Finite temperature lattice QCD with improved Wilson fermions

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Motivation

- Determination of T_c and its continuum and chiral limits

- Study of the heavy quark potential and string breaking at $T < T_c$

- Study of the structure of the vacuum in full lattice QCD at nonzero temperature

Simulation details

The fermionic action we employ is of the form

$$S_F = S_F^{(0)} - \frac{\mathrm{i}}{2}\kappa\, g\; c_{sw} a^5 \sum_x \bar{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}\psi(x),$$

where

$S_F^{(0)}$ is the original Wilson action,
c_{sw} is determined nonperturbatively

Jansen & Sommer '98

Lattice size	$16^3 \times 8$	
β	5.2 and 5.25	
Lattice spacing	~ 0.13 fm	at T_c
$\frac{m_\pi}{m_\rho}$	~ 0.8	at T_c

Check of f.v. effects:

$24^3 \times 8$ $\kappa = 0.1343$

Machines:

HITACHI SR8000 at KEK (8 nodes)
MVS 1000M at Joint Supercomputer Center, Moscow

Performance: 2.4 GFlops per node on SR8000
with HMC code of QCDSF

HMC parameters:

$\delta\tau = 0.0125\,,\ n_\tau = 20$ on $16^3 \times 8$
$\delta\tau = 0.008\,,\ n_\tau = 20$ on $24^3 \times 8$
Acceptance rate: $\sim 70\%$

Critical temperature

Polyakov loop average $\langle L \rangle$

$$L_{\vec{x}} = \mathrm{Tr} P_{\vec{x}}, \qquad P_{\vec{x}} = \prod_{x_0} U_{0;x_0,\vec{x}}$$

Polyakov loop susceptibility χ_L

$$\chi_L = \frac{1}{9N_s^3} \left(\langle L^2 \rangle - \langle L \rangle^2 \right)$$

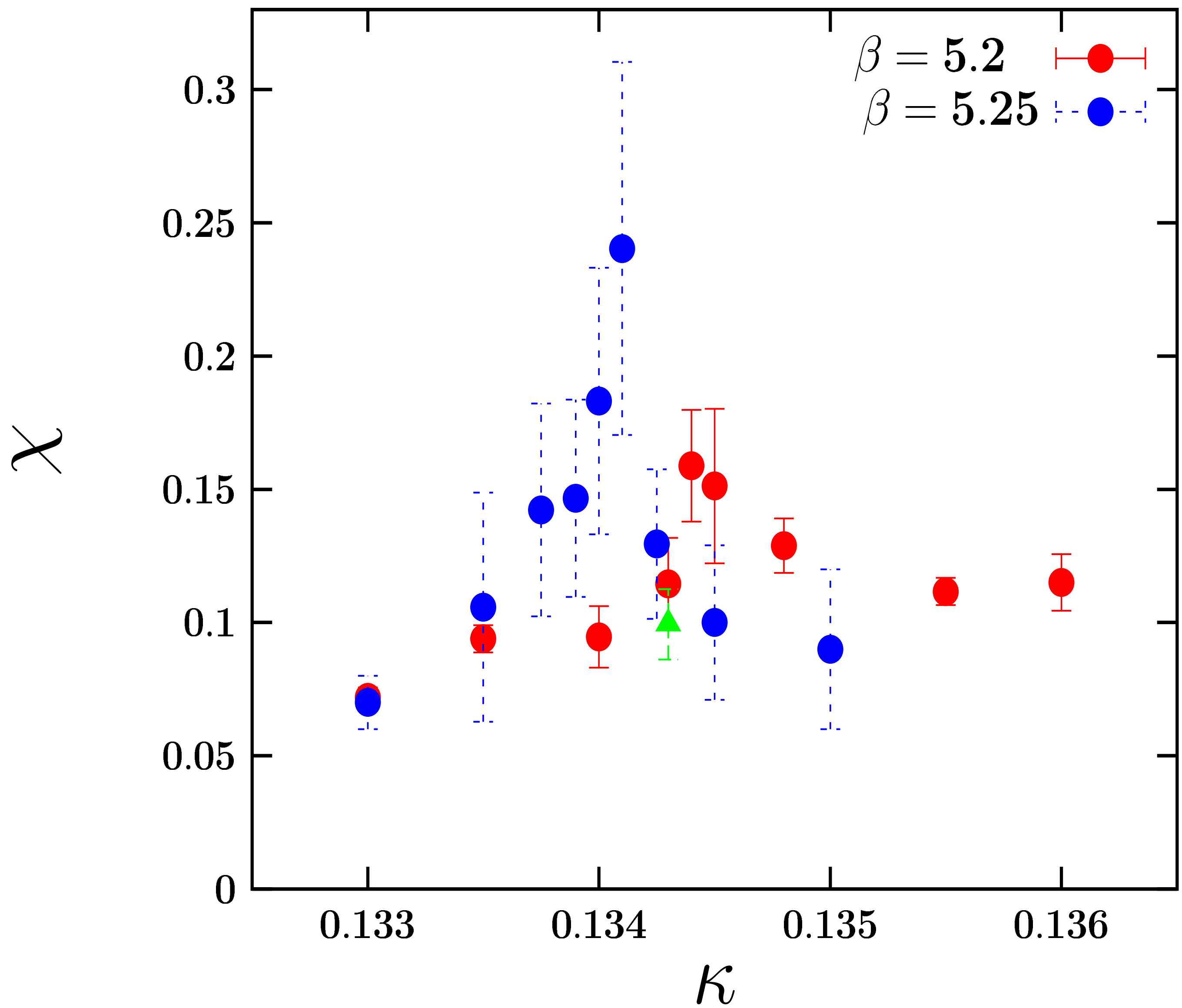
$\langle L \rangle$ is an order parameter in quenched QCD
maximum of χ_L indicates phase transition point

Chiral condensate $\bar{\psi}\psi$ is an order parameter in chiral QCD

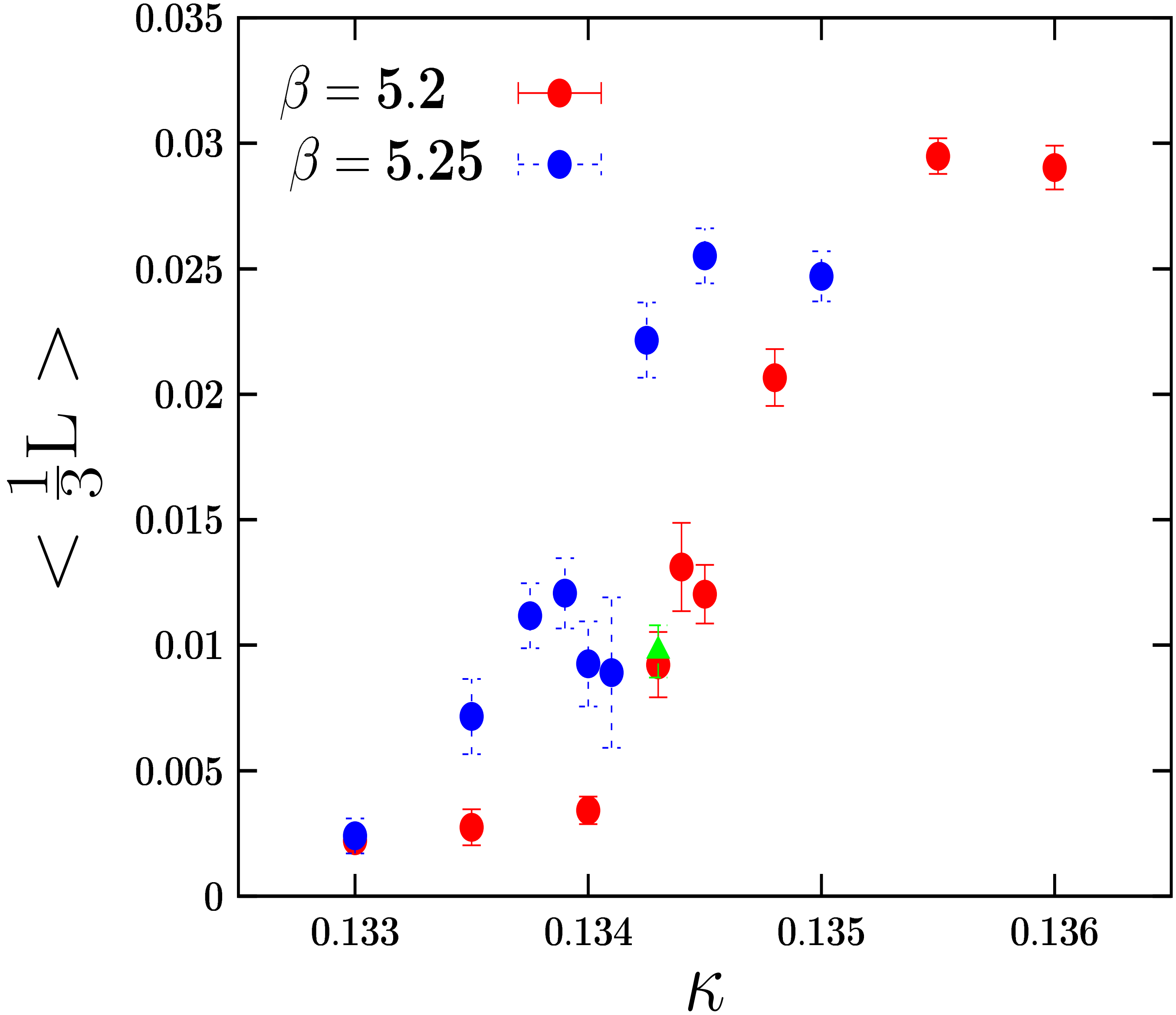
It is found empirically that maxima of χ_L and $\chi_{\bar{\psi}\psi}$ coinside

Karsch, Laermann and Peikert '00

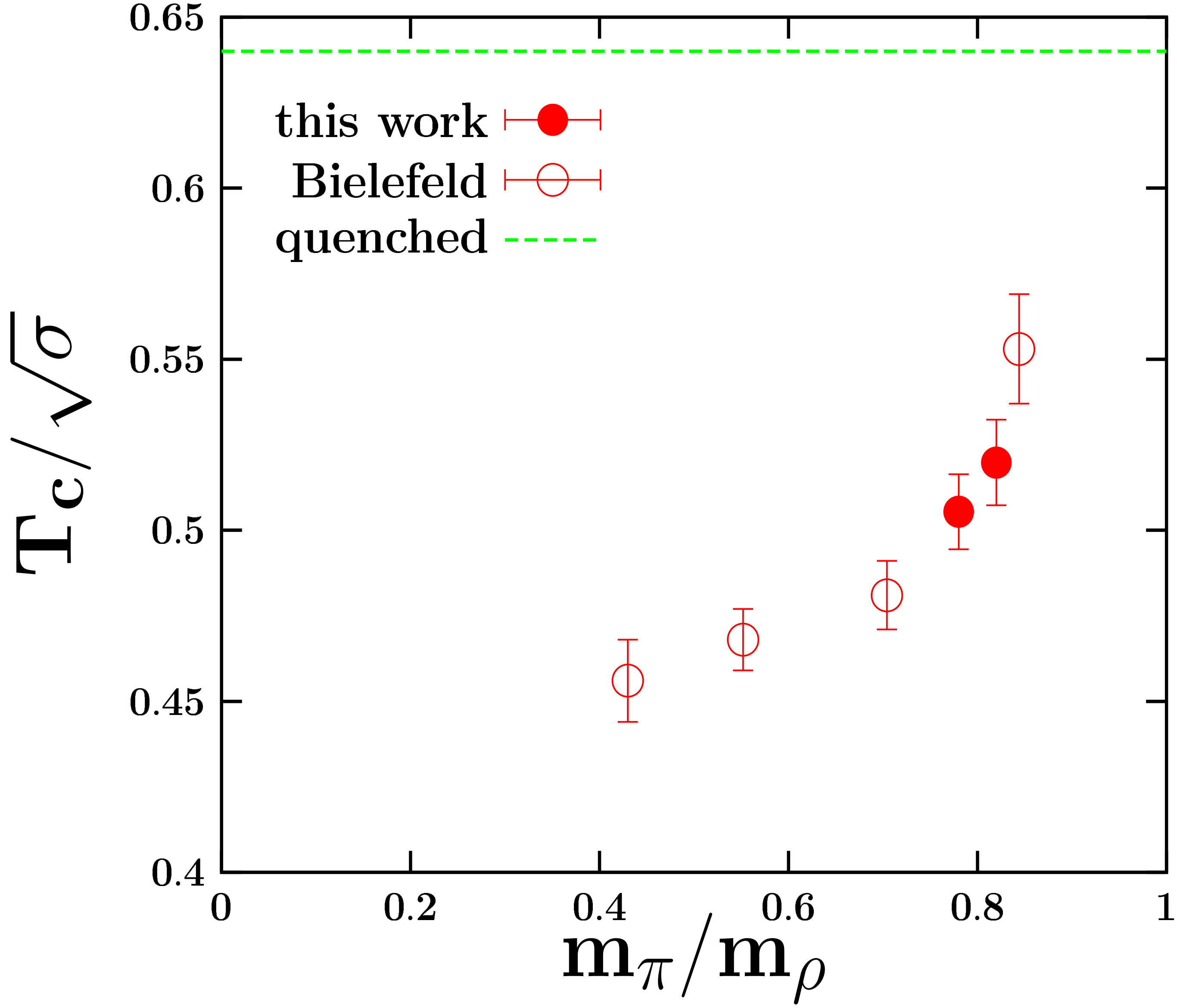
We use χ_L to find T_c



Polyakov loop susceptibility



Polyakov loop average



Critical temperature

String breaking at $T = 0$

Spectral representation for Wilson loop:

$$W(r,t) = C_V(r) e^{-(V_0 + V_{string}(r))t} + C_E(r) e^{-2E \cdot t} + \ldots$$

Numerical studies show that C_E is small

Simple estimate: $C_E(r) \sim e^{-2E \cdot r}$

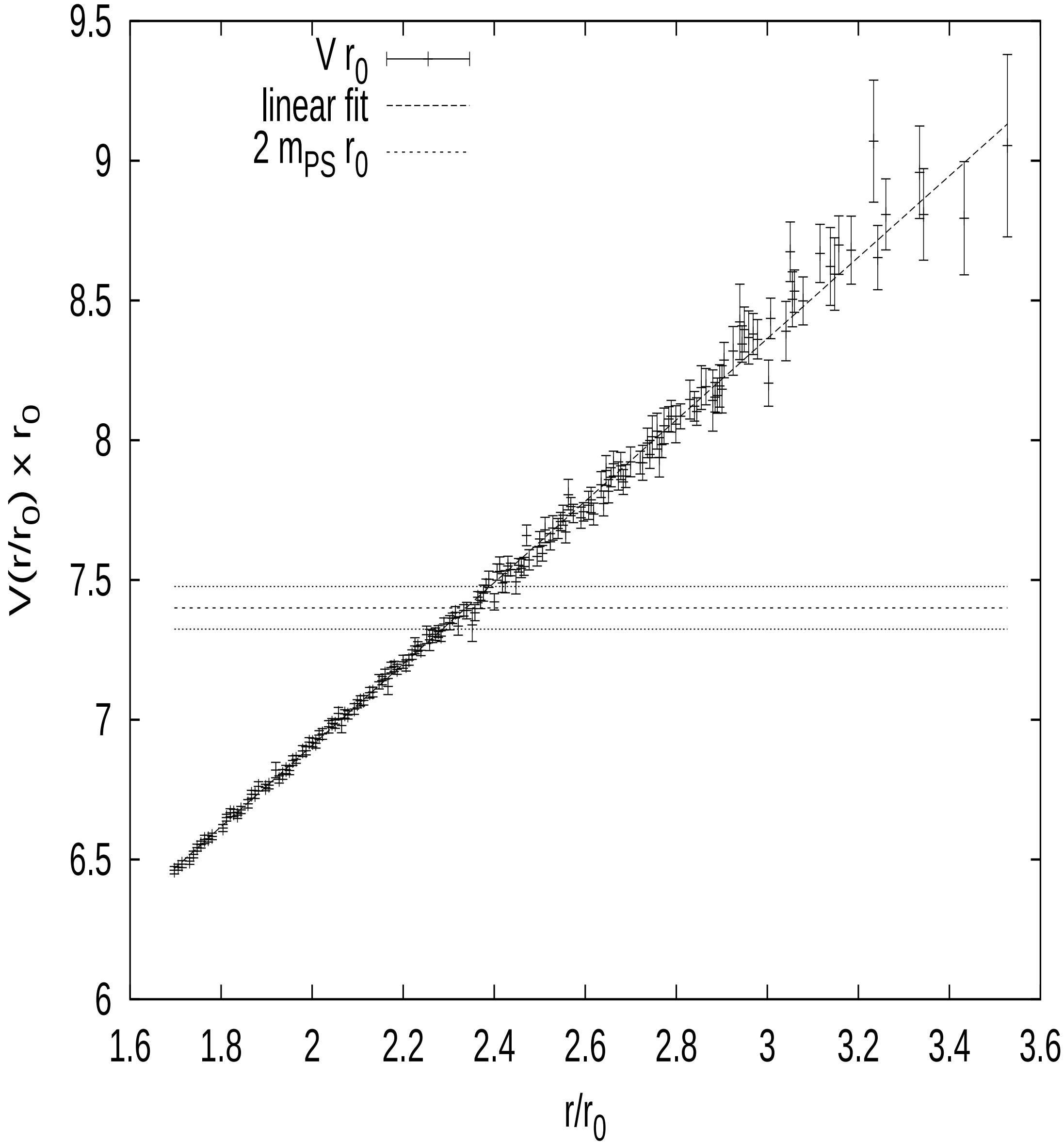
CP-PACS '98
Chernodub & Suzuki '02

Recent results obtained in 3D SU(2) theory

– for fundamental WL in theory with dynamical fermions
Trottier & Wong '02

– for adjoint WL in theory without fermions
de Forcrand & Kratochvila

indicate that overlap might be better



The static potential obtained from Wilson loops at $T = 0$. SESAM collaboration '01.

String breaking distance r_{sb}:

$$2m = V_{string}(r_{sb}),$$

where

$$V_{string}(r_{sb}) = \sigma \cdot r_{sb} - \pi/(12 r_{sb})$$

$$m = E - V_0/2$$

Lattice data for r_{sb} and σ in $N_f = 2$ QCD:

$r_{sb} = 2.2 \sim 2.3 r_0$ SESAM '00, CP-PACS '99

$\sqrt{\sigma r_0^2} = 1.14 \sim 1.16$ SESAM '00, UKQCD '01, JLQCD '02

We find

$$2m \sim 2.9/r_0 \approx 1.1\text{GeV}$$

Good agreement with another estimate:

$$2m = 2(M_D - m_c) \sim 2(M_B - m_b) \sim 1.1\text{Gev}$$

Digal, Petreczky and Satz '01

String breaking at $T > 0$

Clearly demonstrated with the help of Polyakov loops correlator

De Tar, Kaczmarek, Karsch, Laermann '98

In confinement phase of QCD $\langle L \rangle \neq 0$ since global Z_3 is broken by fermions

Polyakov loops correlator

$$\langle L_{\vec{x}} L^{\dagger}_{\vec{y}} \rangle \to |\langle L \rangle|^2 \neq 0, \quad |\vec{x} - \vec{y}| \to \infty$$

This implies flattening of the heavy quark potential at large distances

Some properties of the heavy quark potential at $T < T_c$, including effective quark mass $m(T)$ has been studied

Karsch, Laermann and Peikert '00
Digal, Petreczky and Satz '01

We make an attempt to parameterize heavy quark potential

Spectral representation for the Polyakov loop correlator:

$$\langle L_{\vec{x}} L_{\vec{y}}^{\dagger} \rangle = \sum_{n=0}^{\infty} w_n e^{-E_n(r)/T}$$

Lüscher & Weisz '02

Heavy quark potential

$$V(r,T) \equiv -\frac{1}{T} \log \frac{1}{9} \langle L_{\vec{x}} L_{\vec{y}}^{\dagger} \rangle$$

At $T = 0$ this gives

$$V(r, T = 0) = E_0(r)$$

In contrast, at $T > 0$ $V(r,T)$ gets contributions from all possible states

Our two-state Ansatz:

$$\frac{1}{9}\langle L_{\vec{x}} L_{\vec{y}}^{\dagger}\rangle = e^{-(V_0(T)+V_{string}(r,T))/T} + e^{-2E(T)/T}$$

where

$$\begin{aligned} V_{string}(r,T) \quad = \quad & -\left(\alpha - \frac{1}{6}\arctan(2rT)\right)\,\frac{1}{r} + \\ & \left(\sigma(T) + \frac{2T^2}{3}\arctan\frac{1}{2rT}\right)\,r + \\ & \frac{T}{2}\ln\left(1+4r^2T^2\right), \end{aligned}$$

Gao '89

$$E(T) = V_0(T)/2 + m(T)$$

Another function to describe heavy quark potential in QCD has been suggested long ago

$$V(r,T) = \frac{\tilde{\sigma}}{\mu}(1-e^{-\mu r}) - \frac{\alpha}{r}e^{-\mu r}$$

Karsch, Mehr and Satz '87

In recent phenomenological studies of quarkonium spectrum at finite temperature the following modification was used: Wong '02

$$V^{Wong}(r,T) = \left[-\frac{4}{3}\frac{\alpha_s}{r} - \frac{b(T)}{\mu_0}\right] e^{-\mu_0 r}$$

where $b(T) = b_0\left[1-(T/T_c)^2\right]$, $b_0 = 0.35$ GeV^2 , $\mu_0 = 0.28$ GeV , $\alpha_s \sim 0.32$ for charmonium and ~ 0.24 for bottomonium.

Error reduction

Our task would be prohibitive without error reduction

We employ hypercubic blocking introduced recently

Hasenfratz and Knechtli '01

HCB procedure mixes only links within hypercubes attached to the original link

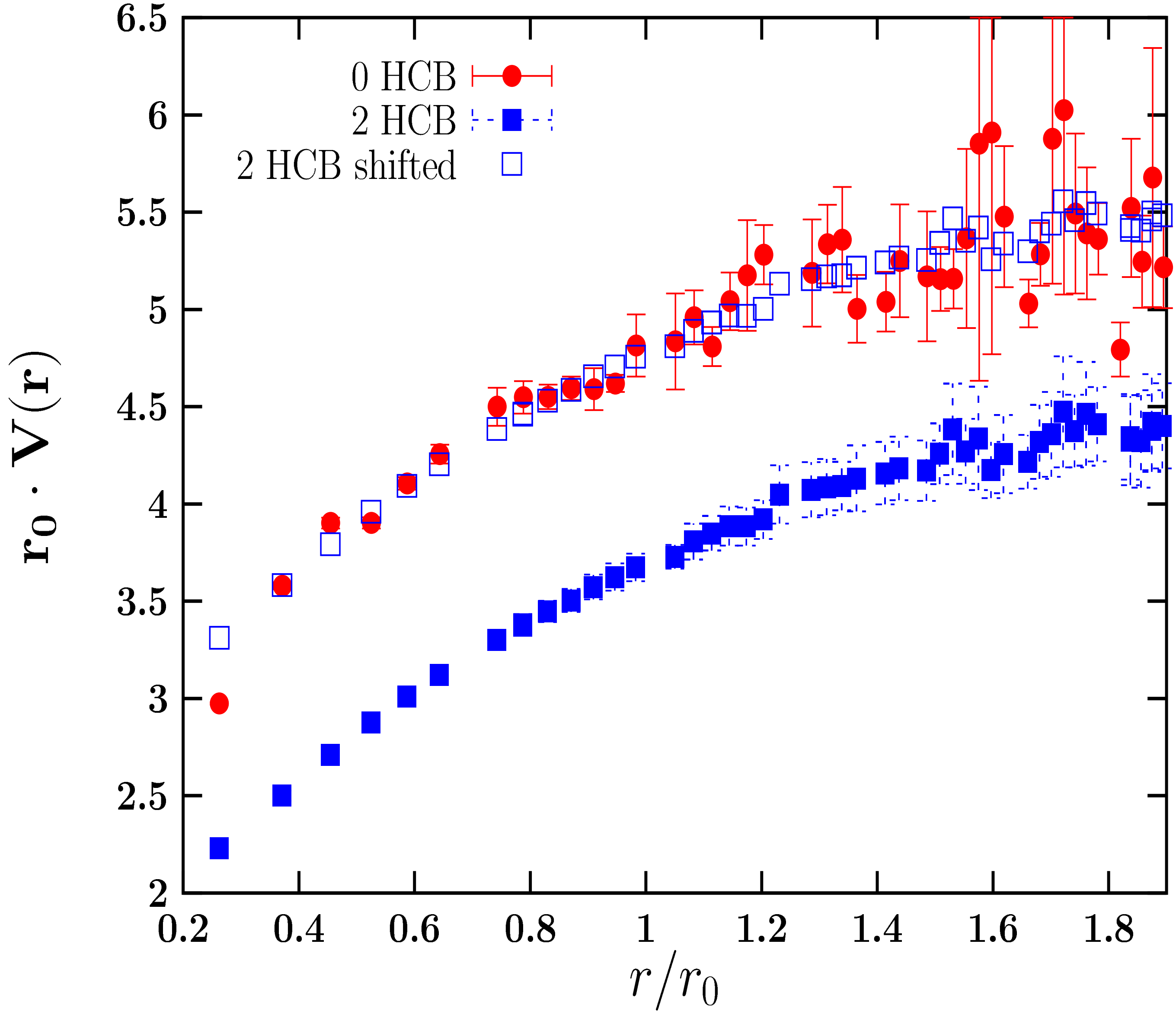
Statistical errors are reduced by about an order of magnitude

Static potential is distorted at very small distances only

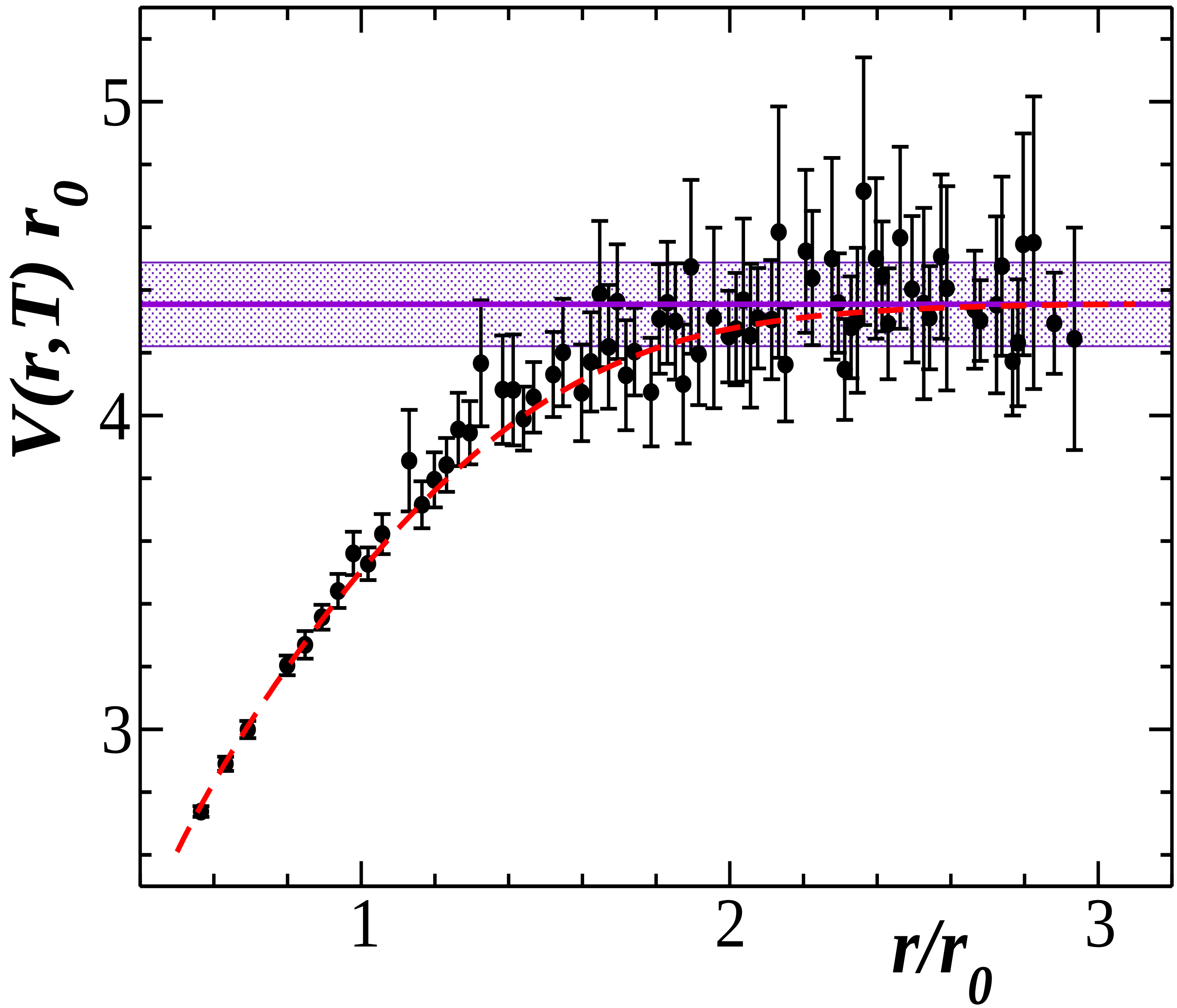
Another group confirmed these properties

Gattringer, Hoffman and Schaefer '02

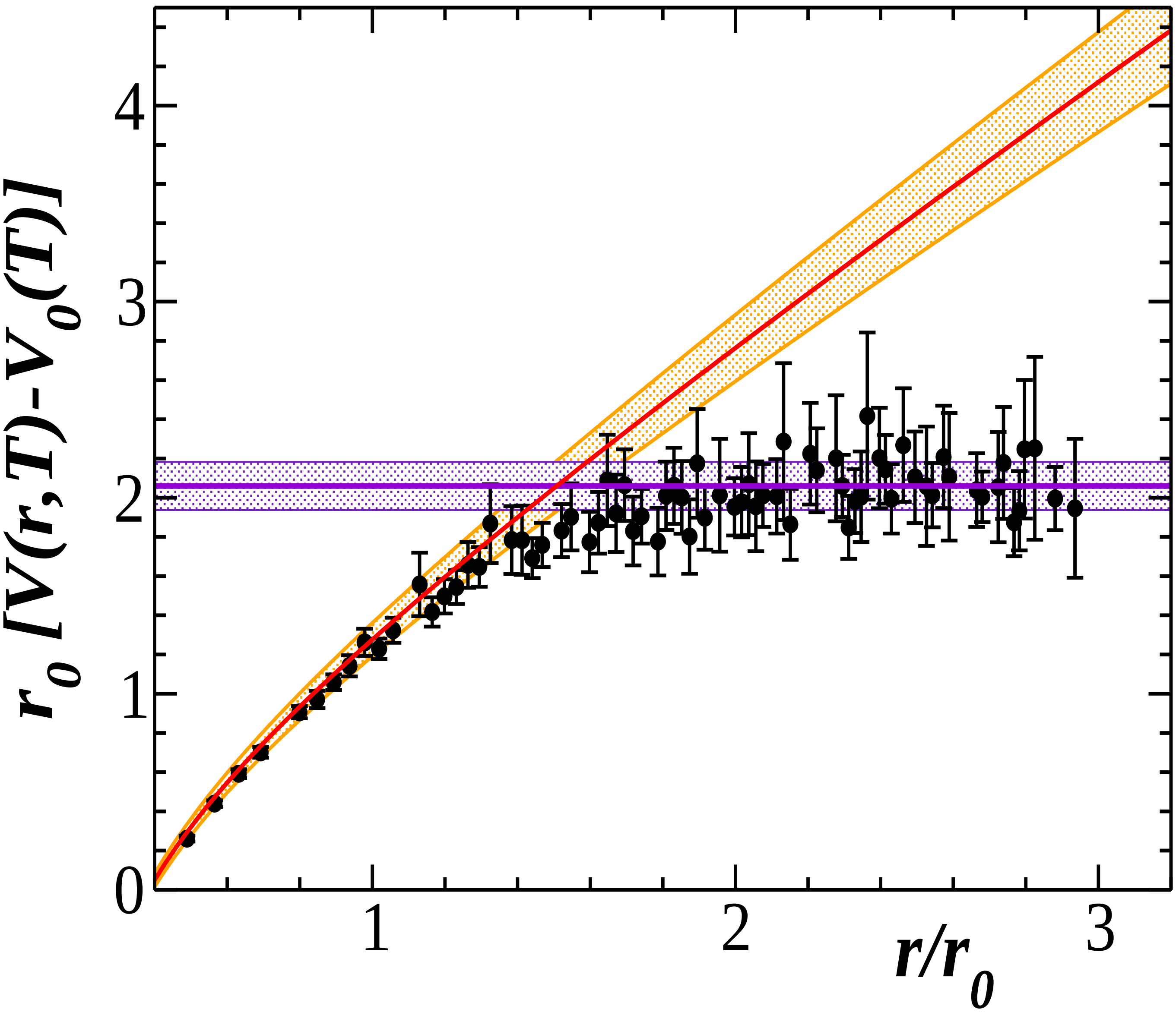
We found these nice features of HCB are preserved in PL correlator computations



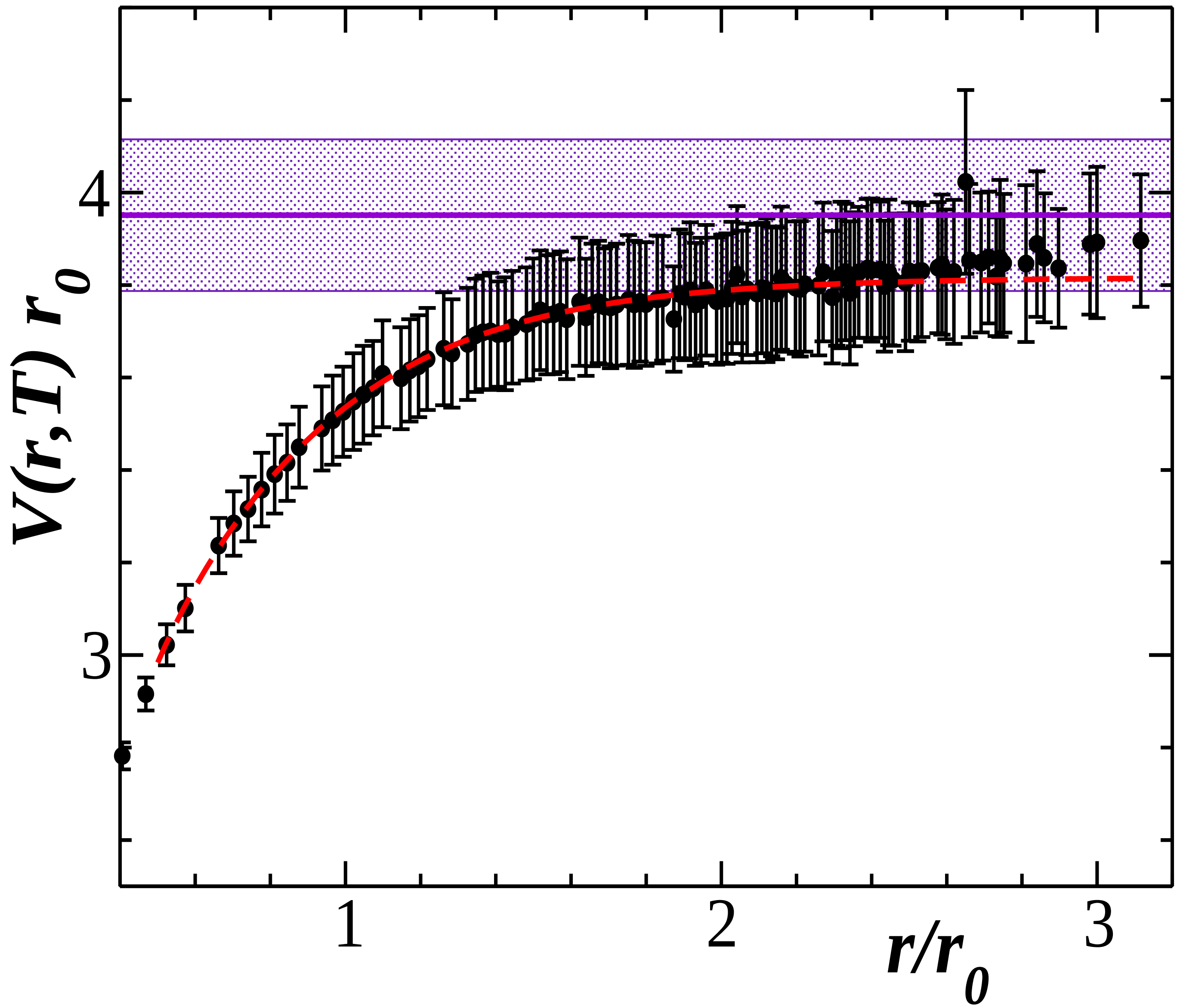
Effects of hypercubic blocking. $T/T_c = 0.87$



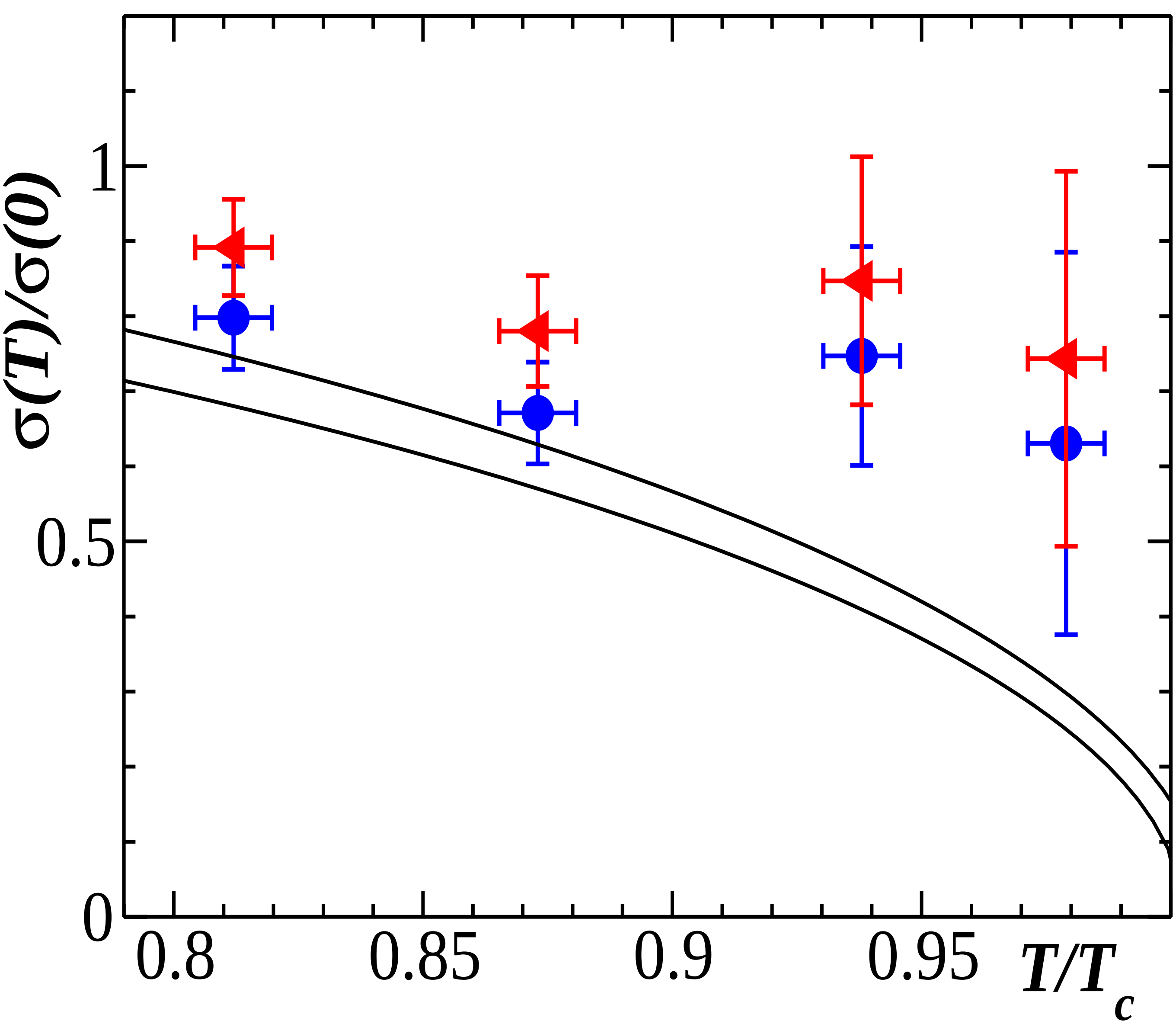
Heavy quark potential at $T/T_c = 0.81$



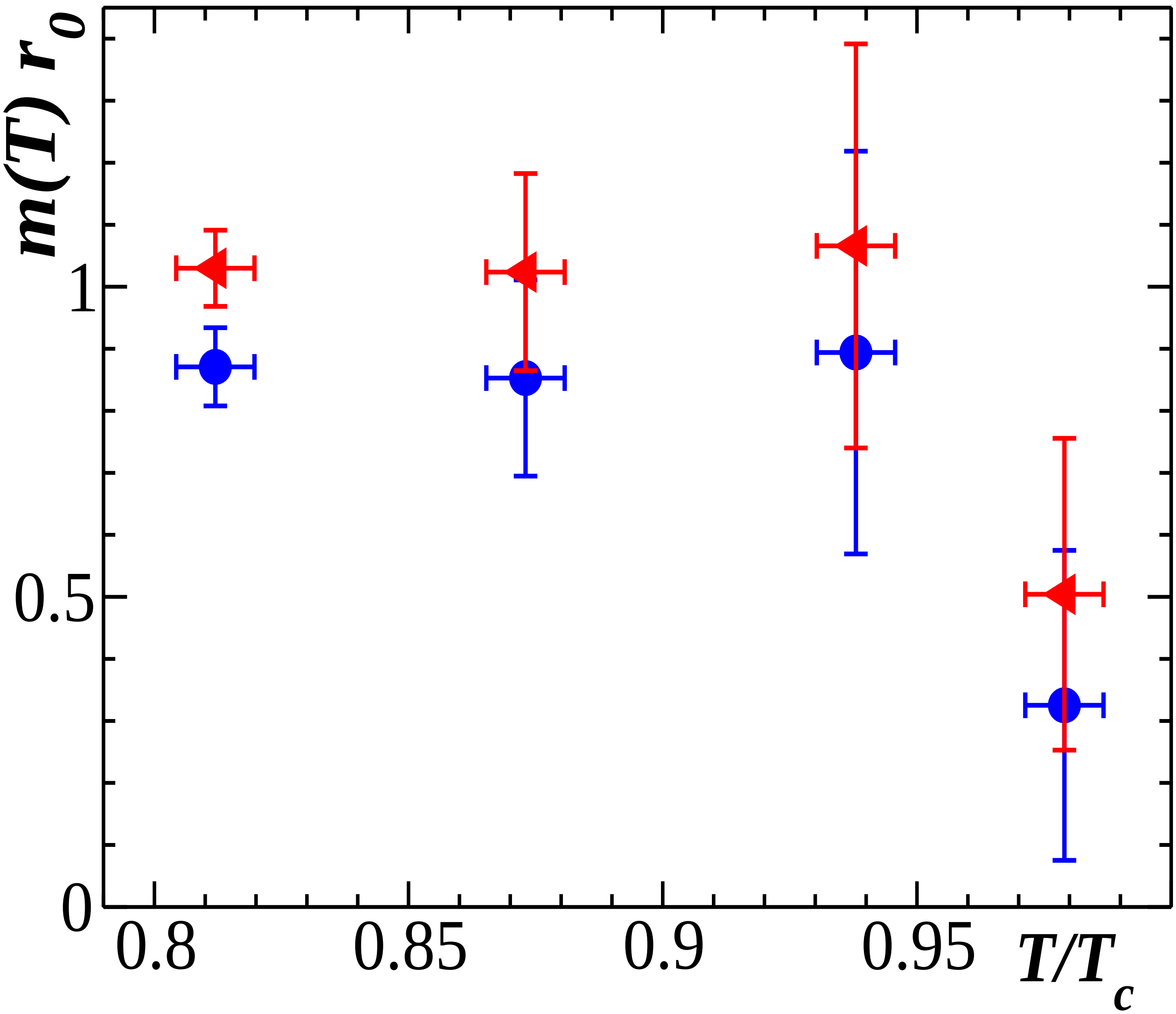
$V_{string}(r,T)$ at $T/T_c = 0.81$



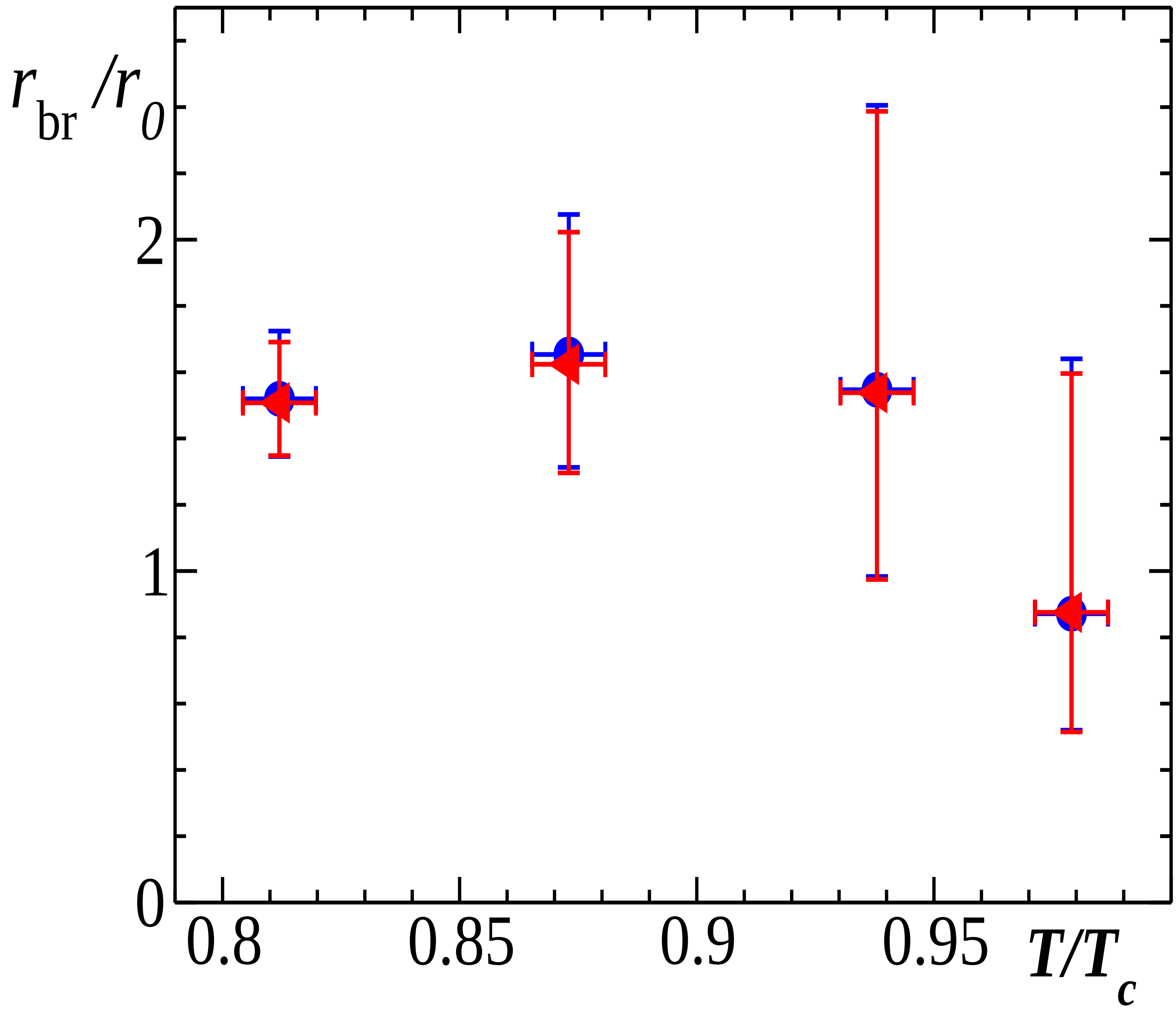
Heavy quark potential at $T/T_c = 0.98$



String tension $\sigma(T)$.
Solid lines show quenched QCD result
from Kaczmarek et al '00.



Effective quark mass $m(T)$



String breaking distance

$V_{string}(r_{sb}, T) = 2m(T)$

Conclusions and Outlook

– T_c is in agreement with other groups results

– evidence supporting two-state Anzatz has been found

– hypercubic blocking was crucial for our computations of the Polyakov loop correaltor

– Ratio $\sigma(T)/\sigma(T = 0)$ at $T/T_c < 0.9$ is in good agreement with quenched results

– Effective quark mass $m(T)$ is in qualitative agreement with earlier result

– Systematic uncertainties due to octet contribution to Polyakov loop correlator should be clarified

– Simulations at $\frac{m_\pi}{m_\rho} \sim 0.7$ on $24^3 \times 10$ are on the way