K_{l3} semileptonic form factor with two-flavors of domain-wall quarks

T. Kaneko[1,2] for the RBC Collaboration

[1]High Energy Accelerator Research Organization (KEK)

[2]Graduate University for Advanced Studies

4th ILFTN Workshop, March 8 – 11, 2006

outline

- introduction
  - motivation : determination of $|V_{us}|$
  - K_{l3} decays
  - previous studies
- simulation method
- extraction of form factor
  - kaon ME w/ $\mathbf{p}=0$ $\Rightarrow$ $f_0(q^2_{\rm max}; m_{ud}, m_s)$
- q^2 interpolation
  - $f_0(q^2_{\rm max}; m_{ud}, m_s)$ $\Rightarrow$ $f_0(0; m_{ud}, m_s) = f_+(0; m_{ud}, m_s)$
- chiral extrapolation
  - $f_+(0; m_{ud}, m_s)$ $\Rightarrow$ $f_+(0; m_{ud,\rm phys}, m_{s,\rm phys})$
- $|V_{us}|$

1. introduction

- determination of $|V_{us}|$
- K_{l3} decays
- previous studies

CKM unitarity

- CKM unitarity in 1st row, *PDG 2004*

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta$$

$$|V_{ud}| = 0.9738(5) \quad \Leftarrow \text{nuclear } \beta \text{ decays}$$
$$|V_{us}| = 0.2200(26) \quad \Leftarrow K_{l3} \text{ decays}$$
$$|V_{ub}| = (3.67 \pm 0.47) \times 10^{-3} \quad \Leftarrow B \text{ meson decays}$$

$$\delta = 0.0033(15)$$

- $|V_{ub}|$ can be ignored
- uncertainty in δ: $\sim$50% from $|V_{ud}|$; $\sim$50% from $|V_{us}|$
  improved accuracy of $|V_{ud}|$ and $|V_{us}|$ $\Rightarrow$ more accurate δ
- ~ 1% accuracy of $|V_{us}|$ is needed

determination of $|V_{us}|$

- toward...
  - precise test of unitarity (δ)
  - λ in Wolfenstein parameterization
- can be determined from several processes:
  - K_{l3} decays: 0.2200(26), *PDG 2004*
  - hyperon decay: 0.2250(27), *Cabibbo et al., 2003*
  - $K_{\mu 2}$ and $\pi_{\mu 2}$ decays + lattice f_K/f_π: 0.2238(30), *Marciano, 2004*
  - hadronic τ decays + QCD sum rule: 0.2208(34), *Gámiz et al., 2004*

K_{l3} decays

- K_{l3} decays: semileptonic decays of kaon

$$K_{l3}^0 : \; K^0 \to \pi^- l^+ \nu_l, \qquad K_{l3}^+ : \; K^+ \to \pi^0 l^+ \nu_l, \qquad (l = e, \mu)$$

(K_{l3}^0 with $m_u = m_d$ in the following)

- decay rate:

$$\Gamma \;=\; \frac{G_F^2}{192\pi^3} M_K^5 \, C^2 \, I \, |V_{us}|^2 \, |f_+(0)|^2 \, S_{\rm ew} (1 + \delta_{\rm em})$$

I = phase space integral

$S_{\rm ew}\,(1 + \delta_{\rm em})$ = raddiative corrections

$f_+(0)$ = vector form factor $(q^2 = 0)$

C = Clebsh-Gordon coefficient $\Rightarrow$ $f_+(0) = 1$ in $SU(3)$ limit

form factor

- $f_+(q^2)$ and $f_-(q^2)$

$$\langle\pi(p')|\bar{s}\gamma_\mu u|K(p)\rangle = (p+p')_\mu f_+(q^2) + (p-p')_\mu f_-(q^2)$$

$$q = p - p'$$

$$f_+(q^2) = f_+(0)\,(1+\lambda_+ q^2 + \lambda' q^4), \qquad \lambda_+ = 0.028(1)\, M_\pi^2$$ *PDG, 2004*

$$f_-(q^2) = f_-(0)\,(1+\lambda_- q^2) \qquad \lambda_- = ?$$

- scalar form factor $f_0(q^2)$, $\xi(q^2)$

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{M_K^2 - M_\pi^2} f_-(q^2), \qquad \xi(q^2) = \frac{f_-(q^2)}{f_+(q^2)}$$

$$\langle\pi(0)|V_0|K(0)\rangle = (M_K + M_\pi)\, f_0(q^2_{\rm max}), \qquad q_{\rm max} = (M_K - M_\pi)$$

$$f_0(0) = f_+(0)$$

phase space integral, raddiative correction

- phase space integral

$$I = \frac{1}{M_K^8}\int d(q^2)\,\lambda^{3/2}\left(1+\frac{M_l^2}{2q^2}\right)\left(1-\frac{M_l^2}{q^2}\right)^2 \times\left\{\frac{f_+(q^2)^2}{f_+(0)^2}+\frac{3M_l^2(M_K^2-M_\pi^2)^2}{(2q^2+M_l^2)\lambda}\frac{f_0(q^2)^2}{f_0(0)^2}\right\},$$

$$\lambda = q^4+M_K^4+M_\pi^4-2q^2M_K^2-2q^2M_\pi^2-2M_K^2M_\pi^2$$

$$K_{e3}:\ I = 0.156 \pm 0.53\Delta\lambda_+$$

1st term: $\Delta\lambda_+ \sim 3$ % $\Rightarrow$ $\Delta I \sim 0.2$ %
2nd term: can be neglected for K_{e3}

- radiative correction $S_{\rm ew}\,(1+\delta_{\rm em})$

$$S_{\rm ew} = 1.0232, \qquad \delta_{\rm em} \lesssim 0.5\%$$

$|V_{us}|$ from K_{l3} decays

- decay rate:

$$\Gamma \quad = \quad \frac{G_F^2}{192\pi^3} M_K^5 C^2 I |V_{us}|^2 |f_+(0)|^2 S_{\rm ew}(1+\delta_{\rm em}),$$

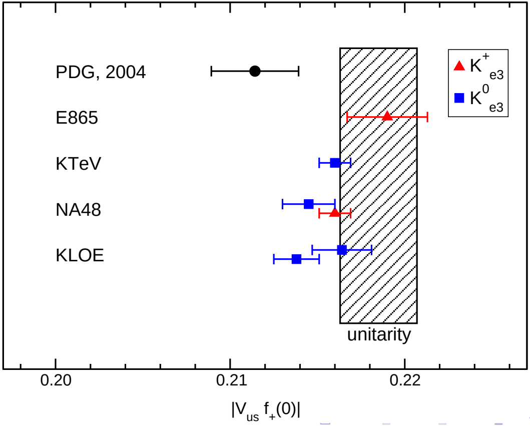
$$\left.\begin{array}{l}\Gamma \text{ from experiment} \\ f_+(0) \text{ from theory}\end{array}\right\} \Rightarrow \text{ precise determination of } |V_{us}|$$

1% accuracy for unitarity test!

recent experiments

PDG 2004: $\delta = 0.0033(15)$ $\Rightarrow$ unitarity violation ?
$\Rightarrow$ recent experiments of Γ

old exp.	~1970
E865	2003
KTeV	2004
NA48	2004
KLOE	2004



recent experiments
$\Rightarrow$ better agreement with CKM unitarity

theoretical studies

- ChPT expansion of $f_+(0)$

$$f_+(0) = 1 + f_2 + f_4 + \cdots , \qquad f_n = O(M_{K,\pi}^n)$$

- f_2

  ChPT: -0.023 *(Gasser-Leutwyler, 1985)*

- f_4

  ChPT $\ni$ LECs in $O(p^6)$ chiral Lagrangian

  (Bijnens et al., 1998; Post-Schilcher, 2002)

  quark model : $-0.016(8)$ *(Leutwyler-Roos, 1984)*

  $\Rightarrow$ used in previous estimates of $|V_{us}|$

NP calculation is desirable $\Rightarrow$ lattice QCD simulation

lattice calculations

- *Bećirević et al., 2004*: first lattice calc.

  $N_f = 0$, plaq. + NP clover
  $L \sim 2.0$ fm, $a^{-1} \sim 2.7$ GeV, $m_q \sim m_s/2 - m_s$
  $f_+(0) = 0.960(5)(7)$

- *JLQCD, 2005*

  $N_f = 2$, plaq. + NP clover
  $L \sim 1.8$ fm, $a^{-1} \sim 2.2$ GeV, $m_q \sim m_s/2 - m_s$
  $f_+(0) = 0.952(6)$

- *Fermilab-MILC-HPQCD, 2004*

  $N_f = 2 + 1$, impr.gauge + Asqtad (impr.Wilson for val. d-quark)
  $L \sim 2.6$ fm, $a^{-1} \sim 1.6$ GeV, $m_q \sim 2m_s/5 - m_s$
  use exp. λ_0
  $f_+(0) = 0.962(6)(9)$

2. simulation method

calculation w/ domain-wall quarks

- chiral symmetry at $a \neq 0$

  $\Rightarrow$ do not need WχPT/SχPT

  $\Leftrightarrow$ ChPT formula of f_2 at $a = 0$ has been used

- automatically $O(a)$-improved

  $\Rightarrow$ do not need NP tuning of "c_V" factors for V_μ

- small scaling violation

  cf. B_k by *CP-PACS 2001*, *RBC 2005*

gauge ensembles

- $N_f = 2$
- DBW2 glue + (standard) domain-wall quarks
- $\beta = 0.80 \;\Rightarrow\; a^{-1} = 1.69(5)$ GeV
- $16^3 \times 32 \;\Rightarrow\; L = 1.86(6)$ fm
- $N_s = 12 \;\Rightarrow\; m_{q,\mathrm{res}} \sim$ a few MeV
- 3 sea quark masses : $m_{s,\mathrm{phys}}/2 \lesssim m_{ud,\mathrm{sea}} \lesssim m_{s,\mathrm{phys}}$
- 4700 trajectories (94 measurements)

measurements

- $m_{ud,\mathrm{val}} = m_{ud,\mathrm{sea}}$
- 3 strange quark masses $\in [m_{s,\mathrm{phys}}/2,\ (5/4)\, m_{s,\mathrm{phys}}]$
- source opr. : exp. smeared ($t=4$),
  sink opr. : local sink ($t=28$) + sequential source method
- boundary condition : (periodic+anti-periodic)/2
- $|\mathbf{p}|=0, 1, \sqrt{2}$ and $\sqrt{3}$
- QCDOC : 1 rack (0.8TFLOPS) $\times$ 24 days

3. form factor

- double ratio method
- $f_0(q^2_{\rm max})$

double ratio method

Hashimoto et al., 2000 : proposed for B meson decays

$$C_\mu^{K\pi}(t,t') = \sum_{\mathbf{x},\,\mathbf{x}'} \left\langle O_\pi(\mathbf{x}',t')\, V_\mu(\mathbf{x},t)\, O_K^\dagger(\mathbf{0},0) \right\rangle$$

$$\to \frac{\sqrt{Z_{K,{\rm src}}\, Z_{\pi,{\rm snk}}}}{4\, M_K M_\pi\, Z_V} \langle \pi | V_\mu^{(\rm R)} | K \rangle\, e^{-M_K\, t - M_\pi\, (t'-t)}$$

$$R(t,t') = \frac{C_4^{K\pi}(t,t')\, C_4^{\pi K}(t,t')}{C_4^{KK}(t,t')\, C_4^{\pi\pi}(t,t')} = \frac{\langle \pi | V_4^{(\rm R)} | K \rangle \, \langle K | V_4^{(\rm R)} | \pi \rangle}{\langle K | V_4^{(\rm R)} | K \rangle \, \langle \pi | V_4^{(\rm R)} | \pi \rangle}$$

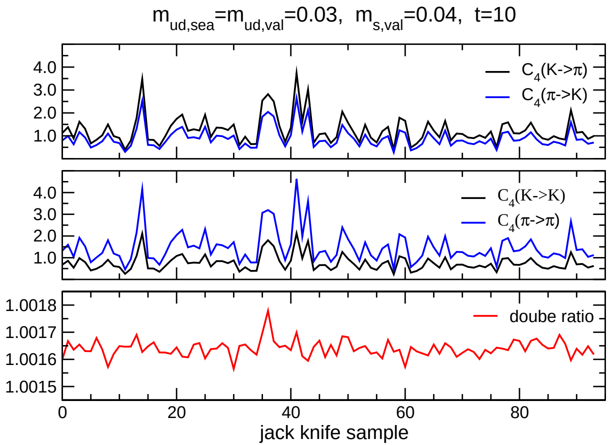
$$\to \frac{(M_K + M_\pi)^2}{4 M_K M_\pi} |f_0(q^2_{\rm max})|^2, \qquad q_{\rm max} = M_K - M_\pi$$

various uncertainties cancels (at least partially) in this ratio

renorm. factor, $\exp[-m\, t]$ factor, statistical fluctuation,...

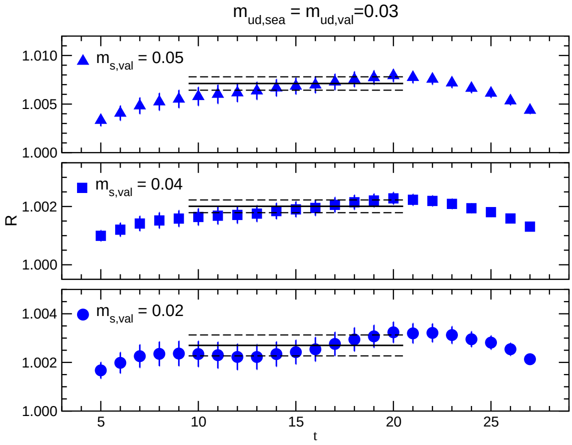
double ratio method

- at each jackknife sample



fluctuation: reduced to 0.03% level

$f_0(q^2_{\max})$ at simulated quark masses



m_{ud}	m_s	$f_0(q^2_{\max})$
0.02	0.03	1.00067**(17)**
0.02	0.04	1.00202**(48)**
0.02	0.05	1.00352**(82)**
0.03	0.02	1.00050**(22)**
0.03	0.04	1.00036**(11)**
0.03	0.05	1.00126**(35)**
0.04	0.02	1.00098**(55)**
0.04	0.03	1.00024**(10)**
0.04	0.05	1.00018**(6)**

larger $m_s - m_{ud}$

$\Rightarrow$ larger error $\lesssim$ 0.1%

remaining steps

- double ratio method ⇒ $f_0(q^2_{\rm max})$ w/ accuracy of 0.1%
  (already seen in *Bećirević et al.,...)*

- $\sqrt{\Gamma} \propto |V_{us}|\,|f_+(0)| = |V_{us}|\,|f_0(0)|$

remaining steps:

1) interpolation to $q^2=0$ at each quark mass

$$f_0(q^2_{\rm max}; m_{ud}, m_s) \;\Rightarrow\; f_0(0; m_{ud}, m_s)$$

2) chiral extrapolation

$$f_0(0; m_{ud}, m_s) \;\longrightarrow\; f_0(0; m_{ud,\rm phys}, m_{s,\rm phys})$$

how large systematic error from these steps?

4. q^2 interpolation

- ratio to study q^2 dependence
- $\xi(q^2)$
- q^2 interpolation

ratio for q^2 dependence

matrix elements w/ $|\mathbf{p}|^2 = 1, 2, 3$ $\Rightarrow$ q^2 dependence of form factor

$$C_\mu^{K\pi}(t, t'; \mathbf{p}, \mathbf{p}') = \sum_{\mathbf{x}, \mathbf{x}'} \left\langle O_\pi(\mathbf{x}', t')\, V_\mu(\mathbf{x}, t)\, O_K^\dagger(\mathbf{0}, 0) \right\rangle e^{-i\mathbf{p}\mathbf{x} - i\mathbf{p}'(\mathbf{x}' - \mathbf{x})}$$

$$\to \frac{\sqrt{Z_{K,\mathrm{src}}\, Z_{\pi,\mathrm{snk}}}}{4\, E_K(\mathbf{p}) E_\pi(\mathbf{p}')\, Z_V} \langle \pi(p') | V_\mu^{(\mathrm{R})} | K(p) \rangle \times e^{-E_K(\mathbf{p})\, t - E_\pi(\mathbf{p}')\, (t' - t)}$$

$$C^{K(\pi)}(t; \mathbf{p}) = \sum_{\mathbf{x}} \left\langle O_{K(\pi)}(\mathbf{x}, t)\, O_{K(\pi)}^\dagger(\mathbf{0}, 0) \right\rangle e^{-i\mathbf{p}\mathbf{x}}$$

$$\to \frac{\sqrt{Z_{K(\pi),\mathrm{src}}\, Z_{K(\pi),\mathrm{snk}}}}{2\, E_{K(\pi)}(\mathbf{p})} e^{-E_{K(\pi)}(\mathbf{p})\, t}$$

$$\frac{C_\mu^{K\pi}(t, t'; \mathbf{p}, \mathbf{p}')}{C^K(t; \mathbf{p})\, C^\pi(t' - t; \mathbf{p}')} = \frac{1}{Z_V \sqrt{Z_{K,\mathrm{snk}}\, Z_{\pi,\mathrm{src}}}} \langle \pi(p') | V_\mu^{(\mathrm{R})} | K(p) \rangle$$

ratio for q^2 dependence

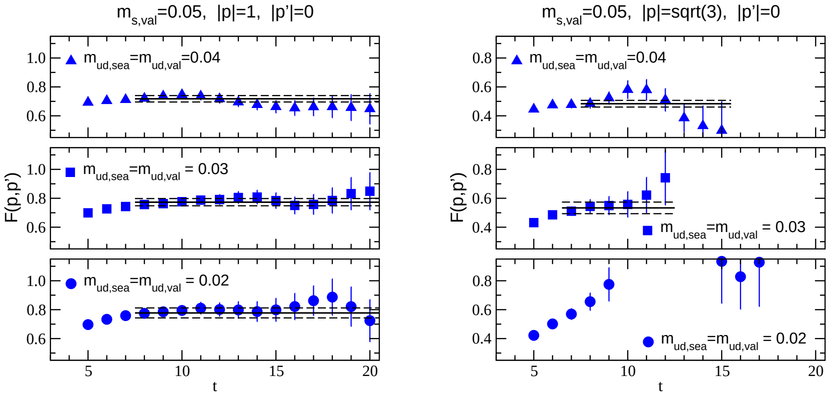
- in this study

$$R' = \frac{\frac{C_\mu^{K\pi}(t,t';\mathbf{p},\mathbf{p}')}{C^K(t;\mathbf{p})\,C^\pi(t'-t;\mathbf{p}')}}{\frac{C_\mu^{K\pi}(t,t';\mathbf{0},\mathbf{0})}{C^K(t;\mathbf{0})\,C^\pi(t'-t;\mathbf{0})}} \rightarrow \frac{\langle\pi(p')|V_\mu^{(\mathrm{R})}|K(p)\rangle}{\langle\pi(0)|V_\mu^{(\mathrm{R})}|K(0)\rangle} = \frac{E_K(\mathbf{p})+E_\pi(\mathbf{p}')}{M_K+M_\pi}\,F(p,p'),$$

($\rightarrow$ double ratio by JLQCD w/ $\mathbf{p}$ or $\mathbf{p}'=\mathbf{0}$)

$$F(p,p') = \frac{f_+(q^2)}{f_0(q^2_{\mathrm{max}})}\left(1+\frac{E_K(\mathbf{p})-E_\pi(\mathbf{p}')}{E_K(\mathbf{p})+E_\pi(\mathbf{p}')}\,\xi(q^2)\right), \qquad \xi(q^2)=\frac{f_-(q^2)}{f_+(q^2)}$$

$F(p,p')$: result



- $|\mathbf{p}|$ or $|\mathbf{p}'| = 1$: clear plateau, $\lesssim 5$% accuracy in $F(p,p')$
- $|\mathbf{p}|$ or $|\mathbf{p}'| = \sqrt{2}$: $\lesssim 5-10$% accuracy in $F(p,p')$
- $|\mathbf{p}|$ or $|\mathbf{p}'| = \sqrt{3}$: poor signal at two smaller $m_{ud,\mathrm{sea}}$ $\Rightarrow$ not used in analysis

$\xi(q^2)$: double ratio

- $F(p,p')$ and $\xi(q^2)$ $\Rightarrow$ $f_+(q^2)$, $f_0(q^2)$

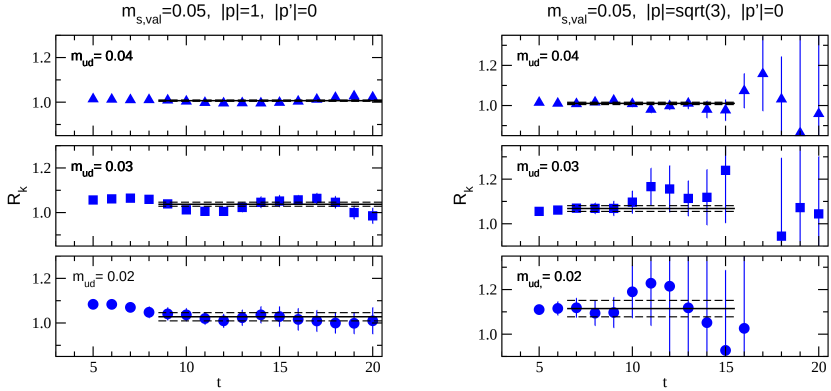
$$F(p,p') = \frac{f_+(q^2)}{f_0(q^2_{\rm max})}\left(1+\frac{E_K(\mathbf{p})-E_\pi(\mathbf{p}')}{E_K(\mathbf{p})+E_\pi(\mathbf{p}')}\,\xi(q^2)\right), \qquad \xi(q^2)=\frac{f_-(q^2)}{f_+(q^2)}$$

- double ratio for $\xi(q^2)$, *Bećirević et al., 2004*

$$R_k(t,t';\mathbf{p},\mathbf{p}') = \frac{C_k^{K\pi}(t,t';\mathbf{p},\mathbf{p}')\,C_4^{KK}(t,t';\mathbf{p},\mathbf{p}')}{C_4^{K\pi}(t,t';\mathbf{p},\mathbf{p}')\,C_k^{KK}(t,t';\mathbf{p},\mathbf{p}')} \qquad (k=1,2,3)$$

$$\xi(q^2) = \frac{-(E_K(\mathbf{p})+E_K(\mathbf{p}'))\,(p+p')_k + (E_K(\mathbf{p})+E_\pi(\mathbf{p}'))\,(p+p')_k\,R_k}{(E_K(\mathbf{p})+E_K(\mathbf{p}'))\,(p-p')_k - (E_K(\mathbf{p})-E_\pi(\mathbf{p}'))\,(p+p')_k\,R_k}.$$

$\xi(q^2)$: result



- $|\xi(q^2)| \lesssim 0.05$ w/ error of $50 - 100$% error
- significant $m_{q,\mathrm{val}}$ dependence

q^2 interpolation: method-1

- q^2 interpolation at each sea quark mass
  1. calculate $f_0(q^2)$ from $F(p,p')$ and $\xi(q^2)$
  2. interpolate $f_0(q^2)$ from step 1) and $f_0(q^2_{\mathrm{max}})$ to $q^2=0$

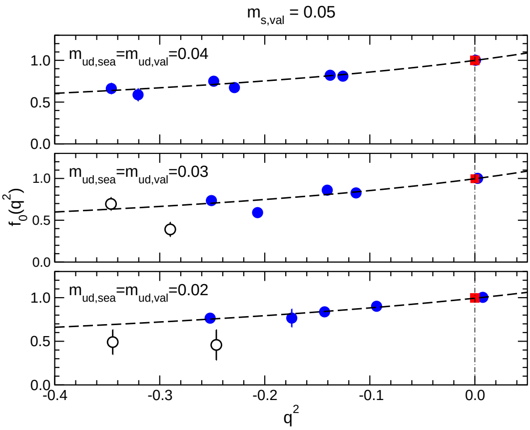
- interpolation form

$$f_0(q^2) = f(0)\,(1+\lambda_0\,q^2) \quad \text{(linear)}$$

$$f_0(q^2) = \frac{f(0)}{1-\lambda_0\,q^2} \quad \text{(polar)}$$

$$f_0(q^2) = f(0)\,(1+\lambda_0\,q^2+\lambda'\,q^4) \quad \text{(quadratic)}$$

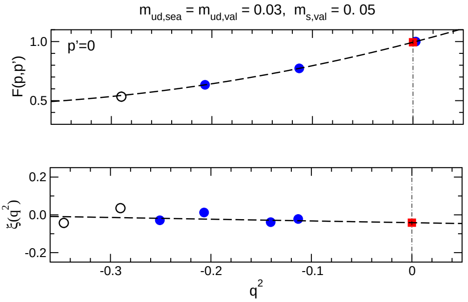
method-1: result



- q^2_{max} is small
  $\Rightarrow$ short interp.
- linear, polar, quad.
  $\Rightarrow$ consistent $f_0(0)$
- linear:
  largest χ^2/dof
- quad.:
  large error of λ'
- employ polar fit

q^2 interpolation: method-2

- *JLQCD, 2005*
  1. interpolate $F(p,p')$ to $q^2=0$ (with $\mathbf{p}$ (or $\mathbf{p}'$) fixed)
  2. extrapolate $\xi(q^2)$ to $q^2=0$
  3. calculate $f_0(0)$ from $F(p,p')|_{q^2=0}$ and $\xi(0)$



- $F(p,p')$
  lin, polar, quad. fits
  $\Rightarrow$ consistent results
- $\xi(q^2)$
  mild q^2 dependence
  $\Rightarrow$ employ linear fit
- $f_0(0)$ consistent with method-1
- similar accuracy

q^2 interpolation: summary

- very accurate $f_0(q^2_{\max})$
- small $q^2_{\max} \simeq 0.01$ with $m_{s,\mathrm{phys}}/2 \lesssim m_{ud} \lesssim m_{s,\mathrm{phys}}$
- reasonably accurate value at $|\mathbf{p}|, |\mathbf{p}'| \neq 0$

  $\Downarrow$
- q^2 interpolation:
  - 1% correction to f_0
  - w/ small sys error

m_{ud}	m_s	$f_0(q^2_{\max})$	$f_0(0)$
0.02	0.03	1.00067(17)	0.9994(5)
0.02	0.04	1.00202(48)	0.9973(14)
0.02	0.05	1.00352(82)	0.9939(24)
0.03	0.02	1.00050(22)	0.9987(5)
0.03	0.04	1.00036(11)	0.9990(2)
0.03	0.05	1.00126(35)	0.9965(8)
0.04	0.02	1.00098(55)	0.9959(9)
0.04	0.03	1.00024(10)	0.9991(2)
0.04	0.05	1.00018(6)	0.9992(2)

$\Downarrow$

- $f_0(0)$ w/ accuracy of $\lesssim$0.3%
- several interp. forms/ method-1 and 2 $\Rightarrow$ consistent $f_0(0)$

5. chiral extrapolation

- fit form
- $f_+(0)$
- $\xi(0)$

fit form

- consider ChPT expansion of $f_+(0)$ $(= f_0(0))$

$$f_+(0) = 1 + f_2 + \Delta f$$

- Ademollo-Gatto theorem: $SU(3)$ breaking $\propto (m_s - m_{ud})^2$
  $\Rightarrow$ no analytic term ($\ni$ LECs in $O(p^4)$ $\mathcal{L}$) in f_2

f_2 in $N_f = 2$ PQChPT, *Bećirević et al., 2005*

$$f_2^{(\mathrm{PQ})} = -\frac{2\,M_K^2 + M_\pi^2}{32\,\pi^2\,f_\pi^2} - \frac{3\,M_K^2\,M_\pi^2 \ln[M_\pi^2/M_K^2]}{64\,\pi^2\,f_\pi^2\,(M_K^2 - M_\pi^2)} + \frac{M_K^2\,(4\,M_K^2 - M_\pi^2)\ln[2 - M_\pi^2/M_K^2]}{64\,\pi^2\,f_\pi^2\,(M_K^2 - M_\pi^2)},$$

f_2 can be calculated precisely from measured $M_{K,\pi}$ and f_π

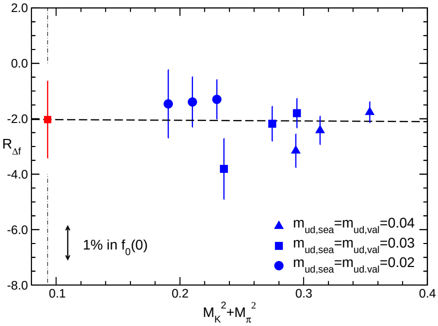
fi t form

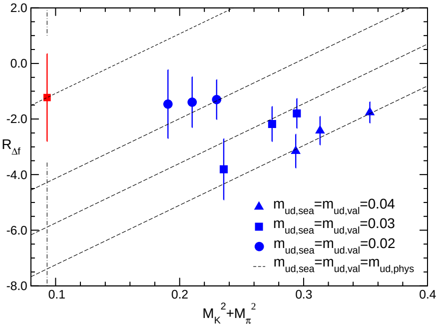
- chiral extrapolation of Δf
  from Ademollo-Gatto theorem:

$$R_{\Delta f} = \frac{\Delta f}{(M_K^2 - M_\pi^2)^2}$$

$$= \begin{cases} c_0 & \Leftarrow \text{ analytic term in } f_4 \\ c_0 + c_{1,v}\,(M_K^2 + M_\pi^2) & \Leftarrow \text{ previous studies} \\ c_0 + c_{1,s}\,M_\pi^2 + c_{1,v}\,(M_K^2 + M_\pi^2) \\ c_0 + c_{1,v}\,(M_K^2 + M_\pi^2) + c_{2,v}\,(M_K^2 + M_\pi^2)^2 \end{cases}$$

$f_+(0)$





$R_{\Delta f} = c_0 + c_{1,v}\,(M_K^2 + M_\pi^2)$:

$\Rightarrow$ χ^2/dof~ 0.4, $\Delta(R_{\Delta f}) \sim 50$%

$\Rightarrow$ $\Delta(f_+(0)) \sim 1$%

$R_{\Delta f} = c_0 + c_{1,s}\,M_\pi^2 + c_{1,v}\,(M_K^2 + M_\pi^2)$

$\Rightarrow$ χ^2/dof~ 0.1, consistent $R_{\Delta f}$

$\Rightarrow$ $\Delta(f_+(0)) \sim 1$%

$f_+(0)$ at physical m_q

- mild m_q dependence $\Rightarrow$ ill-determined quad.term
- 50% error in $R_{\Delta f}$ $\Rightarrow$ 50% error in Δf $\Rightarrow$ 1% error in $f_+(0)$

  $f_2 \sim 2\%$ correction, $\Delta f \sim 1-2\%$ correction
- at physical quark mass

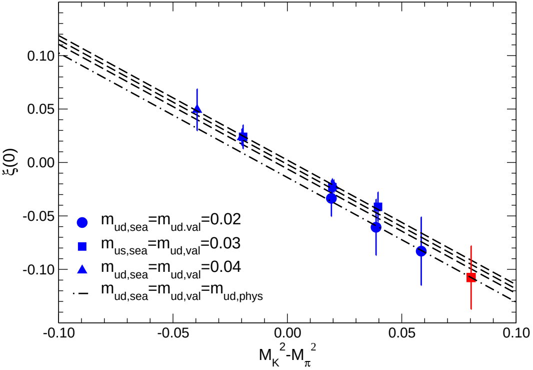
$$f_+(0) = 0.964(9)(5)$$

- previous estimates;

Leutwyler-Roos	quark model	$0.961(8)$
Becirevic *et al.*	$N_f=0$	$0.960(5)(6)$
JLQCD	$N_f=2$	$0.952(6)$
Fermilab-MILC-HPQCD	$N_f=3$	$0.962(6)(9)$

$\xi(0)$

fit form = $c_0 + c_{1,s}\, M_\pi^2 + c_{1,v}\,(M_K^2 - M_\pi^2)$



- $\xi(0) \to 0\,(M_K^2 - M_\pi^2 \to 0)$

  $\Rightarrow\ c_0, c_{1,s} \to 0$

  $c_0 = -0.02(3)$

  $c_{1,s} = 0.10(16)$

- $\xi(0) = -0.107(30)$

  $\Leftrightarrow$ exp. $= -0.125(23)$, *PDG, 2004*

6. $|V_{us}|$

$|V_{us}|$

- $|V_{us}\, f_0(0)| = 0.2239(23)$ from $\Gamma_{K^+_{e3}}$, *E865, 2003*
- $f_+^{K^+\pi^0}(0)/f_+^{K^0\pi^-}(0) = 1.022$, *Leutwyler-Roos, 1984*
- $|V_{us}|$

$$f_+(0) = 0.964(9)(5) \;\Rightarrow\; |V_{us}| = 0.2273(24)(23)$$

- CKM unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta$$
$$|V_{ud}| = 0.9738(5)$$
$$|V_{ub}| = (3.67 \pm 0.47) \times 10^{-3}$$

$$\delta = 0.0001(18) \;\Leftrightarrow\; 0.0033(15),$$ *PDG 2004*

7. summary

summary: what have been done

- K_{l3} form factor in two-flavor QCD with domain-wall quarks

$$f_+(0) = 0.964(9)(5) \;\Rightarrow\; |V_{us}| = 0.2273(26)(23)$$

- ~ 1% accuracy for $f_+(0)$
  - double ratio method $\Rightarrow$ very accurate $f_0(q^2_{\rm max}; m_{ud}, m_s)$
  - q^2 interpolation
    - small $q^2_{\rm max}$
    - reasonably accurate $f_0(q^2)$ w/ $|\mathbf{p}| \neq 0$

      $\Rightarrow$ small sys. error due to the short interpolation
  - chiral extrapolation
    - no LECs in f_2
    - Δf is 1 – 2% correction

      $\Rightarrow$ 50% accuracy in Δf is sufficient

summary: what have to be done

- $|V_{us}|$ from K_{l3} decays
  - scaling violation $\Leftarrow$ consistency with JLQCD($N_f = 2$)
  - finite size effects $\Leftarrow$ ChPT, *Bećirević et al., 2004*
  - extension to three-flavor QCD
    - $\Leftarrow$ consistency among $N_f = 0, 2, 3$
    - $\Leftarrow$ RBC+UKQCD: talks by *C.Maynard, S.Cohen*
  - toward lighter ud sea quark mass $\Rightarrow$ larger $q^2_{\rm max}$
  - byproduct: $F_V^\pi(q^2)$, $\langle r^2 \rangle_\pi$
- $|V_{us}|$ from hyperon decay $\Leftarrow$ talk by *S.Sasaki*
- other CKM elements from heavy meson decays
  - $\Leftarrow$ construction of heavy quark action: talk by *H-W.Lin*

DWQCD on QCDOC $\Rightarrow$ flavor physics