Recent Progress in the Staggered ϵ'/ϵ Project

Weonjong Lee

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Collaboration

1. Seoul National University

   Weonjong Lee

2. University of Washington, Seattle

   Stephen Sharpe

3. Computing Support from Columbia University

   Norman Christ, Guofeng Liu, Robert Mawhinney, Lingling Wu

4. Programming support (QCDSP, HYP)

   George Fleming (OSU / Jefferson Lab)

5. Various contribution in the early stages of this work

   LANL: Tanmoy Bhattacharya, Rajan Gupta

   Ewha Woman's University: Chanju Kim

   OSU: Greg Kilcup

Main Goals

1. Check the results obtained using the quenched Domain-Wall fermions
   - CP-PACS $\rightarrow$ negative ϵ'/ϵ.
   - RBC $\rightarrow$ negative ϵ'/ϵ.

2. Calculate ϵ'/ϵ in the full QCD
   - Quenched Approximation is one of the main systematic errors.
   - One needs to use the improved actions.

3. Compare the results with the observed value in experiment
   - A window for new physics.
   - Confirmation of the standard model.

Unimproved Staggered Fermions

1. Pros:
   - Conserved $U(1)_A$ chiral symmetry.
   - Quark mass is protected (No residual mass).
   - Computationally cheap: $\sim 1/(10 \times 4)$ of DWF.
   - Discretization error: $\mathcal{O}(a^2)$.
   - Easy to improve.

2. Cons:
   - Broken SU(4) flavor symmetry.
   - Operator mixing matrix of 65536×65536.
   - Large $\mathcal{O}(a^2)$ error in pion spectrum.
   - Large one-loop correction for $[P \times P]$.
   - Non-perturbative renormalization is tough (impractical?).

The First Stage:
Perturbation for unimproved staggered operators

1. We must use gauge-invariant operators for ϵ'/ϵ.

2. Penguin diagrams calculated by Patel and Sharpe (1993)

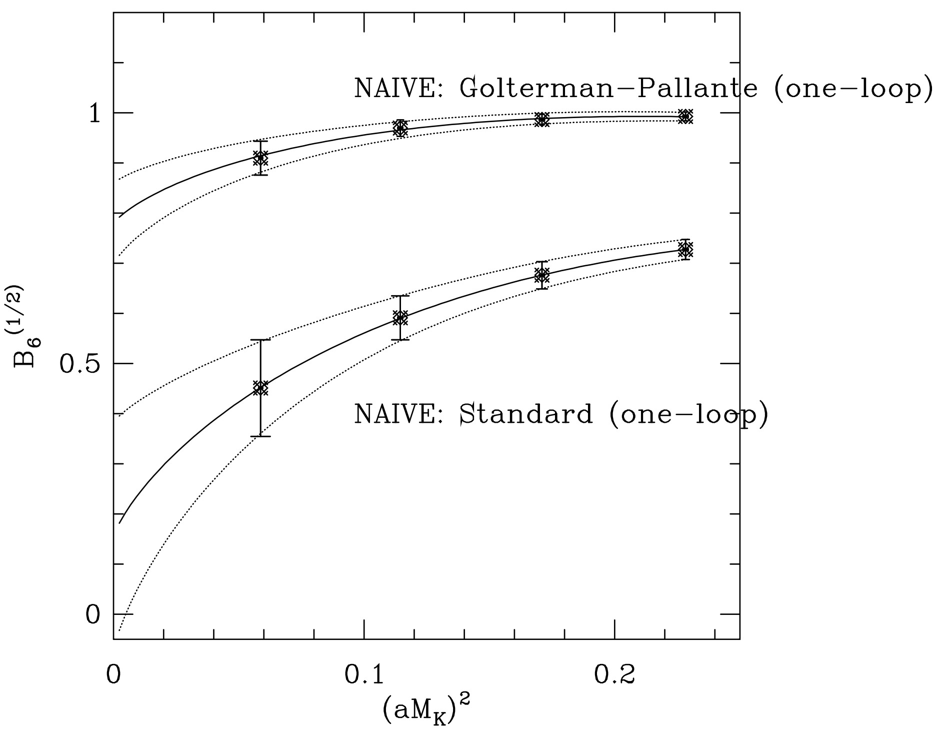
3. Current-current diagrams calculated by W. Lee (2001)

   - Conserve the $U(1)_A$ symmetry.
   - Preserve the gauge invariance.
   - Respect the Fierz identities.

The Second Stage: NPB (Proc.) 106 (2002) 311

Numerical study on QCDSP (unimproved staggered)

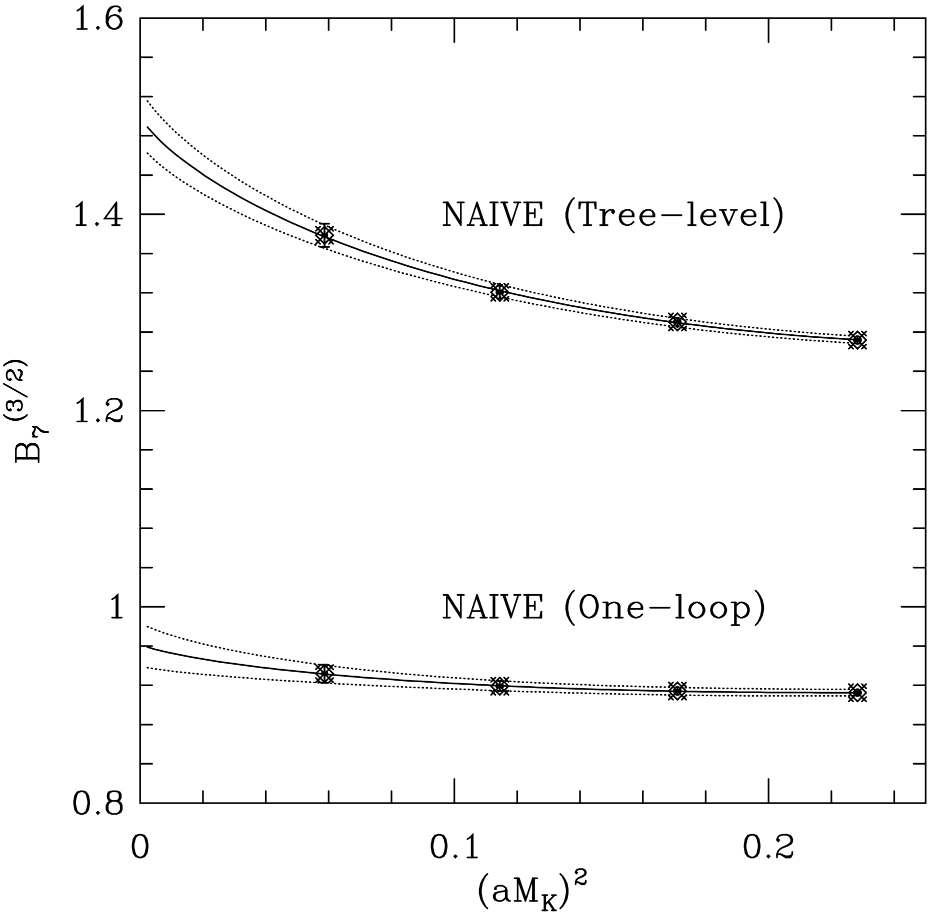
1. Check the previous works (Kilcup, Pekurovsky).
2. Use fully one-loop matched operators.
3. Large one-loop correction.
4. Large quenching uncertainty.



The Second Stage: NPB (Proc.) 106 (2002) 311

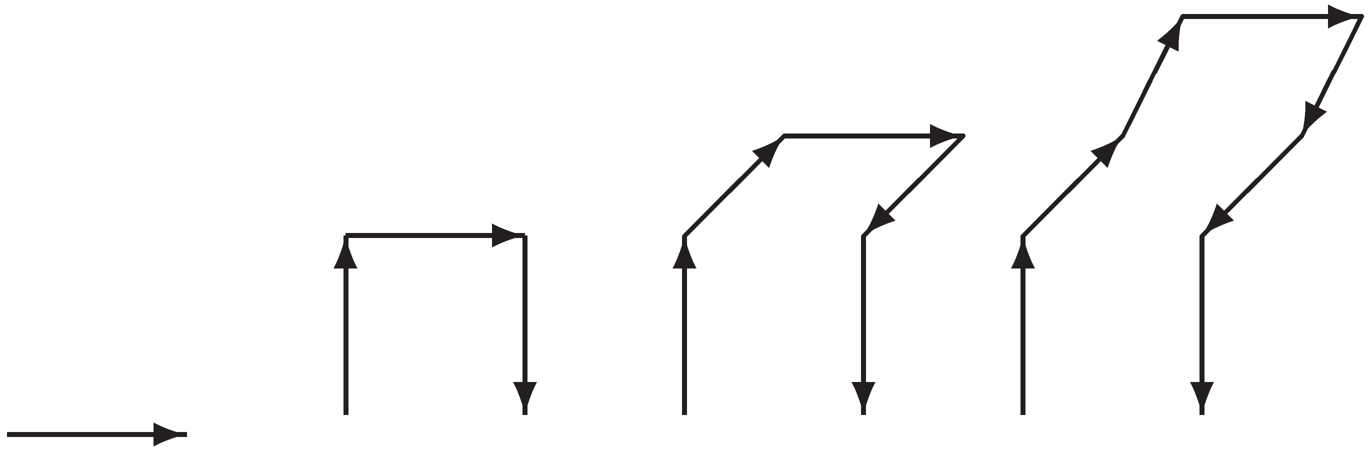
Large Perturbative Correction (unimproved staggered)

1. Problem: matching at $\mu = \pi/a$ instead of $\mu = 1/a$.

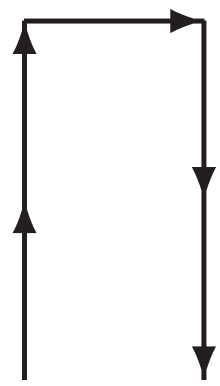


Fat Links

- Fat 7

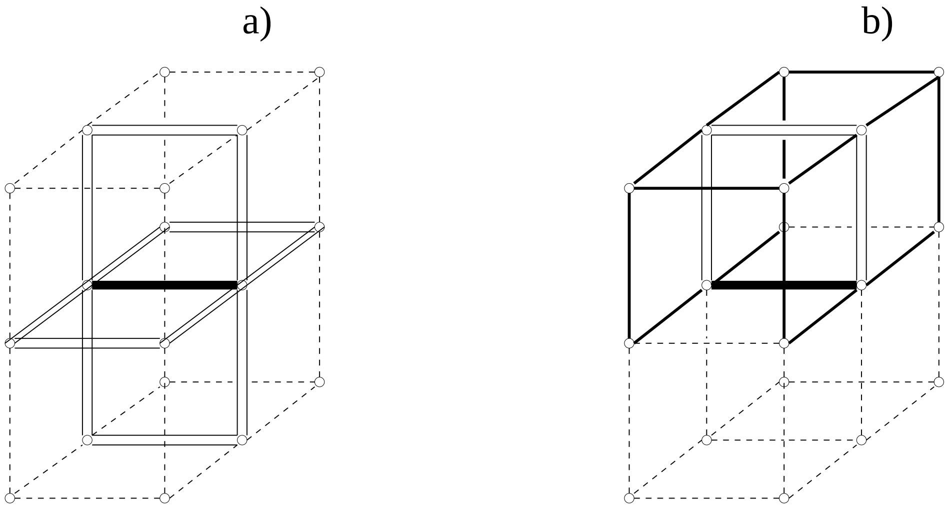


- Lepage term



Hypercubic Links

- Hypercubic (HYP)



The Third Stage:

Improved Staggered Actions and Operators

1. Improved Staggered Fermion Actions.

   - AsqTad(MILC): Fat7 + Lepage + Naik

     the Naik term:

     Little role on the flavor symmetry restoration.

     No role on operator transcription.

     Hard to code with high efficiency.

   - Fat7

   - Fat7 + Lepage

   - Hypercubic (HYP)

   - Other possibilities

2. Key Ideas of Improvement:

   Suppress the flavor changing interaction of high momentum gluons with quarks, using the Symanzik programme or smearing.

3. Numerical studies find

   - Flavor symmetry breaking substantially reduced. (Numerical study of the pion spectrum)

   - Largest Reduction for HYP and APE smearing.

4. Bilinear Operator Renormalization:

(W. Lee and S. Sharpe: PRD 66 (2002) 114501.)

- All choices of smeared links significantly reduce the size of one-loop contributions:
  Fat7 and HYP are the best.

- The one-loop corrections are now comparable to those for Wilson and Domain Wall fermions.

- It is possible to reduce the one-loop correction down to less than 10 % level.

- This leads us to choosing HYP for the numerical study.

The Fourth Stage: hep-lat/0208050

Perturbation For HYP

1. Various studies on improvement:
   $\rightarrow$ We choose HYP for the numerical study.

2. Properties of HYP link:
   (W. Lee: PRD 66 (2002) 114504.)

   Any fat link can be expanded in powers of gauge fields (A_μ):

$$B_\mu = B_\mu^{(1)} + B_\mu^{(2)} + B_\mu^{(3)} + \cdots$$
$$B_\mu^{(n)} = \mathcal{O}(A^n)$$

- Theorem 1. SU(3) Projection

(a) The linear term ($B_\mu^{(1)}$) is invariant under SU(3) projection.

(b) The quadratic term ($B_\mu^{(2)}$) is antisymmetric in gauge fields.

- Theorem 2. Triviality of one-loop renormalization

(a) At one loop level, only $B_\mu^{(1)}$ term contribute to the renormalization of the gauge-invariant staggered fermion operators.

(b) At one loop level, the contribution from $B_\mu^{(n)}$ for any $n \geq 2$ vanishes.

(c) At one loop level, the renormalization of the gauge-invariant staggered operators can be done by simply replacing the propagator of A_μ by that of $B_\mu^{(1)}$.

(d) This theorem is true regardless of the details of smearing.

- Theorem 3. Multiple SU(3) Projection

(a) The linear gauge field term $B_\mu^{(1)}$ in the perturbative expansion is universal.

(b) In general, the quadratic terms may be different from one another. But all of them are antisymmetric in gauge fields.

(c) This theorem is true regardless of the details of smearing.

- Theorem 4. Uniqueness under the improvement programme

  If we impose the perturbative improvement condition of removing the flavor changing interactions on the HYP action, then the HYP fat link satisfies the following:

(a) The linear gauge field term in perturbative expansion is identical to that of the SU(3) projected fat link of Fat7 type.

(b) The quadratic terms are antisymmetric in gauge fields.

- Theorem 5. Equivalence at one loop (Universality Class)

  If we impose the perturbative improvement condition of removing the flavor changing interactions, at one loop level,

  (a) The renormalization of the gauge invariant staggered operators are identical between the HYP link and the SU(3) projected fat links of Fat7 type ($\overline{Fat7}$).

  (b) The contribution of one–loop renormalization can be obtained by simply replacing the propagator of A_μ by that of $B_\mu^{(1)}$.

- Punch line: Using the above five theorems, it is possible to prove that $|C_{HYP}| \ll |C_{thin}|$, for each Feynman diagram.

3. Four–Fermion Operator Renormalization for HYP:
(W. Lee and S. Sharpe in preparation)

$$(\mathcal{O}_3)_{II} = 2([P \times P][P \times P] - [S \times P][S \times P])$$

Table of finite correction to $(\mathcal{O}_3)_{II}$.

Operators	$[S \times P][S \times P]_{II}$
NAIVE	$2 \times (95.6 - 6C_N)$
HYP	$2 \times (14.6 - 6C_H)$

Operators	$[P \times P][P \times P]_{II}$
NAIVE	$2 \times (111.3 - 2C_N)$
HYP	$2 \times (8.3 - 2C_H)$

Tadpole Improvement Correction:
$C_N = 13.159, \quad C_H = 0.7709$

KEY POINT: HYP one-loop correction $\approx 10\%$.

TABLE XI. Renormalization constants for $\mathcal{O}_i = (\mathcal{O}_3^{Latt})_{II} = 2[P \times P][P \times P]_{II} - 2[S \times P][P \times P]_{II}$.

$\mathcal{O}_j^{Latt}$	color trace	$\hat{\gamma}_{ij}$	$\hat{C}_{ij}^{Latt}$ NAIVE	$\hat{C}_{ij}^{Latt}$ HYP(I)	$\hat{C}_{ij}^{Latt}$ $HYP(II)/\overline{\text{Fat7}}$	$\hat{T}_{ij}$
$[S \times T_{\mu\nu}][S \times T_{\mu\nu}]$	I	0	-0.915	-0.004	$+0.033$	0
$[S \times T_{\mu\nu}][S \times T_{\mu\nu}]$	II	0	$+0.305$	$+0.001$	-0.011	0
$[S \times T_{\mu\nu}][S \times T_{\mu\rho}]$	I	0	-0.299	-0.015	-0.012	0
$[S \times T_{\mu\nu}][S \times T_{\mu\rho}]$	II	0	$+0.100$	$+0.005$	$+0.004$	0
$[S \times P][S \times P]$	II	$2 \times (-16)$	$2 \times (+95.6216)$	$2 \times (+14.6233)$	$2 \times (+19.1180)$	$2 \times (-6c_{TI})$
$[V_\mu \times A_\mu][V_\mu \times A_\mu]$	I	0	$+0.682$	$+0.017$	$+0.028$	0
$[V_\mu \times A_\mu][V_\mu \times A_\mu]$	II	0	-0.227	-0.006	-0.009	0
$[V_\mu \times A_\nu][V_\mu \times A_\nu]$	I	0	$+0.395$	$+0.006$	-0.010	0
$[V_\mu \times A_\nu][V_\mu \times A_\nu]$	II	0	-0.132	-0.002	$+0.003$	0
$[V_\mu \times A_\mu][V_\mu \times A_\nu]$	I	0	$+0.191$	$+0.005$	$+0.004$	0
$[V_\mu \times A_\mu][V_\mu \times A_\nu]$	II	0	-0.064	-0.002	-0.001	0
$[V_\mu \times A_\nu][V_\mu \times A_\mu]$	I	0	$+0.191$	$+0.005$	$+0.004$	0
$[V_\mu \times A_\nu][V_\mu \times A_\mu]$	II	0	-0.064	-0.002	-0.001	0
$[V_\mu \times A_\nu][V_\mu \times A_\rho]$	I	0	$+0.089$	$+0.002$	$+0.006$	0
$[V_\mu \times A_\nu][V_\mu \times A_\rho]$	II	0	-0.030	-0.001	-0.002	0
$[A_\mu \times V_\mu][A_\mu \times V_\mu]$	I	0	$+6.015$	$+1.233$	$+1.230$	0
$[A_\mu \times V_\mu][A_\mu \times V_\mu]$	II	0	-2.005	-0.411	-0.410	0
$[A_\mu \times V_\nu][A_\mu \times V_\nu]$	I	0	-0.387	$+0.140$	$+0.139$	0
$[A_\mu \times V_\nu][A_\mu \times V_\nu]$	II	0	$+0.129$	-0.047	-0.046	0
$[A_\mu \times V_\mu][A_\mu \times V_\nu]$	I	0	-1.530	-0.435	-0.385	0
$[A_\mu \times V_\mu][A_\mu \times V_\nu]$	II	0	$+0.510$	$+0.145$	$+0.128$	0
$[A_\mu \times V_\nu][A_\mu \times V_\mu]$	I	0	-1.530	-0.435	-0.385	0
$[A_\mu \times V_\nu][A_\mu \times V_\mu]$	II	0	$+0.510$	$+0.145$	$+0.128$	0
$[A_\mu \times V_\nu][A_\mu \times V_\rho]$	I	0	$+0.833$	$+0.178$	$+0.146$	0
$[A_\mu \times V_\nu][A_\mu \times V_\rho]$	II	0	-0.278	-0.059	-0.049	0
$[P \times T_{\mu\nu}][P \times T_{\mu\nu}]$	I	0	$+0.915$	$+0.004$	-0.033	0
$[P \times T_{\mu\nu}][P \times T_{\mu\nu}]$	II	0	-0.305	-0.001	$+0.011$	0
$[P \times T_{\mu\nu}][P \times T_{\mu\rho}]$	I	0	$+0.299$	$+0.015$	$+0.012$	0
$[P \times T_{\mu\nu}][P \times T_{\mu\rho}]$	II	0	-0.100	-0.005	-0.004	0
$[P \times P][P \times P]$	II	$2 \times (+16)$	$2 \times (+111.266)$	$2 \times (+8.250)$	$2 \times (+6.925)$	$2 \times (-2c_{TI})$

The Fifth Stage:

Numerical Study (HYP staggered, QCDSP)

- $\beta = 6.0$, $16^3 \times 64$ lattice, 218 confs.

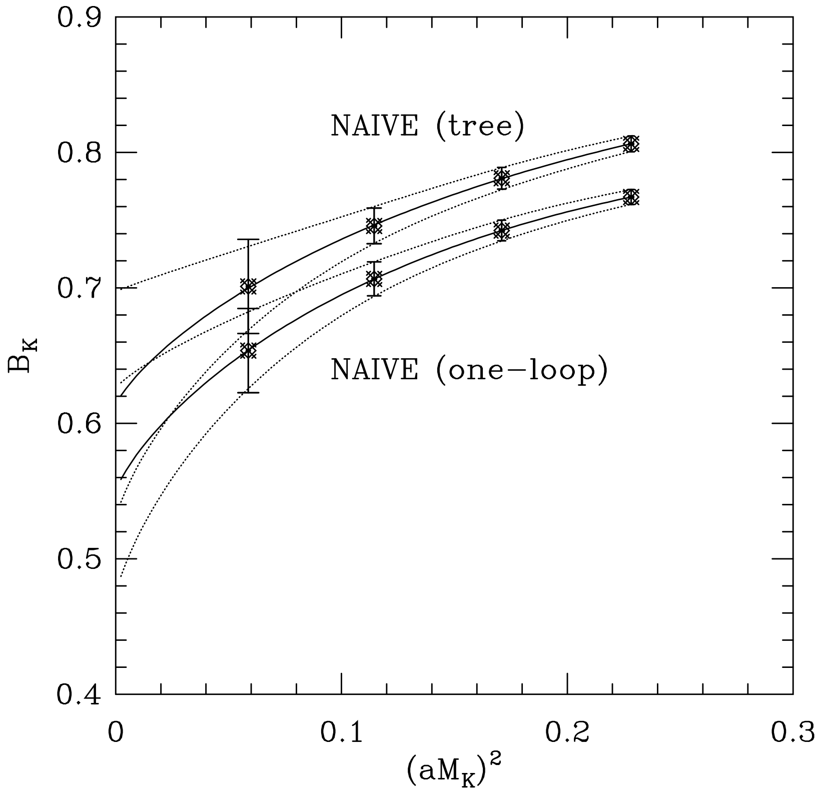
- Bare quark mass shifted by ≈ 2.5.
  $Z_m(Naive) \cong 2.5 \to Z_m(HYP) \cong 1$.

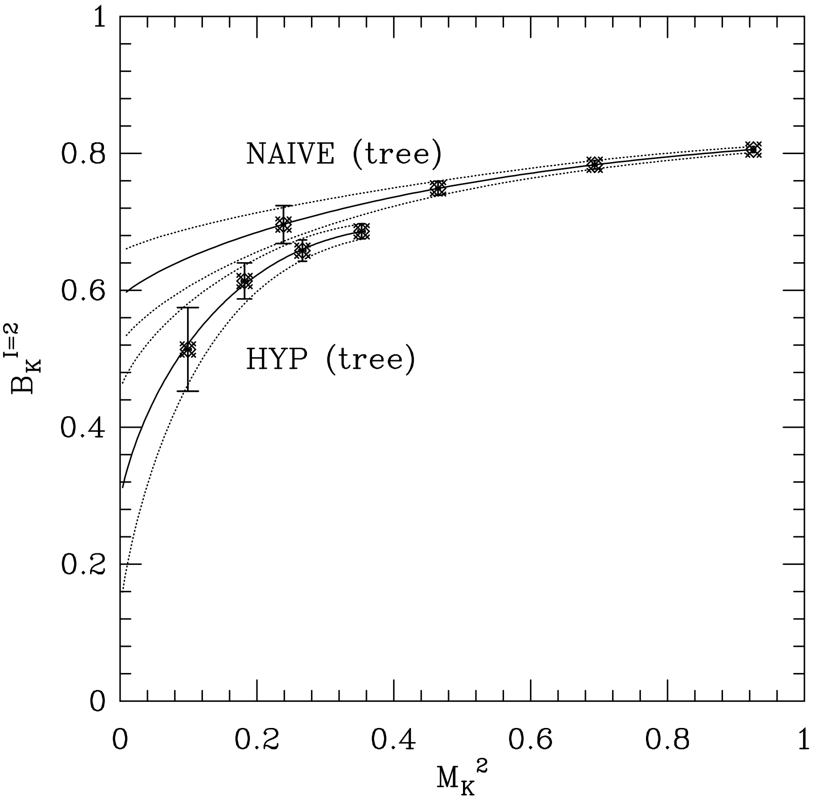
- Small perturbative correction.

- Reduced flavor symmetry breaking.

The Fifth Stage:

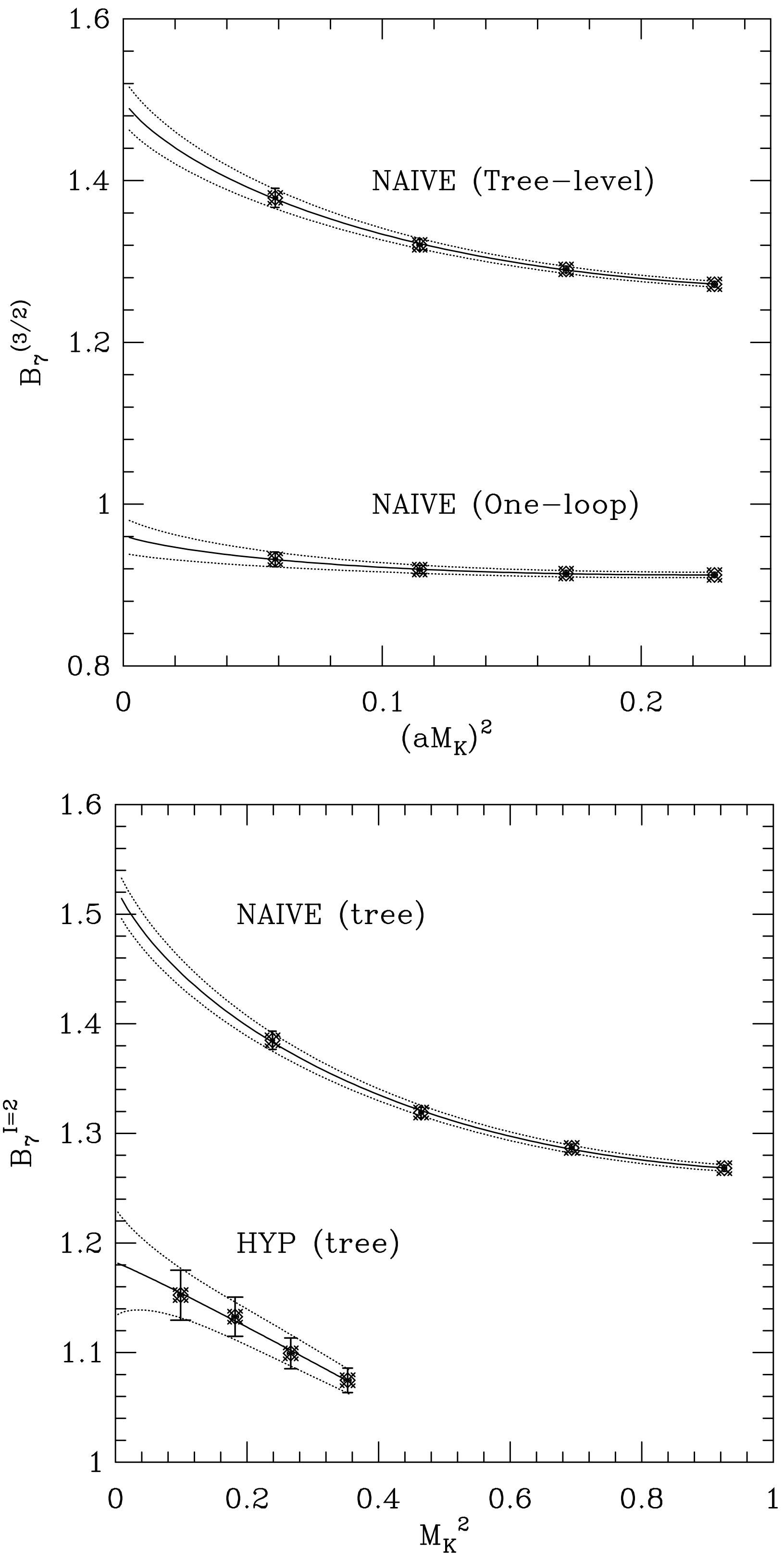
Preliminary B_K (HYP staggered, QCDSP)





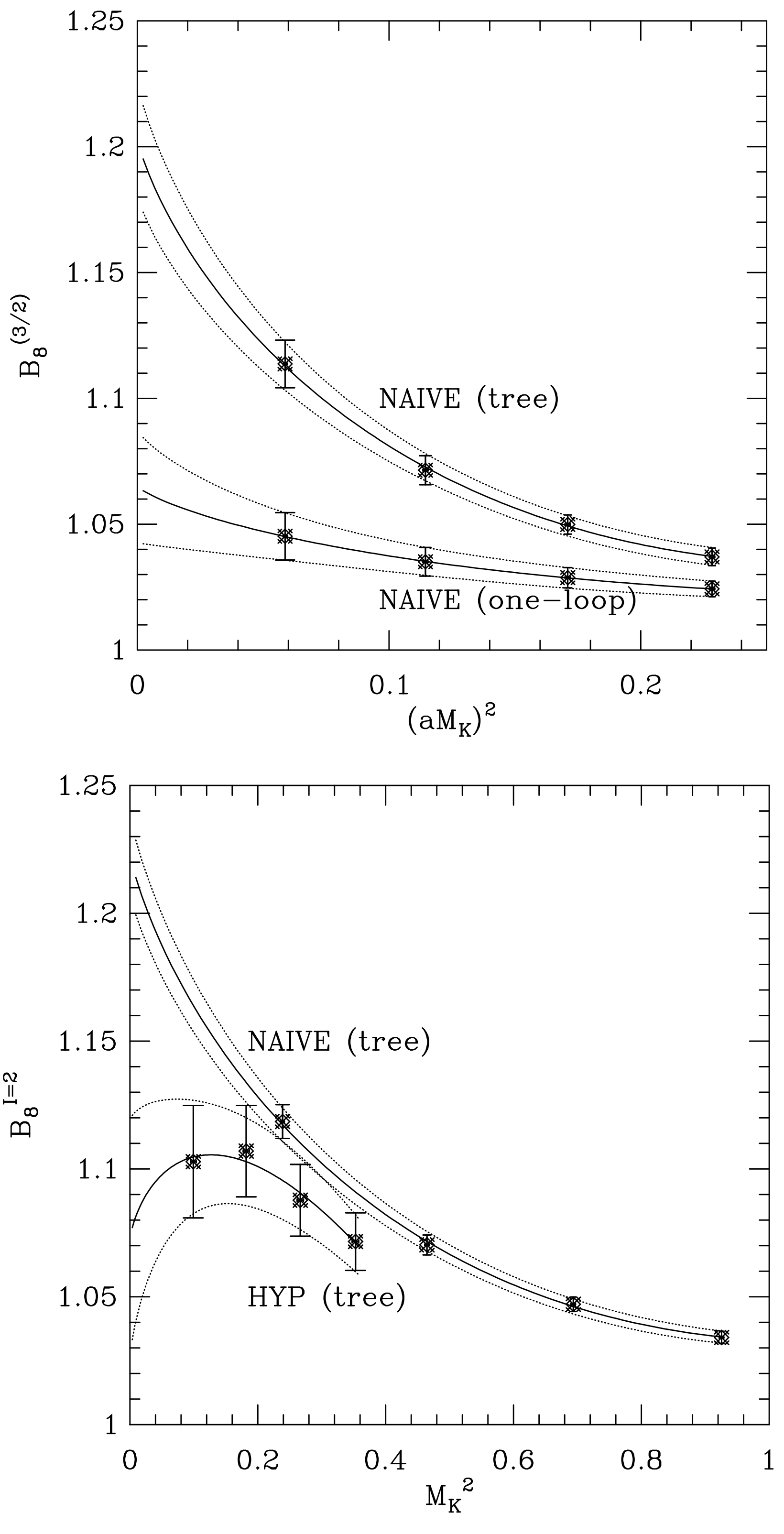
The Fifth Stage:

Preliminary B_7 (HYP staggered, QCDSP)



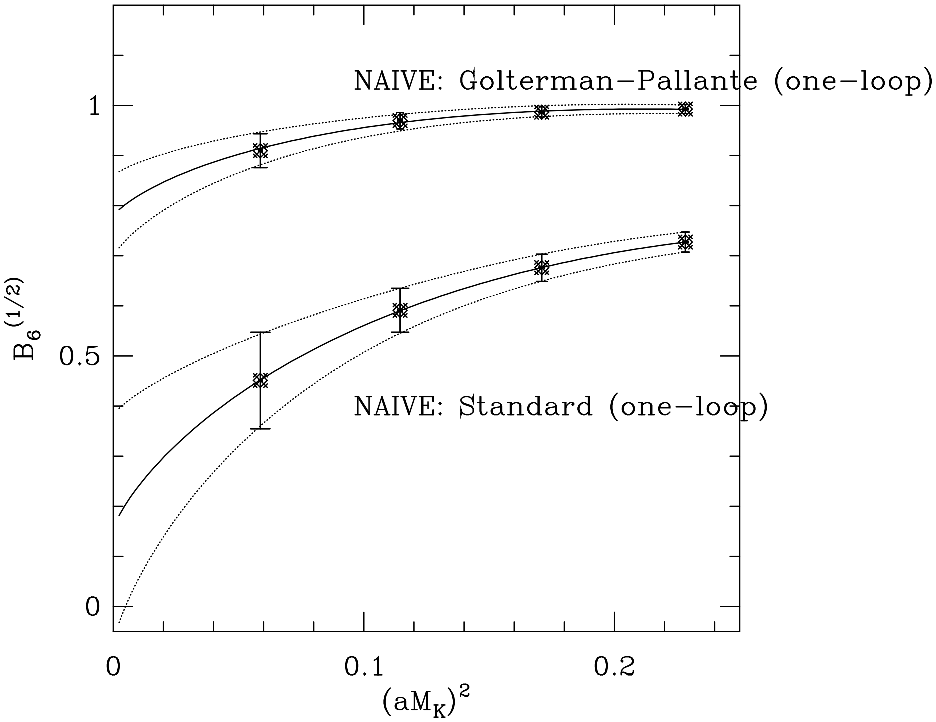
The Fifth Stage:

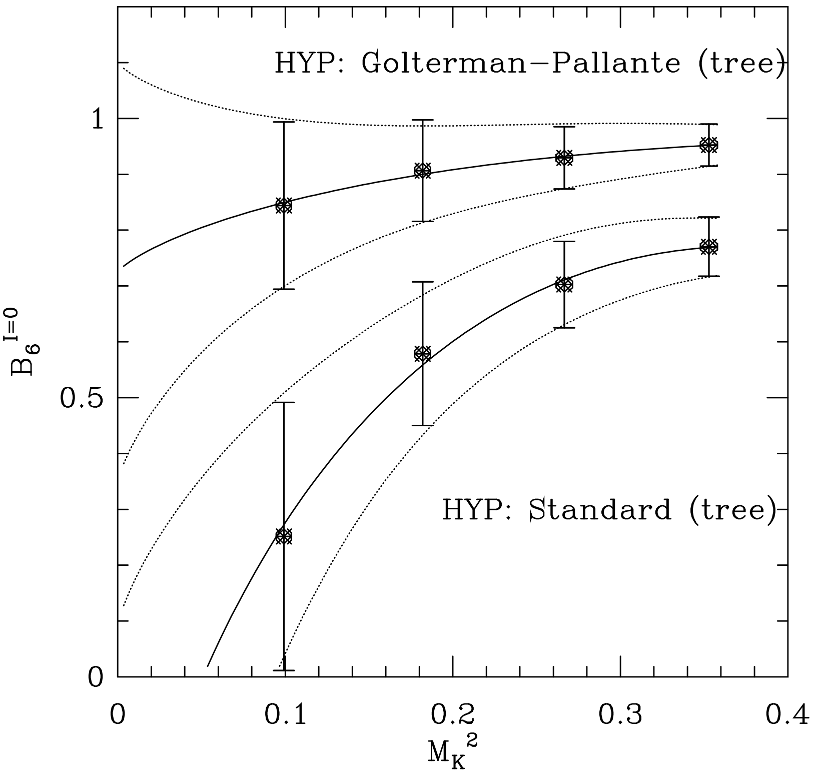
Preliminary B_8 (HYP staggered, QCDSP)



The Fifth Stage:

Preliminary B_6 (HYP staggered, QCDSP)





The Future Plan

1. Current quenched calculation of ϵ'/ϵ using the HYP staggered at $\beta = 6.0$ shows that we need higher statistics.

2. The gap between the Golterman-Pallante and the standard is reduced. ($\rightarrow$ higher statistics)

3. Extend the HYP analysis from tree to one-loop.

4. Calculate the penguin diagrams for B_6.

5. Calculate q^* for operators with anomalous dimension.

6. Extend the calculation to the full QCD (partially quenched QCD).