Heavy Quarks & Lattice QCD

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Lattice 2003

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Outline

- Motivation: CKM matrix; spectroscopy
- ☆ Critical review of methods (as charged)
  - Heavy quark discretization effects
  - New developments
- Tests: quarkonium & heavy-light systems
- *B* and *D* decays: status & chiral extrapolation
- Lessons

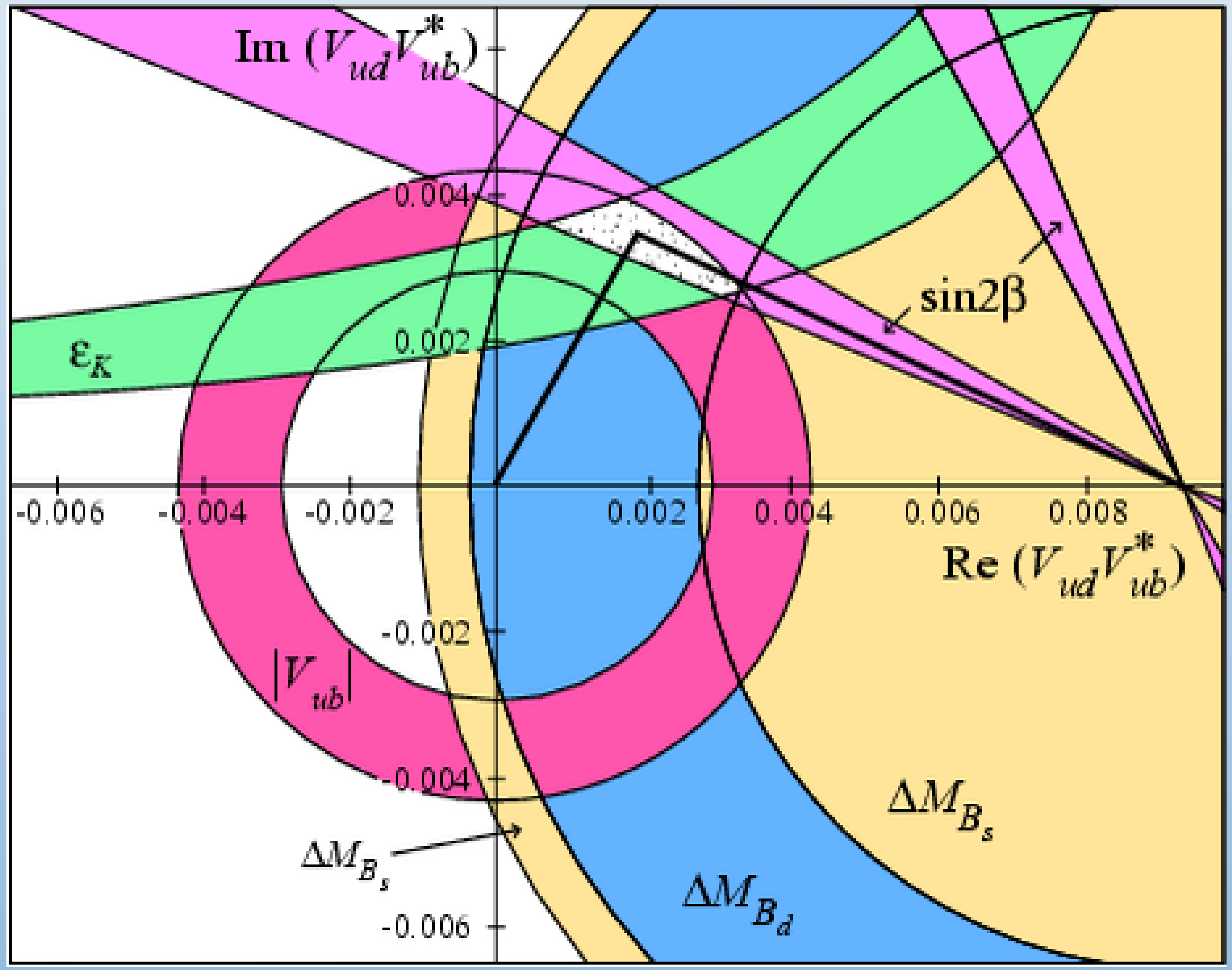
Motivation

Motivation: CKM

- "Standard UT fit is now entirely in the hands of Lattice QCD (up to, perhaps, $|V_{ub}|$)"
  Martin Beneke (Lattice 2001, Berlin)
-
- Are there non-KM sources of *CP*V in *B* and *K* mixing? In rare decays?

Unitarity Triangle

- Are the error bands reliable?
- Are *our* error bands reliable?
- To diagnose new physics?

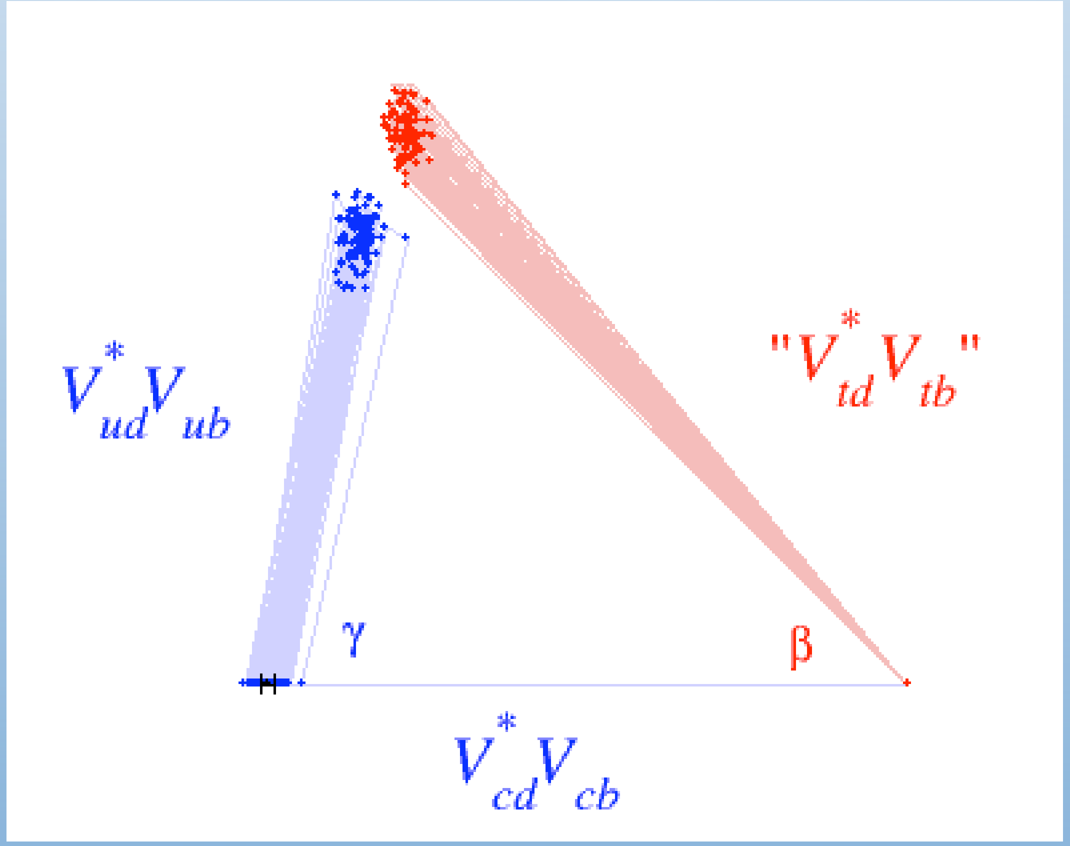


PDG 2002

MATRIX RELOADED

- $|V_{cd}|$ from f_D, $f_+^{D\to\pi}(E_\pi)$
- $|V_{cb}|$ from $\mathcal{F}^{B\to D^*}(1)$
- $|V_{ub}|$ from $f_+^{B\to\pi}(E_\pi)$
- $|V_{ud}|$ from $F_1^{n\to p}$
- $|V_{td}|$ “from” $f_B^2 B_B$

all gold-plated
(up to chiral extrapolation)



Spectroscopy, *etc.*

- D_s spectroscopy ➥ BaBar & CLEO 0^+ & 1^+ states

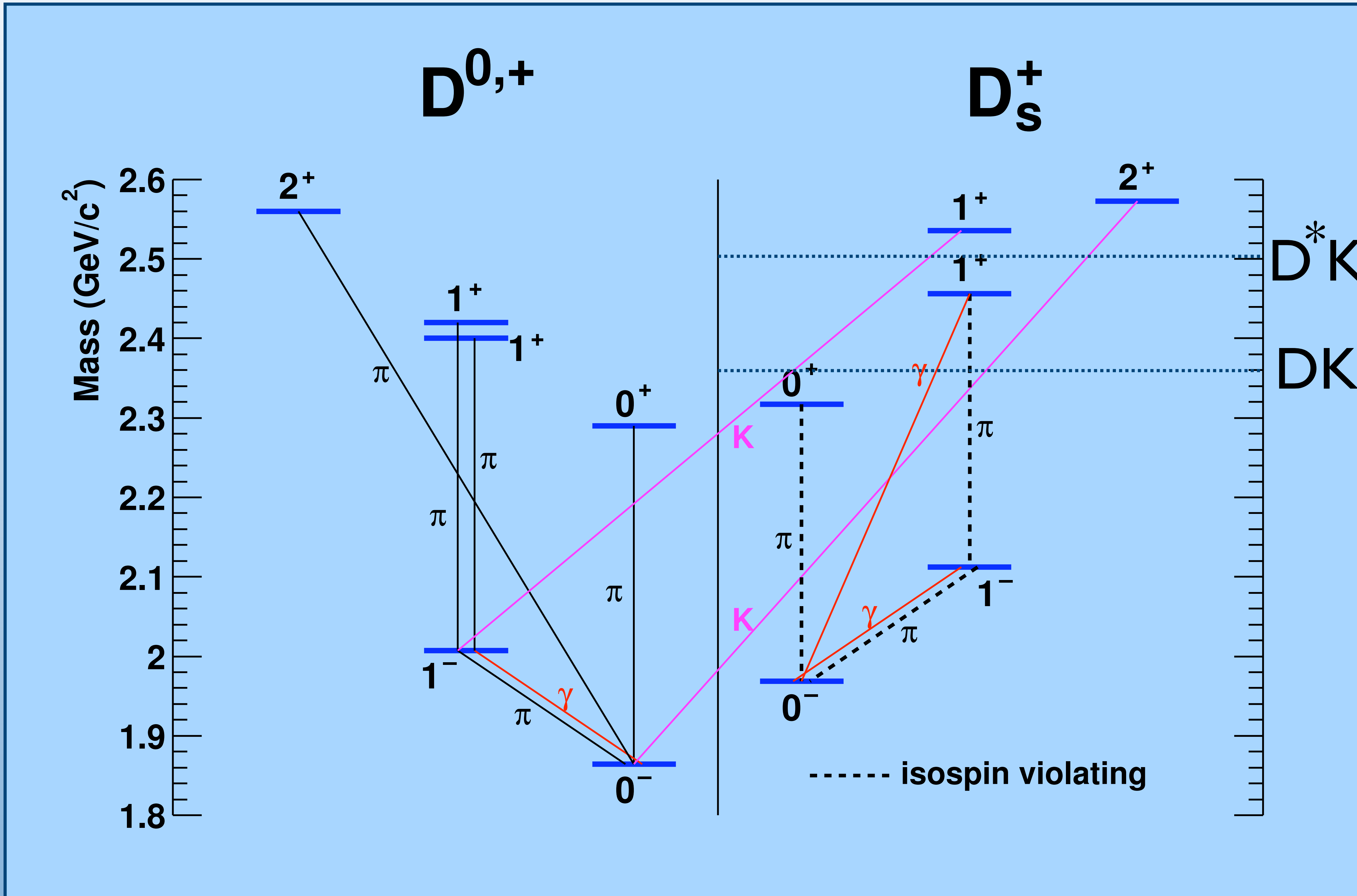
  `Bali, hep-ph/0305209`
  `Dougall et al., hep-lat/0307001`
  `Koponen, HQ.pstr`
  `Mackenzie, HQ.I`

- *ccl* spectroscopy ➥ SELEX states

  `Flynn, Mescia, Tariq, hep-lat/0307025`

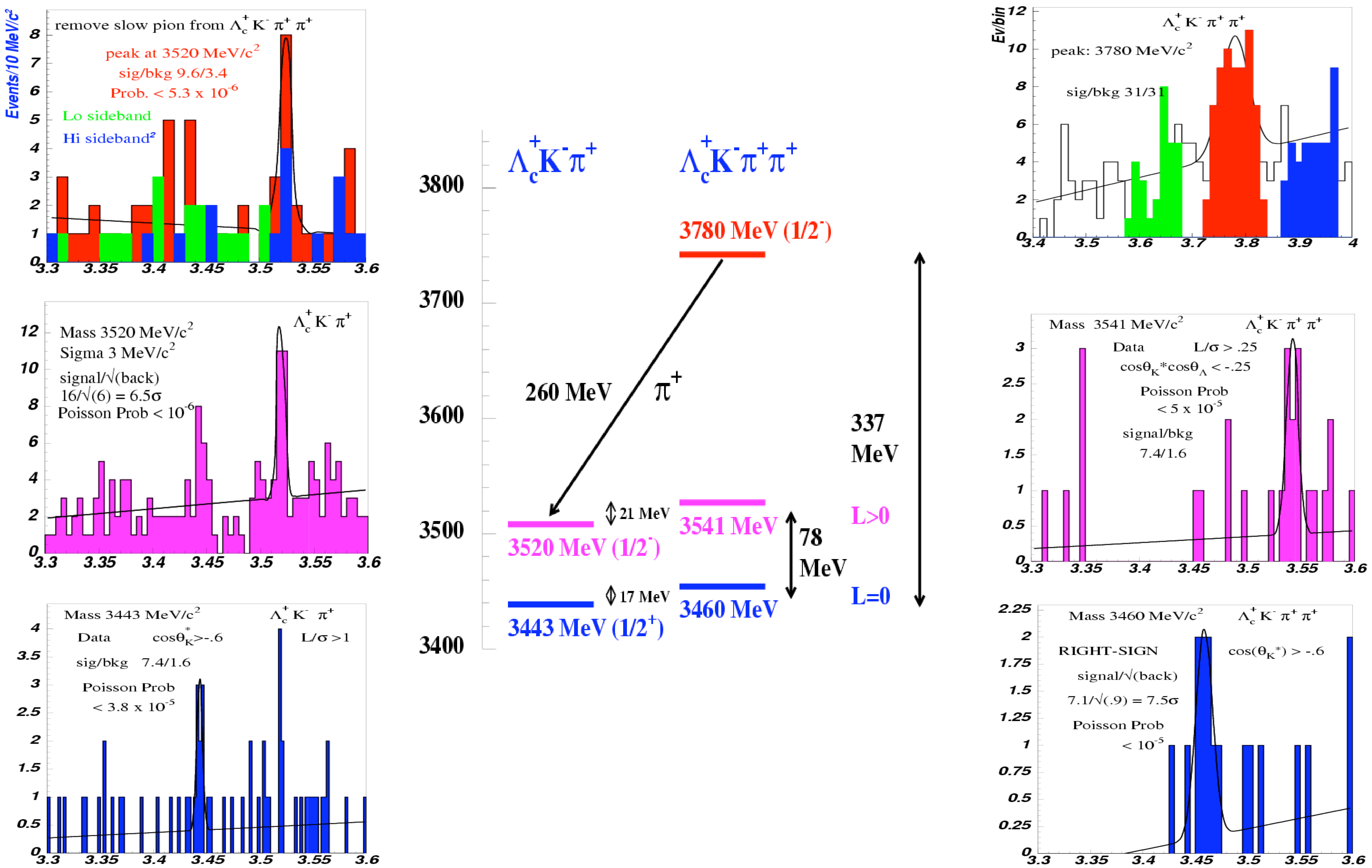
- $\Box_Q$-$\Box_Q$ potential ➥ mid-range deuteron potential

  `Arndt, Beane, Savage, nucl-th/0304004`


$D^{0,+}$
D_s^+
Mass (GeV/c^2)
2.6
2.5
2.4
2.3
2.2
2.1
2
1.9
1.8
2^+
1^+
1^+
0^+
1^-
0^-
2^+
1^+
1^+
0^+
1^-
0^-
π
γ
K
D^*K
DK
isospin violating

SELEX Doubly Charmed Baryon States

An excited state and pair of isodoublets?



psc 13 Jun 2003

Lesson I

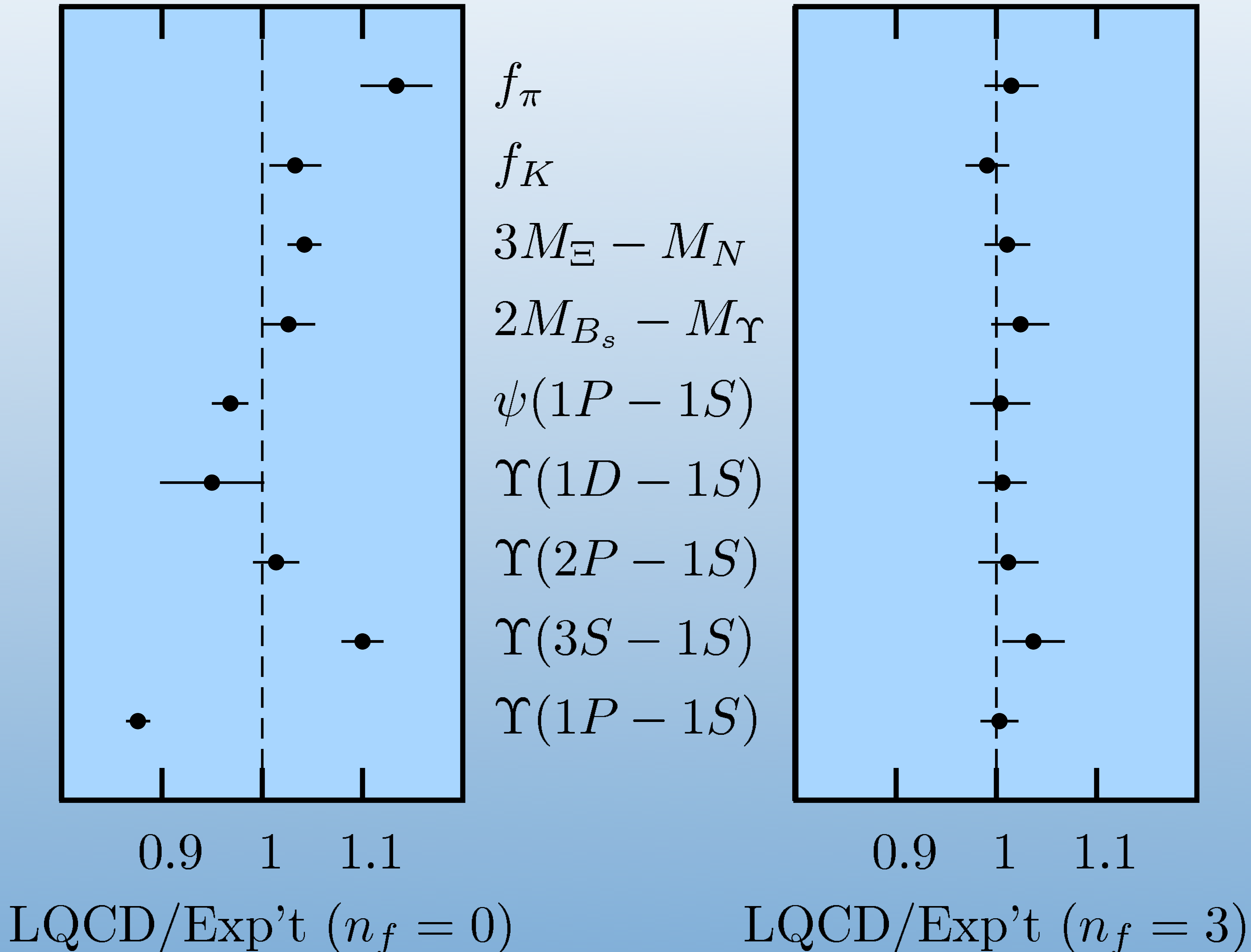
- Flavor physics demands from us full and reliable estimates of all uncertainties, *yet* when we are done, the total error budget must be small.

Three Concerns

- Quenched approximation—going away.
- Discretization effects, because $m_b a \not\ll 1$
- Chiral extrapolations: when is m_q small enough?

Unquenched QCD

Davies et al., hep-lat/0304004



Heavy Quark Methods

Matrix of Methods

Discretization

heavy quark
- (improved) Wilson
- static + insertions
- lattice NRQCD
- anisotropy: $a_t < a_s$
- overlap
- domain-wall

light quark (in $\bar{q}Q$)
- Wilson
- staggered
- Ginsparg-Wilson

EFT Tools

$a \neq 0$

Symanzik LE$\mathcal{L}$
- for $m_Q a \ll 1$
- for $m_Q a \not\ll 1$

HQET (for $\bar{q}Q$)

NRQCD (for $\bar{Q}Q$)

$m_q \gg m_d$

Heavy Meson χPT

Renormalization

or “matching”

Perturbative
- tadpole tree-level
- 1- or 2-loop

Non-perturbative

Combination

$Z_A = \rho_A^{PT}\, Z_V^{NP}$

No tadpoles or KLM

for all $m_Q a$

Extrapolation Method

Discretization

heavy quark
- (improved) Wilson
- static + insertions
- lattice NRQCD
- anisotropy: $a_t < a_s$
- overlap
- domain-wall

light quark (in $\bar{q}Q$)
- Wilson
- staggered
- Ginsparg-Wilson

EFT Tools

$a \neq 0$

Symanzik LE$\mathcal{L}$
- for $m_Q a \ll 1$
- for $m_Q a \not\ll 1$

& extrapolate

HQET (for $\bar{q}Q$)

NRQCD (for $\bar{Q}Q$)

$m_q \gg m_d$

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or “matching”

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Non-perturbative

Combination

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No tadpoles or KLM

for all $m_Q a$

Lattice HQET

Discretization

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- lattice NRQCD
- anisotropy: $a_t < a_s$
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light quark (in $\bar{q}Q$)

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Lattice NRQCD

Discretization

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- static + insertions
- lattice NRQCD
- anisotropy: $a_t < a_s$
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EFT Tools

$a \neq 0$

Symanzik LE$\mathcal{L}$
- for $m_Q a \ll 1$
- for $m_Q a \not\ll 1$

HQET (for $\bar{q}Q$)

NRQCD (for $\bar{Q}Q$)

$m_q \gg m_d$

Heavy Meson □PT

Renormalization or "matching"

Perturbative
- tadpole tree-level
- 1- or 2-loop

Non-perturbative

Combination

$Z_A = □_A^{\mathrm{PT}}\, Z_V^{\mathrm{NP}}$

No tadpoles or KLM

for all $m_Q a$

Fermilab Method

Discretization

heavy quark
- (improved) Wilson
- static + insertions
- lattice NRQCD
- anisotropy: $a_t < a_s$
- overlap
- domain-wall

light quark (in $\bar{q}Q$)
- Wilson
- staggered
- Ginsparg-Wilson

EFT Tools

$a \neq 0$

Symanzik LE$\mathcal{L}$
- for $m_Q a \ll 1$
- for $m_Q a \not\ll 1$

HQET (for $\bar{q}Q$)

NRQCD (for $\bar{Q}Q$)

$m_q \gg m_d$

Heavy Meson □PT

Renormalization or “matching”

Perturbative
- tadpole tree-level
- 1- or 2-loop

Non-perturbative

Combination

$Z_A = □_A^{PT} Z_V^{NP}$

No tadpoles or KLM

for all $m_Q a$

(Perceived) Problems

- Extrapolation method ($m_Q < m_c$; $m_Q^{-1} \quad m_b^{-1}$)
  - ($a \neq 0$) extrapolation amplifies $(m_Q a)^n$ uncertainties
  - ($a = 0$) heavy-quark theory breaks down for $m_Q < m_c$
- Lattice NRQCD
  - perturbative matching
  - power-law divergences as $a \quad 0$

(Perceived) Problems II

- Lattice HQET
  - power-law divergences as $a \quad 0$
  - no non-perturbative matching of $1/m_Q$ yet
- Fermilab method
  - perturbative matching & "renormalon shadows"
  - "$O(a^n)$" effects not yet a^n

Cutoff Effects

- A theory of cutoff effects that applies to all methods is needed.
- Symanzik is not enough.
- A theory based on HQET/NRQCD is available:
  - `hep-lat/0002008`
  - `hep-lat/0112044`, `hep-lat/0112045`
  - `hep-lat/0205021` (*Handbook of QCD*, Vol. 4)

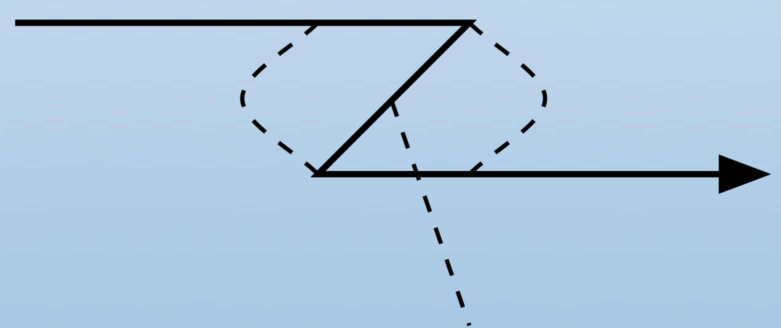
Effective Field Theory

- Elementary-particle theory is imbued with this notion:
  - at energies □ below some scale □, particles with $E >$ □ have small effects, suppressed by (□/E)^n
  - analytic properties of Green functions are impervious to off-shell particles [Coleman-Norton theorem]
  - field theory gives general description respecting analyticity, unitarity, *etc.* [Weinberg]

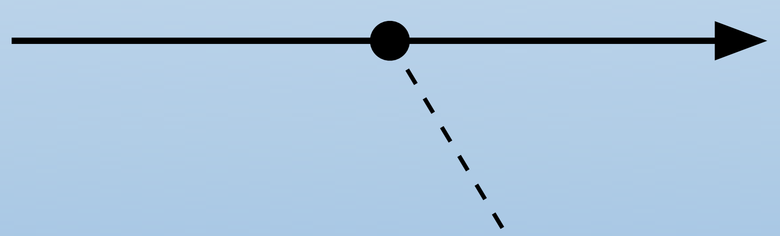
Coleman-Norton

- Singularities in Green functions appear where, and only where, particles go on shell:

≡ diagram reduced diagram



$$= \frac{z(\mu/m_Q)}{m_Q^n} \times$$



- singularities are reproduced if off-shell lines are shrunk to a point: reduced diagrams ~ diagrams of an effective field theory

Heavy Quark Theory

- Heavy quarks have $m_Q \gg \Lambda_{QCD}$ (by definition)
  - zig-zags and pair production suppressed
  - $\Rightarrow$ fields $h_v^{(+)}$, $h_v^{(-)}$
- One heavy quark: static source $\Rightarrow$ HQET
- Two heavy quarks: binary system $\Rightarrow$ NRQCD
- EFT: separate m_Q from soft scales Λ, $m_Q v^n$
- Grinstein established to all orders PT w/o rigor

Local Effective $\mathcal{L}$

$$\mathcal{L}_{\mathrm{QCD}} \doteq \mathcal{L}_{\mathrm{HQ}}$$

$$\begin{aligned}\mathcal{L}_{\mathrm{HQ}} &= \mathcal{L}_{\mathrm{light}} - \bar{h}_v(m_1 + iv\cdot D)h_v \\ &\quad + \frac{\bar{h}_v D_\perp^2 h_v}{2m_2} + z_B(\mu)\frac{\bar{h}_v s_{\mu\nu} B^{\mu\nu} h_v}{2m_2} \\ &\quad + z_{\mathrm{D}}(\mu)\frac{\bar{h}_v D_\perp \cdot E h_v}{4m_2^2} + z_{\mathrm{s.o.}}(\mu)\frac{\bar{h}_v s_{\mu\nu} D_\perp^\mu E^\nu h_v}{4m_2^2} \\ &+ \cdots \\ &= \sum_i \quad \mathcal{C}_i(m_Q, m_Q/\mu) \quad \mathcal{O}_i(\mu/\Lambda)\end{aligned}$$

short distances: $1/m_Q$, a: lumped into coefficients

long distances: 1/□, L: described by operators

but for quarkonium

$$\mathcal{L}_{\rm QCD} \doteq \mathcal{L}_{\rm HQ}$$

$$\mathcal{L}_{\rm HQ} = \mathcal{L}_{\rm light} - \bar{h}_v(m_1 + iv\cdot D)h_v + \frac{\bar{h}_v D_\perp^2 h_v}{2m_2}$$
$$+z_B(\mu)\frac{\bar{h}_v s_{\mu\nu}B^{\mu\nu}h_v}{2m_2} - z_R(\mu)\frac{\bar{h}_v (D_\perp^2)^2 h_v}{8m_2^3}$$
$$+z_{\rm D}(\mu)\frac{\bar{h}_v D_\perp\cdot E h_v}{4m_2^2} + z_{\rm s.o.}(\mu)\frac{\bar{h}_v s_{\mu\nu}D_\perp^\mu E^\nu h_v}{4m_2^2}$$
$$+ \cdots$$

$$\doteq \sum_i \quad \mathcal{C}_i(m_Q, m_Q/\mu) \quad \mathcal{O}_i(\mu/m_Q v^n)$$

short distances: $1/m_Q$, a: lumped into coefficients

long distances: $1/m_Q$□^{n}, L: described by operators

HQET vs. NRQCD

what	HQET	NRQCD
hadrons	heavy-light	quarkonium
leading term	static limit	kinetic energy
heavy-quark symmetries	spin & flavor	spin
power counting	$(\Box/m_Q)^n$	$\Box^n$
m_Q dependence	power law *and* log: $z(m_Q/\Box)$	essentially log: $\Box \sim \Box_s(m_Q)$

Almost nothing is known
about heavy quarks
(in bound states)
without these and allied
ideas for inclusive decays
(OPE, SCET).

Symanzik EFT

- Years ago, Symanzik introduced a (continuum) effective field theory to describe cutoff effects
  - for quarks: fields q satisfying the Dirac equation

$$
\begin{aligned}
\mathcal{L}_{\mathrm{LGT}} &\doteq \mathcal{L}_{\mathrm{Sym}} \\
\mathcal{L}_{\mathrm{Sym}} &= \mathcal{L}_{\mathrm{QCD}}(g^2, m_q; \mu) + K_{\sigma F}\ \bar{q} i \sigma_{\mu\nu} F^{\mu\nu} q + \cdots \\
&= \mathcal{L}_{\mathrm{QCD}} + \textstyle\sum_i K_i(a, g^2, m_q a; \{c_j\}; \mu a) \quad O_i(\mu/\Lambda)
\end{aligned}
$$

short — long, *e.g.*, L

- EFT: separate a^{-1} from soft scales □, m_q
- Reisz Theorem to all orders PT w/ rigor

- For light quarks, or heavy quarks with $m_Q a \ll 1$, one can expand the coefficients in $(m_Q a)^n$
  - then, Symanzik LE$\mathcal{L}$ yields an a expansion
  - but we will not see $m_b a \ll 1$ for a long time
- For $m_b a \not\ll 1$ the $(m_Q a)$-expansion breaks down
  - lattice gauge theory does not break down!
  - the Symanzik LE$\mathcal{L}$ does not break down!!

- The split "QCD + small corrections" does break down!!!

  - Exploit redundant directions, or use the eq'ns of motion, to eliminate $\bar{Q}(\gamma_4 D_4)^n X\, Q$. One finds

$$\begin{aligned}\mathcal{L}_{\text{Sym}} &= \mathcal{L}_{\text{light}} - \bar{Q}\left(m_1 + \gamma_4 D_4 + \sqrt{\frac{m_1}{m_2}}\gamma\cdot D\right) Q \\ &+ \text{ small corrections}\end{aligned}$$

  - The ugly term breaks relativistic invariance.

- This LE$\mathcal{L}$ is not very useful unless $m_1 = m_2$.

HQET & NRQCD II

- LGT with Wilson quarks has the same degrees of freedom and heavy-quark symmetries as QCD
  - ≡ lattice HQET and lattice NRQCD do too
- All 3 may be described by HQET (for heavy-light systems) and 2/3 by NRQCD (for quarkonium).
- Logic and structure is the same for LGT as QCD
- Both $1/m_Q$ & a are short distances, lumped into coefficients: $\mathcal{C}_i^{\text{lat}} = \mathcal{C}_i^{\text{lat}}(m_Q, m_Q a; \{c_j\}; \mu a)$

HQET Matching

hep-lat/0002008
hep-lat/0112044
hep-lat/0112045

$$\mathcal{L}_{\text{QCD}} \doteq \mathcal{L}_{\text{HQET}}$$

$$\begin{aligned}\langle L|v\cdot\mathcal{V}|B\rangle &= -C_{V_\parallel}\langle L|\bar{q}h_v|B_v^{(0)}\rangle - B_{V1}\langle L|v\cdot\mathcal{Q}_{V1}|B_v^{(0)}\rangle - B_{V4}\langle L|v\cdot\mathcal{Q}_{V4}|B_v^{(0)}\rangle \\ &- \mathcal{C}_2 C_{V_\parallel}\int d^4x\langle L|T\,\mathcal{O}_2(x)\bar{q}h_v|B_v^{(0)}\rangle^\star - \mathcal{C}_{\mathcal{B}} C_{V_\parallel}\int d^4x\langle L|T\,\mathcal{O}_{\mathcal{B}}(x)\bar{q}h_v|B_v^{(0)}\rangle^\star \\ &+ O(\Lambda^2/m^2)\end{aligned}$$

$$\mathcal{L}_{\text{LGT}} \doteq \mathcal{L}_{\text{HQET}}$$

$$\begin{aligned}\langle L|v\cdot V_{\text{lat}}|B\rangle &= -C^{\text{lat}}_{V_\parallel}\langle L|\bar{q}h_v|B_v^{(0)}\rangle - B^{\text{lat}}_{V1}\langle L|v\cdot\mathcal{Q}_{V1}|B_v^{(0)}\rangle - B^{\text{lat}}_{V4}\langle L|v\cdot\mathcal{Q}_{V4}|B_v^{(0)}\rangle \\ &- \mathcal{C}^{\text{lat}}_2 C^{\text{lat}}_{V_\parallel}\int d^4x\langle L|T\,\mathcal{O}_2(x)\bar{q}h_v|B_v^{(0)}\rangle^\star - \mathcal{C}^{\text{lat}}_{\mathcal{B}} C^{\text{lat}}_{V_\parallel}\int d^4x\langle L|T\,\mathcal{O}_{\mathcal{B}}(x)\bar{q}h_v|B_v^{(0)}\rangle^\star \\ &- K_{\sigma\cdot F} C^{\text{lat}}_{V_\parallel}\int d^4x\langle L|T\,\bar{q}i\sigma Fq(x)\bar{q}h_v|B_v^{(0)}\rangle^\star + O(\Lambda^2a^2b(ma))\end{aligned}$$

normalize with $Z_V = C_{V_\parallel}/C^{\text{lat}}_{V_\parallel}$ &
adjust $\mathcal{C}^{\text{lat}}_2 = \mathcal{C}_2, \mathcal{C}^{\text{lat}}_{\mathcal{B}} = \mathcal{C}_{\mathcal{B}}, Z_V B^{\text{lat}}_{Vi} = B_{Vi}$

Summary so far

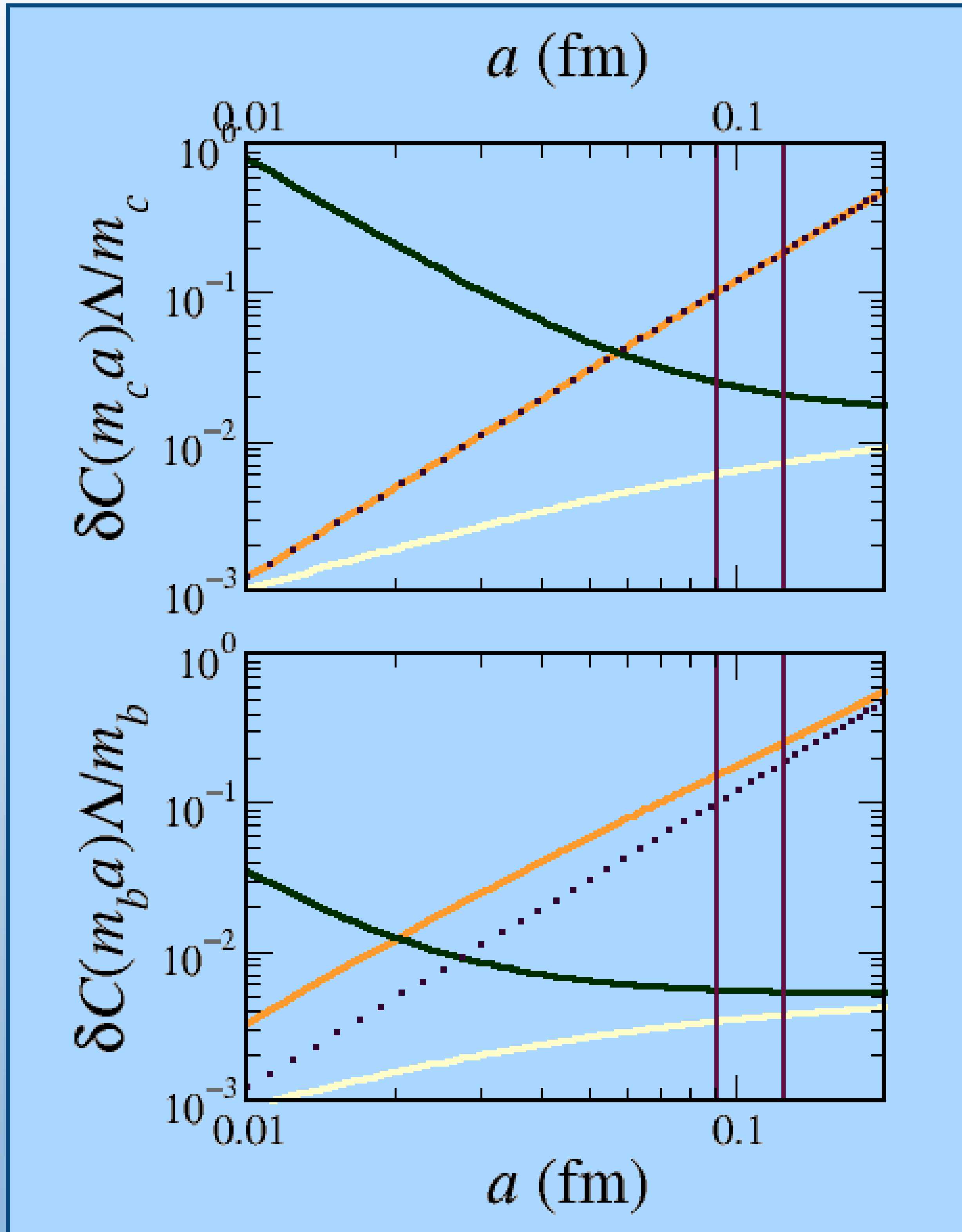
- Symanzik LE$\mathcal{L}$ not so useful when $m_b a \not\ll 1$
- HQET with coefficients $\mathcal{C}^{\mathrm{lat}}(m_Q a)$ is useful
  - latHQET & latNRQCD $\mathcal{C}^{\mathrm{lat}}$ blow up for $m_Q a \ll 1$
  - Wilson (& clover) $\mathcal{C}^{\mathrm{lat}}$ tend to $\mathcal{C}^{\mathrm{cont}}$ for $m_Q a \ll 1$
- Next: analyze the discretization uncertainties in each method studying $\mathcal{C}^{\mathrm{lat}} - \mathcal{C}^{\mathrm{cont}}$

Leading Cutoff Effects

- Clover + Symanzik($m_Q a \ll 1$)
  - $\left[\frac{1}{2m_2} - \frac{1}{2m_1}\right]\Lambda$ from kinetic energy
  - $(m_Q a)^2, \alpha_s (m_Q a)^2, (m_Q a)^3, \ldots$ from currents
- Lattice NRQCD
  - $\alpha_s^2 \left[1 + \frac{1}{4m_Q^2 a^2}\right] \frac{\Lambda}{2m_Q}$ from many sources
- Clover + HQET (Fermilab method)
  - $\alpha_s^2 \frac{\Lambda a}{2(1+m_Q a)}$ from 2-loop mismatch of $\Sigma \cdot \boldsymbol{B}$

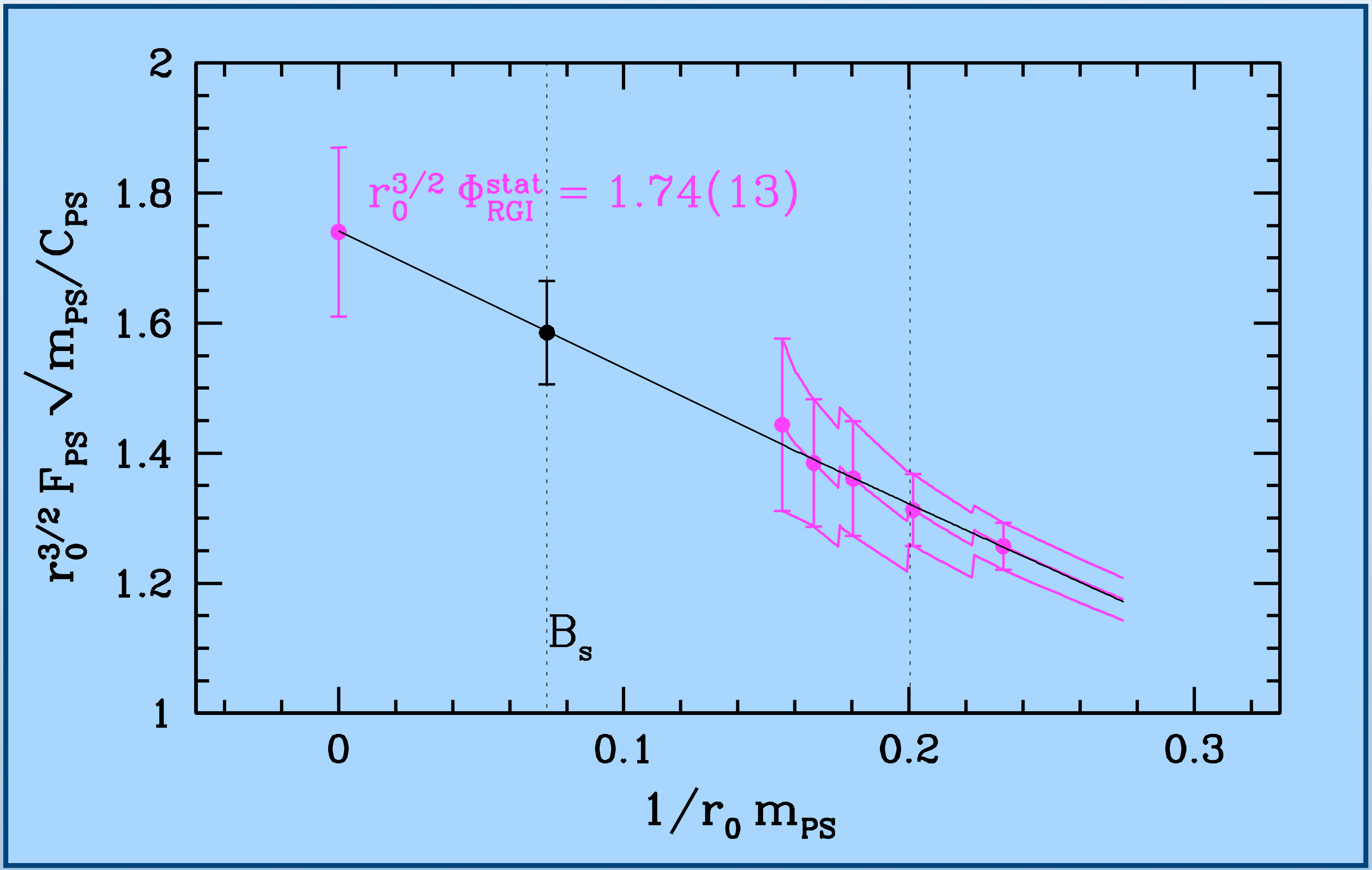
Heavy Quarks

$$\begin{aligned} \Lambda &= 700 \text{ MeV} \\ m_c &= 1400 \text{ MeV} \\ m_b &= 4200 \text{ MeV} \\ \alpha_s &= 0.25 \end{aligned}$$



- clover/SymLE$\mathcal{L}$ beats lattice NRQCD for *b* quark when *a* is 5 times smaller
- ~ 21 years away

First a, then $1/m_Q$



Jüttner, HQ.I
Rolf, HQ.I
Della Morte, HQ.III

Noise reduction
Continuum limit
Inter/Extrapolation

The heavy-quark description will break down when gluons inside the hadron can excite zig-zags (and pairs). The breakdown is smeared out and won't be obvious.

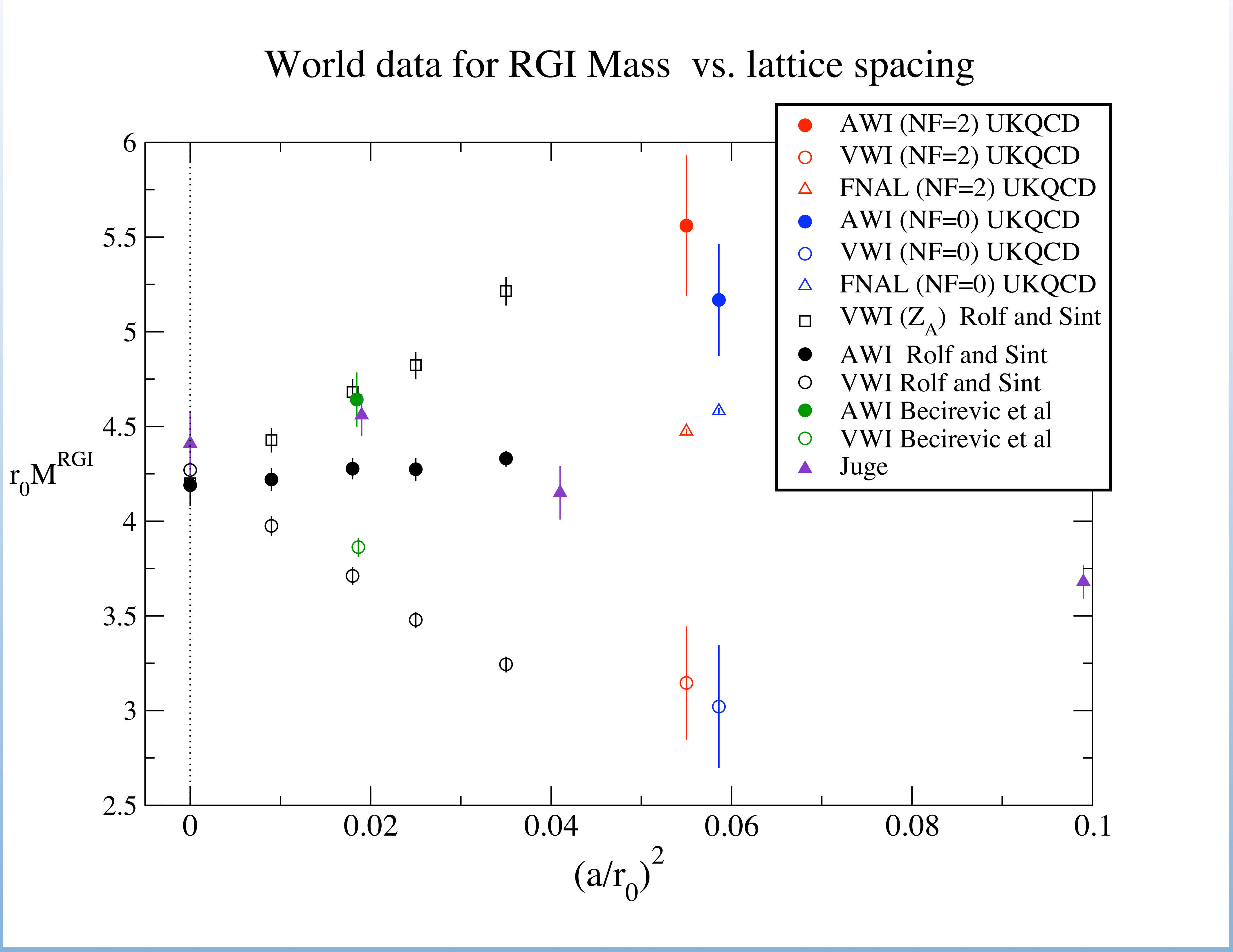
Lesson II

- Extrapolation in m_Q^{-1} is fraught with danger
  - at non-zero a large discretization effects
  - after continuum limit, too close to HQ breakdown
- Dangers are not self-diagnostic.

- The HQET analysis suggests some remedies
  - identify m_Q with m_2, not m_1
  - don't use Alpha's currents (non-leading $m_Q a$ effects are unnecessarily large)
- but this boils down to the Fermilab's "non-relativistic interpretation of Wilson quarks"

Heavy Quarks

Dougall, HSQM.I



Renormalon Shadows

- Renormalons are power-law ambiguities that arise in EFTs and OPEs with mass-independent renormalization schemes: $\mathcal{C}(\mu) \pm \Lambda/\mu$.
  - ambiguities in Wilson coefficients and matrix elements cancel
- At Lattice 2000, Bernard—spurred by work of Martinelli and Sachrajda (M&S)—conjectured that their "shadows" could plague matching conditions in the Fermilab method.

M&S, hep-ph/9605336
Bodwin & Chen, hep-ph/9807492

- M&S consider the following problem:

  - measure (or compute) $\mathcal{P}$ & $\mathcal{Q}$, and then predict $\mathcal{R}$

$$\begin{aligned}
\mathcal{Q}(Q) &= B_1(Q/\mu)\langle f|O_1|i\rangle^{(\mu)} + Q^{-1}B_2(Q/\mu)\langle f|O_2|i\rangle^{(\mu)} \\
\mathcal{P}(Q) &= C_1(Q/\mu)\langle f|O_1|i\rangle^{(\mu)} + Q^{-1}C_2(Q/\mu)\langle f|O_2|i\rangle^{(\mu)} \\
\mathcal{R}(Q) &= D_1(Q/\mu)\langle f|O_1|i\rangle^{(\mu)} + Q^{-1}D_2(Q/\mu)\langle f|O_2|i\rangle^{(\mu)}
\end{aligned}$$

- they ask how well one must compute the Wilson coefficients to attain enough accuracy to make the power corrections worth the bother

- To avoid schemes with renormalons, let us do some simple algebra

$$\mathcal{R} = \frac{1}{2}\left[\frac{D_1}{C_1}\mathcal{P} + \frac{D_1}{B_1}\mathcal{Q}\right] + \left\{D_2 - \frac{1}{2}\left[\frac{D_1}{C_1}C_2 + \frac{D_1}{B_1}B_2\right]\right\}\frac{\mathcal{P}/C_1 - \mathcal{Q}/B_1}{C_2/C_1 - B_2/B_1}$$

"leading twist" "higher twist," formally $O(1/Q)$

- Common sense says to omit higher-twist unless the (renormalon-free) D_1/C_1, D_1/B_1 are accurate enough.

- M & S point out, in effect, that the coefficients in the red numerator must also be accurate enough so that it is $O(1/Q)$ in practice.

- The M&S ambiguity arises from subtracting *non-perturbative* quantities that are normalized by perturbatively calculated coefficients.

- This does not happen in matching calculations.

- Matching poses the following problem

$$\langle J\rangle_0 = C_1^{\text{lat}}(m_Q/\mu)\langle O_1\rangle_0^{(\mu)}$$

$$\langle J\rangle_1 = C_1^{\text{lat}}(m_Q/\mu)\langle O_1\rangle_1^{(\mu)} + m_Q^{-1}C_2^{\text{lat}}(m_Q/\mu)\langle O_2\rangle_1^{(\mu)}$$

$$\langle \mathcal{J}\rangle_0 = C_1^{\text{cont}}(m_Q/\mu)\langle O_1\rangle_0^{(\mu)}$$

$$\langle \mathcal{J}\rangle_1 = C_1^{\text{cont}}(m_Q/\mu)\langle O_1\rangle_1^{(\mu)} + m_Q^{-1}C_2^{\text{cont}}(m_Q/\mu)\langle O_2\rangle_1^{(\mu)}$$

$$\Rightarrow \frac{C_1^{\text{lat}}(c_1)}{C_1^{\text{cont}}} = \frac{\langle J\rangle_0}{\langle \mathcal{J}\rangle_0} \overset{!}{=} 1, \qquad \frac{C_2^{\text{lat}}(c_2)}{C_2^{\text{cont}}} = \frac{\langle J\rangle_0 - \langle J\rangle_1}{\langle \mathcal{J}\rangle_0 - \langle \mathcal{J}\rangle_1} \overset{!}{=} 1$$

- ambiguities from □ and its scheme manifestly cancel and are not inferred onto c_j

- On the other hand, non-perturbative matching calculations do introduce power-law ambiguities.
    - $O(a)$ improvement coefficients (c_{SW}, c_A, c_V, b_A, b_V) inherit ambiguities of order □a, and sometimes a/L, from the $O(a^2)$ errors in the PCAC & CVC relations
- In the end, “renormalon shadow” just expresses the fear that the next order could be unexpectedly large
    - if that’s all you mean, just say so

New Developments

- Roma "*Tor Vergata*" f_B, m_b

```
Guagnelli et al.,   hep-lat/0206023
de Divitiis et al., hep-lat/0305018–Tantalo, HQ.III
                    hep-lat/0307005–Palombi, HQ.II
```

- lattice Lagrangians with $v \neq 0$

```
Foley & Lepage, hep-lat/0209135
Boyle, HQ.II
```

- Heavy-light with staggered light valence quarks

```
Wingate et al., hep-lat/0211014
```

Tor Vergata Method

- A novel application of step-scaling functions

$$\Phi(\infty) = \Phi(L_0)\sigma(L_0)\sigma(2L_0)\sigma(4L_0)$$

$$\sigma(L) = \frac{\Phi(2L)}{\Phi(L)}$$

  - $L_0 = 0.4$ fm: small enough so that $m_b a \ll 1$ is possible
  - $2L_0$: use $1/m_Q$ extrapolation from $m_b/2$
  - $4L_0$: use $1/m_Q$ extrapolation from $m_b/4$; ~ □ volume

- In other words, $m_Q a$ is always small enough to compute continuum limit of each factor.

- How do the uncertainties accumulate?

  - statistics straightforward to combine

  - here we see how extrapolations accumulate

- Basic Ansatz ($\Phi = M$):

  - $M(L) = m + \bar{\Lambda}(L)$: L effects are long distance

This implies

$$\sigma(m, L) = \frac{M(m, 2L)}{M(m, L)} = 1 + \frac{1}{m}[\bar{\Lambda}(2L) - \bar{\Lambda}(L)]$$

but because of extrapolation, one really has

$$\sigma(m, 2^j L) = 1 + \frac{2^j}{\text{“}2^j\text{”}} \frac{1}{m}[\bar{\Lambda}(2^{j+1}L) - \bar{\Lambda}(2^j L)]$$

where 2_j/“2_j” denotes the error in extrapolation.

The beauty of this method is that when extrapolation is worst (larger *j*), the difference between the □s cancels, according to usual asymptotic *L* dependence.

Lessons III & IV

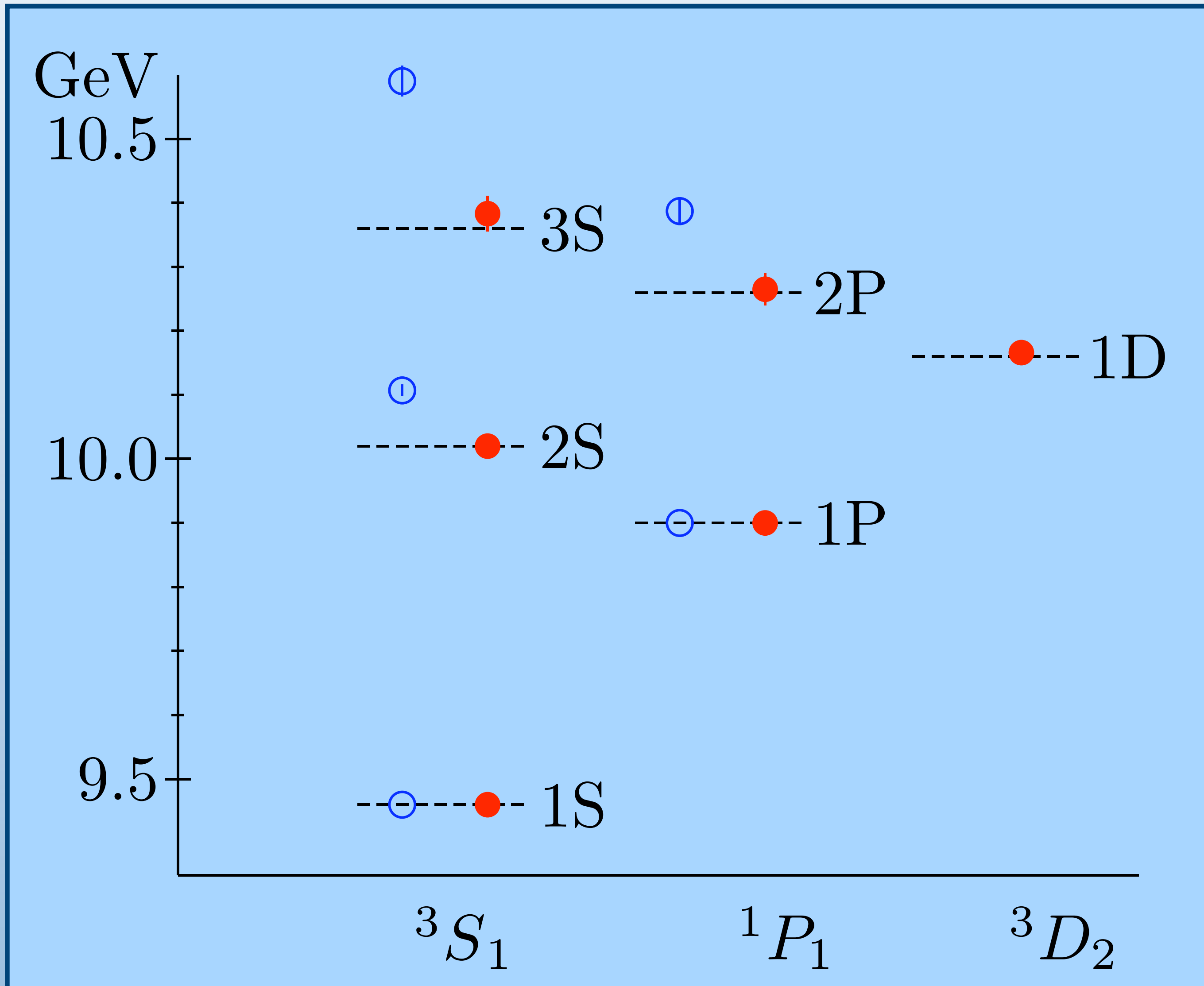
- *Tor Vergata*'s step scaling looks like a relatively safe application of extrapolation.
- No matter where you put HQET/NRQCD into LGT, they are useful for estimating uncertainties.
    - can even teach you that they are smaller than you might have thought.

Quarkonium

Quarkonium as Tests

- Lattice NRQCD and the Fermilab method enjoy the advantage that the parameters (a^{-1}, m_b, m_c) may be tuned & tested with quarkonium, and then applied to heavy-light systems.
- The spectrum of the well-established states test whether we understand the uncertainties.
  - Theory is nice, but explicit calculations are reassuring
- Several slides of spectra follow:

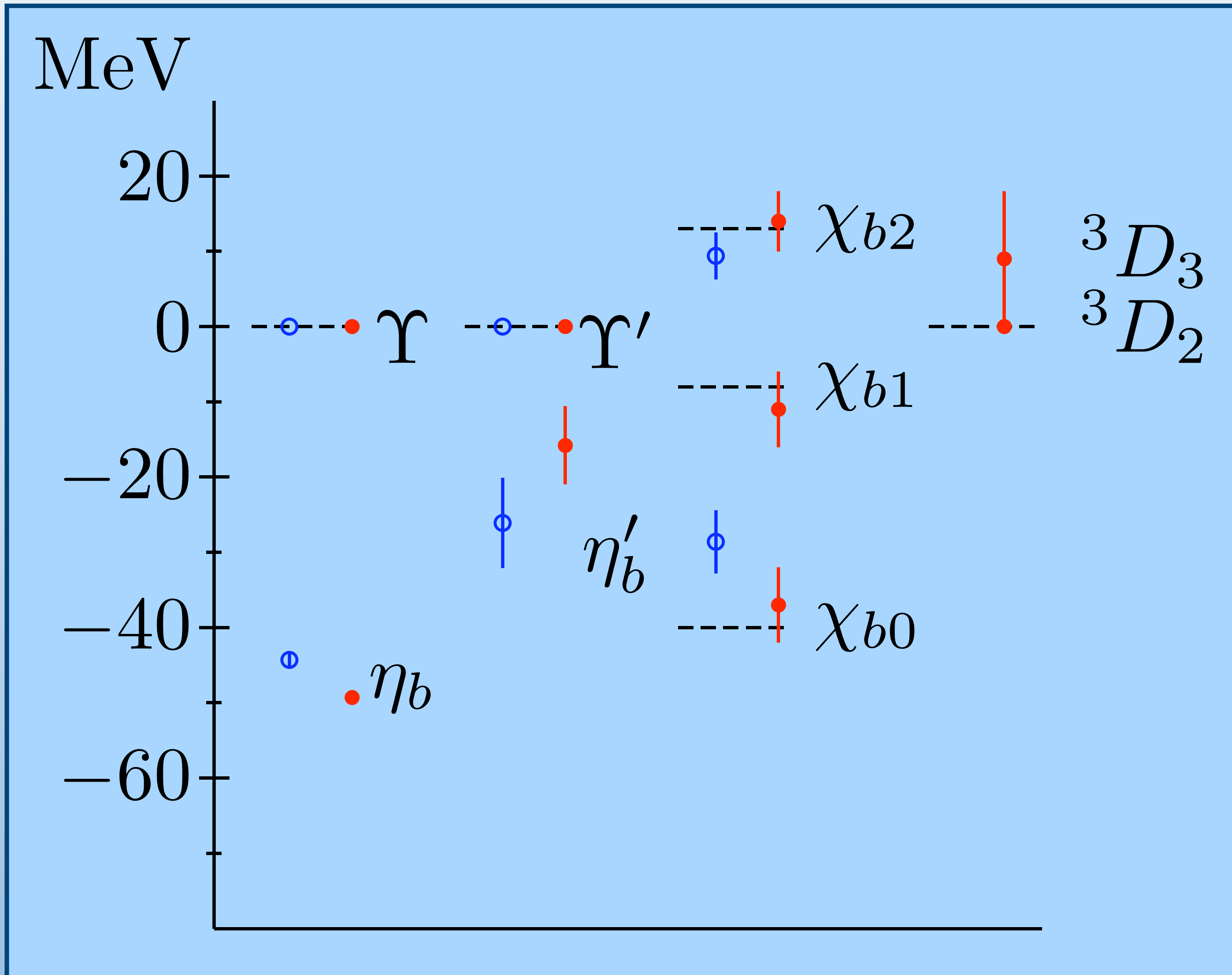
Upsilon Spectrum



--- : Experiment
○ : Quenched MILC
● : 2+1 flavors MILC with $m_{u,d} = m_s/5$.

- HPQCD-Glasgow
- gross structure relatively $O(\square^4)$
- n_f = 2+1 better than quenched
- 3S and 2P states *not* gold-plated

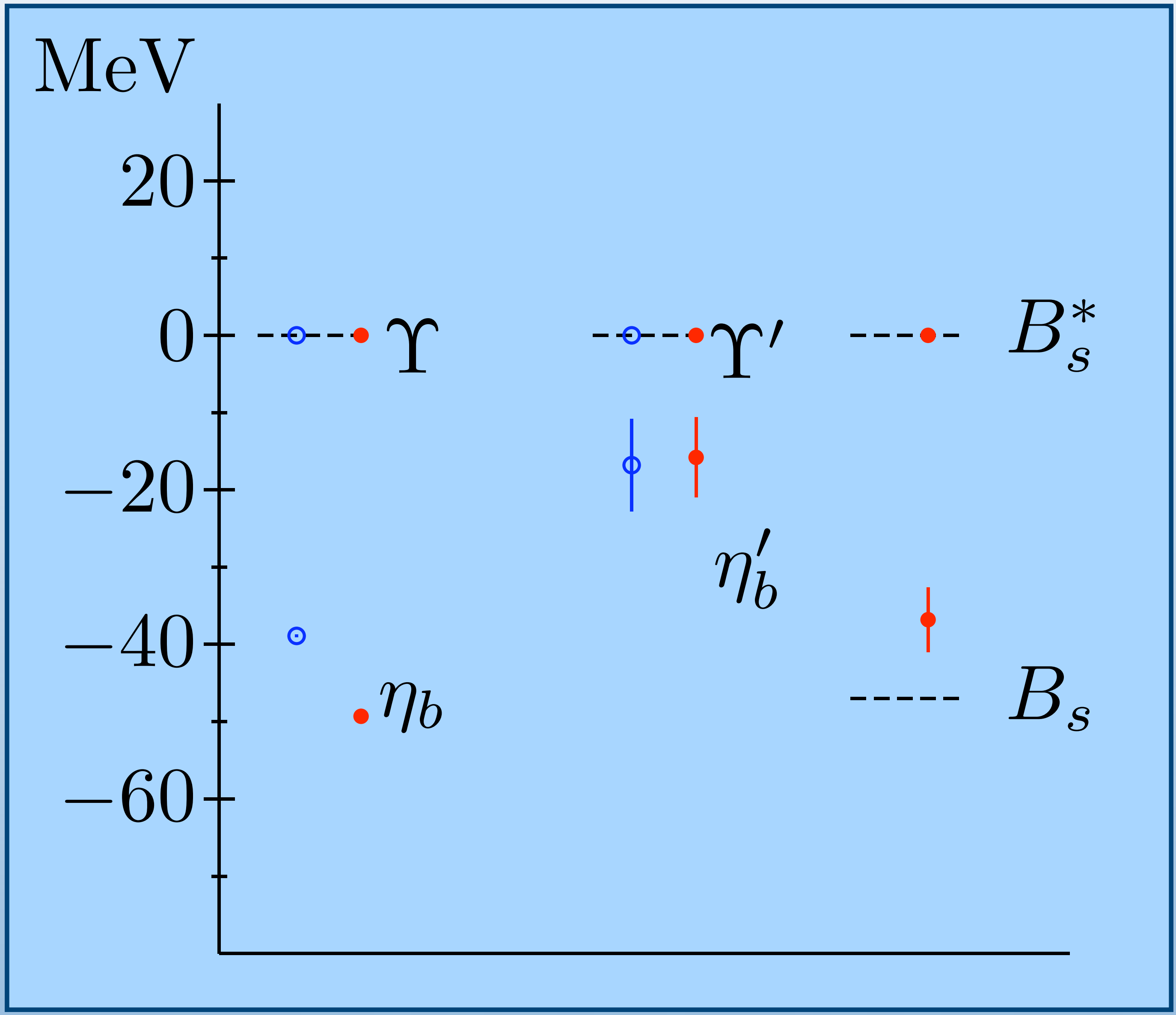
Υ Fine Structure



- HPQCD-Glasgow
- fine structure relatively $O(\square^4)$
- $\square_{bJ}$ states agree better

--- : Experiment
∘ : Quenched
• : 2+1 flavours MILC with $m_{u,d} = m_s/5$.

B_s^*-B_s Splitting



- HPQCD-Glasgow
- $m_\Upsilon - m_{\eta_b}$ is O(□^{4})
- $2m_{B_s} - m_\Upsilon$ :: 1.02
- $m_{B_s^*} - m_{B_s}$ is O(□$_s$□$/m_b$)

≡ need one-loop c_B

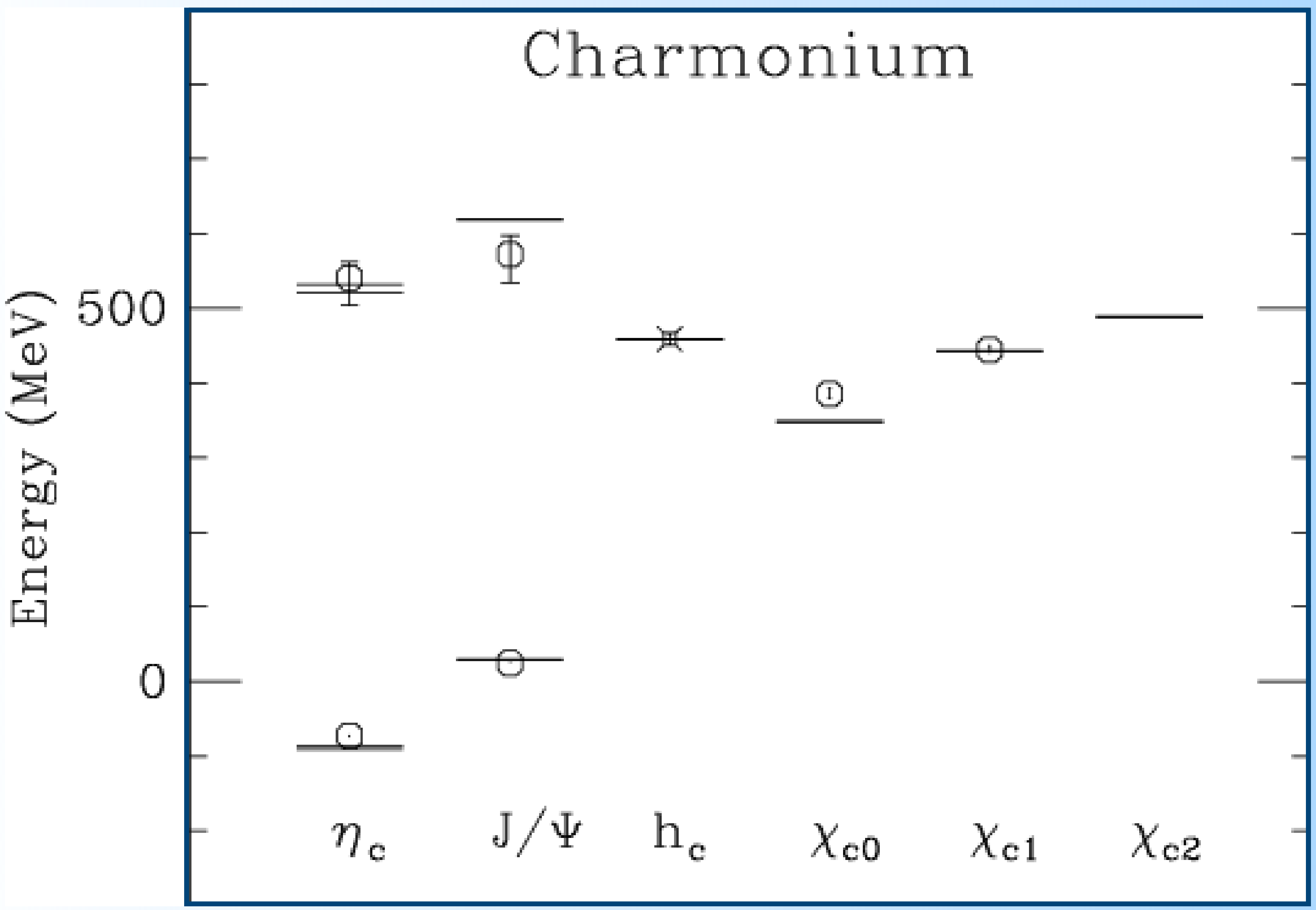
--- : Experiment
∘ : Quenched
• : 2+1 flavours MILC with $m_{u,d} = m_s/5$.

r_0 and r_1

- In the past, the potential scales r_0 and r_1 have been estimated from potential models, *e.g.*, $r_0 = 0.5$ fm.
- The Υ spectrum calculations (indeed everything on the ratio plot) imply different values

$$r_0 = 0.46(1) \text{ fm from } 0.462(9)_{\text{coarse}}, 0.462(9)_{\text{fine}}$$
$$r_1 = 0.36(1) \text{ fm}$$

Charmonium Spectrum



coarse MILC 2+1 again
□(1P-1S) sets a^{-1}

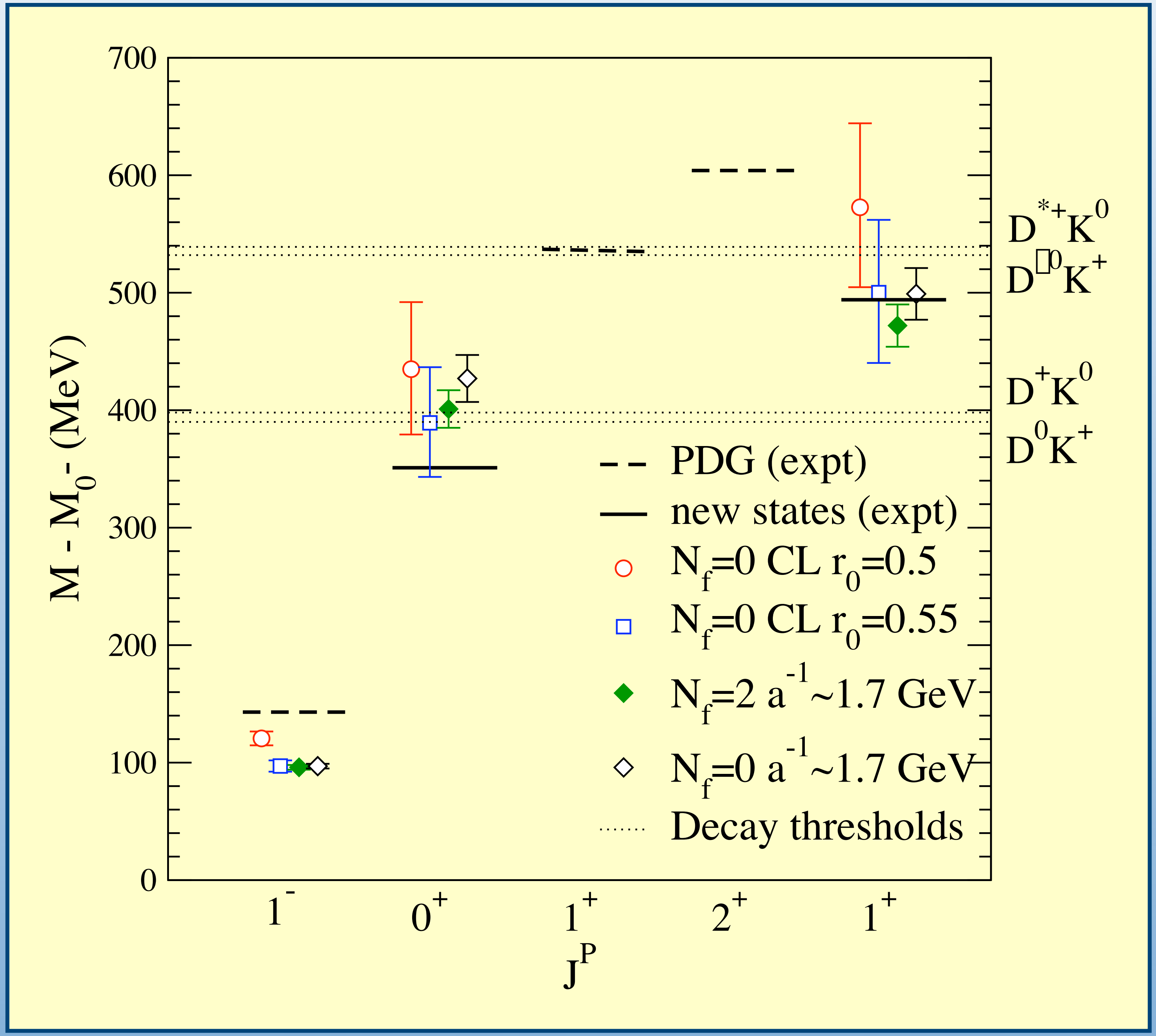
- Fermilab `Simone, HQ.II`
- gross structure $O(\square^4)$
- fine and hyperfine splittings $O(\square^2)$ and t.i. tree-level c_B & c_E
  - more improvement needed `Oktay, HQ.III`
  - one loop needed `Nobes, HQ.III`

Lesson V

- With unquenched gauge fields, the successes and shortcomings of the spectrum make sense using NRQCD/HQET power-counting estimates.
- Further improvements are needed, *e.g.*, one-loop c_B, and Oktay's improvements to Fermilab action

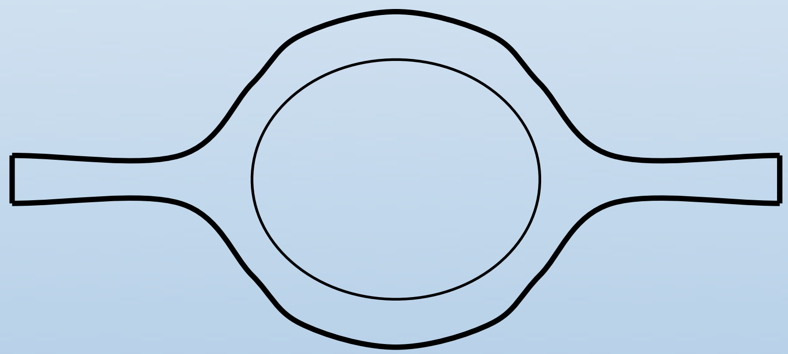
D_s Spectrum

Dougall et al., hep-lat/0307001



Threshold Effects

- $D_s(0+)$ & $D_s(1+)$ are close to open thresholds (similarly for □(2S)). There is some interaction:



$$D_s(0+) \to (DK)_{\text{off shell}} \to D_s(0+)$$

which is weakened when $m_q > m_d$

- m_q dependence of $m_{D_s}(q\bar{q} \text{ sea})$, say, should be flat until $m_{D_q} + m_{K_q}$ approaches and pushes it

Semi-Leptonic Decays

Many Combos

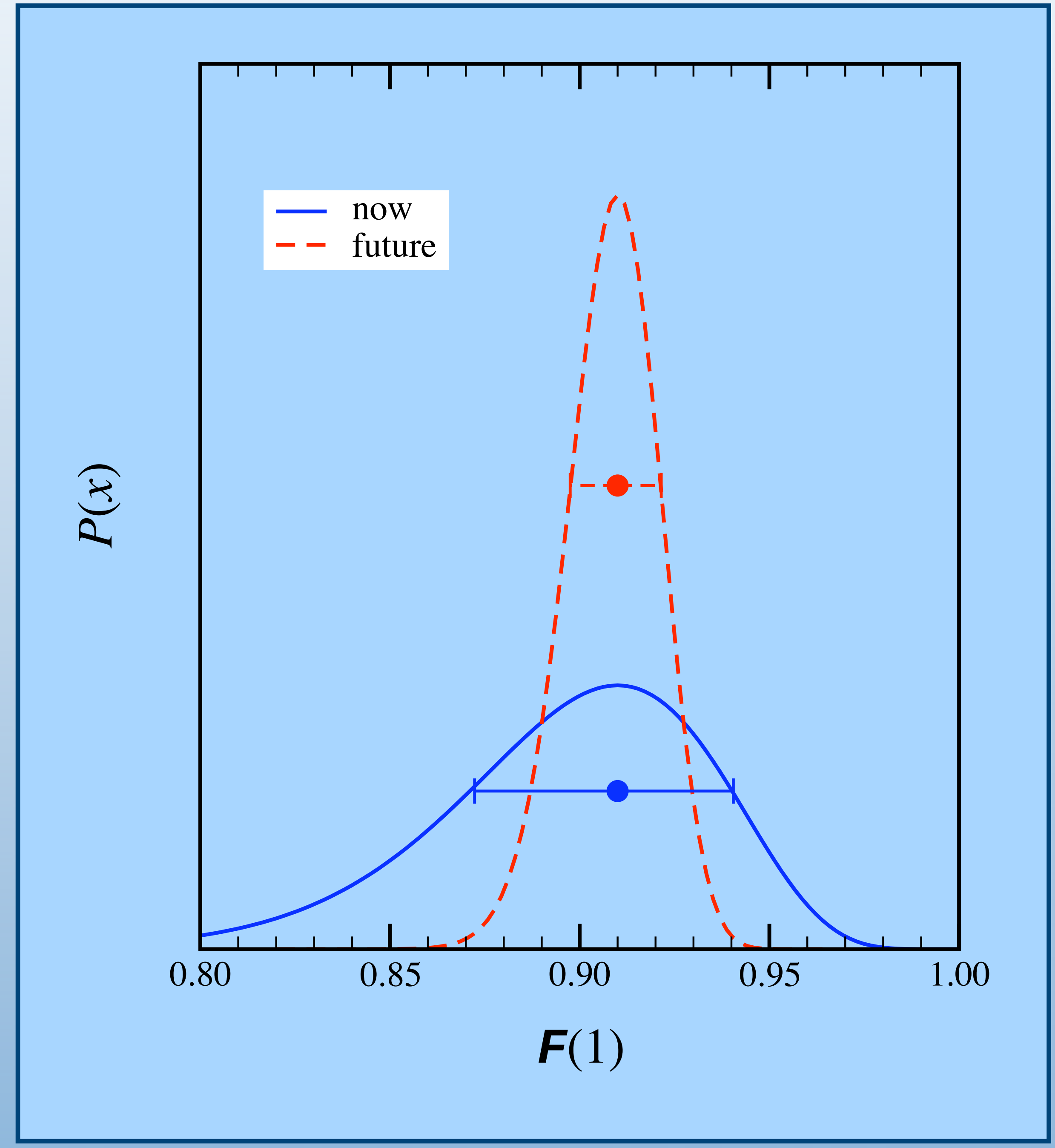
- Semi-leptonic decays are a key way to determine the top two rows of the CKM matrix.
- They are (quark-level) tree decays, so they are unlikely to be sensitive to non-Standard physics.
- Lattice QCD calculates the hadronic form factors, $f_+(q^2)$, $f_+(E_\pi)$, $\mathcal{F}(w)$, *etc.*, from matrix elements $\langle\pi|V^\mu|K\rangle$, $\langle\pi|A^\mu|B\rangle$, and $\langle D^{(*)}|J^\mu|B\rangle$.

B $D^{*}l$□ and $|V_{cb}|$

- hep-ph/0110253 relies on HQET
- Error:

$$\sim 35\% \text{ of } (\mathcal{F}(1) - 1)$$
$$\Rightarrow 4\% \text{ of } \mathcal{F}(1)$$

- $n_f = 2 + 1$ desired



$$\mathcal{F}_{B\to D^*}(1) = 0.913^{+0.024}_{-0.017} \pm 0.016^{+0.003}_{-0.014}{}^{+0.000}_{-0.016}{}^{+0.006}_{-0.014}$$

B □l□ and $|V_{ub}|$

- CLEO says, “B □ is as easy as pie, but B □ is a tough row to hoe!”
- From HQS and □S, it is natural to consider

$$
\begin{aligned}
f_{\|}(E_\pi) &\propto \langle \pi | V^4 | B \rangle \\
f_{\perp}(E_\pi) &\propto \langle \pi | V^j | B \rangle / p_j
\end{aligned}
$$

- CLEO-c will measure $f_+(E_\pi)$ in D decay “soon”.

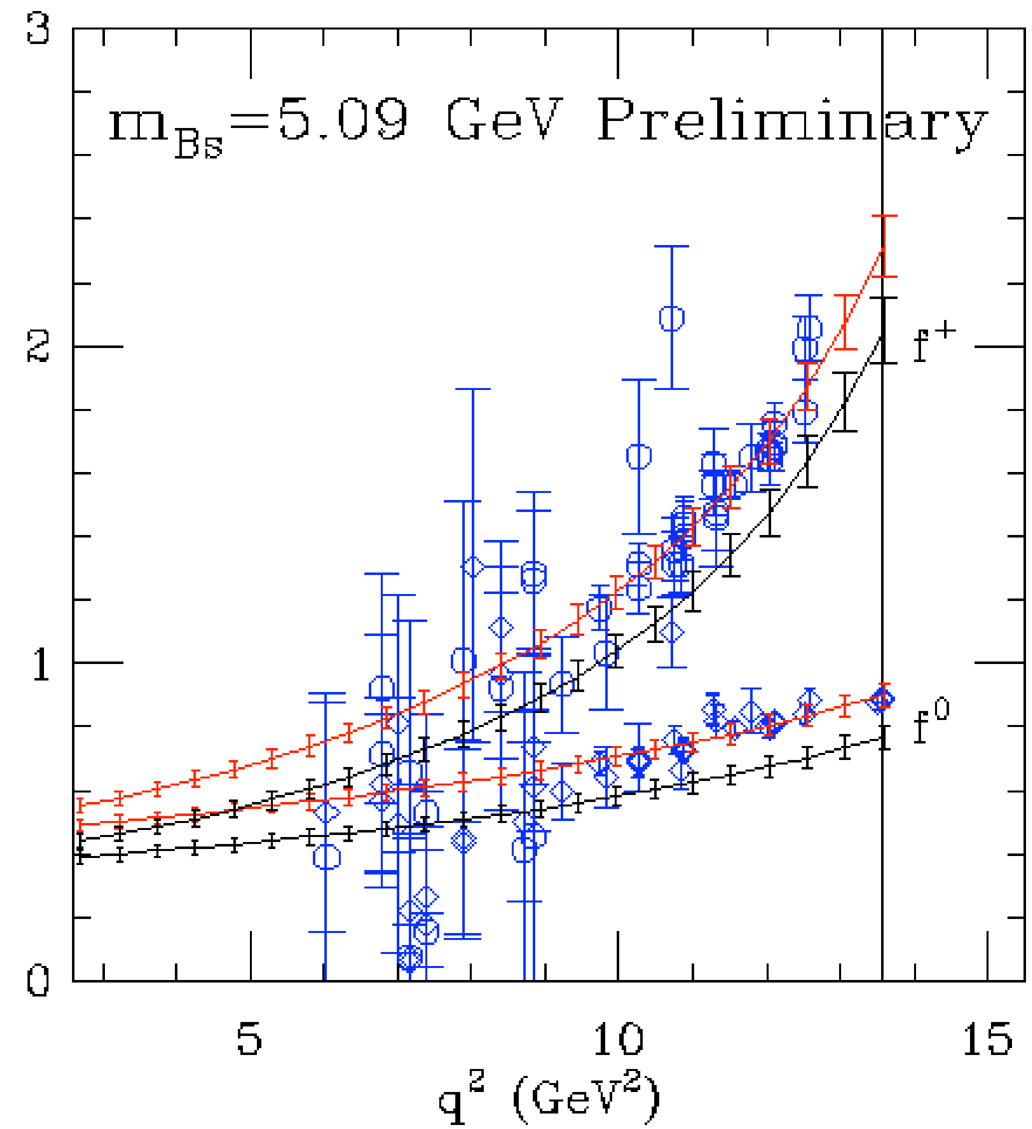
Heavy Quarks

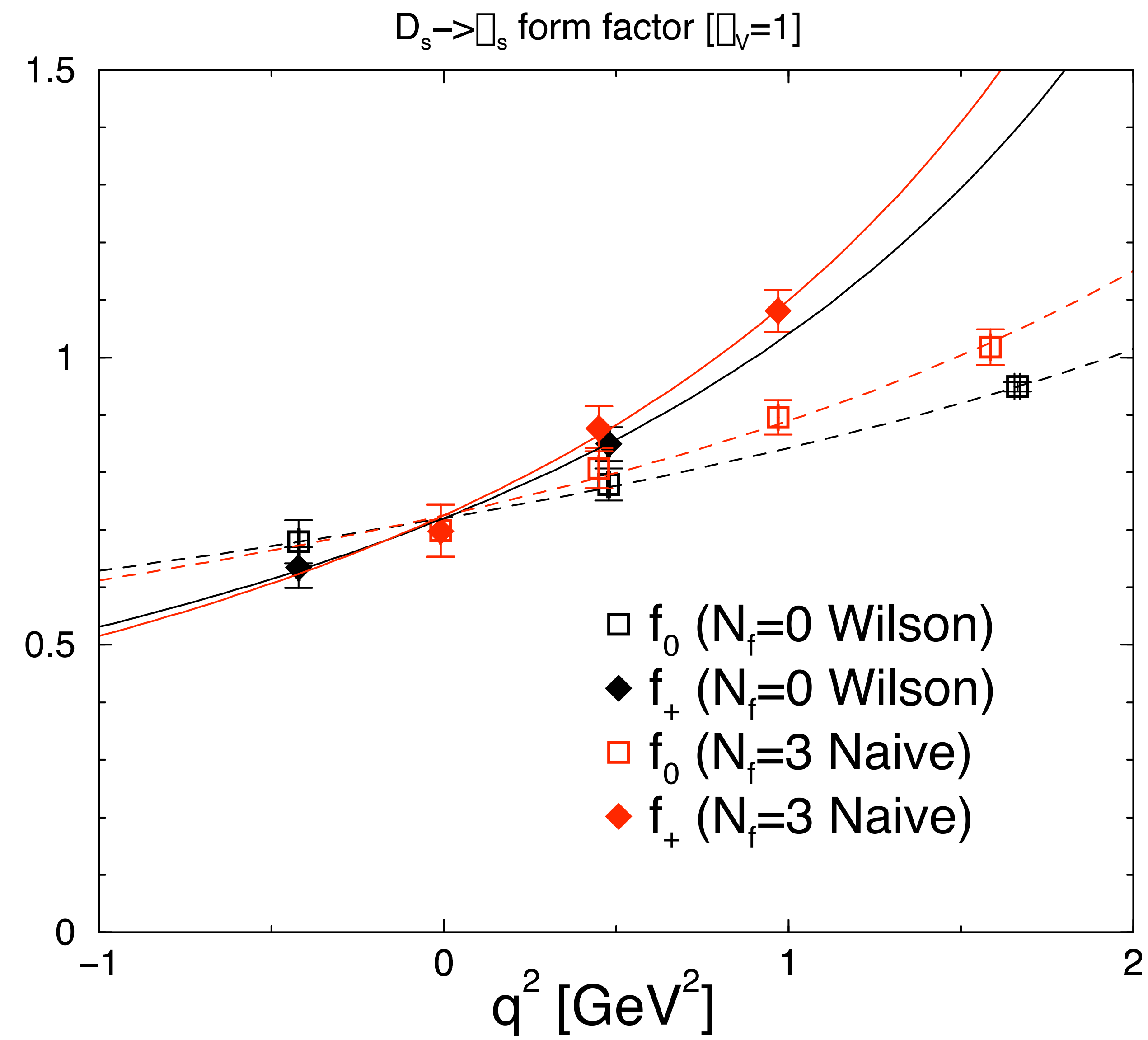
B, D □: $n_f = 2+1$ & 0

clover heavy

clover light

naive light





$B \to$ □$_s$ DeTar, HQ.pstr

$D \to$ □$_s$ Okamoto, HQ.II

- These calculations present several challenges
  - heavy-quark discretization effects
  - energetic pions' discretization effects
  - chiral extrapolation (with energetic pions)
- The last will be easier now, with two papers by Bećirević, Prelovšek, and Zupan [hep-lat/0210048, hep-lat/0305001].
  - partially quenched heavy-meson χPT

f_B and $\overline{B^0_q}$-B^0_q Mixing

Mixing in SM

- In the Standard Model, neutral B mixing gives the "top" side of the unitarity triangle.
- On the other hand, it proceeds through loop diagrams: as in rare decays, non-Standard physics could compete with Standard processes.
- Δm_d is precisely measured
- Δm_s will be measured in about another year

$$\Delta m_q = \frac{G_F^2 m_W^2 S_0}{16\pi^2 m_{B_q^0}} |V_{tb}^* V_{tq}|^2 \eta_B \mathcal{M}_q$$

$$\mathcal{M}_q = \langle \bar{B}_q^0 | [\bar{b}\gamma^\mu(1-\gamma_5)q][\bar{b}\gamma_\mu(1-\gamma_5)q] | B_q^0 \rangle$$

$$= \tfrac{8}{3} m_{B_q^0}^2 f_{B_q^0}^2 B_{B_q^0}$$

- Largely cancel uncertainties with ratio!?
  - old conventional wisdom: Yes!
  - current wisdom (I hope): No!
  - chiral extrapolation, chiral extrapolation, ...

stats	✔
a	✔
m_Q	✔
□	✔
m_q	X

Chiral Extrapolation

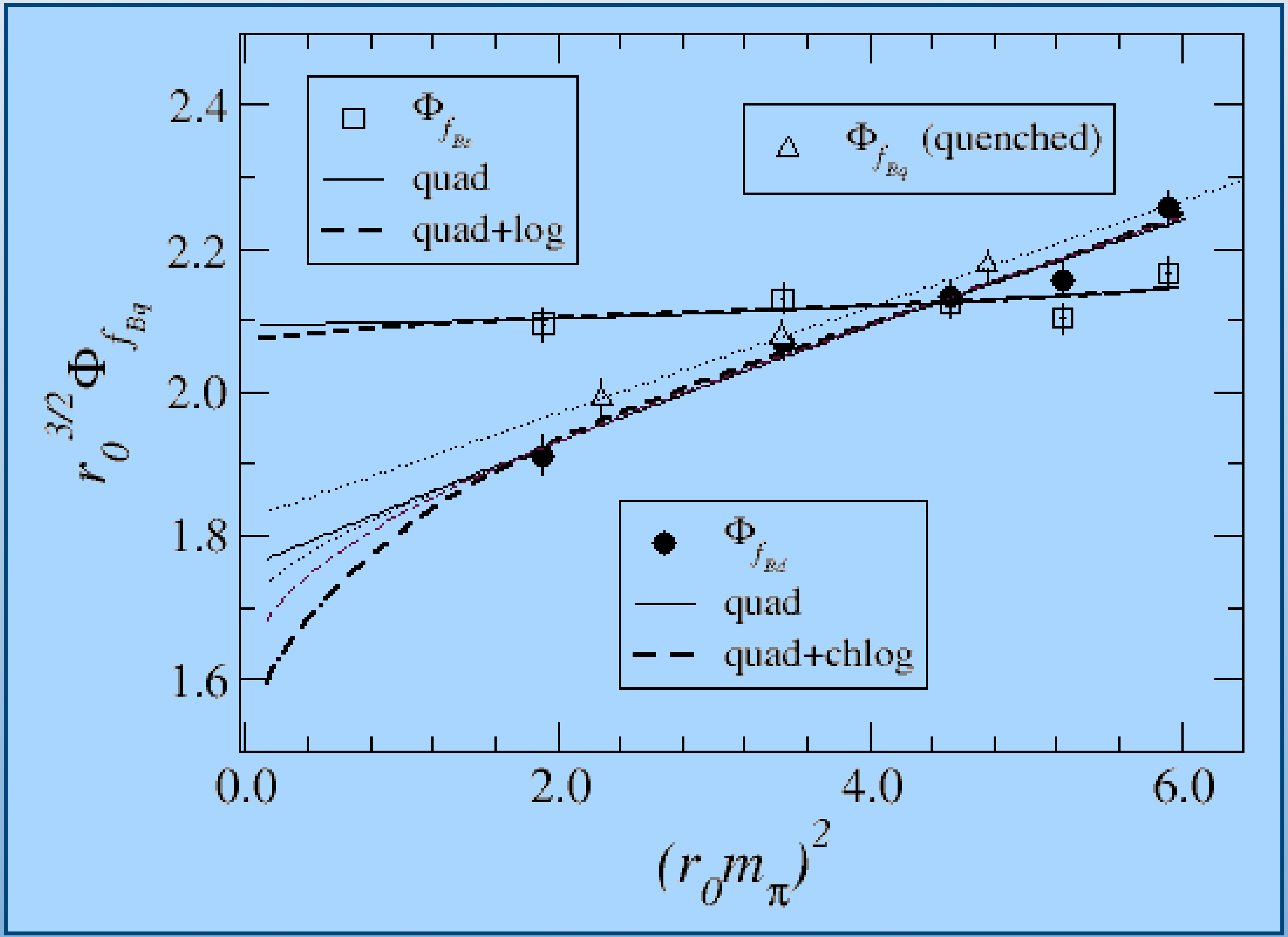
- Despite warnings from Booth and from Sharpe & Zhang, the lattice community concluded that the ratio

$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

had a 5% error.

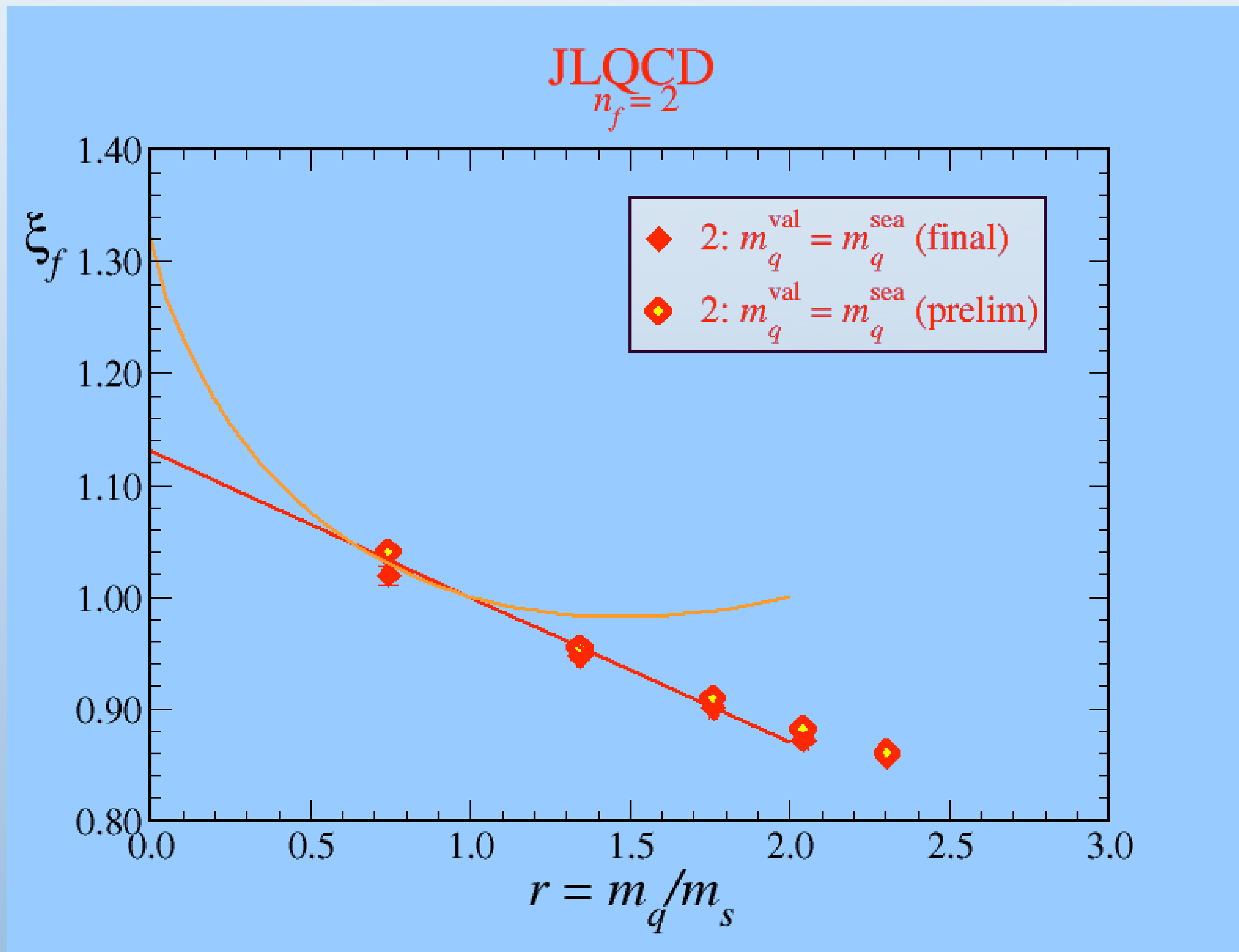
- This picture started to unravel when N. Yamada (JLQCD) showed some evidence for curvature (2001)—a chiral log?

JLQCD, hep-ph/0307039



- To many, the 2001 plot, was an indication that the 5% uncertainty from linear extrapolation was unreliable.
- At small enough quark mass, curvature must set in: the pion cloud contributes $\sim m_\pi^2 \ln(m_\pi^2)$
- Linear chiral extrapolations omit this feature

□log vs linear

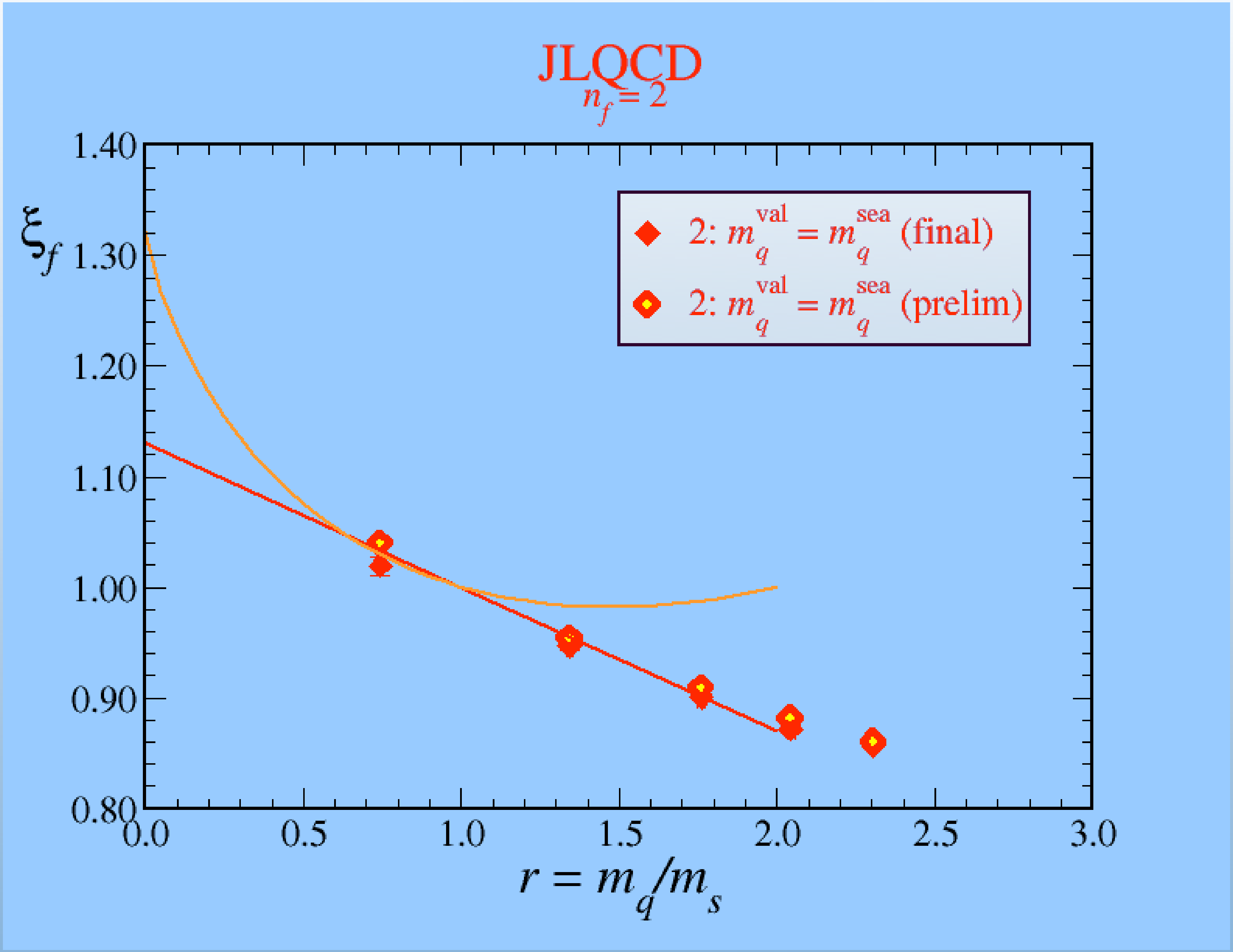


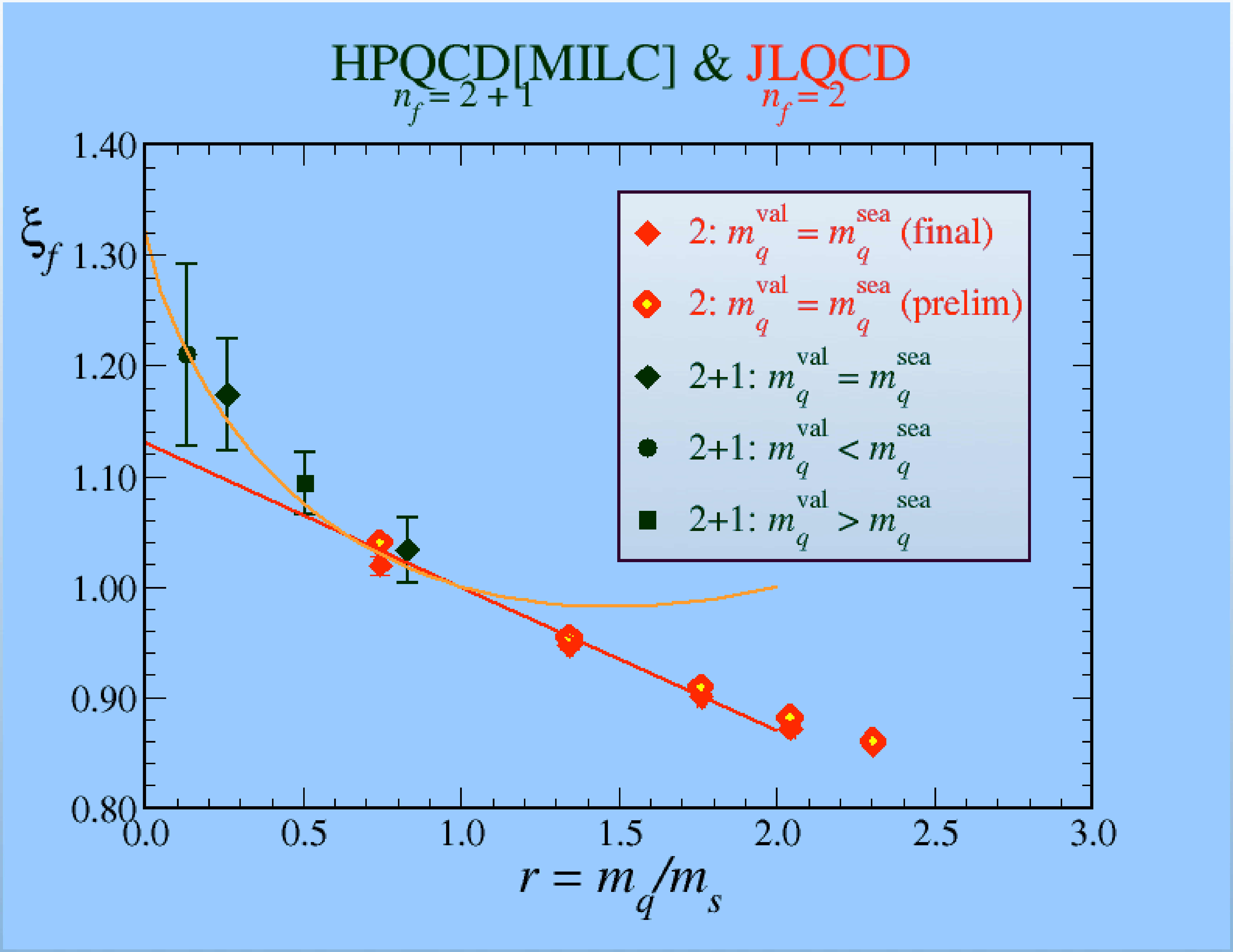
The plot compares JLQCD's linear fit with one that feeds their slope into the □log expression. ASK & Ryan, hep-ph/0206058

Other Ansätze lie between these two.

Thanks to N. Yamada, S. Hashimoto, and T. Onogi

Now add 2+1 (MILC) results from Wingate (HPQCD)




HPQCD[MILC] & JLQCD
$n_f = 2+1$
$n_f = 2$
ξ_f
1.40
1.30
1.20
1.10
1.00
0.90
0.80
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$r = m_q/m_s$
2: $m_q^{val} = m_q^{sea}$ (final)
2: $m_q^{val} = m_q^{sea}$ (prelim)
2+1: $m_q^{val} = m_q^{sea}$
2+1: $m_q^{val} < m_q^{sea}$
2+1: $m_q^{val} > m_q^{sea}$

- What is our best estimate of □ and the decay constants?
  - JLQCD has $n_f = 2$, but final
  - HPQCD has $n_f = 2+1$, but preliminary
- For □ it is better to look at

$$R = \frac{f_{B_s}}{f_{B_d}} \frac{f_\pi}{f_K}$$

Bećirević, Fajfer, Prelovšek, and Zupan, hep-ph/0211271

- Using this method, and $g^2 = 0.35$, which is taken from CLEO's measurement in the D^* decay
  - JLQCD $\Rightarrow \xi = 1.23 \pm 0.05 \pm 0.01_{g^2}$
  - HQCD $\Rightarrow \xi = 1.32 \pm 0.05 \pm 0.01_{g^2}$
  - JLQCD itself finds $\xi = 1.13 \pm 0.03^{+0.13}_{-0.02}{}^{\chi}$

Lessons and Conclusions

- The experimenters need error bars reliable and small.
- $1/m_Q$ extrapolations are dangerous (*Tor Vergata* may evade it).
- Use heavy-quark theory to get a rough guide to uncertainties.
- Quarkonium spectrum supports this.
- Unquenched calculations evolving rapidly.