Lattice superstring and noncommutative geometry

Jun Nishimura (KEK)

at LATTICE 2003, Tsukuba, JAPAN,
July 15, 2003

0. Introduction

Superstring theory

- unified theory

$$\left\{\begin{array}{l}\text{matter particles}\\ \text{gauge particles}\end{array}\right. \Longleftarrow \begin{array}{l}\text{oscillation modes}\\ \text{of strings}\end{array}$$

$$\begin{array}{l}\text{interactions}\\ \text{(incl. gravity)}\end{array} \Longleftarrow \begin{array}{l}\text{joining/splitting}\\ \text{of strings}\end{array}$$

- perturbatively well-defined quantum theory of gravity

$$\left\{\begin{array}{l}\text{no UV divergence}\\ \text{unitarity OK}\end{array}\right.$$

However,

- too many perturbatively stable vacua

$$\left\{\begin{array}{l}\text{space-time dimensionality}\\ \text{gauge group}\\ \text{matter contents}\end{array}\right.$$

- non-perturbative effects crucial for understanding the "true" vacuum

Recall :

understanding the vacuum of QCD

such as $\begin{cases} \text{confinement} \\ \text{chiral symmetry breaking} \end{cases}$

and the dynamics of low energy excitations

$\Longleftarrow$ nonpert. formulation of gauge theory

$=$ lattice gauge theory (Wilson '74)

Likewise :

nonpert. formulation of string theory

$=$ matrix models

$\Longrightarrow$ new insights into

the nonpert. dynamics of string theory

and many related issues

- large N gauge theory
- noncommutative geometry
- super YM theories on the lattice

Plan

0. Introduction

1. How matrix models describe strings

2. Large N reduced models
   equiv. to SU(∞) gauge theories

3. Lattice superstring
   dynamical generation of space-time

4. Lattice noncommutative geometry
   MC studies of NCYM, NC ϕ^4

5. Lattice supersymmetry via orbifolding

6. Summary

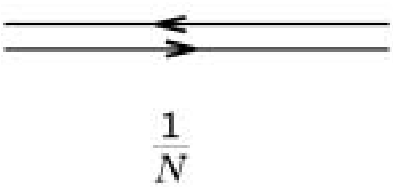
1. How matrix models describe strings

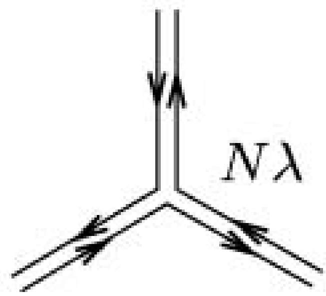
one-matrix model (Brezin-Itzykson-Parisi-Zuber '78)

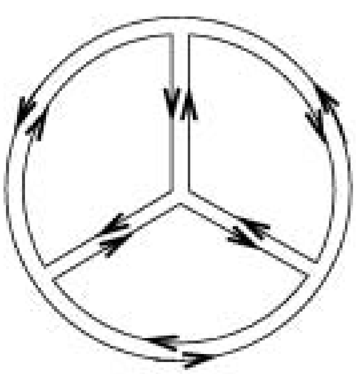
ϕ : $N \times N$ hermitian

$$S = \frac{1}{2} N \mathrm{tr} \phi^2 - \frac{1}{3} N \lambda \mathrm{tr} \phi^3 \quad (1)$$

Feynman diagrams







$$\sim (\tfrac{1}{N})^P (N\lambda)^V N^I = \lambda^V N^{V-P+I} = \lambda^V N^{\chi}$$

V = no. of vertices

P = no. of propagators

I = no. of index loops

$\chi = V - P + I$ (Euler number)

Two parameters (N, λ)

i) Planar limit

$N \to \infty$ for fixed λ

⇨ Only planar diagrams dominate

then $\lambda \nearrow \lambda_c$

⇨ continuum limit $(a \to 0)$

2d quantum gravity

with fixed topology (sphere)

string theory (at the tree level)

ii) Double scaling limit

(Brezin-Kazakov,Douglas-Shenker,Gross-Migdal '90)

$$\left.\begin{matrix} N \to \infty \\ \lambda \nearrow \lambda_c \end{matrix}\right\}$$ simultaneously

⇨ all the topologies contribute

Nonperturbative formulation of
bosonic (non-critical) string theory

2. Large N reduced models

2.1 Eguchi-Kawai equivalence

SU(N) lattice gauge theory on L^D lattice with periodic b.c.

$L=1$ model $\underset{N\to\infty}{\simeq}$ $L=\infty$ model

(planar limit)

⇧

Eguchi-Kawai model (Eguchi-Kawai '82)

$$S = -N\beta \sum_{\mu\neq\nu} \mathrm{tr}(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger)$$

important assumption :

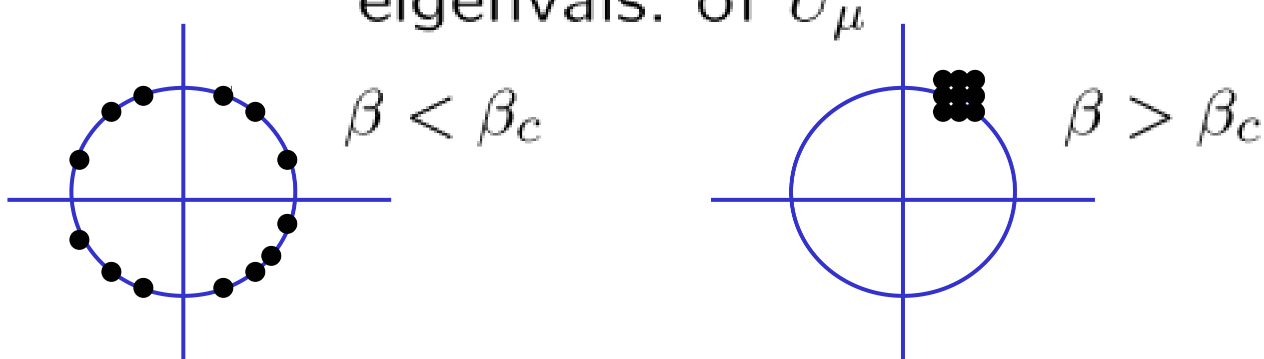
U(1)^D sym. : $U_\mu \to e^{i\alpha_\mu} U_\mu$

is not spontaneously broken

broken at large β, for $D>2$

(Bhanot-Heller-Neuberger '82)

eigenvals. of U_μ



2.2 remedies to the original proposal

a) Quenched Eguchi-Kawai model

(Bhanot-Heller-Neuberger '82)

Constrain the eigenval's to be uniformly distributed on a unit circle

i.e. insert $\int dV_\mu \, \delta(U_\mu - V_\mu Q V_\mu^\dagger)$

$$Q = \mathrm{diag}(1, \omega, \omega^2, \cdots, \omega^{N-1})$$

$$\omega = \exp(2\pi i/N)$$

b) Twisted Eguchi-Kawai model

(Gonzalez-Arroyo-Okawa '83)

Consider L^D lattice with twisted b.c. and then take $L = 1$

$$\Rightarrow \quad S = -N\beta \sum_{\mu \neq \nu} Z_{\mu\nu} \mathrm{tr}(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger)$$

minimum-action config. $U_\mu^{(0)}$

$$(U_\mu^{(0)} U_\nu^{(0)} = Z_{\mu\nu}^* U_\nu^{(0)} U_\mu^{(0)})$$

has uniform eigenval. distribution

$\Rightarrow$ No SSB of $U(1)^D$ at large β

c.f.) Profumo '02, Ishikawa-Okawa

c) Partial reduction

(Narayanan-Neuberger hep-lat/0303023)

In fact, Eguchi-Kawai equiv.
holds for arbitrary *L*

as far as

$U(1)^D$ is not spontaneously broken

$\beta_c(L)$: critical β for $U(1)^D$ SSB

[$\beta_c(L) \to \infty$ as $L \to \infty$]

3 parameters (β, *N*, *L*)

Choose *L* such that $\beta < \beta_c(L)$

extensions of Eguchi-Kawai model :

- matter fields (adjoint, fund.)
  Gross-Kitazawa '82, Levine-Neuberger '82, Das '83
- non-gauge theories
  Parisi '82, Gonzalez Arroyo-Okawa '83,
  Eguchi-Nakayama '83
- finite temperature
  Klinkhamer-van Baal '84, Das-Kogut '84

2.3 "continuum ver." of reduced models

☺ manifest SUSY implementable !

zero-vol. lim. of cont. U(N) gauge theory

➡ $$S = -\frac{1}{4g^2}\,\mathrm{tr}[A_\mu, A_\nu]^2$$

$A_\mu \ (\mu = 1, \cdots, D)$: $N \times N$ hermitian

Monte Carlo studies :

Krauth-Staudacher '98 — finiteness of Z → proved (Austing-Wheater '01)

Hotta-J.N.-Tsuchiya '99 — no SSB of SO(D)

Anagnostopoulos-Bietenholz-J.N. '02 — area law

roughly, $U_\mu \sim \mathrm{e}^{iaA_\mu}$

U(1)^D symmetry : $A_\mu \to A_\mu + \alpha_\mu \mathbf{1}$

spontaneously broken at $D \geq 3$

- twisting : possible only *formally* at $N = \infty$

  Gonzalez Arroyo-Korthals Altes '83

- quenching : *perturbatively* OK, but

  topological charge $Q = \mathrm{tr} F_{\mu\nu}\tilde{F}_{\mu\nu} \equiv 0$

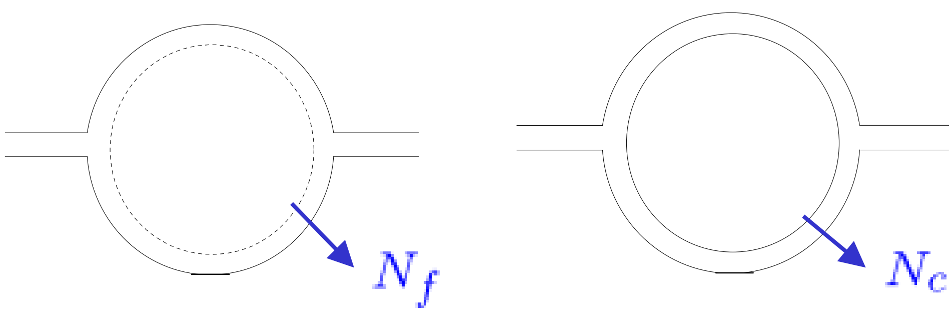
  Gross-Kitazawa '82

2.4 revived interest in solving planar QCD

(Kiskis-Narayanan-Neuberger '02)

"valence quark approx." becomes exact at $N_c \to \infty$ with finite N_f

c.f.) full QCD is still costly...



each fermion loop $\sim O\left(\frac{N_f}{N_c}\right)$

Veneziano limit : $N_f \to \infty,\ N_c \to \infty$

with $r = \frac{N_f}{N_c}$ fixed

quenched QCD $\longleftrightarrow$ full QCD

smooth interpolation !

◆Quenched EK model

c.f.) twisted EK model (only integer *r*)

◆Overlap Dirac operator (Neuberger '98)

topological charge : well-defined

chiral anomalies

Inagaki-Kikukawa-Suzuki hep-lat/0305011

3. Lattice superstrings

zero-vol lim. of cont. 10d $\mathcal{N}=1$ U(N) SYM

$$S = -\frac{1}{4g^2}\,\mathrm{tr}[A_\mu, A_\nu]^2 - \frac{1}{2g^2}\,(\Gamma_\mu)_{\alpha\beta}\mathrm{tr}(\Psi_\alpha[A_\mu, \Psi_\beta])$$

Ishibashi-Kawai-Kitazawa-Tsuchiya ’96

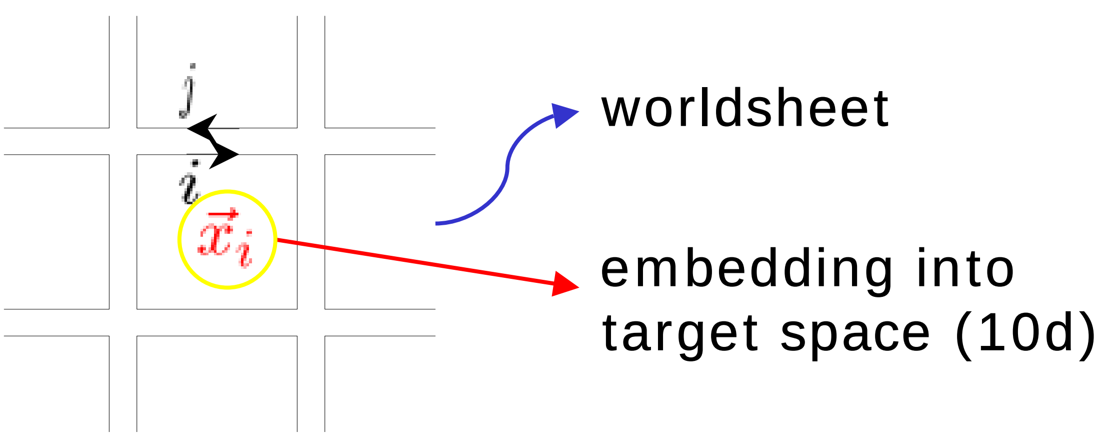
type IIB superstring theory in 10d

Evidences :

- matrix regularization of the worldsheet action
- D-brane interactions (including gravity!)
- derivation of string field theory

$$A_\mu = V_\mu X_\mu V_\mu^\dagger \qquad X_\mu = \mathrm{diag}(x_{1\mu}, \cdots, x_{N\mu})$$

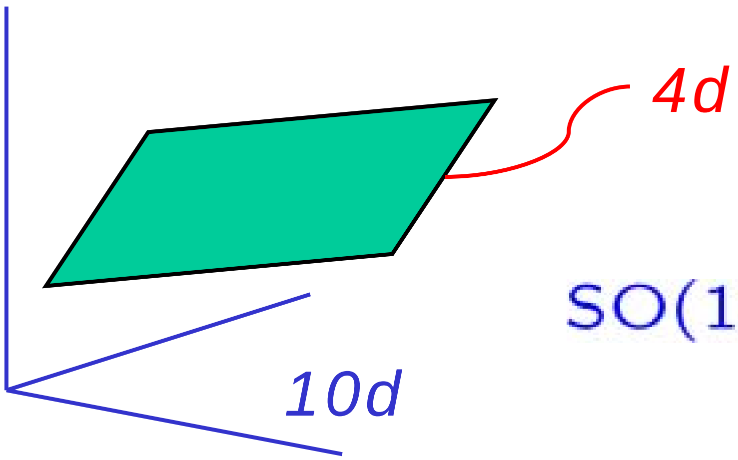
integrate out V_μ first



eigenvalues of A_μ ⟺ space-time coord.

(Aoki-Iso-Kawai-Kitazawa-Tada ’98)

Dynamical generation of 4d space-time



$$SO(10) \xrightarrow{\text{SSB}} SO(4)$$

Order parameter :

$$T_{\mu\nu} = \frac{1}{N}\mathrm{tr}(A_\mu A_\nu)$$ moment of inertia tensor

eigenval's of $T_{\mu\nu}$: $\lambda_1 > \lambda_2 > \cdots > \lambda_{10}$

(principal moment of inertia)

Gaussian expansion method

$\Rightarrow$

$$\langle\lambda_1\rangle, \cdots, \langle\lambda_4\rangle \gg \langle\lambda_5\rangle, \cdots, \langle\lambda_{10}\rangle$$

J.N.-Sugino '01

- higher order calculations

  Kawai-Kawamoto-Kuroki-Matsuo-Shinohara '02

  Kawai-Kawamoto-Kuroki-Shinohara '02

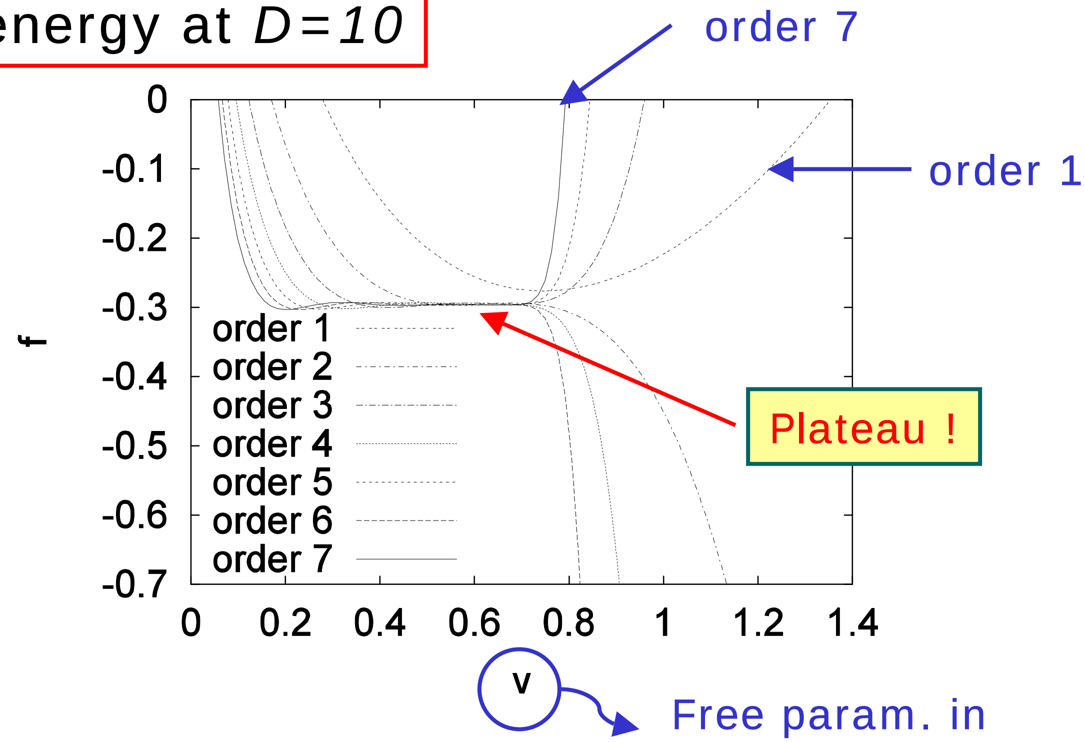
- validity in the bosonic case

  J.N.-Okubo-Sugino '02

Gaussian expansion method in the bosonic case

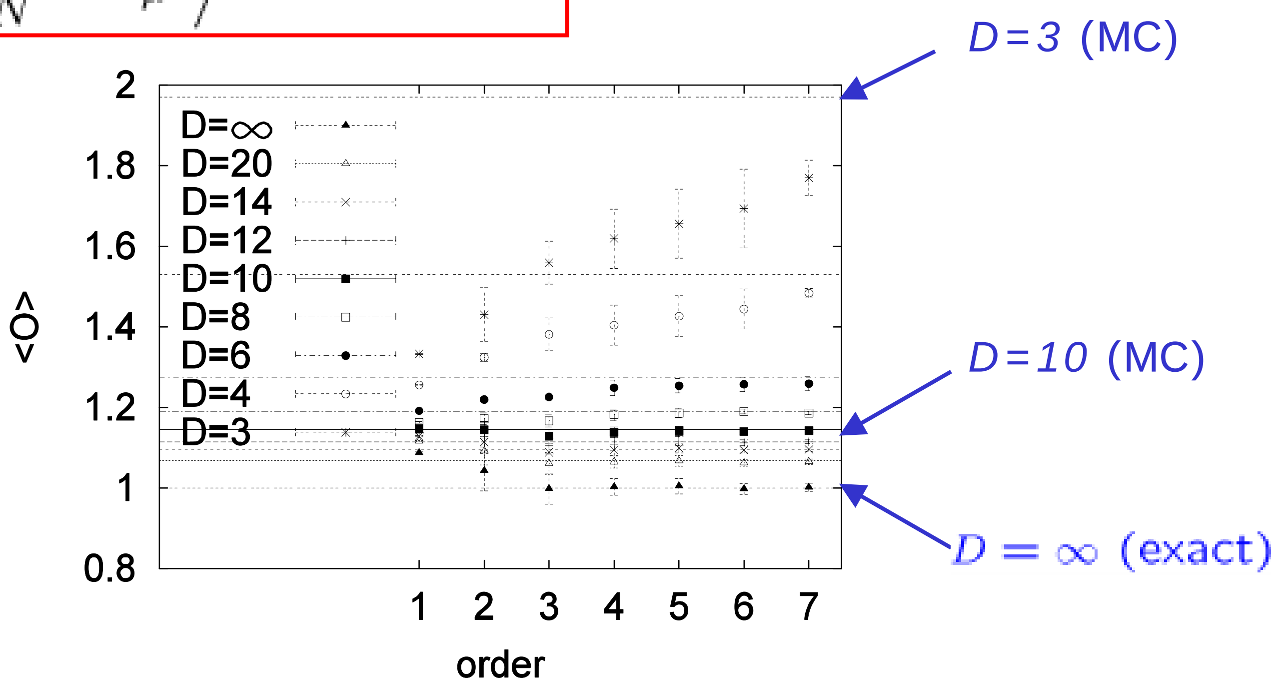
J.N.-Okubo-Sugino '02

Free energy at $D=10$



Free param. in Gaussian action

$\left\langle \frac{1}{N}\mathrm{tr}(A_\mu^2) \right\rangle$ at various D

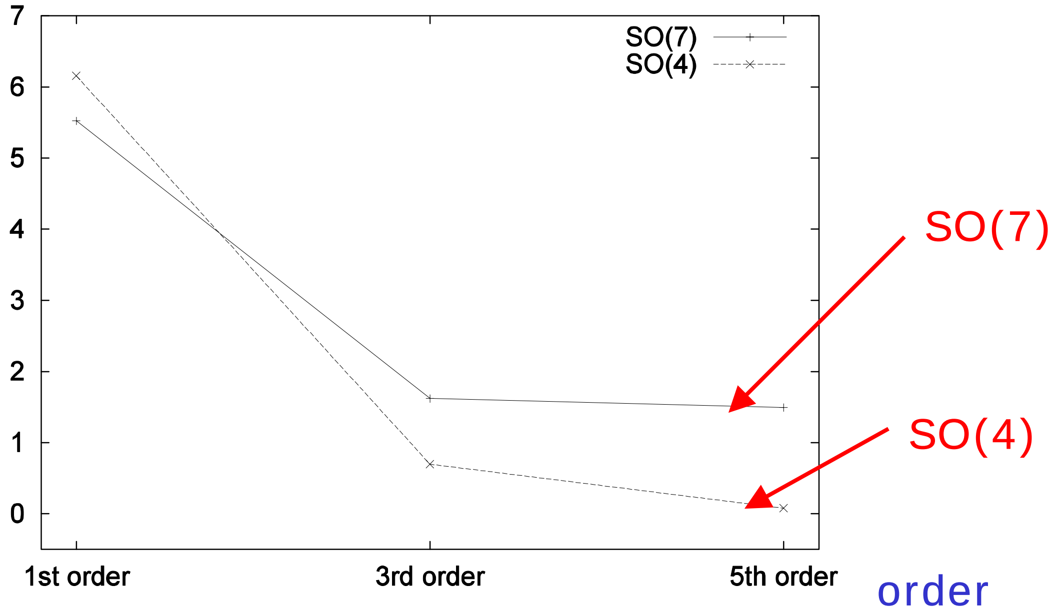


Gaussian expansion *CONVERGES !*

Higher order results in the IKKT model

Kawai-Kawamoto-Kuroki-Matsuo-Shinohara '02

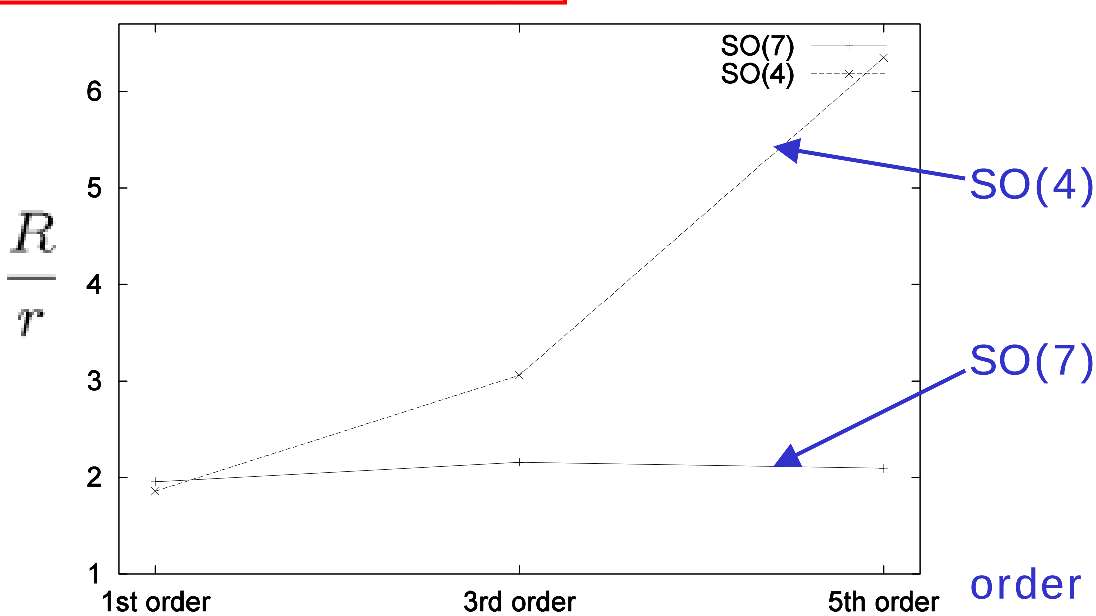
Free energy for SO(4) and SO(7) Ansatz



SO(4) Ansatz gives the smallest free energy

Ratio of the extents $\frac{R}{r}$

R : extent in d dim.
r : extent in $10-d$ dim.



Monte Carlo sim. of IKKT matrix model

fermion det. : $\det M = |\det M| \, e^{i\Gamma}$

(10d Maj.-Weyl fermion)

→ complex-action problem !

phase-quenched simulation ⇒ no SSB

(Ambjorn-Anagnostopoulos-Bietenholz-Hotta-J.N. '00)

Factorization method

(Anagnostopoulos-J.N. '02)

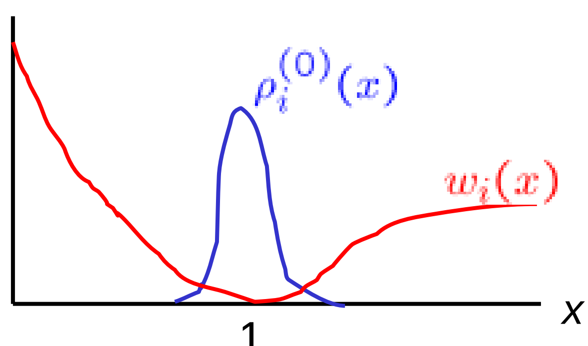
$\rho_i(x) = \langle \delta(x - \tilde{\lambda}_i) \rangle$; $Z = \int dA \, e^{-S_b} \det M$

$\rho_i^{(0)}(x) = \langle \delta(x - \tilde{\lambda}_i) \rangle_0$; $Z_0 = \int dA \, e^{-S_b} |\det M|$

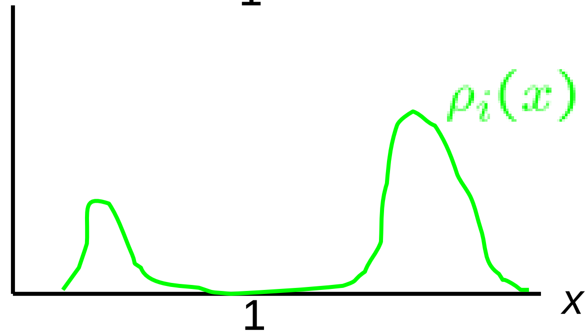
$w_i(x) = \langle e^{i\Gamma} \rangle_i$; $Z_i(x) = \int dA \, e^{-S_b} |\det M| \, \delta(x - \tilde{\lambda}_i)$

$$\rho_i(x) = \frac{1}{C} \rho_i^{(0)}(x) w_i(x)$$

$$\tilde{\lambda}_i \equiv \frac{\lambda_i}{\langle \lambda_i \rangle_0}$$



dominant peak may depend on i



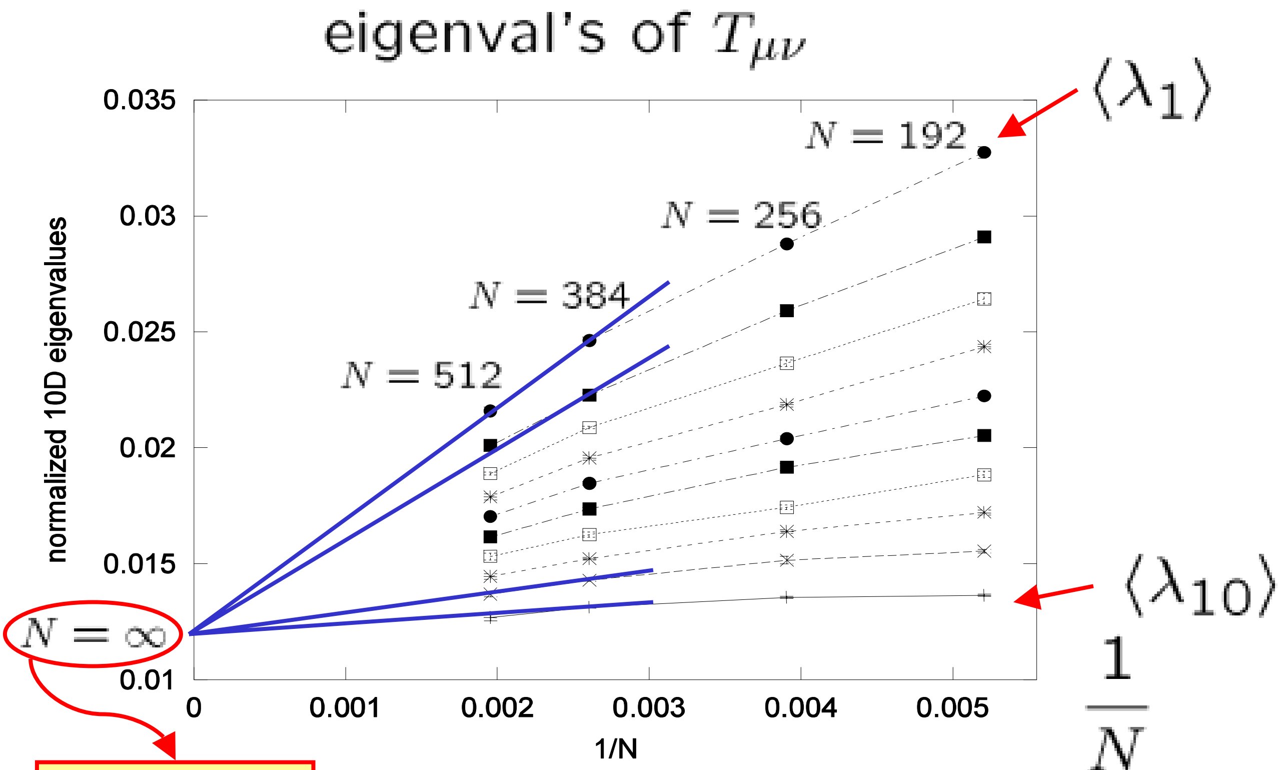
right for $i = 1, 2, 3, 4$

left for $i = 5, 6, 7, 8, 9, 10$

→ $d = 4$

Phase-quenched simulation of IKKT model

(Ambjorn-Anagnostopoulos-Bietenholz-Hotta-J.N. '00)



no SSB !

Effects of the phase

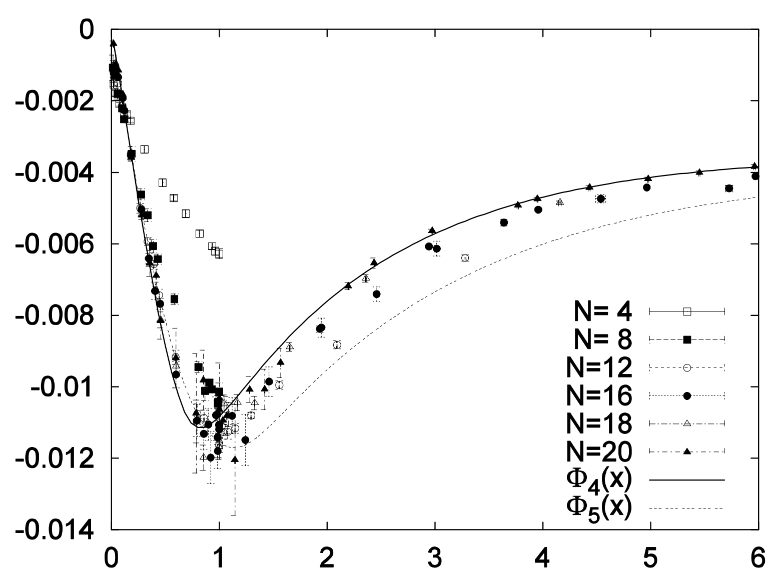
(Anagnostopoulos-J.N. '02)

$$\Phi_i(x) \equiv \frac{1}{N^2} \ln w_i(x)$$

Suppress the peak of $\rho_i^{(0)}(x)$ at $x = 1$

scaling of $\Phi_i(x)$

➡ extrapolation to larger N



4. Lattice noncommutative geometry

twisted reduced models at finite N

⟷ Lattice-regularized field theories

on noncommutative geometry (NCG)

Ambjorn-Makeenko-J.N.-Szabo '99

$$x_\mu \to \hat{x}_\mu \qquad [\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu}$$

$$\phi(x) \to \phi(\hat{x})$$

On the lattice :

$\phi(x)$ (field on L^D lattice) $\Longleftrightarrow$ 1 : 1 $\hat{\Phi}$ ($N \times N$ matrix)

$$L^D = N^2$$

noncommutativity : $\theta = \frac{1}{\pi} L a^2$

size of the torus : $\ell = La$

planar limit ($N \to \infty$ first) ⇨ $\theta = \infty$

$\simeq$ large N field theories

double scaling limit ($N^{2/D} a^2$:fixed) ⇨ finite θ

Effects of nonplanar diagrams

$$\int d^4q \frac{e^{i\theta_{\mu\nu}q_\mu p_\nu}}{q^2+m^2} = F(p)$$

finite !

but singular at $p=0$

$$F(p) \sim \frac{1}{|\theta p|^2}$$

(phase factor due to NCG)

"UV/IR mixing" (Minwalla-van Raamsdonk-Seiberg '00)

Perturbative renormalizability :

difficult even for scalar fields (Chepelev-Roiban '00)

Does cont. lim. of lattice NC theory exist ?

2dNCYM : Yes ! (Bietenholz-Hofheinz-J.N. '02)

("dynamical AB effect" : $B = \theta^{-1}$)

NC scalar field theory (Gubser-Sondhi '01)

dispersion relation : $E^2 = p^2 + m^2 + \frac{\lambda}{|\theta p|^2}$

minimum occurs at $p = p_c \neq 0$

As $m^2 \searrow$ → $\langle\phi(x)\rangle \sim \sin(p_c x)$

striped phase !

MC sim.

(2+1)d : Bietenholz-Hofheinz-J.N. '02,

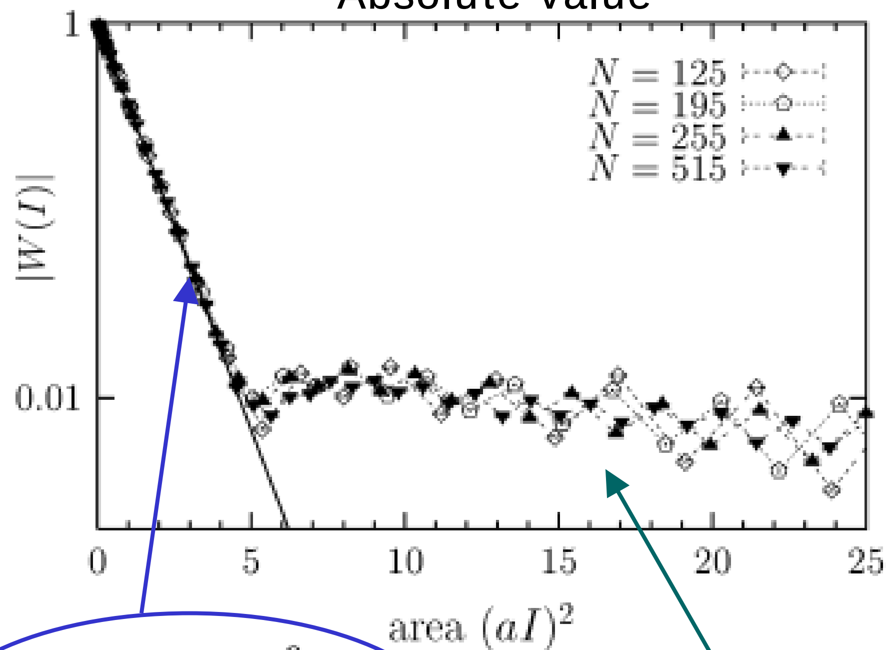
2d : Ambjorn-Caterall '02

2d Yang-Mills theory on NC torus

Bietenholz-Hofheinz-J.N. ’02

$\langle W(I \times I) \rangle$ is complex (θ breaks parity !)

Absolute value



area $\ll \theta$

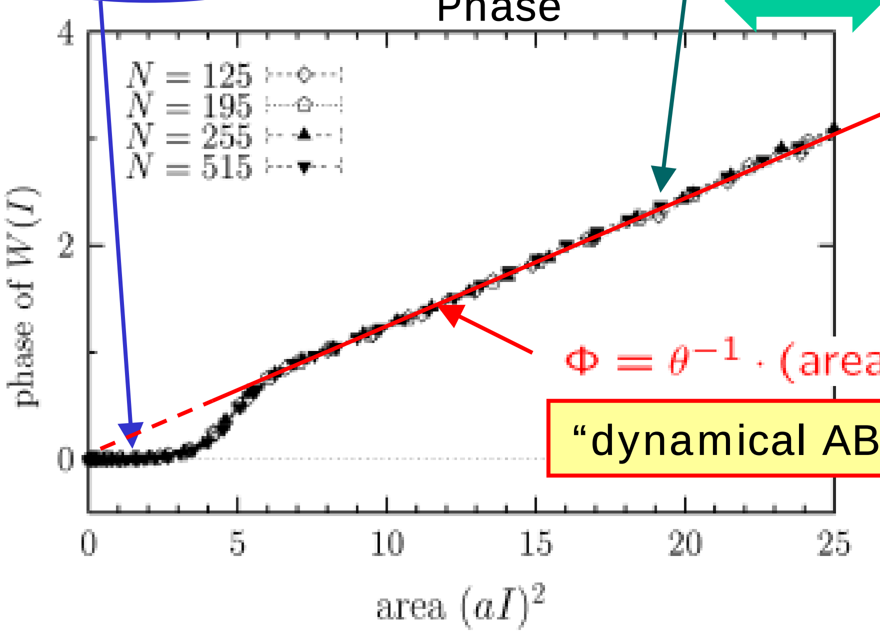
Planar result (Gross-Witten ’80)

area $\gg \theta$

large N scaling with fixed $\frac{N}{\beta}$

finite θ

Phase



$\Phi = \theta^{-1} \cdot$ (area)

“dynamical AB effect”

$(2+1)$d NC ϕ^4 theory on the lattice

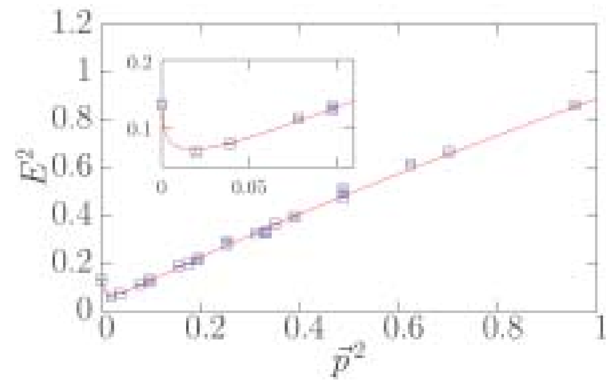
Bietenholz-Hofheinz-J.N. '02

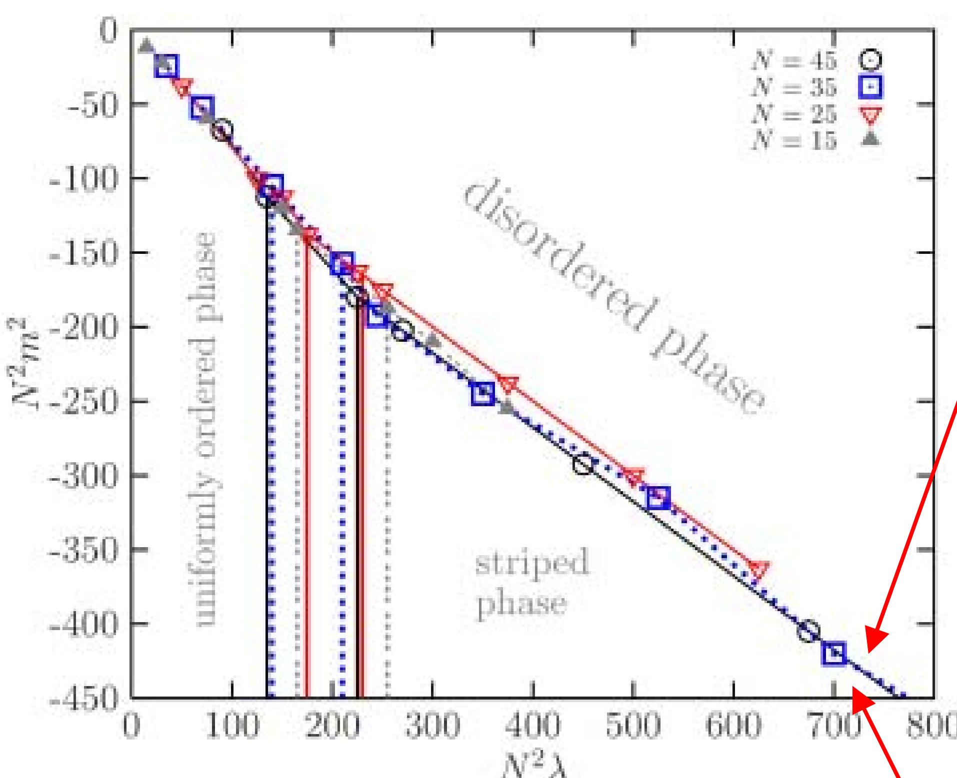
dispersion relation

(disordered phase)

singularity at $p = 0$

minimum occurs at $p = p_c \neq 0$

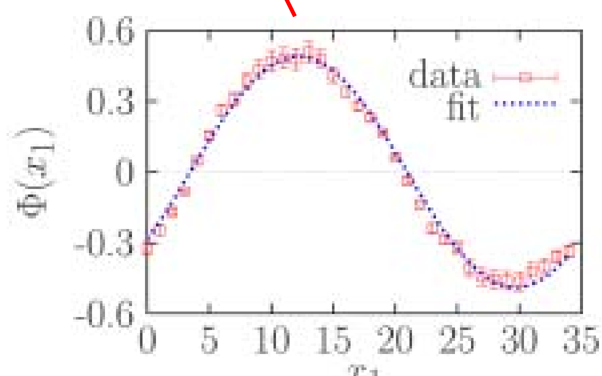




striped phase

$\langle \phi(x) \rangle \sim \sin(p_c x)$

condensation of p_c mode



5. Lattice supersymmetry

Kaplan-Katz-Unsal '02

cont. ver. of reduced models (e.g. IKKT model)

manifest SUSY even at finite N

$$\{Q, \bar{Q}\} \propto P_\mu$$

$$A_\mu \to A_\mu + \alpha_\mu \mathbf{1}$$

Translation : realized in the internal space

(consistent with
eigenval's of A_μ ⟷ space-time coord.)

"orbifolding"

constraint on matrices such as

$$\Omega_j \Phi \Omega_j^\dagger = \mathrm{e}^{ir_j} \Phi \qquad j = 1, \cdots, d$$

→ d-dimensional lattice field theory

Φ ⇒ lattice field on the link connecting $\vec{x}$ and $\vec{x} + \vec{r}$

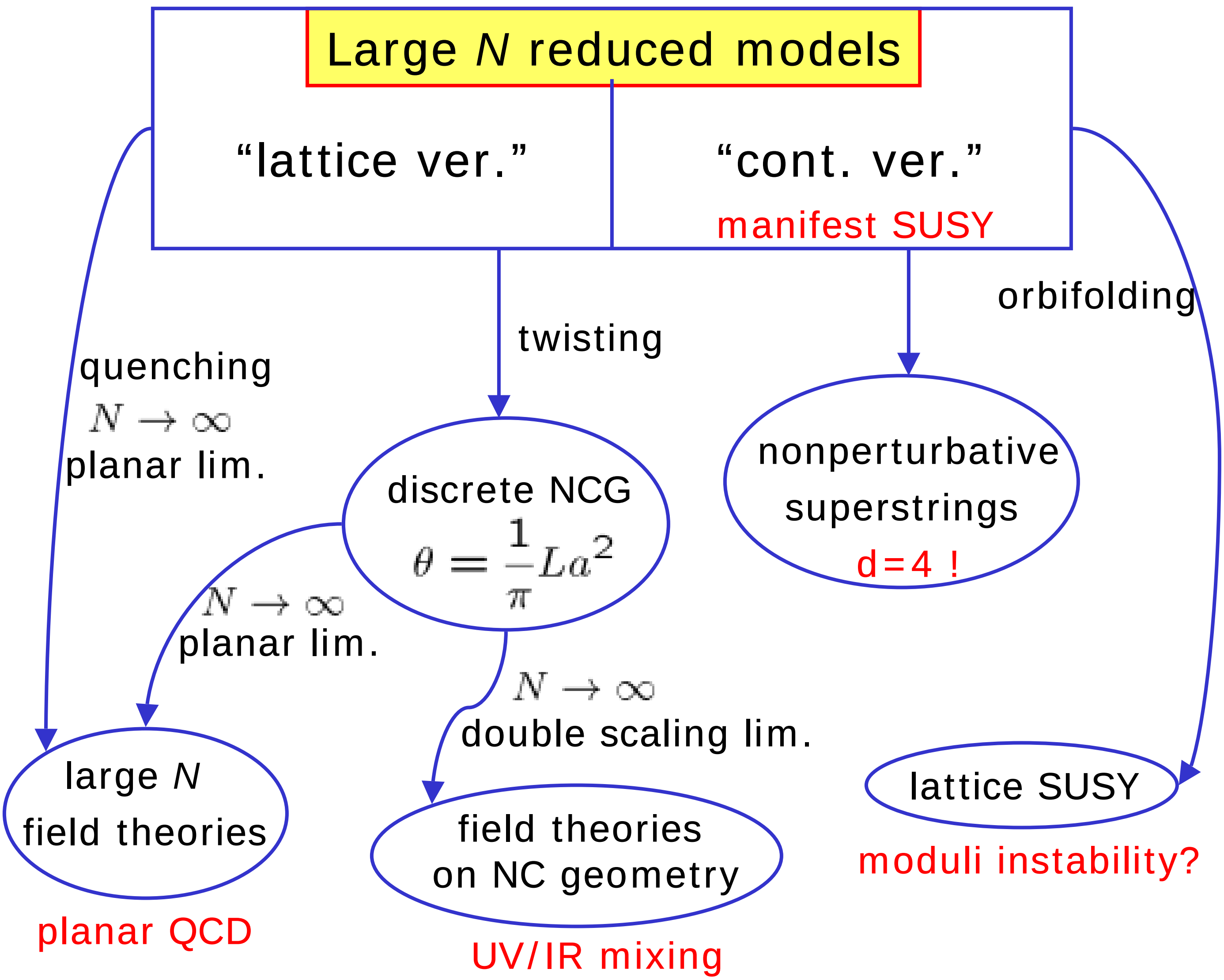
Part of SUSY survive "orbifolding"

→ restoration of full SUSY in the cont. lim. without fine-tuning

Cohen-Kaplan-Katz-Unsal hep-lat/0302017

NC extension : J.N.-Rey-Sugino hep-lat/0301025

6. Summary



Dynamics of eigenvalues

crucial for
- equivalence to large *N* field theories
- dynamical generation of space-time

There are many exciting issues to be explored in this field !