Weak Matrix Elements from Lattice QCD

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Outline

- Simplest weak matrix element: $K_{\ell 3}$ decay
- $K^0 - \bar{K}^0$: lattice estimate of $\hat{B}_K$ [update]
- Getting to grips with ε'/ε
- New results for EW penguin ($K \to \pi\pi$) and QCD penguin (via $K \to \pi$)
- Assorted topics

I apologise to everyone whose work is not cited properly.

Simplest weak matrix element

Decay constant and weak decay form factors

- Recent interest in $K \to \pi\ell\nu$:
  - direct extraction from $K_{\ell 3} \to |V_{us}| = 0.2201(24)$
  - indirectly, i.e. from the CKM unitarity $\to |V_{us}| = 0.2269(21)$
  - which error is underestimated? Examine $K_{\ell 3}$
  - ChPT calculation to NNLO completed (Bijnens, Talavera, 2003)

    $\longrightarrow$ key observation : $\mathcal{O}(p^6)$ LEC's can be related to the scalar form factor
  - Precision calculation of the scalar form factor on the lattice is possible

    $\longrightarrow$ follow the strategy used in $B \to D\ell\nu$ (Hashimoto et al, 2000)
  - Exploratory study by SPQcdR

$K_{\ell 3}$ decay

$$\begin{aligned}\langle \pi^-(p')|\bar{u}\gamma_\mu s|K^0(p)\rangle &= \left(p + p' - q_\mu \frac{M_K^2 - M_\pi^2}{q^2}\right) f^+(q^2) \\ &\quad + q_\mu \frac{M_K^2 - M_\pi^2}{q^2} f^0(q^2)\end{aligned}$$

Use $f^0(0) = f^+(0) \equiv f(0) \oplus$ *Ademollo-Gatto theorem*

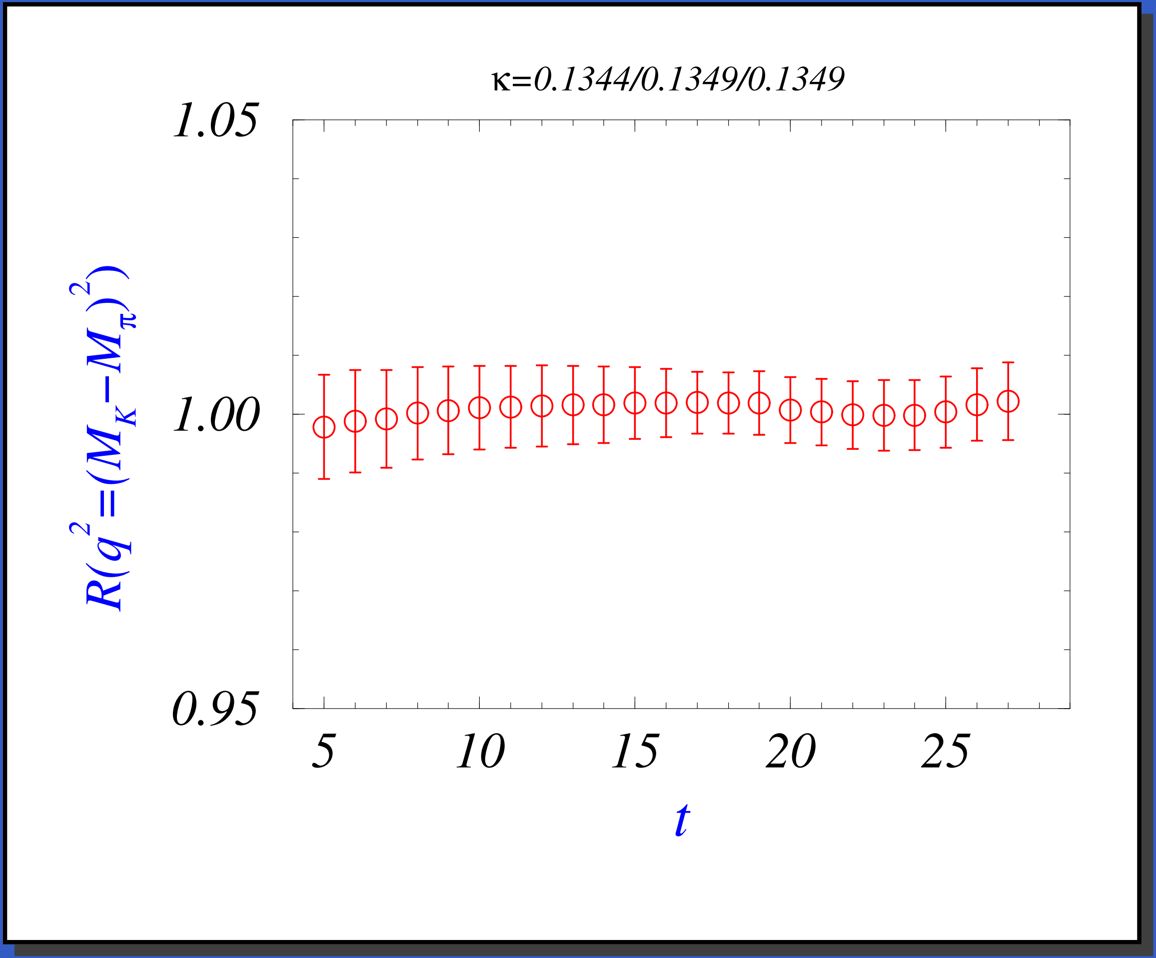
With improved Wilson quarks

$$R(q^2, M_K^2, M_\pi^2) = \frac{\langle \pi|V_\mu^{us}|K\rangle\langle K|V_\mu^{su}|\pi\rangle}{\langle \pi|V_\mu^{uu}|\pi\rangle\langle K|V_\mu^{ss}|K\rangle} \propto f^{0/+}(q^2, M_K^2, M_\pi^2) + \mathcal{O}(a^2)$$

$$f^0(q^2, M_K^2, M_\pi^2) = \underbrace{f^0(0, M_K^2, M_\pi^2)}_{1 - \rho(M_K^2 - M_\pi^2)^2} \left(1 - q^2/M_{V^{us}}^2\right)$$

$$\boxed{\rho = ?}$$

$K_{\ell 3}$ decay - cont.



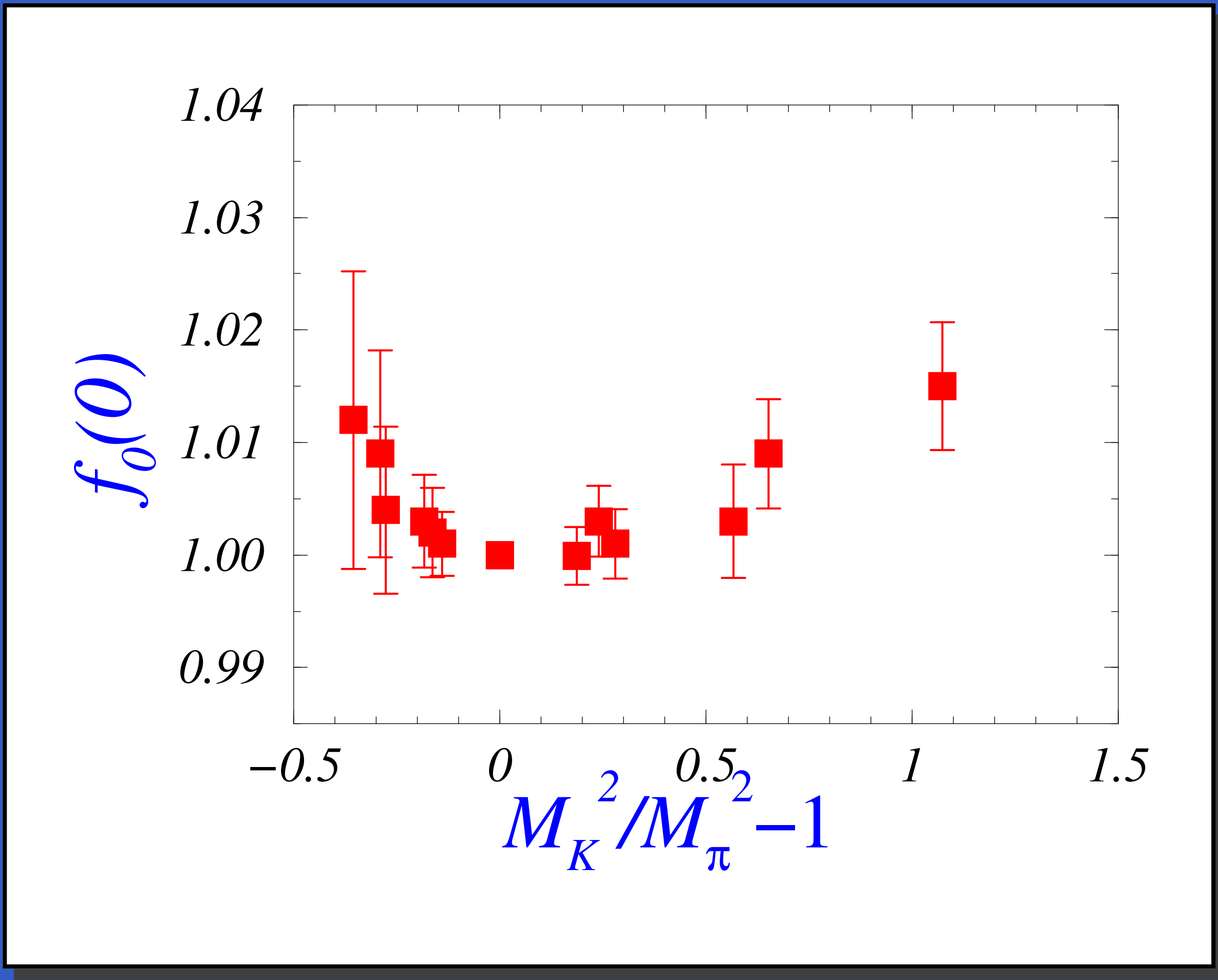
Exploratory (quenched) study by SPQcdR underway.

Unquenching SHOULD be done too!

Statistics poor : errors small $\longrightarrow$ worth pursuing!

$K_{\ell 3}$ decay - cont.

Info on the mass dependence of $f(0) \stackrel{?}{\longrightarrow}$ solution to the "$|V_{us}|$ problem"



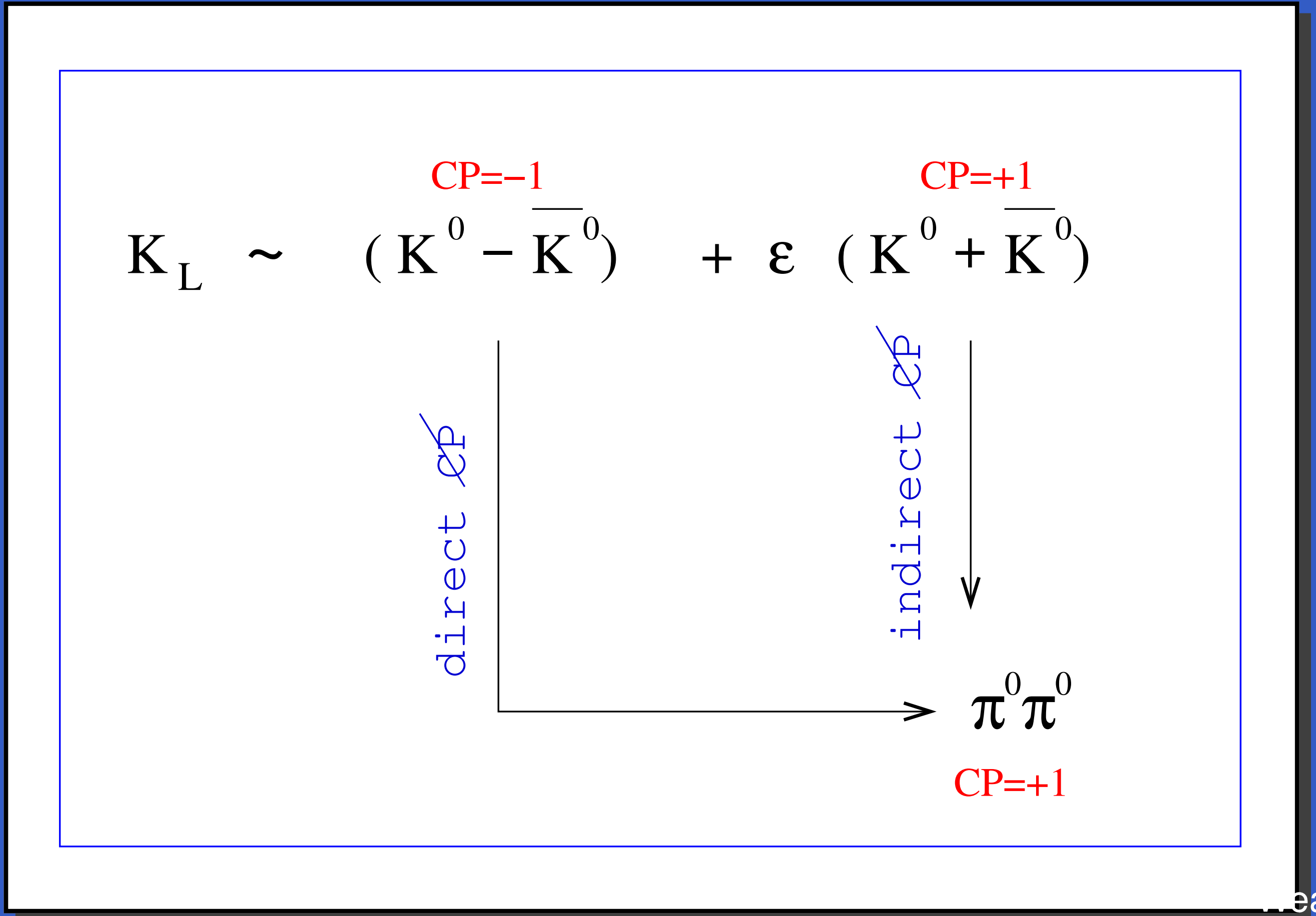
Large statistics is necessary: doable with currently available hardware!

$K^0 - \bar{K}^0$ mixing - recall

$$(K^0, \overline{K}^0) \neq (K_L, K_S) \Rightarrow$$ *the system oscillates.*

Parameter measuring the indirect CP violation

$$\varepsilon_K = \frac{\mathcal{A}(K_L \to (\pi\pi)_{I=0})}{\mathcal{A}(K_S \to (\pi\pi)_{I=0})} \simeq \frac{e^{i\pi/4}}{\sqrt{2}\Delta M_K} \mathrm{Im}\left\{ \frac{1}{2m_K} \langle \overline{K}^0 | \mathcal{H}_{\mathrm{eff}}^{\Delta S=2} | K^0 \rangle \right\}$$



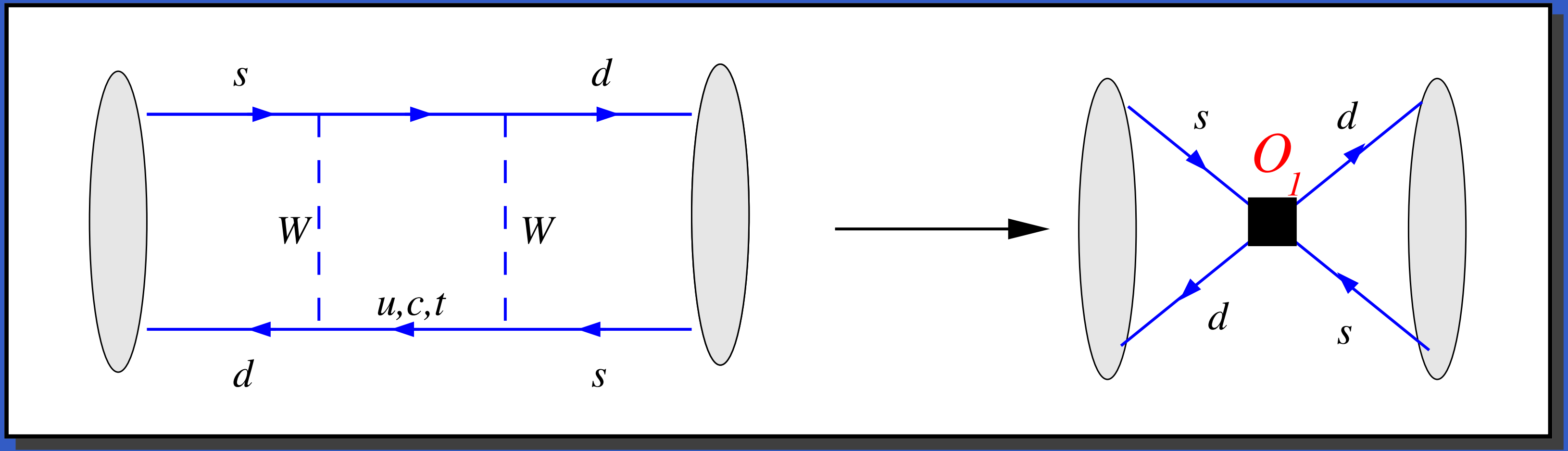
Experimentally (PDG)

$$\Delta M_K = (3.491 \pm 0.009) \times 10^{-15}\ \mathrm{GeV}$$

$$\varepsilon_K^{\mathrm{exp.}} = (2.280 \pm 0.013) \times 10^{-3} e^{i\pi/4}$$

$K^0 - \bar{K}^0$ mixing - setup

$$\varepsilon_K \sim \langle \bar{K}^0 | \mathcal{H}_{\rm eff}^{\Delta S=2} | K^0 \rangle = C(\mu) \cdot \langle \bar{K}^0 | \overbrace{\bar{s}\gamma_\mu(1-\gamma_5)d\; \bar{s}\gamma_\mu(1-\gamma_5)d}^{O_1(\mu)} | K^0 \rangle$$

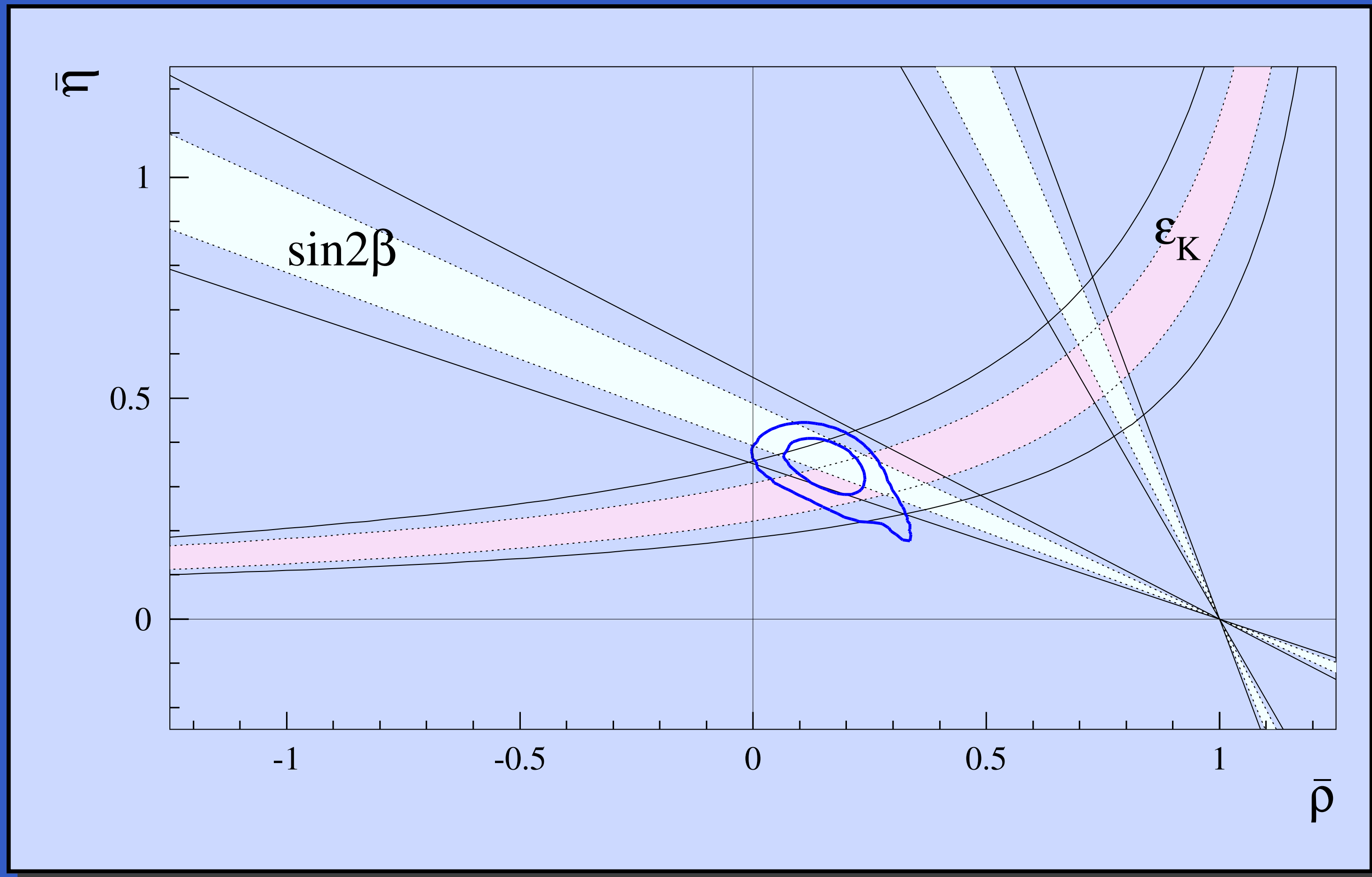


- $C(\mu)$ info on SD dynamics: perturbation theory (A. Buras et al. 1995)
- low energy QCD dynamics

$$\langle \bar{K}^0 | Q(\mu) | K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$$

- AD to 2-loops in $\overline{\rm MS}$ and RI/MOM (Altarelli et al. 1981, Buras et al. 1990)$_{\overline{\rm MS}}$, (Ciuchini et al. 1997, Buras et al. 2000)$_{\rm RI/MOM}$

$K^0 - \bar{K}^0$ in the UTA



- Hyperbola: $|\varepsilon_K| = C\ \hat{B}_K\ A^2\lambda^6\ \bar{\eta}\,[A^2\lambda^4\,(1-\bar{\rho})\ \ F_{tt} + F_{tc}]$
- Combined with $\sin 2\beta$ (direct CP-violation in $B \to J/\psi K_S$): two solutions for the vertex of the UTA (*marked area – B-physics constraints*)
- Crucial input $\hat{B}_K$: dominant uncertainty in UTA!

B_K and lattices

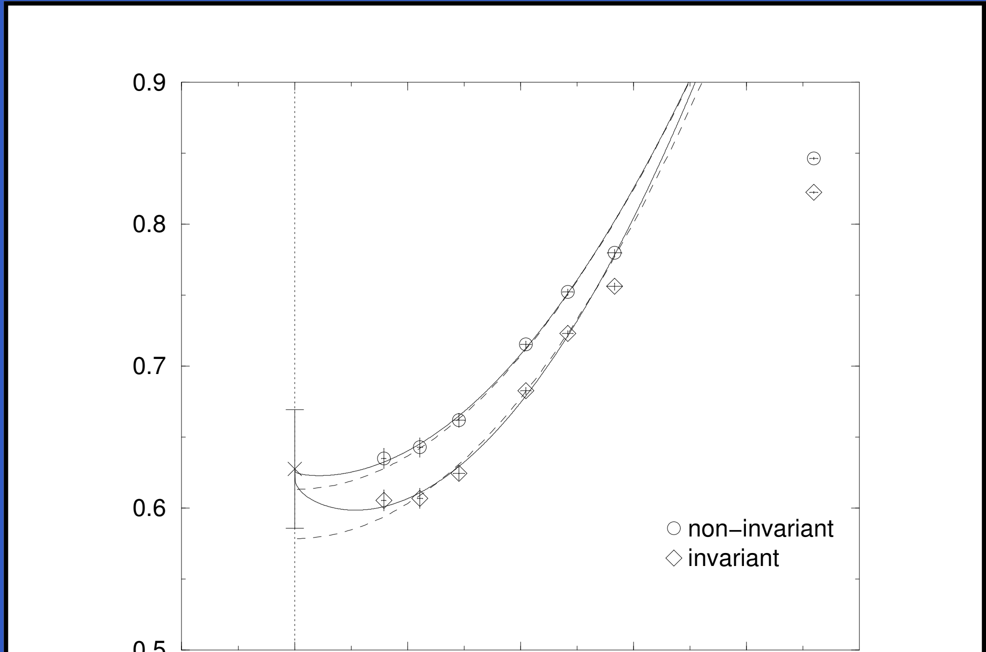
- The only method allowing to compute the matrix elements of 4-f operators from the first principles of QCD : capture the low energy QCD physics, AND unambiguous matching with the continuum renormalisation scheme at high energy scale μ
- Benchmark calculation of the 4-f operators on the lattice. Challenging issues: matching and renormalisation, chirality, quenching
- HEP community is urging the lattice community to reduce the errors; Lattice community is not converging to a consensus.
  Several recent calculations require a critical overview.
  Here is my take!

B_K with staggered quarks

:-) $U(1)_A$ symmetry protects B_K (and m_q) from additive renormalisation, i.e. B_K has a good chiral behavior

$$\langle \bar{K}^0 | O_1(\mu) | K^0 \rangle \big|_{\vec{p}_K=0} = \alpha M_K^2 + \beta M_K^4 + \ldots$$

:-("Staggered flavor" symmetry is broken: non perturbative renormalisation (NPR) unfeasible [$(16 \times 16)^2$ lattice operators to match onto 9 in continuum!] $\Rightarrow$ 1-loop perturbation theory : "wrong tasting" operators give *'large'* contributions. As a result, scaling violation pronounced.



Extensive study by JLQCD `(Aoki et al,1998)`; Impressive lattices up to $56^3 \times 96$ @ $\beta = 6.65^{WP}$!

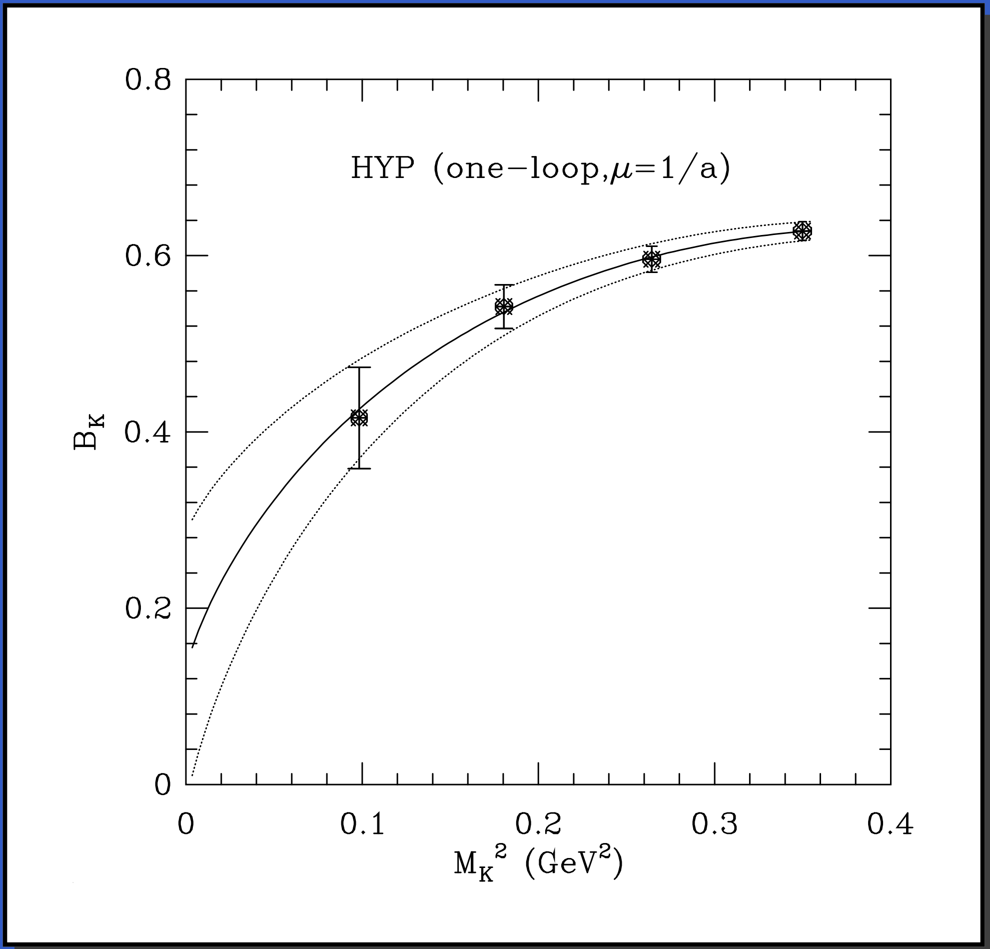
$$\boxed{B_K^{\overline{\rm MS}}(2{\rm GeV}) = 0.63(4)}$$

Set-I : $B_K^{\overline{\rm MS}}(2{\rm GeV}) = 0.67(6)$

Set-II : $B_K^{\overline{\rm MS}}(2{\rm GeV})_{\rm inv} = 0.71(7)$

New: B_K with staggered quarks (W.Lee et al)

- Taste symmetry breaking in the renormalisation gets largely suppressed by fattening the link variable (Lee, Sharpe, 2003). The most efficient is the HYP link fattening (cf. A.Hasenfratz, Knechtli, 2001)



★ at $\beta = 6.0$ and $16^3 \times 64$ and fitting to $B_K = c_0(1 + c_1 M_K^2 \log(M_K^2)) + c_2 M_K^2$, they get

$$\boxed{B_K^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) = 0.59(2)}$$

★ N.B. With similar lattice setup JLQCD had $B_K^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) = 0.68(1)$ (cf. prev.transp.)

★ Does the HYP-ening improve the scaling behavior? Such a study is needed.

B_K with Wilson quark action

:-(Wilson term explicitely breaks chiral symmetry: CP⊗S ($s \leftrightarrow d$) symmetry, but no chirality → additive renormalisation. Mixing with 4 other parity even operators

$$\begin{aligned} O_1 &= V \times V \;+\; A \times A \\ O_2 &= V \times V \;-\; A \times A \\ O_3 &= S \times S \;-\; P \times P \\ O_4 &= S \times S \;+\; P \times P \\ O_5 &= T \times T \end{aligned}$$

$$O_1(\mu) = Z_1(a\mu)\left\{ O_1(a) + \sum_{k=2}^{5} \Delta_k(a) O_k(a) \right\} \qquad (\partial\Delta/\partial\mu = 0)$$

Sandwich this formula by kaons : $\langle \bar{K}^0 | O_k | K^0 \rangle \equiv \langle O_k(a) \rangle$

$$\langle O_1(\mu) \rangle \;=\; Z_1(a\mu)\langle O_1(a)\rangle \underbrace{\left[1 + \sum_{k=2}^{5} \Delta_k(a) \overbrace{\langle O_k(a)\rangle / \langle O_1(a)\rangle}^{\text{VSA(for illustration!)}} \right]}_{1 - \frac{6\Delta_2 - \Delta_3 - \Delta_5}{8} - \frac{4\Delta_2 - 6\Delta_3 + 5\Delta_4 - 8\Delta_5}{8}\left(\frac{M_K}{m_d(a) + m_s(a)}\right)^2}$$

$\lim_{m_q \to 0} (M_P/m_q)^2 \simeq 1/m_q \to \infty \Rightarrow$ smallness of Δ_i is irrelevant when close to $m_q \to 0$!

B_K Wilson - without subtraction

- Non-perturbative method to determine Δ_i and $Z_1(a\mu)$ devised in (Donini et al,1999): RI/MOM(Landau) scheme.
- Better avenue: get rid of subtractions (DB et al,2000). Recall that the spurious mixing in PV sector does not occur due to S-symmetry (Bernard, 1989)
- Ward Identity on the m.e.of the PV operator leads to the m.e. of the PC one. Rotation around the 3^{rd} axis in the isospace:

$$\begin{aligned} \delta u &= \gamma_5 u & \delta d &= -\gamma_5 d \\ \delta \bar{u} &= \bar{u}\gamma_5 & \delta \bar{d} &= -\bar{d}\gamma_5 \qquad\qquad (m_u = m_d \equiv m) \end{aligned}$$

$$2m\langle \sum_{\vec{x},\vec{y},\vec{z},t_z} \Pi(\vec{z},t_z)\ P(\vec{x},t_x)\ O_{VA+AV}(0)\ P(\vec{y},t_y)\rangle = 2\langle\sum_{\vec{x},\vec{y}} P(\vec{x},t_x)\ O_{VV+AA}(0)\ P(\vec{y},t_y)\rangle$$

+ correlators with chirally rotated sources, but they vanish by C-symmetry.

- SPQcdR checked in numerical data the consistency of $\langle O_1\rangle \equiv \langle O_{VA+AV}\rangle$, as obtained by methods with and without subtractions (method with subtr. supplied by Δ_i's fixed NP'ly)! (DB et al,2001 and 2002)

B_K Wilson - how to extract it?

- Strategy-I: Valid when chirality is guaranteed (staggered use this). With Wilson, even with tricks to get rid of subtractions, chirality is still missing ($\mathcal{O}(a)$ effects present!), and we have

$$\frac{\langle \sum_{\vec{x},\vec{y}} P(\vec{x},t_x)\ O_1(0)\ P(\vec{y},t_y)\rangle}{(8/3)\langle \sum_{\vec{x}} P(\vec{x},t_x)A_0(0)\rangle\langle \sum_{\vec{y}} A_0(0)P(\vec{y},t_y)\rangle} \rightarrow \frac{\langle \bar{K}^0|O_1|K^0\rangle}{(8/3)|\langle 0|A_0|K^0\rangle|^2} = \frac{\langle \bar{K}^0|O_1|K^0\rangle + \mathcal{O}(a)}{(8/3)f_K^2 M_K^2}$$

$$= \quad B_K + \mathcal{O}(a/M_K^2)$$

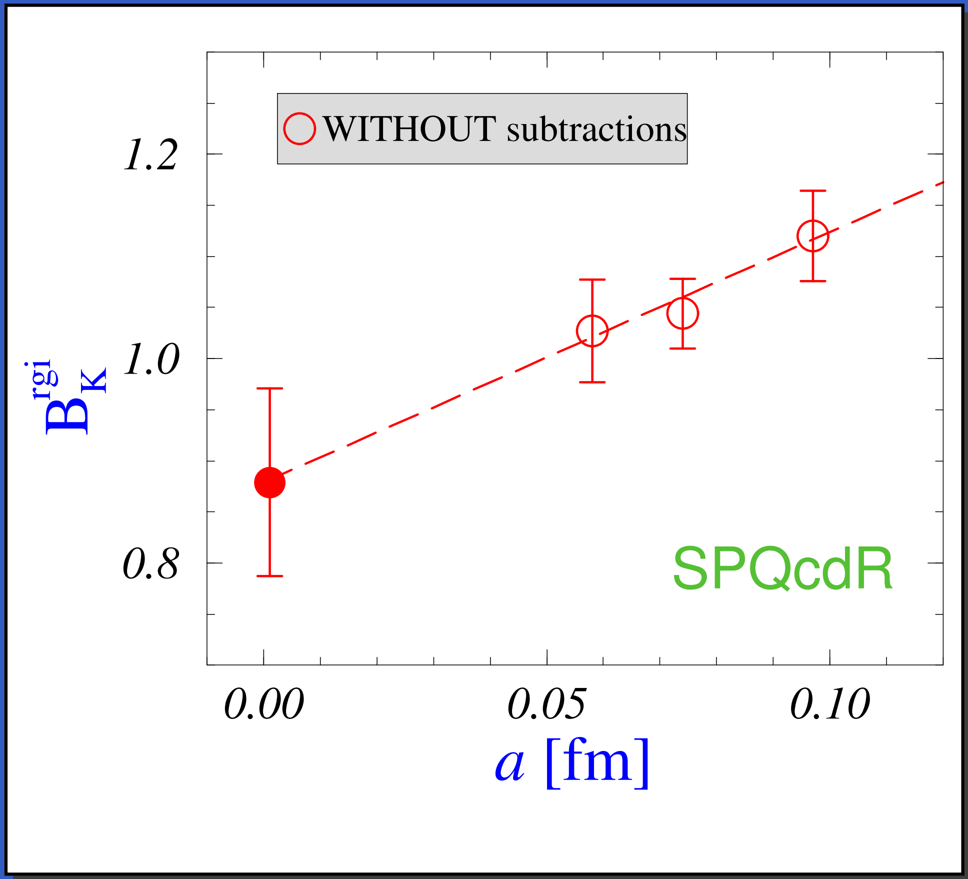
Devilish game! Without the manifest chiral symmetry, results acceptable IFF $M_K^2 \sim m_q$ are large “enough" and by taking $a \rightarrow 0$. N.B. that NOTHING can be said about the change of B_K when approaching the chiral limit!

- Strategy-II: Safe road if assuming exact $SU(3)$ (Gavela et al,1988), and

$$\frac{C^{(3)}_{PO_1P}(x,0,y)}{C^{(2)}_{PP}(x,0)C^{(2)}_{PP}(0,y)} \rightarrow \frac{\langle \bar{K}^0|O_1|K^0\rangle}{|\langle 0|P|K^0\rangle|^2} \stackrel{fit}{=} \alpha + B_K \frac{8}{3}\frac{|\langle 0|A_0|K^0\rangle|^2}{|\langle 0|P|K^0\rangle|^2}$$

This strategy can rescue the current extractions of B_K from DWF action, in which the subtractions of $d=6$ operators were so far ignored (*Will get back to that later!*)

B_K Wilson - result



SPQcdR : without subtractions, high statistics at $\beta^{WP} = 6.0, 6.2$ and 6.4, NPR! Conversion to $\overline{\mathrm{MS}}$ at NLO in perturbation theory gives

$$\boxed{B_K^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) = 0.64(8)}$$

⋆ I don't see this result improving soon

⋆ Consistent with SPQcdR and JLQCD estimates **with** subtraction followed by $a \longrightarrow 0$

- Alternative way of getting the Wilson B_K without subtractions : use tmQCD. `(cf. Frezzotti, @ "Lattice 2002")`

B_K with tmQCD

$$S = \sum_x \left[\bar{\psi}(x) \left(\not{D} + m_q + i\mu_q \gamma_5 \tau_3 \right) \psi(x) + \bar{s} \left(\not{D} + m_s \right) s \right], \qquad \psi = (u\ d)^T \qquad m_u = m_d \equiv m_q$$

is invariant under: $\psi \to \exp(i\alpha\gamma_5\tau_3/2)\psi$, $\bar{\psi} \to \bar{\psi}\exp(i\alpha\gamma_5\tau_3/2)$ for $\tan\alpha = \mu_q/m_q$

- Axial rotation of O_{VV+AA} leads to

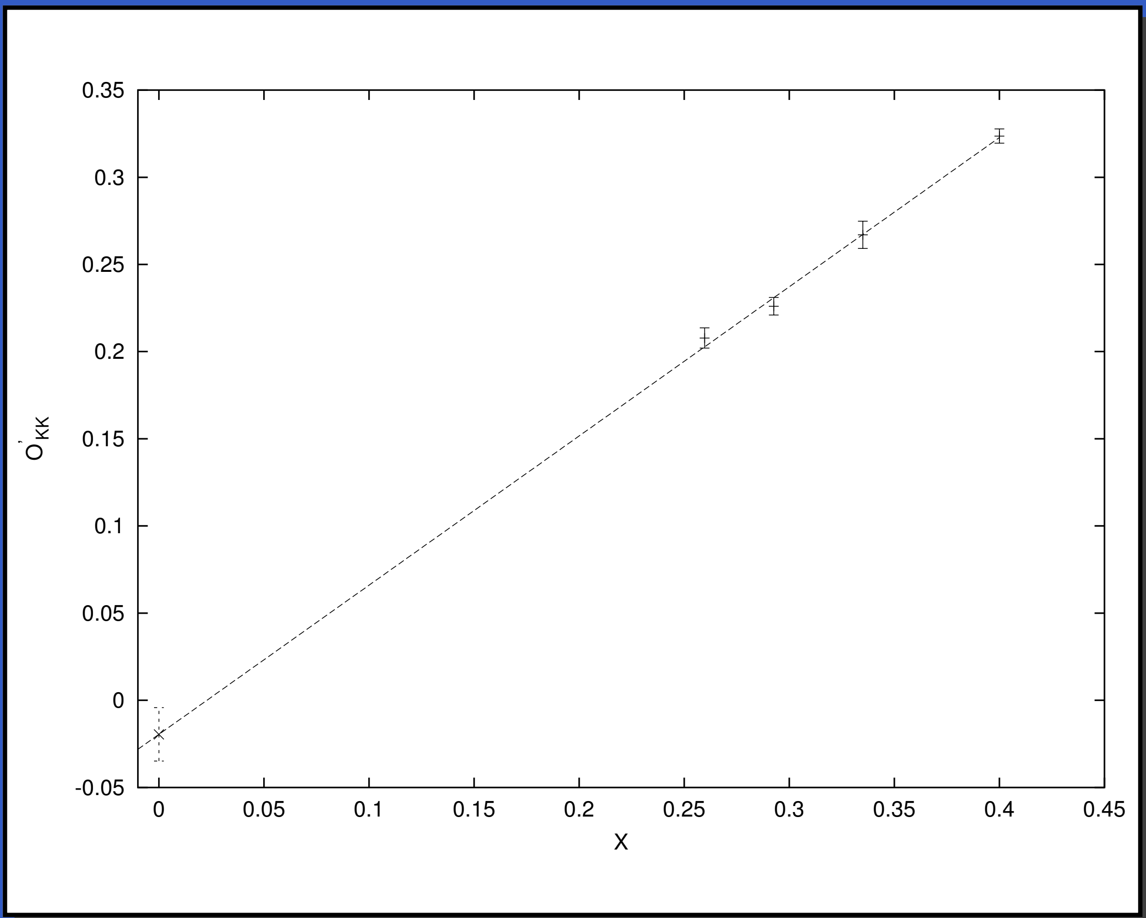
$$\langle \bar{K}^0 | O_{VA+AV} | K^0 \rangle^{\alpha=\pi/2}_{\mathrm{tmQCD}} = i \langle \bar{K}^0 | O_{VV+AA} | K^0 \rangle^{\alpha=0}_{\mathrm{tmQCD}} \equiv i \langle \bar{K}^0 | O_1 | K^0 \rangle_{\mathrm{QCD}}$$

- Important advantage of tmQCD : no exceptional configurations
  $\Rightarrow$ getting closer to the chiral limit.
- Last year NPR implemented in the Schrödinger functional scheme (Guagnelli et al, 2002). NLO anomalous dimension of O_1 in that scheme is missing.

New: B_K with tmQCD by Dimopoulos et al. – Alpha

A PRELIMINARY result at $\beta^{WP} = 6.0$, on $16^3 \times 48$ lattice (cca 200 configs). Strategy-I employed to extract B_K, but they work with $m_P > 600$ MeV.

Strategy-II pushes B_K upwards(!)



Picture : Strategy-II

chiral behavior of $\langle O_1 \rangle$

They quote (PRELIMINARY!)

$$\boxed{B_K^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) = 0.67(2)}$$

⋆ Two loop AD of O_1 in the SF scheme is needed

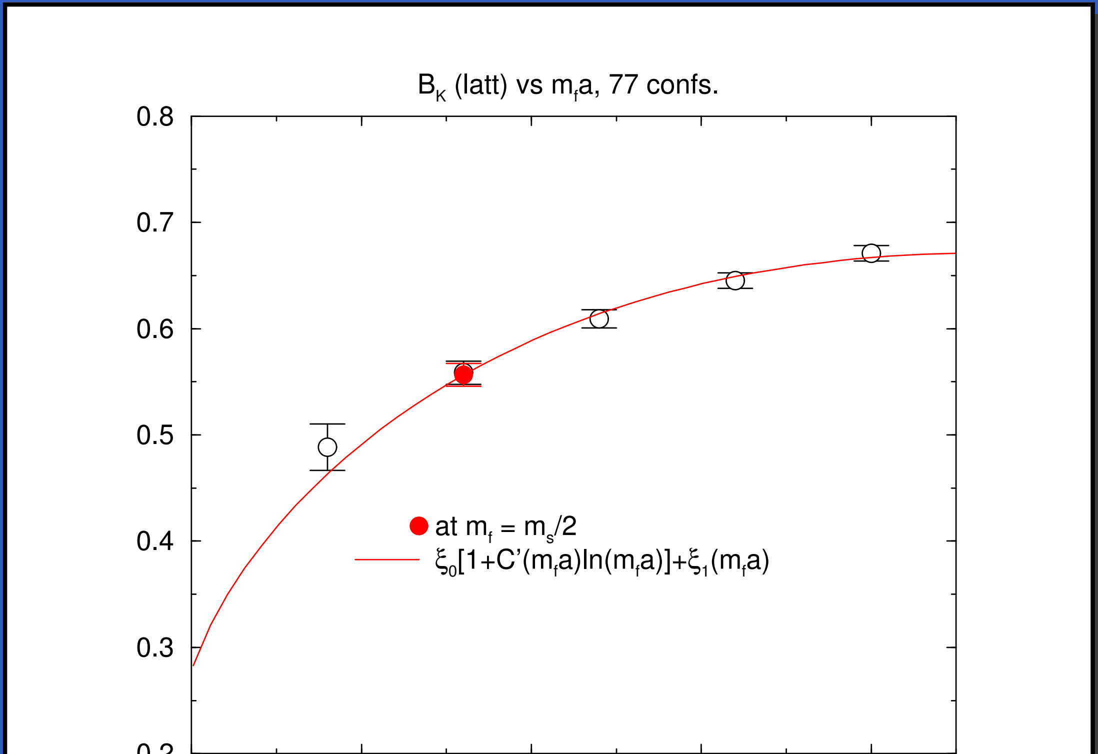
⋆ Perspective:

Getting to lighter meson masses and $a \to 0$.

B_K with DWF (RBC)

DWF action satisfies the Ginsparg–Wilson relation $\{D^{-1}, \gamma_5\} = \gamma_5$
$\Rightarrow$ chiral symmetry guaranteed at finite lattice spacing.
In practice, chiral symmetry not exact : residual mass term in AWI.
Coarse lattices $m_{5q} \not\to 0$, even with $N_5 \to \infty$. On finer lattices ($1/a \sim 2$ GeV), and with WP gauge action, not clear if $\lim_{N_5\to\infty} m_{5q} = 0$. Improved gauge action (IW) reduce m_{5q} by an order of magnitude (Aoki et al, 2001)

- ♣ RBC (WP-action), $16^3 \times 32 \times 16$, 400 configs, (Blum et al, 2001)
  NPR in the RI/MOM scheme; by using Strategy-I, they get $\boxed{B_K^{\overline{\rm MS}}(2{\rm GeV}) = 0.532(11)}$



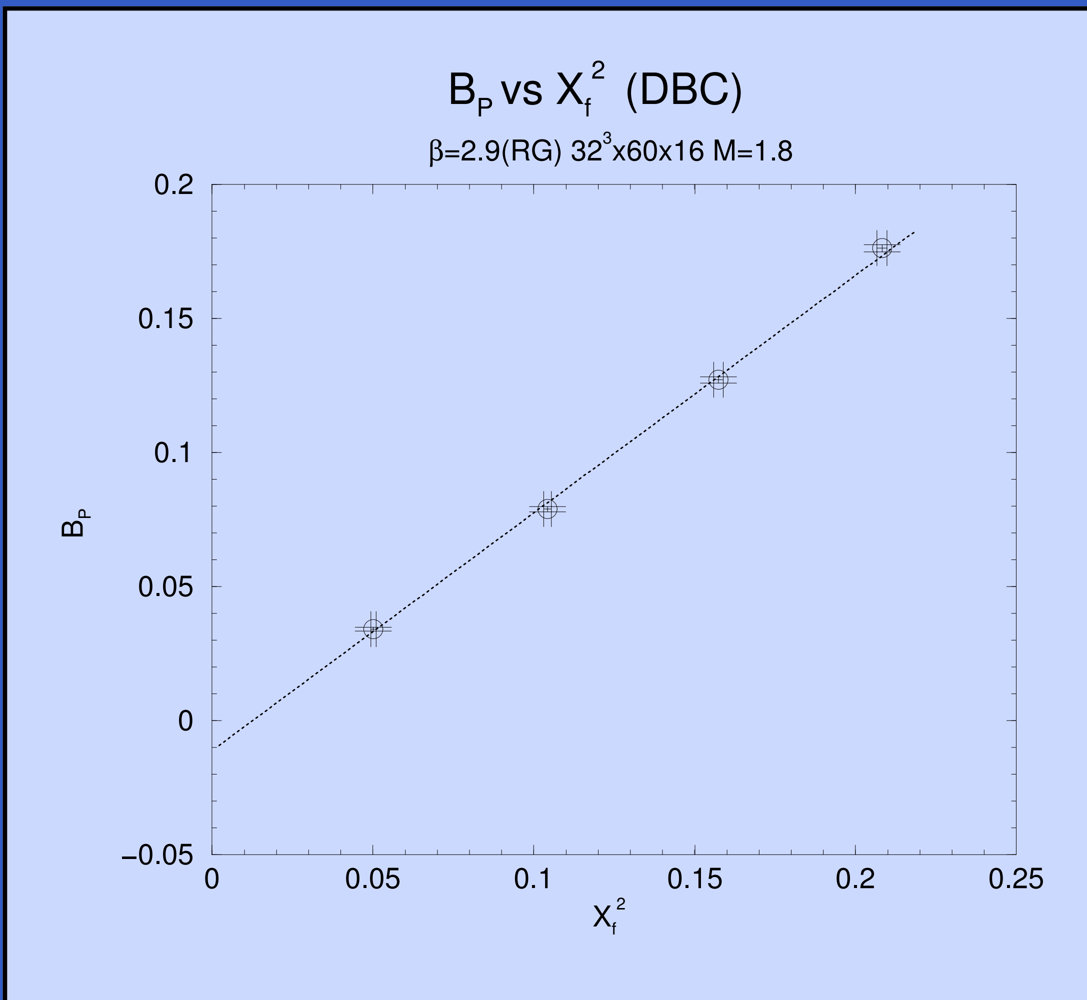
NEW result this year (see J.Noaki's talk), from 77 configs at $\beta^{DBW2} = 1.22$ ($1/a = 2.9(1)$ GeV), their PRELIMINARY value is

$$\boxed{B_K^{\overline{\rm MS}}(2{\rm GeV}) = 0.552(11)}$$

★ Subtractions ignored! Strategy-I may lead to a wrong B_K (b/c of mixing with operators that scale like $1/m_q$)

B_K with DWF (CP-PACS)

- CP-PACS, $\beta^{IW} = 2.6$ $(1/a \simeq 1.9$ GeV) and $\beta^{IW} = 2.9$ $(1/a \simeq 2.8$ GeV). several lattices: as large as $32^3 \times 60 \times 16$! (Ali Khan et al, 2001)
Renormalisation perturbative (S.Aoki et al, 2001); by using Strategy-I, in $a \longrightarrow 0$, they quote $\boxed{B_K^{\overline{\rm MS}}(2{\rm GeV}) = 0.575(6)(19)}$



HOWEVER if the Strategy-II is adopted:
at $1/a \sim 1.9$ GeV

$$B_K^{\overline{\rm MS}}(2{\rm GeV}) = 0.737(6)$$

and at $1/a \sim 2.8$ GeV

$$B_K^{\overline{\rm MS}}(2{\rm GeV}) = 0.723(6).$$

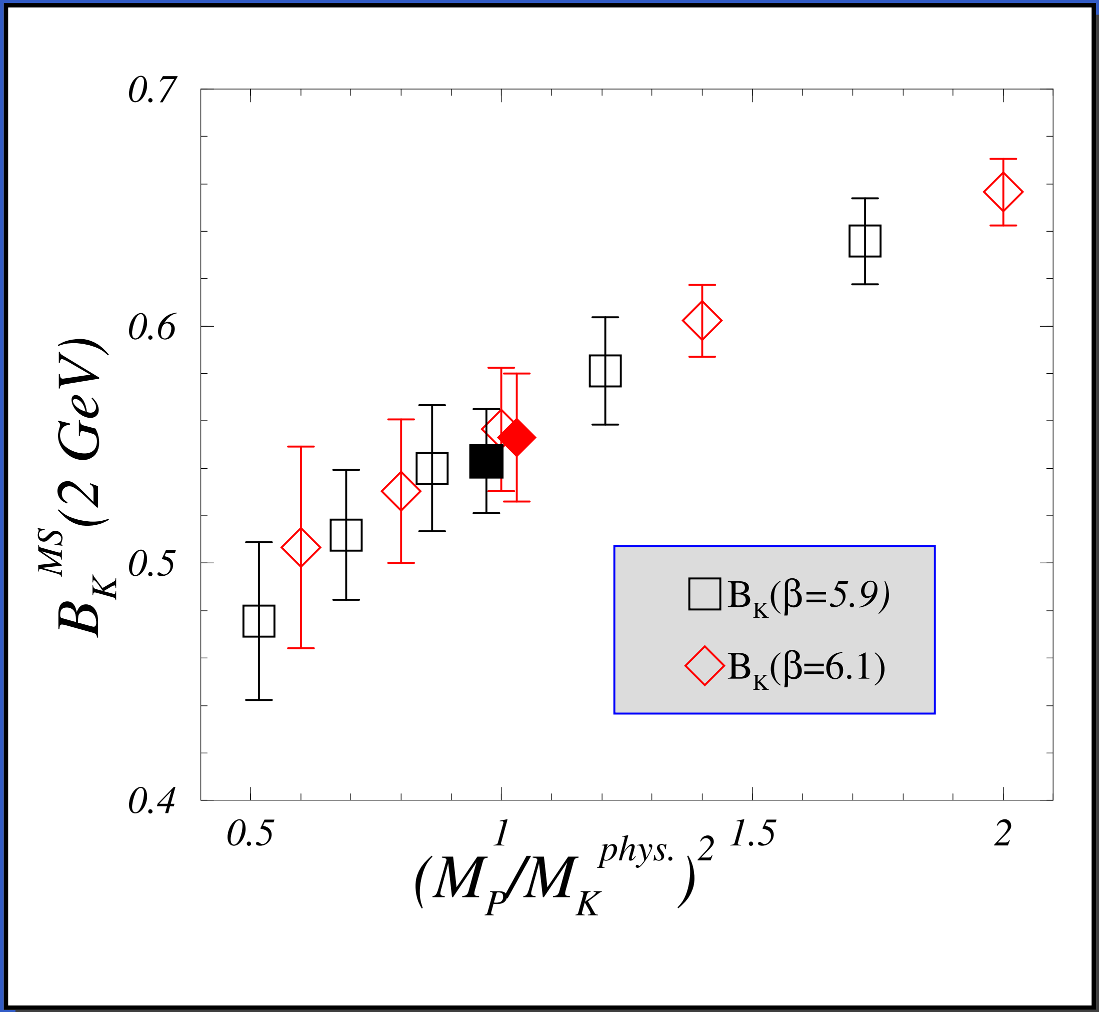
In $a \longrightarrow 0$ this leads to

$$\boxed{B_K^{\overline{\rm MS}}(2{\rm GeV}) = 0.696(7)(23)}$$

Thanks to Yusuke Taniguchi!

B_K with Overlap DeGrand et al. (MILC)

NEW: smaller quark masses than last year, 2 sets of data: [$\beta = 5.9$ ($12^3 \times 36$), 80 configs] and [$\beta = 6.1$ ($16^3 \times 48$), 60 configs] WP + HYP link which kills lattice tadpoles $\rightarrow$ better matching to continuum. Perturbative renormalisation `(T.DeGrand, 2002)`



Strategy-I is appropriate:

at $\beta = 5.9$,

$$B_K^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) = 0.54(2), \text{ and}$$

at $\beta = 6.1$,

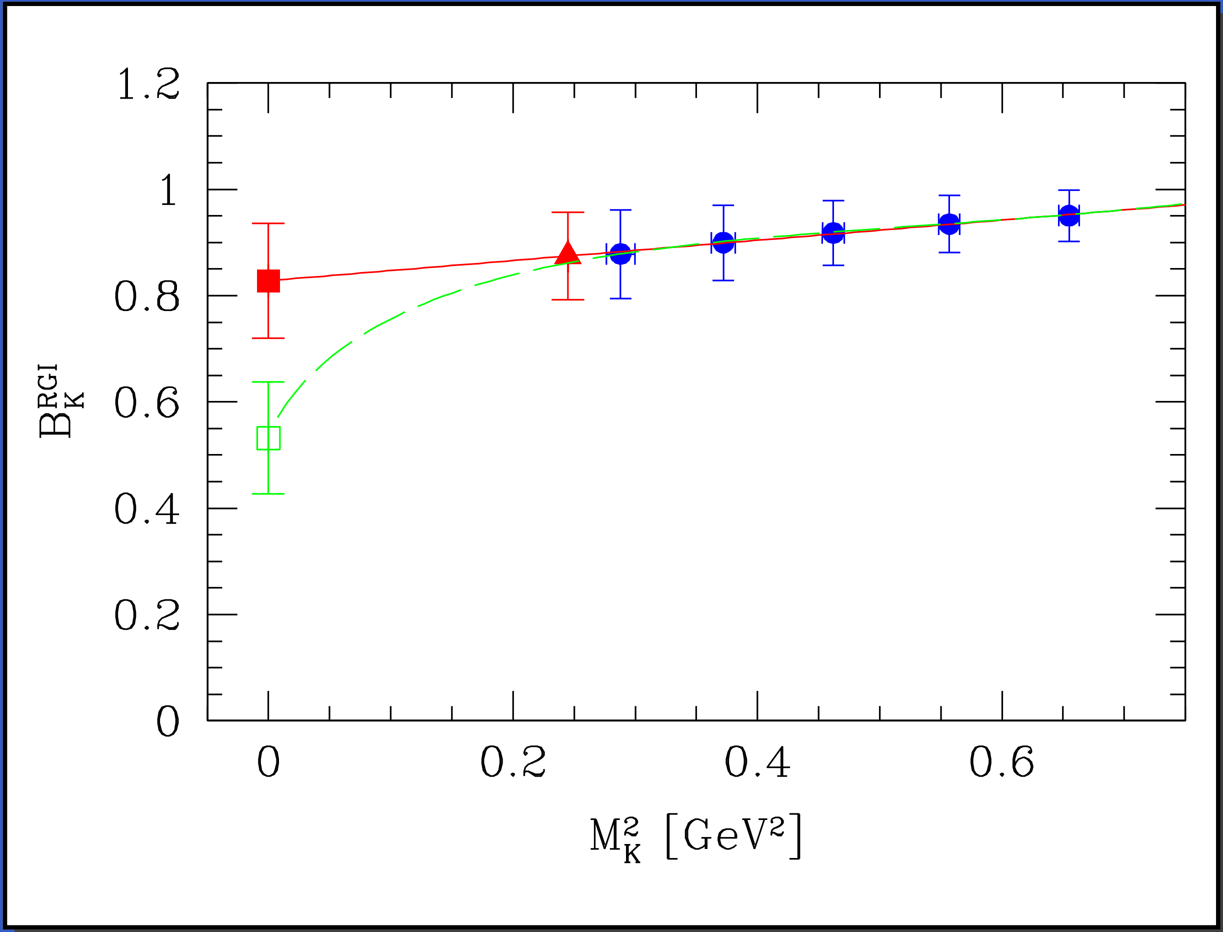
$$B_K^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) = 0.55(3).$$

Attempt $a \rightarrow 0$, gives

$$\boxed{B_K^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) = 0.56(5)}$$

B_K with Overlap Boston-Marseille

Last year preliminary $\rightarrow$ this year definitive result from 80 configs at $\beta^{WP} = 6.0$ ($16^3 \times 32$). NPR in the RI/MOM scheme. They obtain $B_K^{\overline{\rm MS}}(2{\rm GeV}) = 0.62(6)(1)$ (N.Garron et al, 2003)



Speculate that the perturbative coefficient of the chiral log term would reduce B_K to $B_\chi^{\overline{\rm MS}}(2{\rm GeV}) \simeq 0.35$.

B_K conclusion

- Although with unpleasant scaling violations, staggered results for B_K are the most accurate ($a \to 0$ taken)
- Wilson fermions with methods that alleviate problems of mixing with $d = 6$ operators, agree well with staggered result. However, no info on B_K/B_χ! ($a \to 0$ taken)
- DWF : Strategy-II should be used AT LEAST to assess the systematic uncertainty. In practice, the difference between the results from two strategies do not agree: (S-I) mixing gives rise to $1/m_q$ terms; (S-II) exact SU(3) is not verified esp. with smaller m_q.
- Overlap (Neuberger) fermions implemented. Results consistent with more elaborated approaches. Must explore finer lattices and larger volumes (still costly!)
- Unquenching is a major worry! Problems not indicated in (Q)ChPT [log term is the same (deg.case!)]. Sharpe's guesstimate of irreducible 15% of quenching error remains with us! Non-degeneracy vs. degeneracy is a tiny effect in full ChPT $\sim 2\%$
- Finally, $B_K^{\overline{\rm MS}}(2\ {\rm GeV}) = 0.63(4)(\pm 15\%)$, i.e.

$$\boxed{\hat{B}_K = 0.87(6)(13)}$$

ε'/ε

Direct CP-violation $\eta \neq 0$! NA48 and KTeV

$$\varepsilon'/\varepsilon = (16.6 \pm 1.6) \times 10^{-4}$$

- A nightmare for theory: all problems in calculation of the matrix elements appear there
- After decomposing $K \to \pi\pi$ amplitudes into parts of definite isospin ($I = 0, 2$)

$$\begin{aligned}
\mathcal{A}(K^+ \to \pi^+\pi^0) &= \sqrt{3/2}A_2 e^{i\delta_2} \\
\mathcal{A}(K^0 \to \pi^+\pi^-) &= \sqrt{2/3}A_0 e^{i\delta_0} + \sqrt{1/3}A_2 e^{i\delta_2} \\
\mathcal{A}(K^0 \to \pi^0\pi^0) &= \sqrt{2/3}A_0 e^{i\delta_0} + \sqrt{4/3}A_2 e^{i\delta_2}
\end{aligned}$$

one writes

$$\epsilon' = \frac{ie^{i\pi/4}}{\sqrt{2}} \underbrace{\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}}_{\omega} \left(\underbrace{\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}}_{P^2} - \underbrace{\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}}_{P^0} \right)$$

- $(1/\omega)^{\exp} \approx 22$ – famous puzzle: large $\Delta I = 1/2$ w.r.t. $\Delta I = 3/2$. P^2 and P^0 both large, difference small.

ε'/ε

- Again, one employs OPE: Short distance dynamics for these ($\Delta S = 1$) processes encoded in Wilson coefficients, all computed at NLO, 10 years ago by Rome and Munich groups (Ciuchini et al and Buras et al 1993)

  See recent paper by Buras and Jamin

- Relevant operators (out of 10) are: current-current

$$Q_1 = (\bar{s}d)_{V-A}(\bar{u}u)_{V-A} \qquad Q_2 = (\bar{s}^a d^b)_{V-A}(\bar{u}^b u^a)_{V-A}$$

  QCD penguins

$$Q_4 = (\bar{s}^a d^b)_{V-A} \sum_{q=u,d,s} (\bar{q}^b q^a)_{V-A} \qquad Q_6 = (\bar{s}^a d^b)_{V-A} \sum_{q=u,d,s} (\bar{q}^b q^a)_{V+A}$$

  Electroweak penguins

$$Q_8 = (\bar{s}^a d^b)_{V-A} \sum_{q=u,d,s} (\bar{q}^b q^a)_{V+A} \qquad Q_9 = (\bar{s}d)_{V-A} \sum_{q=u,d,s} (\bar{q}q)_{V-A}$$

Need to calculate $\langle (\pi\pi)_I | Q_i | K \rangle$

ε'/ε : matrix elements

Matrix elements can be computed in 3 ways

- INDIRECTLY : In the chiral limit, relate $K \to \pi\pi$ to $K \to \pi$ and $K \to 0$, which are simpler to compute (C.Bernard et al 1985)
  Chirality is crucial!
  Early calculations with staggered fermions (Kilcup, Pekurovsky,1999).
  Heroic effort by CP-PACS and RBC collaborations with DWF (RBC: Blum et al, hep-lat/0110075; CP-PACS: Ali Khan et al, hep-lat/0108013)
- DIRECTLY but with UNPHYSICAL kinematics (to avoid Maiani-Testa theorem), and then extrapolate to the physical point by using ChPT. SPQcdR carries out such a project by using Wilson fermions
  (Boucaud et al, 2001; D.B. et al, 2002)
- DIRECTLY in finite volume, by using the Lellouch–Lüscher formula. For a reasonable discretisation (a), huge lattice volumes are needed to have $L \approx 6$ fm (crucial condition for the applicability of the method).

$\Delta I = 1/2$ rule

- $$(1/\omega)^{\text{exp}} \approx 22 \qquad 1/\omega = \frac{\text{Re}A_0}{\text{Re}A_2} \qquad \{\langle Q_1\rangle, \langle Q_2\rangle\} \in \text{Re}A_{0,2} \qquad \langle Q_6\rangle \in \text{Re}A_0$$

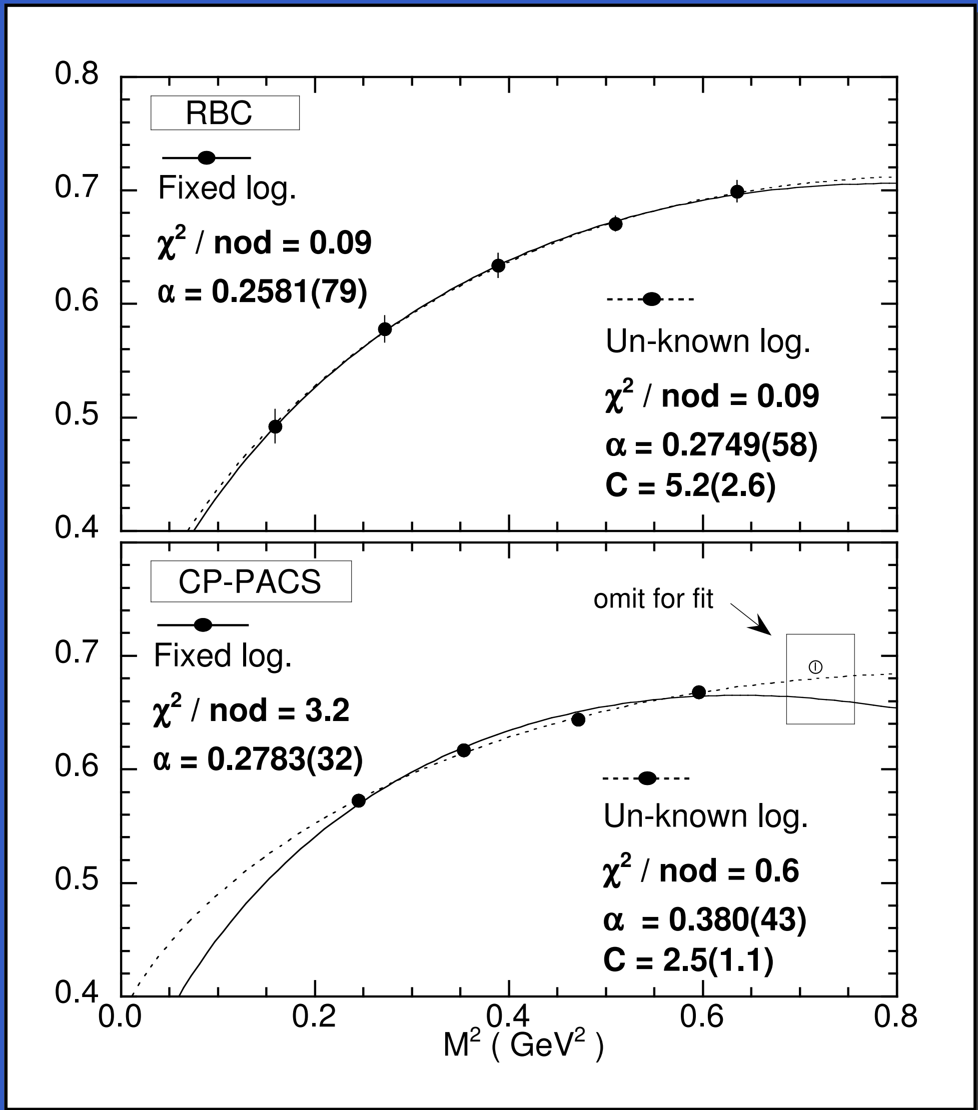
  SD physics $(1/\omega)^{\text{NLO}}_{\text{SD}} \approx 2$. The rest is non-perturbative. ITEP group first pointed out at Q_6, which was/is expected to be the answer to the puzzle (Shifman et al, 1977). Computation of $\langle Q_6\rangle$ very demanding!

- Lattice:
  RBC : $1/\omega = 25.3 \pm 1.8 \pm \text{syst.}$ CP-PACS : $1/\omega = 9.5 \left({}^{+3.2}_{-1.8}\right) \pm \text{syst.}$
  Both see that $\langle Q_6\rangle$ is very small, i.e. irrelevant for $\Delta I = 1/2$ rule.
  How do they see this enhancement of $1/\omega$? B_K^χ -again- is the answer!

- CP-PACS and RBC:
  They agree on $\text{Re}A_0$, but not on $\text{Re}A_2$.

$$\langle \pi^+ | Q_{1,2}^{3/2} | K^+ \rangle = \frac{9}{4} f_K^2 m_K^2 B_K^\chi$$

$\Delta I = 1/2$ rule cont.



Fit to a form ($\alpha \equiv B_K^\chi$)

$$B_K = \alpha\left(1 + \beta M^2 + C\frac{M^2}{(4\pi f)^2}\log(M^2/\Lambda_\chi^2)\right)$$

- RBC-consistent with $C^{\mathrm{ChPT}} = -6$, CP-PACS not consistent
- RBC quote the result by fixing $C = -6$ in the fit, CP-PACS treats C as a free parameter. Thus the difference!

My previous comments on B_K^{DWF} obviously apply here too.

What about $\langle Q_6 \rangle$?

- $Q_i^{1/2}$ have "penguin" contractions: mixing with lower dimensional operators $Q_{\rm sub} = (m_s + m_d)\bar{s}d + (m_s - m_d)\bar{s}\gamma_5 d$, to be subtracted away!
  Use CPS and parity (chirality is a must!), and subtractions are made by imposing
  (C.Bernard et al, 1985)

  $$\langle 0|Q_6 - \alpha_6 Q_{\rm sub}|K\rangle = 0$$

  Likewise for other $\Delta I = 1/2$ operators. Q_6 nightmareish : subtraction almost completely washes out the signal.

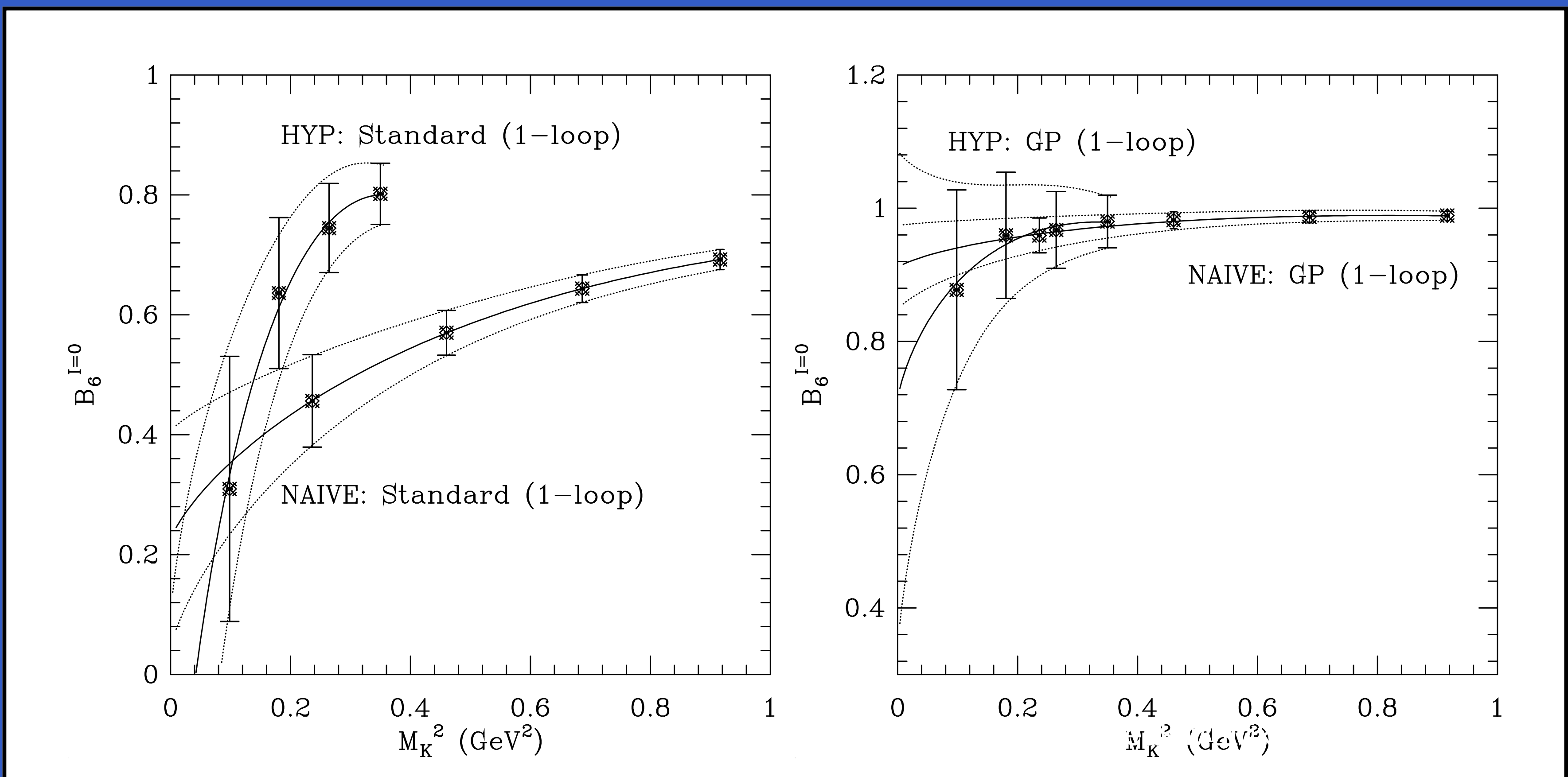
- In full theory $Q_6 \in (8_L, 1_R)$ irrep of $SU(3)_L \otimes SU(3)_R$. In quenched theory, $SU(3) \rightarrow SU(3|3)$, Q_6 is not a singlet under $SU(3|3)_R \Rightarrow Q_6^{\rm full} \neq Q_6^{\rm quench}$
  (Golterman, Pallante, 2001)
  If simply transcribed from full to quenched QCD ($\psi = (q, \tilde{q})^T$, $N = ([\mathbf{1}_{\rm diag}] \oplus [-\mathbf{1}_{\rm diag}]$):

  $$Q_6 = \frac{1}{2} Q_6^S + Q_6^{NS} \quad : \quad Q_6^S = (\bar{s}^a d^b)_{V-A} \sum_q (\bar{\psi}^b \psi^a)_{V+A} \qquad Q_6^{NS} = (\bar{s}^a d^b)_{V-A} \sum_q (\bar{\psi}^b N \psi^a)_{V+A}$$

  A serious problem for all quenched estimates of $\langle Q_6 \rangle$!

NEW: B_6 staggered

- A way out proposed by Golterman & Pallante: kick out Q_6^{NS}. It reduces to omitting the $\bar{q}q$ contractions in "eye" and annihilation diagrams. However, there is no unique way to get rid of Q_6^{NS}.
Golterman & Peris argued that anyway $\alpha^Q_{1\ (8,1)}/\alpha_{1\ (8,1)} < 1$: i.e. Quenched $\langle Q_6\rangle$ smaller than the full one!
- Bhattacharya et al: with HYP-staggered; subtraction is made à la Bernard et al.; same lattice setup as discussed for B_K *(see also talk by W.Lee)*



NEW: B_6 staggered

- They compute B_6 parameter

$$\langle(\pi\pi)_{I=0}|Q_6(\mu)|K\rangle^{(0)} = -4\sqrt{\frac{3}{2}}(f_K - f_\pi)\left(\frac{M_K^2}{m_s(\mu)+m_d(\mu)}\right)^2 B_6(\mu)$$

♣ Result in the $\overline{\mathrm{MS}}$(NDR) of Buras et al is

$$\boxed{B_6^{\mathrm{stand.}}(m_c) = 0.73(9) \qquad B_6^{\mathrm{GP}}(m_c) = 0.98(7)}$$

the value used in the standard ε'/ε analyses for ages.

$\star$ Their "standard" value is more than 2 times larger than CP-PACS $B_6(m_c) \approx 0.3$ (similar situation with RBC – their B_6 even smaller)

$\longrightarrow \langle Q_6\rangle$ is still controversial!

Plugging the B_6^{GP} and $B_8^{3/2} = 1.0(2)^{\mathrm{NEW}}$, in the Buras & Jamin (BJ) formula, they get

$$\varepsilon'/\varepsilon = (10.9 \pm 1.5) \times 10^{-4}$$

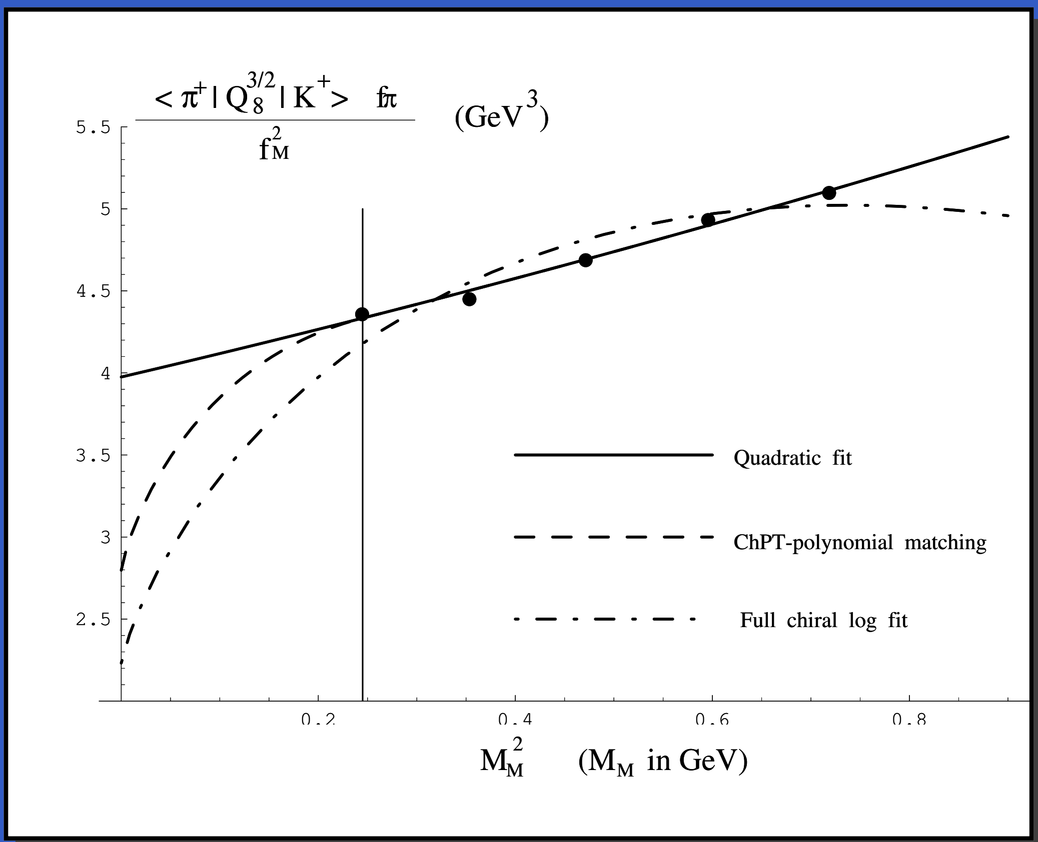
2 comments:

(1) P^0 part of the BJ formula contains the isospin breaking term whose value is also controversial: $\Omega_{IB} = 0.15 \div 0.20$, confirmed recently by Cirigliano et al. On the other side, S.Gardner suggests $\Omega_{IB} \in (0.05, 0.78)$. Be very affraid!

(2) Would be interesting if W.Lee et al. computed $\langle Q_2\rangle$, to see if they agree with findings from DWF.

$K \to \pi\pi$ from $K \to \pi$, $K \to 0$ (remarks)

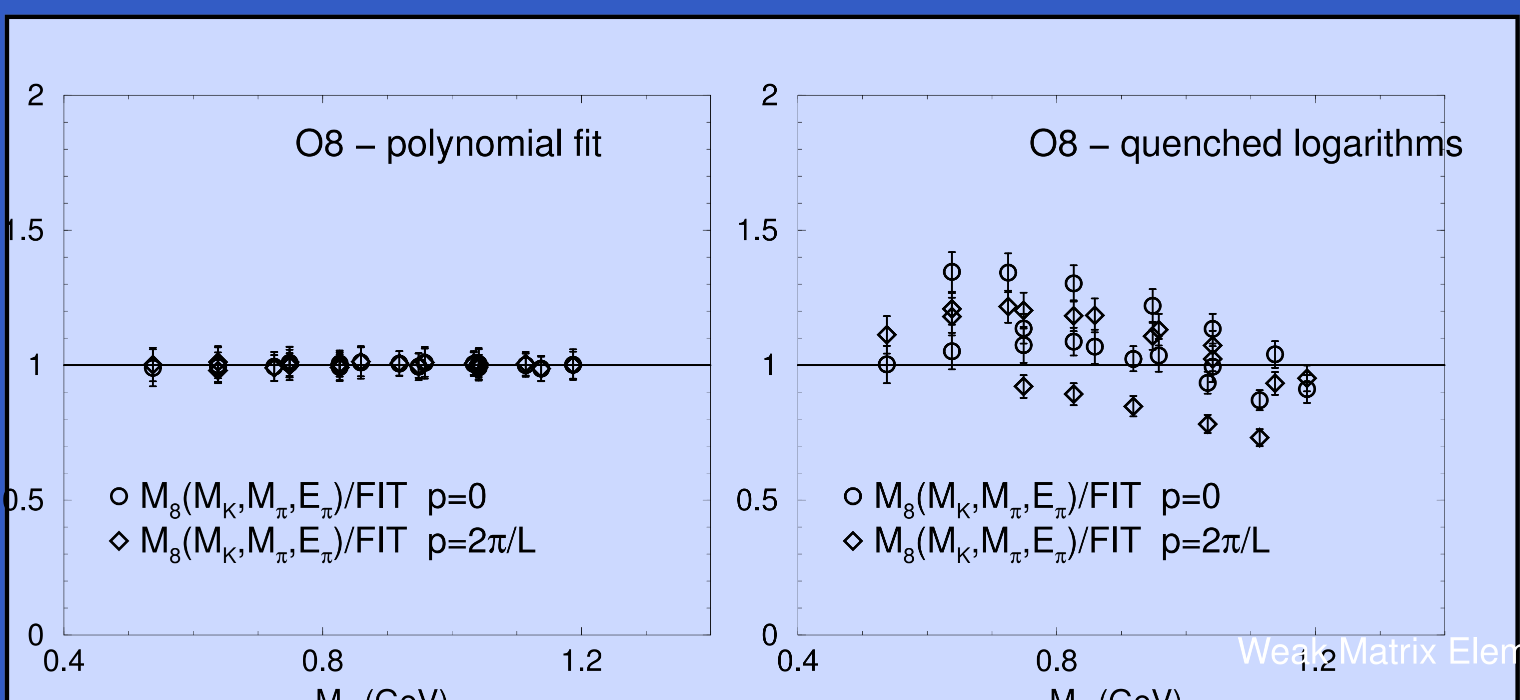
- Problem of subtractions may be tackled by using tmQCD. By a judicous choice of 2 twisting angles (dynamical charm!), in the quenched approximation, it is possible to eliminate the power divergence problem altogether (see S.Sint's talk)



- How to reach the chiral limit with $K \to \pi$ and $K \to 0$ is a difficult problem.
- with RBC results, D.Lin shows that hitting the chiral limit is ambiguous (vertical line, ”arbitrary” point at which the polynomial and log-dominated behavior match)
- FSI ignored
- $\mu = m_c$ is very low for OPE to set in; With dynamical charm, GIM efficient for ReA_0 (but not for ImA_0)
- Unquenching tackled this year for the 1st time: First results from the unquenched study by RBC (see talk by R.Mawhinney).

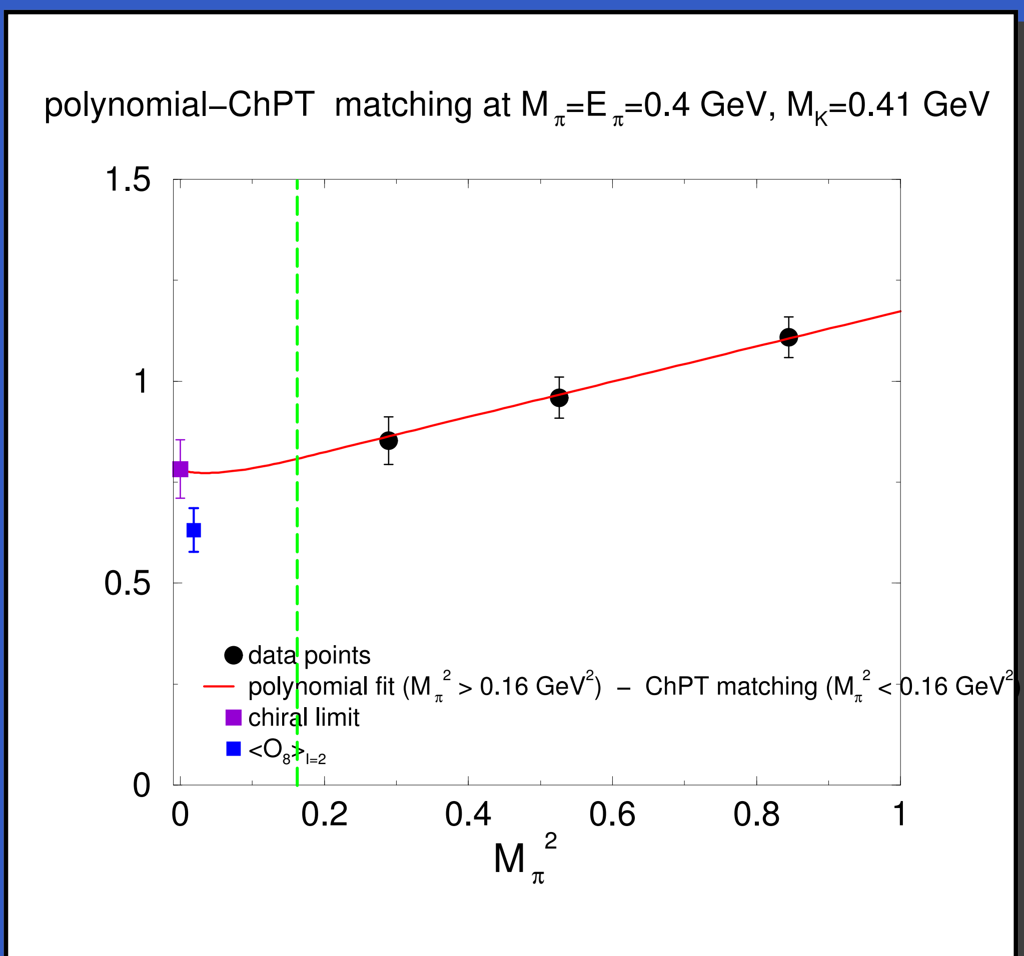
NEW: $\Delta I = 3/2$ by SPQcdR

- Direct computation of $\langle \pi\pi | Q_{7,8}^{3/2} | K \rangle$, but with SPQR kinematics: K and one π in the final state are at rest; the other pion moves ($\vec{p}_\pi = 0, 2\pi/L$, considered). When $\vec{p}_\pi \neq 0$, to have $|(\pi\pi)_{I=2}\rangle$ state important to symmetrise as $[(|\pi^+(\vec{p})\pi^0(\vec{0})\rangle + |\pi^+(\vec{0})\pi^0(\vec{p})\rangle]/2$, to remove $I = 1$ component.
- Quenched study at $\beta^{WP} = 6.0$ ($24^3 \times 64$), 480 configs with Wilson quarks. Matching and renormalisation done nonperturbatively (in RI/MOM).
- Extracted amplitudes are functions of M_K, M_π and E_π. (Q)ChPT expressions worked out at NLO by `(D.Lin et al, 2003)`
- QChPT expression : no suitable description of the data points (simulated pions $M_\pi > 500$ MeV).



NEW: $\Delta I = 3/2$ by SPQcdR cont.

- To go to the physical limit they fit the data to the polynomial form in general kinematics, match to the SPQR kinematics enriched by the (full-physical) chiral logs at some m_M. That fix all the low energy constants, after which they extrapolate to the physical pion and kaon masses.
- The point at which the smooth matching is made (from which the chiral log behavior becomes important in extrapolation) is varied between 0.3-0.5 GeV, and th spread is included in the syst.error.



RESULT in $\overline{\mathrm{MS}}$(NDR) at $\mu = 2\ \mathrm{GeV}$:

$$\langle \pi\pi | Q_8^{3/2} | K \rangle = 0.664(57)\,{}^{40}_{38}\,\binom{50}{40}$$

As a byproduct they also compute

$$\langle \pi\pi | Q_7^{3/2} | K \rangle = 0.111(10)\,\binom{6}{4}\,(6)$$

(see poster by M.Papinutto) N.B. The errors due to the $\pi\pi$-phase shift and the finite volume effects are not included in the final systematics (*i. Lellouch–Lüscher factor applies to the center of mass frame of 2 pions; ii. QChPT formula does not fit the data, so its finite volume version cannot be used*).

Direct calculation of $\Delta I = 1/2$ amplitudes?

- `Lin, Martinelli, Pallante, Sachrajda, Villadoro, 2003`:
  Lack of unitarity in the (partially) quenched theory induces several horrendous problems to SPQcdR:
  (i) no Watson theorem $\rightarrow$ FSI phase is not universal (depends on the operator used to create two pion state);
  (ii) no unambiguous way to form the time independent ratios of correlation functions in order to extract the desired amplitudes from the lattice (η' propagates with other PGB – it does not decouple from the octet)
  (iii) Amplitudes increase with the size of the lattice volume (see also Golterman,Pallante,2000)
  (iv) Lüscher's quantisation condition and LL formula are not valid any more
- `Laiho, Soni, 2003`:
  PQCD may be good enough for $K \rightarrow 0$, $K \rightarrow \pi$ and $K \rightarrow \pi(\vec{0})\pi(\vec{0})$. A specially good is the situation in which $m_{\rm sea} = m_{\rm val.} = m_u = m_d$, in which the finite volume enhancement dissapears *(see J.Laiho's talk)*

Many other issues that I cannot cover today...

- `Young Ross`:
  Adelaide group computed μ_p and μ_{Δ^+}. They observe a strong nonlinearity that is very well fit by the expressions derived in QChPT. (Unambiguous evidence for the quenched chiral log?)
- `A.Schindler & I.Wertzorke`:
  NPR twist-2 $\langle x \rangle_\pi$, strong dependence on the lattice volume is getting under control. First physical result? (see their talks)
- `Y.Aoki (RBC)`:
  Hadronic matrix element of proton decay: feasibility study with DWF
- `T.Yamazaki (JLQCD), K.J.Juge (GRB), C.Kim (RBC)` :
  $\pi - \pi$: a_0^2 and $\delta_2(W)$
- `G.C.Rossi`:
  In what way their proposal to use $S_W(+r)$ and $S_W(-r)$ to reduce all $\mathcal{O}(a)$ effects, may be useful in kaon physics?
- `Y.Aoki (RBC)`:
  Hadronic matrix element of proton decay: feasibility study with DWF

Questions, requests, comments . . .

★ B_K

UNQUENCH, UNQUENCH, UNQUENCH . . .

World average remains:

$$\hat{B}_K = 0.87(6)(13)$$

second error is due to quenching (this afternoon RBC presents B_K^{DWF} with $n_F = 2$).

UNQUENCH, UNQUENCH, UNQUENCH . . .

♠ Props to RBC and CP-PACS for a huge effort with DWF
Important: NLO chiral corrections must be implemented.
What would they obtain if implemented the GP proposal?
Overlap fermions have not been explored yet.

♣ To get to the physical $K \to \pi\pi$, Avenue-1: ChPT. There exists Avenue-2 too: dispersion relations (Bourrely, Caprini, Micu, 2002; Bücher et al 2001) with bouble diagrams resummed à la Omnès (subtraction constants can be fixed from the matrix elements already computed on the lattice!)

Questions, requests, . . .

- ε'/ε:
  Do the collaborations using staggered fermions see large $\langle Q_2 \rangle$ (i.e. large to get $1/\omega$ right)?
  More studies of $\langle Q_4 \rangle$ are needed: if $\langle Q_6 \rangle$ is indeed small (quenching is a scare!), then probably Buras& Jamin should include explicitely $\langle Q_4 \rangle$ in their formula.
- “Directly" obtained value for the EW penguin in quenched approximation (SPQcdR) is by a factor 2-4 smaller than the predicitions based on analytic (phenomenological) approaches by Bijnens et al, Donoghue et al, DeRafael et al. Why?
  Community is strongly encouraged to implement LL proposal and compute $K \to (\pi\pi)_{I=2}$ amplitudes directly.
- $I = 0$ amplitudes seem hopeless in the quenched approximation. Call for new (clever) idea?

♠ Thanks to the organisers, collaborators and to all of you!