Nucleon axial charge from quenched lattice QCD with DBW2 gauge action and domain wall fermions

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Domain wall fermions (DWF) preserves almost exact chiral symmetry on the lattice:

- by introducing a fictitious fifth dimension in which the symmetry violation is exponentially suppressed.

DBW2 ("doubly blocked Wilson 2") action improves approach to the continuum:

- by adding rectangular (2×1) Wilson loops to the action.

By combining the two, the "residual mass," which controls low energy chiral behavior, is driven to

- $am_{\rm res} < O(10^{-4})$ or $\ll$ MeV.

Success in: chiral symmetry and ground-state mass spectrum[1], Kaon matrix elements[2], N^* mass[3], ...

So what about g_A, perhaps the simplest of nucleon electroweak matrix elements?

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[1] T. Blum et al, hep-lat/0007038, to appear in PRD.
[2] T. Blum et al, RBRC Scientific Articles 4; hep-lat/0110075.
[3] S. Sasaki et al, Phys. Rev. D65, 074503 (2002); hep-lat/0102010.

From neutron β decay, we know $g_{_V} = G_F \cos\theta_c$ and $g_{_A}/g_{_V} = 1.2670(30)$[4]:

- $g_{_V} \propto \lim_{q^2\to 0} g_{_V}(q^2)$ with $\langle n|V_\mu^-(x)|p\rangle = i\bar{u}_n[\gamma_\mu g_{_V}(q^2) + q_\lambda \sigma_{\lambda\mu} g_{_M}(q^2)]u_p e^{-iqx}$,
- $g_{_A} \propto \lim_{q^2\to 0} g_{_A}(q^2)$ with $\langle n|A_\mu^-(x)|p\rangle = i\bar{u}_n\gamma_5[\gamma_\mu g_{_A}(q^2) + q_\mu g_{_P}(q^2)]u_p e^{-iqx}$.

On the lattice, in general, we calculate the relevant matrix elements of these currents

- with a lattice cutoff, $a^{-1} \sim$ 1-2 GeV,
- and extrapolate to the continuum, $a \to 0$,

introducing lattice renormalization: $g_{_{V,A}}^{\text{renormalized}} = Z_{_{V,A}} g_{_{V,A}}^{\text{lattice}}$.

Also, unwanted lattice artefact may result in unphysical mixing of chirally distinct operators.

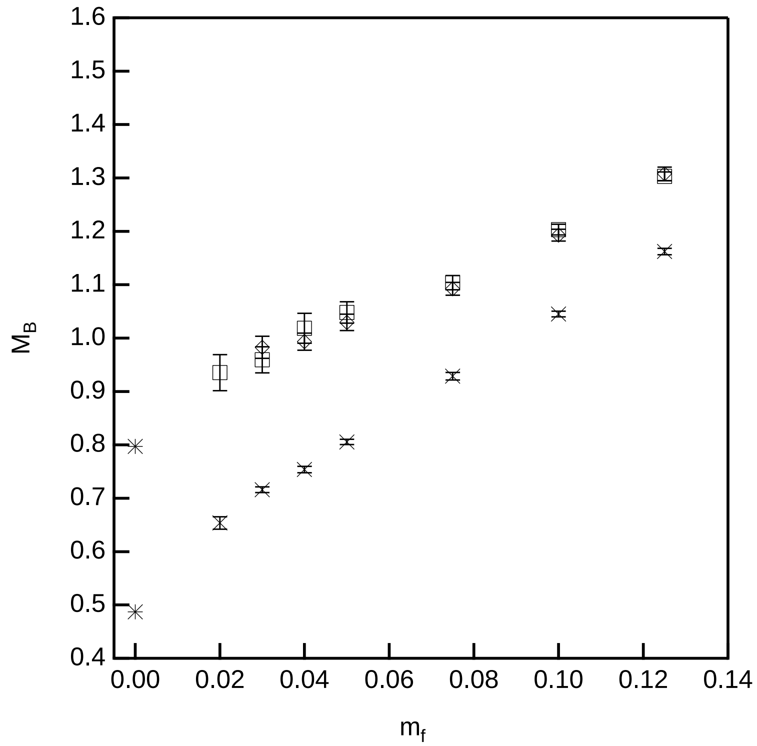
DWF makes $g_{_A}/g_{_V}$ particularly easy, because:

- the chiral symmetry is almost exact, and
- maintains $Z_{_A} = Z_{_V}$, so that $g_{_A}^{\text{lattice}}/g_{_V}^{\text{lattice}}$ directly yields the renormalized value.

[4]The Particle Data Group.

$T = 1/2$ mass spectrum: $N(939)$ and $N'(1440)$ with positive parity, and $N^{*}(1535)$ with negative parity

- NR quark models and bag models fail here.
- Quenched DWF works well for N(939)-N*(1535) parity partner mass splitting [5]:



So DWF seems to have a good prospect for nucleon matrix elements:

- g_{A} is a particularly interesting exercise, because with DWF, $Z_{A} = Z_{V}$ can be and is maintained.

[5]See S. Sasaki et al, Phys. Rev. D65:074503,2002; hep-lat/0102010 for more detail.

While historically

- NR quark model gives 5/3,
- MIT bag model gives 1.07,
- lattice calculations with Wilson or clover fermions typically underestimates by up to 25 %:

type	group	fermion	lattice	β	volume	configs	$m_\pi L$	g_A
quenched	KEK[a]	Wilson	$16^3 \times 20$	5.7	$(2.2\text{fm})^3$	260	≥ 5.9	0.985(25)
	Liu et al[b]	Wilson	$16^3 \times 24$	6.0	$(1.5\text{fm})^3$	24	≥ 5.8	1.20(10)
	DESY[c]	Wilson	$16^3 \times 32$	6.0	$(1.5\text{fm})^3$	1000	≥ 4.8	1.074(90)
	LHPC-SESAM[d]	Wilson	$16^3 \times 32$	6.0	$(1.5\text{fm})^3$	200	≥ 4.8	1.129(98)
	QCDSF[e]	Wilson	$24^3 \times 48$	6.2	$(1.6\text{fm})^3$	O(300)		1.14(3)
			$32^3 \times 48$	6.4	$(1.6\text{fm})^3$	O(100)		
			$16^3 \times 32$	6.0	$(1.5\text{fm})^3$	O(500)		
	QCDSF-UKQCD[f]	Clover	$24^3 \times 48$	6.2	$(1.6\text{fm})^3$	O(300)		1.135(34)
			$32^3 \times 48$	6.4	$(1.6\text{fm})^3$	O(100)		
full($N_f = 2$)	LHPC-SESAM[d]	Wilson	$16^3 \times 32$	5.5	$(1.7\text{fm})^3$	100	≥ 4.2	0.914(106)
	SESAM[g]	Wilson	$16^3 \times 32$	5.6	$(1.5\text{fm})^3$	200	≥ 4.5	0.907(20)

[a] M. Fukugita, Y. Kuramashi, M. Okawa and A. Ukawa, Phys. Rev. Lett. 75, 2092 (1995).

[b] K.F. Liu, S.J. Dong, T. Draper and J.M. Wu, Phys. Rev. D49, 4755 (1994).

[c] M. Göckeler et al, Phys. Rev. D53, 2317 (1996).

[d] D. Dolgov et al, hep-lat/0201021.

[e] S. Capitani et al, Nucl. Phys. B (Proc. Suppl.) 79, 548 (1999).

[f] R. Horsley et al, Nucl. Phys. B (Proc. Suppl.) 94, 307 (2001).

[g] S. Güsken et al, Phys. Rev. D59, 114502 (1999)

– with $Z_A \neq Z_V$ and other renormalization complications.

Our formulation follows the standard one,

- Two-point function: $G_{N}(t) = \mathrm{Tr}[(1+\gamma_t)\sum_{\vec{x}}\langle TB_1(x)B_1(0)\rangle]$, using $B_1 = \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$ for proton,
- Three-point functions,
  - vector: $G_{V}^{u,d}(t,t') = \mathrm{Tr}[(1+\gamma_t)\sum_{\vec{x}'}\sum_{\vec{x}}\langle TB_1(x')V_t^{u,d}(x)B_1(0)\rangle]$,
  - axial: $G_{A}^{u,d}(t,t') = \frac{1}{3}\sum_{i=x,y,z}\mathrm{Tr}[(1+\gamma_t)\gamma_i\gamma_5\sum_{\vec{x}'}\sum_{\vec{x}}\langle TB_1(x')A_i^{u,d}(x)B_1(0)\rangle]$.

  with fixed $t' = t_{\mathrm{source}} - t_{\mathrm{sink}}$ and $t < t'$.
- From the lattice estimate

$$g_{\Gamma}^{\mathrm{lattice}} = \frac{G_{\Gamma}^{u}(t,t') - G_{\Gamma}^{d}(t,t')}{G_{N}(t)},$$

  with $\Gamma = V$ or A, the renormalized value

$$g_{\Gamma}^{\mathrm{ren}} = Z_{\Gamma} g_{\Gamma}^{\mathrm{lattice}},$$

  is obtained.
- Non-perturbative renormalizations, defined by

$$[\bar{u}\Gamma d]_{\mathrm{ren}} = Z_{\Gamma}[\bar{u}\Gamma d]_0,$$

  satisfies $Z_{A} = Z_{V}$ well, so that

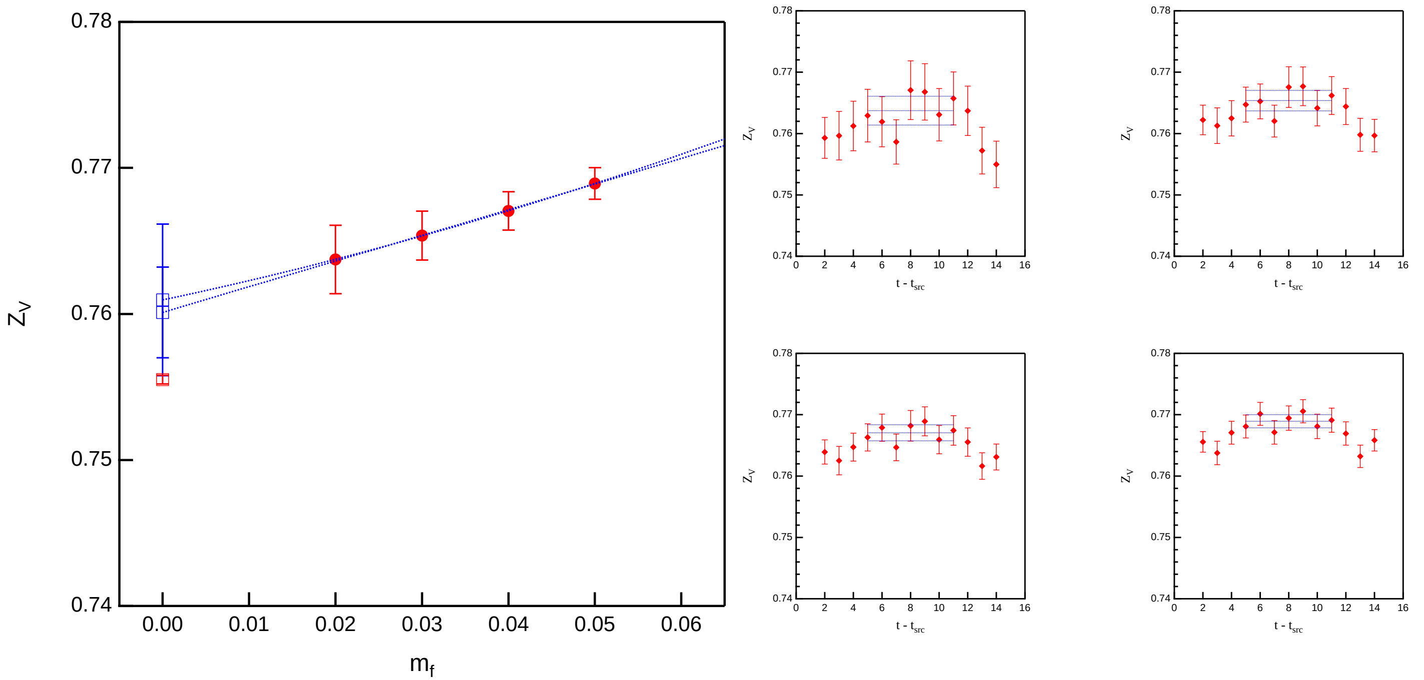
$$\left(\frac{g_{A}}{g_{V}}\right)^{\mathrm{ren}} = \left(\frac{G_{A}^{u}(t,t') - G_{A}^{d}(t,t')}{G_{V}^{u}(t,t') - G_{V}^{d}(t,t')}\right)^{\mathrm{lattice}}.$$

g_{A} is also described as $\Delta u - \Delta d$.

Numerical calculations with Wilson (single plaquette) gauge action:

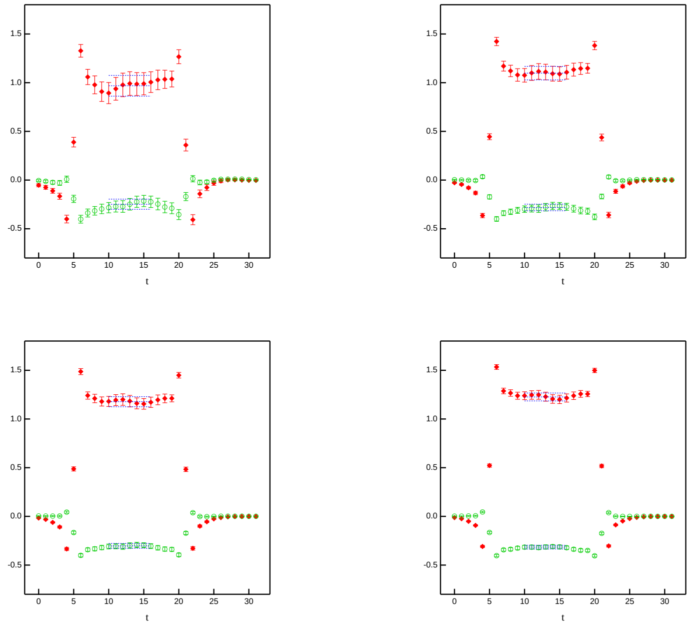
- RIKEN-BNL-Columbia QCDSP,
- 400 gauge configurations, using a heat-bath algorithm,
- $\beta = 6.0$, $16^3 \times 32 \times 16$, $M_5 = 1.8$,
- source at $t = 5$, sink at 21, current insertions in between.

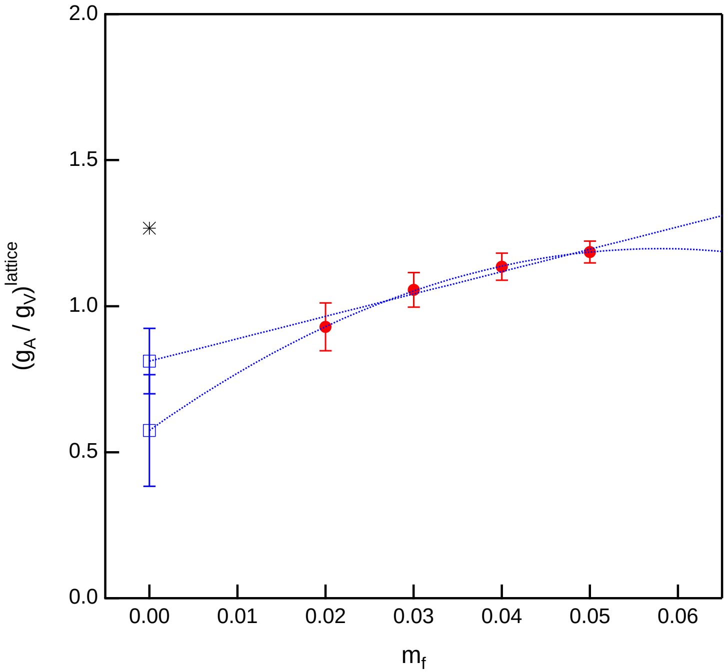
$Z_V = 1/g_V^{\text{lattice}}$ is well-behaved,



- the value 0.764(2) at $m_f = 0.02$ agrees well with $Z_A = 0.7555(3)$ from
  - $\langle A_\mu^{\text{conserved}}(t)\bar{q}\gamma_5 q(0)\rangle = Z_A \langle A_\mu^{\text{local}}(t)\bar{q}\gamma_5 q(0)\rangle$ (RBC hep-lat/0007038, to appear in Phys. Rev. D),
- linear fit gives $Z_V = 0.760(7)$ at $m_f = 0$, and quadratic fit, 0.761(5).

Δu, Δd, and g_A/g_V (averaged in $10 \leq t \leq 16$):





- linear extrapolation yields 0.81(11) at $m_f = 0$, and simlarly small values for
  - $\Delta q/g_V = 0.49(12)$ and
  - $(\delta q/g_V)^{\mathrm{lattice}} = 0.47(10)$ (with a preliminary $Z_T \sim 1.1$).
- While relevant three-point functions are well behaved in DWF, and $Z_V = Z_A$ is well satisfied, 0.760(7) and 0.7555(3).

Why so small?

- finite lattice volume [6],
- excited states (small separation between t_{source} and t_{sink}),
- quenching (zero modes, absent pion cloud, ...).

To investigate size-dependence, we simultaneously need

- good chiral behavior, *i.e.* close enough to the continuum, and
- big enough volume.

Improved gauge actions help both. DBW2[7], in particular,

$$S_G = \beta[c_0 \sum W_{1,1} + c_1 \sum W_{1,2}],$$

with $c_0 + 8c_1 = 1$ and $c_1 = -1.4069$:

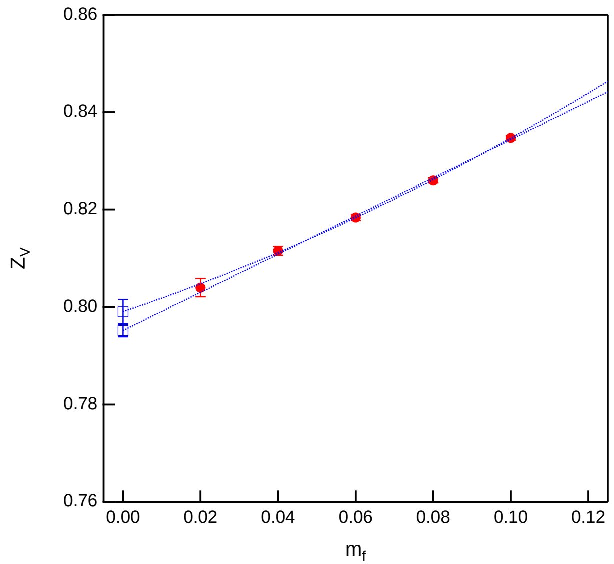
- very small residual chiral symmetry breaking, $am_{\text{res}} < 10^{-3}$,
- at the chiral limit, $am_\rho = 0.592(9)$ (so $a^{-1} \sim 1.3\text{GeV}$), $m_\rho/m_N \sim 0.8$,
- $m_\pi(m_f = 0.02) \sim 0.3a^{-1}$.

[6] R.L. Jaffe, Phys .Lett. B529:105, 2002; hep-ph/0108015. See also T.D. Cohen, Phys. Lett. B529:50, 2002; hep-lat/0112014.
[7] QCD-TARO collaboration, Nucl. Phys. B577, 263 (2000); RBC collaboration, in preparation.

DBW2 calculations are performed at $a \sim 0.15$ fm ($\beta = 0.87$) with both wall and sequential sources on

- $8^3 \times 24 \times 16$ ($\sim (1.2\text{fm})^3$), 400 configurations (wall) and 160 (sequential),
- $16^3 \times 32 \times 16$ ($\sim (2.4\text{fm})^3$), 100 configurations (wall and sequential),
- source-sink separation of about 1.5 fm,
- $m_f = 0.02, 0.04, \ldots$: $m_\pi \geq 390$MeV, $m_\pi L \geq 4.8$ and 2.4.

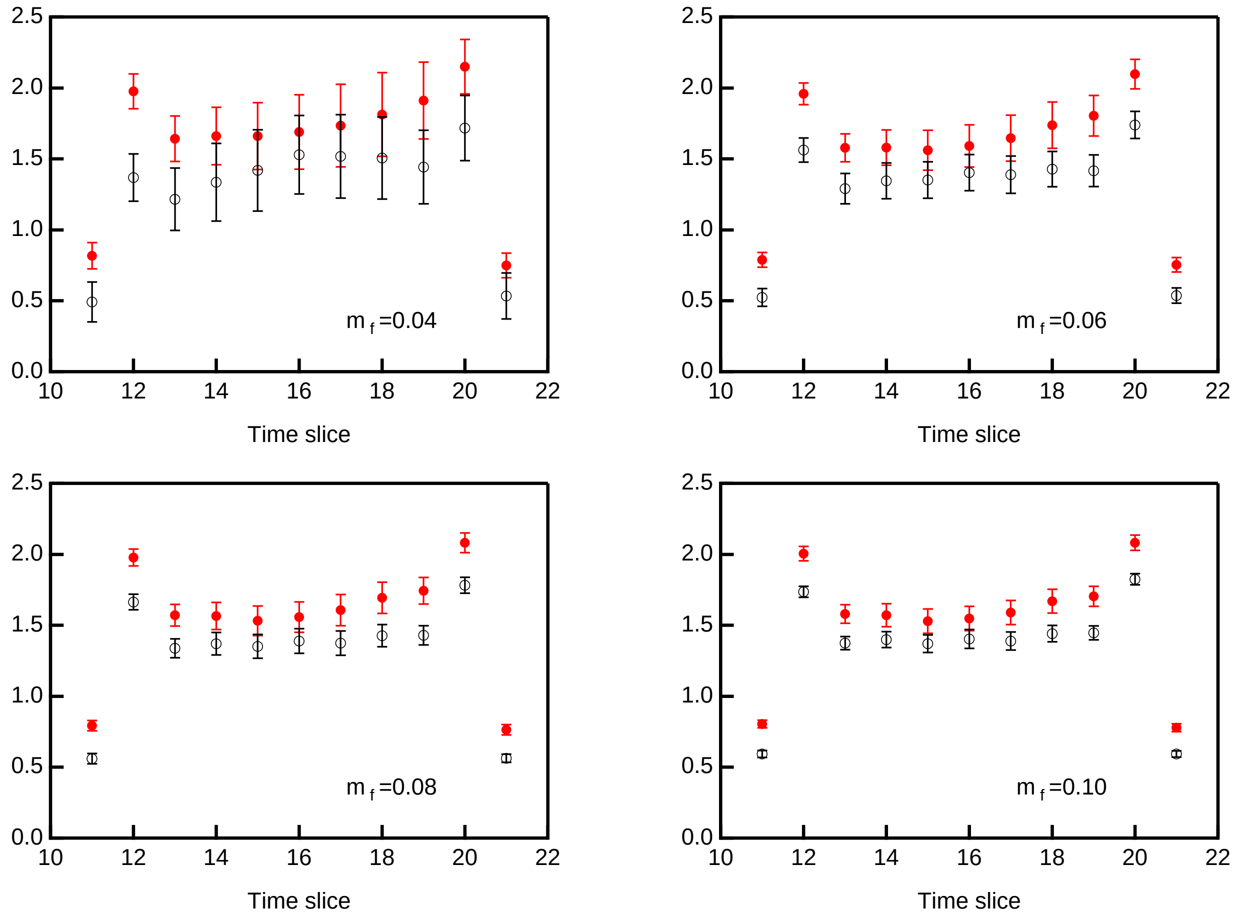
Renormalization factors: $\mathcal{O}^{\text{ren}}(\mu) = Z_{\mathcal{O}}(a\mu)\mathcal{O}^{\text{lattice}}(a)$.



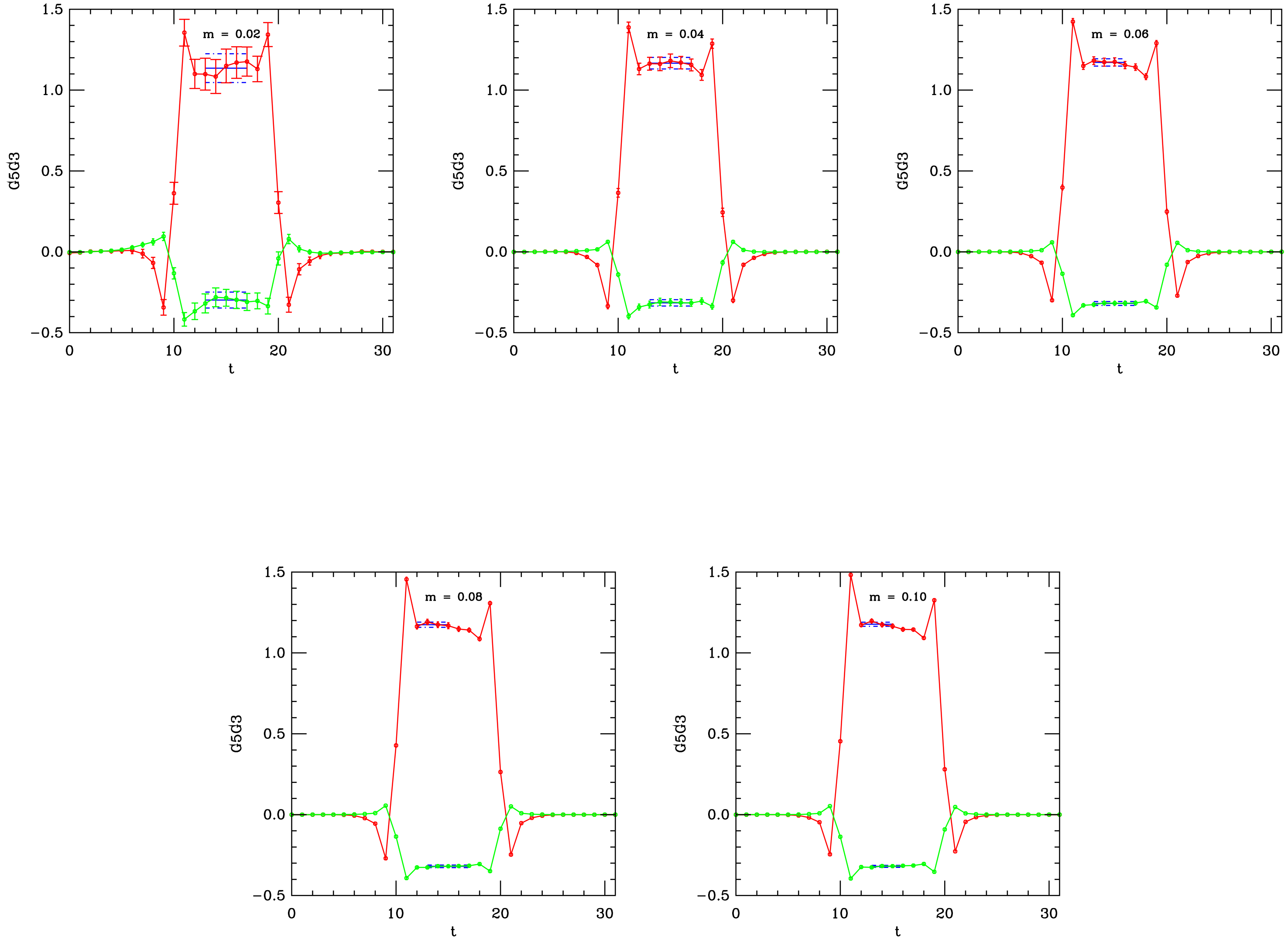
- Z_V shows slight quadratic dependence on m_f as expected: $V_\mu^{\text{conserved}} = Z_V V_\mu^{\text{local}} + \mathcal{O}(m_f^2 a^2)$,
  - yielding a value $Z_V = 0.784(15)$,
  - agrees well with $Z_A = 0.77759(45)$ [8].

[8]RBC Collaboration, in preparation: this value is obtained from a relation $\langle A_\mu^{\text{conserved}}(t)[\bar{q}\gamma_5 q](0)\rangle = Z_A \langle A_\mu^{\text{local}}(t)[\bar{q}\gamma_5 q](0)\rangle$.

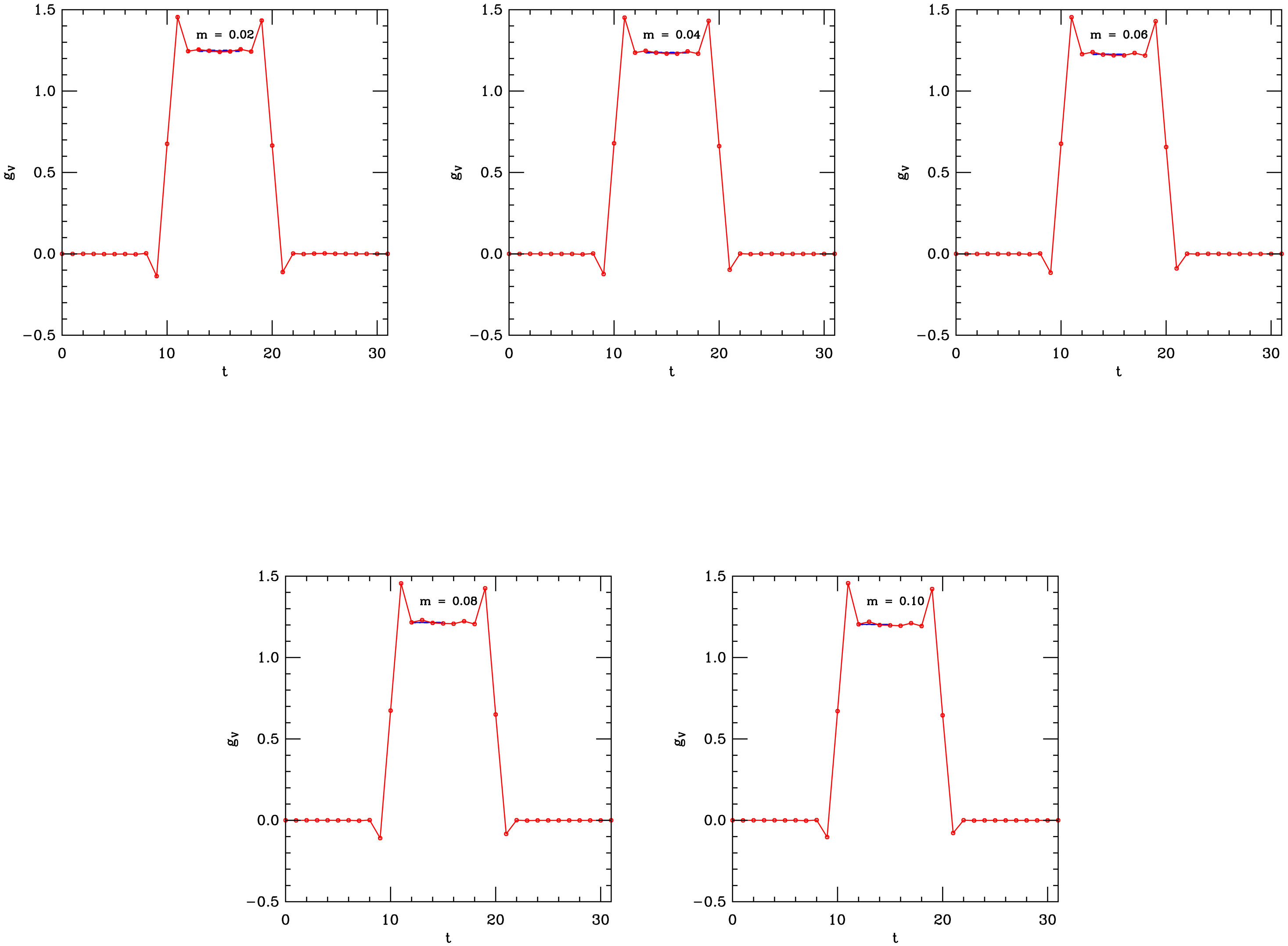
Bare g_A^{lattice} from wall source show volume dependence at medium m_f ($(2.4\mathrm{fm})^3$ (filled) and $(1.2\mathrm{fm})^3$ (open) volumes):



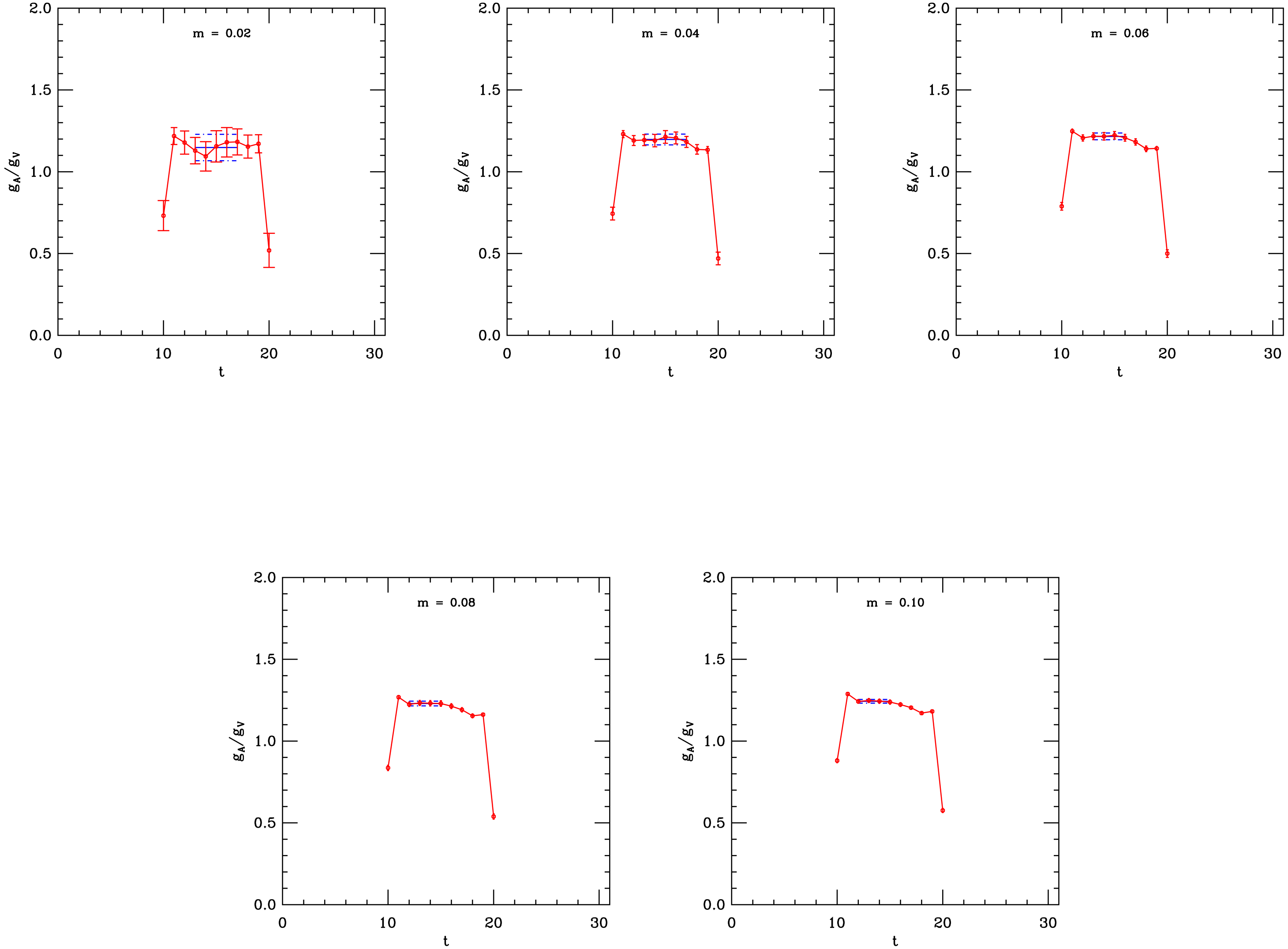
Bare $\Delta u^{\text{lattice}}$ and $\Delta d^{\text{lattice}}$ from sequential source $((2.4\text{fm})^3)$:



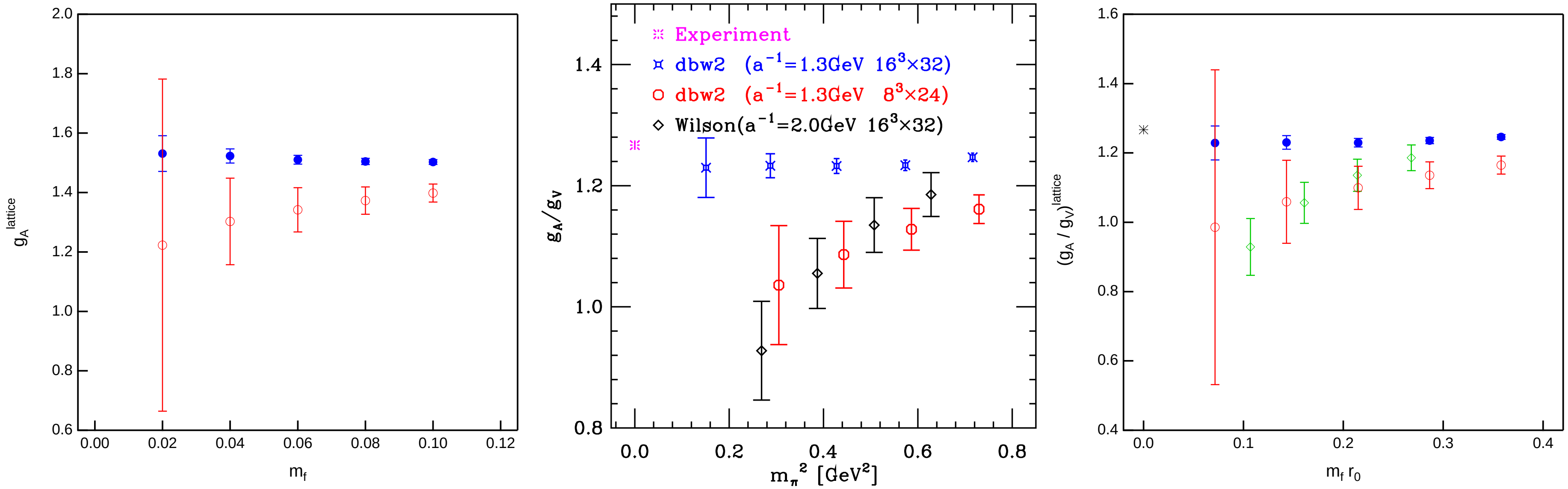
Bare g_V^{lattice} from sequential source $((2.4\mathrm{fm})^3)$:



$(g_A/g_V)^{\rm lattice}$ from sequential source ($(2.4{\rm fm})^3$):

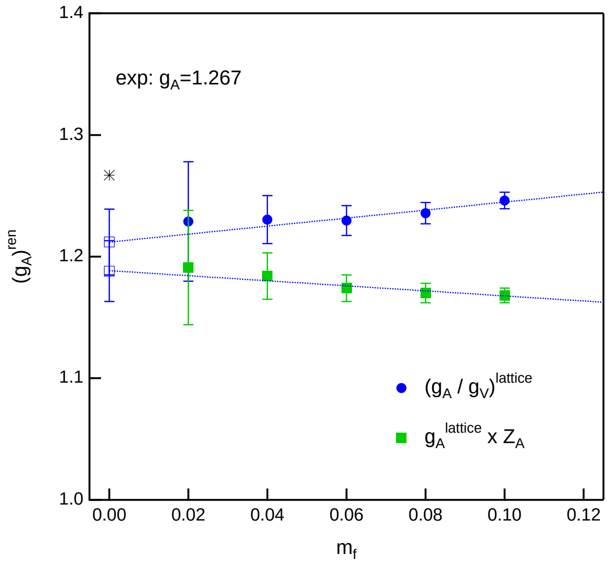


$(g_A/g_V)^{\rm lattice} = (g_A/g_V)^{\rm ren}$: m_f and volume dependence in bare and physical scales (m_ρ and Sommer):



- Clear volume dependence is seen between $(2.4\text{fm})^3$ and $(1.2\text{fm})^3$ volumes.
- The large volume results (sequential)
  - show a very mild m_f dependence,
  - extrapolate to about 8 % under estimation, $g_A = 1.15(11)$.
- The large volume wall source and small volume sequential source calculations still lack in statistics.

Alternatively we can use $g_A^{\text{lattice}} \times Z_A$:



agree well with $(g_A/g_V)^{\text{lattice}}$ in the chiral limit, and and an expected difference seen away from there.

Conclusions: with quenched DBW2 and DWF for nucleon currents, indications are seen for

- good chiral behavior:
  - especially the relation $Z_A = Z_V$ is easily and well maintained,
- milder m_f dependence,
- clear size dependence, 20 % increase from 1.2 fm to 2.4 fm,
- $g_A/g_V = 1.21(3)_{\rm stat.}(3)_{\rm syst.}$, where
  - the systematic error is estimated from Z_A systematics only,
  - and does not include the volume systematics yet.

Future:

- a few more observables, e.g. interesting to see how well quenched calculation works
  - for a well-known example of soft-pion, Goldberger-Treiman relation: $g_A/g_V \simeq f_\pi g_{\pi N}/m_N$,
- larger volume,
- flavor structure,
- full QCD,
- probably using the new QCDOC computer.

Other nucleon observables:

- moments of structure functions (Kostas Orginos),
- form factors (?),
- nucleon decay matrix elements (Yasumichi Aoki).