Recent topics in lattice hadron physics

Speaker: Noriyoshi ISHII (RIKEN)

START

Topics

- ✓ The quark-gluon mixed condensate $g_s\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ in lattice QCD
  - –T.Doi et al., hep-lat/0212025.
  - –T.Doi et al., hep-lat/0211039.
- ✓ The glueballs at finite temperature
  - –N.Ishii et al., Phys.Rev.D66,014507,(2002).
  - –N.Ishii et al., Phys.Rev.D66,094506,(2002).
- ✓ The three quark potential
  - –T.T.Takahashi et al., Phys.Rev.Lett.86,18(2001).
  - –T.T.Takahashi et al., Phys.Rev.D65,114509(2002).
- ✓ SU(3) lattice QCD studies of octet and decouplet baryon spectra
  - –Y.Nemoto et al., hep-lat/0204014.

The quark-gluon mixed condensate $g_s\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ in lattice QCD

Contents from

- ✓ T.Doi, N.Ishii, M.Oka, H.Suganuma, hep-lat/0211039.
- ✓ T.Doi, N.Ishii, M.Oka, H.Suganuma, hep-lat/0212025.

Backgrounds

The condensates can represents the non-perturbative feature of QCD vacuum.

$\langle 0|\bar{q}q|0\rangle$ ⟺ the spontaneous breaking of chiral symmetry

$\langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle$ ⟺ the trace anomaly

Further, the condensates can be used to calculate the hadronic observables in the frame work of QCD sum rule.

QCD sum rule

Wilson coefficient
(perturbatively calculable)

The operator product expansion (OPE)

normal ordered
composite operator

$$\int d^4x e^{iqx} TJ(x)J(0) = \sum_n C_n(q^2) O_n$$
$$= C_I(q^2) I + C_{\bar{q}q}(q^2)\bar{q}q + C_{GG}(q^2) G_{\mu\nu}G^{\mu\nu}$$
$$+ C_{\bar{q}\sigma Gq}(q^2)\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q + \cdots$$

$$\int d^4x e^{iqx}\langle 0|TJ(x)J(0)|0\rangle = \sum_n C_n(q^2)\langle 0|O_n|0\rangle$$

condensates

vacuum expectation
value of the normal
ordered operators
(nonperturbative object)

- adopt some ansatz
- parameterize this with hadronic observables

The values of condensates
have to be supplied as
inputs in QCD sum rule

fit the lhs.(OPE side) with the
rhs.(phenomenological side) to obtain
the hadronic observables.

hadronic masses, couplings,etc.

QCD sum rules requires

- ✓ fundamental parameters

  $m_u,\ m_d,\ m_s,\ \cdots,\ \alpha_s(\mu)$
- ✓ values of condensates

  $\langle 0|\bar{q}q|0\rangle,\ \langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle,\quad g_s\langle 0|\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q|0\rangle,\ \cdots$

→ Hadron masses, coupling constants, etc.

OUTPUT

INPUT

One of the least known condensates (the mixed condensate)

The standard values are determined by phenomenological analysis.

$$\langle\bar{q}q\rangle = -(0.23\,\mathrm{GeV})^3,\ \frac{\alpha_S}{\pi}\langle G_{\mu\nu}G^{\mu\nu}\rangle = (0.33\,\mathrm{GeV})^4,\ m_0^2 \equiv \frac{g_S\langle\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle}{\langle\bar{q}q\rangle} = 0.8\,\mathrm{GeV}^2$$

(For some condensates, Shifman et al. gave estimates from dilute instanton gas approx.)

The mixed condensate $g_s\left\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\right\rangle$

✓ plays important roles in various system in QCD sum rule.

– the nucleon-delta mass difference

$$\lambda_N^2 m_N e^{-m_N^2/M^2} = \frac{1}{(2\pi)^2} M^4\left(-\left\langle \bar{q}q\right\rangle\right) + 0 + \cdots$$
$$\lambda_\Delta^2 m_\Delta e^{-m_\Delta^2/M^2} = \frac{4}{3(2\pi)^2} M^4\left(-\left\langle \bar{q}q\right\rangle\right) - \frac{2}{3(2\pi)^2} M^2\left(-g_S\left\langle \bar{q}\sigma_{\mu\nu}G^{\mu\nu}\right\rangle\right) + \cdots$$

– the light-heavy meson system

– $\pi - N$ sigma term

– etc.

✓ is another chiral order parameter of the second lowest dimension.

✓ represents a direct correlation between quarks and gluons.

The determination of the mixed condensate

- ✓QCD sum rule itself
  – V.M.Belyaev et al., Sov. Phys. JETP 56, 493 (1982).
  – L.J. Reinders et al., Phys. Lett B120, 209 (1983).
  – A.A. Ovchinnikov et al., Sov. J. Nucl. Phys. 48, 721 (1988). (Yad Fiz. 48, 1135 (1988))
  – S. Narison, Phys. Lett. B210, 238 (1988).
- ✓Instanton liquid model
  – M.V. Polyakov et al., Phys. Lett. B387, 841 (1996).
- ✓etc.
- ✓lattice QCD calculation

  There is only one lattice QCD calculation.

  – M. Kremer et al., Phys. Lett. B194, 283 (1987).

The lattice QCD calculation of the mixed condensate has been limited to this preliminary (but pioneering) work for 15 years !!

× KS-fermion (quenched)

× 8^4 lattice

× $\beta = 5.7 \ (a = 0.19\text{fm})$

×1 point ✕ 5 configs.

⇒ total number of data : 5

It is important to perform a new lattice QCD calculation of mixed condensate on a larger and a finer lattice with a better statistics !

Determination of condensates within QCD sum rule framework

OUTPUT

Hadron masses, coupling constants, etc.

INPUT

- ✓ fundamental parameters

  $m_u,\ m_d,\ m_s,\ \cdots,\ \alpha_s(\mu)$

- ✓ values of condensates

  $\langle 0|\bar{q}q|0\rangle,\ \langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle,\quad g_s\langle 0|\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q|0\rangle,\ \cdots$

OUTPUT

INPUT

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(For some condensates, Shifman et al. gave estimates from dilute instanton gas approx.)

Lattice QCD calculation

Lattice Parameter Setup:

- Gauge configuration by Wilson action $\beta = 6.0$
- Lattice spacing: $a = 0.1\,\mathrm{fm}$ (from $\sigma = 0.89\,\mathrm{GeV/fm}$)
- Lattice size: $16^4,\ (1.6\,\mathrm{fm})^4$
  periodic BC for gluon
  anti-periodic BC for quarks
- The pseudo-heat bath algorithm for the update of gauge configuration

thermalization	1000 sweeps
measurement interval	500 sweeps
number of gauge configs.	100 configs.

We use 16 space-time points from each gauge config.

⇨ total number of data = 1600

- Kogut-Susskind fermion (quenched calculation)

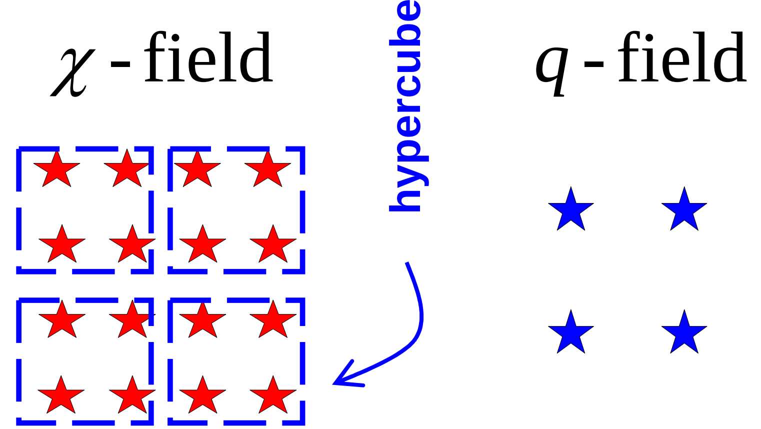
$$S_{\mathrm{KS}} = \tfrac{1}{2}\sum_{s,\mu}\eta_\mu(s)\bar{\chi}(s)\left\{U_\mu(s)\chi(s+\mu) - U_\mu^+(s-\mu)\chi(s-\mu)\right\} + ma\sum_s \bar{\chi}(s)\chi(s)$$

ma	0.0105	0.0184	0.0263
m [MeV]	21	36	52

$g_s\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$

It is important to respect the chiral symmetry, since the mixed condensate is another chiral order parameter. ➡ KS fermion is appropriate.

The operator $g_s\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$



$$q_i^f(x)=\tfrac{1}{8}\sum_\rho(\Gamma_\rho)_{if}\,\chi(2x+\rho)$$

$$\Gamma_\rho\equiv\gamma_1^{\rho_1}\gamma_2^{\rho_2}\gamma_3^{\rho_3}\gamma_4^{\rho_4}$$

$$\rho\equiv(\rho_1,\rho_2,\rho_3,\rho_4);\ \rho_i\in\{0,1\}$$

The KS expression of condensate

strictly local

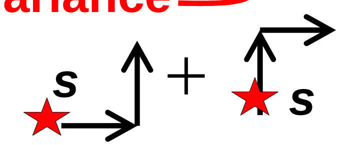
$$a^3\langle\bar{q}q\rangle=-\tfrac{1}{8}\sum_\rho\mathrm{Tr}\left[\Gamma_\rho\Gamma_{\rho'}\langle\chi(2x+\rho)\bar{\chi}(2x+\rho)\rangle\right]$$

$$a^3g_S\langle\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle=-\tfrac{1}{8}\sum_{\rho,\mu,\nu}\mathrm{Tr}\left[U_{\mu,\nu}(2x+\rho)\Gamma_{\rho+\mu+\nu}\Gamma_{\rho'}\langle\chi(2x+\rho+\mu+\nu)\bar{\chi}(2x+\rho)\rangle\sigma^{\mu\nu}G^{\rm lat}_{\mu\nu}(2x+\rho)\right]$$

semi nonlocal

to respect gauge covariance

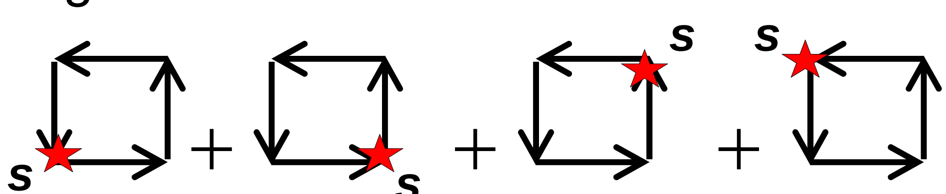
$U_{\mu,\nu}(s)\equiv\tfrac{1}{2}($



$)$

The lattice field strength

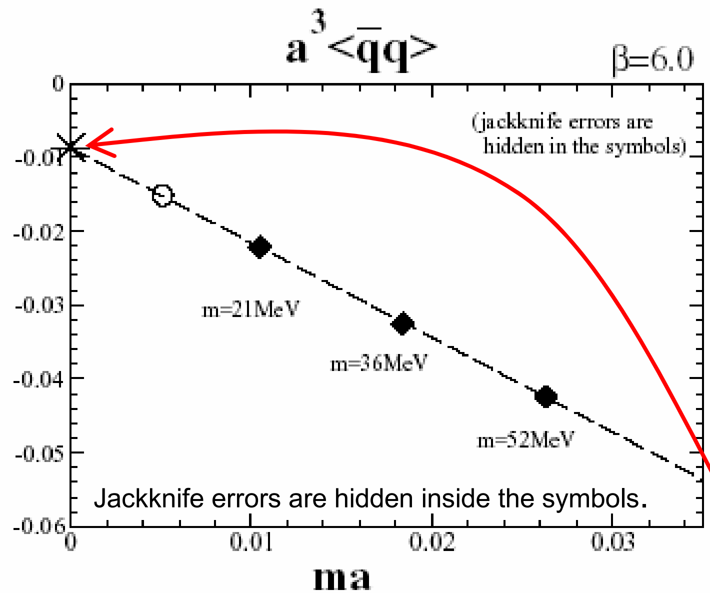
$G^{\rm lat}_{\mu\nu}(s)\equiv\tfrac{i}{8}\Big($

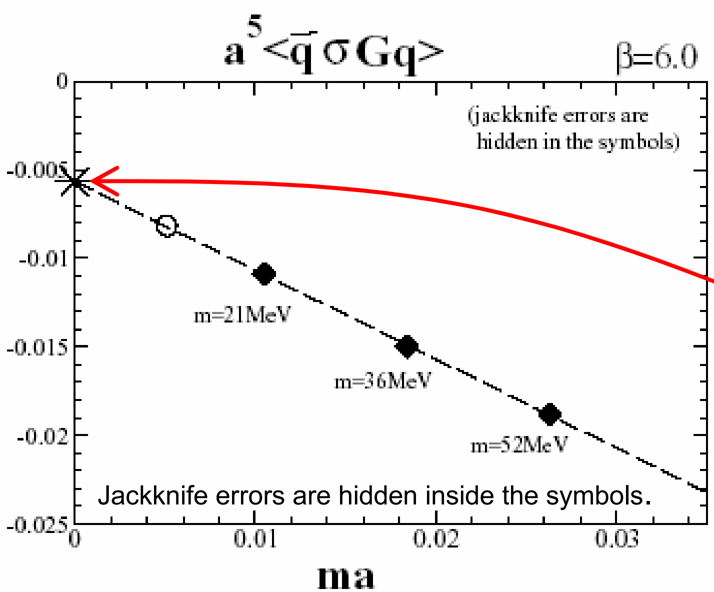


$-(\mu\Leftrightarrow\nu)\Big)$

$$=a^2\left(g_SG^a_{\mu\nu}(s)T^a+O(a^2)\right)$$

Numerical results





Both the condensates behave linearly with the quark mass.

↓

Linear chiral extrapolation.

↓

Values in the chiral limit.

$$a^3\langle\bar{q}q\rangle = -0.008721(39)$$

$$a^5\langle\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle = -0.005652(14)$$

Comparison with the standard value employed in QCD sum rule

Rescaling of the scale: $\mu \approx \pi / a \Rightarrow \mu \equiv 1\mathrm{GeV} \approx \text{scale of QCD sum rule}$

by 1-loop anomalous dimension

$$\langle \bar{q}q \rangle_\mu = \left(\frac{\alpha_S(\mu)}{\alpha_S(\pi/a)} \right)^{-4/b_0} \langle \bar{q}q \rangle_{\pi/a}$$

$$g_S \langle \bar{q}\sigma_{\mu\nu} G^{\mu\nu} q \rangle_\mu = \left(\frac{\alpha_S(\mu)}{\alpha_S(\pi/a)} \right)^{2/(3b_0)} g_S \langle \bar{q}\sigma_{\mu\nu} G^{\mu\nu} q \rangle_{\pi/a}$$

$\alpha_S(\mu) = \frac{4\pi}{b_0 \log(\mu/\Lambda_{\mathrm{QCD}})}$

$b_0 = \frac{11}{3} N_{\mathrm{C}} - \frac{2}{3} N_{\mathrm{F}}; \quad (N_{\mathrm{F}} = 0)$

$\Lambda_{\mathrm{QCD}} = 200 - 300\,\mathrm{MeV}$

$$m_{0;\mu}^2 \equiv \frac{g_S \langle \bar{q}\sigma_{\mu\nu} G^{\mu\nu} q \rangle_\mu}{\langle \bar{q}q \rangle_\mu} \cong 3.5 - 3.7\ \mathrm{GeV}^2 \Longleftrightarrow$$

The standard value:

$$m_0^2 = 0.8 \pm 0.2\ \mathrm{GeV}^2$$

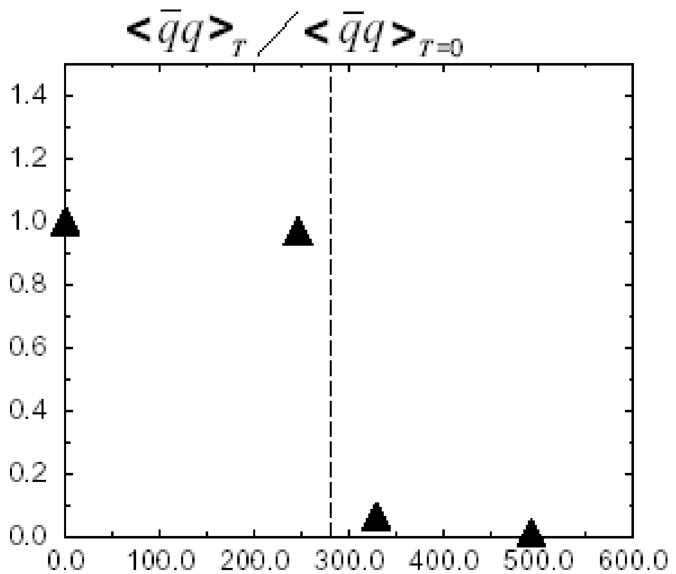
Comparing with the standard value, our calculation results in a rather large value.

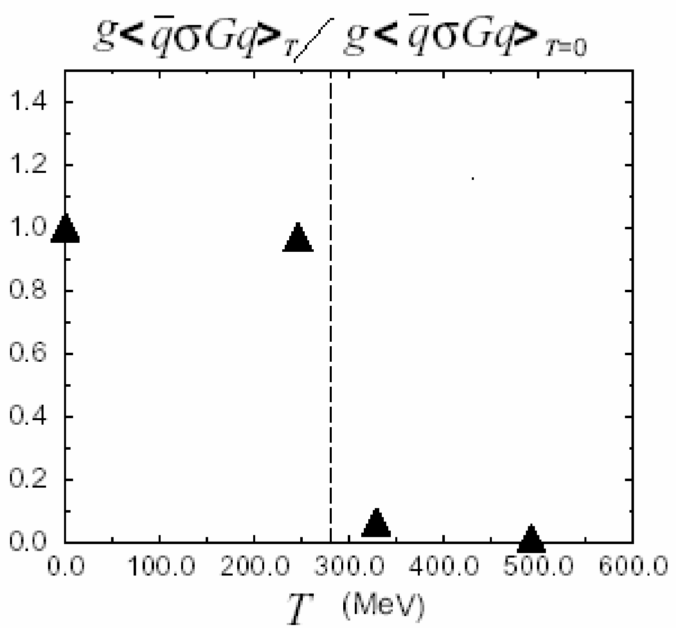
Comments:

✓ Instanton model has made a slightly larger estimate: $m_0^2 = 1.4\ \mathrm{GeV}^2$

✓ For improvement, the renomalization should be performed more carefully.

Temperature dependence of the condensates (Preliminary)





Comments:

✓Quenched level result

✓chiral limit by linear extrapolation

Below Tc,
both condensates reduces by about only 3 % in the vicinity of Tc.

Above Tc,
both condensates almost vanish.
(They are chiral order parameters)

Checks on systematic uncertainties

anti-periodic BS v.s. periodic BS

✓ **The finite volume artifact**

is estimated by imposing **a different boundary condition on quark fields**.

⇨ The deviation is about 1 %.

⇨ The finite volume artifact is small.

✓ **The discretization error of** $g_s \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$

– No O(a) error from the quark propagator after averaging over flavor space

$$S_{\mathrm{KS}} = \frac{1}{2} \sum_{s,\mu} \eta_\mu(s) \overline{\chi}(s) \left\{ U_\mu(s) \chi(s+\mu) - U_\mu^+(s-\mu) \chi(s-\mu) \right\} + ma \sum_s \overline{\chi}(s) \chi(s)$$

– No O(a) error from $G_{\mu\nu}^{\mathrm{lat}}(s) = a^2 \left(g_S G_{\mu\nu}^a(s) T^a + O(a^2) \right)$

– An ambiguity from a particular choice of the gauge link to respect the gauge covariance:

To estimate the size of this ambiguity, we examine a different path.

⇨ The deviation is about 1 %.

⇨ The discretization error is small.

Summary & Discussion

Recalculation of the mixed condensate on a finer and larger lattice with higher statistics using KS-fermion at quenched level.

Our calculation:

$\beta = 6.0,\ 16^4$ lattice

$16\,\text{points} \times 100\,\text{configs} = 1600\,\text{data}$

Older calculation:

$\beta = 5.7,\ 8^4$ lattice

$1\,\text{points} \times 5\,\text{configs} = 5\,\text{data}$

by M.Kremer et al., Phys.Lett.B194,283(1987)

$$\Rightarrow\quad a^5 g_s \langle \bar{q}\sigma_{\mu\nu} G_{\mu\nu} q \rangle = -0.005652(14) : \text{bare value at } a = 0.1\,\text{fm}$$

We have rescaled our result to compare it with the standard QCD sum rule value by using the pertubative anormalous dimension.

$$m_{0;\mu}^2 \equiv \frac{g_S \langle \bar{q}\sigma_{\mu\nu} G^{\mu\nu} q \rangle_\mu}{\langle \bar{q}q \rangle_\mu} \cong 3.5 - 3.7\ \text{GeV}^2 \quad \Leftrightarrow$$

The standard value:

$$m_0^2 = 0.8 \pm 0.2\ \text{GeV}^2$$

Further studies in progress:

- ✓ Finite temperature.
- ✓ Full QCD
- ✓ Renormalization constants.

The large value suggests the importance of the mixed condensate in QCD sum rule.

The Glueballs at Finite Temperature

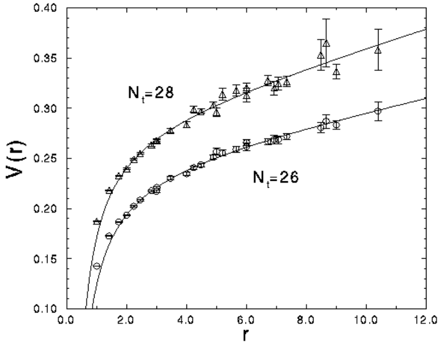
Contents from

- ✓ N.Ishii, H.Suganuma, H.Matsufuru, Phys. Rev. D66, 014507 (2002)
- ✓ N.Ishii, H.Suganuma, H.Matsufuru, Phys. Rev. D66, 094506 (2002)

Backgrounds

At finite temperature/density, QCD vacuum is expected to change its structure (even below Tc)

- The reduction of the confinement force, i.e., the string tension
- The partial restoration of the spontaneous chiral symmetry breaking
- Changes of various vacuum condensates



$q\bar{q}$ - potential

$N_t = 26{,}28$ correspond to

$T = 0.93T_C, 0.87T_C$, respectively.

from H.Matsufuru et al., "Proceedings of Quantum Chromodynamics and Color Confinement", p.216.

- Hadrons are bound states of quarks and gluons.

→ Hadrons change their properties as a consequence of the change in the QCD vacuum.

- The changes of the hadronic properties at T>0 are expected to serve as important precritical phenomena of the QCD phase transition.
- These changes are predicted by various QCD-motivated effective models.
  - T.Hatsuda et al., PRL 55 (1985) 158.
  - T.Hashimoto et al., PRL 57 (1986) 2123.
  - T.Hatsuda et al., PRD 47 (1993) 1225.
  - H.Ichie et al., PRD 52 (1995) 2944.

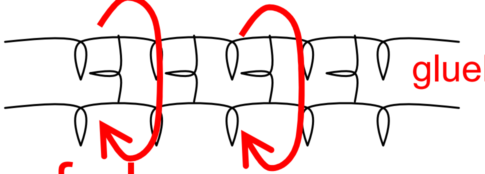
- These studies predict, in the vicinity of Tc, as a direct consequence of
  - the reduction of the string tension
  - the partial restoration of the spontaneous chiral symmetry breaking

the polemass shift of

- charmonium
- light $q\bar{q}$ mesons
- glueball

- Motivated by these results, anisotropic lattice QCD has been used to measure the polemass of various $q\bar{q}$ mesons at finite T (assuming the narrowness of the thermal width) at quenched level.
  - QCD-TARO Collab., PRD 63 (2001) 054501.
  - T.Umeda et al., Int.J.Mod.Phys. A16 (2001) 2215.
- showing (unfortuately) no significant change below Tc
- In this talk, we report the anisotropic lattice QCD studies of thermal glueball at finite T
  - N.Ishii et al., PRD 66 (2002) 014507.
  - N.Ishii et al., PRD 66 (2002) 094506.

Glueballs



- The Glueballs are hadrons mainly consisting of gluons.
- Their existence is predicted by QCD:
  - The non-Abelian nature of the gauge transformation group
  - The self-interaction of the gluons suggests the existence of the glueballs
- There have been quite a lot of theoretical studies:
  - MIT bag model
    - T.Barnes et al., Nucl.Phys. B224 (1983) 241., etc.
  - Constitutent gluon model
    - D.Horn et al., Phys.Rev.D17 (1978) 898., etc.
  - Flux tube model
    - N.Isgur et al., Phys.Lett. B147 (1984) 169., etc.
  - Instanton liquid model
    - T.Schafer et al., Phys.Rev.Lett., 75 (1995) 1707.
  - QCD sum rule
    - M.Shifman, Z.Phys. C9 (1981) 347.
  - Lattice QCD
    - [quench] C.J.Morningstar et al., Phys.Rev. D66 (1999) 034509, J.Sexton et al., Phys.Rev.Lett. 75 (1995) 4563., etc.
    - [full] A.Hart et al., Phys.Rev. D65 (2002) 034502., etc.

Glueball spectrum in quenched SU(3) lattice QCD

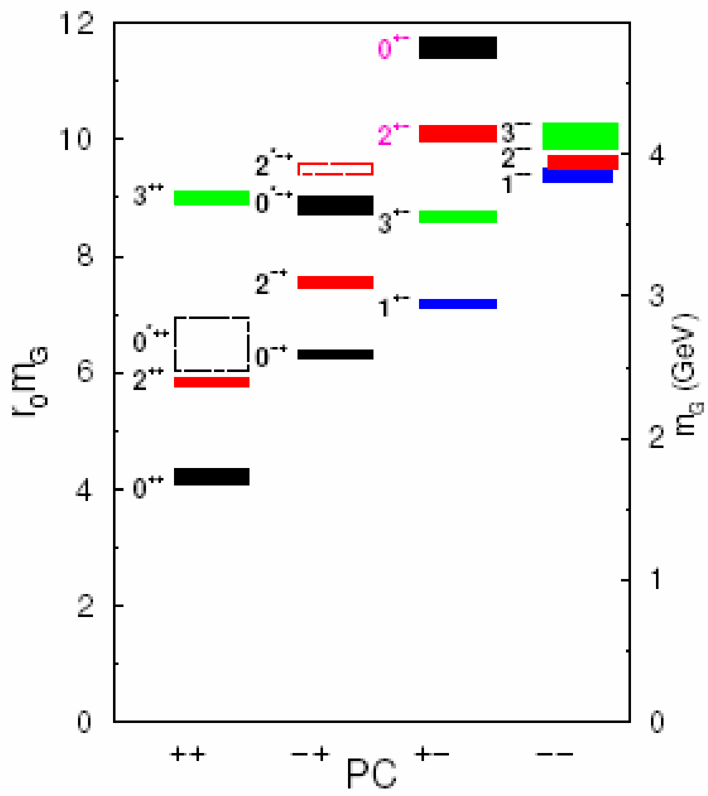


FIG. 8. The mass spectrum of glueballs in the pure SU(3) gauge theory. The masses are given in terms of the hadronic scale r_0 along the left vertical axis and in terms of GeV along the right vertical axis (assuming $r_0^{-1} = 410$ MeV). The mass uncertainties indicated by the vertical extents of the boxes do *not* include the uncertainty in setting r_0. The locations of states whose interpretation requires further study are indicated by the dashed hollow boxes.

from C.J.Morningstar et al., Phys.Rev.D60 (1999) 034509.

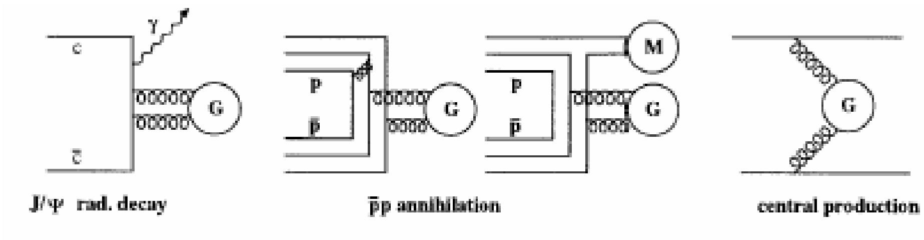
- Except for minor variations, quenched SU(3) lattice QCD predicts

$$m(0^{++}) \cong 1500 - 1700 \text{ MeV}$$

$$m(2^{++}) \cong 2000 - 2400 \text{ MeV}$$

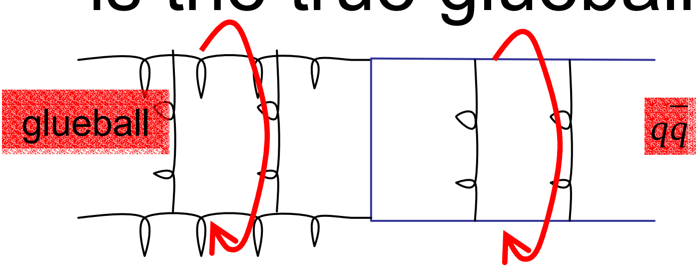
Glueballs in experiment

- Glueballs are created in the glue-rich processes:



from K.Seth, Nucl.Phys.A675(2000)25c.

- Due to the mixing with quarkonium, it is hard to determine which meson is the true glueball.



- The glueball is required to satisfy the following properties:
  - It is created by the glue-rich process.
  - It should be exotic meson.
  - Its decay width should be narrower than the ordinary meson (from OZI rule)
  - Flavor-blind decay
  - It should not decay into two photon.
- The glueball candidates

$$f_0(1500),\ \ f_0(1710)$$

Mass reduction of glueball from effective theory

(from H.Ichie et al., Phys.Rev.D52 (1995) 2944.)

Dual Ginzburg-Landau model

$$L = -\frac{1}{4}\left(\partial_\mu \vec{B}_\nu - \partial_\nu \vec{B}_\mu\right)^2 + \sum_{\alpha=1}^{3}\left[\left|\left(i\partial_\mu - g\vec{\varepsilon}_\alpha \cdot \vec{B}_\mu\right)\chi_\alpha\right|^2 - \lambda\left(\left|\chi_\alpha\right|^2 - v^2\right)\right]$$

λ and v are used as parameters.

$\vec{B}_\mu = \left(B_\mu^3, B_\mu^8\right)$ dual gauge field

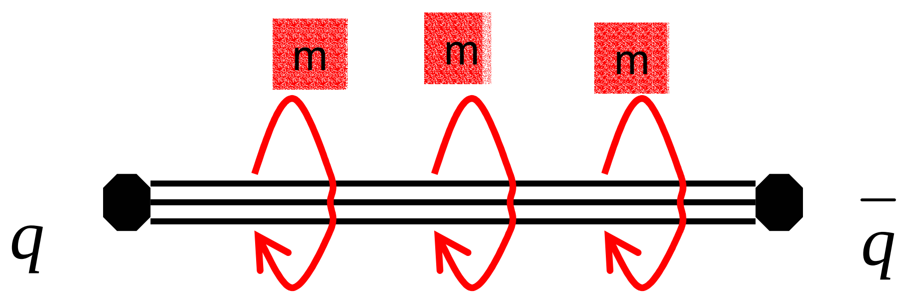
$\chi_\alpha \quad (\alpha = 1,2,3)$ QCD monopole filed

$\vec{\varepsilon}_1 = (1,0)$

$\vec{\varepsilon}_2 = \left(-1/2, -\sqrt{3}/2\right)$

$\vec{\varepsilon}_3 = \left(-1/2, \sqrt{3}/2\right)$

Higgs phase of DGL ⟺ confinement phase of QCD



QCD monopole condensation

↓

spontaneous breaking of dual U(1) symmetry
(Higgs phase of DGL)

↓

color electric flux quantization

Unitary gauge

$$\chi_\alpha = \left(\overline{\chi} + \tilde{\chi}_\alpha\right) e^{i\xi_\alpha}$$

χ - field is shifted.

mean-field

fluctuation

quadratic source term

$$L - J\sum_{\alpha=1}^{3}\left|\chi_\alpha\right|^2$$

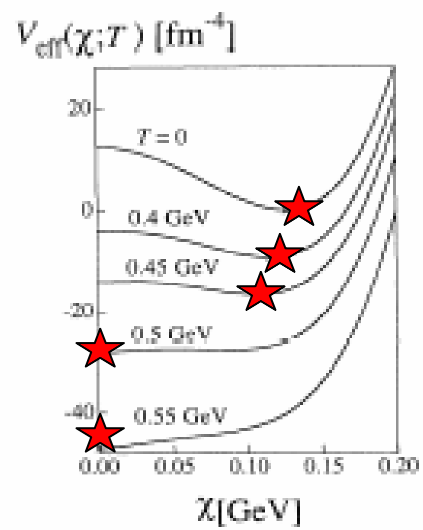
$$= L_{\rm cl}\left(\overline{\chi}\right) - 3J\overline{\chi}^2 - 2\overline{\chi}\left(2\lambda\left(\overline{\chi}^2 - v^2\right) + J\right)\sum_{\alpha=1}^{3}\tilde{\chi}_\alpha$$

$$-\frac{1}{4}\left(\partial_\mu \overrightarrow{B_\nu} - \partial_\nu \overrightarrow{B_\mu}\right)^2 + \frac{1}{2}m_B^2 \overrightarrow{B_\mu}^2 + \sum_{\alpha=1}^{3}\left(\left(\partial_\mu \tilde{\chi}_\alpha\right)^2 - m_\chi^2 \tilde{\chi}^2\right)$$

$$+\sum_{\alpha=1}^{3}\left[g^2\left(\overrightarrow{\varepsilon_\alpha}\cdot\overrightarrow{B_\mu}\right)^2\left(\tilde{\chi}_\alpha^2 + 2\overline{\chi}\tilde{\chi}_\alpha\right) - \lambda\left(4\overline{\chi}\tilde{\chi}_\alpha^3 + \tilde{\chi}_\alpha^4\right)\right]$$

with $m_\chi^2 = 4\lambda\overline{\chi}^2$ $\quad m_B^2 = 3g\overline{\chi}^2$

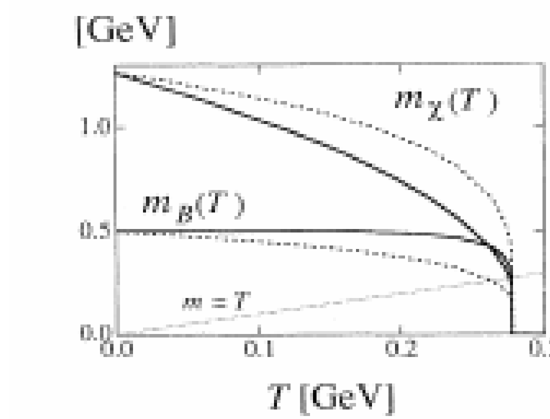
$\overline{\chi}$ is determined from effective action $V_{\rm eff}\left(\chi, T\right)$



After a fine tune of the parameter to adjust Tc=280 MeV

The mass of χ field:

$$m_\gamma = m_\gamma(T) = 4\lambda\overline{\chi}(T)^2$$



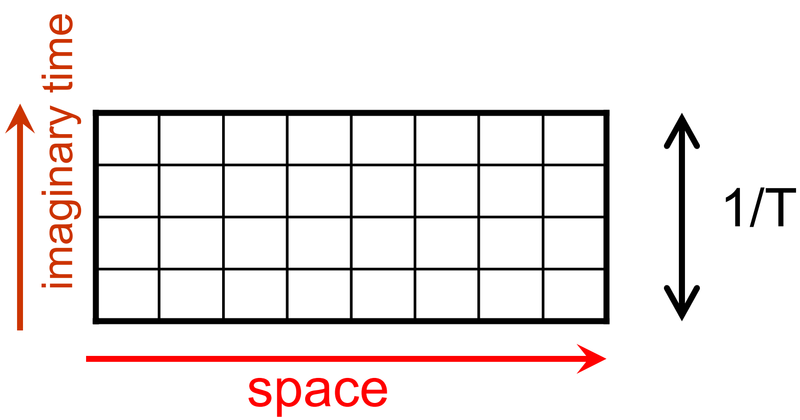
χ is a complicated combination of gluon field.

χ is a glueball.

The significant polemass reduction of glueball near Tc

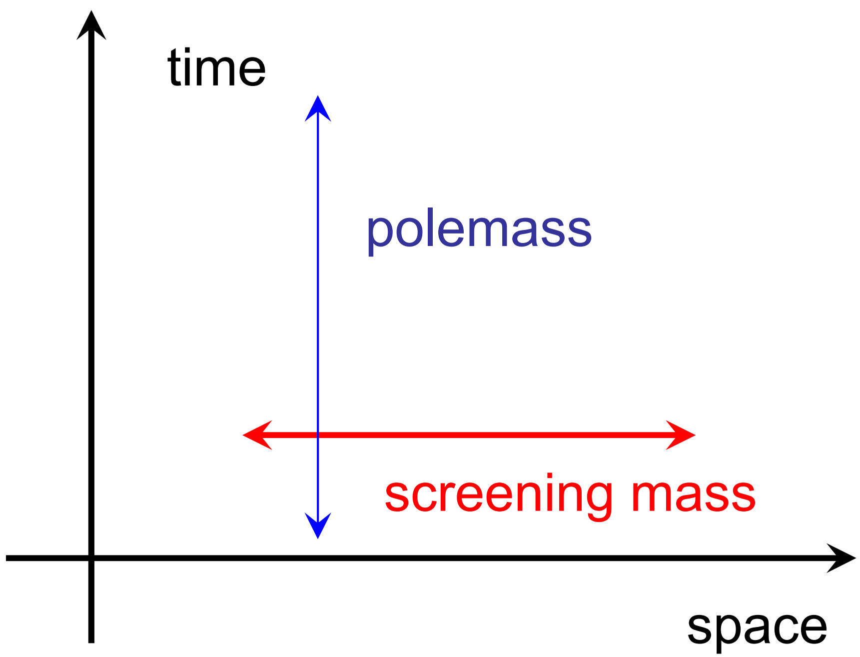
The thermal hadrons in lattice QCD

- Hadronic mass is obtained from the temporal correlation, i.e., the two-point correlator
- At high temperature, the temporal lattice size shrinks as 1/T, number of data decrease.



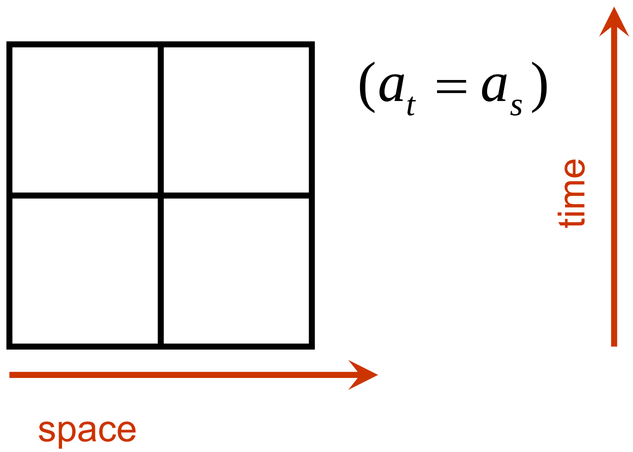
- It is difficult to mesure the polemass at high temperature.

- Due to this technical difficulty, the lattice studies of the thermal hadron mass had been restricted to the spatial correlations, i.e., the screening mass.

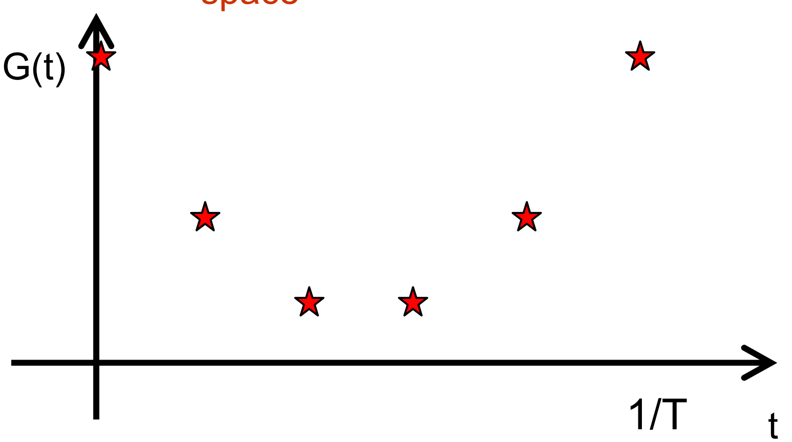


Anisotropic Lattice QCD

- Isotropic lattice



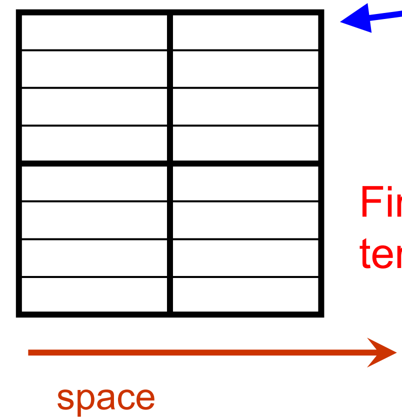
$(a_t = a_s)$



Only 3data are available for mass measurement

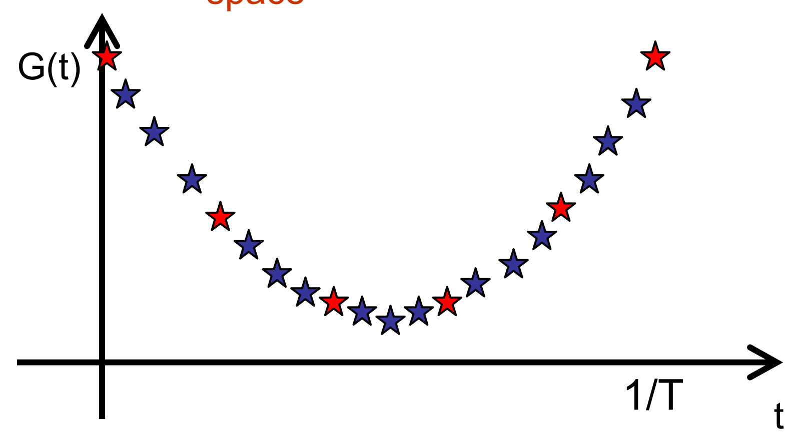
- Anisotropic lattice

$\gamma = 4$ case



$(a_t = \frac{1}{\gamma} a_s, \gamma > 1)$

Finer mesh along the temporal direction.



11 data are available for mass measurement

The technical difficulty can be resolved by the anisotropic lattice !

Glueball correlators and the spectral functions

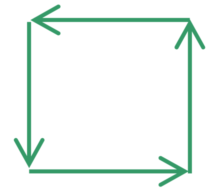
The glueball correlator:

$$G(t) = Z(\beta)^{-1} \mathrm{Tr}\left[e^{-\beta H} \varphi(t)\varphi(0)\right],$$

$$= \int \frac{d\omega}{2\pi} \frac{\rho(\omega)}{2\sinh(\beta\omega/2)} \cosh\left[\omega(\beta/2 - t)\right]$$

The glueball operator:

$$\varphi(t) = e^{tH} \varphi(0) e^{-tH}$$

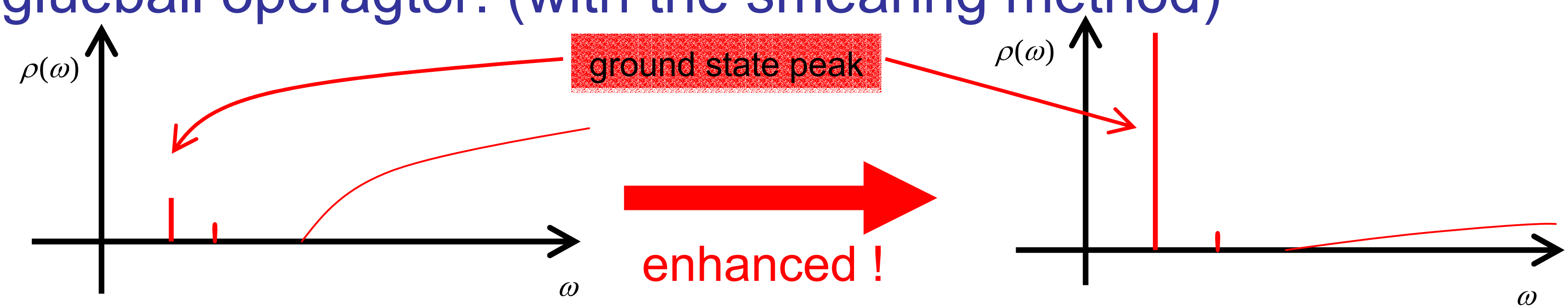


The spectral function

$$\rho(\omega) \equiv \sum_{n,m} \frac{\left|\langle n|\varphi|m\rangle\right|^2}{Z(\beta)} e^{-\beta E_m} \times 2\pi\left[\delta(\omega - \Delta E_{nm}) - \delta(\omega + \Delta E_{nm})\right], \qquad \Delta E_{nm} \equiv E_n - E_m$$

Physical observables (mass, width,…) are extracted by parameterizing $\rho(\omega)$

We enhance the ground state contribution by improving the glueball operagtor. (with the smearing method)



The smearing method

cf) APE Collab., M.Albanese et al., Phys.Lett.B192,163(1987).

In the case of the glueball, the poor overlap problem is known to due to the difference:

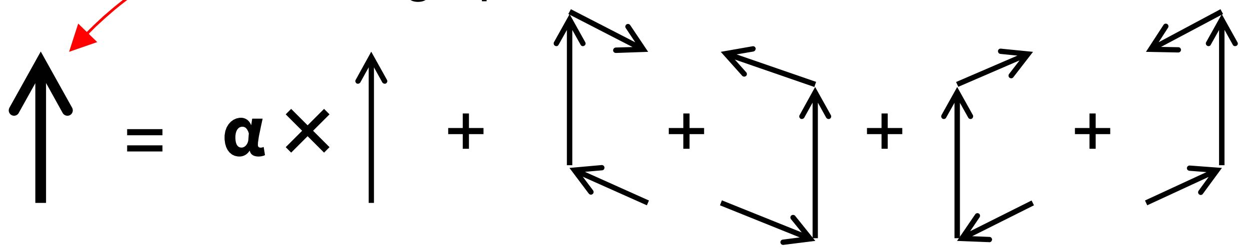
the physical glueball size ⟷ the "size" of the glueball operator

$r \approx 0.2 - 0.5\,\mathrm{fm}$ $\qquad$ $r \approx a_s$

Improvement by systematically providing a spatial size to the glueball operator.

the fat link

<u>Sketch of the smearing operation</u>



Repeat this procedure n-times. → n-th fat link → n-th smeared plaquette

continuum approximation

Diffution eq.

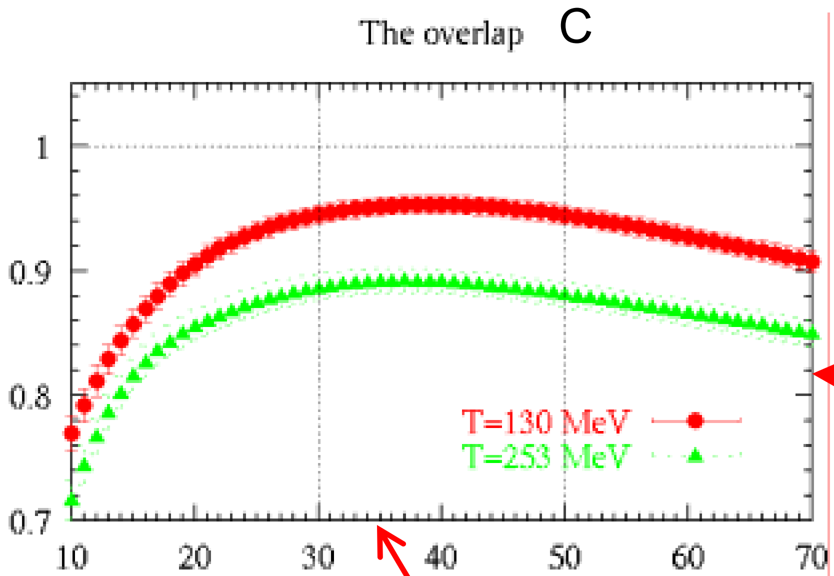
$$\frac{\partial}{\partial n} A_i(\vec{x};n) = D\vec{\nabla}^2 A_i(\vec{x};n)$$

$$D \equiv \frac{a_s^2}{\alpha+4} : \text{diffusion coefficient}$$

$$A_i(\vec{x};n) = \int d^3y \frac{e^{-(\vec{x}-\vec{y})^2/\rho^2}}{\pi^{3/2}\rho^3} \times A_i(\vec{y})$$

$$\rho \equiv 2a_s\sqrt{\frac{n}{\alpha+4}}$$ size of the n-th smeared plaquette

The smearing dependence

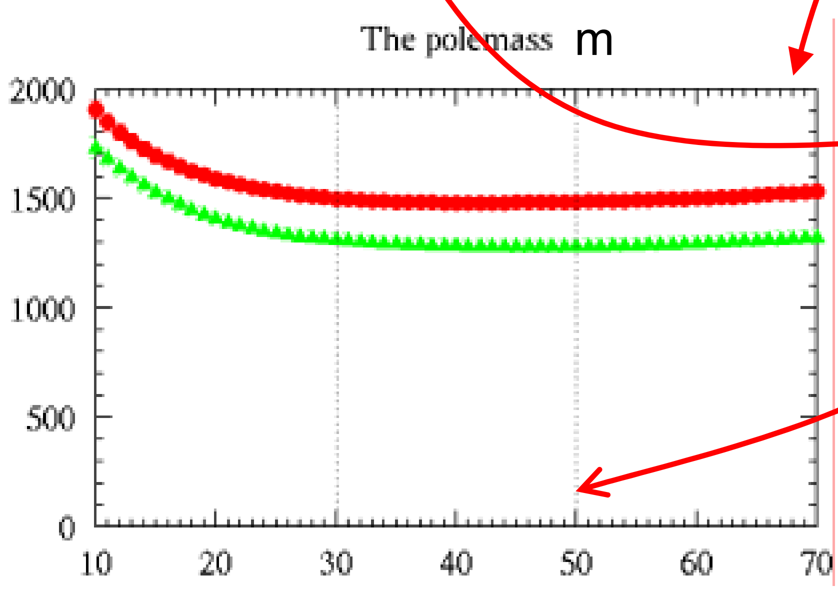


Fit the correlator with

$$\frac{G(t)}{G(0)} = C\left(e^{-mt} + e^{-m(\beta-t)}\right) + \cdots$$

C : ground state overlap

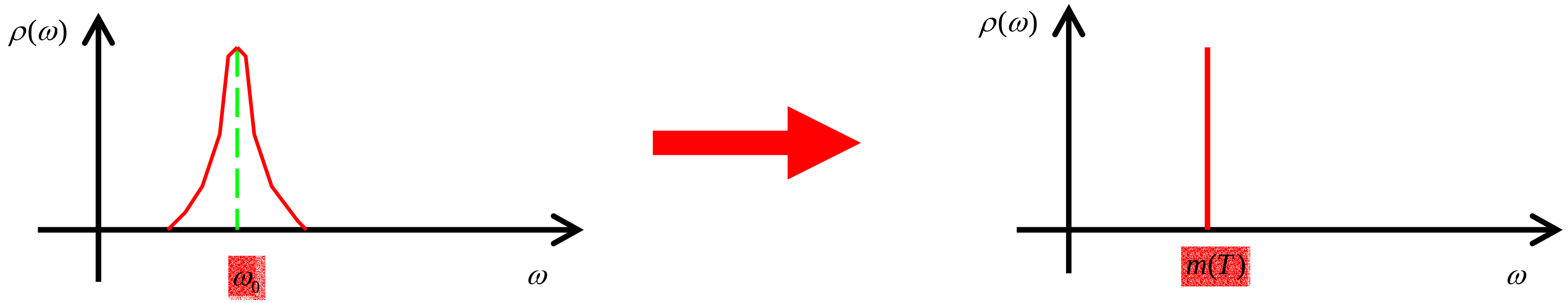
m : ground state mass



Maximum ground state enhancement is achieved in this region.

Narrow peak ansatz

If we assume the narrowness of the peak, the spectral function can be approximated by introducing a temperature dependent "polemass" m(T)



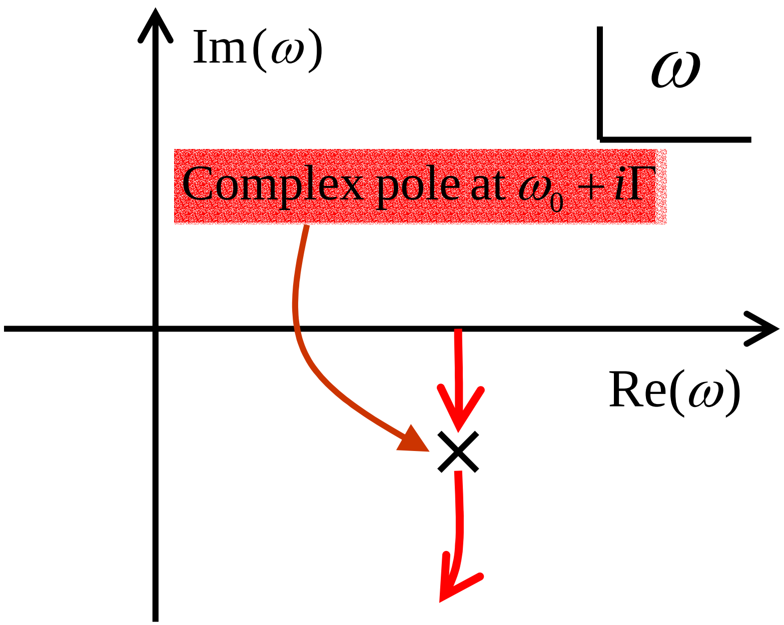
$$\rho(\omega) \cong 2\pi A\left[\delta(\omega - m(T)) - \delta(\omega + m(T))\right]$$

At T>0, each peak acquires a thermal width through the interaction with the heat bath.

It is desirable to respect the presence of thermal width at T>0.

Ansatz with thermal width

- What is the appropriate functional form ?
  - With increasing T (>0), bound state poles of $G_R(\omega)$ are moving off the real ω axis into complex ω plane.



$$G_R(\omega) = \frac{A}{\omega - \omega_0 + i\Gamma} + \cdots$$

$$\rho(\omega) = -2\,\mathrm{Im}(G_R(\omega))$$
$$= 2\pi A\left[\delta_\Gamma(\omega - \omega_0) - \delta_\Gamma(\omega + \omega_0)\right] + \cdots$$

Lorentzian at ω_0 with width Γ

$$\delta_\Gamma(\omega - \omega_0) \equiv \frac{1}{\pi}\,\mathrm{Im}\left[\frac{1}{\omega - \omega_0 + i\Gamma}\right]$$
$$= \frac{1}{\pi}\frac{\Gamma}{(\omega - \omega_0)^2 + \Gamma^2}$$

- The appropriate functiona form is "Breit-Wigner type".

$$g(t) \equiv \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\cosh[\omega(\beta/2 - t)]}{\sinh[\beta\omega/2]} \times 2\pi A\left[\delta_\Gamma(\omega - \omega_0) - \delta_\Gamma(\omega + \omega_0)\right]$$

A, Γ, ω fit parameters

What happens if the thermal width is broad ?

The spectral representation

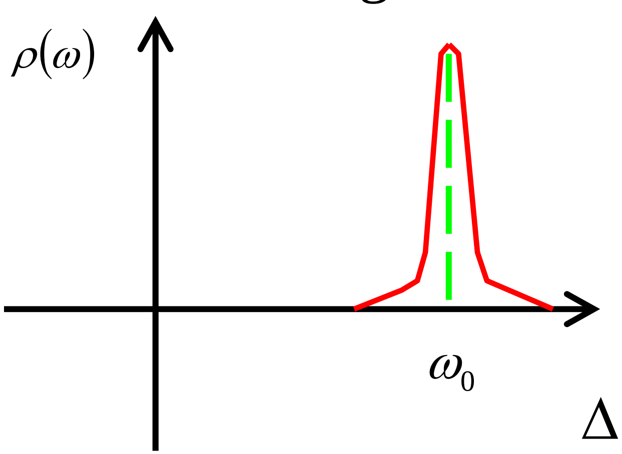
$$G(t)=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{\rho(\omega)}{2\sinh(\beta\omega/2)}\cosh\left[\omega(\beta/2-t)\right]$$

G(t) is an average of $\cosh\left[\omega(\beta/2-t)\right]$ with the weight $\dfrac{\rho(\omega)}{2\sinh(\beta\omega/2)}$

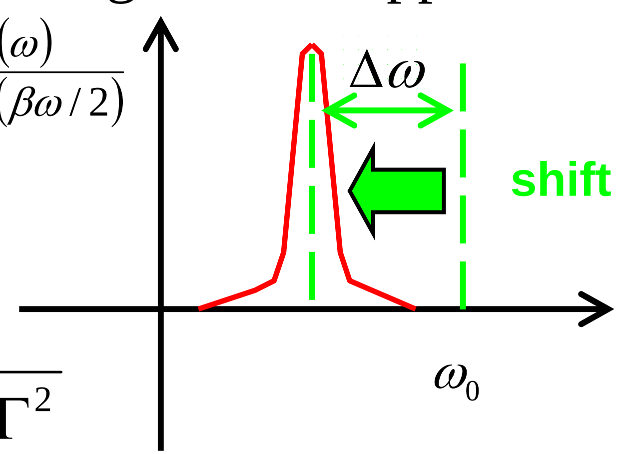
In the narrow peak ansatz, the weight is approximated by the δ function.

The role of the denominator $\left[2\sinh(\beta\omega/2)\simeq e^{\beta\omega/2}\right]$

small ω region $\Leftarrow$ enhanced, large ω region $\Leftarrow$ suppressed



$$\frac{\rho(\omega)}{2\sinh(\beta\omega/2)}$$



$$\Delta\omega\simeq 2T-\sqrt{4T^2-\Gamma^2}$$

The peak position is shifted to below !

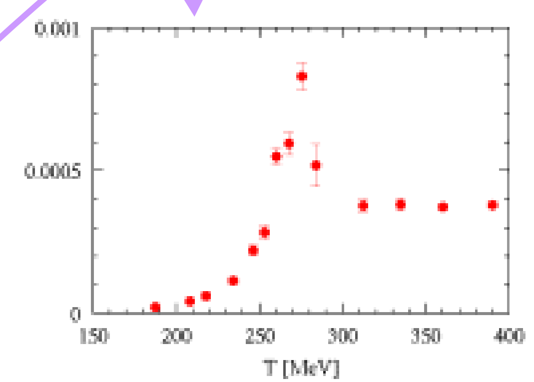
The Numerical Result

$\sqrt{\sigma} = 440\ \mathrm{MeV}$

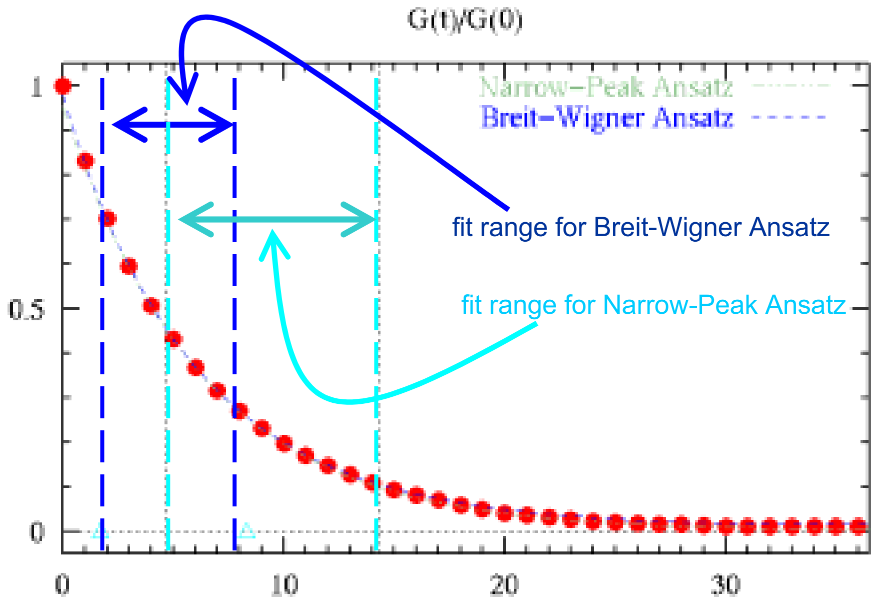
- The Lattice Parameter Setup
  - Gauge config by anisotropic Wilson action: $(\beta_{latt} \equiv 2N_C = 6.25)$
  - Lattice spacings are determined from the string tension:

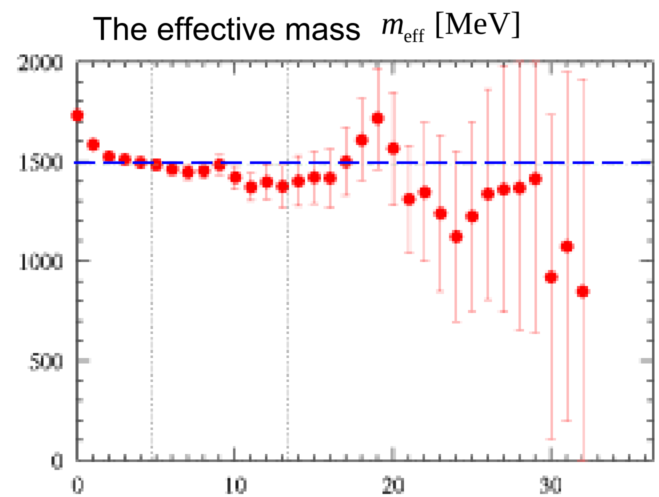
    $a_s = 0.084\ \mathrm{fm},\ a_t = 0.021\ \mathrm{fm} \Rightarrow \text{lattice anisotropy } a_s / a_t = 4$
  - Lattice size: $20^3 \times N_t (N_t = 24, \cdots, 72 \Leftrightarrow T = 390, \cdots, 130\mathrm{MeV})$
  - number of gauge config: 5,000-9,900, (bin size: 100)
  - For each T, we pick up gauge configs every 100 sweeps after skipping 20,000 sweeps for thermalization.
  - An appropriate smearing is adopted to enhance the lowlying peak contribution.
  - The critical temperature is determined from the Polyakov loop susceptibility:

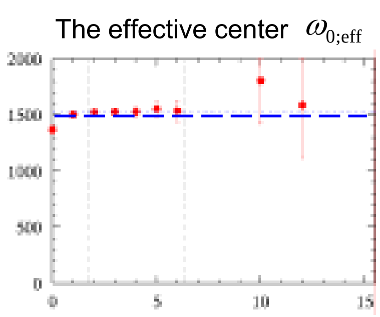
    Tc = 280 MeV

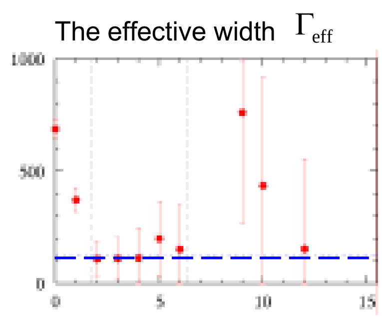


The glueball correlator at low temperature (T=130MeV)









The both ansatz fit the lattice QCD data very well.

It is natural, since the thermal width is still narrow at low temperature.

The effective mass, the effective center and the effective width

The effective mass

$$\frac{G(t)}{G(t+1)} = \frac{\cosh\left[\left(t-\beta/2\right)m_{\mathrm{eff}}(t)\right]}{\cosh\left[\left(t+1-\beta/2\right)m_{\mathrm{eff}}(t)\right]}$$

$\Rightarrow$ The existence of the plateau is a necessary condition for the single pole saturation.

The effective center and the effective width

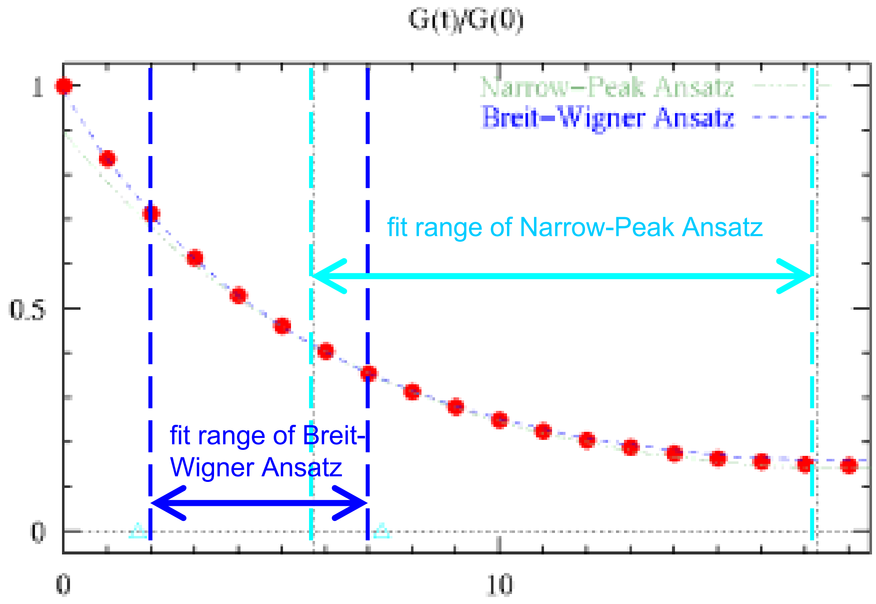
$$\frac{G(t)}{G(t+1)} = \frac{g(t,\omega_{\mathrm{eff}}(t),\Gamma_{\mathrm{eff}}(t))}{g(t+1,\omega_{\mathrm{eff}}(t),\Gamma_{\mathrm{eff}}(t))}$$

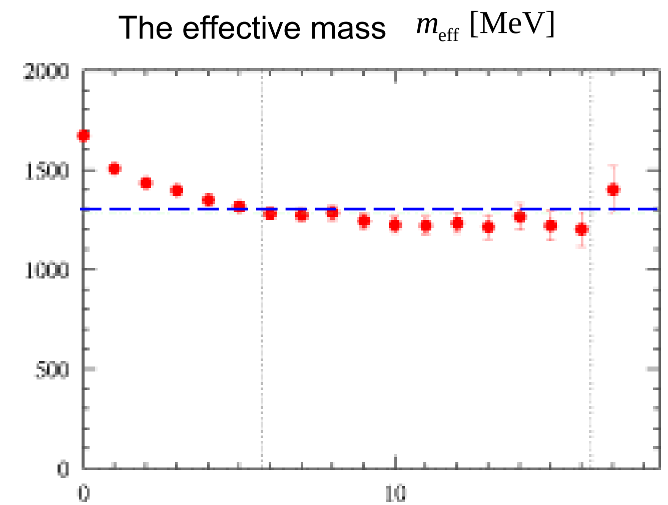
$$\frac{G(t+1)}{G(t+2)} = \frac{g(t+1,\omega_{\mathrm{eff}}(t),\Gamma_{\mathrm{eff}}(t))}{g(t+2,\omega_{\mathrm{eff}}(t),\Gamma_{\mathrm{eff}}(t))}$$

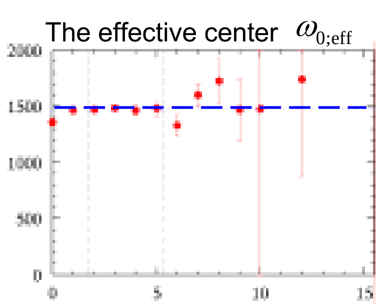
$\Rightarrow$ The existence of the simultaneous plateau is a necessary condition for the single peak saturation.

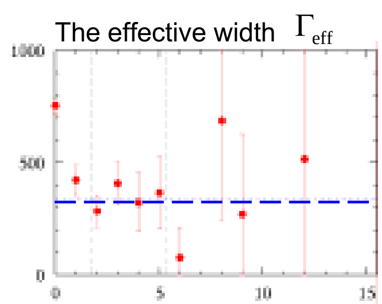
$$g(t,\omega_0,\Gamma) \equiv \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\cosh\left[\omega(\beta/2-t)\right]}{2\sinh(\beta\omega/2)} \times 2\pi\left\{\delta_\Gamma(\omega-\omega_0) - \delta_\Gamma(\omega+\omega_0)\right\}$$

The glueball correlator at hight temperature (T=253MeV<Tc)





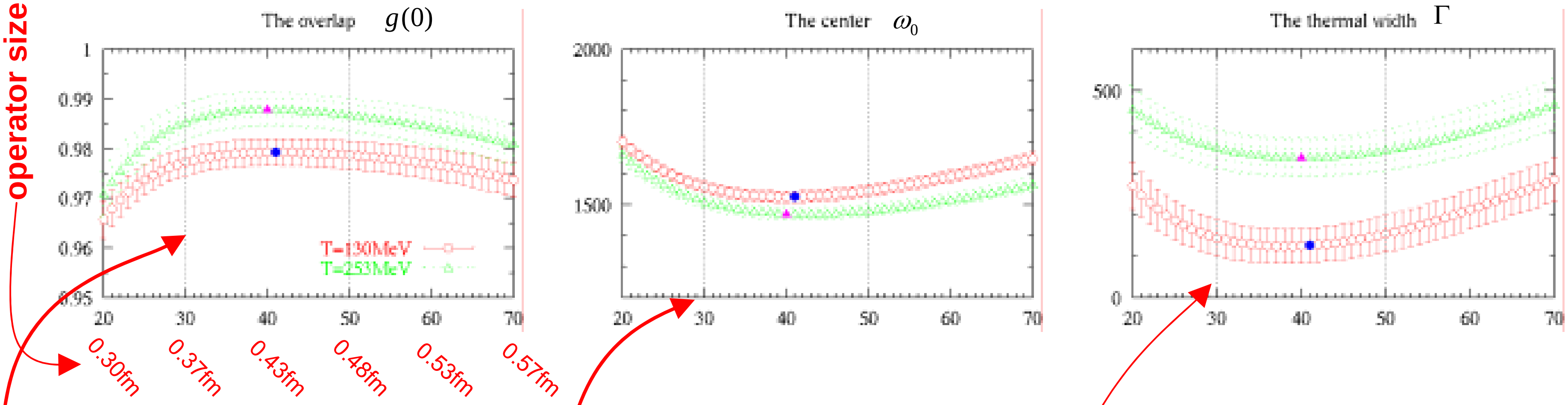




The narrow-peak ansatz fails to fit the lattice QCD data around t=0.

The Breit-Wigner ansatz fits the lattice QCD data in the whole region rather well.

The smearing dependence (T<Tc)



$g(0) \cong 1$

the single-peak saturation of G(t).

ω_0 minimum

contributions from higher spectral components are suppressed.

Γ minimum

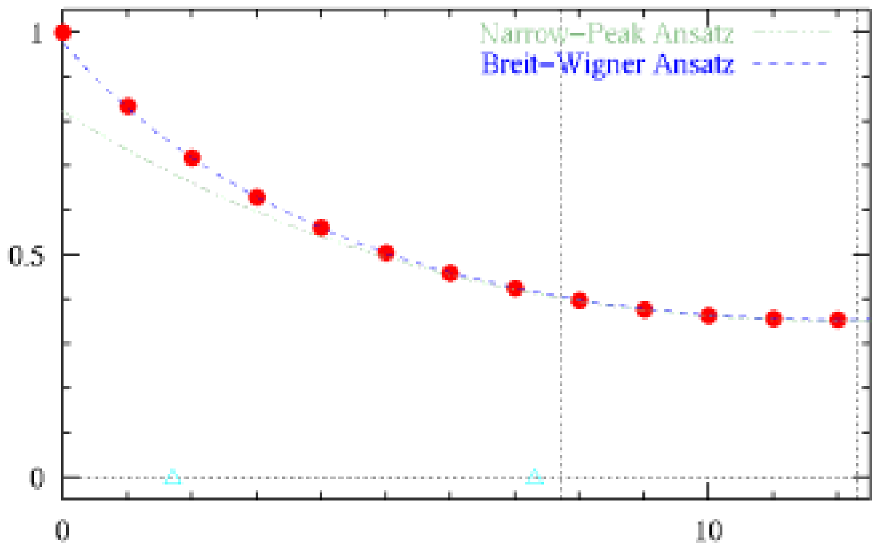
contributions from higher spectral components are suppressed.

The lowest peak contribution seems to be maximally enhanced in this region.

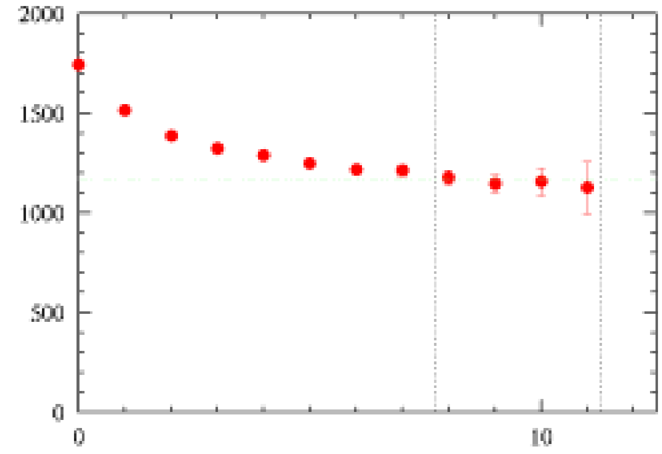
cf) operator size

$$\rho \cong 2a_s\sqrt{\frac{N_{\rm smr}}{\alpha+4}}\ \ (\alpha = 2.1, a_s = 0.84\ \text{fm})$$

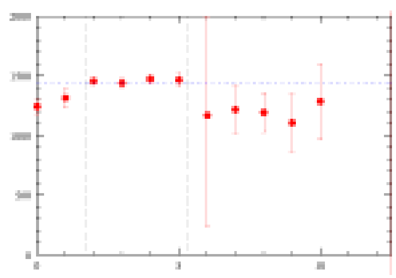
The glueball correlator above Tc (T=390 MeV)



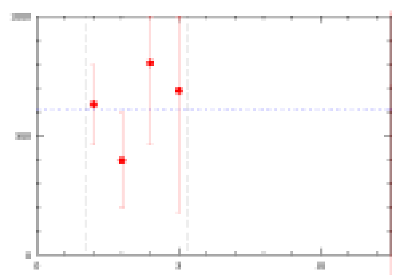
The effective mass m_{eff} [MeV]



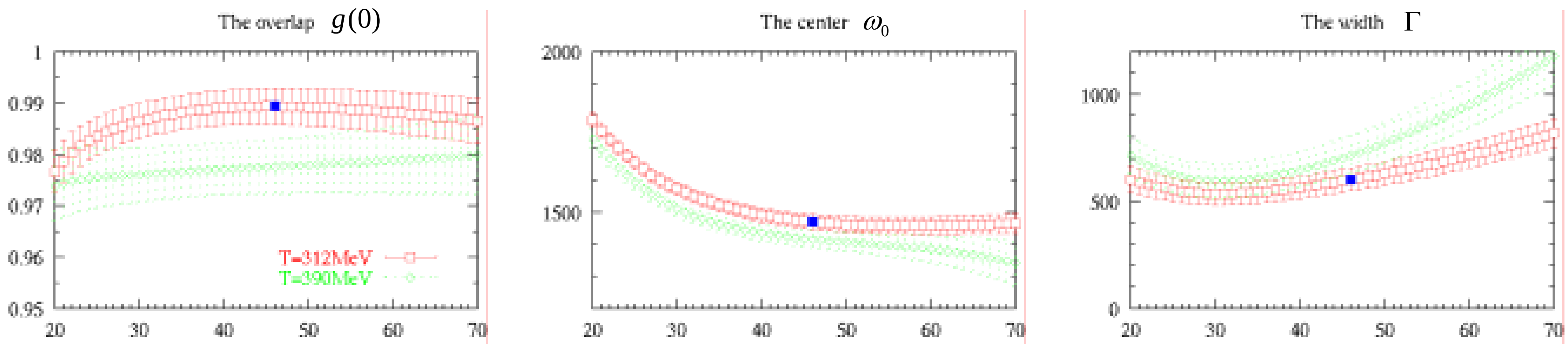
The effective center $\omega_{0;\mathrm{eff}}$



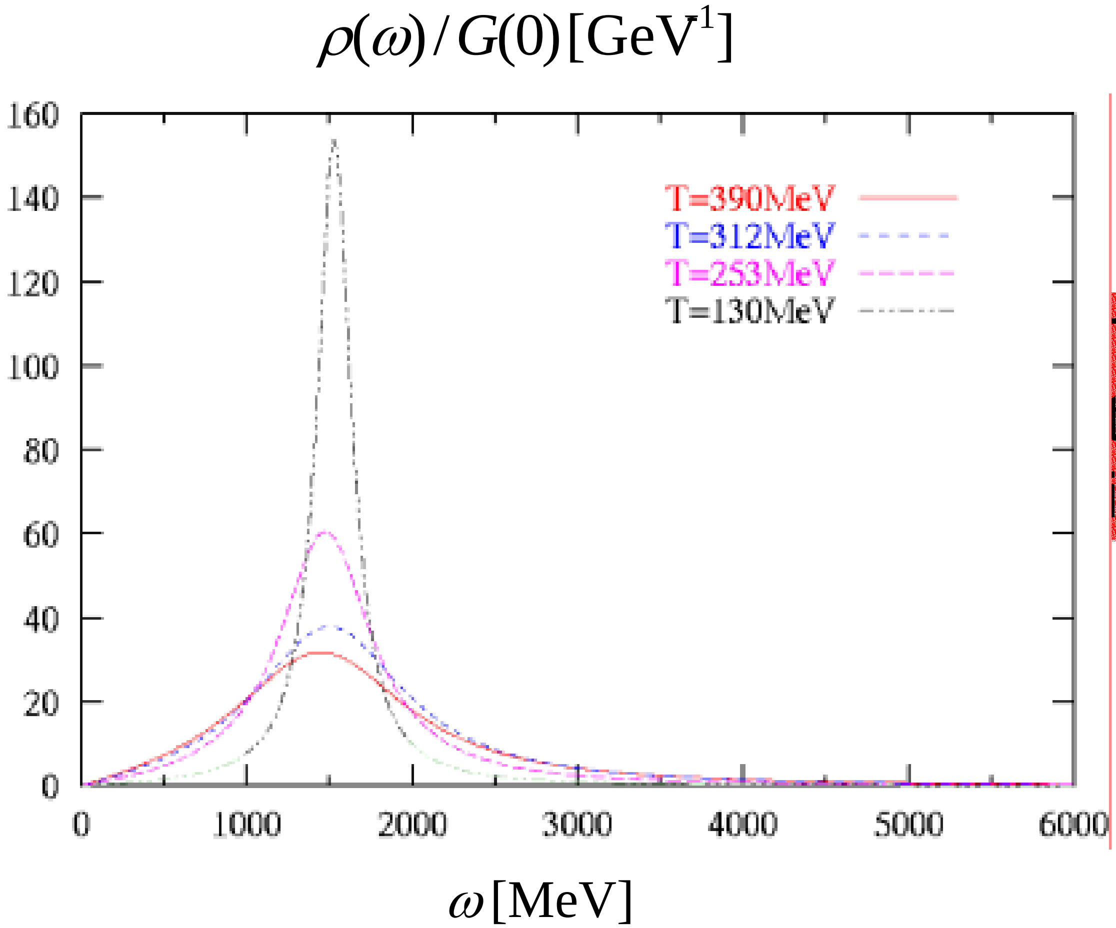
The effective width Γ_{eff}



The smearing dependence (T>Tc)

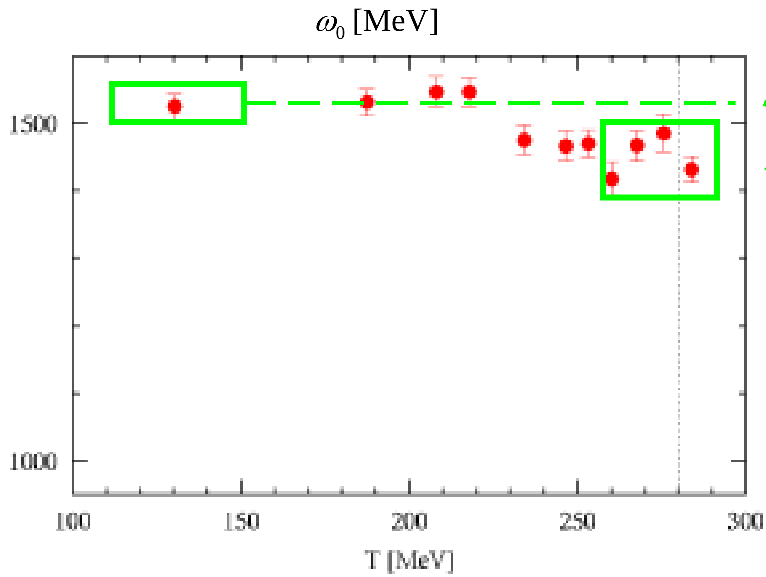


The low lying glueball peak(0++)

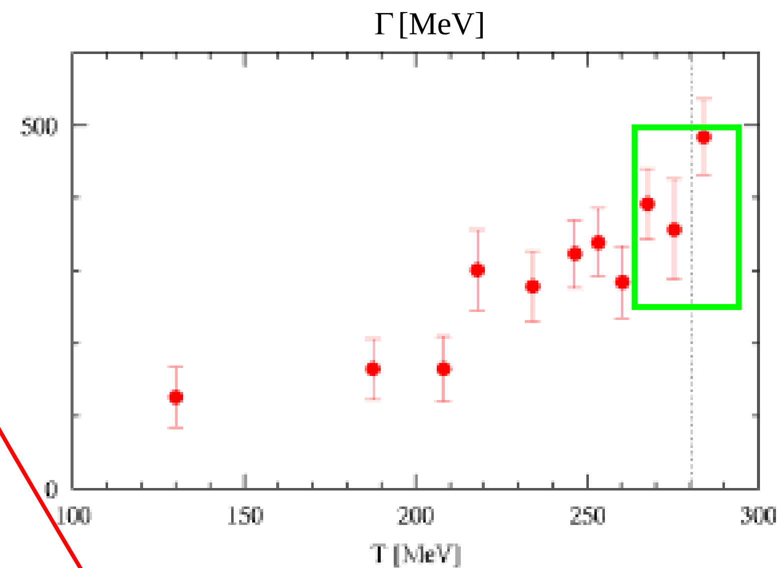


The thermal width is seen to become broader with increasing temperature.

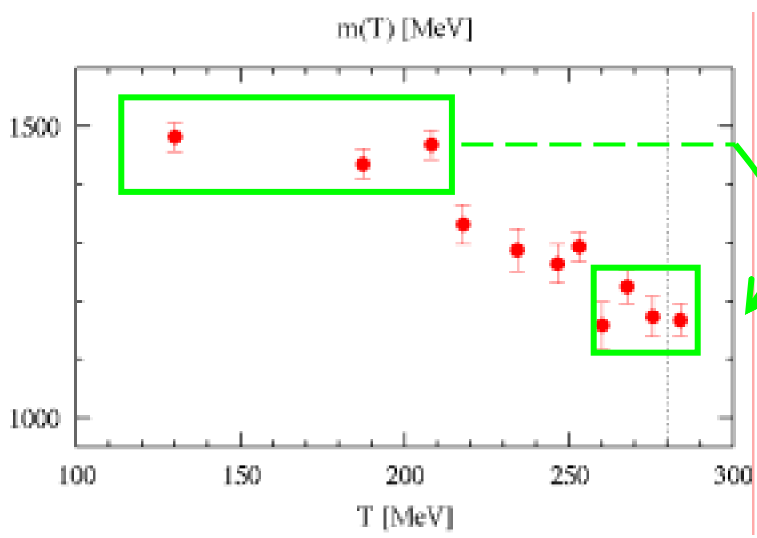
The peak center



The thermal width



The polemass from narrow peak ansatz



- The Breit-Wigner ansatz
  - thermal width broadening near Tc

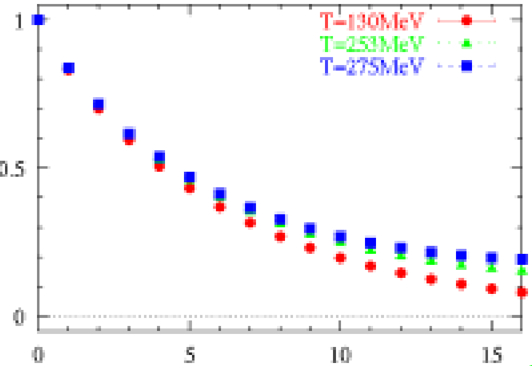
    $\Gamma(T_C) \cong 300\ \text{MeV}$
  - modest reduction in peak center

    $\Delta\omega_0 \leq 100\ \text{MeV}$
- The narrow peak ansatz
  - polemass reduction near Tc

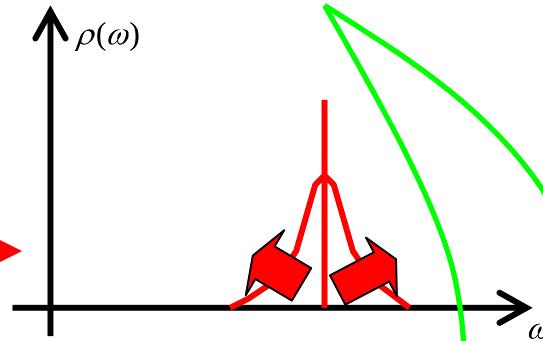
    $\Delta m \cong 300\ \text{MeV}$

Summary & Discussion

SU(3) lattice QCD result of glueball correlator at T>0.



The Breit-Wigner ansatz



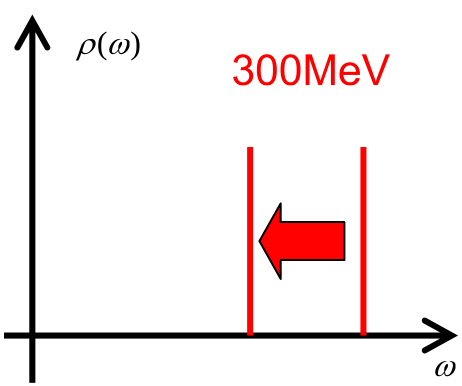
Thermal width broadening

$$\Gamma(T_C) \approx 300 \text{ MeV}$$

Modest reduction of peak center

$$\Delta\omega_0 \approx 100 \text{ MeV}$$

Narrow peak ansatz



Polemass reduction

Comparison with related lattice QCD results (**quench**)

	Polemass	Thermal width	Peak center	Screening mass
glueball	○	○	△	○
light $q\bar{q}$	×	?	?	×
heavy $q\bar{q}$	×	×	×	×

○ **significant**
△ **modest**
× **unchanged**

Ref)
- •QCD-TARO Collab. PRD 63 (2001) 054501.
- •T.Umeda et al., Int.J.Mod.Phys. A16 (2001) 2215.
- •E.Laermann et al., Eur.Phys.J. C20 (2001) 541.
- •S.Datta et al., Nucl. Phys. B534 (1998) 392.

In the confinement phase, the glueball is more sensitive to the thermal effects than the ordinary $q\bar{q}$ mesons.