direction of the RG flow at $\beta \lesssim 6$ is identical to the case $N_F = 240$. Combining this with the perturbative result that, for $N_F \geq 17$, $\beta = \infty$ is the IR fixed point, we conjecture that the RG flow along the massless quark line in the deconfining phase is uniformly directing to smaller β, as in the case of $N_F = 240$. When this is the case, the theory has only one IR fixed point at $\beta = \infty$ — the theory is trivial for $N_F \geq 17$.

6.4. $16 \geq N_F \geq 7$

As shown in Sec. 4, quark confinement is lost for $N_F \geq 7$ at $\beta = 0$. We have intensively simulated the cases $N_F = 12$ and 7. The phase diagram for $N_F = 12$ is shown in Fig. 5(a). At $\beta \lesssim 6.0$, the gross feature of the phase diagrams for $N_F = 7$ and 12 is quite similar to the case $N_F = 18$ shown in Fig. 5(b). Physical quantities at $\beta \lesssim 6.0$ also show behavior similar to that shown in Fig. 7 for $N_F \geq 17$. Therefore, we consider it probable that the direction of the RG flow in the deconfining phase at small β is towards a larger β as in the case of

*) It is known for the pure SU(3) gauge theory, that one has to make a more careful analysis using several types of Wilson loop with many blocking steps to extract a precise value of $\Delta\beta$. We reserve elaboration of this point and a fine tuning of $1/K$ at each β for future works. For $N_F = 240$, because the velocity of the RG flow is large, we will be able to obtain the sign and an approximate value of $\Delta\beta$ by a simple matching.

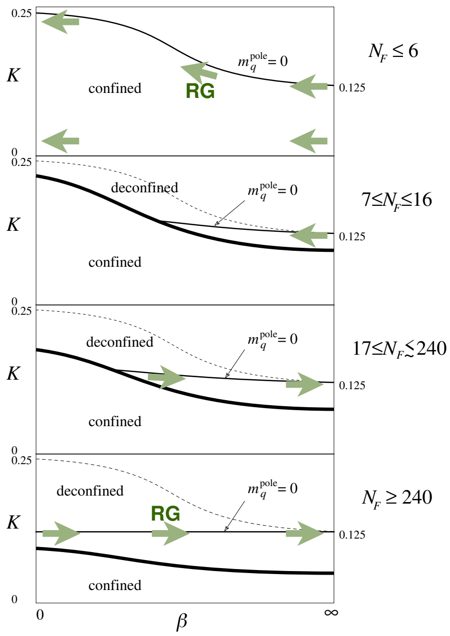

Fig. 8. Phase structure and the RG flow at $T = 0$ for general number of flavors. Thin dashed lines in the phase diagrams for $N_F \geq 7$ represent the location of the chiral limit for $N_F \leq 6$ as a guide for eyes.

$N_F \geq 17$.

On the other hand, when $N_F \leq 16$, the theory is asymptotic free. Therefore, the RG flow at $\beta = \infty$ is opposite to that for $N_F \geq 17$. This means that we have an IR fixed point somewhere at a finite value of β. The continuum limit is governed by this IR fixed point.

On the $m_q = 0$ line around $\beta = 4.5$, we find that m_π is roughly twice the lowest Matsubara frequency. This implies that the quarks are not confined and almost free. Therefore, the anomalous dimensions are small at the IR fixed point, suggesting that the IR fixed point locates at finite β. In order to pin down the position of the IR fixed point, we have to make a detailed study of RG flows in a wider coupling parameter space. We reserve these studies for future works.

In summary, we conjecture that, for $16 \geq N_F \geq 7$, we have an IR fixed point at finite β. The theory in the continuum limit is a non-trivial theory with small anomalous dimensions, however, without confinement.

§7. Conclusions

Based on our numerical data combined with the results of the perturbation theory, we propose Fig. 8 for the phase structure and the RG flow at $T = 0$ for general number of flavors.

When $N_F \leq 6$, quarks are confined for any values of the current quark mass and β. The chiral limit $K_c(\beta)$, where the current quark mass m_q vanishes, belongs to the confining phase.

When $N_F \geq 7$, there is no chiral limit in the confining phase. There is a line of a first order phase transition from the confining phase to a deconfining phase at a finite current quark mass for all values of β. The chiral limit exists only in the deconfining phase.

In order to study the nature of the deconfining phase for large N_F, we first increase N_F, because the region of the deconfining phase becomes larger with larger N_F. For $N_F = 240$, a MCRG study along the massless quark line shows that the direction and the magnitude of the RG flow at small β is consistent with the result of the perturbation theory, suggesting that the direction of the RG flow is identical for all values of β, as shown in Fig. 8. Therefore, there is only one IR fixed point at $\beta = \infty$, so that the theory in the continuum limit is trivial. We also find that the K-dependences of pion screening mass and current quark mass in the deconfining phase are almost identical to those of free Wilson quarks. Also for smaller values of $N_F \geq 7$, the general features of the phase diagram and the K-dependence of physical quantities are quite similar to the case of large $N_F \sim 240$. Therefore, we conjecture that the direction of the RG flow is the same as in the case of $N_F = 240$, at least for $\beta \lesssim 6.0$ we simulate.

For $N_F \geq 17$, because the RG flow in the weak coupling limit has the same direction due to the lack of asymptotic freedom, we conjecture a uniform RG flow towards the trivial IR fixed point, as in the case of $N_F \geq 240$. Thus the theory is trivial for $N_F \geq 17$.

When $7 \leq N_F \leq 16$, because the theory is asymptotically free, we expect a non-trivial IR fixed point at finite β. In this case, the theory in the continuum limit is a non-trivial theory with anomalous dimensions, however, without confinement.

ACKNOWLEDGEMENTS

Numerical simulations are performed with HITAC S820/80 at KEK, and Fujitsu VPP500/30 and QCDPAX at the University of Tsukuba. KK would like to thank the participants of YKIS'97 for valuable comments. KK is also grateful to Robert D. Pisarski, Motoi Tachibana and Bengt Petersson for useful discussions. This work is in part supported by the Grants-in-Aid of Ministry of Education, Science and Culture (Nos. 08NP0101 and 09304029).

References

1) Y. Iwasaki, K. Kanaya, S. Sakai, and T. Yoshié, Nucl. Phys. **B**(Proc. Suppl.)**42** (1995), 502.

2) Y. Iwasaki, K. Kanaya, S. Kaya, S. Sakai, and T. Yoshié, Nucl. Phys. **B**(Proc. Suppl.)**53** (1997), 449.
3) T. Banks and A. Zaks, Nucl. Phys. **B196** (1982), 173.
4) R. Oehme and W. Zimmermann, Phys. Rev. **D21** (1980), 471, 1661. R. Oehme, Phys. Rev. **D42** (1990), 4209.
5) K. Nishijima, Nucl. Phys. **B238** (1984), 601; Prog. Theor. Phys. **75** (1986), 22; *ibid.* **77** (1987), 1053.
6) T. Appelquist et al., Phys. Rev. Lett. **77** (1996), 1214.
7) R.D. Pisarski and D.L. Stein, Phys. Rev. **B23** (1981), 3549; J. of Phys. **A14** (1981), 3341. A.J. Paterson, Nucl. Phys. **B190[FS3]** (1981), 188.
8) J. Kogut, R. Pearson and J. Shigemitsu, Phys. Rev. Lett. **43** (1979), 484.
9) H. Kluberg-Stern et al., Nucl. Phys. **B190[FS3]** (1981), 504. N. Kawamoto, Nucl. Phys. **B190[FS3]** (1981), 617. N. Kawamoto and J. Sumit, Nucl. Phys. **B192** (1981), 100.
10) J. Smit, Nucl. Phys. **B175** (1980), 307. J. Greensite and J. Primack, Nucl. Phys. **B180[FS2]** (1981), 170. H. Kluberg-Stern, A. Morel and B. Petersson, Nucl. Phys. **B215[FS7]** (1983), 527.
11) J.-M. Blairon et al., Nucl. Phys. **B180[FS2]** (1981), 439.
12) K. Wilson, in "New phenomena in subnuclear physics" (Erice 1975), ed. A. Zichichi (Plenum, NY, 1977).
13) Y. Iwasaki, K. Kanaya, S. Sakai, and T. Yoshié, Phys. Rev. Lett. **69** (1992), 21.
14) K. Kanaya, in these proceedings.
15) M. Fukugita, S. Ohta and A. Ukawa, Phys. Rev. Lett. **60** (1988), 178. J.B. Kogut and D.K. Sinclair, Nucl. Phys. **B295[FS21]** (1988), 465. S. Ohta and S. Kim, Phys. Rev. **D44** (1991), 504. F. Brown et al., Phys. Rev. **D46** (1992), 5655. P.H. Damgaard et al., Phys. Lett. **B400** (1997), 169.
16) Y. Iwasaki, K. Kanaya, S. Kaya, S. Sakai, and T. Yoshié, Phys. Rev. **D54** (1996), 7010.
17) M. Bochicchio et al., Nucl. Phys. **B262** (1985), 331.
18) S. Itoh, Y. Iwasaki, Y. Oyanagi and T. Yoshié, Nucl. Phys. **B274** (1986), 33.
19) S. Gottlieb et al., Phys. Rev. **D35** (1987), 2531.
20) Y. Iwasaki, K. Kanaya, S. Sakai and T. Yoshié, Phys. Rev. Lett. **67** (1991), 1494.