Infrared Properties of Landau Gauge QCD *

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1. Introduction

- The Gribov's theory

  Gribov proposed that the integration range of the gauge field should be restricted to the region where its Faddeev-Popov determinants are poitive and that the restriction will cancel the infrared singularity of the gluon propagator and results in the singularity of the ghost propagator which makes the linear potential between coloured sources, and the colour confinement will be realized.

- The Kugo-Ojima's theory

  The Kugo and Ojima proposed colour confinement criterion based on the BRST(Becchi-Rouet-Stora-Tyutin) symmetry without Gribov's problem taken into account.

- Kugo-Ojima two point function

$$(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) u^{ab}(p^2) =$$
$$\frac{1}{V}\sum_{x,y} e^{-ip(x-y)} \langle \mathrm{tr}\left(\lambda^{a\dagger} D_\mu \frac{1}{-\partial D}[A_\nu, \lambda^b]\right)_{xy} \rangle \qquad (1)$$

$u^{ab}(0) = \delta^{ab} u(0)$ and $1 + u(0) = \frac{Z_1}{Z_3} = \frac{1}{\tilde{Z}_3}$

$Z_3, \tilde{Z}_3$: gluon- and ghost- wave function renormalization factor,

Z_1 : gluon vertex renormalization factor

- Kugo expressed the ghost propagator $G(p^2)$ in the infrared asymptotic region as

$$G(q^2) \sim \frac{1}{q^2(1 + u(0) + O(q^2))}$$

- The Zwanziger's theory

- Fundamental modular region Λ.

  The region specified by the **absolute minimum** along the gauge orbits, i.e.,

$$\Lambda = \{A|\ A = A(U),\ F_U(1) = \mathrm{Min}_g F_U(g)\}, \qquad \Lambda \subset \Omega \quad , \tag{2}$$

  where Ω is called as **Gribov region** (local minimum), and

$$\Omega = \{A| - \partial D(A) \geq 0\ ,\ \partial A = 0\}. \tag{3}$$

- Core region Ξ.

$$\Xi_L = \{U|\ I(U) = \mathrm{Min}_{h(\text{periodic}),\theta\in C} J(U^h, \theta)\}, \qquad \Xi_L = \Lambda_L^\infty \tag{4}$$

- The gluon Green function is infrared vanishing.

- The lattice Landau gauge formulation of QCD:

  The gauge invariant action is given in terms of link variables, $U_{x,\mu} \in SU(3)$ as usual. We define gauge field $A_\mu(U)$ and describe the lattice Landau gauge in terms of two possible options of $A_\mu(U)$, $U-$linear (Zwanziger's) and log U (ours) type.

- The definition $A_\mu(U)$ (anti-hermitian):

  1. U-linear type

  $$A_{x,\mu} = \frac{1}{2}(U_{x,\mu} - U^\dagger_{x,\mu})|_{traceless\ part},$$

  2. $\log U$ type(exponential map)

  $$U_{x,\mu} = \exp A_{x,\mu}, \quad A^\dagger_{x,\mu} = -A_{x,\mu}, \quad |\text{eigenvalues} \quad \text{of} A_{x,\mu}| \leq 4\pi/3$$

- The Landau gauge, $\partial A = 0$ and the optimizing functions $F_U(g)$ of g:

  In both cases the Landau gauge, $\partial A^g = 0$, can be characterized such that $\delta F_U(g) = 0$ for any δg.

  1. U-linear definition;

  $$F_U(g) = \sum_{x,\mu} \left(1 - \frac{1}{3}\text{Re tr}U^g_{x,\mu}\right).$$

  2. $\log U$ definition;

  $$F_U(g) = ||A^g||^2 = \sum_{x,\mu} \text{tr}\left({A^g}_{x,\mu}{}^\dagger A^g_{x,\mu}\right),$$

- In both options of the gauge field definition, the Landau gauge is formulated as a local extremum of the optimizing function, $F_U(g)$, and the variation of the optimizing function, $\Delta F_U(g)$, under infinitesimal gauge transformation $g^{-1}\delta g = \epsilon$, reads as

$$\Delta F_U(g) = -2\langle \partial A^g | \epsilon \rangle + \langle \epsilon | - \partial D(A^g) | \epsilon \rangle + \cdots, \tag{5}$$

where $D_\mu(A)$ denotes the covariant derivative in each definition;

- in U-linear version,

$$D_\mu(U_{x,\mu})\phi = \frac{1}{2}\left\{\frac{U_{x,\mu}+U^\dagger_{x,\mu}}{2}, \partial_\mu\phi\right\}\Bigg|_{traceless\ part} + [A_{x,\mu}, \bar{\phi}], \quad (6)$$

$$\partial_\mu\phi = \phi(x+\mu) - \phi(x), \quad \bar{\phi} = \frac{1}{2}\left(\phi(x+\mu) + \phi(x)\right), \quad (7)$$

- in $A = \log U$ version,

$$D_\mu(A_{x,\mu})\phi = \mathcal{S}(\mathcal{A}_{x,\mu})\partial_\mu\phi + [A_{x,\mu}, \bar{\phi}], \quad (8)$$

$\mathcal{A}_{x,\mu} = adj_{A_{x,\mu}} = [A_{x,\mu}, \cdot]$, and $\mathcal{S}(x) = \dfrac{x/2}{\mathrm{th}(x/2)}$.

- BRST symmetry and Gribov copy (Neuberger)

  BRST invariant physical quantity

$$\frac{\int \prod_{Links} dU_{ij} O[U] e^{-S[U]}}{\int \prod_{Links} dU_{ij} e^{-S[U]}} \tag{9}$$

  will become indefinite of 0/0 type due to cancellation of the contributions from Gribov copies.

- The Topological Lattice Field Theory(Shaden)

  The partition function $\mathcal{Z}[U]$ is proportional to the Euler characteristic $\mathcal{A}/\mathcal{G}$ of the manifold, which does not necessarily vanish. Need lattice with the proper boundary condition.

2. The gauge fixing on the lattice

- Mandula-Ogilvie's prescription: physical quantities are measured after gauge fixing non-gauge fixed Boltzman samples.

  (1) $\log U$ version
  $0 = \partial A^g, g = e^\epsilon$ Newton method: We solve $0 = \partial A + \partial \mathcal{D}\epsilon$ treating A as a perturbation. (η is an overrelaxation factor)

$$\epsilon = \eta \sum_{n=0}^{N_{end}} ((-\partial^2)^{-1}\partial\tilde{D})^n(-\partial^2)^{-1}\partial A, \quad \tilde{\mathcal{D}} = \mathcal{D} - \partial.$$

  (2) U-linear version
  We operate local exact solution on even-odd site.

- Smearing gauge fixing

  Via smearing we pick up the unique gauge field and perform the Landau gauge fixing. Simultaneously the same gauge transformation is applied to the original gauge field and then Landau gauge fixing is performed.

  We observe that the unique gauge structure after the smearing is maintained by the same gauge transformation applied on the original in most cases. But the unique gauge does not necessarily have the minimum norm.

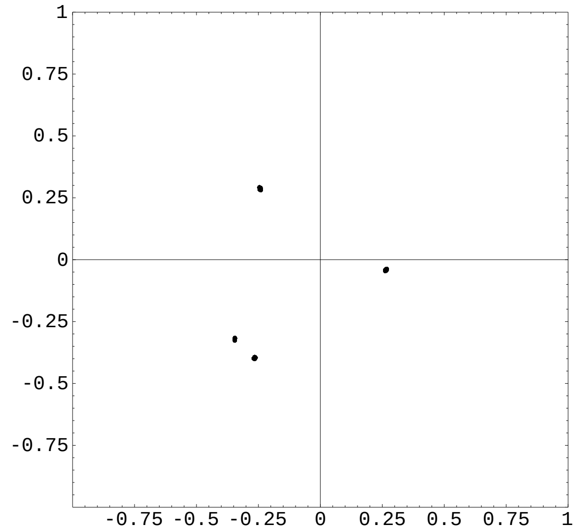


Fig.1: The Polyakov loop along the $x, y, z-$ and t axis after smearing gauge fixing in the accuracy of 10^{-7}. $\beta = 6.0.$ 16^4

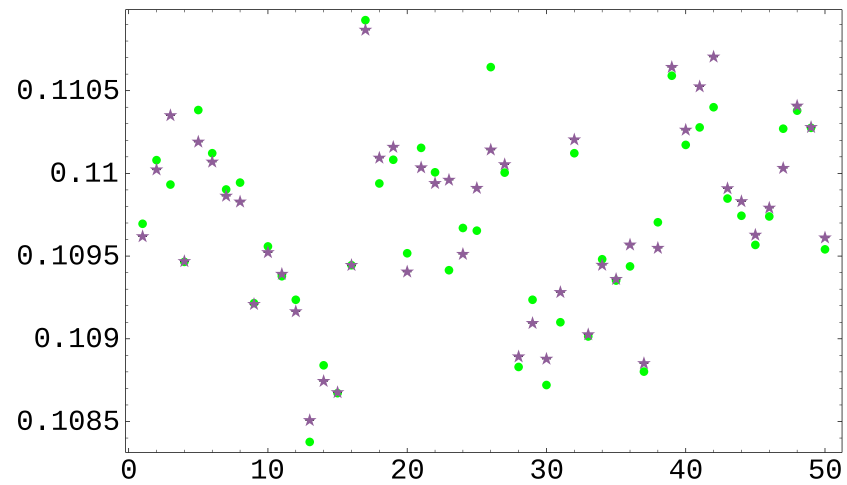


Fig. 2 : The norm of the gauge field $\|A_\mu\|^2$ before(circle) and after(star) the smearing gauge fixing in the accuracy of 10^{-7}. $\beta = 6.0.$ 16^4.

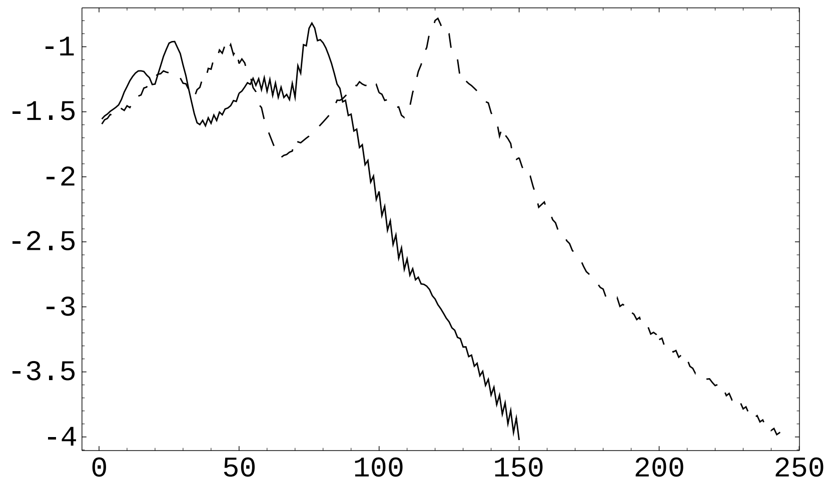


Fig. 3 : The logs of the divergence norm $\|\partial_\mu A_\mu\|$ in the gauge fixing process. In the Green's function method(full line) and in the conjugate gradient method(dashed line). $8^3 \times 16$

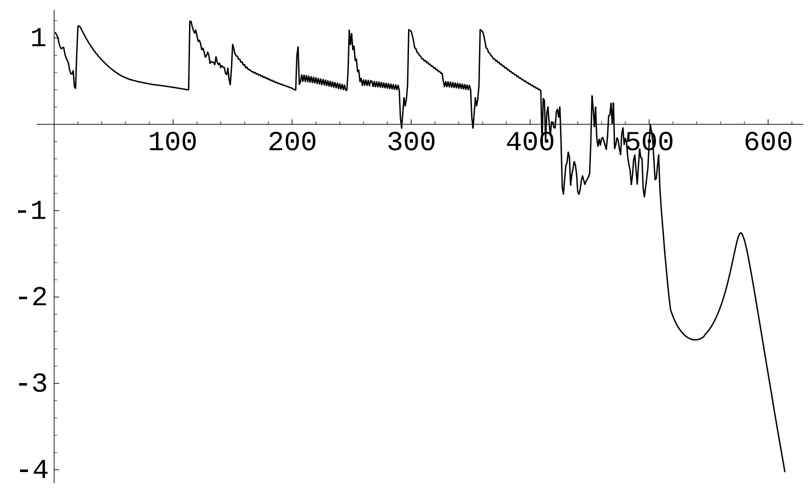


Fig. 4 : The logs of the divergence norm $\|\partial_\mu A_\mu\|$ in the gauge fixing process. In the Multi Grid method. 24^4

- Zero mode of the Faddeev-Popov determinant

$$g = e^{\omega} e^{\theta x} = e^{X} \tag{10}$$

where $\theta = \epsilon \eta_\mu$ and η_μ is an element of Cartan sub-algebra that satisfies $[\eta_\mu, \eta_\nu] = 0$. When the gauge field after the gauge transformation is written as

$$A^g = A + A^{(1)} + \cdots \tag{11}$$

$$X = \epsilon X^{(1)} + \cdots = \omega^{(1)} + \theta^{(1)} x + \cdots \tag{12}$$

where, utilizing the covariant derivative $D(A)$ as

$$A^{(1)} = D(A) X^{(1)} \tag{13}$$

Landau Gauge condition becomes

$$\partial(A + A^{(1)}) = 0 \tag{14}$$

$$\begin{aligned}\|A^g\|^2 - \|A\|^2 &= 2tr\langle X\partial A\rangle + tr\langle X^\dagger(-\partial D)X\rangle \\ &+ \frac{1}{n^2-1}tr\langle X[[A,X],\partial X])\\ &+ \frac{1}{4(n^2-1)}tr\langle[\partial X,X][DX,X]\rangle + \cdots \end{aligned} \tag{15}$$

$$X = \epsilon X_1 + \epsilon^2 X_2 + \cdots \tag{16}$$

where X_1 is zero-mode of Faddev-Popov determinant

$$-\partial D\epsilon X_1 = -\partial D(\omega_{(1)} + \theta^{(1)}x) = 0 \tag{17}$$

In the case of SU(2)

$$\|A^g\|^2 - \|A\|^2 = \epsilon^4(3\langle X_2, -\partial D X_2\rangle - \frac{1}{12}\langle[DX_1,X_1],[\partial X_1,X_1]\rangle) + \cdots . \tag{18}$$

- Zwanziger's Horizon tensor $H_{\mu\nu}$

Zwanziger defind a tensor whose trace is negative when the configuration is in the core region.(global minimum with extended periodic condition)

$$-\partial D\epsilon X_1 = -\partial D(\omega_{(1)} + \theta^{(1)}x) = 0 \tag{19}$$

defines, when the complexity from the boundary condtion is ignored

$$-\partial D\omega_{(1)} = \partial D_\mu \theta^{(1)} x = B\eta^{(1)}\epsilon \tag{20}$$

$$\omega_{(1)} = \frac{1}{-\partial D} B\eta^{(1)}\epsilon \tag{21}$$

The successive variation of the optimizing function in second order with respect to ω and η leads to non-zero results of the horizon function.

$$
\begin{aligned}
\langle {X_1}^\dagger(-\partial D)X_1\rangle &\simeq -(\langle B\eta^\dagger \frac{1}{-\partial D} B\eta\rangle - \langle \eta^\dagger \mathcal{S}(A_\mu^\omega)\eta\rangle) \\
&= -\langle \eta_\mu H_{\mu\nu}\eta_\nu\rangle \qquad (22)
\end{aligned}
$$

- The statistical average of the first term is called Zwanziger's average tensor.

$$
G^{ab}_{\mu\nu xy} = tr(\lambda^{a\dagger} D_\mu \frac{1}{-\partial D}(-D_\nu)\lambda^b)_{xy} \qquad (23)
$$

$$
G_{\mu\nu}(p) = \frac{1}{V}\sum_{x,y} e^{-ip(x-y)} G_{\mu\nu xy}
$$

- Zwanziger's average tensor

$$G_{\mu\nu}(p)\delta^{ab} = \left(\frac{e}{d}\right)\frac{p_\mu p_\nu}{p^2}\delta^{ab} - \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)u^{ab}, \tag{24}$$

where

$$e = \langle E(U)\rangle / V.$$

- in U-linear version,

$$E(U) = \sum_l \frac{1}{N}\mathrm{Re}\ \mathrm{tr} U_l, \tag{25}$$

- in $A = \log U$ version,

$$E(U) = \frac{1}{N^2-1}\sum_{l,a}\mathrm{tr}\left(\lambda^{a\dagger}\mathcal{S}(\mathcal{A}_l)\lambda^a\right), \tag{26}$$

where $\mathcal{A}_l = adj_{A_l}$. Note that $e = d = 4$ if all link $U_l = 1$.

- Average tensor is related with the horizon function as

$$\frac{\langle H(U)\rangle}{V} = (N^2-1)\left[\lim_{p\to 0} G_{\mu\mu}(p) - e\right]. \tag{27}$$

- Putting Kugo-Ojima parameter as

$$u(0) = -c,$$

one finds that the horizon condition is written as

$$h = \left(\frac{e}{d}\right) + (d-1)c - e = (d-1)\left(c - \frac{e}{d}\right) = 0.$$

- Kugo-Ojima's and Zwanziger's arguments appear to be consistent with each other provided the lattice covariant derivative meets with the continuum one.

3. The gluon propagator

$$
\begin{aligned}
D_{\mu\nu}(q) &= \frac{1}{n^2-1}\sum_{x=\mathbf{x},t} e^{-ikx} Tr\langle A_\mu(x)^\dagger A_\nu(0)\rangle \quad (n=3) \\
&= D_A(q^2)(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) \qquad (28)
\end{aligned}
$$

- The gluon dressing function

$$
Z(q^2) = q^2 D_A(q^2) \qquad (29)
$$

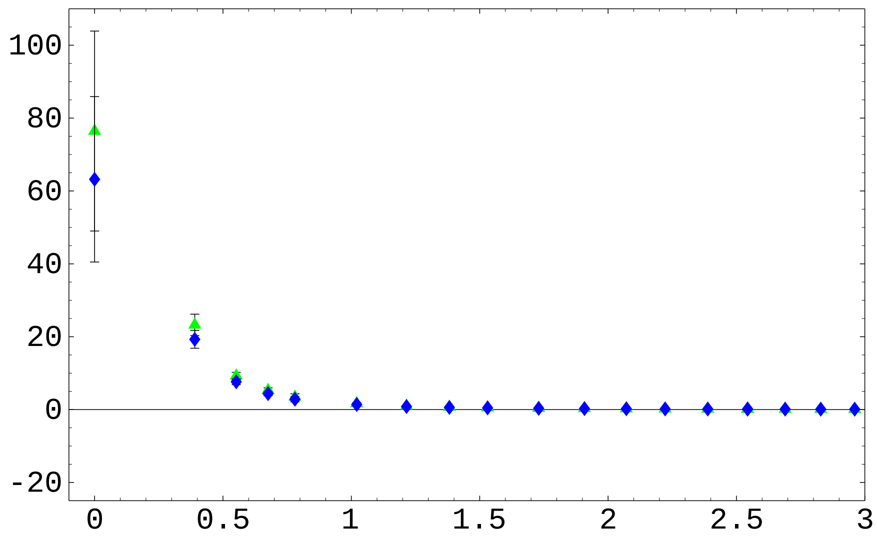


Fig.5 : The gluon propagator as the function of the lattice momentum qa. $\beta = 6.0$, 16^4 in U-linear (diamond) and $\log U$ version (triangle). 50 samples.

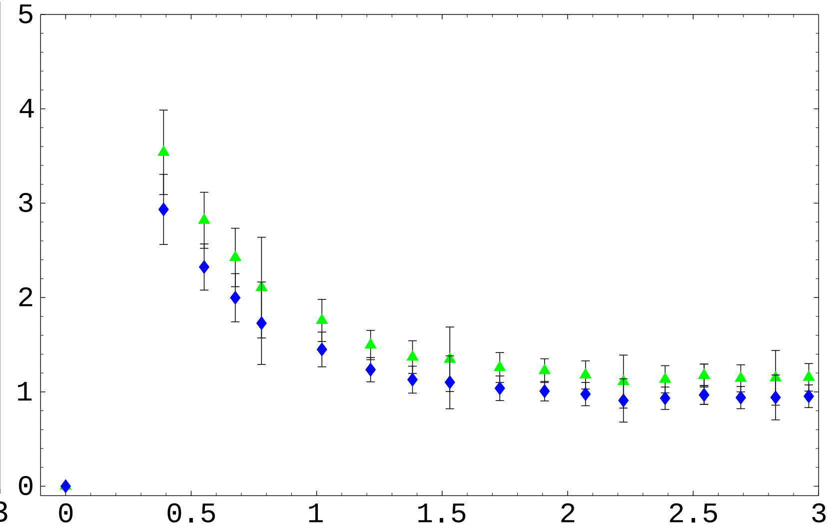
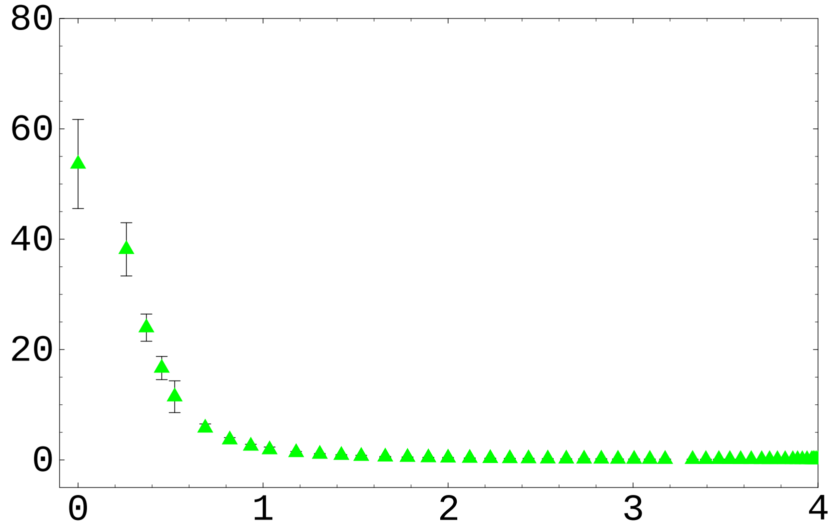


Fig.6 : The gluon dressing function Z as a function of the lattice momentum qa. $\beta = 6.0$, 16^4 in U-linear (diamond) and $\log U$ version (triangle). 50 samples.



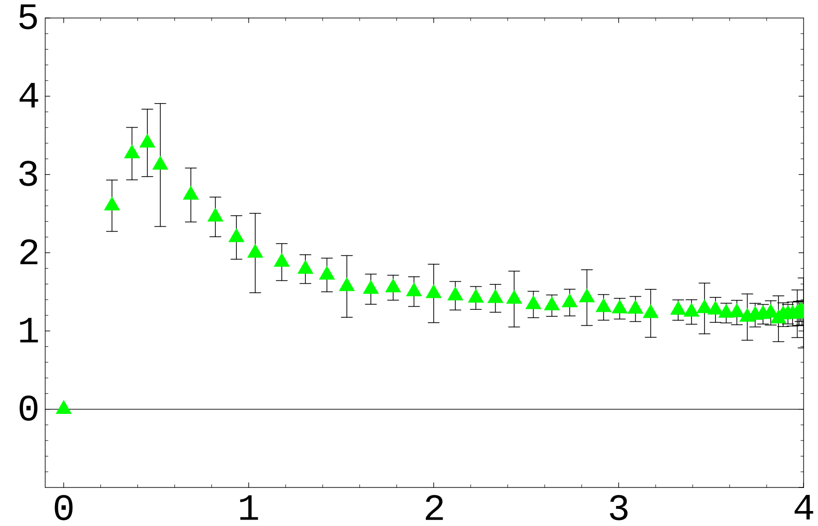
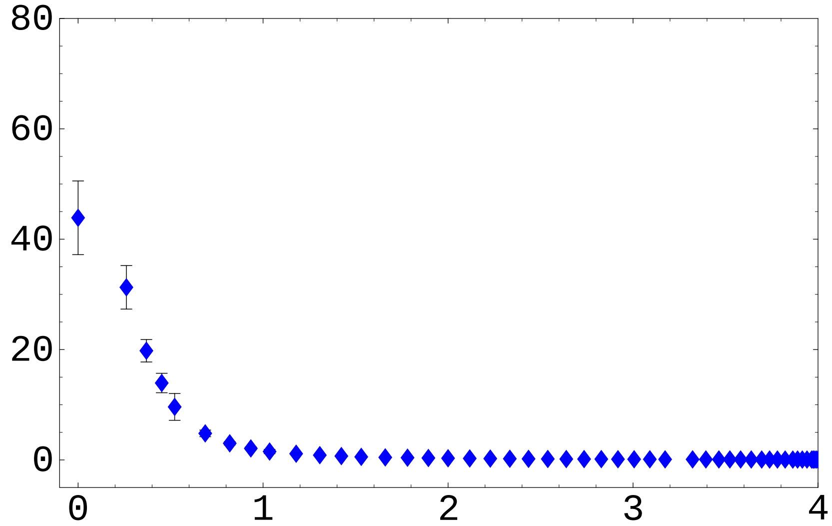


Fig.7 : The gluon propagator as a function of the lattice momentum qa. $\beta = 6.0$, 24^4 in $\log U$ version. 50 samples.

Fig.8 : The gluon dressing function Z as a function of the lattice momentum qa. $\beta = 6.0$, 24^4 in $\log U$ version. 50 samples.



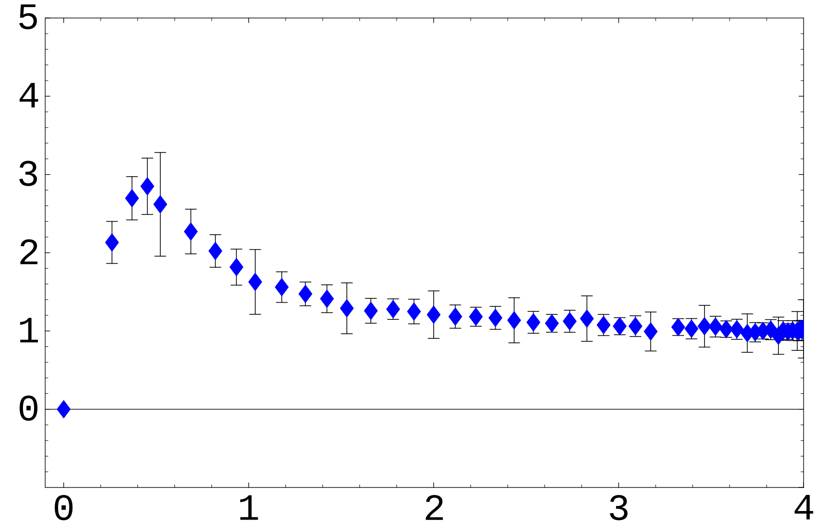


Fig.9 : The gluon propagator as a function of the lattice momentum qa. $\beta = 6.0$, 24^4 in $U-$linear version. 50 samples.

Fig.10 : The gluon dressing function Z as a function of the lattice momentum qa. $\beta = 6.0$, 24^4 in $U-$linear version. 50 samples.

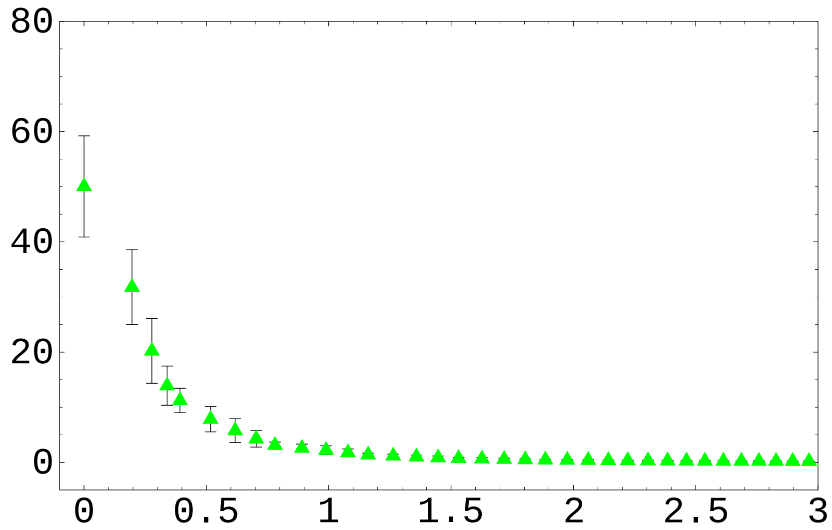


Fig.11 : The gluon propagator of $\beta = 6.0$, 32^4 in $\log U$ version(triangle).

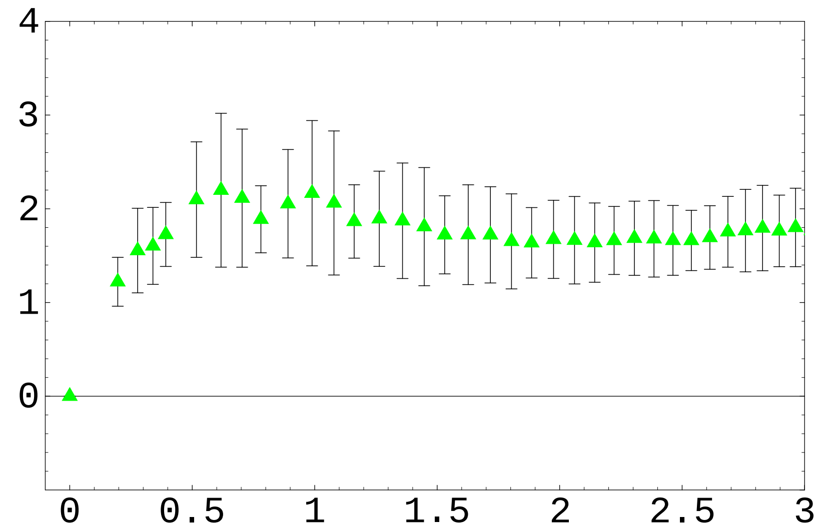


Fig.12 : The gluon dressing function Z as a function of the lattice momentum qa. $\beta = 6.0$, 32^4 in $\log U$ version(triangle).

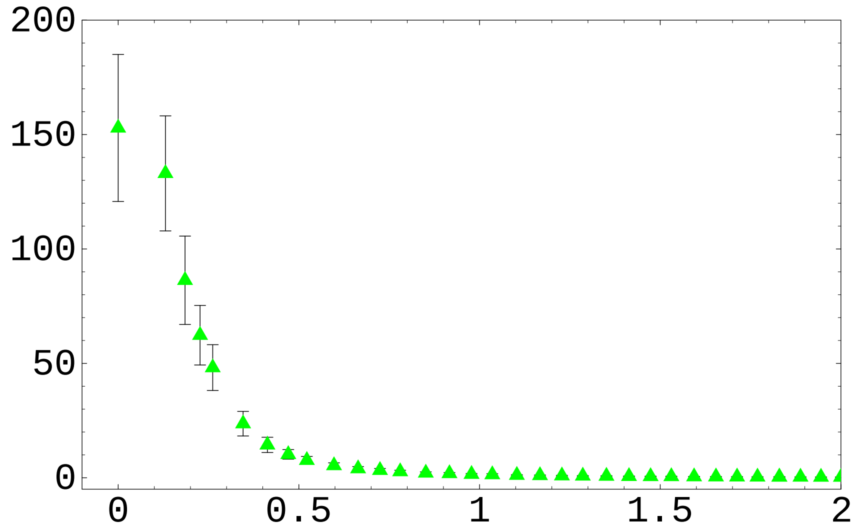


Fig.13 : The gluon propagator of $\beta = 6.4$, 48^4 in $\log U$ version.

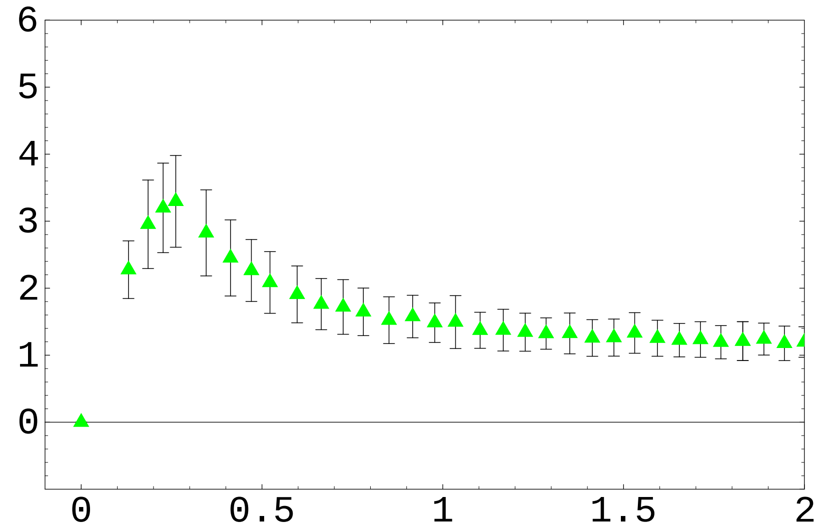
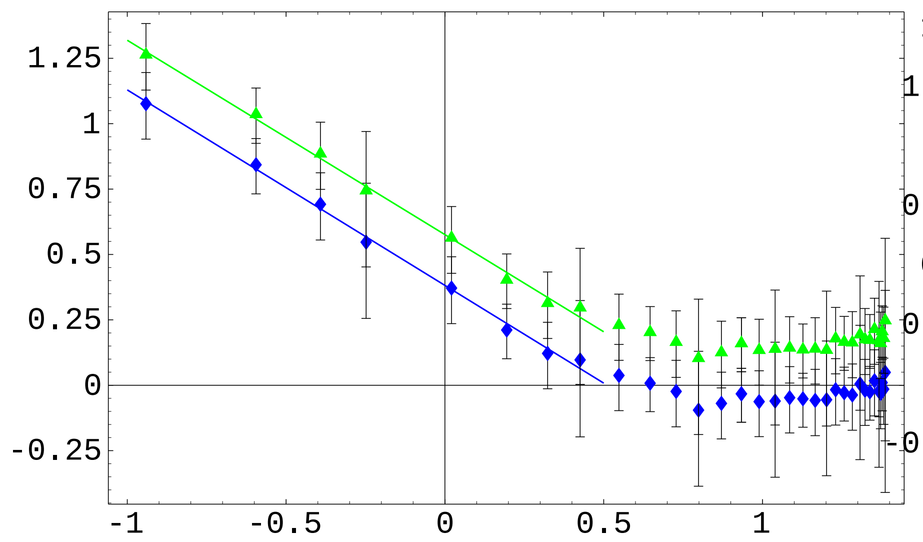


Fig.14 : The gluon dressing function Z as a function of the lattice momentum qa. $\beta = 6.4$, 48^4 in $\log U$ version.



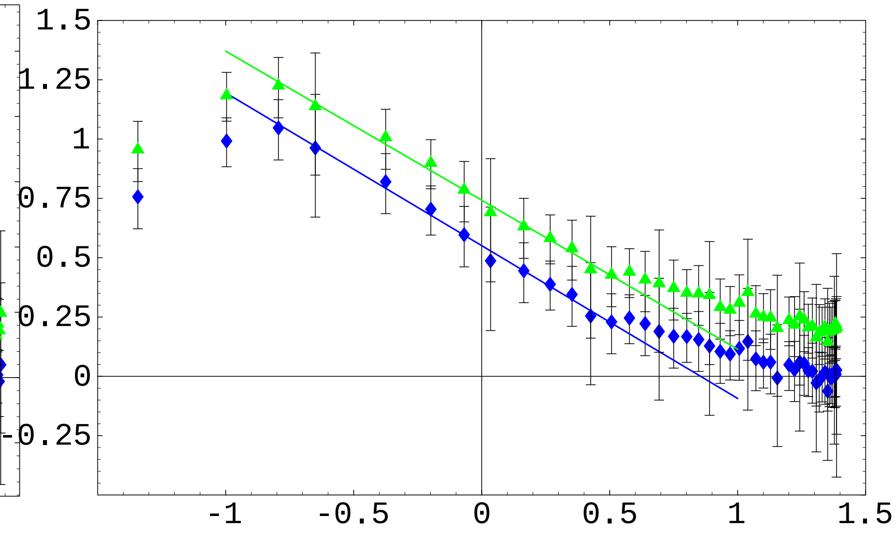


Fig.15 : The log of the gluon dressing function $\log Z$ as a function of $\log(qa)$. $\beta = 6.0$, 16^4 in $\log U$ version(triangle) and U-linear version(diamond). 50 samples.

Fig.16 : The log of the gluon dressing function $\log Z$ as a function of $\log(qa)$. $\beta = 6.0$, 24^4 in $\log U$ version(triangle) and U-linear version(diamond). 50 samples.

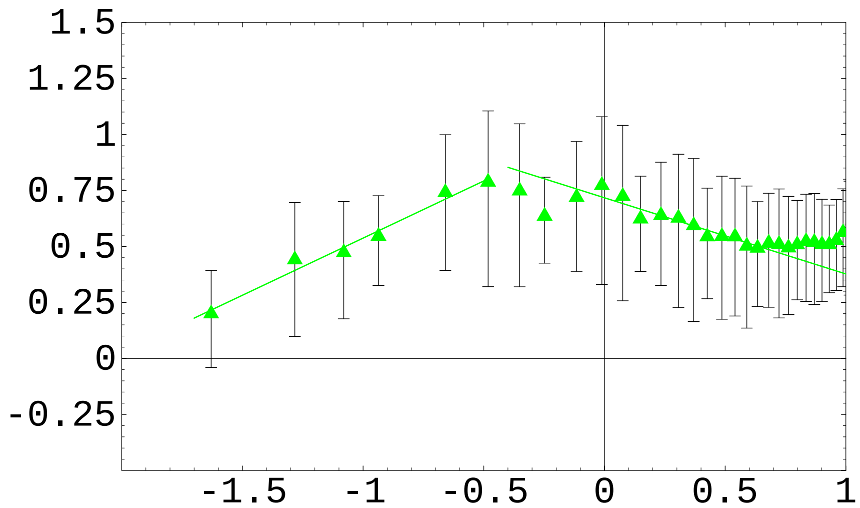


Fig.17 : The log of the gluon dressing function as a function of $\log qa$. $\beta = 6.0$, 32^4 in $\log U$ version. 10 samples.

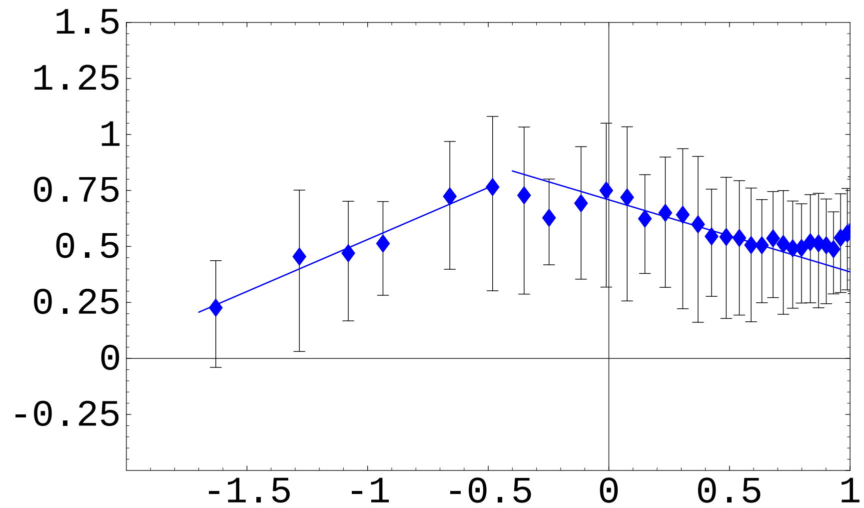


Fig.18 : The log of the gluon dressing function $\log Z$ as a function of $\log qa$. $\beta = 6.0$, 32^4 in U-linear version. 9 samples.

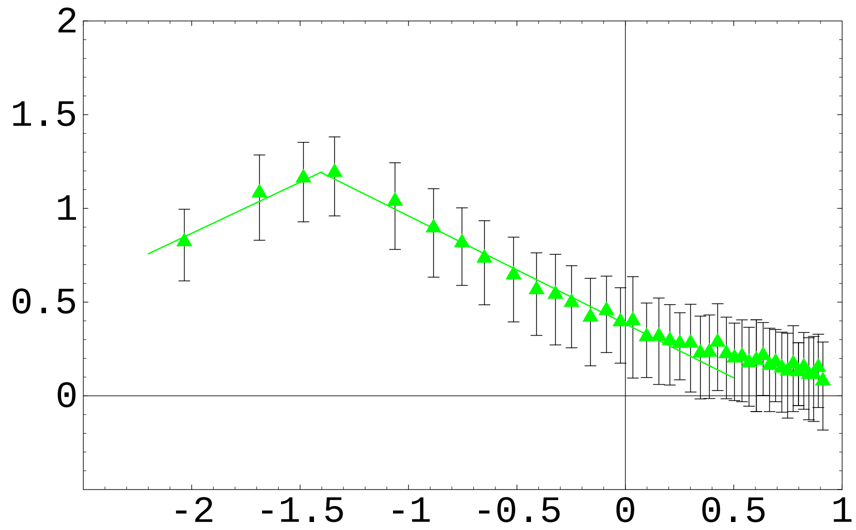


Fig.19 : The log of the gluon dressing function $\log Z$ as a function of $\log qa$. $\beta = 6.4$, 48^4 in $\log U$ version.

- Infrared suppression of the gluon propagator

  Its singularity can be parametrised as $p^{-2(1+\alpha_D)}$, where $p^2=\sum_{k\mu}(4\sin^2\frac{\pi k_\mu}{L})$, $(-L/2<k_\mu\leq L/2)$. $\quad \alpha_D<0$ for $L\geq 24$.

- The norm of the gauge field

  As a kinematical structure of lattice Landau gauge QCD with Gribov problem taken into account, Zwanziger gave a bound on the constant mode of a gauge field $A\in\Omega$,

  $\bar{A}_\mu=\frac{1}{V}\sum_x A_{x,\mu}$ as

  $$|\bar{A}_\mu|\leq 2\tan(\frac{\pi}{L})\frac{1}{\alpha}\equiv\sigma(L), \tag{30}$$

  where L is a lattice size such that $V=L^4$, and $\alpha=\sqrt{\frac{3}{2}}$ in $SU(3)$ case and $\alpha=\sqrt{2}$ in $SU(2)$ case, respectively. This fact can be proven irrespective of the gauge field options.

- The Zwanziger's condition on the norm of gauge field in $\beta = 6.4$ $L = 48$ is satisfied. For each sample $|\bar{A}_\mu| \leq 0.005$.

$$2\tan(\pi/L)\frac{1}{\alpha} = 0.10 \quad (L = 48) \tag{31}$$

- In the gluon propagator factor V needs to be multiplied.

  The gauge field $\sqrt{V}\langle\bar{A}_\mu\rangle = 0.2 \pm 4.2$.

  $\frac{3}{4}V\langle\sum_{\mu,a}\bar{A}_\mu{}^{a2}\rangle \simeq 150 \pm 30$ (averaged over samples).

  The connected part at 0 momentum is large.

  $\frac{3}{4}V\sum_\mu\langle\bar{A}_\mu\rangle^2 = 0.16 \pm 31$

- There is a strong cancellation between samples.

- Connected Green function

It is useful to prove that all the connected n-point Green functions are vanishing at zero momentum in the infinite volume limit. Let a partition function be

$$\mathcal{Z}(J) = \int dA \rho(A) e^{J \cdot A}, \tag{32}$$

and for constant $J_x = h$, it reads that

$$\mathcal{Z}(h) = \int dA \rho(A) e^{h \cdot \tilde{A}(0)} = \int dA \rho(A) e^{V h \cdot \bar{A}}. \tag{33}$$

$$\tilde{A}_\mu(0) = a^d \sum_x A_\mu(x). \tag{34}$$

Let $W(h) = \log \mathcal{Z}(h)$, and then it follows that

$$V G_c^{(n)}(0, ..., 0) = \langle (\tilde{A}(0))^n \rangle|_c \tag{35}$$

and it holds that

$$G_c^{(n)}(0, ..., 0) = \frac{1}{V} \frac{\partial^n}{\partial h^n} W(h). \tag{36}$$

Note that

$$e^{-Vh\sigma(L)} \leq \mathcal{Z}(h) \leq e^{Vh\sigma(L)}, \tag{37}$$

Putting $w(h) \equiv \dfrac{W(h)}{V}$, we obtain

$$|w(h)| \leq h\sigma(L) \to 0 \tag{38}$$

as $L \to \infty$. Hence it proves that $G_c^{(n)}(0, ..., 0) \to 0$ in the infinite volume limit. However, it should be noted that this infinite volume limit is only proved directly for connected Green functions.

4. The ghost propagator

- In use of colour source $|\lambda^a x\rangle$ normalised as $Tr\langle\lambda^a x|\lambda^b x_0\rangle = \delta^{ab}\delta_{x,x_0}$, the ghost propagator is given by

$$D_G^{ab}(x,y) = \langle \mathrm{tr}\langle\lambda^a x|(\mathcal{M}[U])^{-1}|\lambda^b y\rangle\rangle \tag{39}$$

where the outmost $\langle\rangle$ denotes average over samples U.

$$D_G{}^{ab}(q^2) = \frac{1}{V}\sum_{x=\boldsymbol{x},t} e^{-iqx} D_G{}^{ab}(x,0) \tag{40}$$

- The ghost dressing function

$$G^{ab}(q^2) = q^2 D_G{}^{ab}(q^2) \tag{41}$$

- The inverse of the Faddeev-Popov matrix, $\mathcal{M}^{-1}[U] = (M_0 - M_1[U])^{-1}$, is calculated perturbatively by using the Green function of the Poisson equation $M_0^{-1} = (-\partial^2)^{-1}$ and $M_1 = \partial_\mu (D - \partial)_\mu$, as

$$\mathcal{M}^{-1} = M_0^{-1} + \sum_{k=0}^{N_{end}} (M_0^{-1} M_1)^k M_0^{-1}. \qquad (42)$$

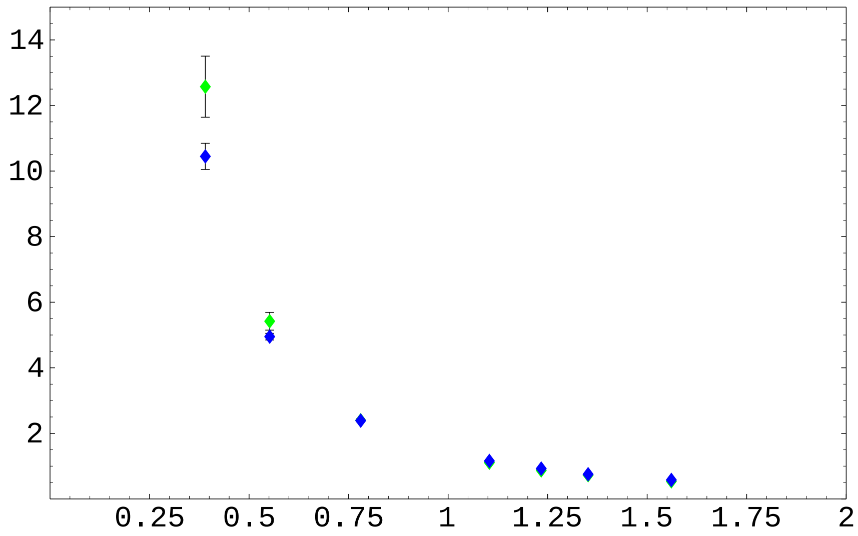


Fig.20 : The ghost propagator as a function of the lattice momentum qa. $\beta = 6.0$, 16^4 in U-linear(diamond) and $\log U$(triangle) version .

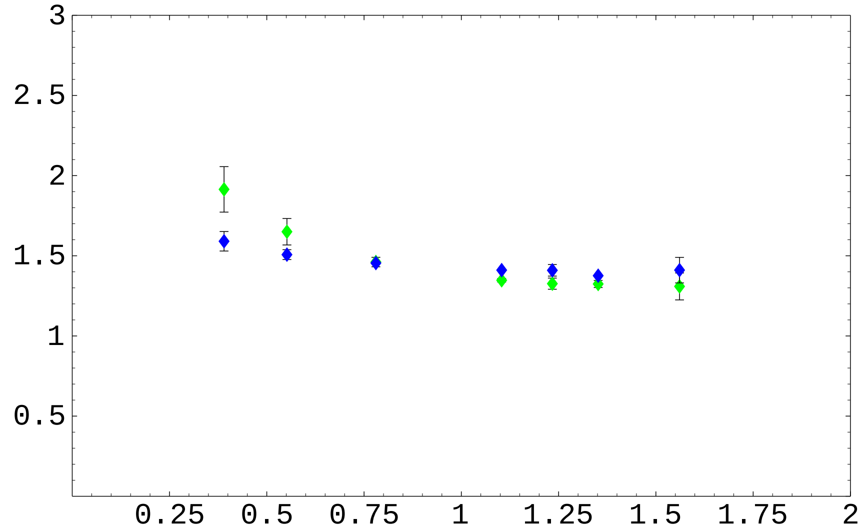


Fig.21 : The ghost dressing function G as a function of the lattice momentum qa. $\beta = 6.0$, 16^4 in U-linear(diamond) and $\log U$ version(triangle).

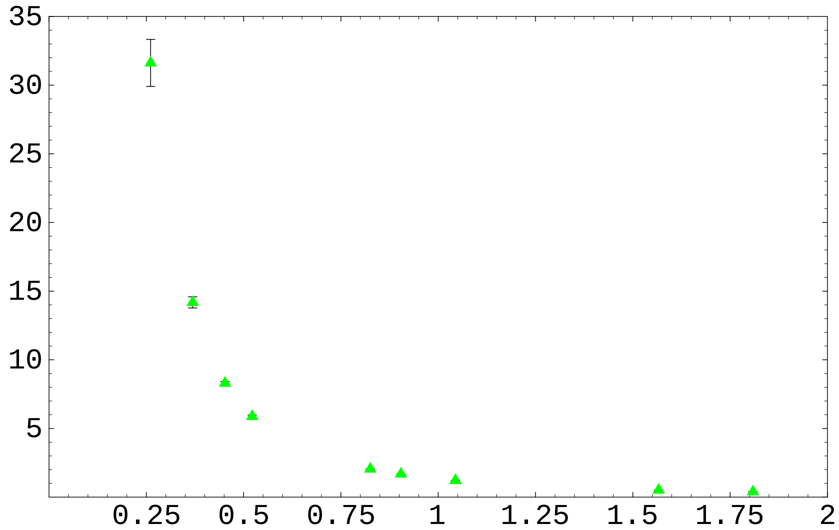


Fig.22 : The ghost propagator as a function of the lattice momentum qa. $\beta = 6.0$, 24^4 in $\log U$ version.

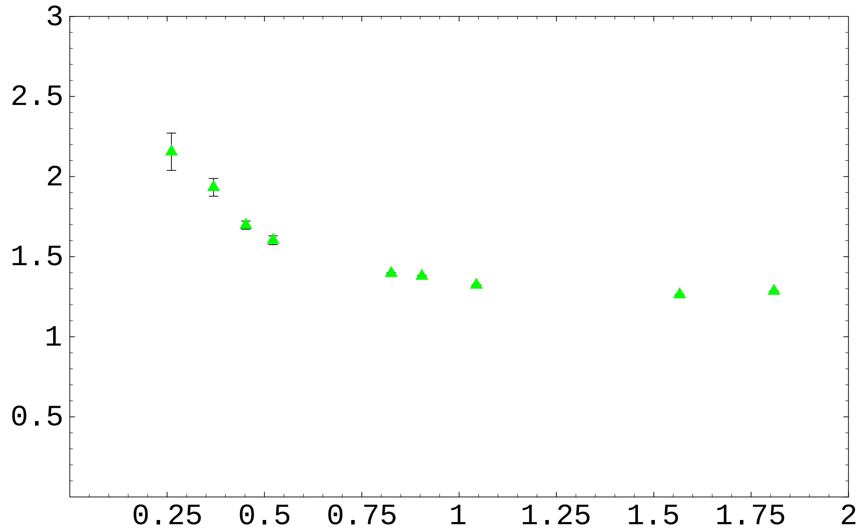


Fig.23 : The ghost dressing function G as a function of the lattice momentum qa. $\beta = 6.0$, 24^4 in $\log U$ version(triangle).

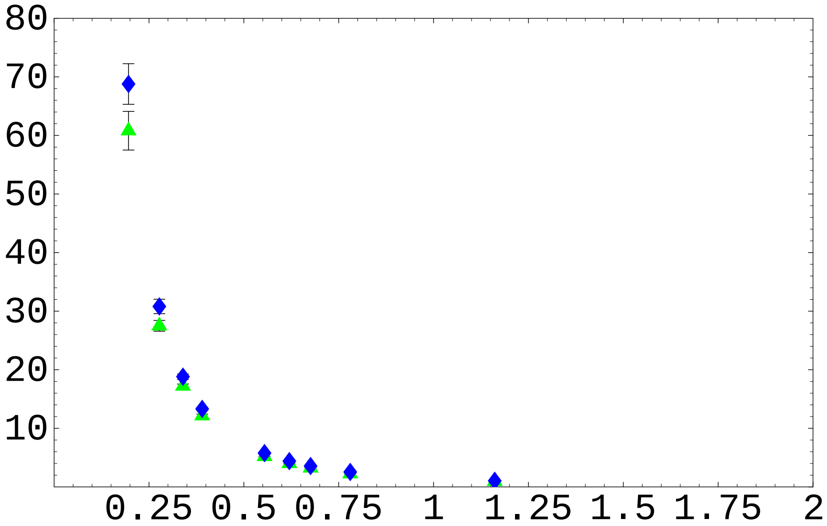


Fig.24 : The ghost propagator of $\beta = 6$, 32^4 in $U-$linear(diamond) and $\log U$(triangle) version.

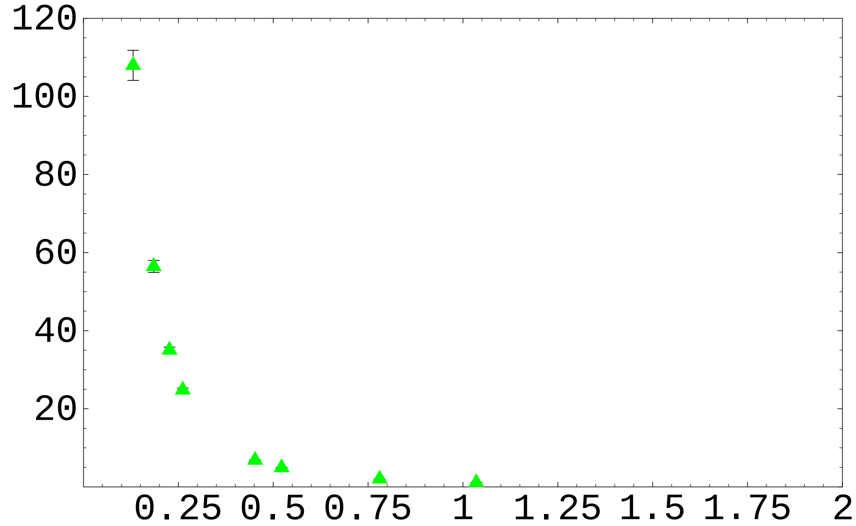


Fig.25 : The ghost propagator of $\beta = 6.4$, 48^4 in $\log U$ version.

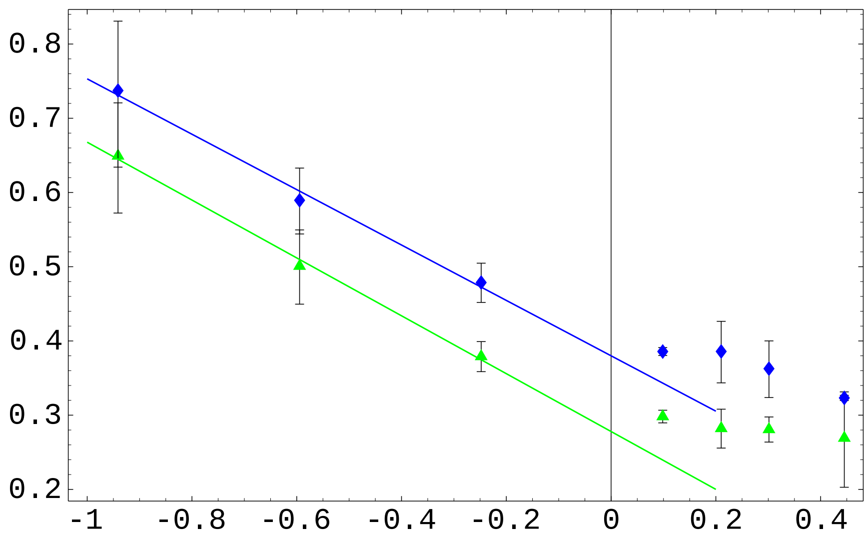


Fig.26 : The log of the ghost dressing function $\log G$ as a function of the log of the lattice momentum $\log(qa)$. $\beta = 6.0$, 16^4 in $\log U$ version(triangle) and in $U-$linear version(circle).

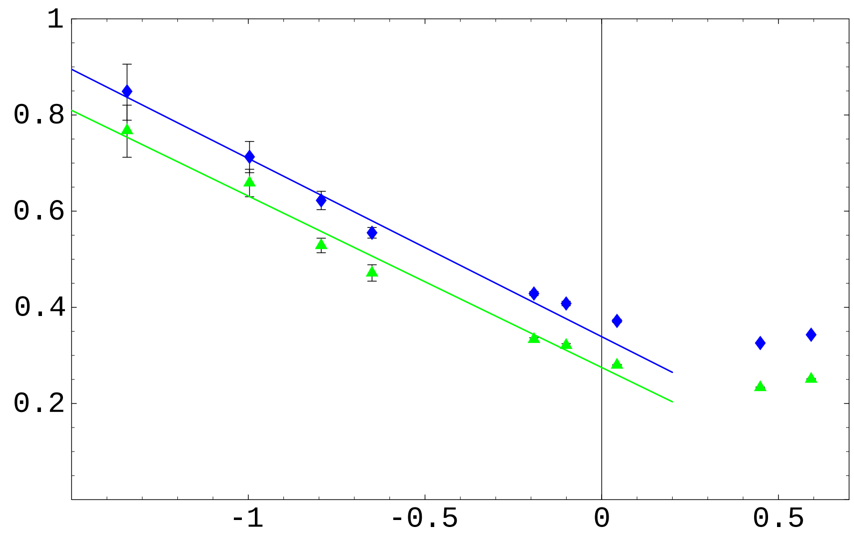
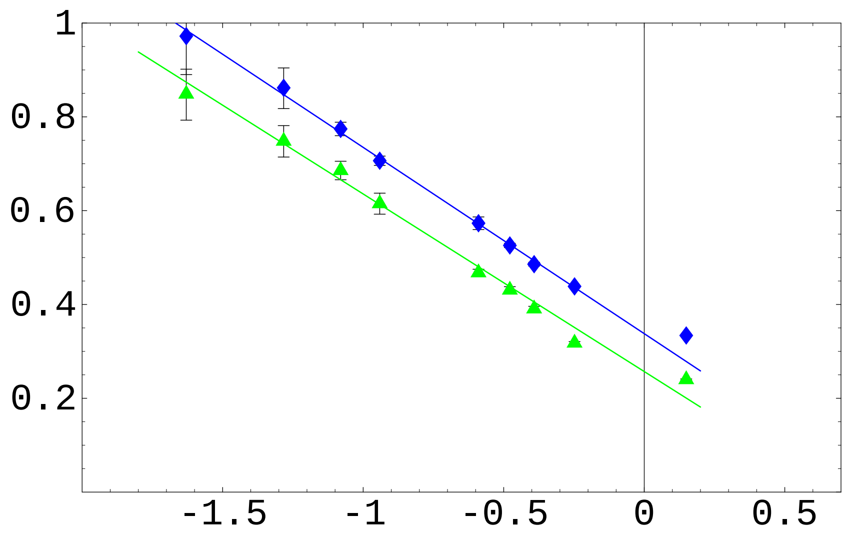


Fig.27 : The log of the ghost dressing function $\log G$ as a function of the log of the lattice momentum $\log(qa)$. $\beta = 6.0$, 24^4 in $\log U$ version(triangle) and in $U-$linear version(diamond).



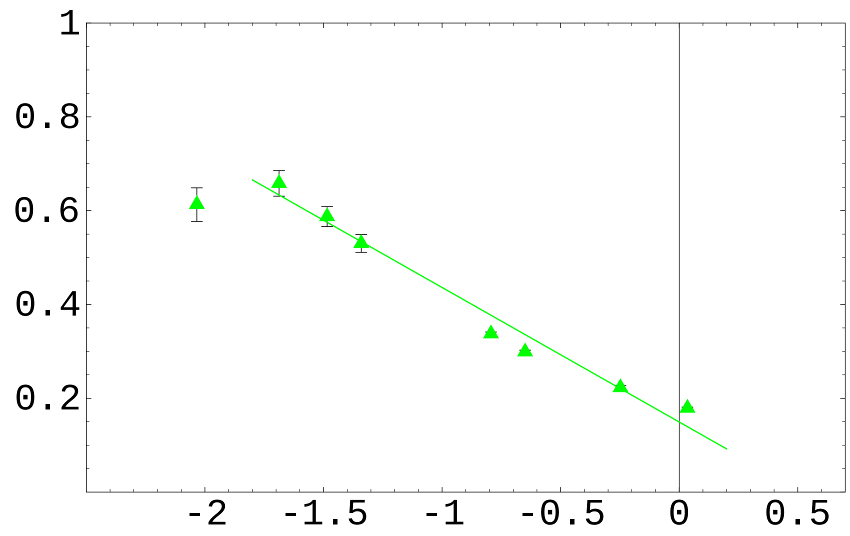


Fig.28 : The log of the ghost dressing function $\log G$ as a function of the log of $\log(qa)$. $\beta = 6.4$, 32^4 in U-linear(diamond) and $\log U$(triangle) version.

Fig.29 : The log of the ghost dressing function $\log G$ as a function of the log of $\log(qa)$. $\beta = 6.4$, 48^4 in $\log U$ version.

- The ghost propagator is infrared divergent and its singularity can be parametrised as $p^{-2(1+\alpha_G)}$, where $p^2 = \sum_{k\mu}(4\sin^2\frac{\pi k_\mu}{L})$, $(-L/2 < k_\mu \leq L/2)$.

- The lowest momentum point of the ghost propagator of $L = 48$ is suppressed. The validity of this point is under investigation.

5. The Kugo-Ojima parameter

L	c_1	e_1/d	h_1	c_2	e_2/d	h_2
16	0.576(79)	0.860(1)	-0.28	0.628(94)	0.943(1)	-0.32
24	0.695(63)	0.861(1)	-0.17	0.774(76)	0.944(1)	-0.17
32	0.706(39)	0.862(1)	-0.15	0.777(46)	0.944(1)	-0.16

Table 1a: Lattice size dependence of the Kugo-Ojima parameter c, the trace e divided by the dimension d and the deviation parameter from the horizon condition h. The suffix 1 corresponds to the u-linear and 2 corresponds to the $\log U$ definition. Data are those of $\beta = 6.0$.

L	c_1	e_1/d	h_1	c_2	e_2/d	h_2
32	0.650(39)	0.883(1)	-0.23	0.700(42)	0.953(1)	-0.25
48				0.720(49)	0.982(1)	-0.26

Table 1b: Same as Table 1a. $\beta = 6.4$

- Kugo-Ojma parameter c and the renormalization scheme.

  We want to find an extrapolation from $\mu = 1/a$ where $\overline{MOM}$ scheme is stable to $\mu = 0$.

- The effective charge method (Grunberg)

  Extension of Gell-Mann-Low renormalization scheme for QED to QCD

$$Z_3(\mu^2, \Lambda^2) = Z_R(\mu^2)/Z_b(\Lambda^2) \tag{43}$$

  Scheme invariant anomalous dimension

$$\gamma_R(\mu^2) = \lim_{M^2 \to \infty} \mu^2 \frac{\partial Z(\mu^2, M^2)}{\partial \mu^2} / Z(\mu^2, M^2) \tag{44}$$

- When in $Z(\mu^2, M^2)$ only constant and logarithmic divergent terms are retained, $\lim_{M^2\to\infty}$ can be omitted and we obtain

$$\gamma_R(\mu^2) = \mu^2 \frac{\partial Z_R}{\partial \mu^2} / Z_R(\mu^2) \tag{45}$$

$$\log Z_R(\frac{k^2}{\Lambda^2}, g, \alpha) = -\int_{\alpha}^{\alpha(k^2)} \frac{\gamma(x) dx}{\beta(x)} \tag{46}$$

$$\log Z_b(\frac{k^2}{\Lambda^2}, g, \alpha(1/a)) = -\int_{\alpha}^{\alpha(k^2)} \frac{\gamma_0 d(1/a)}{\beta_0 \alpha(1/a)} \propto \log(k^2/\Lambda^2) \tag{47}$$

- The $\overline{MOM}$ and $\overline{MS}$ scheme

$$Z_3(\mu^2, \Lambda_{\overline{MOM}}) = Z_R(\mu^2)/Z_b(\Lambda_{\overline{MOM}}) \tag{48}$$

$$Z_3(\mu^2, \Lambda_{\overline{MOM}}) = Z_3(\mu^2, \Lambda_{\overline{MS}})(Z_b(\Lambda_{\overline{MS}})/Z_b(\Lambda_{\overline{MOM}})) \tag{49}$$

When there is an infrared fixed point $Z_3(0, \Lambda_{\overline{MS}}) = 1$, and

$$\frac{Z_b(\Lambda_{\overline{MOM}})}{Z_b(\Lambda_{\overline{MS}})} = \frac{\Lambda_{\overline{MOM}}}{\Lambda_{\overline{MS}}} = 1/0.346 = 2.89 \simeq e \tag{50}$$

$$1 - \log Z_3(0, \Lambda_{\overline{MOM}}) = 0 \tag{51}$$

When the value c is plotted as a function of $\log Z(1/a)$ for different L and for U-linear and $\log U$ definitions of the gauge field, the c in the continuum limit $c = \frac{Z_1}{Z_3} - 1$ will be obtained as a crossing point of the extrapolation of each definition.

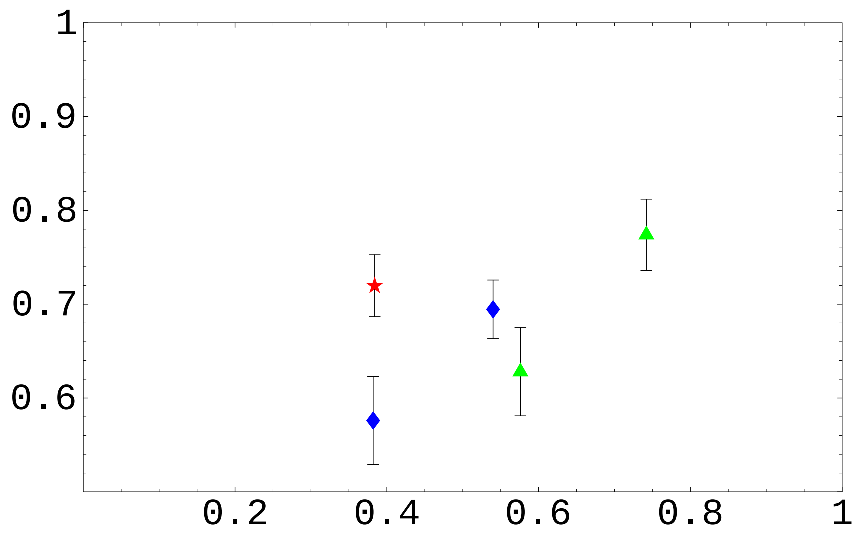


Fig.30 : The Kugo-Ojima parameter c as the function of $\log Z(1/a)$. $\beta = 6.4$, 48^4 in $\log U$(star), $\beta = 6.0$, 16^4 and 24^4 in $\log U$(triangle) and U-linear version(diamond).

6. The QCD running coupling

- The QCD running coupling α_s is a gauge- and scheme-dependent quantity and it defines the scale of the system.

- The three-point functions (three gluon vertex, ghost-gluon-ghost vertex) in $\overline{MOM}$ scheme.

$$g(\mu^2) = \frac{G_A^{(3)}(q_i^2, q_f^2, q_c^2) Z_3^{3/2}(\mu^2)}{D_A(q_i^2) D_A(q_f^2) D_A(q_c^2)}. \tag{52}$$

$$g(\mu^2) = \frac{G_g^{(3)}(q_i^2, q_f^2, q_c^2) Z_3^{1/2}(\mu^2) \tilde{Z}_3(\mu^2)}{D_G(q_i^2) D_G(q_f^2) D_A(q_c^2)}. \tag{53}$$

where $Z_3(\mu^2) = D_A(\mu^2)\mu^2$ and $\tilde{Z}_3(\mu^2) = D_G(\mu^2)\mu^2$.

- The derivation of running coupling in the $\overline{MOM}$ scheme is valid near $qa \simeq 1$ only. In the infrared region α_s from the three point function decreases to 0 due to the loss of correlation of the gauge field.

- The derivation of running coupling from the two point function (gluon- and ghost-dressing functions)

$$\alpha(p^2)_s = \frac{g_0^2}{4\pi} Z(p^2) G(p^2)^2 \simeq (pa)^{-2(\alpha_D + 2\alpha_G)}. \tag{54}$$

  suggests $\alpha_s(0) \simeq 1$

- Dyson-Schwinger approach predits

  $\alpha_G = 0.5953, \alpha_D = -1.1906$ and $\alpha_s(0) = 2.972$ (Fischer,Alkofer,Reinhardt hep-ph/0202195)

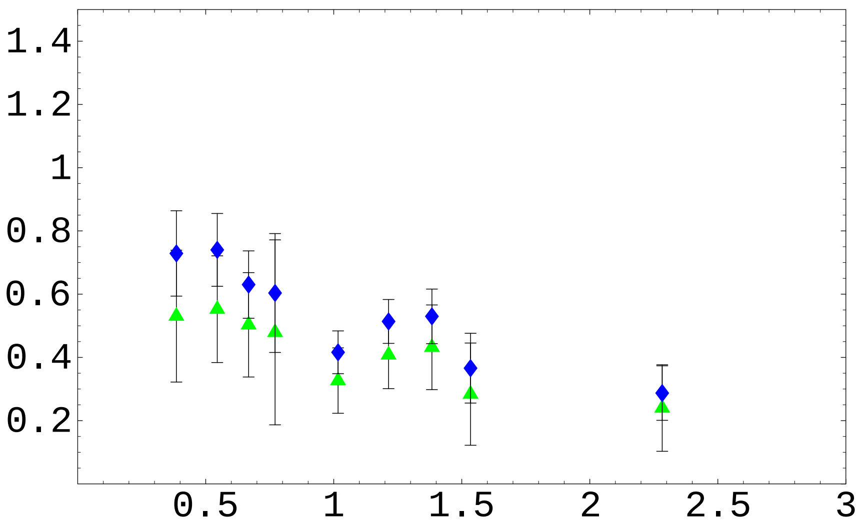


Fig.31: The running coupling $\alpha(\mu)$ of $\beta = 6.0$, 32^4 in $\log U$(triangle) and U-linear version(diamond).

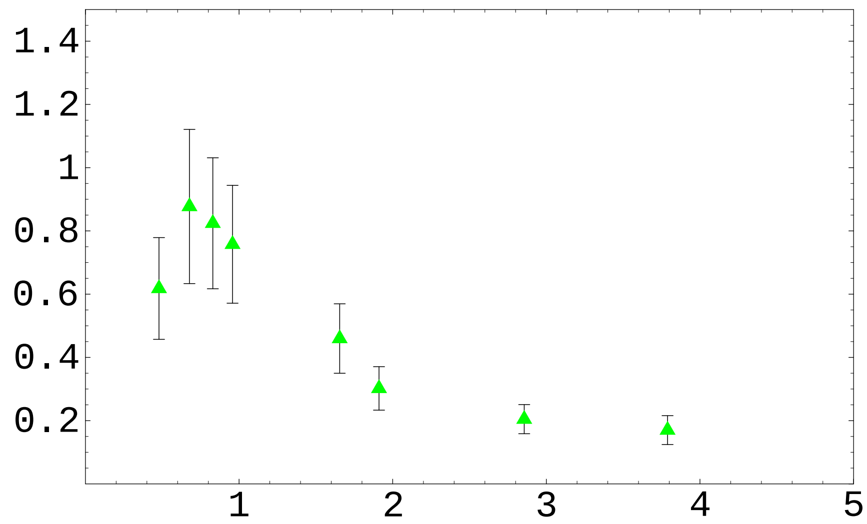


Fig.32: The running coupling $\alpha_s(\mu)$ as a function of the lattice momentum q(GeV). $\beta = 6.4$, 48^4 in $\log U$ version.

- Suppression at the lowest momentum of 48^4 is due to the suppression of the ghost dressing function.

• Extrapolation to high momentum.

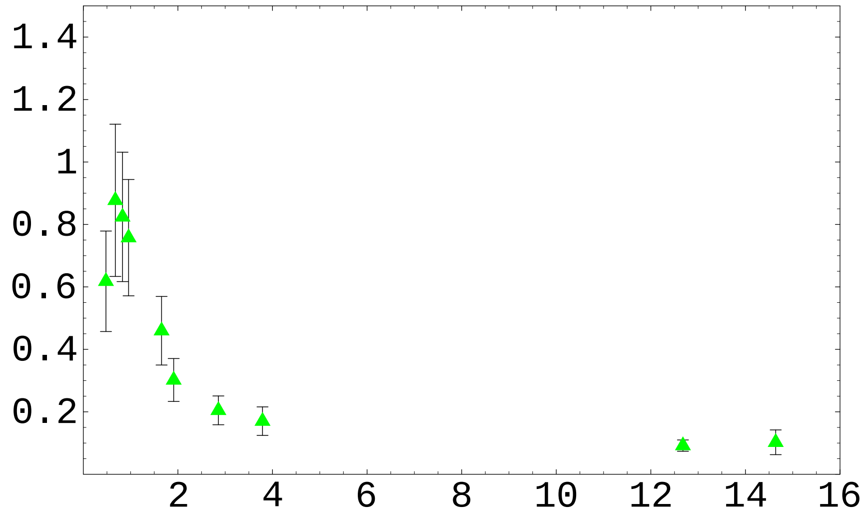


Fig.33: The running coupling $\alpha_s(\mu)$ as a function of the lattice momentum q(GeV). $\beta = 6.4$, 48^4 in $\log U$ version.

• At $q = 14.64 GeV$ $\alpha = 0.102(40)$.

L	α_D	α'_D	α_G	$\alpha_D + 2\alpha_G$
32	-0.232	0.161	0.174	0.12(10)
48	-0.273	0.288	0.109	-0.06(10)

Table 2: The exponent of gluon dressing function near the zero momentum α_D, near the $pa = 1$ α'_D, and the exponent of ghost dressing function near the zero momentum α_G. $\beta = 6.4$ $\log U$ type.

- Derivation of scheme independent running coupling in the Schrödinger functional method. (ALPHA colaboration)

  In this method the infrared coupling constant could be obtained from the Fourier transform of the long-range behaviour of the QCD coupling in the force between static quarks.

  $\alpha_s(0)$ is either a) finite or b) 'slightly singular '$\simeq 1/Q$' (Shirkov)

7. Discussion and Conclusion

- The sample average $\langle \frac{1}{V}\sum_x A_\mu(x)\rangle = 0$ but for each sample $\frac{1}{V}\sum_x A_\mu(x) \neq 0$.

- In the space $\mathcal{A}/\mathcal{G}$, colour symmetry is preserved.

- The QCD running coupling $\alpha_s(0) \simeq 1$.

- Consistency check between $\overline{MOM}$-scheme and the Schrödinger functional scheme is necessary.

- The decrease of the gluon dressing function and the gluon three point function as the momentum decreases suggests that the correlation of the gluon field is lost in the infrared region.

- The Polyakov loop after smearing gauge fixing does not converge to a point. But there is an oscillating canncellation structure which manifests itself as a suppression of gluon propagator in large lattice with fine resolution. Our lattice simulation suggsests that the threshold is $L \geq 24$.

- Numerical calculation of Faddeev-Popov zero-mode will clarify the structure.

- More sophisticated gauge fixings(simulated annealing or parallel tempering) are worth trying.

- Inclusion of fermions (Earth simulator).

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