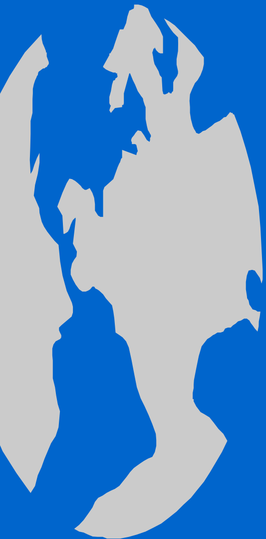
I=2 $\pi\pi$ scattering length on small anisotropic lattices



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Outline

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why

$\pi\pi$ scattering ： a playground for studying strong interactions

- Major players:
  - Experiment (K_{e4})
  - Chiral perturbation theory (+something else)
  - Dispersion relations
  - Lattice:
    - Sharpe, Gupta et al (92,93)
    - Fukugita et al (95)
    - JLQCD (00,02)
    - CP-PACS(02)

How

- Luescher formula
  - relates the energy shift δE in a *finite volume* with elastic scattering phase shift $\delta(E)$ in *infinite volume*:

$$\pi \cot \delta_0(E) = \frac{Z_{00}(1; q^2)}{\pi^{1/2} q}$$

$$E^2/4 = k^2 + m_\pi^2 \quad ; \quad q = \frac{kL}{2\pi}$$

$$Z_{lm}(1; q^2) = \sum_{\mathbf{n}} \frac{Y_{lm}(\mathbf{n})}{\mathbf{n}^2 - q^2}$$

How

- Measure δE from correlation functions and plug into Luescher formula to get phase shift $\delta(E)$
- At small relative momentum k, scattering phase is:

$$\tan\delta_0(k) = a_0 k + b_0 k^3 + O(k^5)$$

where a_0 is scattering length, which is our prey.

How

- For the scattering length in large volume, we have:

$$E_{\pi\pi} - 2m_\pi = -\frac{4\pi a_0}{m_\pi L^3} \times$$
$$\times \left(1 + c_1 \left(\frac{a_0}{L} \right) + c_2 \left(\frac{a_0}{L} \right)^2 \right) + O(L^{-6})$$

where numerical coefficients:

c1=-2.837297,

c2= 6.375183

History

- Related works done before
  - First scattering lengths calculation in quenched QCD by S. Sharpe et al (1992,1993).
  - scattering phase in linear sigma model (1994).
  - scattering lengths in quenched QCD by Fukugita et al (93,95); by JLQCD (00,02).
  - Scattering phase in quenched QCD by CP-PACS (02).
- Using Wilson gauge action and conventional Wilson fermions.
- Large lattices are needed
- Is it possible at all to do it on small lattices?

anisotropic lattice actions

- Improved gauge action on anisotropic lattices:

$$S_g = -\beta \sum_{i>j} \frac{5}{9} \frac{TrP_{ij}}{\xi u_s^4} - \frac{1}{36} \frac{Tr\ (R_{ij} + R_{ji})}{\xi u_s^6}$$

$$- \beta \sum_{i} \frac{4\xi}{9} \frac{TrP_{0i}}{u_s^2} - \frac{\xi}{36} \frac{TrR_{i0}}{u_s^4}$$

- Improved fermion action on anisotropic lattices:

$$S_f = \overline{\psi}\left(m_0 + D_0 + \frac{c_{SW}}{8} \sum_{\mu\nu} \sigma_{\mu\nu} F_{\mu\nu} \frac{a_\mu + a_\nu}{a_\mu a_\nu} \right)\psi$$

$$D_0 = \frac{1}{2} \sum_{\mu} \nu_\mu \left[(\nabla_\mu + \nabla_\mu^*)\gamma_\mu - \nabla_\mu \nabla_\mu^* \right]$$

Tuning of parameters

- Anisotropy $\xi = a_s / a_t$: using the tadpole improvement, renormalization effect is small
- Tadpole improvement factor u_s: using gauge invariant version from plaquette value
- Clover coefficient c_{SW}: tree level value is taken
- Bare velocity of light ν : tuned using pseudo-scalar meson dispersion relation

β controls a_s, κ controls m_0.

Why improved?
Why anisotropic?

- Anisotropic lattice will facilitate the mesurement of δE (increase resolution)
- Improved actions will enable us using smaller lattices with same physical volume
- Combination of the two will give us a chance to perform this calculation (with a similar precision) that were only possible on large computers

Simulation details

- operators & correlators

- Meson operators and correlation functions (adopted from Fukugita et al 1995):

$$\pi^+(\mathrm{x},t) = -\bar{d}(\mathrm{x},t)\gamma_5 u(\mathrm{x},t)$$

$$\pi^-(\mathrm{x},t) = \bar{u}(\mathrm{x},t)\gamma_5 d(\mathrm{x},t)$$

$$\pi^0(\mathrm{x},t) = \frac{1}{\sqrt{2}}[\bar{u}(\mathrm{x},t)\gamma_5 u(\mathrm{x},t) - \bar{d}(\mathrm{x},t)\gamma_5 d(\mathrm{x},t)]$$

$$\pi_0^a(t) = \frac{1}{L^{3/2}}\sum_{\mathrm{x}}\pi_0^a(\mathrm{x},t)$$

$$O_{I=0}^{\pi\pi} = \frac{1}{\sqrt{3}}\begin{bmatrix}\pi_0^+(t)\pi_0^-(t+1) + \pi_0^-(t)\pi_0^+(t+1) \\ -\pi_0^0(t)\pi_0^0(t+1)\end{bmatrix}$$

$$O_{I=2}^{\pi\pi} = \pi_0^+(t)\pi_0^+(t+1)$$

Simulation details

- *operators & correlators*

$$C^{\pi\pi}_{I=2}(t) = < O^{\pi\pi}_{I=2}(t) O^{\pi\pi}_{I=2}(0) >$$

$$C^{\pi}(t) = < \pi^{a}_{0}(t) \pi^{a}_{0}(0) >$$

$$R_{I=2}(t) = C^{\pi\pi}_{I=2}(t) / [C^{\pi}(t) C^{\pi}(t)]$$

- When temporal separation is large, the ratio $R(t)$ yields the energy shift δE directly:

$$R(t) \rightarrow Z\left(1 - \delta E_{\pi\pi} t\right) + O(\delta E_{\pi\pi}{}^{2})$$

Simulation details

- how to measure

- *two-pion correlation functions* are constructed from *quark propagators.*
- Measure quark propagators, solving a linear equation (we used multi-mass Minimal residual algorithm):

$$M_{x,a,\alpha;y,b,\beta} \cdot X^{(z,c,\delta)}_{y,b,\beta} = \delta^{(z,c,\delta)}_{x,a,\alpha}$$

- Wall sources are used for better signal.

Simulation details

- *measured results*

- Simulation parameters:
  - Lattice spacing: 0.18-0.39fm
  - Lattice volume: 0.7-3.2fm
  - Statistics: a few hundred for each set of parameters.
- One meson energy m_π and m_ρ
- Tuning of ν
- Two pion energy shift δE.

Chiral extrapolations of $a^{(2)}_0$

- Chiral extrapolation

$$a_0^{(2)} m_\rho^2 / m_\pi$$ is a suitable choice?

- Comparison with ChPT:

$$a_0^{(2)} = -\frac{1}{16\pi}\frac{m_\pi}{f_\pi^2}, \quad a_0^{(2)} m_\rho^2 / m_\pi \approx -1.3638$$

- Typical results

Infinite volume extrapolations of $a^{(2)}_0$

- According to Luescher formula, $a_0^{(2)} m_\rho^2 / m_\pi$ should receive corrections of the form L^{-3} (scheme I)。
- However, quenching might spoil this by corrections of the form L^{-2} (scheme II).
- Unfortunately, our data was not accurate enough to disentangle these two cases (scheme I, scheme II).

Continuum limit extrapolations of $a^{(2)}_0$

- Finite lattice spacing correction is $O(a_s)$
- Physical value of a_s can be obtained from quark-antiquark potential parameter r_0
- Linear extrapolation in lattice spacing (both schemes)
- Results: $a^{(2)}_0 m_\pi = -0.0342(75)$, compatible with χPT, previous lattice results and experiment

Outlooks

- We have calculated pion scattering length using anisotropic actions on small lattices. Results are encouraging.
- Calculation for the $I=0$ channel can be difficult since it is badly contaminated by quenching effects (Bernard et al)
- Measure phase shifts with non-zero momenta in $I=2$ channel is possible. Need larger lattices and hence larger computers
- Scattering with other hadrons
- Domain wall? Overlap? un-quenching ?